Correcting Credences with Chances*

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Abstract

Lewis's Principal Principle is widely recognized as a rationality constraint that our credences should satisfy throughout our epistemic life. In practice, however, our credences often fail to satisfy this principle because of our various epistemic limitations. Facing such violations, we should correct our credences in accordance with this principle. In this paper, I will formulate a way of correcting our credences, which will be called the Adams Correcting Rules and then show that such a rule yields non-commutativity between conditionalizing and correcting. With the help of the notion of 'accuracy', then, I attempt to provide a vindication of the Adams Correcting Rule and show how we can respond to the non-commutativity in question.

1 Introduction

Many philosophers of probability think that there are some epistemic norms governing how objective chances ought to be related to our credences. The most famous chance-credence norm is what is called the Principal Principle, which is formulated and named by David Lewis (1980). This principle and its variants are widely recognized as a rationality constraint that our credences should satisfy throughout our epistemic life.

In practice, however, our credences often fail to satisfy this principle because of our various epistemic limitations. For example, some may have no idea what chance

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functions are possible, some may have no idea how chance functions should be reflected in their credences, and so forth. Of course, if the Principal Principle is a rationality constraint and such agents happen to know this, then they should correct their credences in accordance with the principle. Then, is there any way of correcting our credences? Is such a way, if any, helpful in achieving our epistemic goal?

In order to make the point clear, let me consider an agent who, at a given time, has no idea about the objective chances and their relationship to her credences, and so violates the Principal Principle. After a while, she finds that the principle is an epistemic norm that she should satisfy, and then thinks through a way of correcting her credences in accordance with the principle. Meanwhile, she also happens to know that she will receive several pieces of evidence sometime later. (However, she does not know exactly what evidence she will receive.) The agent, who is placed in this doxastic situation, deliberates how to correct her credences when she receives the evidence. In this regard, we can consider two possible ways of correcting her credences in accordance with the Principal Principle. The first is to correct the credences after adjusting them according to the evidence; the second is to adjust the credences according to the evidence after correcting them. If these two kinds of correction make no difference in her final credences, then there will be no reason to prefer one to the other, and so either of the two may be good to her. Even if her new credences vary according to how she corrects, we do not have to consider the difference seriously if they are all expected to succeed in achieving the epistemic goal.

As will be shown, however, the agent's new credences may be different depending on which of the two ways of correction she adopts. Moreover, it will also be argued in what follows that there is an epistemic reason to prefer one way to the other. At first glance, we might say, "For a credence function to respect properly the evidence, the credence function should be rational in advance. So, if the Principal Principle is a requirement of rationality, then the agent should plan to adjust the credences according to the evidence after correcting them." However, considering an epistemic value, i.e., accuracy, of our credences, we will arrive at the opposite conclusion—that is, the agent should plan to correct the credences after adjusting them according to the evidence.[1]

[1]A caveat needs to be stated from the start. This paper concerns ways of *correcting* our credences. Someone may think that this kind of correction leads us to be committed to doxastic voluntarism. Indeed, readers can find several voluntaristic expressions like 'adopt the credence' in this paper. However, this paper is not committed to such voluntarism. My argument in this paper may be regarded as criticizing or evaluating, rather than blaming, agents whose credences evolve in violation of the rele-

This paper is structured as follows: In the next section, I will give some preliminary notes about chances and credences. In Section 3, I will formulate a Bayesian rule of correcting our credences, which will be called the Adams Correcting Rule, and then show in Section 4 that such a rule yields non-commutativity between conditionalizing and correcting. Section 5 will be devoted to vindicating, with the help of the notion 'accuracy', the Adams Correcting Rule. In a similar way, it will also be argued in that section that an agent, who faces a decision problem due to the non-commutativity in question, should plan to make a correction after adjusting our credences according to evidence.

2 Stage Setting

Before I proceed further, some notes about assumptions, terminology, and notations are in order. First, I assume that credences and chances are all probabilities. That is, credence and chance functions are assumed to satisfy the standard probability axioms. It is also assumed that those credence and chance functions are defined on the same outcome space $\mathcal{W}$, which is a set of possible worlds. For the sake of mathematical simplicity, it will be assumed that $\mathcal{W}$ has only finitely many possible worlds. Propositions in this paper are defined as a subset of $\mathcal{W}$.[2]

In this paper, I will discuss several kinds of credence functions that are placed in various evidential situations. Some have no empirical evidence, others have several pieces of evidence. The credence functions, which have no empirical evidence whatsoever, can be regarded as ones that agents have at the beginning of their credal life. In this sense, I will call them the 'initial credence functions'. The agent who has such a credence function is often called a 'superbaby'. (See, for example, Hájek (2012) and Pettigrew (2016b).) Superbabies are rational, and so, by definition, satisfy all the rationality constraints.

I will also assume that the credence functions that have evidence are updated from the initial credence functions by means of Conditionalization on the evidence. That is, it is assumed that if an agent with credence function C receives total evidence E,

vant epistemic norms. I should thank to Alan Hájek and Jaemin Jung for helping me make this point clear.

[2] The unit set $\{w\}$ is a proposition, but I will use interchangeably '$\{w\}$' and w if there is no danger of confusion. When A is a proposition, '$\neg A$' refers to the complement of A with respect to $\mathcal{W}$—namely, the negation of A. Similarly, when A and B are propositions, the conjunction 'AB' and the disjunction '$A \vee B$', respectively, refer to the intersection and union of A and B.

then it holds that:

Conditionalization. For any $A \subseteq \mathcal{W}$, $C_E(A) = C(A|E)$ where $C(E) > 0$.

Here, C_E refers to the agent's credence function that has total evidence E. In what follows, I will divide credence functions into two different types—*Chance-free* and *Chance-fed* credence functions. Roughly speaking, the former is the credence function that reflects no impact of chances on the credal system and so may violate the Principal Principle, while the latter is the function that does reflect such impact in accordance with the principle. Then, the correction that will be discussed in this paper may be regarded as a kind of updating from a chance-free credence function to a chance-fed one.

In this paper, I will follow many theorists in assuming that the Principal Principle is a rationality constraint and do so without any arguments for such rationality of the principle.[3] In order to formulate this principle, we need to introduce some additional assumptions and notations. First of all, I assume that each possible world has only one ur-chance function. Here, 'an ur-chance function at a particular world' refers to a chance function that does not reflect any history of that world. I will not rule out the metaphysical possibility that two different worlds have the same ur-chance function. As assumed, there are only finitely many possible worlds. Thus, there are only finitely many ur-chance functions. Following Hall (2004) and Pettigrew (2014), I will formulate the Principal Principle by means of ur-chance functions.[4] 'U_i' stands for the proposition that ch_i is the ur-chance function. This kind of proposition will be called 'an ur-chance proposition'. U_i is true at all and only possible worlds whose ur-chance function is ch_i. Thus, U_i is equivalent to the disjunction of all possible worlds whose ur-chance function is ch_i. It is noteworthy that the set of U_is is a partition, which will be denoted by '$\mathcal{U}$'.

[3]Indeed, there may be many philosophical debates on what it is for an epistemic principle or rule to be a requirement of rationality. However, as the purpose of this paper is concerned, it is not necessary to discuss such a problem. Rather, I will assume that there are some reasons that the Principal Principle should be regarded as a requirement of rationality, and such reasons do not undermine my discussion that follows. A similar assumption goes with other principles appearing in this paper. In what follows, I will formulate a decision-theoretic principle that is related to credence correction. See Sections 5.2 and 5.3. Regarding such a principle, I assume a similar thing to the rationality of the Principal Principle. I should thank an anonymous referee for making this point clear.

[4]Someone may think that this formulation cannot be applied to the world that has no beginning—an infinite past. I agree. However, this feature has little to do with the discussion that follows. Indeed, if there is any proposition characterizing chance functions that one should respect, then my discussion will go through.

With these assumptions and notations in hand, we can formulate Lewis's original version of the Principal Principle:

PP0. Suppose that C is an initial credence function. Then, it should hold that:

$$C(A|U_i) = ch_i(A), \text{ for any } A \subseteq \mathcal{W} \text{ and } U_i \in \mathcal{U}.$$

Here, I will assume that the initial credence functions are *regular* in the sense that the function assigns a non-zero value to any non-empty proposition. Note that if the initial credence function is irregular, then there may be an evidential situation in which Conditionalization does not place any constraint on the way of updating the function. Suppose that an initial credence function C assigns zero to a non-empty proposition E, and then receives evidence E. In this case, the condition of Conditionalization is not satisfied, and so it cannot help keeping silence. The assumption is intended to rule out such a possibility.[5]

As is well known, however, PP0 suffers from the so-called 'Big Bad Bug'. Roughly speaking, the bug in question is that Lewis's original Principal Principle is incompatible with his Humean Supervenience. It can be shown, in particular, that if $i \neq j$ and $ch_i(U_j) > 0$, then PP0 yields a contradiction—that is, $0 = C(U_j|U_i) = ch_i(U_j) > 0$. So, some philosophers—e.g., Lewis (1994), Thau (1994), and Hall (1994)—provided a modified version of PP0, which is often called the New Principle:[6]

NP0. Suppose that C is an initial credence function. Then, it should hold that:

$$C(A|U_i) = ch_i(A|U_i), \text{ for any } A \subseteq \mathcal{W} \text{ and } U_i \in \mathcal{U}.$$

We can hardly deny, I think, that NP0 is more plausible than PP0. However, it is also noteworthy that, when the following is assumed:

Self-esteem. $ch_i(U_i) = 1$, for any $U_i \in \mathcal{U}$.

PP0 is equivalent to NP0, and so can circumvent the above contradiction.[7] My discussion that follows has little bearing on Humean Supervenience and does not attempt to

[5]This argument for regularity is found in Lewis (1980). However, Hájek (2012) points out that there are various problems that beset regularity. I agree. So, regularity is not regarded, in this paper, as a requirement of rationality.

[6]There are other attempts to overcome the bug. For example, see Robert (2001) and Ismael (2008).

[7]The expression 'Self-esteem' is intended to express that each chance function is certain that it is the only one true chance function. Pettigrew (2013) calls this feature of chance functions 'Immodesty'. I do not use this name since I will use it in another context.

examine the metaphysical plausibility of PP0 and/or NP0. For the sake of notational and mathematical simplicity, thus, I will assume Self-esteem and regard PP0 as a norm governing the relationship between credences and chances. My main result remains the same even if we accept NP0 without assuming Self-esteem. (See Appendix.)

It is natural, on the other hand, that credence-chance norms like the Principal Principle should not rule out any kind of credences. In particular, such norms should be able to constrain any credences no matter what evidential situation the credences are placed in. However, PP0 *itself* concerns only a special kind of evidential situations, in which the credences have no evidence whatsoever.

In this regard, it is noteworthy that chances are often regarded as an analytic-expert for credences.[8] Thus, the credence functions with total evidence E should respect the chance functions that also have total evidence E. Here, let us assume that the ur-chance function is also updated by means of Conditionalization—that is, ch_i is updated to $ch_i(\cdot|E)$ when ch_i receives the total evidence E. Then, we have the following version of the Principal Principle:

> **PP+.** Suppose that C_E is a credence function that has total evidence E. Then, it should hold that:
>
> $$C_E(A|U_i) = ch_i(A|E), \text{ for any } A \subseteq \mathcal{W} \text{ and } U_i \in \mathcal{U},$$
>
> where the relevant conditional credences and chances are all well defined.

Here, C_E is a credence function that is updated from the initial credence function C by means of Conditionalization on the total evidence E. Note that PP+ follows from PP0 and Conditionalization—to put it another way, the Principal Principle is preserved under Conditionalization.[9] Thus, superbabies, who are born rational, will satisfy the principle throughout their credal life as long as they update their credences by means of Conditionalization.

[8] For example, see Hall (2004). In our context, the chance functions are analytic-experts who satisfy Self-esteem. Using Hall's terminology, our chance functions are analytic-experts who know they meet the (analytic) expert conditions.

[9] Suppose that an initial credence function C satisfies PP0 and is conditionalized on total evidence E. Then, it holds that, for any $A \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$, $C_E(A|U_i) = C(A|U_iE) = C(AE|U_i)/C(E|U_i) = ch_i(AE)/ch_i(E) = ch_i(A|E)$. Thus, C_E satisfies PP+.

3 Chances in the Realistic Credal Life

However, we are not born super. At the beginning of our realistic credal life, we may have no information regarding objective chances and their relation to our credences, and so our credences may not satisfy the Principal Principle. Be that as it may, we do not have to say that objective chances cannot guide our real life via the Principal Principle. Although we form our credences at the beginning without considering objective chances, we will *correct* our credences in accordance with the Principal Principle if we obtain some relevant information about chances and its relationship to credences.

This kind of correction raises a question: Is there any way of correcting our credences in accordance with the Principal Principle? In this section, I will attempt to answer this question in a Bayesian way. That is, I will suggest a way of correcting credences using a variant of the Bayesian belief updating rule, which is often called Adams Conditionalization. For this purpose, I will introduce two special kinds of credence functions, which I will call 'chance-free' and 'chance-fed' credence functions.

3.1 Chance-free and Chance-fed Credence Functions

Suppose that you do not know how objective chances should be related to our credences and so violate the Principal Principle. Then, it should be said that your credence function does not reflect properly the impact of the chances on your credal system. I will call this kind of credence function 'a chance-free credence function'. Note that the chance-free functions can have some evidence E even if they violate the principle. Of course, it is also doxastically possible that the chance-free functions do not have any evidence whatsoever.

Now, suppose that you happen to know that you should satisfy the Principal Principle in order to be rational—that is, you happen to know that the chances are given by one of the functions $ch_1, \cdots, ch_n$ and new values should be assigned to the relevant conditional credences according to such chance functions. So, you form a new credence function by correcting the old one. This new credence function will be called 'a chance-fed credence function' in the sense that the correction can be regarded as feeding the impact of the chances into the chance-free credence function via the Principal Principle[10]. As stated, this chance-fed credence function may be placed in vari-

[10] In our context, all and only chance-free credence functions violate the Principal Principle, and all and only chance-fed credence functions satisfy the principle. In this paper, however, I am not committed to the claim that nothing but the Principal Principle properly governs the relationship between

ous evidential situations. Some may have several pieces of evidence, others may have no evidence except the aforementioned information about chances.

We can think of this correction as a belief update from a chance-free credence function to a chance-fed one. Let C be a chance-free credence function with no evidence. C may be called the chance-free *initial* credence function since it has no evidence. Let C_{CH} be the chance-fed credence function that is updated from C in accordance with the Principal Principle. Here, the subscript 'CH' is intended to express that the impact of chances is fed to the chance-free credence function C via the chance-credence norm.

What is the difference between C and C_{CH}? Note that the correction of C to C_{CH} is made *in accordance with* the Principal Principle, which says that the conditional credence function, given that ch_i is the ur-chance function, should be equal to the chance function ch_i. In this context, thus, PP0 could be regarded as a source from which C receives new conditional credences. To put it another way, PP0 could be thought of as a constraint stating what values should be newly assigned to the conditional credences in question: For any $A \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$, $ch_i(A)$ should be newly assigned to the conditional credence in A given U_i. Equivalently, it requires that, for any $w \in \mathcal{W}$ and $U_i \in \mathcal{U}$, $ch_i(w)$ should be newly assigned to the conditional credence in w given U_i.[11]

Note that this requirement does not directly impose any constraint on credences other than the conditional credences in question. Then, is there any Bayesian updating rule governing how other credences should be updated through this kind of correction? In other words, is there any Bayesian rule stating how our credence function should evolve when the new conditional credences are obtained, and nothing else? Fortunately, yes.

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chances and credences. As shown in Appendix, the main arguments in this paper will go through even if the impact of chances is fed into the chance-free credence function *via the New Principle*. Thus, the terminology 'chance-fed' (and 'chance-free') may be applied to a credence function that satisfies the New Principle if the principle is accepted as a plausible chance-credence norm.

[11] Note that it is assumed that ch_i is a probability function. Thus, the following two propositions are equivalent to each other: (i) For any $A \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$, $C_{CH}(A|U_i) = ch_i(A)$, if defined ; (ii) For any $w \in \mathcal{W}$ and $U_i \in \mathcal{U}$, $C_{CH}(w|U_i) = ch_i(w)$, if defined.

3.2 Correcting Chance-free Credences

What is often called Adams Conditionalization plays such a role.[12] Let $\mathcal{E}$ be a partition that consists of E_is. Suppose that a course of experience directly changes an agent's conditional credences in some members in $\mathcal{E}$ given F, and nothing else. Let C be a credence function that the agent has before undergoing the experience, and $C_{\mathcal{E}|F}$ be a credence function that she will have after the conditional credences in question are incorporated into her credal system. Then, Adams Conditionalization is formulated as follows:

Adams Conditionalization (AC)

$$C_{\mathcal{E}|F}(A) = \sum_{E_i \in \mathcal{E}} C(F) C_{\mathcal{E}|F}(E_i|F) C(A|E_iF) + C(A\neg F),$$

for any $A \subseteq \mathcal{W}$.

Note that Adams Conditionalization cannot handle a case in which the conditional credence in E given $\neg F$, as well as the conditional credence in E given F, is directly changed. That is, this kind of Conditionalization *itself* is not a rule that governs the way of updating our credence function when conditional credences with various conditioning propositions are directly changed.

However, we need a rule that can be applied to such a case. Note that the correction in accordance with the Principal Principle requires that, for any $w \in \mathcal{W}$ and $U_i \in \mathcal{U}$, $ch_i(w)$ should be newly assigned to the conditional credence in w given U_i. That is, the correction requires us to directly change the conditional credences that have various conditioning propositions U_1, U_2, $\cdots$. Fortunately, we can easily generalize Adams Conditionalization so that it can handle such a case. Let $\mathcal{E}$ and $\mathcal{F}$ be two partitions, whose members are E_is and F_js, respectively. Suppose that a course of experience directly changes an agent's conditional credences in some members $\mathcal{E}$ given F_1, some members in $\mathcal{E}$ given F_2, $\cdots$, and nothing else. Let C be a credence function that the agent has before undergoing the experience, and $C_{\mathcal{E}|\mathcal{F}}$ be a credence function that she will have after the conditional credences in question are incorporated

[12] Adams Conditionalization is named by Bradley (2005). A similar discussion is also found in Wagner (2003). There are some attempts to respond to several difficulties of Bayesian epistemology by means of this kind of Conditionalization. For example, Douven and Romeijn (2011) suggest a solution for the so-called Judy Benjamin Problem using Adams Conditionalization.

into her credal system. Then, Adams Conditionalization is generalized as follows:[13]

Generalized Adams Conditionalization (GAC)

$$C_{\mathcal{E}|\mathcal{F}}(A) = \sum_{E_i \in \mathcal{E}, F_j \in \mathcal{F}} C(F_j) C_{\mathcal{E}|\mathcal{F}}(E_i|F_j) C(A|E_i F_j),$$

for any $A \subseteq \mathcal{W}$.

It is noteworthy that GAC is equivalent to AC when $\mathcal{F} = \{F, \neg F\}$ and the conditional credences given $\neg F$ remain the same.

Now, we can provide a Bayesian way of correcting chance-free credence functions. In other words, we can formulate a rule governing the way of updating a chance-free credence function to a chance-fed one in accordance with the Principal Principle. I will call such a rule the Adams Correcting Rule in the sense that it is formulated by means of Adams Conditionalization.

Let me first consider the Adams Correcting Rule for a chance-free initial credence function that has no evidence whatsoever. As explained, the correction in accordance with PP0 requires that, for any $w \in \mathcal{W}$ and $U_i \in \mathcal{U}$, $ch_i(w)$ should be newly assigned to the conditional credence in w given U_i. Then, we can derive a way of correcting in accordance with the Principal Principle. Let C be a chance-free initial credence function, and C_{CH} be a chance-fed credence function that is obtained by correcting C in accordance with the Principal Principle. Then, a version of Adams Correcting Rule is formulated as follows:[14]

ACR0. $C_{CH}(A) = \sum_{U_i \in \mathcal{U}} C(U_i) ch_i(A)$, for any $A \subseteq \mathcal{W}$.

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[13] To understand the way of generalizing Adams Conditionalization, let me consider the following equation:

$$C(A) = \sum_{E_i \in \mathcal{E}} C(F) C(E_i|F) C(A|E_i F) + C(A \neg F),$$

which follows from the probability calculus. It is noteworthy that, according to Adams Conditionalization, the new credence function $C_{\mathcal{E}|F}$ is obtained by replacing $C(E_i|F)$ in the above equation with $C_{\mathcal{E}|F}(E_i|F)$. Now, consider the following equation, which follows from the probability calculus:

$$C(A) = \sum_{E_i \in \mathcal{E}} C(F_1) C(E_i|F_1) C(A|E_i F_1) + \sum_{E_i \in \mathcal{E}} C(F_2) C(E_i|F_2) C(A|E_i F_2) + \cdots.$$

Then, we can obtain a generalized version of Adams Conditionalization by replacing $C(E_i|F_j)$ with $C_{\mathcal{E}|\mathcal{F}}(E_i|F_j)$ for any E_i and F_j.

[14] The derivation of ACR0 from PP0 and GAC is given in Appendix. Similarly, ACR+, which is formulated below, is derived from PP+ and GAC. This derivation is also given in Appendix.

This rule is related to a chance-free initial credence function. However, we can suggest a similar rule that is related to chance-free credence functions with total evidence E. Let C_E be such a function. Let $C_{E,CH}$ be the chance-fed credence function updated from C_E in accordance with PP+. Similar to PP0, PP+ can be thought of as a norm stating what values should be newly assigned to some conditional credences—that is, it requires that for any $w \in \mathcal{W}$ and $U_i \in \mathcal{U}$, $ch_i(w|E)$ should be newly assigned to the conditional credence in w given U_i. Then, the rule, which governs the way of feeding the impact of chances to C_E in accordance with the Principal Principle, can be formulated as follows:

ACR+. $C_{E,CH}(A) = \sum_{U_i \in \mathcal{U}} C_E(U_i) ch_i(A|E)$, for any $A \subseteq \mathcal{W}$.

Heretofore, I have formulated some ways of correcting chance-free credence functions in accordance with the Principal Principle. As explained, realistic agents are not born super, and so they hardly satisfy the principle at the beginning of their credal life. Nevertheless, if such agents update or correct their credences by means of ACR0 and ACR+, then they can form the credences in which objective chances are reflected in accordance with the Principal Principle. So far, so good. However, it seems that an interesting but undesirable result is drawn from such a kind of credence updating or correcting. In the next section, I will explain and discuss this result.

4 When Credences Are Corrected

Let C be a chance-free credence function without any evidence. Then, we may say that $C_{E,CH}$ is obtained through a sequential belief updating from C, in which C is first updated to C_E by means of Conditionalization, and then C_E is updated to $C_{E,CH}$ by means of ACR+. Note that, in the sequential belief updating from C to $C_{E,CH}$, the credences are corrected in accordance with the Principal Principle *after* the credences are conditionalized on the total evidence E. In other words, $C_{E,CH}$ is obtained by *conditionalizing first and then correcting*. However, this order can be reversed. That is, the chance-free credence function can be corrected in accordance with the Principal Principle *before* the credences are conditionalized on E. Let $C_{CH,E}$ be the credence function so obtained. That is, $C_{CH,E}$ is obtained by *correcting first and then conditionalizing*.

Note first the following propositions:

Proposition 1. Suppose that C is a chance-free credence function without any evidence. Suppose also that our credences are updated by means of Conditionalization, ACR0, and ACR+. Then, it holds that, for any $A \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$,

$$C_{E,CH}(A|U_i) = C_{CH,E}(A|U_i) = ch_i(A|E),$$

where the relevant conditional credences and chances are well defined.

According to this proposition, when credence functions are updated by means of the relevant rules, PP+ is satisfied no matter when the chance-free credence function is corrected. That is, the order in question—i.e., the timing of the correction—does not affect whether the final credences satisfy the Principal Principle.

Note also that:

Proposition 2. Suppose that C is a chance-free credence function without any evidence. Suppose also that our credences are updated by means of Conditionalization, ACR0, and ACR+. Then, the following two propositions are equivalent to each other:

(a) $C_{E,CH}(A) = C_{CH,E}(A)$, for any $A \subseteq \mathcal{W}$.

(b) $C_{E,CH}(U_i) = C_{CH,E}(U_i)$, for any $U_i \in \mathcal{U}$.

According to this proposition, $C_{E,CH}$ and $C_{CH,E}$ would be different from each other exactly when the timing of the correction affects our credences in the ur-chance propositions.

Then, are there any evidential situations in which the timing in question has no influence on the credences in the ur-chance propositions and so $C_{E,CH}$ is equal to $C_{CH,E}$? In this regard, we have that:

Proposition 3. Suppose that C is a chance-free initial credence function. Suppose also that our credences are updated by means of Conditionalization, ACR0, and ACR+. Suppose that:

(1) E is a disjunction of some U_is, or

(2) there is an U_i such that $E \subseteq U_i$.

Then, it holds that $C_{E,CH}(A) = C_{CH,E}(A)$, for any $A \subseteq \mathcal{W}$.

Here, $C_{E,CH}$ and $C_{CH,E}$ have the same total experience, and they are updated from the same initial credence function. The only difference between them is the timing of correcting the credences in accordance with the Principal Principle. It is not the case, however, that the results in question are always the same as each other. That is,

Proposition 4. Suppose that our credences are updated by means of Conditionalization, ACR0, and ACR+. Then, there is a chance-free credence function C and total evidence E such that $C_{E,CH}(A) \neq C_{CH,E}(A)$, for some $A \subseteq \mathcal{W}$.

This proposition says that the final credence functions depend on *when our credences are corrected*. Let me consider the following example for the proof of this proposition.

Example

There are six possible worlds, $w_1, \cdots,$ and w_6. The ur-chance function of w_1, w_2, and w_3 is ch_1; the ur-chance function of the other worlds is ch_2. Thus, $U_1 \equiv w_1 \vee w_2 \vee w_3$ and $U_2 \equiv w_4 \vee w_5 \vee w_6$. Let us assume that $E \equiv w_1 \vee w_2 \vee w_4 \vee w_5$ and $\neg E \equiv w_3 \vee w_6$. I will describe the relevant probability assignments by means of the following 2×3 tables.

Possible Worlds

w_1	w_4
w_2	w_5
w_3	w_6

Each cell of this table refers to the associated possible world. The tables whose cells are filled with a numerical value express the relevant probability assignments. Here are such tables.

C

1/10	2/10
2/10	2/10
2/10	1/10

ch_1

1/3	0
1/3	0
1/3	0

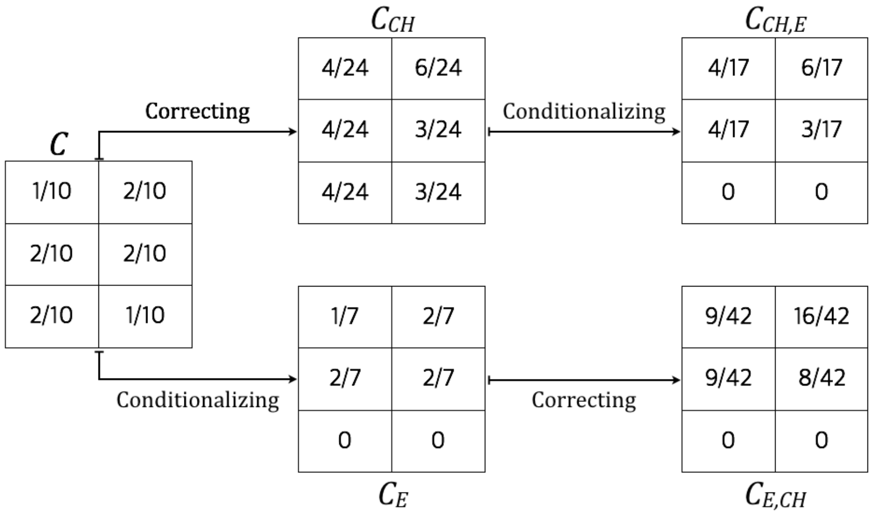
ch_2

0	2/4
0	1/4
0	1/4

These tables express, for example, that $C(w_1) = 1/10$, $ch_1(w_2) = 1/3$, and $ch_2(w_6) = 1/4$. Recall that I assumed Self-esteem. Thus, $ch_1(U_1) = ch_1(w_1 \vee w_2 \vee w_3) = 1$. A similar point goes with ch_2.

Now, we can calculate the credence assignments of $C_{CH,E}$ and $C_{E,CH}$ using Conditionalization, ACR0 and ACR+. Figure 1 displays two different sequential belief updates. Note that $C_{CH,E}(w_i|U_1) = C_{E,CH}(w_i|U_1) = ch_1(w_i|E)$ and $C_{CH,E}(w_i|U_2) =$

Figure 1: $C_{CH,E}$ vs. $C_{E,CH}$



$C_{E,CH}(w_i|U_2) = ch_2(w_i|E)$ for any w_i. That is, both of $C_{CH,E}$ and $C_{E,CH}$ satisfy PP+. However, $C_{CH,E} \neq C_{E,CH}$. It holds, for example, that $C_{CH,E}(w_1) = 4/17 \neq 9/42 = C_{E,CH}(w_1)$. Note also that $C_{CH,E}(U_1) = 8/17 \neq 18/42 = C_{E,CH}(U_1)$. This result means that the final results are different depending on *when* our credences are corrected in accordance with the Principal Principle.

As is well known, Bayesian belief updating by means of Jeffrey Conditionalization is non-commutative in the sense that the final credence function is sensitive to the order in which *the new credences* are incorporated into the credal system. In this regard, several philosophers argue that this kind of non-commutativity is not problematic since the impacts of the new credences on our credal system cannot help being different according to the order in question, and this non-commutativity does not happen at all when the impacts are properly represented using parameters like Bayes factors.[15] Note that Proposition 4 and Figure 1 reveal non-commutativity between conditionalizing and correcting. Then, someone may attempt to respond, in a similar way, to this kind of non-commutativity. That is, it may be argued that the impact of correction through the belief update from C to C_{CH} on a credal system is different from the impact of correction through the belief update from C_E to $C_{E,CH}$ on the system, and

[15]See, for example, Jeffrey (2004), Lange (2001), and Wagner (2003). This point can be made in a different way. One's credences should be sensitive to one's *total* evidence. The *order* in which one had the relevant experiences is part of one's total evidence. If the order is changed, then one changes the total evidence and so does his credences. Therefore, such non-commutativity is not problematic.

so that it is epistemologically intuitive that $C_{CH,E}$ is different from $C_{E,CH}$.

However, this kind of epistemological consideration seems to be of little help in achieving our epistemic goal. In particular, there might be doxastic situations in which we face a decision problem due to non-commutativity between conditionalizing and correcting. In such a situation, we deliberate which of $C_{CH,E}$ and $C_{E,CH}$ should be adopted as the corrected function—in other words, when our credences should be corrected. However, the epistemological fact that the impact of correction on the credal system will be different depending on the timing of correction does not provide helpful guidance on how we should plan to update our credences.[16]

5 Correcting and Accuracy

In the previous section, I have suggested the Adams Correcting Rule and shown that such a correcting rule yields non-commutativity between conditionalizing and correcting. This suggestion and observation raise two questions: (a) Why should we correct our credences by means of the Adams Correcting Rule?; (b) How should we plan to update our credences if there is a decision problem due to the non-commutativity in question and we face such a problem? In this section, I will attempt to respond to these two questions. For this purpose, I will focus on the notion of 'accuracy', which is often regarded as one of the main epistemic values of our credences. According to Accuracy-centred Probabilism, rational agents should strive to have credences as accurate as possible. Interestingly, such probabilism provides a way of vindicating the Adams Correcting Rule, and gives a clue to a decision problem due to non-commutativity between conditionalizing and correcting.

5.1 Accuracy-centred Probabilism

Before I proceed, a brief note on measuring accuracy is in order. Roughly speaking, the accuracy of a credence function at a world is regarded as something like the distance between the credences and the truth values at the world. Let $\mathcal{X}$ be a partition that consists of X_is, and C be a credence function that assigns each member of $\mathcal{X}$ to a

[16] As noted in footnote 15, the order in which we receive the relevant pieces of information may be taken as a part of our total evidence. By the same token, it may also be said that $C_{CH,E}$ has different total evidence from $C_{E,CH}$. In what follows, I will assume that, in some relevant situations, we could decide when our credences are corrected. Under this assumption, the decision between $C_{CH,E}$ and $C_{E,CH}$ may be regarded as the decision between two different pieces of total evidence.

particular real value. Moreover, let V_i be a truth function such that $V_i(X_j) = 1$ when $i = j$, and $V_i(X_j) = 0$ otherwise. We can say that V_i represents worlds at which X_i is true and the other members of $\mathcal{X}$ are false. Lastly, let $\mathfrak{I}_{\mathcal{X}}(C, V_i)$ be a function that measures the *inaccuracy* of a credence function C with respect to $\mathcal{X}$ at V_i. Suppose, for example, that the inaccuracy is measured by the Brier score. Then, the inaccuracy of C with respect to $\mathcal{X}$ at V_i is as follows:

$$\mathfrak{I}_{\mathcal{X}}(C, V_i) = \sum_{X_j \in \mathcal{X}} \left(C(X_j) - V_i(X_j)\right)^2 .$$

Now, we can define the expected inaccuracy of C with respect to $\mathcal{X}$. The expected values in the decision theory are weighted averages with the weights being a credence assigned by a particular credence function. That is, the expected values are always determined by the light of a particular credence function. Thus, the expected inaccuracy of a credence function C' with respect to $\mathcal{X}$ is also determined by the light of a credence function C. Then, the expected inaccuracy of C' with respect to $\mathcal{X}$ by the light of C, which will be referred to as '$EI_C[C', \mathcal{X}]$', is formulated as follows:

$$EI_C[C', \mathcal{X}] = \sum_{X_i \in \mathcal{X}} C(X_i) \mathfrak{I}_{\mathcal{X}}(C', V_i).$$

Note that, besides the Brier score, there are various candidates for the inaccuracy measure.[17] Regarding such candidates, several conditions are assumed—for example, Truth-directedness, Convexity, Extentionality, and so forth. Among them, what is called Immodesty is intimately related to our discussion. Here is the assumption:

> **Immodesty.** Suppose that the inaccuracy of any credence function with respect to a partition $\mathcal{X}$ is measured by $\mathfrak{I}_{\mathcal{X}}$. Then, it holds that, for any credence functions C and C',
>
> $$EI_C[C, \mathcal{X}] \leq EI_C[C', \mathcal{X}],$$
>
> where equality holds if and only if $C(X_i) = C'(X_i)$ for any $X_i \in \mathcal{X}$.

In words, Immodesty says that any credence function has the unique minimum expected inaccuracy by the light of its own credence assignment.

[17] For the candidates, see Joyce (2009) and Pettigrew (2016a) for example.

5.2 An Argument for Adams Correction

With these definitions and assumptions in hand, we can argue for the Adams Correcting Rule. Note that, besides the Adams Correcting Rule, there are many ways of correcting our credences in accordance with the Principal Principle. What is required of such ways is just that the corrected credence functions satisfy the principle. So, we should say that the Adams Correcting Rule is not a unique way of correcting our credences.[18] Interestingly, however, we can show that, in view of Accuracy-centred Probabilism, the Adams Correcting Rule has an epistemic merit that the other correcting rules do not have.

To see this, let me consider an agent who has a chance-free credence function C at time t. (Here, I do not assume that C is an initial credence function. That is, C would be a chance-free credence function with total evidence E.) The agent happens to know at $t'(> t)$ that she should satisfy the Principal Principle in order to be rational. (It is assumed that she does not obtain any other information between t and t'.) Thus, she decides to correct her credences immediately. Then, what rule should she adopt at t' for the correction? As mentioned, there are many rules that can correct C in accordance with the Principal Principle. Let **R** be such a rule for her credence function C. In addition, let $C^{\mathbf{R}}$ be the corrected credence function from C by means of **R**. Lastly, let $\mathbb{R}$ be the set of such rules. Then, which of members of $\mathbb{R}$ should the agent adopt at t'?

In response to this question, Accuracy-centred Probabilists may require the agent to adopt the correcting rule that leads her to a chance-fed credence function that has the minimal expected inaccuracy. It is noteworthy here that, when the agent deliberates what correcting rule should be adopted, she has the chance-free credence function C, and thus the expected inaccuracy in question is determined by the light of C. In this regard, someone may say, "C violates the Principal Principle, which is a requirement of rationality. So, the expected inaccuracy in question, which is determined by the light of the irrational credence function C, cannot provide any guidance on what correcting rule should be adopted. In other words, we should not adopt any correcting rule that is recommended by the irrational function C."

[18] Consider the example in the previous section. Suppose that the chance-free initial credence function C is corrected to a credence function C^* whose credence assignment is as follows: $C^*(w_1) = C^*(w_2) = C^*(w_3) = 1/12$ and $C^*(w_4) = 2C^*(w_5) = 2C^*(w_6) = 3/8$. Note that $C^*(w_i|U_1) = C^*(w_i|U_1) = ch_1(w_i|E)$ and $C^*(w_i|U_2) = C^*(w_i|U_2) = ch_2(w_i|E)$. That is, this function satisfies the Principal Principle. However, C^* is not equal to C_{CH}, which is corrected by means of the Adams Correcting Rule, namely ACR0.

How can we respond to this worry? First of all, we need to note that C recommends a correcting rule in $\mathbb{R}$, which leads the agent to a credence function satisfying the Principal Principle. Thus, the credence function, which is led by a rule that C recommends, is at least rationally *permissible*. That is, the recommendation of C does not lead us to any outrageously irrational credence function. So, we may conclude that the worry in question is not as serious as it may sound. Secondly, I would like to point out that the agent seeks a way of doing the best within her epistemic limitations. When the agent happens to know the irrationality of her credence function, she has no idea what rational credence function should guide the correction of her credences. If she knows that, then she would not need to consider how to correct her credences. This being the case, there seems to be no other way but to evaluate the expected inaccuracy by her current function C. Admittedly, these responses are not enough. Indeed, it needs to be argued that the correcting rules recommended by C are all what rationality requires us to adopt. However, I will not provide such an argument in this paper. Rather, I will assume that.[19]

On the other hand, there is another point that we should address regarding the expected inaccuracy in question. Note, in particular, that Proposition 2 can be generalized. That is, it holds that if C_1 and C_2 are credence functions that satisfy the Principal Principle, then the following two propositions are equivalent to each other: (i) $C_1(A) = C_2(A)$, for any $A \subseteq \mathcal{W}$; (ii) $C_1(U_i) = C_2(U_i)$, for any $U_i \in \mathcal{U}$.[20] Thus, we can say that the decision among the correcting rules in $\mathbb{R}$ depends on what credences each corrected function assigns to the ur-chance propositions. Then, when we evaluate the correcting rules according to Accuracy-centred Probabilism, it may be sufficient to consider the expected inaccuracy *with respect to the partition* $\mathcal{U}$. To put it another way, Accuracy-centred Probabilists may require the above agent to adopt a rule that is expected to lead her to a chance-fed credence function that has the minimally inaccurate opinion *about the ur-chance function*.

Now, we can formulate clearly a principle to which Accuracy-centred Probabilists may appeal in order to evaluate the correcting rules in question. Here is such a principle:

Minimizing Expected Inaccuracy with the Principal Principle (MIPP). Let $\mathbb{R}$ be a set of updating rules for a chance-free credence function C. Suppose that

[19]This line of response to a similar problem is also found in Pettigrew (2016a, 199-200). I owe a debt of gratitude to an anonymous referee for helping me make this point clear.

[20]The proof of this is basically the same as the proof of Proposition 2, which is given in Appendix.

each rule in $\mathbb{R}$ corrects C to a chance-fed credence function. Suppose also that an agent with C is to correct her credences so as to abide by the Principal Principle. Then, it is a requirement of rationality that she adopts a rule in $\mathbb{R}$ that corrects C to a chance-fed credence function that has the minimal expected inaccuracy with respect to $\mathcal{U}$ by the light of C. That is, it is a requirement of rationality that any agent with C adopts **R** in $\mathbb{R}$ such that, for any $\mathbf{R}^* \in \mathbb{R}$,

$$EI_C[C^{\mathbf{R}}, \mathcal{U}] \leq EI_C[C^{\mathbf{R}^*}, \mathcal{U}].$$

As the following proposition states, on the other hand, the Adams Correcting Rule has an epistemologically desirable feature that the other rules do not have.

Proposition 5. Let $\mathbb{R}$ be a set of updating rules for a chance-free credence function C_E, which has total evidence E. Suppose that each rule in $\mathbb{R}$ corrects C_E to a credence-fed function. Suppose also that **R** is the Adams Correcting Rule, i.e., ACR+. Then, it holds that, for any $\mathbf{R}^* \in \mathbb{R}$,

$$EI_{C_E}[C_E^{\mathbf{R}}, \mathcal{U}] \leq EI_{C_E}[C_E^{\mathbf{R}^*}, \mathcal{U}],$$

where equality holds if and only if $C_E^{\mathbf{R}} = C_E^{\mathbf{R}^*}$.

Roughly speaking, this proposition states that the chance-fed credence function obtained by the Adams Correcting Rule is expected to be less inaccurate than any other chance-fed functions—that is, such a chance-fed function has the minimal expected inaccuracy with respect to $\mathcal{U}$ by the light of C_E.

In order to prove this proposition, we should pay attention to Immodesty, according to which any credence function has the unique minimum expected inaccuracy by the light of its own credence assignments. Thus, Immodesty implies that $EI_{C_E}[C_E^{\mathbf{R}}, \mathcal{U}]$ has the unique minimal value if and only if $C_E^{\mathbf{R}}(U_i) = C_E(U_i)$ for any $U_i \in \mathcal{U}$. Interestingly, when the chance-free function C_E is corrected to $C_{E,CH}$ by means of the Adams Correcting Rule, it holds that $C_E(U_i) = C_{E,CH}(U_i)$ for any $U_i \in \mathcal{U}$. Thus, we can conclude that the expected inaccuracy of the corrected function with respect to $\mathcal{U}$ by the light of C_E has the minimal value exactly when the chance-free function is corrected by means of the Adams Correcting Rule. (A detailed proof is given in Appendix.)

As a result, it follows from MIPP and Proposition 5 that:

Adams Correction. Suppose that an agent has a chance-free credence function

C_E, which has total evidence E. Suppose also that she happens to know (and nothing else) that she should satisfy the Principal Principle in order to be rational. Suppose even that she is to correct her credences so as to abide by the principle. Then, it is a requirement of rationality that she should correct her credences by means of ACR+.

We can also provide a similar argument for ACR0. Thus, this result could be regarded as an epistemic vindication of the Adams Correcting Rule.

As shown above, my argument for the Adams Correcting Rule heavily depends on the notion of accuracy and a decision-theoretic framework. This notion and framework shed light on a decision problem due to non-commutativity between conditionalizing and correcting.

5.3 Non-commutativity between Conditionalizing and Correcting

Consider again the agent who has a chance-free credence function C_E. She is to correct her credences in accordance with the Principal Principle. If she corrects her credences by means of ACR+, then she will have $C_{E,CH}$ after correcting. (Hereafter, I will use 'C_{CH}', '$C_{E,CH}$', and '$C_{CH,E}$' as the notations that stand for the credence functions that are updated by means of Conditionalization and Adams Correcting Rule.) Here, it is worth noting that there may be other ways of correcting her credences. For example, she may consider first *what her initial credence function should have been in order to abide by the Principal Principle*, and then conditionalize the corrected function on E. In our context, the corrected function in question is C_{CH}. So, if she corrects her credences in this way, then she will have $C_{CH,E}$ after correcting. Indeed, this kind of correction is not entirely new—for example, Meacham (2016, 451-452) discusses this way of correcting an agent's credences that violates the Principal Principle.[21] Then, the agent faces a decision problem: Which of $C_{E,CH}$ and $C_{CH,E}$ should she adopt her new credence function?

As Proposition 5 states, the relevant expected inaccuracy of $C_{E,CH}$ must be greater than of $C_{CH,E}$ if the two credence functions are different from each other. Thus, we can say, with the help of MIPP, that it is irrational that the agent, who has the chance-

[21] Meacham (2016, 451-452) regards a very similar example as a motivation for adopting what he calls Ur Prior Conditionalization. He formulates Ur Prior Conditionalization as follows: If a subject has ur-priors up and current evidence E, her credence cr should be $cr(\cdot) = up(\cdot|E)$, if defined. Here, 'ur-priors' corresponds to our 'initial credences'.

free credence function C_E, corrects her credence function to the chance-fed credence function $C_{CH,E}$. That is, rationality seems to favor $C_{E,CH}$ over $C_{CH,E}$, in this situation.

In this regard, someone may think that the way, which leads the agent to $C_{CH,E}$, is not always available to her. For example, if the agent entirely forgets her past credence function, i.e., the chance-free initial credence function C, then she may have no idea how to determine her corrected credence function, i.e., C_{CH}. In such a case, she may regard the way in question as useless, and so does not have to wrestle with the non-commutativity between conditionalizing and correcting.

However, we are able to imagine another doxastic situation in which the non-commutativity at issue yields a decision problem. Suppose that an agent has a chance-free initial credence function C at time t. Suppose also that she happens to obtain the following two pieces of information at $t'(> t)$ and nothing else: (i) she should satisfy the Principal Principle in order to be rational; (ii) she will receive the information about the truth value of E at $t''(> t')$. Suppose even that the agent is to update and correct her credences at t''—that is, she is to update and correct her credence function C when she receives the information.

Having the non-commutativity at issue in mind, we may find that there are at least two kinds of credence function that she will have at t''. Suppose that the agent gets to know at t'' that E is true. Then, she may update C to C_E by means of Conditionalization on E, and then correct C_E to $C_{E,CH}$ by means of the Adams Correcting Rule. Or, she may correct C to C_{CH} by means of the Adams Correcting Rule, and then update C_{CH} to $C_{CH,E}$ by means of Conditionalization on E. That is, when she receives the information about the truth value of E, she may update her credence function by means of a rule that requires to *conditionalize first and then correct*, or by means of a rule that requires to *correct first and then conditionalize*. Let me call the former 'the Conditionalizing-first Rule' and the latter 'the Correcting-first Rule'. Then, which of the two rules should the agent adopt at t''?[22]

Interestingly, we can argue, in the spirit of Accuracy-centred Probabilism, that it is a requirement of rationality that the agent should plan to adopt the Conditionalizing-

[22] Some readers may think that there is another way of incorporating the two pieces of information into the credal system. Note that the Conditionalizing-first and Correcting-first Rules might be regarded as sequential updating rules. However, the two pieces of information might be *simultaneously* incorporated into the credal system. I do not rule out this possibility in this paper. Be that as it may, we do not have to consider separately such a way. This is because the following discussion shows that the Conditionalizing-first Rule has an epistemic merit that *any other relevant rules* do not have.

first Rule. That is, it can be shown that the Conditionalizing-first Rule has the minimal expected inaccuracy with respect to $\mathcal{U}$ by the light of C.[23] To see this, we should formulate first the expected inaccuracy of *the relevant rules themselves*—not the credence functions obtained by means of such rules. Let $\mathcal{E}$ be a partition whose elements E_is represent possible new evidence. Let $\mathbf{R}_{\mathcal{E}}$ refer to an updating rule on $\mathcal{E}$ that leads the agent to a chance-fed credence function that respects evidence from $\mathcal{E}$. This rule can be regarded as a function that associates a particular piece of evidence with a chance-fed function in question. Let $C^{\mathbf{R}}_{E_i}$ refer to the chance-fed function that is obtained by means of $\mathbf{R}_{\mathcal{E}}$ when evidence E_i is received.[24]

Now, we can formulate the expected inaccuracy of $\mathbf{R}_{\mathcal{E}}$ with respect to a partition $\mathcal{U}$ by the light of C. Suppose that the agent updates her credence function by means of $\mathbf{R}_{\mathcal{E}}$. Then, the inaccuracy of her new credence function with respect to $\mathcal{U}$ depends on *not only* the evidence that she receives, *but also* the truth values of U_is. Suppose, for example, that she receives E_i, and that U_j is true. Then, her credence function will be updated to $C^{\mathbf{R}}_{E_i}$, and its inaccuracy will be equal to $\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j)$. So, the expected inaccuracy of $\mathbf{R}_{\mathcal{E}}$ with respect to $\mathcal{U}$ by the light of a credence function C is defined as follows:

$$EI_C[\mathbf{R}_{\mathcal{E}}, \mathcal{U}] = \sum_{E_i \in \mathcal{E}, U_j \in \mathcal{U}} C(E_i U_j) \mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j).$$

With this kind of expected inaccuracy in hand, we can prove the following proposition:[25]

Proposition 6. Let $\mathbb{R}_{\mathcal{E}}$ be a set of updating rules on $\mathcal{E}$ for a chance-free credence function C. Suppose that each rule in $\mathbb{R}_{\mathcal{E}}$ updates C to a credence-fed function that respects evidence from $\mathcal{E}$—that is, $C^{\mathbf{R}}_{E_i}$ satisfies the Principal Principle and assigns E_i to 1, for any $\mathbf{R}_{\mathcal{E}} \in \mathbb{R}_{\mathcal{E}}$ and $E_i \in \mathcal{E}$. Suppose also that $\mathbf{R}_{\mathcal{E}}$ is the Conditionalizing-first Rule on $\mathcal{E}$ for C such that $C^{\mathbf{R}}_{E_i} = C_{E_i, CH}$, for any $E_i \in \mathcal{E}$.

[23] The following argument is basically similar to Accuracy-centred Probabilists' argument for a synchronic version of Conditionalization. For example, see Easwaran (2013), Pettigrew (2016a), Wallace and Greaves (2006).

[24] Here, we should note that this rule (or function) takes as arguments only the evidence that will be received later. Thus, the rule in question has nothing to do with the credence function that the relevant agents have before receiving evidence. In other words, $\mathbf{R}_{\mathcal{E}}$ says nothing about what credence function the agents should have before receiving evidence from $\mathcal{E}$.

[25] The proof of Proposition 6 is very similar to the proof of Proposition 5. In particular, both proofs rely on Immodesty and the fact that the credences in the ur-chance propositions remain the same when a credence function is corrected by means of the Adams Correcting Rule. The detailed proof is given in Appendix.

Then, it holds that, for any $\mathbf{R}^*_{\mathcal{E}} \in \mathbb{R}_{\mathcal{E}}$,

$$EI_C[\mathbf{R}_{\mathcal{E}}, \mathcal{U}] \leq EI_C[\mathbf{R}^*_{\mathcal{E}}, \mathcal{U}],$$

where equality holds if and only if $C^{\mathbf{R}}_{E_i} = C^{\mathbf{R}^*}_{E_i}$ for any $E_i \in \mathcal{E}$.

Roughly speaking, this proposition states that, in the relevant doxastic situation, the Conditionalizing-first Rule is expected to be less inaccurate than any other rules in $\mathbb{R}_{\mathcal{E}}$—that is, such a rule has the minimal expected inaccuracy with respect to $\mathcal{U}$ by the light of C. As a result, MIPP and this proposition imply, *mutatis mutandis*, the following principle:[26]

Plan Conditionalizing-first. Suppose that an agent has a chance-free credence function C at time t. Suppose also that she happens to knows at $t'(> t)$ the following two things and nothing else: (i) she should satisfy the Principal Principle in order to be rational; (ii) she will receive evidence from $\mathcal{E}$ at $t''(> t')$. Suppose even that she is to update her credences at t'' so as to abide by the Principal Principle and respect evidence. Then, it is a requirement of rationality that she adopts the Conditionalizing-first Rule $\mathbf{R}_{\mathcal{E}}$ on $\mathcal{E}$ for C such that $C^{\mathbf{R}}_{E_i} = C_{E_i,CH}$, for any $E_i \in \mathcal{E}$.

This result, I think, is somewhat interesting. In particular, this principle provides an guidance on how agents in the relevant doxastic situation should incorporate the two pieces of information, i.e., (i) and (ii), into her credal system at t''. That is, this principle says that such agents should update and correct her credences by means of the Conditionalizing-first Rule, rather than any other rules, including the Correcting-first Rule.

I should, however, add some caveats. Consider a doxastic situation in which you have no idea from what partition you will receive evidence. Plan Conditionalizing-first cannot apply to this kind of doxastic situation. What the principle says is just that, when you know the partition from which you will receive evidence, you should plan to incorporate the two pieces of information, i.e., (i) and (ii), into your credal

[26] In order for MIPP and Proposition 6 to imply Plan Conditionalizing-first, MIPP needs to be slightly modified. Note that MIPP appearing in Section 5.2 is related to the expected inaccuracy of a particular credence-fed function. However, what is needed to imply Plan Conditionalizing-first is a principle about the expected inaccuracy of an updating rule. Of course, such a modified version can be readily formulated in a similar way of MIPP.

system according to the Conditionalizing-first Rule. For this reason, we should say that Plan Conditionalizing-first can apply to only a few doxastic situations.

Moreover, there is another reason that we should think the principle has a very narrow scope of application. To see this, we need to note that Plan Conditionalizing-first can only apply to agents who are to update their credences at t'' so as to abide by the Principal Principle and respect evidence. In other words, the principle can only apply to agents who are to correct her credences when they receive evidence. Suppose that an agent, who has a chance-free credence function, happens to know (i) and (ii) at a given time. Suppose also that she is to correct her credences immediately regardless of what evidence will be received. Then, Plan Conditionalizing-first *itself* cannot apply to such an agent, and so provides no guidance on her correcting plan. In this connection, we should also note that this principle does not require you to postpone the correction until the evidence is received. Some readers might think that if you adopt the Conditionalizing-first rule in the relevant doxastic situation, then your credence function between t' and t'' should stay uncorrected. However, the principle *itself* does not require that. Rather, what the principle considers is just *the credence function at* t'' that you will have when the evidence is received. Thus, we should say that Plan Conditionalizing-first has nothing to do with the credence function between t' and t''.[27]

It is hard to deny that these caveats disclose that Plan Conditionalizing-first has a very narrow scope of application, and so is too weak. I agree. However, this point does not make the principle useless. Indeed, Plan Conditionalizing-first shows a substan-

[27] Then, is there any rational way of deciding what credence function we have between t' and t''? Let me consider the following argument. Suppose that, at time t', an agent, whose credence function C violates the Principal Principle, gets to know (i) and (ii) appearing in Plan Conditionalizing-first. Suppose also that she knows at t' that she always updates her credences by means of Conditionalization on evidence. At time t', she deliberates whether she corrects her credences immediately or puts off the correction until receiving evidence from $\mathcal{E}$. Note that she knows at t' that she is a conditionalizer. So, she also knows at t' that if she corrects her credences immediately, then she will have one of C_{CH,E_i}s at time t''. However, Proposition 6 says that, by the light of her current credence function C, this result does not have the minimal expected inaccuracy. Thus, it seems rational that the agent does not correct her credences immediately. In this paper, however, I am not committed to this conclusion. This is partially because of the assumption that the agent in question knows that she always updates her credences by means of Conditionalization. There may be some arguments for having such knowledge. However, I leave the matter open for further investigation. Be that as it may, I would like to emphasize here that what credence function we should have between t' and t'' does not undermine the main point I wish to argue for here. This is because Proposition 6, which my main results heavily depend on, concerns only the credence function at t'', and the proposition can still follow no matter what credence function we should have between t' and t''. Many thanks to anonymous referees for encouraging me to make these points clear.

tial difference between the Conditionalizing-first and the Correcting-first Rule. We could formulate another relevant principle that may be called Plan Correcting-first, which corresponds to the Correcting-first Rule. This principle, unlike Plan Conditionalizing-first, cannot be the case, however. For this reason, we could think that the Conditionalizing-first Rule has an epistemic merit that any other relevant rules, including the Correcting-first Rule, do not have. In regard to non-commutativity in question, we can conclude that there is a doxastic situation in which we should favor the Conditionalizing-first Rule over the Correcting-first Rule.

6 Concluding Remarks

Admittedly, my discussions have little to do with superbabies who are born rational. Such agents satisfy the Principal Principle throughout their credal life as long as they are conditionalizers. Thus, they do not face any situation in which they have to correct their credences in accordance with the principle. However, we are not born rational and so we should correct our credences at some time or other using our information about chances. In this regard, I have argued that such a realistic agent should adopt the Adams Correcting Rule. Moreover, it is also argued that, when an agent faces a decision problem due to non-commutativity between conditionalizing and correcting, the agent should plan to adopt the Conditionalizing-first Rule, given the relevant assumptions. As mentioned, objective chances are often regarded as an expert that our credences should respect. Note that the way of respecting an expert is basically the same as the Principal Principle. Then, we may derive similar conclusions.

Accuracy-centred Probabilists have recently suggested several interesting arguments for various epistemic norms—for example, Probabilism, the Principle of Indifference, Reflection, a synchronic version of Conditionalization, and so forth. In doing so, they focus on the epistemic notions like accuracy and use the relevant decision-theoretic principles. Similarly, my arguments in this paper heavily depend on such notions and principles. Admittedly, the rules and principles that I have suggested here are rather weak. Nevertheless, my suggestion, I think, may contribute to extending the scope of application of Accuracy-centred Probabilism.

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Appendix

The following proofs and derivations have various conditional credences and chances. For presentational simplicity, I will assume in what follows that such conditional credences and chances are all well defined. I think this assumption yields no confusion.

A derivation of ACR0 from PP0 and GAC:

Suppose that C_{CH} satisfies PP0, and that C_{CH} is updated from C by means of GAC. Then we have that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned}
C_{CH}(A) &= \sum_{U_i \in \mathcal{U}, w_j \in \mathcal{W}} C(U_i) C(A|w_j U_i) C_{CH}(w_j|U_i) \\
&= \sum_{U_i \in \mathcal{U}, w_j \in \mathcal{W}} C(U_i) C(A|w_j U_i) ch_i(w_j) \\
&= \sum_{U_i \in \mathcal{U}} C(U_i) \sum_{w_j \in \mathcal{W}} C(A|w_j U_i) ch_i(w_j) \\
&= \sum_{U_i \in \mathcal{U}} C(U_i) \sum_{w_j \in A} C(A|w_j U_i) ch_i(w_j) \\
&\quad + \sum_{U_i \in \mathcal{U}} C(U_i) \sum_{w_j \in \neg A} C(A|w_j U_i) ch_i(w_j) \\
&= \sum_{U_i \in \mathcal{U}} C(U_i) \sum_{w_j \in A} ch_i(w_j) = \sum_{U_i \in \mathcal{U}} C(U_i) ch_i(A),
\end{aligned}$$

as required.

A derivation of ACR+ from PP+ and GAC:

Suppose that $C_{E,CH}$ satisfies PP+, and that $C_{E,CH}$ is updated from C_E by means of GAC. Then we have that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned}
C_{E,CH}(A) &= \sum_{U_i \in \mathcal{U}, w_j \in \mathcal{W}} C_E(U_i) C_E(A|w_j U_i) C_{E,CH}(w_j|U_i) \\
&= \sum_{U_i \in \mathcal{U}, w_j \in \mathcal{W}} C_E(U_i) C_E(A|w_j U_i) ch_i(w_j|E) \\
&= \sum_{U_i \in \mathcal{U}} C_E(U_i) \sum_{w_j \in \mathcal{W}} C_E(A|w_j U_i) ch_i(w_j|E) \\
&= \sum_{U_i \in \mathcal{U}} C_E(U_i) \sum_{w_j \in A} C_E(A|w_j U_i) ch_i(w_j|E) \\
&\quad + \sum_{U_i \in \mathcal{U}} C_E(U_i) \sum_{w_j \in \neg A} C_E(A|w_j U_i) ch_i(w_j|E) \\
&= \sum_{U_i \in \mathcal{U}} C_E(U_i) \sum_{w_j \in A} ch_i(w_j|E) = \sum_{U_i \in \mathcal{U}} C_E(U_i) ch_i(A|E),
\end{aligned}$$

as required.

A proof of Proposition 1:

Suppose that our credences are updated by means of Conditionalization, ACR0, and ACR+. It is straightforward that $C_{E,CH}(w|U_i) = ch_i(w|E)$ for any $w \in \mathcal{W}$, and so $C_{E,CH}(A|U_i) = ch_i(A|E)$ for any $A \subseteq \mathcal{W}$. Similarly, we have that $C_{CH}(w|U_i) = ch_i(w)$, and so that, for any $A \subseteq \mathcal{W}$,

$$C_{CH}(A|U_i) = \sum_{w \in A} C_{CH}(w|U_i) = \sum_{w \in A} ch_i(w) = ch_i(A).$$

Then, Conditionalization and the above equation imply that, for any $A \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$,

$$C_{CH,E}(A|U_i) = C_{CH}(A|U_i E) = \frac{C_{CH}(AE|U_i)}{C_{CH}(A|U_i)} = ch_i(A|E).$$

Then, we have Proposition 1. Done.

A proof of Proposition 2:

Note that $U_i \subseteq \mathcal{W}$ for any $U_i \in \mathcal{U}$. Thus, it is straightforward that (a) implies (b). Now, let me prove that (b) implies (a). For this purpose, let's assume (b)—that is, $C_{E,CH}(U_i) = C_{CH,E}(U_i)$ for any $U_i \in \mathcal{U}$. From this assumption and Proposition 1, then, it follows that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned} C_{E,CH}(A) &= \sum_{U_i \in \mathcal{U}} C_{E,CH}(U_i) C_{E,CH}(A|U_i) = \sum_{U_i \in \mathcal{U}} C_{E,CH}(U_i) ch_i(A|E) \\ &= \sum_{U_i \in \mathcal{U}} C_{CH,E}(U_i) ch_i(A|E) = \sum_{U_i \in \mathcal{U}} C_{CH,E}(U_i) C_{CH,E}(A|U_i) \\ &= C_{CH,E}(A), \end{aligned}$$

as required.

A proof of Proposition 3:

According to ACR+ and Conditionalization, we have that, for any $A \subseteq \mathcal{W}$,

$$C_{E,CH}(A) = \sum_{U_i \in \mathcal{U}} C(U_i|E) ch_i(A|E). \tag{1}$$

Similarly, it follows from Conditionalization and ACR0 that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned} C_{CH,E}(A) = C_{CH}(A|E) &= \frac{C_{CH}(AE)}{C_{CH}(E)} \\ &= \frac{\sum_{U_i \in \mathcal{U}} C(U_i) ch_i(AE)}{\sum_{U_i \in \mathcal{U}} C(U_i) ch_i(E)}. \end{aligned} \tag{2}$$

First, let me prove that if E is a disjunction of some U_is, then it holds that $C_{E,CH}(A) = C_{CH,E}(A)$ for any $A \subseteq \mathcal{W}$. Suppose that E is a disjunction of some U_is. Let $\mathcal{U}_E$ be the set of the disjuncts in question. Then, it holds that U_i implies E when $U_i \in \mathcal{U}_E$, and U_i implies $\neg E$ otherwise. Moreover, $ch_i(E) = 1$ when $U_i \in \mathcal{U}_E$, and $ch_i(E) = 0$ otherwise. (Note that Self-esteem was assumed.) From this, it follows that $ch_i(AE) = ch_i(A|E) = ch_i(A)$ when $U_i \in \mathcal{U}_E$, and $ch_i(AE) = 0$ otherwise. Then, we have that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned}
C_{CH,E}(A) &= \frac{\sum_{U_i \in \mathcal{U}} C(U_i)ch_i(AE)}{\sum_{U_i \in \mathcal{U}} C(U_i)ch_i(E)} \\
&= \frac{\sum_{U_i \in \mathcal{U}_E} C(U_i)ch_i(AE) + \sum_{U_i \notin \mathcal{U}_E} C(U_i)ch_i(AE)}{\sum_{U_i \in \mathcal{U}_E} C(U_i)ch_i(E) + \sum_{U_i \notin \mathcal{U}_E} C(U_i)ch_i(E)} \\
&= \frac{\sum_{U_i \in \mathcal{U}_E} C(U_i)ch_i(AE)}{\sum_{U_i \in \mathcal{U}_E} C(U_i)} = \frac{\sum_{U_i \in \mathcal{U}_E} C(U_i)ch_i(A|E)}{\sum_{U_i \in \mathcal{U}_E} C(U_i)} \\
&= \frac{\sum_{U_i \in \mathcal{U}} C(U_i E)ch_i(A|E)}{C(E)} = \sum_{U_i \in \mathcal{U}} C(U_i|E)ch_i(A|E) = C_{E,CH}(A),
\end{aligned}$$

as required.

Now, let us prove that if there is a U_i such that $E \subseteq U_i$, then it holds that $C_{E,CH}(A) = C_{CH,E}(A)$ for any $A \subseteq \mathcal{W}$. Suppose that there is an ur-chance proposition that is a subset of E. Let U_k be such a proposition. Then, it holds that $ch_i(AE) = ch_i(E) = 0$ when $U_i \neq U_k$, and that $C(U_k|E) = 1$. Then, we have that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned}
C_{CH,E}(A) &= \frac{\sum_{U_i} C(U_i)ch_i(AE)}{\sum_{U_i} C(U_i)ch_i(E)} \\
&= \frac{\sum_{U_i = U_k} C(U_i)ch_i(AE) + \sum_{U_i \neq U_k} C(U_i)ch_i(AE)}{\sum_{U_i = U_k} C(U_i)ch_i(E) + \sum_{U_i \neq U_k} C(U_i)ch_i(E)} \\
&= \frac{C(U_k)ch_k(AE)}{C(U_k)ch_k(E)} = ch_k(A|E);
\end{aligned}$$

$$\begin{aligned}
C_{E,CH}(A) &= \sum_{U_i} C(U_i|E)ch_i(A|E) \\
&= \sum_{U_i = U_k} C(U_i|E)ch_i(A|E) + \sum_{U_i \neq U_k} C(U_i|E)ch_i(AE) \\
&= C(U_k|E)ch_k(A|E) = ch_k(A|E).
\end{aligned}$$

Thus, it holds that $C_{E,CH}(A) = C_{CH,E}(A)$ for any $A \subseteq \mathcal{W}$. Done.

Several calculations related to Proposition 4 and Figure 1:

As assumed in the example related to Figure 1, $\mathcal{U} = \{U_1, U_2\}$. Then, it follows from (1) and (2) in the proof of Proposition 3 that, for any $A \subseteq \mathcal{W}$,

$$C_{CH}(A) = C(U_1)ch_1(A) + C(U_2)ch_2(A), \tag{4a}$$

$$C_{CH,E}(A) = \frac{C(U_1)ch_1(AE) + C(U_2)ch_2(AE)}{C(U_1)ch_1(E) + C(U_2)ch_2(E)}, \tag{4b}$$

$$C_E(A) = \frac{C(AE)}{C(E)}, \text{ and} \tag{4c}$$

$$C_{E,CH}(A) = C(U_1|E)ch_1(A|E) + C(U_2|E)ch_2(A|E). \tag{4d}$$

Note that $E \equiv w_1 \vee w_2 \vee w_4 \vee w_5$, $U_1 \equiv w_1 \vee w_2 \vee w_3$, and $U_2 \equiv w_4 \vee w_5 \vee w_6$. With the help of (4a)-(4d), the chance assignments of ch' and ch^*, and the initial credence assignment of C, we have that, for any $A \subseteq \mathcal{W}$,

$$\begin{aligned} C_{CH}(A) &= \frac{1}{2}ch_1(A) + \frac{1}{2}ch_2(A), \\ C_{CH,E}(A) &= \frac{12}{17}\left(ch_1(AE) + ch_2(AE)\right), \\ C_E(A) &= \frac{10}{7}C(AE), \text{ and} \\ C_{E,CH}(A) &= \frac{9}{14}ch_1(AE) + \frac{16}{21}ch_2(AE). \end{aligned}$$

Now, we can derive the probability assignments of C_{CH}, $C_{CH,E}$, C_E, and $C_{E,CH}$. For example,

$$\begin{aligned} C_{CH}(w_1) &= \frac{1}{2}ch_1(w_1) + \frac{1}{2}ch_2(w_1) = \frac{1}{2}ch_1(w_1) = \frac{1}{6}; \\ C_{CH,E}(w_1) &= \frac{12}{17}\left(ch_1(w_1E) + ch_2(w_1E)\right) = \frac{12}{17}ch_1(w_1) = \frac{4}{17}; \\ C_E(w_1) &= \frac{10}{7}C(w_1E) = \frac{10}{7}C(w_1) = \frac{1}{7}; \text{ and} \\ C_{E,CH}(w_1) &= \frac{9}{14}ch_1(w_1E) + \frac{16}{21}ch_2(w_1E) = \frac{9}{14}ch_1(w_1) = \frac{3}{14}. \end{aligned}$$

These results conform with the probability assignments in Figure 1.

Proofs of Propositions 5 and 6:

Let me start with noting that it follows from GAC and the probability calculus that: for any $F_k \in \mathcal{F}$,

$$\begin{aligned}
C_{\mathcal{E}|\mathcal{F}}(F_k) &= \sum_{E_i \in \mathcal{E}, F_j \in \mathcal{F}} C(F_j) C_{\mathcal{E}|\mathcal{F}}(E_i|F_j) C(F_k|E_i F_j) \\
&= \sum_{E_i \in \mathcal{E}, F_j = F_k} C(F_j) C_{\mathcal{E}|\mathcal{F}}(E_i|F_j) C(F_k|E_i F_j) \\
&\quad + \sum_{E_i \in \mathcal{E}, F_j \neq F_k} C(F_j) C_{\mathcal{E}|\mathcal{F}}(E_i|F_j) C(F_k|E_i F_j) \\
&= \sum_{E_i \in \mathcal{E}} C(F_k) C_{\mathcal{E}|\mathcal{F}}(E_i|F_k) = C(F_k).
\end{aligned}$$

That is, when some experience directly changes the relevant conditional credences, and so C is updated to $C_{\mathcal{E}|\mathcal{F}}$ by means of GAC, the credences in the conditioning propositions, namely F_is, remain the same. As explained above, the correction of C_E to $C_{E,CH}$ by means of ACR+ can be regarded as a belief updating by means of GAC, in which the conditioning propositions are the ur-chance propositions U_is. So, the credences in U_is remain the same through the correction in question. More formally, it holds that: for any $E \subseteq \mathcal{W}$ and $U_i \in \mathcal{U}$,

$$C_{E,CH}(U_i) = C_E(U_i) = C(U_i|E). \tag{A}$$

Now, with this mathematical feature of the Adams Correcting Rule in hand, we can prove Propositions 5 and 6.

A proof of Proposition 5: Let $\mathbb{R}$ be a set of updating rules for a chance-free credence function C_E. All members of $\mathbb{R}$ correct C_E to a chance-fed function. Let **R** be the Adams Correcting Rule, i.e., ACR+. Then, it holds that $C_E^{\mathbf{R}} = C_{E,CH}$. Moreover, let **R*** be a correcting rule in $\mathbb{R}$. Then, Immodesty implies that,

$$EI_{C_E^{\mathbf{R}}}[C_E^{\mathbf{R}}, \mathcal{U}] \leq EI_{C_E^{\mathbf{R}}}[C_E^{\mathbf{R}^*}, \mathcal{U}], \tag{5a}$$

where equality holds if and only if $C_E^{\mathbf{R}}(U_i) = C_E^{\mathbf{R}^*}(U_i)$ for any $U_i \in \mathcal{U}$. Note that $C_E^{\mathbf{R}} = C_{E,CH}$. Thus, it follows from (A) that

$$\begin{aligned} EI_{C_E^{\mathbf{R}}}[C_E^{\mathbf{R}}, \mathcal{U}] &= \sum_{U_j \in \mathcal{U}} C_{E,CH}(U_j) \mathfrak{I}_{\mathcal{U}}(C_E^{\mathbf{R}}, V_j) \\ &= \sum_{U_j \in \mathcal{U}} C_E(U_j) \mathfrak{I}_{\mathcal{U}}(C_E^{\mathbf{R}}, V_j) = EI_{C_E}[C_E^{\mathbf{R}}, \mathcal{U}]. \end{aligned} \tag{5b}$$

Similarly, we also have that:

$$EI_{C_E^{\mathbf{R}}}[C_E^{\mathbf{R}^*}, \mathcal{U}] = EI_{C_E}[C_E^{\mathbf{R}^*}, \mathcal{U}]. \tag{5c}$$

Now, (5a), (5b), and (5c) jointly imply that

$$EI_{C_E}[C_E^{\mathbf{R}}, \mathcal{U}] \leq EI_{C_E}[C_E^{\mathbf{R}^*}, \mathcal{U}],$$

where equality holds if and only if $C_E^{\mathbf{R}}(U_i) = C_E^{\mathbf{R}^*}(U_i)$ for any $U_i \in \mathcal{U}$. In a similar way to the proof of Proposition 2, on the other hand, we can prove that the following two propositions are equivalent to each other: (i) $C_E^{\mathbf{R}}(U_i) = C_E^{\mathbf{R}^*}(U_i)$ for any $U_i \in \mathcal{U}$; (ii) $C_E^{\mathbf{R}} = C_E^{\mathbf{R}^*}$. Finally, we have that

$$EI_{C_E}[C_E^{\mathbf{R}}, \mathcal{U}] \leq EI_{C_E}[C_E^{\mathbf{R}^*}, \mathcal{U}],$$

where equality holds if and only if $C_E^{\mathbf{R}} = C_E^{\mathbf{R}^*}$. Done.

A proof of Proposition 6: We can prove Proposition 6 in the very similar way to the proof of Proposition 5. Let $\mathbb{R}_{\mathcal{E}}$ be a set of updating rules on $\mathcal{E}$ for a chance-free credence function C. All members of $\mathbb{R}_{\mathcal{E}}$ correct C to a chance-fed function such that $C_{E_i}^{\mathbf{R}}$ satisfies the Principal Principle and assigns E_i to 1, for any $\mathbf{R}_{\mathcal{E}} \in \mathbb{R}_{\mathcal{E}}$ and $E_i \in \mathcal{E}$. Let $\mathbf{R}_{\mathcal{E}}$ be the Conditionalizing-first Rule on $\mathcal{E}$ for C such that $C_{E_i}^{\mathbf{R}} = C_{E_i,CH}$ for any $E_i \in \mathcal{E}$. Lastly, let $\mathbf{R}_{\mathcal{E}}^*$ be an updating rule in $\mathbb{R}_{\mathcal{E}}$. Then, Immodesty implies that: for any $E_i \in \mathcal{E}$,

$$EI_{C_{E_i}^{\mathbf{R}}}[C_{E_i}^{\mathbf{R}}, \mathcal{U}] \leq EI_{C_{E_i}^{\mathbf{R}}}[C_{E_i}^{\mathbf{R}^*}, \mathcal{U}], \tag{6a}$$

where equality holds if and only if $C_{E_i}^{\mathbf{R}}(U_j) = C_{E_i}^{\mathbf{R}^*}(U_j)$ for any $U_j \in \mathcal{U}$, which is equivalent to $C_{E_i}^{\mathbf{R}} = C_{E_i}^{\mathbf{R}^*}$. Note that $C_{E_i}^{\mathbf{R}} = C_{E_i,CH}$ for any $E_i \in \mathcal{E}$. Similar to (5b) and

(5c), it follows from (A) that, for any $E_i \in \mathcal{E}$,

$$
\begin{aligned}
EI_{C^{\mathbf{R}}_{E_i}}[C^{\mathbf{R}}_{E_i}, \mathcal{U}] &= \sum_{U_j \in \mathcal{U}} C_{E_i,CH}(U_j)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j) \\
&= \sum_{U_j \in \mathcal{U}} C_{E_i}(U_j)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E}, V_j) \\
&= \sum_{U_j \in \mathcal{U}} C(U_j|E_i)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E}, V_j). \qquad (6b)
\end{aligned}
$$

By the same token, it also holds that: for any $E_i \in \mathcal{E}$,

$$
EI_{C^{\mathbf{R}}_{E_i}}[C^{\mathbf{R}^*}_{E_i}, \mathcal{U}] = \sum_{U_j \in \mathcal{U}} C(U_j|E_i)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}^*}_{E}, V_j). \qquad (6c)
$$

Then, (6b) and the relevant definitions imply that:

$$
\begin{aligned}
EI_C[\mathbf{R}_{\mathcal{E}}, \mathcal{U}] &= \sum_{E_i \in \mathcal{E}, U_j \in \mathcal{U}} C(E_iU_j)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j) \\
&= \sum_{E_i \in \mathcal{E}} \left(\sum_{U_j \in \mathcal{U}} C(E_iU_j)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j) \right) \\
&= \sum_{E_i \in \mathcal{E}} C(E_i) \left(\sum_{U_j \in \mathcal{U}} C(U_j|E_i)\mathfrak{I}_{\mathcal{U}}(C^{\mathbf{R}}_{E_i}, V_j) \right) \\
&= \sum_{E_i \in \mathcal{E}} C(E_i) EI_{C^{\mathbf{R}}_{E_i}}[C^{\mathbf{R}}_{E_i}, \mathcal{U}]. \qquad (6d)
\end{aligned}
$$

Similarly, it follows from (6c) and the relevant definition that:

$$
EI_C[\mathbf{R}^*_{\mathcal{E}}, \mathcal{U}] = \sum_{E_i \in \mathcal{E}} C(E_i) EI_{C^{\mathbf{R}}_{E_i}}[C^{\mathbf{R}^*}_{E_i}, \mathcal{U}]. \qquad (6e)
$$

Now, (6a), (6d), and (6e) imply that:

$$
EI_C[\mathbf{R}_{\mathcal{E}}, \mathcal{U}] \leq EI_C[\mathbf{R}^*_{\mathcal{E}}, \mathcal{U}],
$$

where equality holds if and only if $\mathbf{R}_{\mathcal{E}} = \mathbf{R}^*_{\mathcal{E}}$. Done.

Correcting chance-free credences in accordance with the New Principle

As mentioned in Section 2, the original version of the Principal Principle suffers from the Big Bad Bug—however, the New Principle does not. That said, my discussions go through even if we accept the New Principle rather than the original version. In particular, we can derive the new versions of ACR0 and ACR+, which require us to correct in accordance with the New Principle. Let C and C_E be chance-free credence functions, and C_{CH} and $C_{E,CH}$ be chance-fed credence functions that are updated from C and C_E, respectively, in accordance with the *New Principle*. Then, we can formulate such versions, as follows:

New ACR0: $C_{CH}(A) = \sum_{U_i \in \mathcal{U}} C(U_i) ch_i(A|U_i)$, for any $A \subseteq \mathcal{W}$.

New ACR+: $C_{E,CH}(A) = \sum_{U_i \in \mathcal{U}} C_E(U_i) ch_i(A|EU_i)$, for any $A \subseteq \mathcal{W}$.

Note that these versions follow, in a similar way to ACR0 and ACR+, from GAC and the New Principle. In particular, we can derive these versions by replacing the unconditional chance function $ch_i(\cdot)$ in ACR0 and ACR+ with the conditional chance function $ch_i(\cdot|U_i)$. In a similar way, moreover, the Propositions corresponding to Propositions 1-6 also follow from New ACR0 and New ACR+. Therefore, we can say that my discussions do not depend on which version of the Principal Principle we accept.

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