WHY MATHEMATICAL SOLUTIONS OF ZENO's PARADOXES MISS THE POINT: ZENO's ONE AND MANY RELATION AND PARMENIDES' PROHIBITION.

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MATHEMATICAL RESOLUTIONS OF ZENO's PARADOXES of motion have been offered on a regular basis since the paradoxes were first formulated. In this paper I will argue that such mathematical "solutions" miss, and always will miss, the point of Zeno's arguments. I do not think that any mathematical solution can provide the much sought after answers to any of the paradoxes of Zeno. In fact all mathematical attempts to resolve these paradoxes share a common feature, a feature that makes them consistently miss the fundamental point which is Zeno's concern for the one-many relation, or it would be better to say, lack of relation. This takes us back to the ancient dispute between the Eleatic school and the Pluralists. The first, following Parmenide's teaching, claimed that only the One or identical can be thought and is therefore real, the second held that the Many of becoming is rational and real.[1] I will show that these mathematical "solutions" do not actually touch Zeno's argument and make no metaphysical contribution to the problem of understanding what *is* motion against immobility, or multiplicity against identity, which was Zeno's challenge.

I would like to point out at this stage that my contention is not with the *mathematics* of the particular solutions—I am sure that they are correct just as I have no doubt that such mathematical advances

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[1]"Parmenides of Elea, improving upon the ruder conceptions of Xenophanes, was the first to give emphatic proclamation to the celebrated Eleatic doctrine, Absolute Ens as opposed to Relative Fientia: *i.e.* the Cogitable, which Parmenides conceived as the One and All of reality, *en kai pan*, enduring and unchangeable, of which the negative was unmeaning, and the Sensible or Perceivable, which was in perpetual change"; George Grote, *Aristotle,* vol. 2(London:John Murray, 1872), 243.

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will find appropriate uses, for example in making a jet go faster. What I wish to show instead is that no metaphysical sense can be made out of mathematical sense and any claim to the contrary is unjustified. And further that *any* resolution to Zeno's paradoxes, if it is to "hit the point", must indeed make metaphysical sense.

I

A Summary of Zeno's Paradoxes [2]

1. *Achilles and the Tortoise.* Achilles is to run a race against the tortoise who has a head start. Zeno argues that Achilles will never be able to catch up with the tortoise no matter how fast he runs. In order to overtake the tortoise he must first make up the distance that separated them at the start of the race. When he has accomplished this the tortoise will have moved ahead from his own starting point to a new point. Now Achilles will have to arrive at this new point by which time the tortoise will again have moved ahead to a new position and so on *ad infinitum*. Whenever Achilles arrives at a point where the tortoise was, the tortoise has already moved ahead. The gap can be narrowed but Achilles will never actually catch up with the tortoise.

2. *The Dichotomy.* This paradox has two forms. The first considers an object moving in a straight line from point A to B. Before covering the whole distance and arriving at point B an object must first cover half that distance. Then it must cover half of the remaining distance, and so on *ad infinitum* leading Zeno to conclude that the destination, point B, will never be reached. In the second form the conclusion is that motion can not even begin! The moving object, before moving half the distance must move a quarter of the distance etc. *ad infinitum* so that the object can not even begin to move.

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[2]For detailed discussion see, Wesley C. Salmon ed., *Zeno's Paradoxes* (Indianapolis: Bobbs-Merrill, 1970).

3. *The Arrow.* This paradox denies motion to a moving object, arguing that at any point in time a "moving" arrow must be at rest. Thus at any given instant, the arrow must occupy a portion of space equal to itself. During the instant it cannot move for that would require the instant to have parts, and an instant is by definition a minimal and indivisible element of time. As Russell says, "It is never moving, but in some miraculous way the change of position has to occur *between* the instants, that is to say, not at any time whatever"[3].

4. *The Stadium.* "Half the time may be equal to double the time".4 The last of Zeno's arguments considers three rows of objects arranged in parallel in a staggered formation. Row 1 remains at rest while rows 2 and 3 move in opposite directions until all rows are lined up. Due to the arrangement of the rows of objects and their movement, one object of e.g. row 3 will pass twice as many objects in row 2 than in row 1. Zeno's conclusion was that "double is sometimes equal to half".

We can confidently say that the first two paradoxes are concerned with the passage from many to one (infinite divisibility of a quantity) whereas the second two paradoxes are concerned with the impossibility to accomplish the passage from one to many, from identity to a concrete multiplicity.[5]

For the sake of my argument I will highlight two recent examples of proposed "solutions": one which exploits the concept of

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[3] Salmon, *Zeno's Paradoxes*, 11

[4]John Burnet; see Bertrand Russell, "The Problem of Infinity considered Historically," in Salmon, ed., *Zeno's Paradoxes*, 51.

[5]What I mean by "concrete multiplicity" will appear clear in the following discussion where I will point out that the multiplicity of mathematics is an abstract one insofar as it is purely a reiteration of the unit or the identical one. Mathematics is not able to show, nor is it concerned with showing, the real passage from one unit to the next which is what Zeno was concerned in pointing out with his paradoxes. As long as this passage is not conceptualised, it will be impossible to talk of concrete change or movement against the immobility and identity of each successive and reiterated unit.

"indeterminate forms"[6] and the other which utilises "infinitesimals" and "Internal Set Theory" in its endeavours.[7]

II

Indeterminate Forms Mark Zangari[8] uses indeterminate forms as a key feature in his response to Zeno's arguments and he concentrates mainly on the third paradox, the Arrow:

> This strange term [indeterminate form] plays a key part in the paradox and its misunderstanding has been responsible for much of the confusion surrounding Zeno's "motionless" arrow. The "paradox" in Zeno's "Arrow" rests on being able to show ... that, at any instant during its flight, a moving arrow is actually at rest. Yet the reasoning that yields this conclusion can be shown to be fallacious ... Zeno's argument does *not* establish what he claims about the arrow—that it is stationary at each instant. Rather, the speed evaluates to $v = 0/0$, which is an *indeterminate form*, at any instant; in other words, the value of v is consistent with *any* real number. Therefore, the velocity as determined instantaneously cannot contradict any finite speed that the arrow may possess over a time interval, be it zero or otherwise.[9]

Suppose that the arrow is travelling at a constant, finite velocity, v. In a finite time interval, δt, the arrow travels a distance, δx.

> So that $\delta x/\delta t = v$. Let δt get indefinitely small. Because any interval on the real line can be subdivided into smaller intervals, the ratio is well behaved and constant for all finite values of δt and it is assumed that this remains true no matter how small δt actually gets ... Therefore, some sense can be made of the concept of "motion at an instant" ... so, it seems, Zeno's arrow "paradox" has missed its target.[10]

So $v = \delta x/\delta t$. But, at an instant both δx and δt are zero. Therefore: $v = 0/0$.

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[6]See Mark Zangari, "Zeno, Zero and Indeterminate Forms: Instants in the Logic of Motion", *Australasian Journal of Philosophy* 72 (1994): 187-204.

[7]William I. McLaughlin and Sylvia L. Miller, "An Epistemological Use of Nonstandard analysis to Answer Zeno's Objections Against Motion", *Synthese* 92 (1992): 371-84.

[8]See Zangari, "Zeno, Zero and Indeterminate Forms."

[9] Ibid., 187 (author's emphasis)

[10] Ibid., 191

> Now, 0/0 is not a well defined expression and is what is known as an *indeterminate form*. This is not the same as 1/0 or ∞ which are *undefined forms*. This distinction is an important one and has, in general, been overlooked in most discussions of Zeno's arrow paradox to date. If U is an undefined form, then there are *no* solutions for x of the equation x = U that are not also undefined. However, if I is an indeterminate form, x = I has (possibly) *infinitely many* finite solutions, at least in the sense that there are infinitely many well-defined expressions that are equivalent to I, yet each of which sets x to a different value.[11]

Zangari does not try to prove the indeterminacy of 0/0 as he states that this is well known. He wishes to illustrate the properties of 0/0 and how Zeno's paradox can thus be "resolved". In Zeno's arrow, I = 0/0 and Zangari's argument proceeds as follows:

> Let x be expressed as the ratio of two real numbers y and z, or that
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> $$x = y/z.$$
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> Substituting this into $x = 0/0$ yields
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> $$y/z = 0/0$$
>
> which is equivalent to
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> $$0z = 0y.$$
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> Now [this] equation must be true for all finite values of y and x so there are an infinite number of finite solutions to it, and hence, an infinite number of possible velocities to which the expression [v =0/0] corresponds. In fact, [v =0/0] establishes only that v can be any number whatever. So if we assume that velocity is an acceptable means of making the concept of motion quantitative then, at an instant, the state of motion of a body is mathematically underdetermined, since there is an infinite number of possible states of motion that are consistent with the information provided.[12]

So according to Zangari Zeno is wrong to say that the arrow is at rest at an instant since

> the velocity ... at each instant is indeterminate and, therefore, *cannot* contradict any finite velocity because v = 0/0 is consistent with v = *any*

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[11] Ibid., 193.

[12] Ibid.

> velocity. So the arrow's non-zero velocity, as determined over finite time intervals, is not in the least bit paradoxical, nor does it contradict anything about the state of the arrow at each instant.[13]

Zangari claims therefore that Zeno's conclusion that the arrow is stationary at each instant appears fallacious.

> For if Zeno were correct, then the velocity at each isolated instant would have a determinate value, namely zero. It seems, therefore, that the arrow paradox rests on the tacit but incorrect assumption that, necessarily, 0/0 = 0. But since this is not the case, we have no paradox at all—just a poorly posed problem.[14]

III

Infinitesimals. The main thrust of the argument of McLaughlin and Miller[15] is that motion occurs in infinitesimals—that between each Zenonian instant are the undetectable infinitesimally small "instants" in which an object moves by equally small distances.
The infinitesimal is:

> an interval of space or time that embodies the quintessence of smallness. An infinitesimal quantity..... would be so very near zero as to be numerically impotent; such quantities would elude all measurement, no matter how precise.[16]

Such an entity is "greater than 0 and less than every possible standard real [number]"[17]. For the purposes of devising a theory of motion this concept of infinitesimal is extended such that each positive real number is flanked on either side by these infinitesimals—which are non standard real numbers—and that motion takes place in them. This can be applied to Zeno's second paradox, the Dichotomy, where the problem is that motion can not

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[13] Ibid., 194

[14] Ibid.

[15] See McLaughlin and Miller, "An Epistemological Use of Nonstandard Analysis."

[16] William I. McLaughlin, "Resolving Zeno's Paradoxes," *Scientific American,* no. 5 (1994): 69.

[17] McLaughlin and Miller, "An Epistemological Use of Nonstandard Analysis," 376

start without taking a first step and that the distance of the first step cannot be traversed without traversing half the distance and so on *ad infinitum*. To get around this, motion can begin by taking a first step of infinitesimal length starting from a point P. This step, being infinitesimal, is therefore empirically inaccessible and so not subject to scrutiny and thus escapes the above Zenonian constraints. McLaughlin & Miller attempt to show that such infinitesimal steps can account for motion inasmuch as "The fact of motion of an object is established if the object has been located at two distinct points of space"18. Motion can be accounted for in this way because it can take place in infinitesimal steps but within a *finite* set. Therefore the theory represents motion as being a finite series of infinitesimal steps.

IV

Why mathematical solutions fail. These and other attempts at resolving Zeno's paradoxes may make perfect mathematical sense and yield equations that are of great use in that domain. Nevertheless, in metaphysical terms they do not even scratch the surface of the problem which was at the heart of Zeno's formulation of his paradoxes: the impossibility to conceptualise the passage from One to Many.

With its manipulation of the unit mathematics finds "ways out" of the immobility of the arrow, condemned, according to Zeno, by the self-identity of its position at any moment, never to accomplish the transition from rest to motion. But the point, quite generally put, is that using Zeno's rules of the game, the unit cannot be manipulated, and furthermore a manipulation of the unit does not resolve the problem of the passage from one to many, from the unit to concrete plurality. Zeno's rules of the game were the acceptance of the Parmenidean prohibition[19] that only one or the identical being can be thought of whereas the many of becoming as non-being yet

[18]Ibid., 378.

[19]In the *Sophist* we read: "You see, then, that in our disobedience to Parmenides we have trespassed far beyond the limits of his prohibition...He says you remember, 'Never shall this be proved that things that are not, are, but keep back thy thought from this way of inquiry.'": *Sophist* 258c-d. in *The Collected Dialogues of Plato*, ed. Edith Hamilton and Huntington Cairns (Princeton University Press, 1961), 1005

or non-being anymore is unthinkable. Only being is real. Only the identical with itself is thinkable. The "way out" of this imperative is the position of the pluralists who denied reality to the identity of one altogether and declared that in fact the process of becoming is real. In this way they did not have the problem of how to attain the multiplicity starting from the identity because they simply did not start from it, as they privileged life over logic. But this alternative position that privileges the reality of becoming over that of Being does not offer a way out of Zeno's paradoxes for it simply dismisses the rules in which they are generated. These two positions, in fact, even though historically associated, are incommensurable.

Zeno's target, then, seems more likely to be the Pythagorean pretence to get the many of the Universe by multiplication or addition of the unit. As Kathleen Freeman writes regarding this matter:

> Zeno's attack was on the idea of the Many, that is, of multiplication.....multiplication in itself is useless....It is useless because you are bound to start with either a Nothing or an Infinite, and by its means you get only what you start with, either a nothing or an Infinite.[20]

In other words Zeno argued that One (a non divisible) is one and can never become many and that Many (a divisible) will always be a quantity and, therefore, can never be exhausted by division in order to make of it a One. If you accept this logic, you are hooked and you can easily see how this assumption hinders the conceptualisation of change and movement, when this is intended as a passage from one to many, from the identical of a resting position to the concrete plurality of movement. When, in other words, one tries to find a way out of the simple logic of the identity in whose framework Zeno's paradoxes arise. Many is either empirically or phenomenally given (experienced) or it cannot be conceptualised as a passage from the identical or being in which we think any existent, to the many of movement or change. If you think in terms of being what you are left with is always a new being, without so being able to capture the whom through which a certain being becomes a new different being. But if you think in terms of a given change, you never conceptualise

[20]Kathleen Freeman, *The Pre-Socratic Philosophers*, (Oxford: Blackwell, 1946), 156.

the identity of being as you are at another level, that of life rather than logic. In this sense we must also stress that Zeno's paradoxes do not add anything to the Parmenidean prohibition to think only of the identity. One is One and cannot be Many. If we want to think logically, we can only think the identical, because identity is the form of our thought[21]: our thought can only be identical with itself when it thinks, that is it cannot think two things at the same time[22]. The response of the pluralists and all those who embraced a similar philosophical creed (see in more recent times Hegel and Bergson[23]) was to refuse to think of the existent as being, but to think of it as becoming. This as I said, though, was not a solution to Zeno's paradoxes as it simply embraces a new "logic", the logic of becoming that denies the identity. But if you acknowledge the logic in which Zeno's paradoxes arose, you cannot but accept them as a description of an impasse that is constitutional to our thought. Our thought is self-identical and when it thinks of an existent crystallises it in a self-identical thought withdrawing it from its natural process and becoming. If you accept the identity as the first and universal law of thought, you cannot explain or allow that thought could conceptualise movement. Intrinsic dynamism is alien to the constitution of our thought, for every time thought thinks of movement, of an object moving, this precipitates in the identity of being, necessary for thought to think of that object. Thought for the impossibility to be nothing but self-identical, cannot mimic and so really understand movement. Movement as a sort of hegelian synthesis of two positions *A* and *B* in which thought thinks of an object at different times is incompatible with the logic of the identity. It is in fact this excluded middle, this no-man's land between two identities, the "space and time" where change and movement must be placed. But, in fact, the logic of the identity excludes this middle from what is rationally thinkable.

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[21]"...for the same thing can be thought and can exist", *Parmenides*, trans. Leonardo Taran (Princeton: Princeton University Press, 1965), 41.

[22]"....but also from this, on which mortals who know nothing wander, double-headed........a horde incapable of judgement, by whom to be and not to be are considered the same and yet not the same, for whom the path of all things is backward turning"; ibid., 54.

[23]See Henri Bergson, *An Introduction to Metaphysics* (London: Macmillan, 1913), and G. W. F Hegel, *Science of Logic* trans. A.V. Miller, (Atlantic Highlands, NJ: Humanities Press International, 1969).

It is by misunderstanding Zeno's position, by assuming that Zeno has said more than what Parmenides had taught him, or that he has denied somehow the factuality of movement, that one can be disturbed by his paradoxes and think that one should try to solve them. But failure is unavoidable, because they express a tautology: one is one, many is many. You cannot get from one to many simply by addition (Arrow, Stadium) nor from many to one simply by reduction (Achilles, Dichotomy). Of the real scope of Zeno's paradoxes, Plato, unlike Aristotle, seemed to be perfectly aware when in the "Parmenides' he expresses forcefully this opinion through a young Socrates:

> I see Parmenides, said Socrates, that Zeno's intention is to associate himself with you by means of his treatise no less intimately than by his personal attachment. In a way, his book states the same position as your own; only by varying the form he tries to delude us into thinking that his thesis is a different one. You assert in your poem that the all is one, and for this you advance admirable proofs. Zeno, for his part, asserts that it is not a plurality, and he too has many weighty proofs.[24]

And Zeno's answer confirms this claim:

> Yes, Socrates, Zeno replied, but you have not quite seen the real character of my book. ... The book makes no pretence of disguising from the public the fact that it was written with the purpose you describe, as if such deception were something to be proud of.[25]

To acknowledge, and even maybe to understand, Zeno's paradox it is necessary to take into account that the premise of his argument is that the arrow always occupies a place equal to itself (*kata' to' ison*).

> For everything is either at rest or in motion, but nothing is in motion when it occupies a space equal to itself.[26]

Can we conceive of anything that does not occupy a space equal to itself at any moment? Hardly (in an ordinary logic, at least). This is the real premise, apparently an innocuous one, of the argument, on which he steals the easy agreement of his interlocutor, and from which it really follows that the arrow must be thought of at a

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[24] *Parmenides* 128a-c, in *The Collected Dialogues of Plato*, 923

[25] *Parmenides* 128a-c, in *The Collected Dialogues of Plato*, 923.

[26] Hermann Diels, *Vorsokratiker*, see H. Lee, *Zeno of Elea* (Cambridge: Cambridge University Press, 1936), 80.

durationless instant (*en to' nun*). This durationless instant is in fact the effort to conceptualise the identity with itself of the arrow. Whenever you think of the arrow, this must occupy a place equal to itself, this can only happen tautologically in a non-duration (in a framework in which time is change, of course). It should be clear, then, why the premise is only apparently innocuous, and it assumes, in fact, in a way, the very thing that should be demonstrated. I say "in a way" because, on one hand there is no possible demonstration for the identity, and, on the other, most of his interlocutors would easily agree on this premise though being unable to accept its logical consequences. Aristotle was one of them. He would, then, focus his criticism[27] not on the identity but on the Zenonian instant as the last atom of time, and claim that the paradox would not subsist if we considered time as infinitely divisible. Again he would start from a given or presumed dynamism and so dismiss Zeno's problem: the conceptualisation of change in the framework of the identity. But if you accept Zeno's premise, his conclusion is inescapable. The paradox is, in fact, I repeat, a tautology. One is always one and can only be one. As Parmenides had argued, you cannot bring movement or change into what is identical.[28] Likewise the two paradoxes founded on the infinite divisibility of time, the dichotomy and the Tortoise, are a tautology. Many is always many, and as a quantity, it can never be exhausted in order to finally conceptualise movement at the end of the regressive series in the search for it. If one accepts these premises, one can acknowledge the paradoxes, but if one doesn"t, one is not even able to reason within Zeno's framework.

This, I believe, is what happens in the solution to the "Arrow" proposed by Zangari.[29] He argues that Zeno's is not really a paradox but a "poorly posed problem" and "The "Arrow" is a chimera bred by a misinterpretation of the indeterminate form 0/0". Now 0/0 expressing the velocity evaluated as a ratio at an instant, according

[27]For a detailed discussion of this point see Alba Papa-Grimaldi, "The Paradox of Phenomenal Observation"., *Journal of the British Society for Phenomenology* 27, no. 3 (October 1996): 294-312.

[28]As Hegel pointed out many centuries later: "It is just as impossible for anything to break forth from it as to break into it; with Parmenides as with Spinoza, there is no progress from being or absolute substance to the negative, to the finite."; Hegel*, Science of Logic*, 94-95.

[29]See Zangari, "Zeno, Zero and Indeterminate Forms."

to Zeno is resolved as 0, but this, as we have seen previously, is wrong according to Zangari.

The crux of the argument here is what the two take an instant to be. Zeno's instant is not a mathematical convention, an entity whose value can be modified as a function of a certain mathematical formula. Zeno's instant is a logical absolute. It is, as I said, the effort to conceptualise the identity with itself of the arrow when we think of it. It is in one word the Parmenidean One, the identical being our thought can only think of. The Zenonian instant, then, is the One and movement is a plurality that cannot be accomplished by starting with this One. A manipulation of this unit like the one accomplished by the Pythagoreans will not give us a plurality, a real dynamism, but simply a repetition of the identical unit. That is, many other self-identical positions in which the arrow is found "at rest". But how the transition from one position to the next has been accomplished remains for our thought a mystery. Zangari's instant on the other hand is the mathematical, not logical nor metaphysical, device where this transition is accomplished. But for those who keep in mind the logical coordinates of Zeno's paradox and the essence of his challenge, these mathematical claims are irrelevant, even a nuisance. We all know that the transition is in fact accomplished, that movement is a fact, and hardly need a mathematical device that once again adumbrates this transition, without being able to show us a way to conceptualise it.

Without expanding further details of Zangari's argument, against whose mathematical formulas, I repeat, I have nothing to object, we can say that Zangari's solution appears very clearly as a refusal, possibly unaware, of the premise we have previously pointed out: that everything occupies a space equal to itself, that everything also when in movement must be identical with itself. This apparently banal premise, once accepted, makes movement as an intrinsic property impossible, and the most one can achieve in terms of rescuing the dynamism of the arrow, is to explain movement as the actual being of the arrow at different times at different places, but this falls short of conceptualising motion which was Zeno's challenge.[30] Anyway Zangari sets out to achieve more than this with

[30] About this I do agree with Zangari that "the standard solution that seems to be currently accepted by most philosophers rests on what is often called the 'at-at' theory of motion. According to this, the 'motion' of an object does no more than correlate the position of the object to the time at

his solution. Approaching the paradox from a mathematical point of view, he concludes that there is no mathematical reason why the arrow has to be stationary at an instant. In disputing the validity of Zeno's premise through a mathematical operation that once again manipulates the unit without showing the transition from this (the unit) to a concrete plurality or change, he completely misses the purely logical point of Zeno's paradox.

In showing that in a mathematical framework which does not acknowledge the logical coordinates of the paradox, the arrow at an instant can move, does Zangari say anything about the transition from one to many, which was the one and only concern of Zeno? This question should be by now a rhetorical one. This kind of argument rather says: since from a mathematical point of view we can make a perfect sense of the velocity at an instant, we needn"t be concerned about the logical *aporia* suggested by Zeno. But the problem is that in a mathematical framework this *aporia* cannot be understood. The manipulation of the unit is purely abstract insofar as it simply assumes what has not yet found a metaphysical proof: that there exist these entities, the instants, which are neither the indivisible one nor a knowable quantity that as such can always be further divided. However, from a philosophical point of view, these mathematical entities are too swiftly obtained. Parmenides' and Zeno's scrutiny bears exactly on the logical conceptualisation of these entities within a logic whose first law is the identity. But the mathematical manipulation of the unit cannot be concerned with the objection that Zeno already moved to the Pythagoreans: that this manipulation does not yield concrete plurality or, like the pluralists, takes the concrete plurality of life for granted and just does not

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which it had that position. So it is at a particular place at a particular time. If the object has the same location in the instants immediately neighbouring, then we say it is at rest; otherwise it is in motion ... According to the most commonly accepted view, instantaneous velocity is not an intrinsic property of the object, but a supervenient relation based on the correlation between position and time over a neighbourhood of *T"*; ibid., 192. This theory cannot explain dynamism as it never operates the synthesis that could intrinsically correlate different points in time and space. This was essentially Russell's solution of the paradox. As he wrote "motion can be understood as the position occupied by an object in a continuous series of points in a continuous series of instants."; Bertrand Russell, *I Principi della Matematica*, (Milano: Einaudi, 1963), 637.

acknowledge the Eleatic problematisation of movement. That is, if you accept that the arrow occupies a position always equal to itself, you still have to explain how these abstract mathematical values[31] can become a concrete movement of the arrow. Zeno would not have been impressed by these solutions because they assume as unproblematical the very positions he was historically attacking,[32] namely, the Pythagorean pretense, as we have seen, that a manipulation of the unit can resolve the logical *aporia* of the passage from one to many or from identity to change, or the Pluralistic understanding of many as real and thinkable, whereas he held with Parmenides that only One is thinkable and real.

To say that $v = 0/0$ means v = any velocity, means that the arrow has a velocity at an instant. Now this can either be interpreted as saying that the arrow occupies at one time different positions, an Hegelian sortie that Zangari would hardly cherish, or as saying that the instant is not durationless, but in this latter case the paradox would propose itself all over again. The point that Zeno makes with his paradox following Parmenides' prohibition is very simple: we

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[31]Or maybe one should say "these concrete mathematical values". In fact the blunder of which Zeno's paradox is susceptible appears to be of a double nature. Either movement needs to be conceptualised in abstract, strictly logical terms for which these mathematical solutions simply assuming the factuality of movement fall short of providing a model, or the passage from one to many needs to be shown to yield a concrete plurality and not simply what I have called a mathematical reiteration of the unit which does not reach the concreteness of movement, change and plurality, as all you have is a repetition of an identity. This latter expresses the shortcomings of the Pythagorean position, whereas the previous one expresses those of the Pluralists. Zeno's paradoxes are a challenge for both of them. More fundamentally I believe that these mathematical solutions contain both these ancient positions, since on one hand they simply assume the factuality of movement in their values and formulas, and on the other their values remain purely abstract and so incapable of describing concrete plurality, insofar as they do not show the passage from one to many and vice versa, except as a reiteration of the unit (Pythagorean) or an assumption of the many as immediately intelligible (Pluralists).

[32]And this can be seen as ironical since Zangari declares, with temerity, that "However, the historical facts are not the focus of my discussion. The arrow paradox, no matter how it began, has evolved into its modern form and it is with this that I am concerned." Zangari, "Zeno, Zero and Indeterminate Forms," 190.

cannot unproblematically think that the mathematical multiplicity, as manipulation of a unit, is real and concrete plurality, that is, that it can mimic in our mind the passage from one or identity to many or change. The consequence of this impossibility is that the plurality made up of these units always precipitates when you think of it, into an identity or immobility. The only way "out" not of the paradox, but of the immobility to which the identity tautologically forces the arrow, would be to claim that the arrow does not have to be thought of as occupying a space always equal to itself, but that we should Hegelianly rise above the "thinking that belongs to the understanding alone"[33] and have an intuition of the arrow as never occupying a space equal to itself. This is the Hegelian key to the interpretation of reality and movement: to deny the identity as a constraint on our reasoning and rather opt for the speculative Reason that raises itself above "the mere logic of the understanding"[34] and so has an immediate apprehension of the synthesis of *A* and *B*, in our specific case, of two different points in time and space, two otherwise unbridgeable identities.

The only way to "conceptualise" (but the Hegelian one is no ordinary concept) change and to conceive of the plurality as concrete rather than abstract, that is, as a pure sum of the unit, is the Hegelian synthesis or any other doctrine that privileges an experience of movement over an aseptic attempt to understand it. But these doctrines do not acknowledge the paradox as a "poorly posed problem", they do not acknowledge it at all.

On the other hand it is impossible and it really results in an *aporia* to try and conceptualise movement as concrete, intrinsic plurality while keeping the logic of the identity. But mathematical "solutions" of Zeno's paradoxes are hardly giving up the identity and agreeing on embracing an Hegelian logic of becoming. There would be no point in doing that anyway, for someone who wants to approach Zeno's paradox, because the Hegelian logic is not a solution of the paradox but a dismissal of the logical coordinates that generate it. I think it is worth considering that mathematical solutions of Zeno's paradoxes insofar as they illegitimately assume the abstract plurality of their manipulation of the unit to be a

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[33]G. W. F. Hegel, *The Encyclopaedia Logic*. (Indianapolis: Hackett, 1991), 35.

[34]Ibid., 131.

concrete plurality, are unconsciously Hegelian, for at some critical moment they privilege becoming as a given experience and so they are never really confronted or never really address Zeno's paradoxes in the right logical perspective.

Similar objections, I feel, should also be moved to McLaughlin and Miller's mathematical solution: the attempt to solve Zeno's paradox with the recourse to the "infinitesimals". In William I. McLaughlin's recent article35 we read that the strength of the infinitesimals consists in that being infinitesimal intervals they:

> can never be captured through measurement; infinitesimals remain forever beyond the range of observation.[36]

In fact he argues:

> So how can these phantom numbers be used to refute Zeno's paradoxes?...it is clear that the points of space or time marked with concrete numbers are but isolated points. A trajectory and its associated time interval are in fact densely packed with infinitesimal regions. As a result, we can grant Zeno's third objection: the arrow's tip is caught "stroboscopically" at rest at concretely labelled points of time, but along the vast majority of the stretch, some kind of motion is taking place. This motion is immune from Zenonian criticism because it is postulated to occur inside infinitesimal segments. Their ineffability provides a kind of screen or filter.[37]

All we can say, again, is that if one argues that the arrow is moving in these infinitesimal segments which are presumably different from 0, the absolutely indivisible, we are still faced with an abstract plurality that has not even slightly addressed the problem of the conceptualisation of change. As vanishing quantities, on the other hand, they seem to actually mimic the effort of our mind in grasping this passage from one to many. But all they can do is to be the mathematical counterpart of this effort, not the mathematical solution of it. In fact McLaughlin & Miller themselves write:

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[35]William McLaughlin, "Resolving Zeno's Paradoxes," *Scientific American,* 271, no 5 (1994): 66-71.

[36] Ibid., 69.

[37] Ibid.

> The theory explains the fact of motion but does not describe the nature of "present motion". If there is a concept of "present motion", it must refer to a process taking place during the infinitesimal open intervals ... of time. It cannot be established, in fact, what process of "present motion" is operative within the infinitesimals ... The object could jump instantaneously from one end of an interval to the other, or it could move nonuniformly within an interval, or it could move uniformly within an interval ... More generally, the object might not be, during these time intervals, in any kind of spacetime.[38]

And later:

> Basically, the theory represents motion as a finite series of infinitesimal steps. ... If one wishes to define "present motion", it is possible to do so in a manner consistent with this theory of motion. The fact that motion has occurred is verifiable without encountering Zeno's objections, but the fact of present motion does not appear to be verifiable, since it takes place inside unobservable infinitesimal intervals. The process of change is hidden but the effects of change are visible.[39]

Seeing the infinitesimal as an alternative to both an indivisible unit and to a knowable quantity divisible *ad infinitum*, and so, as the "hidden" place and time in which motion can finally happen, suggests a strong analogy with what Plato says in the *Parmenides* about the instant:

> ...that queer thing, the instant. The word "instant" appears to mean something such that *from it* a thing passes to one or other of the two conditions [sc. rest and motion] there is no transition *from* a state of rest so long as the thing is still at rest, nor *from* motion so long as it is still in motion, but this queer thing, the instant, is situated between the motion and the rest; it occupies no time at all, and the transition of the moving thing to the state of rest, or of the stationery thing to being in motion, takes place *to* and *from* the instant.[40]

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[38]McLaughlin and Miller, "An Epistemological Use of Nonstandard Analysis," 382

[39]Ibid., 383

[40]Plato, *Parmenides* 156d-e, in *The Collected Dialogues of Plato*, 947.