Rejecting Mathematical Realism while Accepting Interactive Realism

Abstract

Indispensablists contend that accepting scientific realism while rejecting mathematical realism involves a double standard. I refute this contention by developing an enhanced version of scientific realism, which I call interactive realism. It holds that interactively successful theories are typically approximately true, and that the interactive unobservable entities posited by them are likely to exist. It is immune to the pessimistic induction while mathematical realism is susceptible to it.

Keywords

Indispensability Argument, Interactive Realism, Mathematical Realism, Pessimistic Induction, Scientific Realism

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1. Introduction

Can we accept scientific realism while rejecting mathematical realism? Willard V. O. Quine (1980: 45) and Hilary Putnam (1979: 347) argue that we cannot on the grounds that doing so involves a double standard. In the same vein, Alan Baker (2005: 225) and Mark Colyvan (2006: 226–227) claim that we ought to accept mathematical realism, if we accept scientific realism. These eminent philosophers of mathematics maintain that mathematical entities, such as numbers and circles, are epistemically on a par with theoretical entities, such as electrons and X-rays, i.e., the evidence for the existence of the former is as powerful as that for the existence of the latter. Accordingly, it is intellectually dishonest to believe that theoretical entities exist, while not believing that mathematical entities exist.

This paper aims to refute the preceding charge of the double standard. I proceed as follows. In Section 2, I analyze scientific realism, mathematical realism, the Quine-Putnam indispensability argument, and Baker's enhanced indispensability argument. In Section 3, I assume for the sake of argument that the pessimistic induction is correct, and then show how it demolishes the two indispensability arguments for mathematical realism. In Section 4, I develop an enhanced version of scientific realism, which I call interactive realism, and show how it avoids the pessimistic induction. In Section 5, I reply to possible objections to interactive realism. In the end, it will become clear that the aforementioned charge of the double standard does not apply to interactive realists, who believe that some theoretical entities, such as X-rays and NDA, exist, but that mathematical entities do not.

2. Indispensability Arguments

Scientific realism is a view about successful scientific theories, such as evolutionary theory, the theory of relativity, and the germ theory of diseases. What does it mean for a theory to be successful? Laudan says that "a theory is 'successful' so long as it has worked well, i.e., so long as it has functioned in a variety of explanatory contexts, has led to confirmed predictions, and has been of broad explanatory scope" (Laudan, 1981: 23). Successful theories are the ones that help to explain puzzling phenomena, to predict future phenomena, and to manipulate things in the world. Such theories are the products of inference to the best explanation (IBE). IBE is a rule of inference that we use when we infer that a theory is true on the grounds that it best explains phenomena.

Putnam (1975: 73) defines scientific realism as the view that successful theories are (approximately) true, and that their key terms (typically) refer. He presents the no-miracles argument to justify scientific realism. It says that the success of a theory would be a miracle if the theory were false, and that the truth of a theory best explains its success. For example, evolutionary theory explains many biological phenomena. The high explanatory power of evolutionary theory would be a miracle, if it were false. It has such explanatory power because it is true. Note that the no-miracles argument is built upon the assumption that IBE is a reliable rule of inference. The best hypothesis is warranted, whether it is scientific or philosophical.

Mathematical realism (Balaguer, 2014) asserts that mathematical objects exist. Where do they exist? They exist not in the concrete world but in the abstract world. The concrete world is the world in which spatiotemporal objects, such as stones and electrons, exist. The concrete objects are causally efficacious. They can interact with one another. By contrast, the abstract world is the world in which aspatial and atemporal objects, such as mathematical objects and propositions, exist. The abstract objects are causally inert. So they can interact neither with one another nor with concrete objects. It does not even make sense to say, for example, that a particle accelerator propelled number one almost to the speed of light and smashed an electron and a triangle. This paper operates under Balaguer's definitions of mathematical realism and abstract entities because as Baker (2005: 223–224) notes, Balaguer's definitions are built into the Quine-Putnam indispensablist argument and Baker's enhanced indispensability argument for mathematical realism.

The Quine-Putnam indispensability argument is defended by Quine (1948, 1992), Putnam (1971), Michael Resnik (1997), and Colyvan (2001). It says that mathematical entities play indispensable roles in our best scientific theories, and that observations confirm not only concrete components but also mathematical components of our best scientific theories. It is intellectually dishonest to believe that theoretical entities exist, but mathematical entities do not. This view of confirmation is called confirmational holism. It has received lethal criticisms from Penelope Maddy (1992), Elliott Sober (1993), and Jacob Busch (2012).

Fortunately, Baker (2005, 2009) constructed a more sophisticated indispensability argument that does not rely on confirmational holism. He calls it an enhanced indispensability argument. It says that a mathematical hypothesis can best explain a concrete phenomenon, just as a concrete hypothesis can best explain a concrete phenomenon. He offers the famous cicada example to show that a mathematical hypothesis can best explain a concrete phenomenon. He argues that if we believe that a concrete hypothesis is true because it is successful, we should also believe that a mathematical hypothesis is true because it is successful.

Like Putnam's no-miracles argument, Baker's enhanced indispensability argument takes IBE to be a reliable rule of inference. The no-miracles argument presupposes that IBE is trustworthy, whether the best hypothesis is scientific or philosophical. Analogously, the enhanced indispensability argument presupposes that IBE is trustworthy, whether the best hypothesis is concrete or mathematical. It appears that scientific, philosophical, and mathematical uses of IBE are all on the same boat. It is natural that Baker uses scientific realism as a stepping stone for mathematical realism.

Baker's enhanced indispensability argument does not invoke confirmational holism to establish mathematical realism. It rather rests on the idea that a mathematical hypothesis can be *directly* confirmed by observations. By contrast, the Quine-Putnam indispensability argument relies on the idea that a mathematical hypothesis can be *indirectly* confirmed by observation, i.e., by being a part of a scientific theory which is confirmed *in toto* by observations. As a result, Maddy's, Sober's, and Busch's refutations of confirmational holism do not affect Baker's enhanced indispensability argument.

Baker's enhanced indispensability argument elicited critical responses from mathematical antirealists. For example, Sorin Bangu (2008) argues that the explanandum of Baker's cicada example is a mathematical statement, so the example is not a mathematical explanation of a concrete phenomenon but a mathematical explanation of a mathematical phenomenon. Bangu's criticism invited Baker's response (2009). This paper does not jump into the debate between Baker and Bangu. It simply grants for the sake of argument that Baker's example is a genuine mathematical explanation of a concrete phenomenon, and explores instead another objection to Baker's enhanced indispensability argument. My objection makes use of the pessimistic induction, to which I turn now.

3. The Pessimistic Induction

Different pessimistic inductions have been propounded by Henri Poincaré (1905/1952: 160), Larry Laudan (1977: 126), Putnam (1978: 25), P. Kyle Stanford (2006: 20), and K. Brad Wray (2013: 4327). This paper, however, need not be embroiled in their differences. In the pessimists' spirit, I define the pessimistic induction as the reasoning that since successful past theories, such as the phlogiston theory of combustion and the caloric theory of heat, turned out to be false, successful present theories, such as the oxygen theory of combustion and the kinetic theory of heat, will also turn out to be false. John Worrall (1982: 216), Philip Kitcher (1993: 149), Jarrett Leplin (1997: 136), P. D. Magnus and Craig Callender (2004: 322), and Wray (2013: 4321) regard the pessimistic induction as the most serious threat to scientific realism. This paper operates under these philosophers' view about the pessimistic induction.

In my view, the two indispensability arguments are on the same boat as scientific realism vis-à-vis the pessimistic induction. The pessimistic induction suggests that our best current theories will be thrown out, although they best explain phenomena, and that as a result, the mathematical entities embedded in them will also be discarded, just as past theoretical entities, such as phlogiston, caloric, and ether, have been discarded. So indispensablists have every reason to fight against the pessimistic induction, just as scientific realists do, but no indispensablist has yet confronted it in the literature. Indispensablists have only operated under the assumption that scientific realism is a defensible position.

Mathematical realists might reply that an empirical refutation of a scientific theory does not mean that all unobservable entities of the theory are discarded. Specifically, they might claim that mathematical entities of present theories will survive scientific revolutions, although theoretical entities of present theories will not, just as mathematical entities of past theories survived scientific revolutions, although theoretical entities of past theories did not. Thus, the pessimistic induction does not spell disaster for mathematical realism.

The preceding reply, however, is not available to indispensablists. Recall that indispensablists claim that the mathematical components of a scientific theory are confirmed by observations, just like the concrete components of the theory. If, however, the mathematical components are retained, indispensabilists owe us an account of why the mathematical components are retained, when concrete components are discarded. They cannot say that mathematical statements are not discarded because they are analytic, whereas concrete statements are discarded because they are synthetic, for such an explanation clashes with the indispensablist spirit that mathematical components are confirmed by observations.

Quine (1992: 15), Resnik (1997: 125–126), and Colyvan (2001: 126) argue that when our belief system conflicts with experience, we discard a concrete belief, not a mathematical one, because discarding a mathematical belief would require a major revision in our belief system, whereas discarding a concrete belief only requires a minor revision. In other words, we negate a concrete belief in preference to a mathematical belief because it is convenient to do so.

Park (2016a) calls these three philosophers' position mathematical convenientism. He argues that mathematical convenientism commits the consequential fallacy, and that it has three disastrous consequences for indispensablists. Let me introduce only one of the consequences in the interest of saving space. On the mathematical convenientist account, we choose one belief system over another not because it is epistemically better but because it is convenient to do so. Suppose that the former includes the belief that 1+1=2, whereas the latter includes the belief that 1+1=3. We believe that 1+1=2, as opposed to that 1+1=3, not because we have a good epistemic reason for believing so but because we have a good pragmatic reason for believing so. So we do not have a good epistemic reason for believing that a mathematical object has a property that we attribute to it. In sum, mathematical realism falls under the axe of mathematical convenientism.

Let me turn back to the pessimistic induction. It has evoked many responses from scientific realists. Of interest in this paper are those built upon the observation that present theories meet a higher epistemic standard than past theories. For example, Leplin (1997) and Juha Saatsi (2009: 358) argue that although past theories were successful, they did not make novel predictions, but some successful present theories make novel predictions, and hence the pessimistic induction does not refute the position that successful theories making novel predictions are true. Their position might be called novel realism. Novel realism faces the criticism from Timothy Lyons (2003: 898–899) and Stanford (2009: 384). They present counterexamples, past theories that made novel predictions, such as Fresnel's wave theory of light and Dalton's atomic theory. In the spirit of novel realism, however, I develop an enhanced version of scientific realism in the next section that overcomes the pessimistic induction and the objection to novel realism.

4. Interactive Realism

Putnam (1973: 210–211), Michael Friedman (1981: 7), and Stathis Psillos (1999: 205) recognize that when two scientific theories are conjoined, they might yield an observational consequence that neither yields individually. To take an example, the electromagnetic theory and the theory of DNA are confirmed independently of each other, and they jointly explain a third phenomenon, viz., skin cancer. Human skin develops cancer, if exposed to the sunshine for a long time. According to the two theories, skin cancer occurs because ultraviolet rays cause extensive damage to DNA (Trefil and Hazen, 1998: 156). The electromagnetic theory cannot yield the explanation alone. Nor can the theory of DNA. They should work together to generate the explanation. The electromagnetic theory previously explained phenomena other than skin cancer. So did the theory of DNA.

Park (2011a: 23–25) claims that the theory of DNA and the electromagnetic theory will not go the way of the past theories on Laudan's list (1981: 33) because the past theories explained only the phenomena in their domains and none of them explained a third phenomenon in conjunction with another successful theory. Unlike the past theories, the theory of DNA and the electromagnetic theory jointly explain a third phenomenon, after they were successful independently of each other. The present theories thus meet a higher epistemic standard than the past theories. Consequently, the former will not be abandoned, although the latter were.

This paper distinguishes between two kinds of successful theories: separately successful theories and interactively successful theories. A separately successful theory is one that is successful but does not explain a third phenomenon in conjunction with another successful theory. The past theories on Laudan's list are some examples of separately successful theories. An interactively successful theory is one that is successful on its own, and also explains a third phenomenon in conjunction with another successful theory. The electromagnetic theory and the theory of DNA are two examples of interactively successful theory.

This paper also distinguishes between two kinds of unobservable entities, viz., interactive unobservable entities and non-interactive unobservable entities. Two unobservable entities are interactive unobservable entities, if and only if the interaction between them explains a third phenomenon. DNA and ultraviolet rays are interactive unobservable entities, whereas phlogiston and caloric are non-interactive unobservable entities. No interaction of either phlogiston or caloric with any other unobservable entity was invoked to explain a third phenomenon in the history of science. To take another example, spacetime and a black hole are interactive unobservable entities. Material objects, including even light, are pulled into a black hole, if they are close enough to it. Such phenomena are third phenomena, which can be explained by the interaction between spacetime and the black hole. The general theory of relativity claims that spacetime interacts with material objects, including black holes. Material objects create the curvature of spacetime, and the curvature of spacetime in turn affects the motion of material objects. Spacetime is not a passive arena.

The interaction between two unobservable entities is to be understood in terms of causality between them. An unobservable entity undergoes a certain change after it interacts with another unobservable entity. For example, after the interaction between DNA and ultraviolet rays, DNA is deformed and ultraviolet rays change their directions, or are absorbed by the atoms constituting the DNA. These changes occur because of the interaction between them. They would not occur, if DNA and ultraviolet rays did not interact with each other. The same goes for spacetime and a black hole. Spacetime causally affects the behavior of the black hole, and the black hole causally affects spacetime. They undergo changes because they interact with each other. They would not undergo the changes, if they did not interact with each other.

Let me now unpack interactive realism. It holds that interactively successful theories are typically approximately true, and that the interactive unobservable entities posited by them are likely to exist.

Interactive realism is immune to the pessimistic induction. Pessimists need to present a new list of past theories to refute interactive realism. The new list should be composed of past theories which were interactively successful. Or pessimists should dig deeper into the past theories on Laudan's list and show that they were interactively successful. The prospect of accomplishing such tasks is dim, given that the interactions of ideas from different scientific domains are by and large a distinctive feature of present science. Different fields of science were isolated from one another, i.e., scientists belonging to different fields of science did not share their research results with one another, until around the end of the nineteenth century (Park, 2011b: 80–81). Over this period, the high walls separating different fields of science began to fall. For example, biologists began to communicate with chemists at the end of the nineteenth century, when Louis Pasteur persuaded biologists that a disease can be understood in chemical terms.[1]

Interactive realism is similar to novel realism in that both advance higher epistemic standards for trustworthy theories than scientific realism, as defined by Putnam (1975: 73), does. Scientific antirealists, who believe that successful theories are empirically adequate, might also have to advance a higher epistemic standard. After all, their belief falls prey to the pessimistic induction that successful past theories turned out to be empirically inadequate, so successful present theories will also turn out to be empirically inadequate (Park 2001: 78; Lange, 2002: 282; Lyons, 2003: 898). In response, scientific antirealists might have to revise their position to the effect that successful theories making novel predictions are empirically adequate. This new antirealist position might be called novel antirealism. Interactive realism, novel realism, and novel antirealism share the view that we can avoid the pessimistic induction by raising the epistemic standard for trustworthy theories.

What is the difference between the ontology of interactive realism, and that of scientific realism as defined by Putnam? The ontology of interactive realism is more austere than that of scientific realism. The ontology of interactive realism is composed of interactive unobservable entities, such as X-rays and DNA. It does not include non-interactive unobservable entities, such as phlogiston and caloric. By contrast, the ontology of scientific realism includes both interactive and non-interactive unobservable entities, although Putnam did not intend to include past theoretical entities in the ontology of scientific realism.

Should we accept mathematical realism, if we accept interactive realism? My answer is "Not necessarily." *Prima facie*, no double standard is involved in rejecting mathematical realism while accepting interactive realism. The epistemic status of interactive unobservable entities is higher than that of mathematical entities. Recall that an interactive unobservable entity explains not only phenomena in its domain but also a third phenomenon in conjunction with another interactive unobservable entity which explains phenomena in its domain. By contrast, a mathematical entity at best explains phenomena in its domain, as Baker's (2005, 2009) cicada example illustrates. So the evidence for the existence of interactive unobservable entities is more powerful than that for the existence of mathematical entities.

My objection to mathematical realism differs from Paul Benacerraf's classic objection (1973) to it. Benacerraf objects that we cannot have knowledge about mathematical objects because they are abstract entities and abstract entities cannot have any causal influence on epistemic agents who exist in the concrete world.[2] In contrast, interactive realism does not say anything about whether there should be a causal relationship between a target object and an epistemic agent. It rather says that the epistemic status of mathematical entities is lower than that of interactive unobservable entities. So it is wrong to reject interactive realism on the grounds that our knowledge about a target object does not require that there should be a causal relationship between the target object and an epistemic agent. In sum, interactive realists reject mathematical realism not on the grounds that there cannot be a causal relationship between a target object and an epistemic agent but on the grounds that the evidence for the existence of mathematical entities is not as powerful as that for the existence of interactive theoretical entities.

Interactive realists would accept the existence of mathematical entities as long as the epistemic status of mathematical entities is as high as that of interactive unobservable entities.

[1] See an introductory science text (Trefil and Hazen, 1998) for numerous examples illustrating how the ideas of different fields are connected with one another in contemporary science. Interactions are such pervasive phenomena in contemporary science that Park (2016b) proposes that to be scientific is to be interactive in his attempt to demarcate science and religion.

[2] See Colin Cheyne (1998) for a similar objection to mathematical realism.

Of course, mathematical realists cannot say that since the interaction between a mathematical entity and another interactive unobservable entity explains a third phenomenon, mathematical entities are epistemically on a par with interactive unobservable entities. This, however, does not mean that there is no other method for mathematical realists to show that the epistemic status of mathematical entities is as high as that of interactive entities. It is possible that there is such a method, and interactive realism would be compatible with the existence of such a method. In other words, interactive realism does not claim that the explanatory role of the interaction between unobservable entities is a necessary condition for the belief that the unobservable entities exist. The explanatory role of the interaction is merely a means to increase the probability that the unobservable entities exist. There might be an alternative means to increase this probability. But the burden falls on mathematical realists to specify this alternative method.

5. Objections and Replies

5.1. Ad Hoc Position

Indispensablists might now object that interactive realism is an *ad hoc* position. It is developed purely for the sake of diverting the charge that a double standard is involved in accepting scientific realism while rejecting mathematical realism. There is no independent justification for interactive realism. Why should we accept interactive realism except for the reason that it allows scientific realists to believe that some present theoretical entities exist without believing that mathematical entities exist?

My answer to this question is that interactive realism is developed not only to divert the charge of the double standard but also to get around the pessimistic induction. It is legitimate to develop an enhanced version of scientific realism in response to the pessimistic induction. If it were illegitimate to develop such a position, novel realism and novel antirealism would also be illegitimate positions. Indispensablists would disagree on the way interactive realists avoid the pessimistic induction. They are reminded, however, that they have the burden of finding an alternative way to get around the pessimistic induction because as we have seen before, it spells doom for mathematical realism as well as for scientific realism.

Suppose for the sake of argument that interactive realism is an *ad hoc* position. Even so, interactive realism still rebuts indispensablists' contention that it is intellectually dishonest to believe that some theoretical entities exist, while not believing that mathematical entities exist. After all, interactive realists are not holding to a double standard when they reject mathematical realism. Moreover, interactive realism is no worse off than mathematical realism with respect to independent justification. Mathematical realists do not have an independent justification for mathematical realism either. Their only justification for it is that a mathematical hypothesis explains a concrete phenomenon. Thus, mathematical realism should be rejected, if interactive realism is rejected.

Should we reject a philosophical position, if there is no independent justification for it? An answer to this question can be extracted from the debate between scientific realists and antirealists. Scientific antirealists take an epistemic risk when they infer that successful theories are empirically adequate. Scientific realists object that it is arbitrary to believe that successful theories are empirically adequate while not believing that it is true. Bas van Fraassen replies that "it is not an epistemological principle that one might as well hang for a sheep as for a lamb" (1980: 72). In other words, although scientific antirealists take some epistemic risk, it does not follow that they should risk as much as scientific realists do, i.e., that they should believe that successful theories are true. Van Fraassen's reply implies that no independent justification is required for the choice of scientific antirealism over scientific realism. The success of a scientific theory confers enough justification on the belief that the theory is empirically adequate, and no additional justification is required. Thus, scientific antirealism is a legitimate philosophical position, even if there is no independent justification for it. Interactive realists can adopt van Fraassen's insight. Interactive realism is a legitimate philosophical position, even if there is no independent justification for it.

5.2. Attacking the Pessimistic Induction

Indispensablists might now set out to attack the pessimistic induction with a view to reviving mathematical realism and Putnam's scientific realism. Putnam's scientific realism needs to be resurrected so that it can be used as a means to arrive at mathematical realism. With this goal in mind, indispensablists might tap into the philosophy of science literature and employ the heavy weapons that philosophers developed to combat the pessimistic induction. For example, they might appeal to Ludwig Fahrbach (2011a: 148), Park (2011b: 79), and Moti Mizrahi (2013: 3220) who argue that the pessimistic induction based upon Laudan's list (1981: 33) commits the fallacy of biased sample because the list includes distant past theories but not recent past theories and because the former are superior to the latter.[3]

To attack the pessimistic induction, however, is to play a new game, and this new game falls outside the scope of this paper. Suffice it to say that fruitful discussions will ensue over the issue of whether indispensablists can avoid the pessimistic induction. Indispensablists are reminded, however, that the destruction of the pessimistic induction only means the evaporation of the reason that I cited in Section 5.1. as an independent justification for interactive realism. It does not refute interactive realism. Interactive realism is refuted not when the pessimistic induction is destroyed but when counterexamples are provided, viz., interactive unobservable entities that turned out to be nonexistent in the history of science. Moreover, there might be another independent justification for interactive realism which I have not introduced in this paper. Even if there is no such justification for it, it still rebuts indispensablists' charge of the double standard, and it is still a legitimate position, just as novel realism and novel antirealism are legitimate positions despite the absence of any independent justifications for them.

Indispensabilists might object that interactive realism is not the only way for scientific realists to circumvent the pessimistic induction, i.e., there are other realist strategies to the pessimistic induction, and that the other realist strategies might be friendly to mathematical realism.

Indispensabilists are right on that account. I leave it, however, to them to specify such a realist strategy. The competition between their strategy and interactive realism will generate fruitful discussions about what epistemic attitude we should take towards mathematical and theoretical entities.

5.3. Other Positions

In this section, I respond to the anonymous referees' requests to distinguish interactive realism from other positions, such as entity realism, selectivism, and Busch's (2011) pessimism about mathematical theories.

How does interactive realism differ from entity realism? Entity realism denies that successful theories are (approximately) true, but asserts that "under conditions in which one can demonstrate impressive causal knowledge of a putative (unobservable) entity, such as knowledge that facilitates the manipulation of the entity and its use so as to intervene in other

[3] See Park (2018) for a list of problems with the pessimistic induction.

phenomena, one has good reason for realism regarding it" (Chakravartty, 2017).[4] This view is defended by Ian Hacking (1983), Nancy Cartwright (1983: Chapter 5), and Ronald Giere (1988: Chapter 5). Hacking's famous slogan is that if you can spray electrons, then they are real (Hacking, 1983: 24). Entity realism, thus defined, and interactive realism both affirm that we can have knowledge about theoretical entities, but deny that we can have knowledge about mathematical objects.

The two positions, however, have two important differences. First, entity realism denies, while interactive realism affirms, that interactively successful theories are (approximately) true. So interactive realism makes a greater commitment than entity realism to what interactively successful theories say about the world. Second, entity realism is open to a historical objection, viz., past realists were wrong to think that scientists manipulated past theoretical entities, such as phlogiston and caloric, so present realists are also wrong think that scientists manipulate present theoretical entities, such as electrons. By contrast, interactive realism is not open to such an objection, since no interactive theoretical entity has been discarded in the history of science, as we noted earlier.

How does interactive realism differ from selectivism? Selectivism is the view that only some components of a successful scientific theory are warranted. It is endorsed by central participants in the scientific realism debate, such as John Worrall (1989), Philip Kitcher (1993), Stathis Psillos (1999, 2009), Anjan Chakravartty (2008), Patrick Enfield (2008), Peter Godfrey-Smith (2008), David Harker (2008), Juha Saatsi (2009), Samuel Ruhmkorff (2011: 882), and Peter Vickers (2016). According to these prestigious philosophers, warranted components are confirmed by observations, whereas unwarranted components are not, and the former survive scientific revolutions, whereas the latter do not.[5] Selectivism and interactive realism are similar in that both are attempts to circumvent the pessimistic induction.

Selectivism and interactive realism differ in the following respect. Selectivism implies, as Park (2017a: 65, 2017b: 102, 2017c: 8–9) observes, that successful present theories will be replaced by alternatives, just as successful past theories have been replaced by alternatives. By contrast, interactive realism implies that interactively successful theories will not be superseded by alternatives. Interactive realists' justification for this prediction is that no interactively successful theory has ever been superseded by an alternative in the history of science. In short, selectivists claim that scientific revolutions will affect the target of their epistemic attitude, viz., successful present theories, whereas interactive realists assert that scientific revolutions will not affect the target of their epistemic attitude, viz., interactively successful present theories.

How does interactive realism relate to Busch's (2011) position? Busch observes that the theory of quaternions and the theory of infinitesimals contributed to the success of scientific theories, but that they have been unstable across theory change. They were "applied at one time in successful scientific theories and then effectively replaced, yet later applied in other successful theories" (Busch, 2011: 315). In other words, they have been adopted, dismissed, and then adopted again in the history of science. This observation leads him to the

[4] Chakravartty's conception of entity realism is different from that of Alan Musgrave (2017). Musgrave takes entity realism to be the view that we can know that a theoretical entity exists but cannot attribute any property to it, and then says, "To believing in an entity, while believing nothing else about that entity, is to believe nothing or next to nothing (2017: 88). If Musgrave is right, 'property-antirealism' or 'entitism' is a better nomenclature than 'entity realism' is. In any event, entity realism is much closer to scientific antirealism than to scientific realism.

[5] It is controversial whether the warranted constituents are rich enough for selectivists to attribute the realist predicate 'approximate truth' to successful past theories,' as Stanford (2015: 876) notes, so this paper uses 'selectivism' instead of 'selective realism.'

view that “a unique kind of pessimistic meta-induction can be developed regarding the application of mathematics in scientific theories” (Busch, 2011: 317). In other words, relying on the examples of quaternions and infinitesimals, he constructs a pessimistic induction against mathematical theories according to which, since mathematical theories as used in past scientific theories were untrustworthy, those in present scientific theories will also prove to be untrustworthy. Busch’s pessimistic induction and interactive realism are similar in that both deny that mathematical objects exist.

There is, however, an important difference between them. Busch’s pessimistic induction is not available to scientific realists, while interactive realism is. In Busch’s spirit, pessimists might construct a pessimistic induction against the success-generating parts of scientific theories. Many hypotheses have been accepted, rejected, and then accepted in the history of science. For example, Democritus’s atomism was rejected by Aristotle, and then accepted by modern scientists, such as Galilei and Dalton. Aristotle’s insight that a property of space can affect the motion of a material object was rejected by Newton, and then accepted by Einstein. Anaximander’s view that humans have evolved from fish was ignored for about two thousands of years, and then revived by Darwin. As Alexander Bird (2007: 95) observes, the hypothesis that the Sun is at the center of the universe was proposed by Aristarchus of Samos, then rejected by Aristotle and Ptolemaic scientists, and later embraced by Copernicus. Wegener’s idea that continents move around was dismissed for more than a half century, and then was enshrined in the theory of plate tectonics in the 1960s. If Busch’s two examples of quaternions and infinitesimals entitle him to construct a pessimistic induction against mathematical theories, then these five examples entitle us to construct a pessimistic induction against the success-generating parts of scientific theories. Thus, it is self-undermining for scientific realists to endorse Busch’s pessimistic induction against mathematical theories.

6. Conclusion

DNA, ultraviolet rays, spacetime, and black holes are interactive unobservable entities. In contrast, mathematical entities are not interactive unobservable entities. The former meet a higher epistemic standard than the latter. Therefore, interactive realists reject mathematical realism without holding to a double standard with respect to theoretical and mathematical entities.

Interactive realism is not a position that is developed solely for the sake of avoiding indispensablists’ charge of the double standard. There is an independent reason for accepting interactive realism, viz., it enables us to overcome the pessimistic induction. Moreover, it is not legitimate to reject interactive realism on the grounds that there is no independent justification for it any more than it is legitimate to reject novel realism and antirealism on the grounds that there is no independent justification for them.

This paper can be summed up in one sentence: Mathematical entities are not on a par with interactive theoretical entities.

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