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Externality, Convexity and Institutions*

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Economic theory has generally acknowledged the role that institutions have in shaping economic space. The distinction however between physical and institutional descriptions of economic activity has not received adequate attention within the mainstream paradigm. In this paper I show how a proper distinction between the physical and institutional space in economic models will help clarify the concept of externality and provide a better interpretation of the relationship between externality and nonconvexity. I argue that within the Arrow-Debreu framework externality should be viewed as incongruence between the physical and institutional descriptions of the economic space. Accordingly the only consistent characterization of externality is to view it as synonymous with missing markets or missing property rights. All that distinguishes a conventional commodity from an externality is the extent to which a market exists for the 'good' in question. The common expression 'markets for externalities' is misleading since it tends to imply that there is more to externality than simply the non-existence of a market. It is most likely the result of confounding certain kinds of jointness in production with externality.

I also argue that contrary to conventional wisdom; detrimental externality has no special association with nonconvexity. Starrett's (1972) fundamental nonconvexity has to do with the specific institutional structure of Arrow markets rather than the detrimental nature of externality. Indeed, Arrow markets will not in general eliminate externalities. In a similar vein it is not detrimental externality, however intense, that causes the production possibility set to become nonconvex as argued by Baumol and Bradford (1972), but the particular interpretation of intensity that would make even conventional production possibility sets nonconvex. These points become apparent when one distinguishes between the convexity of the physical and institutional production sets. An understanding that 'externality' is purely an institutional construct will assist in a proper appraisal of the critical economic function of institutions.

1. Introduction: The Physical and the Institutional

Mainstream models of individual decision making allow a great degree of flexibility in the defining of the economic space. Commodities can be defined to capture any objects of desire or input requirements for productive activities. A commodity could be an apple to be consumed in Piccadilly Square at 10:00 a.m., the warm feeling associated with holding hands or the malicious desire to see a colleague fail to get a promotion. An input for production could be a worker with a particular skill and effort level, a particular quality of the atmosphere, or good spiritedness of the workforce. This definitional flexibility is no doubt a strength of the theory in that it allows an enormous reach of economic modelling. It also, however, entails a danger of overlooking the significance of alternative ways of defining the economic space.

A central motivation of economic analysis is to discern the physical possibilities of fulfilling human ends and to investigate the extent to which economies can be

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organized to exploit these. The contrast between the possible and the feasible is crucial. Objects of desire may be physically unattainable and it is certainly important to know that, but the more interesting situations are those where ends are physically reachable but the economy seems unable to exploit the possibilities. Production and consumption possibility sets and the commodity space with which they are associated are usually defined in both physical and institutional terms. Indeed it is precisely the ease with which the physical and institutional descriptions can be conflated that can lead to problems. It is particularly important because institutions are an economy's means of organizing activity, and thus are part of the tools that we have to exploit the possible.

It is standard in microeconomic textbooks to state that consumption and production possibility sets besides being determined by physical or technological constraints are also determined in part by institutional factors. Familiar explicit reference to institutional factors affecting consumption possibility sets include legal restrictions on work hours, overtime pay beyond the 8 hour work day, limits on overall number of hours worked, child labour, etc. It is less common to see how legal or institutional factors affect production technology.[1]

Despite recognition that institutions have their role in determining the economic space, mainstream economics does not explicitly address their role and significance. More importantly, while physical constraints can be said to be in a real sense exogenous, or beyond control, institutional factors are virtually defined as artefacts of humans' attempt to socially control physical reality. It is thus, particularly disturbing that they should not be explicitly modelled in economic analysis.

Though it is often difficult to distinguish institutional and physical constraints impinging on production and consumption sets, it is important to do so, as it provides a starting point for what can and cannot be controlled by human agency. Market institutions whether formed by evolutionary processes or by conscious design are artefacts of human interaction seen to assist the co-ordination of activity. Property rights and entitlements more generally, are critical elements of our institutional devices. When describing a production technology we often take for granted that the institutional rules align perfectly with the physical reality, so that institutions almost drop out of our description. We say that the production of computers is a function of so many inputs that can be combined in a number of different ways. It is easy to forget that there are two conceptually separable layers to a production function: a pure description or blueprint of the physical possibilities of combining inputs in certain ways to generate an output (physical menu), and a description of the institutional rules circumscribing the way that 'institutionally' defined rights can be combined to produce an 'institutionally' defined output. The most powerful example of this distinction is with the input of labour. A physical description might refer to the level of effort of labour of a particular skill required (in combination with other inputs) to produce another good. An institutional description would modify the physical 'possibilities' by the socially determined possibilities: labour input would be measured in time of labour offered and there would be several restriction on the number of hours worked, conditions of work, etc. A firm may require water from a river to cool down certain production processes. There is a physical description of the possible combinations of

[1] "The set of feasible production plans is limited first and foremost by technological constraints. However, in any particular model, legal restrictions or prior contractual commitments may also contribute to the determination of the production set" (Mas-Colell, p. 128).

inputs with water that will lead to the different output levels, and there are institutional rules that circumscribe where a firm can be located along a river, quantities of water it can exact within specified time periods, etc. What is physically possible and what is institutionally possible are two different things.

The idea of a separate institutional and physical description of the economic space may seem quite obvious, yet the failure to explicitly distinguish the two can obfuscate some important analytical concepts. Indeed I believe that the concepts and the relationship between externality, jointness and nonconvexity can be substantially clarified with a more explicit understanding of the distinction between the physical and the institutional. The next section describes some of the difficulties in characterizing externality and suggests an unambiguous way of characterizing externality within the context of Arrow-Debreu frictionless models that relies on the separate modelling of the physical and institutional economic space. Section 3 points to the need to distinguish between the physical and institutional space when we deal with the convexity of production sets; a production function can be convex in one space and nonconvex in the other. Section 4 reinterprets the well-known discussions of Baumol and Bradford (1972) and Starrett (1972) linking detrimental externality to nonconvexity and argues instead that there is no special link between nonconvexity and externality. It also points out that Arrow and/or Lindahl markets may not eliminate (or address) externalities.

2.Externality as incongruence between physical and institutional descriptions of economic space

2.1. Many characterizations of externality

There has never been a clear and widely accepted definition of externality.[2] To an extent the multiplicity of meanings associated with externality depend on the context within which it is used.[3] Much of the difficulty with the concept stems from transplanting a characterization that may be meaningful in one context to one in which it is not. Two contexts that are particularly prone to this ‘sensitive’ exchange are economic models that do not incorporate transaction costs and ones that do. Part of the reason for this is that these contexts themselves are often inadequately distinguished. In a frictionless general equilibrium model to say that market failure results from 'missing markets' can have a clear analytical meaning. In a model with transaction costs, missing markets may not be a problem at all because there may be alternative 'less costly' institutions that allocate resources more effectively.

Many questions that are meaningful in one context are not in another. The question “Why are there areas of economic activity where markets do not exist?” is only meaningful within a context where the formation of institutions can be explained. Models with transaction costs attempt to answer such questions by referring to the costs of setting up and running institutions. To be able to explain the realm of markets, or the extent of a firm's activities, and thus the boundaries of market and non-market activity, requires an approach that treats firms and markets (and maybe

[2] "There is a strong temptation to avoid giving an explicit definition of externality, since even this first step has been a fertile source of controversy, and instead to approach the matter obliquely by putting to work various models in each of which an externality is obviously present" (Cornes and Sandler p. 39).

[3] In my book Externality and Institutions (1998) I try to clarify the various meanings associated with externality.

institutions more generally) as endogenous. In this context the relevant efficiency concept is *constrained* Pareto optimality that incorporates *institutional* feasibility constraints. If a market is too expensive, or impossible to implement, its non-existence may not signal inefficiency since the outcome may be the best possible.

The Arrow-Debreu framework cannot tell us why markets do not exist. The extent of missing markets is a definitional matter to be determined at the outset by the modeller. It does not have the means to explain why markets (or firms for that matter) exist. In this framework the extent of externalities will generally be determined by the way economic units (firms) and markets are defined. In these frictionless models the failure arising from externality consists in the shortfall of a hypothetical set of incomplete and costless markets from attaining Pareto optimal outcomes. One can also compare allocations arising from complete and incomplete markets. There can be circumstances where models of incomplete markets attain Pareto improvements relative to models with a complete set of markets, e.g., incomplete markets may fruitfully obscure certain nonconvexities.

I emphasize the distinction between models with and without transaction costs, because I believe that much of the confusion over the characterization of externality and its association with other important concepts such as market failure and nonconvexity, arise from an inadequate recognition of how context-sensitive these concepts are. Because this paper focuses on conceptual issues that arise within the axiomatic structure of general equilibrium theory, I will focus on the definition of externality as 'non-market activity' or 'missing markets' that happens to be the most consistent within this framework. In this discussion I will not be concerned with questions of feasibility since I abstract completely from transaction costs.[4] In this respect the difficulty of forming a market or determining the scope of property rights is not considered.

Even within the confines of frictionless Arrow-Debreu general equilibrium models many economists have viewed externality as being something more than just missing markets. One often sees expressions such as "markets for externalities" as if externalities continue to have some special status even when markets have been formed for the activity in question. I will argue, however, that the only consistent characterization of externality within the general equilibrium context is to view it as synonymous with ‘missing’ markets or ‘missing’ property rights. In this view all that distinguishes a conventional commodity from an externality is the extent to which a market exists for the 'good' in question. Once property rights have been defined for a good and markets are present for the exchange of that good the externality ceases to be. In this sense "markets for externalities" is an oxymoron. It is *either* markets *or* externality.

The presence of externality is purely a function of how institutions (property rights and markets) are defined. There is nothing intrinsic about the characteristics of 'goods' that bestows the property of externality. A loaf of bread will involve externality if no one owns it. One person's decision to eat it will impact another 'directly' without the mediation of exchange through parametric prices. Likewise, the use of air to exhale smoke will not involve externality if the air in question is privately owned and the

[4] This does not mean that I consider these issues unimportant. On the contrary, I believe that a clearer understanding of these concepts in the frictionless framework will go a long way in clarifying conceptual issues in models with transaction costs.

smoker must purchase the right to emit smoke. Rights to air turn the externality space into conventional commodity space.

2.2 Externality reinterpreted

There is certainly a very rich variety of goods and 'bads' in the real world with characteristics for which it is extremely difficult to imagine that well defined property rights can actually be formed over: air space for flying, genetic codes, sense of security, ideas, biodiversity. Buchanan and Tullock (1962) have vividly pointed this out by contemplating markets in the colour of underwear others' wear. The cost of setting up institutions is critical in determining the extent of the market, but in frictionless models we simply define institutions into being or not. In fact we must do so, because we have no systematic way of incorporating transaction costs, as they are not part of the axiomatic structure of these models.

The literature on externalities in frictionless models, nonetheless attests to the many insights that can be gained by comparing models with marketed goods in which the only constraint facing agents are those imposed by budgets and production functions, with models that include non-marketed or environmental commodities the quantities of which are exogenous to some agents. Cornes and Sandler (1996, pp. 51-67) provide a good overview of the many ways of modelling alternative types of externalities or non-marketed commodities: general externality, public goods, impure public goods or bads, club goods, etc. The motivation in forming models with different types of non-marketed goods usually lies in capturing features or characteristics of actual goods to see what implications these have for market interaction, as well as to consider ways of improving on market outcomes. But as they point out, one must take care not to impart the fact that they are modelled as non-marketed goods to something intrinsic in their nature. "Rather than seeing the various [types of externality] as synonymous with particular goods or services, we prefer to think of them as *incentive structures* [my italics]" (Cornes and Sandler p. 64). [5]

While I agree that one should not look for externality in particular goods, characterizing externality as incentive structures, begs the question as to the relevance of a separate concept called 'externality'. A more useful interpretation (and maybe this is what Cornes and Sandler have in mind), is that each different type of non-market activity may, in combination with market activity, give rise to a different incentive structure. In order to zero in on a useful characterization of externality (within the Arrow-Debreu framework) it is important to clearly distinguish between the 'physical' and 'institutional' economic space.

A good way to characterize the presence of externality is as a mismatch or lack of isomorphism between a model of the physical economic space and a model of the institutional economic space. The institutional space determines the means of control of the physical environment. It entails the myriad forms of legal and other rules that

[5] The quote continues: "Which of these structures, if any, characterize the production and consumption of a particular commodity is a matter that may be influenced not only by technological considerations but also by such considerations as the way in which institutions have evolved, the nature of individual preferences, and, indeed, the distribution of those preferences" (Cornes and Sandler p. 64). This suggests that if we want to understand why we have different kinds of non-market activity we need to answer the question of how institutions evolved. A very important matter, but as mentioned earlier, it takes us outside the Arrow-Debreu framework, and requires modelling of transactions costs and/or information costs.

assign ownership, exchange and other rights. A misalignment between the physical and institutional space implies that some aspects of the physical world may not be adequately controlled through interactions in the institutionally determined economic space.

For simplicity I will present this characterization of externality in production space alone. Envisage an underlying netput vector $(y_1, y_2, \cdots y_L)^P \in R^L$ that describes production possibilities at a purely physical level. The superscript *P* symbolizes that this is a physical description, devoid of any institutional elements. If the institutional description of production possibilities aligns perfectly with the physical description, so that we have a complete set of property rights and markets for all relevant physical activities, we can say that there are no externalities. The netput vector in the institutional space would be $(y_1, y_2, \cdots y_L)^I \in R^L$, with superscript *I* indicating that all inputs and outputs can be expressed in terms of rights over resources or activities.

Externality would be present if there is incongruence between the physical and institutional descriptions. A simple instance of externality would arise if there were no property rights over valued resources, so that their use might effectively be characterized as open access. In this case the institutional description of production possibilities would be of a lower dimension than the physical description: $(y_1, y_2, \cdots y_K)^I \in R^K$ where $K \leq L$. The actual description of production possibilities should be viewed as a combination of physical and institutional elements so that a netput vector might be $(y_1^I, \cdots y_k^I; y_{k+1}^P, \cdots y_L^P) \in R^L$ where some inputs and outputs are mediated through the market while others are not. The idea is that the firm has control over *k* inputs and outputs through the market, but also uses, is affected by, or has some form of control over other inputs and outputs without market exchange. This would be one way of viewing 'missing markets'. Indeed, I will argue that this is a far more useful characterization of externality than the standard one that views one firm's activity as being directly influenced by another firm's activity.

The notational form I am using here is not meant to be definitive but should be seen more as a heuristic for the underlying conceptual issues. A more complex form of externality arises when institutional control over valued resources is present but is somehow incomplete or imprecise. This is the kind of externality present when there are qualitative characteristics of certain commodities that cannot be fully monitored or controlled, and thus exchanged in the marketplace. An implicit assumption of the fundamental welfare theorems is that market participants can observe all the characteristics of commodities. The 'unobservability' of some characteristics means that markets cannot be complete. All of the 'new' information economics literature that deals with adverse selection, the principal agent problem, and more broadly mechanism design, addresses issues that arise when certain relevant characteristics are wholly or partly unobservable. Typical examples are used cars with qualitative characteristics that cannot be observed by prospective buyers, driver skills that cannot be observed by insurance markets, and worker skills that cannot be observed by firms.

Looking at the case of unobserved worker characteristics, production may be a function of a number of these characteristics, such as skill, diligence, effort, etc. A model without externality would have markets for each of these attributes, so that a firm could pick the optimal combination. Indeed the firm could determine the exact distributions of these attributes for its workforce. If however, as is likely, the institutional setup does not offer such complete and precise control over workers' characteristics, there will be certain production activities taking place that are not monitored by market input decisions. In terms of the previous notation the physical netput vector would include the following inputs: $(y_1, ..., \theta_1, ..., \theta_s, ... y_L)^P \in R^L$ where θ_i reflect all possible 'types' of labour (capturing all possible combinations of valuable characteristics). The institutional netput vector would be $(y_1, ..., l_1, ..., l_m, ..., y_K)^I \in R^K$ with $K < L$ and $m < s$. Essentially the markets for worker types would be fewer than the physically possible worker types, but also the markets for worker types would be less precise in characteristics. The ability to discriminate among types of workers is restricted (usually for informational reasons). A market may exist for an educated worker, l_i, and this may partly reflect another characteristic, say 'effort', but a specific market for workers with a certain effort level will not exist. The existing markets may provide some form of control over non-market characteristics, but the control will be imprecise or incomplete. The externality is present because the physical production functions entail inputs that are not fully captured by the institutional setup.

The standard way of representing production functions with externalities are deceptively similar to the way I have suggested. A typical representation would have $y = f(z_1, ..., z_n; e_1, ..., e_m)$ where y is the output of a firm, z_i's are marketed inputs used by the firm, and e_i's non-marketed inputs that are treated as exogenous by the firm though they are decision variables of other firms. In particular, the e_i's are generally viewed as being input or output variables of other firms rather than some other form of non-market activity, say open access resources. In this respect they are not exactly non-marketed goods, in the sense that they appear as standard marketed inputs or outputs in the firm seen as 'causing' the externality, while they show up as non-marketed in the firm affected by the externality. For instance, apple production may be seen to be *directly* influenced by the neighbouring production of honey. Honey that is a marketed good is produced jointly with the fertilizing of apple trees (a non-market activity). It appears that the use of certain inputs or production of certain outputs by some firms have a *joint* physical impact on other firms. Just as in the previous examples we saw that the externalities could be eliminated by forming property rights and markets for the 'missing' resources or characteristics, in a similar fashion property rights can be formed for these joint physical impacts in the form of Lindahl or Arrow markets.

This conventional representation should be seen as a *particular* form of externality rather than the *general* form. In the broadest sense, I contend, externality is present whenever the institutional structure does not align with, or control for, some valued physical activity or resource. It could be that there are no property rights for some resource so that it is effectively treated as an open access resource, it could be that there is no market for some important characteristic, e.g., effort, or it could be that the use or output of marketed goods is jointly associated with non-market activity. This last form of externality is one possible form rather than the general form. It is also a form that is intimately linked with the particular theoretical institutional construct known as Lindahl or Arrow markets. Indeed the possible forms of externality may be unlimited, however, the point I want to make, and which I will pursue in the following sections is that this representation is far less general than assumed, and that it potentially prevents a richer understanding of the externality issue.

The common representation incorporates both physical and the institutional aspects, distinguishing marketed from non-marketed, environmental or physical inputs. It would be useful to try to separate these aspects by trying to describe an underlying production technology associated with physical reality that is in a sense prior to the institutionally determined production function. Such a description would not make any reference to who is controlling the use of inputs; it would simply provide a complete description of the physical inputs required. In this representation there can be no externality since institutions have not been introduced. The production function is telling us the combination of physical inputs that are required for the production of a commodity. There is no distinction between marketed and non-marketed commodities, as markets have not come into play. The description is not telling us anything about how these inputs will be controlled, it is only giving us the physical relationships or 'blueprint' required for the output to be produced. The typical representation of externalities can be viewed as arising from the superimposition of an institutional structure that turns some of the physical relationships into marketed commodities while others not.

Meade's (1952) orchard-apiary example can be reinterpreted by distinguishing the physical and institutional descriptions. His original representation took the form:

$$y_a = f(z_{a1}, \ldots, z_{an}; y_h)$$
$$y_h = f(z_{h1}, \ldots, z_{hn}; y_a)$$

Where y_a and y_h stand for apple and honey output, z_{ai} and z_{hi} stand for marketed inputs, and y_a and y_h within the production functions stand for non-marketed 'inputs' that are exogenous to the respective firms. The non-marketed inputs represent positive externalities where the apple production provides nectar for the bees and honey production provides fertilization services to the orchard. The subscripts *a* and *h*, can be interpreted as indicating that the inputs are 'controlled' by the respective firm. In the case of marketed inputs the control derives from market purchases, while with the non-market 'inputs' the other firm 'controls' the activity. (Of necessity there is ambiguity as to who controls the output of honey and apple). The intuition behind this representation is clear, but it is a valuable exercise to try to consider the possible interpretations of the apple-apiary story in terms of physical and institutional descriptions.

Closer scrutiny of Meade's original representation suggests that it was not necessarily his intent to describe an underlying physical relationship but to find a proxy of some apparent interconnection between the two activities. Indeed a literal reading of the original production functions make them look as if there is almost mysterious link between apple output and honey production, i.e., it avoids providing the underlying physical description that might explain in precise terms possible connections between the two processes.

A purely physical description of production possibilities in the apple-apiary story might take the following netput form:

$$(y_a, y_n, y_p, \ldots, y_L)$$
$$(y_h, y_n, y_p, \ldots, y_K)$$

The first vector shows apple production (y_a) that is jointly produced with nectar output (y_n) and requires pollination service as an input (y_p). The second vector represents honey production (y_h) that is jointly produced with pollination service output (y_p), and requires nectar input (y_n). The inputs are not distinguished as marketed or non-marketed since the description is prior to institutions. The physical description sees apple and honey production as a function of a number of inputs and services, where apple production would include a description of fertilization services as an input and the apiary production function would include nectar as a requisite input into its production. This description would include an exhaustive list of inputs so that any potential means of fertilizing apples or providing nectar to bees would be incorporated. As a purely physical description of production possibilities there is no externality.

Externality will arise as soon as we superimpose an institutional structure that does not align with this physical description. The following model might be a likely candidate for externality:

$$(y_a^J, y_n^P, y_p^P, \ldots, y_L^J)$$
$$(y_h^J, y_n^P, y_p^P, \ldots, y_K^J)$$

In this case there is externality because property rights and markets have not been formed for nectar and pollination services. If we established conventional markets for these goods then the externality would cease to be. It could be that there are many ways to fertilize apple production, with 'bee fertilizing services' being one of many. If the price is right, and it turns out that bee fertilizing services can be jointly produced with honey then we would expect the apple producer to purchase the service through the market. It may be that alternative fertilizing services come more cheaply (and may or may not be jointly produced with other products). The fact that apples are jointly produced with nectar and that there are potentially two outputs to be sold, while bees are jointly produced with 'fertilizing services', would mean that the more appropriate notational format would involve a netput vector with more than one output. Jointness in outputs does not imply externality.

It is also important to distinguish the two kinds of jointness present in the original Meade representation and in the alternative representation. In the original form there appears to be a fundamental jointness between the outputs of both goods (honey and apples), while in the revised version the jointness appears between two outputs in the separate production functions (honey and pollination; apples and nectar), i.e., not between production functions but within.[6] There are implications also for the nature of property rights envisaged if one wants to set up markets for the relevant activities. In the first case (Meade representation) one would need to have Lindahl markets so that the apple firm could pay for the observation in honey production, while in the second case conventional markets for nectar and pollination services would be required. I will argue later that there will be many cases where the appropriate institutional device to control for physical activity may be Lindahl or Arrow markets, but that in many instances it would be better to use conventional markets.

[6]One possible form for the underlying physical production possibility set in Meade's example is:$(y_1, \ldots, y_a, y_h, \ldots, y_L)$. Where y_i's are inputs or outputs depending on their sign. This representation reflects the possibility that one of the means of fertilizing honey is through joint production with apples. Jointness may simply reflect the possibility of several production processes ultimately being produced by a single firm, or an 'intrinsic jointness' whereby several outputs are a part of a single production process. It also suggests that the single output representation may result from the incorporation of institutional factors that have the activities undertaken by separate entities (firms). It is important to keep in mind that firms are institutional constructs, so that the separation of apple and honey production may be a result of the particular institutional structure. The physical description would treat apple and honey production as joint but the superimposition of an institutional structure that separates their production brings about the externality.

In Meade's representation of externality, joint production and externality are not adequately distinguished. The representation is set up so that joint production appears to be tied to externality, i.e., one firm's production appears to be directly influenced by another's input or output choice. This indeed could be the case, but that is not what makes it an externality. In fact joint production simply describes an underlying feature of the production technology. Externality arises because institutional devices are not in place to adequately account for interactions. By separating the institutional from the physical description, one is able to separate the question of jointness from that of 'inadequate' institutional devices. The physical production set may be riddled with certain kinds of jointness, but there is no sense in talking of missing markets or property rights, as institutions have not been introduced in the model.[7]

The standard 'externality' representation involves a superimposition of a particular institutional structure that turns certain inputs into marketed commodities while others are treated as non-marketed. The space and nature of marketed and non-marketed goods, as well as the space of activity taking place within firms, is a matter for the modeller to determine and is usually motivated by an attempt to resemble an existing institutional structure. In this case representing apple and honey production as taking place within two separate firms as opposed to modelling the activity as joint production within one firm is one of the elements that may turn the joint activity into externality. As it is unlikely, however, that there is an intrinsic jointness in apple and honey production, the externality is more likely to result from an institutional description that does not envisage markets for certain inputs, e.g., fertilizing activity, nectar, etc. An institutional description that sets up markets for all inputs present in the physical description would by definition do away with any externality.[8]

3 Production functions in physical and institutional space

By distinguishing between a physical and institutional description of technology it becomes apparent that the form of a production function (or utility function) will depend on whether we are considering the space of institutions or the underlying physical space. While the physical description must be unique there can be all kinds of institutional structures associated with an underlying physical description. A production function in physical space may involve increasing returns to scale, while its institutional counterpart may be convex depending on how property rights and markets have been defined over inputs. Take the following example of a beneficial externality:

$$y = z^a e^b$$

[7] For the record, apple blossom is not a source of nectar for the production of honey, and fruit pollination often decreases honey production because the bees tend to consume some of their stored honey while pollinating (Cheung 1973, p. 15). Furthermore, there is evidence that nectar and pollination services are controlled effectively through negotiations, i.e., it could almost be said that there are markets for these inputs. See Cheung (1973), Johnson (1973) and Gould (1973) for empirical and theoretical discussions that confound conventional wisdom about this well-known example. The evolution of institutions to deal with complex allocation problems easily surpasses our imagination. Some nice examples can be found in Ostrom (1990) and Eggertsson (1990).

[8] Externality has often been viewed as a technological phenomenon as the following quote suggests: "[T]he presence of an externality is not merely a technological phenomenon but also a function of the set of markets in existence" (Mas Collel et al. 1995). The discussion here argues that externality is purely a function of institutions (or how they are defined relative to a physical or technological structure).

where z is a marketed input raised to the power of *a* while *e* is a beneficial externality (raised to the power of *b*) which is determined by other firms. Note that in the space of marketed inputs, the production technology is convex as long as $a < 1$. A physical description would involve all inputs and thus e^b would be represented and the convexity would depend on *a* + *b*. If $a + b > 1$, then the underlying physical technology would involve increasing returns to scale while the institutionally determined space would be convex in marketed commodities.

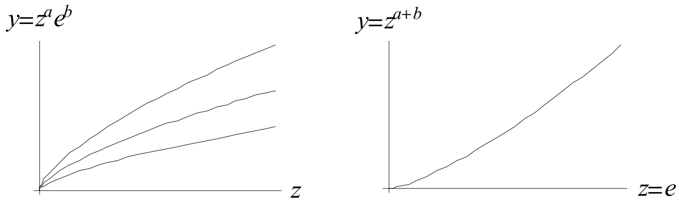

Figure 1

Figure 1(left) shows the production function in the institutionally determined space where e^b is taken as exogenous and we see the output of *y* as function of the marketed input at different levels of the beneficial externality (*e*). Figure 1 (right) shows the underlying physical relationship when the two inputs are scaled by equal amounts (*z* = *e*). While the physical description must be unique, the institutionally determined production function will depend on the particular institutional structure. In the apiary-orchard example the traditional representation may confront both producers with convex technologies in the inputs under their control though the underlying physical production function may involve increasing returns to inputs. In fact, if all inputs were marketed it could turn out that due to increasing returns in inputs under the control of firms it would no longer be profitable to produce privately. [9]

We could also have a situation where the physical space of inputs is convex but the institutionally defined space appears to be nonconvex. Imagine that a firm producing *y*, requires air (*r*) to deposit wastes. Its physical production function (abstracting from institutional rules) is defined by the relationship:

$$y = z^a r^b$$

With *a* + *b* < 1, we have a convex technology. The firm does not perceive the dumping of wastes into the atmosphere as the use of an input, i.e., it treats it as a free good. It could be that the firm effectively uses air in such a way that the perceived

[9] Mas-Collell, Whinston and Green (1995, p. 375 footnote 25), have suggested that a production function of the form shown above might be associated with Marshall's original external economies, where one could think of "an industry composed of many firms and that the externality is produced and felt by all firms in the industry (eg., with [*e*] viewed as an index, correlated with output, of accumulated know-how in the industry)." So that each firm is contributing to the know how of the industry by imperceptible amounts but is benefiting from all others' contributions to know how which enter the production function as non-marketed inputs. Attempting to capture the underlying physical technology involved in this example is less straightforward. It would have to show how each firm's output is a function of certain developed skills which themselves may be a function of particular divisions of labour in the firm. If transaction costs did no matter and firms were fully aware of the underlying physical technology one could envisage firms paying for the specific skills to be developed by other firms. See Chipman (1970) for his attempt to 'rehabilitate' Marshall's original discussion on external economies and Predergast (1993) for an alternative interpretation.

technology is nonconvex. In particular, envisage a relationship between use of the marketed good and the non-marketed good described by the following equation:

$$r = z^c$$

So that the perceived relationship in the institutionally defined space is:

$$y = z^{a+bc}$$

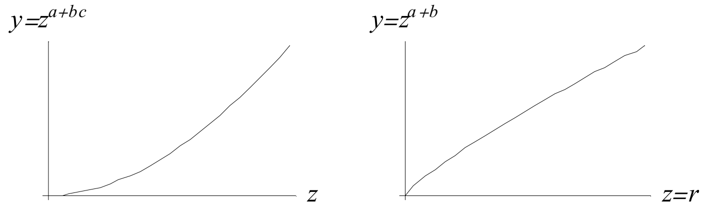

Figure 2

If $a + bc > 1$, the technology in the space of institutionally defined inputs appears nonconvex. The marketed input is capturing the impacts of another input that is not 'controlled' by institutions. Figure 2 (left side) shows the technology as perceived by the firm when air use is free but is actually being used in conjunction with the marketed good, while Figure 2 (right side) shows the underlying physical technology when inputs are scaled up proportionately.

4 No special link between nonconvexity and detrimental externality

Nonconvexities and externalities have always been seen as standard foes of the decentralized private-ownership economy. Arrow's thought experiment of setting up artificial commodities for externality activity seemed to provide a way to reconcile the presence of externalities with the two Fundamental Welfare Theorems. Commodities could be defined as an agent's observations of every other agent's consumption and production choices. As long as markets could be defined for all activity, failure remained the sole prerogative of nonconvexity.

The work of Starrett (1972) and Baumol and Bradford (1972) came to dispel the notion that the problem of externalities could be overcome by the ingenious technique of appropriate redefining of the institutional space. In different ways, both showed that detrimental externalities had a special relationship to nonconvexities. Starrett argued that with Arrovian markets for detrimental externalities, nonconvexities would be unavoidable, that they are in a sense "fundamentally" linked to nonconvexities. Baumol and Bradford on the other hand argued that increasing intensity of detrimental externality will eventually make the production possibility frontier nonconvex. I will argue, instead, that there is no special link between nonconvexities and detrimental externality and that if the production space is nonconvex in the presence of detrimental externalities it is for the same reasons that the production space would be nonconvex for conventional goods. This insight has been hindered by an inadequate delineation between the institutional and the physical parameters of the models. I will use the familiar example of a laundry firm detrimentally affected by smoke from a nearby steel firm to reinterpret the well-known discussions by Baumol and Bradford and Starrett.

4.1 Fundamental nonconvexities and bounds

Starrett (1972) has argued that whenever there are detrimental externalities, the formation of Arrow markets in detrimental externalities unavoidably lead to nonconvexities in the extended production space. Arrow type commodities induce a kind of artificial independence of production sets. Consider an economy with M firms and N marketed goods. The firms' ordinary or *institutionally* defined production sets will be subsets of the R^N space. Net outputs are now distinguished not only by which firms produce them, but also by which firms are affected by them. Accordingly, y_{ijk} stands for the net output of commodity *k* by the firm *j* as observed by firm *i*. For any amount of commodity *k* that a firm produces (or uses) it generates *m* joint commodities which are all other firms' observations of its net output, so that $y_{ijk} = y_k$ for all $i = 1, ..., M$. All firms' production sets can be seen as contained in the *nm*-dimensional net output set with commodities distinguished according to the firm that produced them, or they may be contained in the extended nm^2 dimensional net output set distinguished according the firm that is being affected as well. If firm *j*'s production of steel *k* is associated with smoke that harms firm *i*'s laundry production, firm *j* will be *jointly* producing two commodities y_{jjk} and y_{ijk}.

The well-known example discussed by Starrett (1972) is that of a laundry firm suffering from smoke coming from a nearby steel firm. A standard representation has the laundry production depend on the production of steel standing as a proxy for smoke.

$y_s = h(l_s)$
$y_L = f(l_L; y_s)$ or $f(l_L; l_s)$

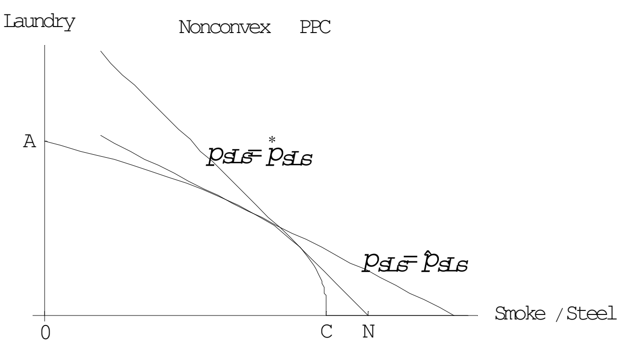

Figure 3

The laundry firm's production will be a decreasing function of smoke (or steel output); as smoke increases the output of laundry for given factor inputs must either fall to zero or level off at some point (see Figure 3). Either way the production function appears to be nonconvex with respect to the detrimental activity. If the laundry firm were allowed to sell pollution permits to the steel firm no equilibrium could arise. At a positive price for the pollution permits, it would always be in the interest of the laundry firm to sell an infinite amount of permits while the steel firm would want a finite quantity. At a zero price the laundry firm would not sell any permits while the steel firm would want to purchase a positive quantity. In Figure 3, levels of smoke beyond point C cannot further reduce laundry production but appear to be part of the production frontier thus making the production function nonconvex. The production function thus appears to be fundamentally nonconvex in the observation "smoke emitted by the steel firm".

With Arrovian markets in smoke permits or observations of steel output by the laundry firm, the profits of the two firms would be:

$\pi_L = p_L\, y_L - w_L\, l_L + p_{SLS}\; y_{SLS}$

$\pi_S = p_{SSS}\; y_{SSS} - w_S\; l_S - p_{SLS}\; y_{SLS}$

with $y_{SSS} = y_{SLS}$

Where p_{SSS} stands for the price of steel output produced and observed by the steel firm, and p_{SLS} the price of steel output produced by the steel firm as observed by the laundry firm. The positive sign for the price of smoke permits in the laundry firm's profit function reflects the fact that for the laundry firm the selling of smoke permits becomes a source of revenue, while for the steel firm the right to pollute is similar to any input. As long as the laundry firm can sell an infinite number of pollution permits there is no chance of equilibrium in this market. For a positive price it will always pay to either shut down or continue to produce (if the production function is of the shape AC) and sell an indefinite number of permits.

A number of authors[10] have pointed out that this problem seems to rest largely on the fact that there are no bounds on the number of permits to be sold. If one were to set bounds on the number of permits, even arbitrarily, it could be that the Arrovian market for detrimental externalities would equilibrate at least for some price ranges. For instance, if the bound were set at N in Figure 3, the market would come to an equilibrium for permit prices lower than $\overset{*}{p}_{SLS}$ such as $\hat{p}_{SLS}$. It would also allow for a general representation of initial endowments of the finite number of permits, from pollutee rights to smoke free air, to polluter rights to soil the air, or any intermediate endowments:

$\pi_L = p_L\, y_L - w_L\, l_L + p_{SLS}(\overline{y}_{SLS} - y_{SLS})$

$\pi_S = p_{SSS}\; y_{SSS} - w_S\; l_S - p_{SLS}(\overline{y}_{SLS} - y_{SLS})$

where $\overline{y}_{SLS}$ are the total number of permits allowed

Along these lines Boyd and Conley (1997) recently argued that the "fundamental" nature of nonconvexity stems, not from an inherent feature of detrimental externality, but from the unboundedness of Arrovian markets *per se*. Unboundedness in any input would unavoidably lead to nonconvexity: "Suppose, for example, that one agent is endowed with an infinite quantity of any input, but that there are no externalities in the economy whatsoever. Suppose also that the first unit of the input has positive value to both the agent with the endowment, and at least one other agent. Then again at any positive price, supply is infinite, and at price zero, supply is zero and there is excess demand. Thus, there do not exist supporting prices. In other words, it is really the unboundedness of the endowment, and not the presence of externalities per se, which drives Starrett's market failure" (Boyd and Conley, p. 394). Nonetheless, they argue, Starrett's conclusion that Arrow markets for detrimental externality lead to fundamental nonconvexity is strictly correct because the structure of Arrovian

[10] Baumol and Oates (1988), Cornes and Sandler (1996), Cornwall (1984) , Papandreou (1994), Boyd and Conley (1997).

markets does not foresee bounds on externality markets. "It is very hard to see...how... a bound could be introduced. It is not very appealing to claim that there is a limit on the capacity of firm *i* to observe the production by firm *j* of commodity *k*. In any event, it would be completely counter to the well-established tradition in general equilibrium theory to incorporate such bounds in the production sets themselves. We usually define production sets such that it is possible to contemplate arbitrarily high levels of input. The fact that there exist only a finite amount of possibilities to pollute in the world does not enter into their definition. Such constraints are more appropriately treated through the endowments of agents. The Arrow model does not seem to have sufficient flexibility to allow for this" (p. 396).

Boyd and Conley also dispel the notion that the presence of externalities may imply fundamental unboundedness: "The land has an inherent limit on the amount of waste that can be stored. It is important to emphasize that *this limit is not imposed by human agency* [my Italics]. A stream can only hold so much effluent. An airport can generate noise pollution at most 24 hours in a given day. Even the atmosphere is finite. The most ambitious factory cannot pollute an infinite quantity of air, because only a finite amount exists to be polluted" (p. 394).

While it is true that Starrett's 'fundamental' nonconvexity has to do with the structure of Arrovian markets, the point is more general. The 'fundamental' nonconvexity is a result of the particular institutional structure used in the model rather than an underlying feature of physical reality, or detrimental interaction however institutionally modelled. Whether bounds are compatible with Arrovian markets or not, it is this aspect of the institutional structure that is causing the 'fundamental' nonconvexity, and lack of bounds would generate nonconvexity for any activity (detrimental or not).

The discussion raises two interrelated issues. First, the question of bounds raises a number of critical issues for institutional design. We know that problems arise when property rights do not align well with physical reality, as when property rights are not defined for scarce resource, what if property rights have been defined for resources, but the total number of rights do not align well with the underlying physical bounds? In the same way that it can be said that property rights have not been adequately or completely defined, one can say that the institutionally determined total quantity of rights have not been adequately defined. Second, is there any compelling reason why one should consider Arrovian markets as a natural candidate for the resolution/removal of externalities? Though bounds may seem to fit uncomfortably with an Arrovian market structure, this problem may not arise under alternative institutional descriptions of the economic space. The next section deals with this issue.

4.2 Resource markets versus Arrow markets

By construction the Arrovian artificial market setup implicitly assumes that non-market activity (externality) is necessarily jointly produced with market activity. For instance, smoke is a spillover that is jointly produced with steel. The Arrovian institutional device would be a consistent means of commodifying externalities if they indeed displayed this intrinsic jointness. However, as argued earlier, externalities and jointness are separate attributes and there is no inherent reason why non-market activity should necessarily be viewed as a by-product of some marketed activity.

Indeed, many of the activities traditionally associated with externality do not display the kind of jointness envisaged in Arrow markets.

If a steel firm is emitting smoke into the air, it is more likely that there are several ways for it to abate the pollution than by reducing steel output. Arrovian markets force a fixed relationship between the steel output and the effluent that may not bear a relationship to the actual *physical* production possibilities. If the steel firm has several means of abating or substituting away from smoke, a market for the observation of steel production will not create the appropriate incentives for selection of the most efficient pollution abatement. Likewise, if the steel firm has several alternative means of disposing of its effluents such as reducing atmospheric deposits by increasing water effluents, the markets for steel observations would not capture this. In this respect, if Arrow markets do not align with the underlying physical production possibilities, they will not reconcile the extended market model with either of the Two Welfare Theorems. In fact Arrow markets will not have eliminated the externality, as the property rights envisaged will not correspond fully with the underlying physical production functions. That Arrow markets have been seen as a theoretical device to eliminate or 'commodify' externalities is due to the restricted way of representing externalities.

Viewing 'observation' markets as natural ways of commodifying externalities derives from thinking of externalities as joint by-products of market activity. Once this perception is abandoned conventional markets (and property rights) become obvious candidates for certain non-market activities. In the case of smoke and laundry, rather than establishing a mysterious link between steel output and laundry, we can envisage a 'conventional' market for the resource 'air' which should have figured in the underlying physical production possibility sets of laundry and steel production. Importantly, this conventional market does a much better job of capturing substitution possibilities in the output of both goods, indeed it even allows for potential 'defensive' activity on the part of the laundry firm. A more realistic description may involve a full listing of potential residual by-products resulting from steel production and waste deposit inputs reflecting the necessity to deposit residuals somewhere in the environment.

Setting benchmarks or limits on detrimental activity also has a more conventional counterpart when treated as missing resource markets. Bounds are not an issue when considering the production of goods. Any ultimate limits on output will be determined by bounds in the inputs required for production. Part of the conceptual difficulty with Arrow markets in detrimental externality is that it seems strange to incorporate bounds or benchmarks on the 'output' of pollution or 'observations' of steel output. By reinterpreting detrimental externality as the use of some previously missing input, bounds can be more easily envisaged. Instead of the *production* of a 'bad' we have the *use* of a 'good'. Pollution 'output' and the corresponding pollution 'observations' are redefined as air 'inputs' or 'resources' and thus bounds are placed in the endowment set.

While Arrovian 'observation' markets do not appear to be appropriate means of commodifying certain kinds of non-market activity, it is no coincidence that they would seem to be a natural candidate for pure public goods or 'bads'. The idea of a Lindahl equilibrium from which the Arrow construction derives its inspiration, was meant to address the problem of determining the appropriate level of production of (and revenue for) public goods. In the case of pure public goods (or 'bads') we could argue that there is intrinsic jointness in physical consumption and production possibilities. Much as we reinterpreted pollution as the use of an input 'air' that allowed a more natural incorporation of physical bounds, it also seems more appropriate to reinterpret public 'bads' as the private (or public) use of goods that may have other public uses. The use of 'air' as a waste sink excludes other uses that require air to be clean. Clean air could have many public or joint uses, e.g., healthy environment, wildlife protection, laundry output, etc. A firm that requires air as a waste sink would have to bid for its use. Any potential owners' of air would weigh the profits from selling air services to polluters (indeed using air as a waste sink may also have public good attributes in the sense that one polluter's use may not exclude another') against the profits from selling air services to those that benefit from air being kept unpolluted (public use). If many agents jointly benefit from activities that require the air to be clean the Lindahl prices will register these benefits. Furthermore, bounds can be more easily envisaged for the good 'air' (that may have public uses) than the public bad 'pollution'.

In Starrett's example laundry production is modelled as a function of labour input and steel output standing as a proxy for smoke. Alternatively, the way to proceed is to envisage an underlying physical description of the production process and then consider alternative institutional descriptions (including the one envisaged by Starrett). A realistic physical description of steel production would incorporate the joint production of residuals. Some residuals might be marketable, but it will often be the case that some residuals need to be disposed of. In this case the production possibility set should include all possible means of disposal as inputs into the production of steel, i.e., steel cannot be produced without having to dispose some residuals into the environment and thus use up inputs for waste disposal. The physical description of laundry production would incorporate air (of certain qualities) as an essential input.

To simplify matters I will just assume that the underlying physical production possibilities for steel incorporate the use of air for waste disposal (this captures the essential difference with Starrett's representation), and laundry production requires inputs of clean air. The physical production functions are:

$$y_S = h(l,\ a)$$
$$y_L = f(l,\ a)$$

where *l* represents labour and *a* represents air

The lack of subscripts is meant to highlight the fact that the means of institutional control over resources have not been established yet. It appears more natural to view the cleaning of laundry as requiring clean atmosphere than viewing laundry as dependent on steel production. In essence we are recognizing that there are physical attributes of air that affect laundry production and that if these are altered by some means the laundry production will be affected. Indeed this representation leaves open all the potential means of maintaining clean air, e.g., locational separation, pollution abatement on the part of the steel firm, defensive action on the part of the laundry firm, etc.

Until we specify the institutional structure describing laundry and steel production we are not in a position to say whether externality is present. If markets exist for all physically possible activity then we appear to have a totally conventional production model. We can incorporate subscripts to indicate that each firm can control the inputs in question through market exchange. With a full set of markets we have a conventional model for the two institutionally defined production functions:

$$y_S = h(l_S,\ a_S)$$
$$y_L = f(l_L,\ a_L)$$

Externality can be introduced into our model by defining an incomplete set of markets. In this case air would be the non-marketed activity. This representation treats the problem of externality as a missing input (air, or air of a certain constituency) that is not explicitly modelled, rather than viewing one firm's input as a function of another's output. Incomplete markets can be represented with subscripts indicating that control over the use of air is now determined by the steel firm (air is now a non-marketed good):

$$y_S = h(l_S,\ a_S)$$
$$y_L = f(l_L;\ a_S) \text{ or } f(l_L;\ \overline{a} - a_S)$$
and $a_S \leq \overline{a}$

where $\overline{a}$ reflects the total amount of air available where the firms are located

The latter representation of the laundry production function is more informative in that it suggests that the steel firm's use of air as a waste deposit is diminishing available air for laundry production. In this case it would not be unnatural to incorporate endowment bounds into the production function since we are explicitly assuming that one firm is in control of a resource, and the other firm is using what is left over. Bounds become part of the description of the non-market activity. While appearing identical with the Meade form where one firm' input choice is an input into another firms production function there is a difference in that now we have acknowledged the presence of a new input (the total number of commodities in our model has increased). Rather than assuming there is some 'mystical' link between one firm's use of an input (say labour) and another firm's output, we are explicitly modelling the physical form of interaction by suggesting that there is a missing input, air, that is an input in both production processes. In this view, the question of bounds comes up naturally as a question of underlying endowments in "air". The introduction of bounds on total quantity of clean "air" is analogous to setting bounds or benchmarks to pollution permits. Detrimental pollution is simply redefined as reduction in clean air. In the Arrovian extended market model setting benchmarks seem arbitrary, largely because external activity is modelled as an *output* being jointly produced with a good. Why should there be limits on how much to produce? However, when viewing the externality as an "open access resource" that is being used up as an *input* into a production process, it seems natural that bounds should be present.

A question arises as to the mechanism of control of the air input. In the standard representation of externality, 'external' activity is a by-product of market activity (in a sense it has no separate status). This seems necessary in the general equilibrium context since the only means of activity envisaged is through the buying and selling of commodities. As a separate mechanism for physical interaction to take place outside the market is not envisaged, externality has to show up as being jointly determined with some market activity. Once externality is modelled as a potentially separate physical activity a proper appraisal of its ramifications would require an explicit modelling of this non-market activity. How does the steel firm decide which residuals to spew out and where? If smoke is not simply tied to output by a fixed relationship, how will the firm decide how much smoke to use or how much air to use as a waste sink? Our standard view of the production function would have the firm use up the entire amount of 'open access' air for waste disposal, but this would not capture important features of the disposal process that would be costless only up to a point.

Viewing externality as a missing input provides greater degrees of freedom in terms of the possible forms of production functions, but can replicate any functional form of a lower dimension. By appropriate assumptions over the functional form, and by incorporating the constraint in steel production that air and labour must be used in fixed proportions, the representation with air becomes effectively identical with Starrett's representation. Starrett's model is seen as a limiting case of a broader set of missing input models.

$$y_S = h(l_S, a_S) = \text{Min}\,\{h(l_S), h(a_S)\}$$

which appears as $h(l_S)$ to the firm that treats the air as an open access resource

Importantly, this representation makes explicit the need to incorporate bounds that are seen not as arbitrary constructs but as anchored in physical reality. In Figure 4 we reinterpret the 'smoke' axis from Figure 3 and show laundry output as a function of reduced availability of the open access resource air. At the origin the laundry firm has full use of clean air. As we move to the right, the steel firm is increasingly using up the clean air available. Eventually the laundry firm does not have enough clean air to produce. The benchmark explicitly represents the total amount of air available. It is not an arbitrary construct, but should be part of the physical description.

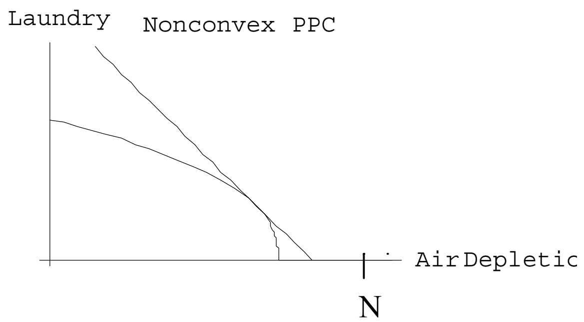

Figure 4

The nonconvexity is not due to the detrimental nature of externality but derives from the underlying implicit physical production possibility set. Laundry requires a minimum quantity of clean air for it to begin production. The natural way to overcome the externality is to form property rights for the resource air that was previously treated as an open access resource.

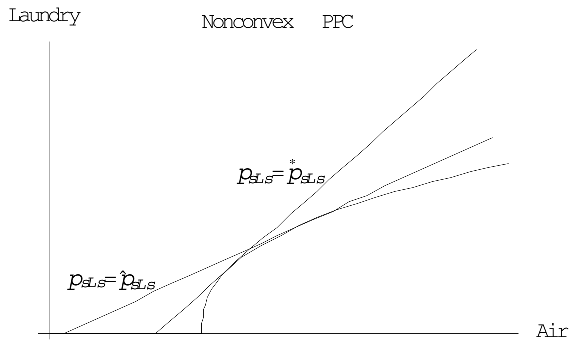

Figure 5

Figure 5 above shows laundry production as a function of input "air" which was previously treated as an 'open access' resource. Essentially, all we have done is to flip the horizontal axis around. Now laundry output is seen as an increasing function of air quality rather than a decreasing function of smoke. Indeed the production appears to entail setup costs in air, i.e., for the laundry firm to produce a minimum quality of air is essential. What appeared to be nonconvexity due to detrimental interaction is now simply a manifestation of the underlying production technology. Furthermore, in this model we find it natural that there are some underlying limits on the resource "air of a certain quality". Externalities are no longer present, and the markets are quite conventional. The profit functions of steel and laundry firms are:

$$\pi_s = p_s\, y_s - w_l\, l_s - w_a\, a_s$$
$$\pi_L = p_L\, y_L - w_l\, l_L - w_a\, a_L$$

where $l_s + l_L = \bar{l}$

$$a_s + a_L = \bar{a}$$

In Figure 6 the first row of graphs places a model with laundry output as a function of detrimental externality (smoke, or depletion of air) on the left, next to the model of laundry output as a function of air. With Arrovian markets in smoke, nonconvexity appears to result because the benchmark is to the right of the point where production function meets the horizontal axis. While it is reasonable that there should be some benchmark of total smoke determined by physical constraints, resource bounds do not seem to fit well with Arrovian markets. On the other hand, when we model the detrimental interaction as a missing market for air it is quite natural that we should specify bounds for the resource. Any nonconvexities will now take on a conventional interpretation: there are setup costs in the production of laundry. It is clear that the nonconvexity is not due to the presence of detrimental externality (once bounds have been incorporated), but the underlying nature of physical production technology.

The second row shows the case where the production set in convex in the detrimental externality or in the input air. The production of laundry declines steadily as pollution increases, or as the quality of air decreases until we reach the bounds of pollution or deplete all of the air input (use in the production of steel). In the conventional model this is simply a reflection of a decreasing returns to scale in the input air. The detrimental externality is no different than flipping the air axis around, i.e., rather than the origin representing a point of zero inputs it represents a point of all air going to laundry production, with air diminishing along the axis as steel production uses it up. The convexity of the detrimental externality function is a reflection of the convexity of the conventional function. Both Arrovian markets with bounds and conventional markets would clear. Simply setting up property rights and conventional markets in air quality would solve the externality in air use.

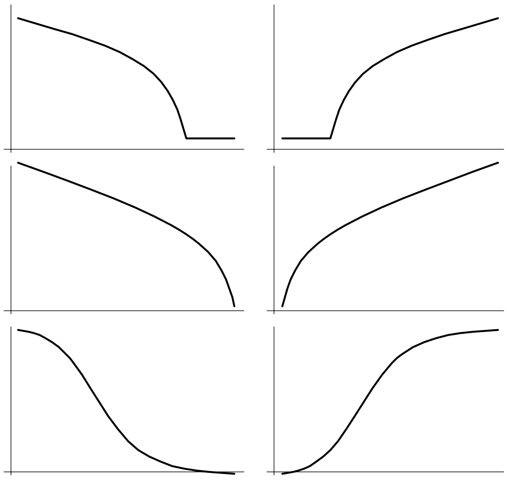

Figure 6

The third row shows the case of a production function that is nonconvex in detrimental externality as well as in the input air despite there being no setup costs.

The two columns of production function correspond to several interpretations. In all cases the shape of the production functions are determined by underlying physical attributes. The left column shows laundry output as a function of smoke or depletion of air. If air is treated as an open access resource then we have externality. If a market for air or smoke is formed, then the externality vanishes. An Arrovian market with pollution permits (or permits to deplete air) would be depicted on the left side. It seems far more natural to set up rights in the use of air as shown on the right side.

Treating a detrimental externality as a missing input has the advantage of making explicit the restrictions in the Arrovian market setup, i.e., of a fixed relationship between the output of the steel firm and the externality. The incorporation of the missing input air allows us to treat the Starrett model as a limiting form of externality, where pollution is jointly produced with steel. A more realistic model of externality will recognize that air may be an input in both steel and laundry production, with numerous substitution possibilities. The laundry firm may be able to take defensive measures and thus reduce its need for air including its capacity to locate elsewhere, i.e., exchange air in one locale for air in another. Likewise the steel firm may be able to use other waste sinks for its residuals, or find ways to reduce smoke without influencing its steel output. Importantly, detrimental externality cannot be a cause of nonconvexity. If there is nonconvexity in the production function, it will appear whether we allow markets for all inputs or leave some activity as non-marketed. The problem of defining benchmarks in Arrovian markets acquires a natural interpretation when the externality is viewed as missing property rights for a conventional inputs.

4.3 Intensity of detrimental interaction and convexity

Baumol and Bradford have argued that if a detrimental externality is 'intense' enough, eventually the production possibility curve will become nonconvex. I will argue that this is not due to some special feature of detrimental externality but has to do in part with an interpretation of 'intensity' of input use that would turn any production possibility curve nonconvex. They consider a two-output, one-input economy in which each output is produced by a single industry, and in which each industry has a convex technology in its own inputs. There is also detrimental externality whereby one firm's increases in output lead to increases in the other firm's costs or increases in inputs required to produce given levels of output. The two firms have convex production sets in the inputs under their control.

If two firm's technologies are linear in their own inputs, the slightest presence of detrimental externality will make the production possibility set nonconvex. Without 'interference' of the two production activities, the production possibility set would be linear. The slightest negative interference can only lead to a reduction in total possible output combinations, leaving only the two extreme points unaffected where one firm does not produce, thus pulling inward the production possibility set. For the more 'general' case an algebraic example is used where each firm has a decreasing returns to scale technology in the inputs it controls. Let y_S and y_L be the output of electricity and laundry, l_S and l_L the amounts of labour (negative leisure) used, and *w* a measure of intensity of the detrimental interaction between the two firms. The labour requirements as well as the implicit equation for the laundry-steel possibility frontier are:

$$l_s = \frac{y_S^2}{2}$$
$$l_L = y_L^2 + w\, y_S\, y_L$$
$$\bar{l} = l_L + l_s = \frac{y_L^2}{2} + w\, y_S\, y_L + \frac{y_S^2}{2}$$

Each industry separately is subject to strictly diminishing returns to scale in terms of the input's each 'controls'. The production possibility frontier is convex for values of *w* less than or equal to one, for higher values it becomes nonconvex. Figure 7 below shows the production possibility curve for different values of *w*.

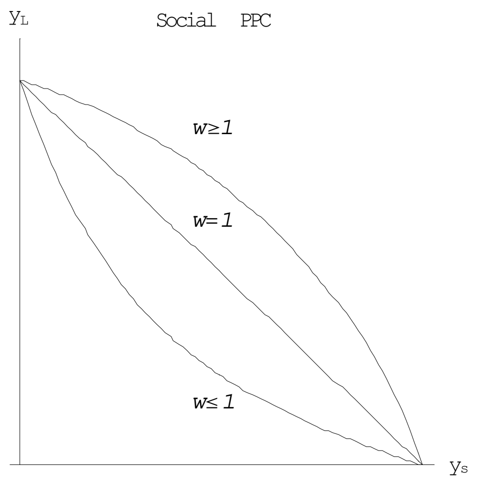

Figure 7

If we present the production functions in a more conventional way with output as a function of inputs, rather than input requirements as a function of outputs, we would have:

$$y_L = \sqrt{y_s w^2 + 2 l_L} - \sqrt{y_s w^2}$$

$$y_s = \sqrt{2 l_s}$$

This latter representation suggests that the functional form of the detrimental externality is far from intuitive. It is also less clear what the precise meaning of 'intensity' of detrimental interaction is. The easiest way to see what is happening here is to reformulate the problem as a missing input model in the same way we dealt with Starrett's fundamental nonconvexity. Indeed we can do so while preserving the identical production structure. Let's assume that both steel and laundry firm require the services of 'air'. By replacing y_s with $\sqrt{2(\overline{a} - a_L)}$ we can write the underlying physical production functions as:

$$y_L = \sqrt{\sqrt{2(\overline{a} - a_L)}\, w^2 + 2 l_L} - \sqrt{\sqrt{2(\overline{a} - a_L)}\, w^2}$$

$$y_S = \text{Min}\left\{\sqrt{2 l_S}, \sqrt{2 a_S}\right\}$$

This tells us that production of laundry is a function of its use of labor and air. Note that there is no externality in the physical representation. The production of steel is a function of labor and air use, which have to be used in equal proportions, this way we imitate the precise production function represented without the input air. If we now want to superimpose an institutional description with a missing market in air we could represent the nature of control that replicates Baumol and Oates detrimental externality by replacing $(\overline{a} - a_L)$ by a_s.

$$y_L = \sqrt{\sqrt{2 a_s}\, w^2 + 2 l_L} - \sqrt{\sqrt{2 a_s}\, w^2}$$

This is one way of showing that the steel firm determines the control of air use/quality. More generally, any detrimental externality can be reinterpreted as a physical production function that is ‘conventional’ and an institutional production function where property rights are missing for some input. This reinterpretation allows us to see that the nonconvexity is already present in the underlying physical production function, even though the individual institutional production functions are convex in the institutionally determined control variables. In this case the institutional setup does not allow the laundry firm to control atmosphere use.

It is not the 'detrimental' nature of externality that leads to nonconvexity in the social production function, but the underlying nonconvexity inherent in the laundry's production function that is 'hidden' by the institutional structure. The underlying physical production function for laundry is an increasing function in air irrespective of the intensity of air use w. It is the lack of a market that turns the conventional two input model into a detrimental externality and masks the nonconvexity present in laundry production. The nonconvexity in the social production possibility set will arise for certain values of w whether we view the model as one of detrimental externality or a standard two input model with jointness in use of labor and air in steel production. As long as the social production set relates to the underlying physical possibilities, its shape will be determined by the underlying physical production functions, and whether and how we introduce externality will not in any way affect the shape of the social production possibility set. Indeed, in this case, externality is totally irrelevant in determining its shape. Even in the case of "linear" production functions, it is not detrimental externality that gives rise to nonconvexity but the underlying structure of the production function.[11]

Baumol and Oates argue that the point that the intensity of detrimental externality causes nonconvexity in the social production possibility set has "generality and rigor. The point is that, with any pair of commodities at least one of which interferes with the production of the other, there will be no such interference if one or the other is not produced. On the other hand, if the interference is sufficiently great, the maximal output of the activity suffering the external damage will approach zero for any nonzero level of output of the other and a nonconvexity in the feasible set is unavoidable" (p. 116).

As we saw in the model above the 'intensity of interference', w, did eventually turn the social production function from convex to nonconvex. However it is not the result of 'interference' (which is associated with the institutional setup - externality), but 'intensity' (and the way it is interpreted) that is giving rise to nonconvexity. Indeed, if we look at the same model in its conventional garb and imagine that air is a commodity to be bought and sold by both steel and laundry firm, then it is the 'intensity of air use' that eventually leads to nonconvexity. On this interpretation of intensity, the intense enough use of any input in an individual production function will turn the social production set nonconvex. The production possibility set associated with a conventional two Cobb-Douglas production function model for laundry and steel, $y_L = l_L^a k_L^{w\,b}$ and $y_s = l_s^c k_s^d$, where w captures the 'intensity of use', will eventually become nonconvex for any fixed positive values of *a,b,c,d.* That intensity of input use causes nonconvexity depends on the interpretation given to intensity. It could be that the intensity of input use for a Cobb-Douglas production function is captured by w in $y_L = l_L^{(1-w)} k_L^w$, for values of $w \in (0,1)$, so that the production function remains convex however strong the intensity of input use is (or of externality if we replace k_L by k_s and assume that it is controlled by another firm). If 'intensity' of input use is strong enough, one firms' production of even a small amount output may prevent the possibility of production by the other firm. In other words, the specific input is so 'essential' that the firm in question either cannot, or can barely, produce without a significant amount of the input in question. This would be the case either if production involved significant setup costs in the specific input, or there were substantial increasing returns to scale in the specific input. It is not a result of interference as suggested by Baumol and Oates.

Although the example used suggests a particular technological form of interdependence the point is general. Jointness of production though often closely associated with externality is not itself the defining feature of externality. By increasing the complexity of the underlying physical production sets, jointness adds to the demands that institutions are called to address.

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[11]Here is a simple linear system (like the one envisaged by Baumol and Bradford (1972)) where even the slightest interference between steel and laundry production would lead to a nonconvex social production set: $y_s = l_s$, $y_L = l_L/(1 + w\,y_s)$. Here is an underlying physical production function with the addition of a missing resource and its externality counterpart that have the identical shapes: $y_s = l_s$ and $l_s = a_s$, $y_L = l_L/(1 + w(\overline{a} - a_L))$.

5 Conclusion: On physical complexity and the social definition of scarcity

Economic theory has generally acknowledged the role that institutions have in shaping the economic space. The distinction however between physical and institutional descriptions of economic activity has not received adequate attention within the mainstream paradigm. While physical constraints are in a sense immutable, institutional structures are artefacts of human design and interaction. Though at one level their function is often to constrain (and thus channel) human activity, at another level they are mechanisms to be designed in accordance with social goals. The distinction is thus important in determining the extent of feasible social action.

By modelling the economic space as a combination of physical and institutional descriptions a number of theoretical insights can be gained and conceptual problems overcome. Within the context of a frictionless Arrow-Debreu setting, externality can be unambiguously defined as incongruence between physical and institutional models of economic space. Externality cannot be present in the physical world and cannot inhere in resources or goods. It is purely a reflection of a mismatch between the institutional and physical models, and is thus best viewed as a purely institutional construct.

Properties of production sets are also clarified by a crisper distinction between the physical and institutional. Production functions can be convex in the institutional space yet nonconvex in the physical space. A more novel point is that the converse also applies. The way inputs and outputs are institutionally defined will determine the shape of the production set in institutional space. The presumed special link between detrimental externality and nonconvexity dissolves when it becomes apparent that externality cannot bring about nonconvexity in the physical production space. The shape of the physical production space is unaffected by how institutions are defined. Starrett's fundamental nonconvexity arises in the space of Arrovian markets and is due to the unboundedness of 'detrimental activity' rights. If bounds were set in these markets there would be no fundamental nonconvexity in this institutional space. With bounds in place, any remaining nonconvexity is neither a function of 'detrimentallness' nor of 'externality', but the underlying shape of the physical production function. This can be seen as replacing Arrovian markets over joint detrimental activity with conventional markets over non-marketed resources, i.e., rather than rights in the steel firm's polluting activity, standard rights over resources such as air. By thus defining institutions one defines away detrimental externality and nonconvexity becomes conventional, i.e., it turns out not to be a function of the institutional description in this instance. In a similar fashion, it is in not the intensity of detrimental externality that brings about nonconvexity in the production possibility curve, but appropriately defined *intensity* of input use in general. In short, there is no special link between detrimental externality and nonconvexity.

The very term 'markets for externalities' tends to imply that there is more to externality than simply the non-existence of a market. This is most likely the result of confounding certain kinds of jointness with externality. Most representations of externality treat it as being a joint by-product of some market activity, rather than simply an open access resource. The use of Arrovian or Lindahl markets as means of overcoming externality is a symptom of that view. Numerous externalities can best be eliminated by conventional markets. In fact the use of Arrovian markets in cases where conventional markets are required will cause inefficiency by reducing the degrees of freedom in input substitution or 'defensive' activity. This suggests that clearer description of the underlying physical relationships is important in order to determine the 'appropriate' or corresponding institution. Generalizing this point, we would expect that different physical structures would require alternative institutional structures.

In this light we can also see the question of bounds. The possibility that bounds have not been set can be viewed as another kind of incongruence (externality) between physical and institutional descriptions of economic space. While property rights may be defined for some resource, it may be that the total quantities of rights do not align with the underlying physical availability. In the case of pollution or 'air' use rights we would like the benchmark for total allowable pollution (or air use) to align with physical possibilities. If there are more or less pollution permits than physically possible it is quite likely that a competitive equilibrium will not be Pareto optimal. Universality of markets, a prerequisite for the first fundamental welfare theorem, must also mean that bounds are adequately determined. This poses a potentially difficult task for public policy: the social definition of scarcity. To the extent that physical bounds are not self-evident, society must find a means discovering these if endowments are to register physical scarcity.[12]

When pollution is modelled as one firm's output entering directly into another's as an input, Arrow markets seem to be a natural choice yet they confront the problem of setting bounds that correspond with our intuition that there are physical limits to the environment's waste absorbing capacity. An attempt to better represent the underlying physical conditions suggests that many detrimental externalities are a combination of the (joint) production of residuals and the use of some media to dispose of the residuals. A firm may have many degrees of freedom in the choice of residuals to produce as well as the place to dispose them. Some residuals may have value in other uses and thus may command a positive price (for consumption or input use). If the residuals cannot be sold for a positive price they need to be disposed of. If disposal involves the use of private resources, e.g., my backyard to dispose of certain kinds of garbage, then these can be purchased in conventional markets. Oftentimes waste disposal requires the use of resources that have public uses such as disposal in air or the sea. In this case the firm would bid to privately use a resource (for the disposal of waste) that could have public uses (such as amenity value). Resources with public uses would require Lindahl prices, and endowments would register total availability. One could envisage a firm purchasing the right to dispose of wastes into a river by outbidding the local community's total valuation of its public uses.[13] To the extent that

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[12] Many of the more intriguing and important questions have to do with how institutions are formed and the normative counterpart how institutions *should* be formed. How are bounds to be determined? There is no doubt that there are physical limits to external activity. This may not be helpful, however, if we have difficulty recognizing what these limits are. By not explicitly modelling institutions we tend to take for granted that institutions align automatically to the contours of physical reality. Endowment limits in standard models are well defined and property rights fully assigned to resources. But now we are confronted with physical limits we know to be present but are unable to immediately discern. The role is far from trivial even if we try to portray it as a scientific endeavour to discover the environment's limits. Furthermore, the role is different than that of trying to discover the optimal levels of pollution.

[13] Boyd and Conley (p. 397) suggest a model with four types of goods: private consumer goods, directed externalities, public goods and public externality rights. Directed externalities are meant to capture private bads (and are included for expositional purposes though they could essentially be subsumed under the heading of private goods). Public externality rights markets are meant to divide the total endowment of externality rights between abatement and generation of public bads. I find it more consistent to simply envisage resources that may be used publicly for amenity and other environmental

resources have physical qualities that give rise to certain public uses, we have to envisage corresponding institutions if we are to eliminate externalities. Lindahl markets are an institutional device that deals with these kinds of jointness.

As long as the world is modelled as a granular world nicely divisible into parcels that can be individually owned and consumed, conventional markets for private property rights can go a long way in allocating resources. Physical jointness and/or indivisibilities raise the demands on institutional devices. Lindahl markets are a complex (theoretical) institutional device meant to deal with certain kinds of physical jointness. Physical complexity can entail, however, much more than joint use and consumption of commodities. It can become increasingly difficult to envisage theoretical institutional devices to align with physical reality. Management of the environment raises so many challenges precisely because the environment departs so starkly from the conventional model of a portable world. The fundamental continuity of environmental media and ecosystems raises the spectre of large-scale indivisibilities and a complex tangle of joint and multiple uses. Many environmental services are better viewed as outputs of natural production processes than as resources to be allocated. The difficult task of designing institutions for managing our environment demands institutional ingenuity rooted in a deep understanding of its physical structure.

By viewing externality as an incongruence between the physical and institutional economic space we see that the kinds of externality as well as the institutional means to overcome them will vary according to how the physical and institutional space are defined. Arrow markets are adequate theoretical institutional devices to eliminate (address) certain kinds of externality; they do not eliminate 'missing resource market' externalities. Likewise Arrow or Lindahl markets could not overcome 'missing characteristic markets'. The more kinds of physical interaction we try to model the more we will have to seek corresponding institutional forms to efficiently allocate resources. This paper does not seek to find an all-encompassing means of eliminating externalities. Instead it offers an interpretation of externality that recognizes the potentially rich variety of physical interdependencies in the world and the institutional challenges that these pose.

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services, or for waste disposal. This avoids the inclusion of an additional type of good. It simply recognizes that public goods may also be provided by nature (rather than produced). Furthermore, it avoids treating externality as something inhering in goods, i.e., 'markets for externalities’. Once resources are protected by rights (public or private) they are no longer 'externalities'.

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