Abstract

This paper investigates the use of theories of mechanics (classical and quantum) to provide answers to questions in the metaphysics of spatial location and persistence. Investigating spatial location, I find that in classical physics bodies pertend the region of space at which they are exactly located, while a quantum system spans a region at which it is exactly located. Following this analysis, I present a 'no-go' result which shows that quantum mechanics (conventionally interpreted) restricts the available options for locational persistence theories in an interesting way: it demonstrates that the spatiotemporal path of a persisting thing is discrete (or discontinuous) in time. This leads to unpalatable consequences for both perdurantists and endurantists. In particular, I argue that Butterfield's 'anti-pointilliste' perdurantism is ruled out, and show that endurantists relying on immanent causation run into trouble. I conclude by suggesting the revival of Whitehead's alternative mode of persistence called 'reiteration.'

How *Do* Things Persist? Location Relations in Physics and the Metaphysics of Persistence*

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In the introduction to her monograph *How Things Persist*, Hawley (2004) writes that: "Metaphysical reflection can help us discover what kinds of empirical facts are relevant to questions about persistence and identity, what kind of facts should be the focus of our investigation" (2). Recent metaphysical reflections about persistence have suggested that the kind of facts that are relevant to these questions have to do with *location*. That is, facts about how things are spatiotemporally located determine how those things persist.[1] I will call this the *locational approach* to persistence. Although some controversies remain, there appears to be a general agreement that the classic conflicting accounts of persistence—endurance and perdurance—can be precisely characterized in locational terms without prejudicing the outcome of the debate. To choose between theories of persistence on the locational approach requires, therefore, input from elsewhere.

The consideration of persistence in the context of relativistic spacetime has provided a major motivation for the development of locational accounts of persistence. Roughly, the idea is that a persisting object (like a table, banana, etc.) occupies its worldtube (a region of spacetime) and different theories of persistence correspond to different ways that the object can be said to be located at that four-dimensional region. Since endurance requires persisting objects to be three-dimensional (i.e., spatial), the failure of

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[1] Relevant authors include Balashov (2000, 2002, 2010), Donnelly (2011), Gibson and Pooley (2006), Gilmore (2006, 2008, 2013), Hudson (2005) and Parsons (2007, 2008).

relativistic spacetimes to provide a unique foliation into spacelike hyperplanes of simultaneity has been thought to provide an argument in favor of perdurance, according to which persisting objects are four-dimensional (Balashov 2000). Replying to such arguments on behalf of the endurantist has provided one impetus for the sophistication of locational accounts of persistence (Gibson and Pooley 2006; Gilmore 2006).

This paper is concerned with the import of physics for the locational approach, but not with relativistic spacetime. Instead, I investigate the metaphysics of location (and thereby persistence) by examining the location relations of theories of mechanics, both classical and quantum. The guiding idea is that to reach a verdict on how physical objects persist in actuality we should pay close attention to how the location relations invoked by metaphysicians compare with their cousins in physics, at least where spatial location is concerned. Both classical and quantum mechanics supply location relations that are well-suited to play this role. By examining the way that these location relations set up a map between physical objects (point particles, extended bodies, quantum systems) and their locations (spatial regions) at a time, I argue that classical bodies *pertend* the regions at which they are exactly located (Hudson 2005), whereas a quantum system *spans* a region at which it is exactly located (McDaniel 2007; Simons 2004).

By extending this account of location to spacetime (and so persistence) one might hope to supply necessary and sufficient conditions for a physical object to exactly occupy a spatiotemporal region, and so answer Gilmore's (2006) 'Location Question.' Since different theories of persistence require incompatible answers to this question, with a satisfying answer from physics in hand one could decide which of the rival theories of persistence holds in the actual world. Unfortunately, though, in considering classical mechanics I find myself in agreement with the verdict of Butterfield (2005) that both endurance and perdurance are equally well-supported. However, I demonstrate that quantum mechanics places a severe restriction on the sort of spatiotemporal regions at which a system can be located. This restriction serves at least to narrow down the available metaphysical options, if not in quite the way that Gilmore may have hoped.

The potential relevance of quantum theory for the persistence debate has been often acknowledged but seldom heeded. For example, in concluding an account of relativistic persistence, Gilmore (2008) wrote:

> [This] paper has had nothing to say about the impact of extant quantum theories themselves on issues about the metaphysics of persistence. For all I have said here, e.g., quantum field the-

> ory in its current form may decisively settle these issues in favor of one of the views already on the table or, alternatively, it may show that the current range of options is incomplete or somehow ill-formulated. (1247)

While in the introduction of his recent book on the locational approach to persistence, Balashov (2010) wrote:

> It is not immediately clear how to extrapolate the notions central to the debate about persistence to general relativistic spacetime [...] More importantly, it is even less clear how to think about persistence in the context of (non-relativistic) quantum mechanics. [...] This is not to suggest that questions of this sort should not be raised. (9)

In the following, I show that there is a clear way to think about persistence in (non-relativistic) quantum mechanics by adapting definitions from the metaphysics of location to the standard way of describing the location of a quantum system in terms of a *localization scheme*.[2] Following this path, I am led to conclude that the second alternative considered by Gilmore turns out to be the case: considerations from quantum theory suggest that the set of options considered thus far by the locational approach to persistence is ill-formulated or incomplete.

The paper is organized as follows. In Section 1 I show how both classical and quantum mechanics supply location relations suitable to play the role of exact location, as considered by Gilmore (2013); Parsons (2007). I then give precise characterizations of two opposing metaphysical views of spatial location: pertension and spanning. Considering classical and quantum mechanics in turn, I show how the location relation of classical mechanics satisfies the definition of pertension, whereas a quantum system must span a region at which it is exactly located, thus confirming the suspicions of Simons (2004).

Section 2 begins with a brief account of Gilmore's (2008) use of exact occupation to classify locational theories of persistence. I then show how both the endurantist and perdurantist can use the location relation of classical mechanics to define satisfactory accounts of persistence in those terms. I also suggest a precise characterization of Butterfield's (2006c) 'anti-pointilliste' perdurantism (which I dub *B*-perdurance) that allows only non-instantaneous, temporally extended temporal parts. This leads me to intro-

[2]This contrasts with an earlier approach to quantum theory where location relations were not taken into account (Pashby 2013).

duce two ways for a persisting object to have temporal extension: (i) having temporal *extent* (by existing at two or more instants), and (ii) being temporally *extended* (by, e.g., existing over a entire temporal interval).

With this distinction in hand, I demonstrate in Section 3 that the dynamical law of quantum mechanics (the Schrödinger equation) limits the spatiotemporal regions that a physically reasonable system can exactly occupy to be those with (at most) mere temporal extent. More generally, this limitation on the sort of regions at which a quantum system can be said to exactly occupy serves to rule out the possibility that such a system can be located at a continuous spatiotemporal path, which has been a common (although often implicit) assumption of the locational approach to persistence.

In Section 4 I draw out the implications of this result for the locational approach to persistence. While it seems that the perdurantist, who maintains that persisting things are four-dimensional and persist by having temporal parts, should be particularly inconvenienced by my result, definitions of endurance and perdurance have been largely insensitive to the distinction between temporal extension and extent. Not so Butterfield's *B*-perdurantist, however, who definitely runs into trouble here. I also contend that an endurantist who relies on a relation of *immanent causation* to secure the identity through time of persisting things in terms of their state at the regions they occupy faces an uncomfortable dilemma. This section concludes with some remarks on the interpretation of quantum mechanics.

Faced with the difficulties of existing accounts of persistence to accommodate the discontinuous spatiotemporal path of a quantum system, Section 5 briefly outlines an alternative account of persistence not currently under active consideration by metaphysicians. That is, if we are to account for the persistence of things that follow a discontinuous spatiotemporal path, I suggest that we should expand our options to include a distinctive mode of persistence which Whitehead (1925) called *reiteration*.

1 Location and Physics

Let us begin with the classical mechanics of point particles (or point masses), the persistence of which is analyzed at length by Butterfield (2005). If a particle p_i follows a path determined by the equations of motion then in general its path is (the image of) a continuous curve through spacetime. Such a curve can be thought of as a (partial) function $f_i : \mathbb{R} \to \mathcal{M}$ from a domain of times to spacetime. Let's assume that $\mathcal{M}$ is non-relativistic so that there is a privileged way of dividing up the set of all point-events $e \in \mathcal{M}$ into disjoint simultaneity slices.[3] Let's also assume that time is composed of instants, in which case an instant of time, I_t, is a three-dimensional (i.e., spatial) hyperplane that includes all and only the point-events in some simultaneity slice.

In that case, the spatiotemporal path of a point particle assigns to each time t that the particle exists a (spatial) point $f_i(t) \in I_t$.[4] It seems fair to say that the particle is located at each of these points, and also that it is located at their union: its (spatio-temporal) path. The persistence debate enters as follows. Endurantists will want to say that the very same object is located at each spatial point in this path. Perdurantists, on the other hand, maintain that an object persists by having distinct temporal parts located at different times; for them, the points $f_i(t)$ are the locations of distinct objects (albeit objects mereologically related as parts to a whole).

There is much more to say about the expression of theories of persistence in locational terms. Before doing so, however, I would like to draw attention to the distinctive ways that *spatial* location is treated in classical and quantum mechanics. Now, considering only point particle mechanics on the classical side, there is not much to say: at a time, a particle is located at a point. Nonetheless, there is also continuum mechanics, which describes the behavior of extended classical bodies, and quantum mechanics. In this section, I will argue that the metaphysicians' distinction between two ways for objects to be spatially located—pertension and spanning—corresponds precisely to the way that matter is described as being located in space by classical mechanics on one hand, and quantum mechanics on the other.

1.1 Metaphysics of Spatial Location

Confining our attention to a single instant of time, I_t, we can divide up our simultaneity slice into spatial regions in various ways. These regions will give the possible locations of a physical object. Since not every physical theory will agree about the kinds of regions at which an object can be located, we require a general notion of a collection of regions that will apply in every case. A suitable notion is provided by a *Boolean algebra* of

[3] Simultaneity is an equivalence relation on point-events; a simultaneity slice is an equivalence class of point-events under the relation of simultaneity.

[4] There is a further complication introduced if our spacetime respects Galilean relativity (i.e., treats all inertial frames as equivalent), in which case these paths must be given relative to an inertial frame. This will not be particularly relevant to our concerns here.

regions, $\mathcal{S} = \langle S, \vee, \wedge, \neg \rangle$, where S is a collection of subsets of I_t, and '$\vee$', '$\wedge$' and '$\neg$' are, respectively, operations of disjunction, conjunction and negation. A Boolean algebra supplies a (minimal) notion of parthood since it comes equipped with an associated partial order '$\leq$' where $r \leq s$ just in case $r = r \wedge s$. Thus $\langle S, \leq \rangle$ is a partially ordered set (or *poset*). When one region is part of another, $r \leq s$, we will say that r is a subregion of s; a subregion $r \leq s$ is *proper*, $r < s$, if $r \wedge s \neq s$.

As an example of a Boolean algebra, consider the Borel sets of the real line. To form the Borel sets of $\mathbb{R}$, we can begin with the collection of all open intervals $(a, b) \subseteq \mathbb{R}$ and close this collection under complementation, countable union and countable intersection. Taking this as our set of subsets S, with disjunction, conjunction and negation given by set-theoretic union, intersection and complementation (respectively), we obtain a Boolean algebra (more specifically, the Borel σ-algebra of $\mathbb{R}$). Repeating this procedure but beginning with open 'cubes' $(a, b) \times (c, d) \times (e, f) \subseteq \mathbb{R}^3$ leads us to a Boolean algebra of regions of space (viz., the Borel σ-algebra of $\mathbb{R}^3$). Unless otherwise stated, this will serve as our algebra $\mathcal{S}$.

One reason to use the Borel sets as spatial regions is that they each have a well-defined volume, i.e., they are (Lebesgue) measurable sets. This would not be the case if we were to take the power set of $\mathbb{R}^3$ as our Boolean algebra, for example. However, there are more measurable sets than there are Borel sets and so we may choose instead to use the (Lebesgue) measurable sets to define a Boolean algebra of regions, in the much same way. As we will see, in quantum mechanics the set of viable spatial regions is obtained by first identifying measurable sets that differ only by a set of measure zero,[5] which means that null sets (e.g., countable sets of points) are identified with the empty set. In continuum mechanics, the spatial regions are given in topological terms as regular open sets which also excludes (countable collections of) points from being viable regions. In both cases the underlying space is assumed to be a point-set and it is only the regions that ignore point-based distinctions; space itself is not thereby forced to be 'gunky'.[6]

[5]For example, the closed interval $[a, b]$ has measure $|b - a|$, and so too does the open interval (a, b) since it differs from $[a, b]$ by a set of measure zero, $\{a, b\}$. Considered with respect to Lebesgue measure on $\mathbb{R}$, a set of reals has measure zero *iff* it can be entirely covered by a countable set of open intervals $(a_i, b_i) \subset \mathbb{R}$ of arbitrarily small length, $|c_i - d_i| < \varepsilon$ for any $\varepsilon > 0$.

[6]Arntzenius and Hawthorne (2005, 446) regard this as an unstable position since they distinguish between the set of 'gunky' regions at which an object can be located and the 'genuine' regions of space, given presumably by the power set of $\mathbb{R}^3$. However, they pro-

Having arrived at a suitably general definition for the spatial regions, $\mathcal{S}$, we now require a precise notion of what it is for an object to be located at a region. Consider again a classical point particle p_i located at the spatial point $f_i(t) \in I_t$ (at time t). Although at each time this point particle is located at a single point, it also passes through any spatial region that intersects the point $f_i(t)$ non-emptily. The former notion is called *exact* location; the latter is *weak* location. Parsons (2007) contends that either can be taken as fundamental.

Following Parsons (2007), and taking exact location as primitive, an object x is weakly located at region $r \in \mathcal{S}$ just in case x is exactly located at a region $s \in \mathcal{S}$ that has a subregion $q \leq s$ such that $q \leq r$, i.e., when s and r overlap. Parsons further analyzes exact location by saying that an object x is exactly located at a region r *iff* x is *entirely* located at r and x is *pervasively* located at r. Essentially, Parsons says that an object x is entirely located at a region r when it is weakly located at r and x is not weakly located at the spatial complement $\neg r$, while an object x is pervasively located at (or *pervades*) a region r if x is weakly located at every subregion of r. Given our algebra of regions $\mathcal{S}$, and taking exact location as primitive, I adapt these definitions as follows:

Exact Location. If an object x is exactly located at region $r \in \mathcal{S}$ then:

- x is weakly located at every subregion $q \leq r$ and at every region $s \in \mathcal{S}$ such that $q \leq s$ for some $q \leq r$
- x is not weakly located at any region $q \leq \neg r$.

Thus if an object x is exactly located at r then it is entirely located at r and pervades r. In order to match the other direction of Parsons' biconditional I further maintain:

Entire Location. If an object x is entirely located at a bounded region[7] r then there exists some subregion $q \leq r$ at which x is exactly located.

Pervasion. If an object x pervades r then there exists no proper subregion $q < r$ at which x is exactly located.

vide no reason to introduce an *additional* algebra of spatial regions beyond the algebra of regions at which material objects can be located. For a detailed discussion of the virtues of an approach that takes space to be a point-set but regards regions as the bearers of properties, see Butterfield (2006a).

[7] I have restricted the definition to bounded regions alone since I want to avoid the possibility that an object could be exactly located at an unbounded region merely as a result of being entirely located there. For our purposes, a region is bounded *iff* it has finite Lebesgue measure.

Therefore, if an object is entirely located at a bounded region r that it pervades then it is exactly located at r.

We are now in a position to consider the ways that an object can be said to occupy a spatial region, which involves parthood relations for both objects and regions. For this purpose, let us define a partial order '$\preceq$' on the domain of objects such that when two objects x, y stand in the relation of parthood, $x \preceq y$, we will say that x is a part of y. This is a minimal notion of mereology, but it will suffice for now.

When it comes to the metaphysics of occupation there are essentially two options: either an object x occupies a spatial region r at which it is exactly located by having additional parts $w \preceq x$ exactly located at every subregion $q \leq r$, or it does not.[8] If not, then an object x can occupy a region r in virtue of being exactly located at r without having any parts exactly located at the (proper) subregions of r. Such objects are called *spanners* by Hudson (2005, 101), in that here x *spans* the spatial region r by pervading each subregion without having any proper parts (exactly) located at those subregions.[9]

The first option, however, corresponds to a mode of occupation that has become known as *pertension*.[10] Inspired by Parsons, Hudson (2005) says that an object x *pertends* a region r if it has a part $w \preceq x$ (exactly) located at each subregion $q \leq r$. In order to understand this relationship better, it will be useful to consider the relation of exact location as a set-theoretic map from a domain of material objects $\mathcal{O}$ to a codomain of spatial regions $\mathcal{S}$.

In particular, if $x \in \mathcal{O}$ is exactly located at r then the extension of the binary relation of exact location includes the ordered pair $\langle x, r \rangle \in \mathcal{O} \times \mathcal{S}$.[11] If every object $x \in \mathcal{O}$ is exactly located at (at most) a single region then this relation defines a (partial) function $f : \mathcal{O} \to \mathcal{S}$. Therefore, when an object $x \in \mathcal{O}$ pertends a region $r \in \mathcal{S}$ the relation of exact location defines an surjective function from a domain of parts of x to a codomain of subregions of r.[12] (That is to say, every subregion $r \leq s$ has some part $w \preceq x$ mapped to it by exact location.)

However, since $\mathcal{O}$ and $\mathcal{S}$ are posets, partially ordered by '$\preceq$' and '$\leq$'

[8] The first case is called the Geometric Correspondence Principle by Simons (2004); the second is his Extended Simples Principle.

[9] This terminology is due to McDaniel (2007). Spanners are called 'extended simples' by Simons (2004).

[10] As the name suggests, this is intended to be the spatial analogue of perdurance.

[11] I will continue to use $\mathcal{S}$ rather than the set S, an abuse of notation for which I apologise.

[12] Here I adopt standard set-theoretic language, according to which a function is surjective (or onto) when every element of the codomain is mapped to by an element of the domain.

(respectively), there is a further condition that we should impose: that the parthood relations of objects mirror the parthood relations of the regions at which they are exactly located.[13] In particular, we will maintain that:

Pertension An object x pertends a region r if exact location defines a function $f : \mathcal{O} \to \mathcal{S}$ from the parts of x to the subregions of r that is (i) surjective, and (ii) order-preserving, i.e., if $x \preceq y$ then $f(x) \leq f(y)$.

Just in case such a map exists, we will say that x pertends r. If x is exactly located at r without the existence of such a map then we will say that x spans r. I will now argue that the bodies of continuum mechanics pertend the regions at which they are exactly located, whereas quantum systems merely span the regions at which they are exactly located.

1.2 Location in Continuum Mechanics

In continuum mechanics, matter is described not as a collection of point masses but as a continuum of extended massive bodies located at overlapping three-dimensional spatial regions.[14] On the side of matter, a continuum body is described as a collection of massive bodies, partially ordered by parthood. On the side of space, we have a collection of regions of three-dimensional Euclidean space. Essentially, a continuum body occupies a region of space by virtue of a mapping between a collection of parts and a collection of spatial regions, called a *placement*. If we take the placement to give the extension of the exact location relation, then this fits very well with our definition of pertension.

In Truesdell's (1991) classic exposition, the collection of bodies form a Boolean algebra known as a *universe of bodies*, Ω_M, so (as explained above) a body can be considered as a poset ordered by parthood, i.e., as our domain of objects $\mathcal{O}$. A prototypical example of a universe of bodies is given by the σ-algebra of regular open sets of a topological space $\mathcal{T} = (X, \tau)$, where $M \in \tau$ is regular if M is equal to the interior of its closure (Truesdell 1991, 15–18).[15] A body, $\mathcal{B}$, is a regular open set of $\mathcal{T}$ and the parts of the body are the regular open sets of $\mathcal{T}$ that are subsets of $\mathcal{B}$. These sets also form a Boolean algebra comprising $\mathcal{B}$ and its parts, which we can take to

[13]This idea is called *mereological harmony* by Uzquiano (2011).

[14]For recent commendable attempts to introduce into contemporary metaphysics some considerations from continuum mechanics, see Smith (2007) and Wilson (2008).

[15]This definition ensures that points are not parts of bodies. For more on the relationship of topology, spatial regions and Boolean algebras see Aiello et al. (2007).

correspond to our collection of objects $\mathcal{O}$.[16]

The instantaneous location of a continuum body is given by a *placement*, $\chi(\cdot, t)$. At a time t, the placement $\chi(\mathcal{B}, t)$ maps the points of $\mathcal{B} \in \tau$ to points of three-dimensional Euclidean space, $\mathcal{E}$ (which comes equipped with the usual topology). The range of this mapping (a set of spatial points) is called the *shape*, S_t, of $\mathcal{B}$. The placement at a time can also be considered as the restriction of a homeomorphism from $\mathcal{T}$ onto $\mathcal{E}$, in which case it maps regular open sets of $\mathcal{T}$ to regular open sets of $\mathcal{E}$. Thus each part of $\mathcal{B}$ (itself a regular open subset of $\mathcal{B}$) is mapped to a regular open subset of S_t (its shape at time t). The Boolean algebra of subregions of S_t is called the *universe of shapes*, Ω_S, which corresponds to (a subalgebra of) our algebra of regions $\mathcal{S}$.[17]

To see that this amounts to an order-preserving surjection from the parts of $\mathcal{B}$ to the subregions of S_t, note that $\chi(\mathcal{B}, t)$ defines a Boolean homomorphism between Ω_M and Ω_S, which preserves Boolean operations and is thus order-preserving. Moreover, since the placement $\chi(\cdot, t)$ has an inverse homeomorphism (defined similarly), a body cannot be located at a shape S_t without the existence of a complete collection of parts located at the subregions of S_t (Truesdell 1991, 87). Restricting the domain of $\chi(\mathcal{B}, t)$ to just the parts of $\mathcal{B}$ a placement thus defines an order-preserving surjective function between the universe of bodies and the universe of shapes.

In that case, so long as we take this mapping to correspond to the extension of the two-place relation 'exact location,' we have satisfied the definition of pertension given above. Furthermore, a collection of point particles trivially satisfies the definition of pertension[18] and so it seems that classical mechanics returns the definitive verdict that matter pertends the spatial regions at which it is exactly located.[19]

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[16]In addition, we have a *mass function* that assigns to each body (or part) a non-negative mass. This function defines a measure over we integrate in order to find the total mass that lies within some spatial region at which the body is (weakly) located. Because only regions with nonzero volume have nonzero mass, there are no massive points in continuum mechanics. See (Truesdell 1991, 92).

[17]For technical reasons, Truesdell further restricts the subregions to the Boolean algebra of 'fit' regular open subsets of S_t Truesdell (1991, 87–91). For our purposes this distinction is unimportant, but see Smith (2007, §4) for a discussion of these reasons.

[18]The placement here (at a time) is a function from the set of particles to the set of their point-locations. It is obviously surjective, and the partial order (given by set-theoretic inclusion in each case) will be preserved.

[19]Unfortunately we cannot claim full generality since classical field theories are excluded from this analysis. Since field theories do not lend themselves to the definition of location relations in the sense discussed here it seems likely that a new approach would be needed.

1.3 Location in Quantum Mechanics

In a certain sense, the location relation of quantum mechanics resembles that which we found in continuum mechanics: each quantum system comes equipped with a *localization scheme* that associates regions of space with states of the system. However, quantum mechanics is at heart a probabilistic theory and so it will require some work to provide a suitable definition of exact location. I will argue that for any possible location of the system there is a condition on the state of the system given in terms of the localization scheme such that, when satisfied, the system can be said to be located (or *localized*) within that region.[20] This condition is given through the association of projection operators with regions of space defined by the localization scheme, $\Delta \mapsto \hat{P}_\Delta$, which associates to a region of space $\Delta \in \mathcal{S}$ a projection operator $\hat{P}_\Delta$.[21]

As von Neumann originally demonstrated, any observable of the system corresponding to a self-adjoint operator (like position, $\hat{Q}$) defines a family of associated projection operators (through its spectral measure).[22] In the standard von Neumann–Mackey interpretation, these projection operators are thought of as representing propositions about the system that may be true or false (or neither). Mathematically, to each such projection operator there corresponds a subspace of $\mathcal{H}$, the Hilbert space of the system; operationally, for each projection there is an experimental question that can (by performing an appropriate measurement) be 'asked of' the system to find out whether or not the proposition is true. The probability of finding on measurement of $\hat{P}_\Delta$ that the corresponding proposition is true is given by the Born Rule.[23]

Each projection operator $\hat{P}_\Delta$ corresponds to the experimental question: Is the system (now) located within region Δ? On this interpretation, if this experimental question receives an affirmative answer then the proposition that the system is located within Δ is true (at the time the question was asked). Unfortunately, this interpretative posit is controversial since it seems to require the 'collapse of the wavefunction' on measurement of $\hat{P}_\Delta$. However, it is widely accepted that if the probability of finding an af-

[20]Wightman (1962) demonstrates in general terms that a unique localization scheme can be found for both relativistic and non-relativistic quantum systems.

[21]More generally, this is a Projection-Valued Measure (PVM) which maps an element Δ of the σ-algebra of Borel sets of $\mathbb{R}$ to the projection operator $\hat{P}_\Delta$. See Teschl (2009) for technical details.

[22]Again, see Teschl (2009) for details.

[23]For more details see Mackey (1963); Von Neumann (1955). For a philosophically oriented introduction to this formalism see Hughes (1989).

firmative answer to the question *before* measurement of $\hat{P}_\Delta$ is unity then the corresponding proposition is true, and no measurement is required to confirm this.[24]

This idea suggests a relatively uncontroversial condition for a quantum system to be located at a given spatial region. For a quantum system with (pure) state $\psi \in \mathcal{H}$, the probability of finding the system to be located in a region Δ is given by the Born Rule:

$$\Pr(\Delta) = \langle \psi | \hat{P}_\Delta \psi \rangle, \tag{1}$$

where $\langle \cdot | \cdot \rangle$ is the inner product for $\mathcal{H}$, and $\psi \in \mathcal{H}$ is a unit vector. If this probability is one, $\Pr(\Delta) = 1$, then the proposition 'the system is located within Δ' is true. The antecedent holds just in case $\hat{P}_\Delta \psi = \psi$, which is to say: a system in state ψ is located at Δ when ψ is an eigenstate of $\hat{P}_\Delta$ (with eigenvalue 1). As I now explain, this condition also ensures that the system is (weakly) located nowhere else and thus serves as a suitable definition of entire location for a quantum system.

Recall that, as I defined it, if an object is entirely located at a region r then it is not weakly located at any disjoint region $q \leq \neg r$. Now, if a quantum system is located at region Δ with probability zero then it is surely not (weakly) located there. And if a system is not (weakly) located in the complement of Δ (that is, the region $\neg\Delta$), then it is not (weakly) located in any region disjoint from Δ. Therefore, so long as the condition $\hat{P}_\Delta \psi = \psi$ suffices to ensure that $\Pr(\neg\Delta) = 0$ then it provides a suitable notion of entire location. It is easily seen that this is the case.

In quantum mechanics, if we are concerned just with questions about location at a time (and not with incompatible questions about momentum, say) then the probabilities for location in disjoint regions simply add:

$$\Pr(\Delta \text{ or } \Sigma) = \langle \psi | \hat{P}_\Delta \psi \rangle + \langle \psi | \hat{P}_\Sigma \psi \rangle = \langle \psi | \hat{P}_{\Delta \vee \Sigma} \psi \rangle. \tag{2}$$

Furthermore, since ψ is a unit vector the probability of finding the system *some*where in space is always one,

$$\Pr(S) = \langle \psi | \hat{P}_S \psi \rangle = \langle \psi | \psi \rangle = 1, \tag{3}$$

[24]Here I mean to endorse the forward implication of the *eigenstate-eigenvalue link*, a biconditional which says that a system is in a eigenstate of a self-adjoint operator (such as a projection) *iff* it is found to have the corresponding (eigen)value upon measurement. The forward implication follows simply from the Born Rule. The reverse implication requires taking a stand on the notorious measurement problem, which I hope to avoid.

which is to say that the projection operator associated with all of space is the identity, $\hat{P}_S = \mathbb{I}$. Therefore, if $\hat{P}_\Delta \psi = \psi$ then the probability of being located at a region disjoint from Δ is zero:

$$\begin{aligned} \Pr(S) = 1 &= \langle \psi | \hat{P}_{\neg\Delta} \psi \rangle + \langle \psi | \hat{P}_\Delta \psi \rangle \\ 1 &= \langle \psi | \hat{P}_{\neg\Delta} \psi \rangle + 1 \\ \implies \Pr(\neg\Delta) &= \langle \psi | \hat{P}_{\neg\Delta} \psi \rangle = 0. \end{aligned}$$

Thus $\hat{P}_\Delta \psi = \psi$ provides a sufficient condition for a system in state ψ to be entirely located in region Δ.

Unfortunately, though, essentially the same argument shows that the condition $\hat{P}_\Delta \psi = \psi$ can hold without the system pervading the region Δ, so that $\hat{P}_\Delta \psi = \psi$ is necessary but not sufficient for exact location at Δ.[25] To reach a sufficient condition, we need to specify explicitly that the system is both entirely and pervasively located at the region in question. We can do this by ensuring that a system is exactly located at a region only if it is not entirely located at any proper subregion, as follows:

Quantum Location. For a quantum system O in state ψ and any bounded[26] spatial region Δ assigned projection $\hat{P}_\Delta$ by the localization scheme of the system, O is exactly located at Δ *iff* (i) $\hat{P}_\Delta \psi = \psi$; (ii) there is no proper subregion $\Sigma < \Delta$ such that $\hat{P}_\Sigma \psi = \psi$.

When both conditions hold we have $0 < \langle \psi | \hat{P}_\Sigma \psi \rangle < 1$ for all $\Sigma < \Delta$ and so Δ is the minimal region at which the system is entirely located. If we accept that a quantum system is weakly located at Σ when $0 < \Pr(\Sigma) < 1$ then it therefore pervades Δ.

Since this definition of quantum location makes no reference to parts of quantum systems, I claim that quantum systems are spanners. That is, a

[25]To see that $\hat{P}_\Delta \psi = \psi$ is not sufficient, consider a proper subregion region $\Sigma < \Delta$ at which we assume the system is entirely located, $\hat{P}_\Sigma \psi = \psi$. In that case, the system is still entirely located at Δ since $\hat{P}_\Delta \hat{P}_\Sigma \psi = \hat{P}_\Delta \psi = \psi$. However, there exists a subregion of Δ, namely $\Delta \setminus \Sigma$ (the complement of Σ in Δ), at which the system is not (weakly) located since

$$\begin{aligned} \Pr(\Delta) = 1 &= \langle \psi | \hat{P}_\Sigma \psi \rangle + \langle \psi | \hat{P}_{\Delta \setminus \Sigma} \psi \rangle \\ 1 &= 1 + \langle \psi | \hat{P}_{\Delta \setminus \Sigma} \psi \rangle \\ \implies \Pr(\Delta \setminus \Sigma) &= \langle \psi | \hat{P}_{\Delta \setminus \Sigma} \psi \rangle = 0. \end{aligned}$$

[26]Again, I restrict the definition of exact location to bounded regions since I want to avoid allowing that a system could be exactly located an unbounded spatial region simply as a result of being entirely located there.

quantum system can be exactly located at a region Δ without having any (proper) parts exactly located at subregions of Δ. This deserves further comment. I have previously suggested that the localization scheme provides a way to assign a 'spatial part' ψ_Δ to a region Δ for a system in state ψ by operating on the state with the corresponding projection operator, i.e., $\psi_\Delta = \hat{P}_\Delta \psi$ (Pashby 2013). This defines a quantum state ψ_Δ that, were it the *entire* quantum state of the system, would be entirely located at Δ. Furthermore, the projection operators $\hat{P}_\Delta$ inherit a (minimal) parthood relation from the subspaces of $\mathcal{H}$ to which they are associated that mirrors the mereological relations of the regions.

Therefore, a quantum system *could* pertend a region Δ by having a complete collection of spatial parts ψ_Σ exactly located at the proper subregions $\Sigma < \Delta$. However, unlike the placement of classical mechanics, the location relation of quantum mechanics makes no reference to any such parts, and a quantum system exactly located at Δ is (by definition) not exactly located at any proper subregion $\Sigma < \Delta$. Furthermore, compound systems in quantum mechanics are formed by taking the direct product of their associated Hilbert spaces. As a result, Caulton (2015) goes so far as to argue that the axioms of mereology fail for quantum systems of electrons. While Bigaj (2016) replies that spatial properties can be used to individuate quantum particles, saving mereology for collections of electrons (but not their parts), on neither account could quantum systems be said to pertend their exact locations.

This, then, confirms Simons' suspicion that quantum mechanics provides motivation for taking spanners (his 'extended simples') seriously:

> We have to think of P's occupation of R [a region that it spans] as a holistic fact: all of P occupies all of R, and that's all there is to it. This does not rule out variations in what we might call the intensity of P's occupation. Some parts of R might be more intensely occupied by than others. This would accord quite well, for example, with the idea that [...] a fundamental [i.e., quantum] particle's relationship to its voluminous locus is not uniform, in that there is a probability density defined over the volume, the probability being that of the particle's being located at a position upon collapse. The probability density, which is derived from the wave equation for the particle [i.e. its quantum state ψ], need not be uniform over the locus, but its integral over the whole locus is equal to one. (Simons 2004, 377)

This idea is broadly right: when a quantum system is exactly located at a region Δ it need not have a uniform probability density across the region. To see how this works, we first write the projection $\hat{P}_\Delta$ in Dirac 'bra-ket' notation:

$$\hat{P}_\Delta = \int_\Delta |q\rangle\langle q|\, dq.$$

This expresses $\hat{P}_\Delta$ in terms of the so-called improper position eigenstates, $|q\rangle$, associated with spatial points $q \in \mathbb{R}^3$.[27] The probability of a system in state ψ being entirely located at a region Δ is then given as an integral over a (properly normalized) probability density $f(q)$:

$$\Pr(\Delta) = \langle\psi|\hat{P}_\Delta\psi\rangle = \int_\Delta \langle\psi|q\rangle\langle q|\psi\rangle\, dq = \int_\Delta f(q)\, dq.$$

Evidently this integral may be equal to one without $f(q)$ (the 'intensity of occupation') being uniform over Δ, and in general it will not be uniform. Note, however, that $f(q)$ is a probability density, and as such does not give the probability for being located at a point q. To turn a probability density into a probability requires integration, but the probability of being located at a point given by this integral is always zero since a point has zero volume. Thus Simons' idea that a probability density somehow gives the probability for a quantum particle's "being located at a position upon collapse" is misleading unless position is taken to mean *region*.

At a first pass, the sort of regions at which a quantum system can be located are those in the domain of the localization scheme, $\Delta \mapsto \hat{P}_\Delta$. Let's take these regions to be the Borel sets of $\mathbb{R}^3$ (discussed above) which form a Boolean algebra $\mathcal{S}$. However, some Borel sets have (Lebesgue) measure zero, in which case the probability of a quantum system being located at such a region is zero and thus a quantum system could never be (entirely) located there. Reflecting this fact, such sets are mapped by the localization scheme to the null projection (whose range is the null vector). Therefore, a quantum system cannot be entirely located at a point, nor a line (nor any m-dimensional subspace of $\mathbb{R}^3$, where $m < 3$). Furthermore, since probabilities are formed by integration, there can be no observable difference between predictions concerning two spatial subsets differing by (at most) a set of measure zero. This justifies the contention that the more appropriate Boolean algebra of regions for quantum mechanics, $\mathcal{S}_Q$, is to be formed by first identifying such regions. In that case, it can be shown that entire

[27] It is important to note that, as $|q\rangle$ is improper, $|q\rangle \notin \mathcal{H}$ and so the state of the system cannot be $|q\rangle$, which is another way to say that a quantum system cannot be located at a spatial point. See below for further details.

location at a bounded region Δ implies exact location at some subregion $\Sigma \leq \Delta$.

Quantum Entire Location. If a quantum system O is entirely located at a bounded region $\Delta \in \mathcal{S}_Q$ then there exists some subregion $\Sigma \leq \Delta$ at which O is exactly located.

Proof. Assume for reductio that there is no subregion $\Sigma \leq \Delta$ at which the system is exactly located. By assumption, the system is entirely located at Δ. But it is not exactly located at Δ, in which case there exists some proper subregion $\Sigma < \Delta$ such that $\hat{P}_\Sigma \psi = \psi$. However, since the system is not exactly located at Σ either, there must exist a proper subregion of Σ at which the system is entirely located. Evidently this process can be continued indefinitely, each time removing a region of finite measure from consideration. Therefore, in the limit we reach a region of Lebesgue measure zero at which the system is entirely located. But the probability of being found at any such region is zero. Contradiction. □

Given the interest of metaphysicians of persistence in relativity theory, it is worth commenting here on results concerning relativistic localization. First note that, restricting one's attention to a particular inertial frame, Wightman (1962) shows that a relativistic quantum system does have a localization scheme of exactly the type discussed here, and so everything I have said about quantum location so far carries over unchanged. However, a relativistic localization scheme displays an alarming dependence on the choice of inertial frame. In particular, a state ψ_Δ that is exactly located at region Δ according to the localization scheme of some inertial frame will not be exactly located at *any* spatial region according to the localization scheme of a distinct inertial frame.

Intuitively, the reason for this is not hard to discern: a three-dimensional region Δ of a relativistic spacetime that is spatial according to inertial frame A will be spatio*temporal* according to inertial frame B (related by a Lorentz boost). Therefore, a region $\Delta \in \mathcal{S}_A$ at which a system is located according to the localization scheme A is not a *spatial* region at all according to localization scheme B, i.e., $\Delta \notin \mathcal{S}_B$. Still, one might expect that every nonempty intersection of that region Δ with some region that is spatial according to the inertial frame B would be a region at which the system can be located. But here we encounter a problem of dimensionality: the intersection of two three-dimensional spatiotemporal regions is two-dimensional. A spatial region, yes, but not the sort of region at which a system can be located—the probability of being located at a region of zero volume is always zero.[28]

Some have taken results along these lines to suggest that any relativistic quantum theory must be a field theory, not a particle theory (Halvorson and Clifton 2002; Malament 1995). My inclination is to suppose that the answer to these worries about relativistic localization is to define a Lorentz invariant notion of localization at a four-dimensional spatiotemporal region. However, there are severe difficulties that arise when attempting to define a notion of localization within a region that is extended in time as well as space. In Section 3.2, I present a result which can be interpreted as ruling out this possibility. Since this result also has implications for the locational approach to the metaphysics of persistence, I turn to this topic first.

2 Location and Persistence

The basic question at the heart of the debate over persistence is this: how can the self-same thing be said to persist through time while its properties change?[29] The debate has often been framed as a dispute between the endurantist, who believes that properties predicated of a persisting object at different times are possessed by the same three-dimensional thing, and the perdurantist, who believes that these properties belong to distinct temporal parts of a four-dimensional thing. In recent years, motivated by attempts to frame the positions in relativistic terms,[30] or perhaps seeking ways to avoid complaints that these two views amount to essentially the same thing,[31] many participants have sought to couch the debate in terms of location in spacetime.

In the above discussion of the metaphysics of spatial location we considered the sort of location relation that might hold between an physical

[28]The discussion here is somewhat superficial: it is not just that the region Δ fails to be a spatial volume but also that the probability density $f_A(q,t)$ associated with a given spatiotemporal point by localization scheme A will not be equal to the probability density $f_B(q,t)$ assigned by localization scheme B. This is because the improper eigenstate $|q\rangle$ of the Heisenberg picture (Newton-Wigner) position operator of a given inertial frame is not an improper eigenstate of the position operator for another inertial frame. For further discussion, see Butterfield and Fleming (1999); Debs and Redhead (2003).

[29]This is sometimes called 'The Problem of Change.' A major desideratum for a theory of persistence is that it should make sense of the claim that a persisting object changes its properties over time without logical inconsistency.

[30]See (e.g.) Balashov (2000, 2010), Gibson and Pooley (2006), Gilmore (2006, 2008).

[31]For such complaints see (e.g.) McCall and Lowe (2003), Miller (2005), Hirsch (2005, 2009).

object (as described by physics) and a spatial region. However, physical theories do more than specify location at a time: in the classical mechanics of point particles, for example, a solution of the equations of motion for the system is a spatiotemporal curve which describes the location of the system at all times. In the metaphysics literature, (the image of) this curve is known as its spatiotemporal *path*. Locational theories of persistence specify what it is for a material object to endure, perdure, etc. in terms of the way that a persisting object is located at its path.

So let us begin with the idea that any material object O has a spatiotemporal path P.[32] Informally, the spatiotemporal path of an object O includes all the spatiotemporal regions at which O can be found, and there is no region included in P at which O can't be found. Gilmore presents this idea as follows:

> a region R is an object's path (my term) just in case R has a subregion in common with all and only those regions at which the object is weakly located. This captures the thought that a thing's path is the region that exactly corresponds to the thing's complete history or career. (Gilmore 2008, 1228)

Note that this definition is satisfied if one simply forms the union of all the (time-indexed) regions at which an object is exactly located. In that case, it seems tempting to say that a persisting object is exactly located at its four-dimensional path. Endurantists, however, will want to resist this idea since they maintain that an enduring object can only be exactly located at a three-dimensional region.[33] But what should the endurantist say instead? This is a controversial matter.

In discussing spatial location I made use of Parsons' (2007) relation of exact location, which holds between an object and a region only when every other region is completely free of that object. Since an object can be exactly located at (at most) one spatial region at a time, this seems to make *multiple* location impossible. We saw this in the way that the relation of exact location defines a map between objects and spatial regions that is a (partial) function: every object is mapped to (at most) one region. In contrast, an object multiply-located at two distinct regions would be mapped to two regions, defining instead a so-called 'multifunction' or set-valued map.

[32] This assumes that being located in spacetime provides a necessary condition for an object to be a material object.

[33] Donnelly (2011) sees this as the major issue separating endurantists and perdurantists.

Although some have considered the metaphysical possibility of multiple location in space, it does not seem to be required by physics: the location relations we have encountered thus far provide a notion of exact location that defines a function between objects and spatial regions.[34] When it comes to considering endurance in space*time*, however, the idea of multiple location seems to naturally arise. For example, the endurantist would seem to want to say that the *same* point particle p_i occupies different spatial points $f_i(t)$ at distinct times. On the face of it, then, endurantism appears to require multiple location: at distinct times, the same enduring object can be located at distinct spatial regions.

This is the approach taken by Gilmore, who uses a location relation of *exact occupation* to classify possible theories of persistence in locational terms. Gilmore (2008) introduces this relation as follows:

> Intuitively, a thing exactly occupies a region just in case the thing has exactly the same shape and size as the region and stands in all the same spatiotemporal relations to things as does the region. Moreover, on the intended interpretation of the predicate, there should be nothing contradictory or obviously false about the claim that a single thing exactly occupies each of two or more regions without exactly occupying their sum or any of their proper subregions. (Gilmore 2008, 1228–9)

Thus Gilmore's endurantist can maintain that an enduring object is located at its path in virtue of exactly occupying each (time-indexed) region at which it is exactly located while failing to exactly occupy their union, its path.

We can use the (extension of the) relation of exact occupation to define a map between a domain of objects and a codomain of spatiotemporal regions. Since the same enduring object exactly occupies distinct regions at distinct times, the same enduring object $O \in \mathcal{O}$ is mapped to multiple time-indexed spatial regions. In that case, the map defined by the exact occupation relation isn't a true function but is instead a set-valued map (or 'multifunction'). A perduring object, on the other hand, can have temporal parts that each exactly occupy (at most) one spatial region—the spatial region at which the object is exactly located at that time—, in which case

[34] To be fair, one could object that the avoidance of multiple location within quantum mechanics is essentially Pickwickian. That is, given the definition above it is perfectly possible for a quantum system to be exactly located at a spatial region composed of two disjoint regions. Nonetheless, by definition the system is not exactly located at either of the disjoint regions separately; thus it is not multiply located.

exact occupation defines a genuine (partial) function from objects to time-indexed spatial regions.

This contrast is neat, and appears to do justice to Lewis's claim that:

> Perdurance corresponds to the way a road persists through space; part of it is here and part of it is there, and no part of it is wholly present at two different places. Endurance corresponds to the way a universal, if there are such things, would be wholly present wherever and whenever it is instantiated. (Lewis 1986, 202)

One would think that a universal can be multiply-located, if anything can. And it seems like a reasonable interpretation of endurance to suggest that *exactly* the same enduring thing exactly occupies distinct spatial regions at different times. This seems to fit naturally with our usual way of talking about things: 'the tennis ball was on my side of the net and now it is on yours,' we might say, which seems to impute to the same object (the tennis ball) two distinct locations.[35]

However, the idea that endurance involves (temporal) multiple location has its detractors, most notably Parsons (2007, 2008). To define endurance, Parsons (2007) makes use of the relation 'wholly located' instead. This relation mixes mereology and location: something is wholly located at a region if none of its parts are missing from that region (ibid., 212). Thus endurantism becomes the thesis that an object persists by having all of its parts located at two disjoint times (ibid., 218). Parsons' perdurantist asserts that an object persists by having disjoint temporal parts (weakly) located at disjoint spatiotemporal regions.

I am not convinced that Parsons' alternative definitions of endurance and perdurance present any advantages. One of Parsons' complaints about Gilmore's use of exact occupation is that:

> Because Gilmore's "exactly occupies" is not explained in terms of location and mereology—and indeed cannot be—it would make the question of "endurantism" vs "perdurantism" independent of the question of whether objects are divisible into arbitrary temporal parts. (Parsons 2007, 220)

But that is precisely the advantage of Gilmore's approach: by cleanly separating out the commitments of the perdurantist and endurantist into mere-

[35] Hirsch (2009) argues that the better fit of endurantism with our verbal habits counts in its favor.

ological and locational components he makes space for hybrid views of the sort considered by Parsons (2000) himself.

In particular, Gilmore provides a classifying scheme for theories of persistence in along two independent directions: mereological endurance/perdurance, and locational endurance/perdurance. A mereological perdurantist (endurantist) believes that persisting objects have (no) temporal parts; a locational perdurantist (endurantist) believes that an object can exactly occupy (more than) one spacetime region. Therefore, locational endurantism requires multiple location. In 'Must a Four-dimensionalist believe in Temporal Parts?,' Parsons (2000) explores the idea that an object could persist by being temporal extended but without having distinct temporal parts at distinct times. Expressed in terms of exact occupation, such a theory is mereologically endurantist but locationally perdurantist (and thus four-dimensionalist).

So let us embrace exact occupation, and with it Gilmore's framework for classifying and articulating different ways to persist. The question then arises: how might we choose between them? At the root of the difficulties in attempting to arbitrate between popular accounts of persistence is what Gilmore (2006) calls the Location Question:

> What is the general principle that determines, for any given material object, *which subregions of that object's path are exactly occupied by the object*?
>
> A correct answer to this question will tell us how to fill in the following blank in such a way as to make the resulting principle both true and informative:
>
> For any material object *O* and spacetime region *R*, *O* exactly occupies *R* if and only if [blank]. (Gilmore 2006, 208, original emphasis)

Because theories of persistence such as endurance and perdurance appear to require incompatible answers to this last question, if we could fill in the blank in an independent, non-question-begging manner we could potentially find out whether material things endure, perdure, or neither. In Section 1, I showed how both quantum and classical mechanics provide satisfying answers to the question: how does a physical object occupy a spatial region? The question now arises of whether the same location relations can brought to bear on the question of persistence.

2.1 Continuum Mechanics and Persistence

Unfortunately, when it comes to arbitrating between endurantism and perdurantism through an examination of physics, we face a problem of underdetermination. Whereas both classical and quantum mechanics seemed to provide quite specific answers about how the physical objects of the theory come to be located at spatial regions, they seem to be open to interpretation concerning their spatiotemporal locations. I will illustrate this first by consideration of continuum mechanics.

In continuum mechanics, the successive spatial locations of an object are given by the placement $\chi(\cdot, t)$. At each time, this defines a function whose domain is a set of parts (the 'universe of bodies') and whose codomain is a set of regions (the 'universe of shapes'). As we saw in Section 1.2, at each time a continuum body has a well-defined location: its shape S_t, which is the spatial region at which the body is exactly located at time t. Thus a placement also defines a function of time, $t \mapsto S_t$, whose domain is a set of instants and whose codomain is a set of spatiotemporal regions exactly occupied by the body.

But does the *same* object exactly occupy each of those regions? It is hard to say: the function is silent about the existence of temporal parts since its domain is not objects but times. Now, the endurantist may think that this helps her case: if temporal parts are not given explicitly by physics then perhaps they are not necessary for the purposes of physics. She may also point to the fact that the placement only explicitly gives a coordination of parts with spatial regions, which suggests an endurantist reading. Furthermore, she may take advice like the following to count in her favor:

> The student must recall always that *a body in assuming various shapes never loses its identity and the properties assigned to it*. (Truesdell 1991, 87, original emphasis)

Nonetheless, the perdurantist may plausibly interpret the data in sympathetic terms by taking the successive instantaneous shapes, S_t, of a body O to be nothing but the locations of its instantaneous temporal parts. In particular, she may say that the placement, considered as a function of time, does not map the *same* body to many instantaneous locations but rather maps each distinct (instantaneous) *temporal* part of the body $O_t \preceq O$ to a single spatial region, and the same for its parts. By maintaining that each temporal part of the body O_t exactly occupies one spatial region, S_t, she avoids multiple location.

As we might expect, this perdurantist view is a close temporal analog of pertension (considered in Section 1.1): since these temporal parts are singly-located, this perdurantist takes the binary relation 'exact occupation' to define a function from a domain of (temporal) parts of $O \in \mathcal{O}$ to a codomain of subregions of its path, $R \leq P$. If we want this perdurantist location function to be surjective then we need only restrict the codomain to the maximal spatial slices of P, which in this case are the shapes S_t. This seems like a fair reading of what the perdurantist might want to say in this case; indeed, these parts (if they exist) would precisely satisfy Sider's (1997) definition of an instantaneous temporal part.

However, it is notable that the only perdurantist to take seriously the import of continuum mechanics for the metaphysics of persistence argued *against* the existence of instantaneous temporal parts. Instead, Butterfield (2006a,b,c) advocates an 'anti-pointilliste' perdurantism which maintains that persisting objects have all and only temporally extended temporal parts. His reasons for adopting this position are complex, but Butterfield (2006c) recommends this version of perdurantism as a handy way of avoiding the negative conclusion of the rotating disks argument.

There is, however, an additional reason to prefer this version of perdurantism in the context of continuum mechanics: it respects a symmetry between time and space (often taken to be a significant motivation for perdurantism). In particular, a continuum body and its parts are only ever exactly located at regions of nonzero spatial volume. In that case, Butterfield's anti-pointilliste perdurantist maintains that a continuum body occupies regions of spacetime in the same way that an endurantist would maintain that a continuum body pertends the region of space at which it is exactly located.

In more detail, the definition of pertension satisfied by continuum mechanics involved an order-preserving surjection from a partially ordered set of (spatial) parts to a partially ordered set of spatial regions. I will define Butterfield's version of perdurance, which I call B-perdurance, similarly but with a codomain of spatiotemporal regions. For this purpose, consider the set of spatiotemporal regions of $\mathcal{M}$ formed by taking only sets of point-events $e \in \mathcal{M}$ of definite spatiotemporal volume, i.e., only subsets Lebesgue measurable in $\mathbb{R}^4$. As in the case of quantum mechanics, we will identify sets that differ only by a set of measure zero and use the set-theoretic operations to define a Boolean algebra of spatiotemporal regions $\mathcal{U} = \langle U, \vee, \wedge, \neg \rangle$.[36]

[36]This algebra of regions is given uniquely if $\mathcal{M}$ is Newtonian, otherwise it is to be given

Defining the path of a continuum body to be the union of its shapes,

$$P := \bigcup_{t\in\mathbb{R}} S_t,$$

we find that P corresponds to some region $R \in \mathcal{U}$ with nonzero spatiotemporal volume. Temporally extended subregions of P (with maximal spatial extent) can be formed by restricting the union to an open interval of times:

$$S_{(t_1,t_2)} := \bigcup_{t\in(t_1,t_2)} S_t.$$

The set of all such subregions $S_{(t_1,t_2)} \leq P$ corresponds to a partially ordered subset of U (here, ordered by subset inclusion of the temporal intervals).

In our terms, Butterfield's anti-pointilliste B-perdurantist maintains that every subregion of $P \in \mathcal{U}$ with nonzero volume is exactly occupied by a (temporal) part of $O \in \mathcal{O}$. We can express this more precisely:

B-perdurance. An object O *B-perdures* if exact occupation defines a (parital) function $f : \mathcal{O} \to \mathcal{U}$ from its temporal parts $O_t \preceq O$ to the temporally extended maximal subregions $S_{(t_1,t_2)} \leq P$ of its spatiotemporal path P that is (i) surjective; (ii) order-preserving.

This is a precise analogue of the definition of pertension of Section 1.1 and, again, it applies equally well to a collection of classical point particles.

Note, however, that for B-perdurance to hold in continuum mechanics we must divorce exact occupation from the relation of exact location that holds at an instant. That is, while the instantaneous location relation considered in Section 1.1 as exact location would suffice to define exact occupation for a pointilliste perdurantist (who believes in instantaneous temporal parts) it cannot do so for the B-perdurantist (who does not). Instead, the B-perdurantist maintains that temporal parts of a continuum body exactly occupy only the temporally *extended* regions formed by taking the union of spatial regions S_t at which the continuum body is exactly located (according to its placement). Before continuing to address persistence in quantum mechanics, which will pose some problems for this view, I first explore this idea of temporal extension.

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relative to an inertial frame. Note that a perdurantist who believes in instantaneous temporal parts would not want to identify measure zero regions in this way.

2.2 Temporal Extension vs. Extent

Lewis's (1986) definition of a persisting thing as something that is wholly present at two distinct times suggests that an object can persist merely in virtue of exactly occupying two spatial regions belonging to disjoint instants of time. As I will put it, such an object exactly occupies a region with mere temporal *extent*. According to classical mechanics, however, the path of a persisting object is given by a continuous function of time whose range is a temporally *extended* region of spacetime. Existing at two times is necessary for something to persist, but is it sufficient?

To illustrate what is at stake here, it will be helpful to consider the analogy with spatial extension. Take the example of a system of eight classical point particles, located at opposite vertices of a cube of volume l^3. What is the dimensionality of the spatial region in which the point particles are located? There are two plausible answers: (a) they are located in a cubical three-dimensional region of volume l^3, or (b) they are located at eight spatial points, collectively having zero volume, i.e. a zero-dimensional region. The answer (a) corresponds the locations of the particles having three-dimensional spatial *extent*, and (b) corresponds to the fact that the region at which the particles are (exactly) located is not spatially *extended*.[37]

Here is a recent explication of 'being spatiotemporally extended' that conflates these two quite distinct notions:

> Say that an entity is *extended* just in case it is a spatiotemporal entity and does not have the shape and size of a point. In this sense of 'extended', a solid cube would count as extended, but so would the mereological sum of two point-particles that are one foot apart. Although such a sum would have zero length, it would be a *scattered* object and so would not have the shape of a point. (Gilmore 2013, §5, original emphasis)

I want to suggest that the two senses of extension mentioned here deserve to be sharply differentiated. There is an important sense in which a solid cube counts as spatially extended whereas a collection of point particles has only spatial extent: the region at which a cube is (exactly) located has nonzero volume, i.e., finite Lebesgue measure on $\mathbb{R}^3$, whereas any finite collection of point particles is located at a region with zero volume, i.e., Lebesgue measure zero.

[37] Note that if those same eight particles were arranged in a plane the first answer would, presumably, now be that they are located in a two-dimensional region. Thus the answer (a) is contingent on the locations of the particles, whereas (b) is not.

When it comes to temporal extension, we must restrict the spatiotemporal path of an object to its temporal dimension alone. Let us say, then, that the path of a persisting object is temporally extended *iff* its temporal extension is a set of instants with finite Lebesgue measure; and if its temporal extension has Lebesgue measure zero then it has mere temporal extent.[38] Thus the temporal extension of a finite collection of instantaneous temporal parts has measure zero (and so has mere temporal extent) whereas a B-perdurantist's temporal part $S_{(t_1,t_2)}$ has measure $|t_2 - t_1|$, and is, therefore, temporally extended.

It is standardly assumed that if something occupies a region of space for a single instant then it does not persist. Yet Lewis would allow that an object located at two instants persists in virtue of having mere temporal extent. But can an object that flickers into existence at only two instants in the history of the universe really be considered to be a persisting thing? Although most of the metaphysics literature would seem to view having mere temporal extent as sufficient to persist, I suspect that this is essentially because the question has not been seriously considered by its authors.[39] This question will become pressing in light of the result I present in the next section, which implies that the path of a quantum mechanical system has (at most) mere temporal extent.

3 Quantum Mechanics and Persistence

As with classical continuum mechanics, the formalism of quantum mechanics seems to provide no definitive answer as to whether objects are multiply- or singly-located at their path. In that case, there would be little evidence here to help us choose between Gilmore's locational perdurantism/endurantism. However, I have already argued that there is a case to be made against mereological perdurantism in quantum mechanics: on one interpretation of what it would be for a quantum system to have temporal parts, a result called Pauli's Theorem rules them out (Pashby 2013). This suggests that the perdurantist may want to pursue a mereologically endurantist but locationally perdurantist theory of persistence instead, according to which a quantum system persists by exactly occupying its spa-

[38] Considered with respect to Lebesgue measure on $\mathbb{R}$, a set of instants has measure zero *iff* it can be entirely covered by a countable set of open intervals $(c_i, d_i) \subset \mathbb{R}$ of arbitrarily small length, $|d_i - c_i| < \varepsilon$ for any $\varepsilon > 0$.

[39] Aside from Butterfield, however, we find Hudson (2001) arguing that a person cannot be identified with an instantaneous object since a finite time span is required to secure their identity through continuity of thought. For him, a person must be temporally extended.

tiotemporal path. I have called this theory of persistence *temporal holism* (Pashby 2013), and here I further note that this position can be motivated by applying the perdurantist analogy between location in space and location in time to quantum mechanics.

That is, in continuum mechanics, the spatial regions at which bodies are located are not pointlike and so, reasoning by analogy, we found an argument in favor of Butterfield's 'anti-pointilliste' B-perdurantist, who rejects instantaneous temporal parts of perduring objects. If, however, we run the same argument from analogy using the mode of spatial location of quantum mechanics—spanning—the perdurantist should conclude that a quantum object perdures without having temporal parts. That is, the argument from analogy with spatial location in quantum mechanics leads to mereological endurantism.

However, this argument against mereological perdurantism in quantum mechanics is not watertight since the perdurantist may choose to ignore the argument from analogy and adopt a distinct definition of temporal parts (that is not threatened by Pauli's Theorem). In particular, one could simply import the definition of temporal parts from continuum mechanics by using (quantum) exact location to define the path of a quantum system in the same way, namely by forming the union of its instantaneous spatial locations. Then one simply posits the existence of an appropriate set of temporal parts that are related to subregions of the path by a relation of exact occupation. (I explain exactly how this works in the next subsection.)

Unfortunately for the B-perdurantist, however, the path of a quantum system cannot be temporally extended, as I now argue. This implies that this version of mereological perdurantism is ruled out by quantum mechanics. It also shows that the scope of both locational perdurantism/endurantism, too, is limited to (what I will term) a discontinuous spatiotemporal path, so presenting problems for locational persistence theories in general.

3.1 Quantum Location in SpaceTime

To define a quantum system's spatiotemporal path, we need to consider time development of the system. Given the Hamiltonian operator for the system, $\hat{H}$, the time-dependent Schrödinger equation determines how the state of the quantum system changes with time. It is first-order in time, so that if we know the state of a system at some instant of time we can apply the Schrödinger equation to find the state at *every* instant of time. Thus for a system in state $\psi \in \mathcal{H}$ at time $t = 0$ the Schrödinger equation returns a family of states $\psi_t \in \mathcal{H}$, one for every $t \in \mathbb{R}$. To find out what is true of the system at time t, we apply the Born Rule to the state ψ_t. This way of thinking about time evolution is known as the *Schrödinger picture*.[40]

This family of states ψ_t is related to the Hamiltonian through a corresponding family of unitary operators[41] parameterized by $t \in \mathbb{R}$ that is *generated* by $\hat{H}$, $\hat{U}_t = e^{-i\hat{H}t}$. This unitary family implements the group of *time shifts* on $\mathcal{H}$, which is to say that the operator $\hat{U}_t$ corresponds to a time shift of t seconds from $t = 0$. So, if the state of a system at time $t = 0$ is ψ, then the state at time t is obtained by shifting ψ forward by t seconds, that is $\psi_t = \hat{U}_t\psi$.[42]

In this picture, the localization scheme is the same at each time, and the condition we used to define (entire) location in space will suffice as a definition of location in spacetime (at an instant). Applying the Born Rule to $\hat{P}_\Delta$, the projection associated with region Δ, we obtain:

$$\begin{aligned} &Pr(\Delta \text{ at } t) = 1 = \langle \psi_t | \hat{P}_\Delta \psi_t \rangle \\ &\implies \hat{P}_\Delta \psi_t = \psi_t. \end{aligned}$$

That is to say that a system is entirely located at Δ at time t *iff* $\hat{P}_\Delta \psi_t = \psi_t$, which is essentially just the definition from Section 1.3, and the definition of exact location at a time follows in the same way by requiring that the system is not entirely located at any subregion of Δ.

In analogy with continuum mechanics, when a quantum system is exactly located at a (bounded) region Δ_t at time t we will say it has the shape Δ_t. As with continuum mechanics, we define the path of a quantum system as the union of its instantaneous shapes:

$$P := \bigcup_{t \in \mathbb{R}} \Delta_t.$$

The crucial difference, however, is that a quantum system need not have a shape *at all* at a given time. That is, although the normalization condition $\langle \psi_t | \psi_t \rangle = 1$ ensures at every time the system is entirely located at the universal region, there may be no bounded region at which it is entirely located, in which case it is not exactly located anywhere.

[40] There is also the *Heisenberg picture*, in which the state is constant in time and the localization scheme evolves with time. These two pictures of time development are distinct, but they return the same probabilities for instantaneous measurements of an observable, such as $\hat{P}_\Delta$. We will only use the Schrödinger picture.

[41] A unitary operator $\hat{U}$ leaves the inner product unchanged, i.e. $\langle \hat{U}\psi | \hat{U}\phi \rangle = \langle \psi | \phi \rangle$ for all $\psi, \phi \in \mathcal{H}$, and has the property that $\hat{U}^\dagger \hat{U} = \hat{U}\hat{U}^\dagger = \mathbb{I}$.

[42] This statement is, in fact, completely equivalent to the Schrödinger equation $-i\hbar(d/dt)\psi_t = \hat{H}\psi_t$, which is just the infinitesimal form of this relationship.

The crucial distinction between locational endurantism/perdurantism hinges on whether or every shape is exactly occupied by the same object, or by a temporal part. This question cannot be decided by restricting the path alone, which is merely the image of the (multi)function defined by the relation of exact occupation. However, the allowed path of a quantum mechanical system can constrain the viable options for the locational persistence theorist. In particular, note that B-perdurance requires that the range of the function defined by exact occupation includes at least one temporally extended region (else it is empty). If the path of a quantum system cannot include any such region then a quantum system cannot B-perdure. I now argue that this is indeed the case: quantum mechanics places just such a restriction on the path of any physically reasonable quantum system.

3.2 No Temporally Extended Paths

To say precisely what 'physically reasonable' means here, we need to introduce the dual role of the Hamiltonian as *energy* observable. That is, applying the Hamiltonian operator $\hat{H}$ to the state of the system ψ returns the *expectation value* of the energy $\langle H \rangle = \langle \psi | \hat{H} \psi \rangle$, which is (something like) the most likely value found if $\hat{H}$ were to be measured. We can also think of the allowed energies of the system as corresponding to projections, in exactly the same way as the possible locations of the system correspond to projections in a localization scheme.[43] So, associated with the proposition 'the energy of the system lies within range $\alpha = (e_1, e_2)$', there is a projection $\hat{P}_\alpha$ determined by $\hat{H}$ such that if $\hat{P}_\alpha \psi = \psi$ then the energy of a system in state ψ lies within α. Similarly, if $\hat{P}_\alpha \psi = 0$ then the value of the energy of the system certainly does not lie within α.

In quantum theory, it has become standard to assume that *any* physically possible system will have a Hamiltonian with the characteristic that there is a value of energy below which the energy of the system cannot drop. This is known as the *spectrum condition*, which thus says that the spectrum of the Hamiltonian of every physical system (which specifies the possible values of energy of the system) is bounded from below.[44] In terms of the projections $\hat{P}_\alpha$, the spectrum condition says that there is some value of energy e_0 such that the projection corresponding to the energy interval

[43] This is through the unique association of a self-adjoint operator $\hat{H}$ and a so-called Projection Valued Measure mentioned in the Appendix.

[44] This condition lies at the root of related results of Halvorson and Clifton (2002) concerning relativistic localization.

$(-\infty, e_0]$ is zero in every state, i.e. $\hat{P}_{(-\infty,e_0]}\psi = 0$ for *all* $\psi \in \mathcal{H}$.[45]

Therefore, so long as the spectrum condition holds it is impossible for a system to have energy less than this value e_0 (where the particular value depends on the form of the Hamiltonian). Note how strange a system would be that didn't meet this condition. Such a system would have no lower bound on its energy. That means that one could extract today enough energy from the system to power the city of Pittsburgh for 24 hours, and then tomorrow extract enough energy to power Pittsburgh for 24 hours, and so on ad infinitum. We have certainly never come across such a system in nature. It seems safe to assume that such systems are not physically possible.

Surprisingly, this rather mild restriction on the Hamiltonian operator suffices to put a quite severe restriction on the way that a quantum system can be located at a spatial region over time. This is captured by the following proposition (see the Appendix for a proof).

Proposition 1. *Let $\psi \in \mathcal{H}$, a unit vector, be the state of a quantum system. Let $\hat{P}_\Delta$ be the projection associated with a bounded spatial region Δ by the localization scheme. Let $\psi_t = \hat{U}_t\psi$, with $\hat{U}_t = e^{-i\hat{H}t}$ the unitary group of time translations generated by $\hat{H}$, a self-adjoint operator with spectrum bounded from below.*

Then either

1. *the system is always entirely located at Δ, that is, $\hat{P}_\Delta\psi_t = \psi_t$ for* all $t \in \mathbb{R}$, or
2. *the set of times at which $\hat{P}_\Delta\psi_t = \psi_t$ has Lebesgue measure zero, in which case there is no time interval $(t_1, t_2) \subseteq \mathbb{R}$ such that the system is entirely located at Δ at every time $t \in (t_1, t_2)$.*

Furthermore, the Hamiltonian $\hat{H}$ includes an infinite potential on the boundary of Δ if and only if *every state entirely located at Δ at a time is confined to Δ for all t by $\hat{U}_t$, that is, when $\hat{P}_\Delta\hat{U}_t\hat{P}_\Delta\psi = \hat{U}_t\hat{P}_\Delta\psi$ for all $\psi \in \mathcal{H}$, for all $t \in \mathbb{R}$.*

This proposition says that, given a spatial region Δ and a Schrödinger picture family of states ψ_t whose evolution is determined by a semi-bounded Hamiltonian $\hat{H}$, there are two exclusive possibilities covered by Case 1 and Case 2.[46] It also says something about the conditions under which Case 1 might apply. In particular, if the region in question is surrounded by an

[45] This is to say that the range of the projection is empty. Since this is a spectral projection of $\hat{H}$, this condition is equivalent to the spectrum of $\hat{H}$ being bounded from below.

[46] This result is closely related to the phenomenon of 'instantaneous wavepacket spreading' (Hegerfeldt 1998). See the Appendix for the key result of Hegerfeldt's paper, from which my proposition follows.

infinite potential then a system entirely located in that region at *any* time is entirely located there at *every* time (and thus Case 1 applies).

My interest here is in using these results to place restrictions on the allowed spatiotemporal path of a quantum system. In particular, now argue that we can rule out the idea that a quantum system persists by being located at a temporally *extended* path. The argument proceeds by showing that the only way a quantum system can have such a path is if it forcibly confined to some particular region over all time (by an infinite potential). Therefore, if the path of a quantum system is to be temporally extended then some very special situation must obtain. The unsatisfiable nature of this necessary condition demonstrates that no actual quantum system persists by having a temporally extended path.

Above, the path of a quantum system was defined as the spatiotemporal region that results from taking the union of all the instantaneous regions Δ_t at which it is exactly located. At each time t at which the system has an exact location, we can find a bounded spatial region $\Gamma \in \mathcal{S}_Q$ that includes Δ_t (considered as a spatial region). In particular, we will choose Γ so that at *any* time that the system that has an exact location Δ_t, the system is entirely located at Γ. Thus whenever the system is entirely located at Δ_t it is also entirely located at Γ, i.e., $\Delta_t \leq \Gamma$ for all t.

If the path of the system is temporally extended then there exists a set of times T with finite Lebesgue measure such that the system is entirely located at Γ at each time $t \in T$. For a system in state ψ we apply Proposition 1 to Γ and conclude that the system is entirely located at Γ at *every* time t, i.e., $\hat{P}_\Gamma \psi_t = \psi_t$ for all $t \in \mathbb{R}$. This demonstrates that if a quantum system has a temporally extended path then there exists some bounded region of space at which it is always entirely located. Furthermore, I claim that there is some subregion $\Sigma \leq \Gamma$ at which the system is almost always *exactly* located, where 'almost always' means for every $t \in \mathbb{R} \setminus \{t_k\}$ with $\{t_k\}$ a set of measure zero.

Confinement. If there is a bounded region $\Delta \in \mathcal{S}_Q$ at which a quantum system O is always entirely located (i.e., to which Case 2 applies) then, for almost all $t \in \mathbb{R}$, the system is exactly located at some particular subregion $\Sigma \leq \Delta$.

Proof. Consider the set of times T at which the system is entirely located at a proper subregion $\Sigma < \Delta$. By Quantum Entire Location, for each $\tau \in T$ there exists a region $\Sigma_\tau < \Delta$ at which is the system is exactly located. If T has measure zero then the system is almost always exactly located at Δ

and we are done. Otherwise, apply Proposition 1 to each Σ_τ: either the system is entirely located at Σ_τ for all t (Case 1) or the system is located at Σ_τ for a set of times $T' \subseteq T$ with measure zero (Case 2). If Case 2 applies to *all* Σ_τ then T has measure zero and we are done. Otherwise, there exists some nonempty set of regions $F = \{\Sigma_\tau\}_{\tau \in T}$ at which the system is always entirely located. Assume for reductio that F has no minimal element when ordered by Lebesgue measure. But then the system is never entirely located anywhere, and so a minimal element of F, Σ_{min}, must exist. Therefore, Case 2 of Proposition 1 applies to any $\Omega < \Sigma_{min}$ and the system is almost always exactly located at $\Sigma_{min} \leq \Delta$. □

Given these results, it is reasonable to assert that that a quantum system will have a temporally extended path only if it is forcibly confined to some particular region by an infinite potential.[47] Although this situation makes sense in terms of the theory, it is unphysical: we know of no infinite potential wells in nature, and we do not know how to create one, since to do so would apparently require a source of infinite energy. Clearly, no objects around us are confined to their path by an infinite potential well. Therefore, we should not suppose that any actual quantum system persists by having a temporally extended path.

Given the purportedly fundamental status of quantum mechanics, this result could be interpreted as saying that no physically possible objects persist by being located at a temporally extended spatiotemporal path.[48] Therefore, if one were also persuaded by the idea that having a temporally extended path is necessary to persist, then one would be forced to conclude that nothing actually persists. I doubt that this a conclusion that many would welcome. More positively, someone who accepts the significance of this result but regards mere temporal extent as sufficient to persist could see this as a demonstration that the spatiotemporal path of a persisting thing has mere temporal extent and is thus necessarily discontinuous (or discrete) in time.

[47] Note that I have not shown that a system confined to a region in state ψ *must* be confined there by a infinite potential, since there could exist other states of the system where it is not so confined. It seems likely that one could arrive at a result that excludes this possibility by restricting the domain of the Hamiltonian to a set of 'well-behaved' states along the lines of Berndl et al. (1995).

[48] Admittedly, quantum mechanics loses out to relativistic quantum field theory in the fundamentality stakes. However, the locational persistence theorist will not have an easier time finding a well-defined spatiotemporal path in that context due to the problem of relativistic localization (Butterfield and Fleming 1999; Debs and Redhead 2003; Halvorson and Clifton 2002).

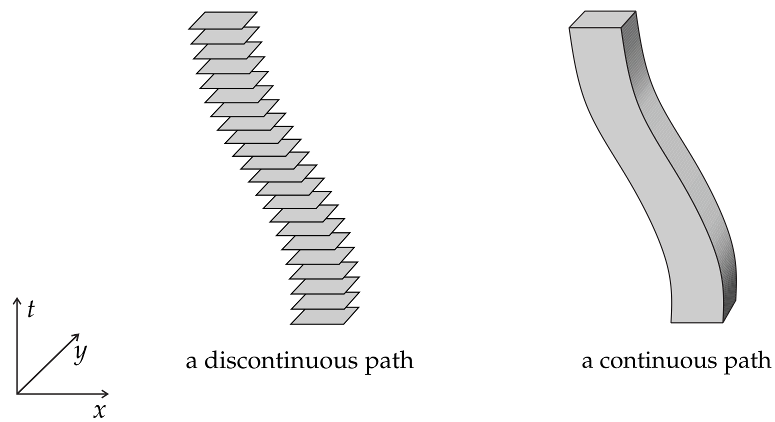

Figure 1: Illustrating a discontinuous spatiotemporal path

4 Implications for Persistence

For those who believe quantum mechanics to be a fundamental theory of matter, the results presented here apparently demonstrate that the path of a persisting material thing cannot be temporally extended. This is surprising, since it is commonly presumed (implicitly if not explicitly) that persisting things can, and do, follow continuous spatiotemporal trajectories. Certainly, according to classical mechanics they do. But if the analysis of persistence in quantum mechanics given here is correct, however, then they do not. That is, while it appears at a macroscopic level that persisting objects such as bananas and chairs follow a continuous path through time, at the microscopic level of their quantum constituents (electrons, protons, etc.) the path is in fact discontinuous. (Illustrated in Figure 1.)

Note, then, that we have a conclusive result regarding Butterfield's *B*-perdurantist: a quantum system cannot *B*-perdure since its path includes no temporally extended regions. For a perdurantist who allows instantaneous temporal parts, however, the situation is less dire. If this perdurantist can accommodate herself to the idea that a perduring object occupies a spatiotemporal path with mere temporal extent then there is nothing to stop her asserting that quantum systems perdure by having a collection of instantaneous temporal parts exactly located at a collection of spatial regions $\Delta_t \leq P$.

This would be a significant departure from the idea that a perduring object is a 'space-time worm' that occupies a worldtube of space-time,[49] but

[49] A worldtube is (the image of) a continuous function of time and so the discontinuous

perhaps we should not be surprised that proper consideration of quantum mechanics requires a significant departure from the usual options considered by metaphysicians of persistence. Indeed, a necessarily discontinuous spatiotemporal path also presents a problem for the endurantist.

4.1 Immanent Causation, Endurance and Location

Some endurantists have thought that the identity of the three-dimensional thing located at different spatial regions at distinct times is grounded in a relation of spatiotemporal continuity, which Proposition 1 says the path of a persisting thing cannot have.[50] The idea is that, rather than positing by fiat the relation of identity which holds between successive three-dimensional things, we should find an appropriate relation that holds between them which suffices to ground these facts about identity through time. However, consideration of so-called *immaculate replacement* scenarios, according to which, say, an enduring thing could be annihilated and replaced by another qualitatively identical but distinct enduring thing created ex nihilo at a successive moment of time, has led to something of a consensus that spatiotemporal continuity is neither necessary nor sufficient for identity over time.

Instead, it is often thought that what is required by the endurantist is a relation of *immanent causation* between the successive states of an instantaneous thing. This seems to weaken the need for spatiotemporal continuity. As Zimmerman (1997) puts it:

> [S]patiotemporal continuity is, at best, an epiphenomenon of persistence. What is absent in immaculate replacements is causal dependence of the later stages upon the earlier stages; the later stages are not the way they are because the earlier stages were the way they were; the later stages do not "evolve out of" the earlier stages, as they should in the case of a genuine persisting thing. Once the importance of causal relations among stages is recognized, the significance of spatiotemporal continuity begins to fade. Stories involving objects that jump discontinuously start to sound plausible, as long as the right kind of causal connections are preserved among the stages of a given jumping object. (435)

spatiotemporal paths under consideration here are not worldtubes.

[50]For example, (Zimmerman 1997, 434–435) reports that C. D. Broad thought that the spatiotemporal continuity of qualitatively similar three-dimensional stages was sufficient for a series of stages to constitute the same thing persisting through time.

Thus it seems as though the endurantist could resist the problem posed by a discontinuous spatiotemporal path by suggesting that temporally disjoint locations of a quantum system could be related by immanent causation.

What is more, locational persistence theorist Gilmore (2006, 2008) contends that a principle he calls MURIC (MUlti-location Requires Immanent Causation) should be viewed as a necessary condition for a persisting object to endure by being multiply located.[51] Gilmore (2008) explains MURIC as follows:

> [I]n order for [the same] material object to exactly occupy distinct spacetime regions R and R^*, a causal relation of the appropriate sort (often called 'immanent causation') must hold between the contents of R and the contents of R^*. (1244)

Applied to the analysis of exact location in quantum mechanics of Section 1.3, interpreted as exact occupation, MURIC appears to demand that the state of the system at each three-dimensional spatiotemporal region at which the system is exactly located be the cause of the state at subsequent regions at which it is located. But since, by Proposition 1, the intervening instants of time will (in general) be ones at which the system is not located at *any* bounded spatial region, it seems hard to see what relation linking these two regions could support a meaningful notion of immanent causation.

Plausibly, the relation that grounds the identity of a quantum system over time is the lawlike relation that the family of states ψ_t bear to one another in virtue of being related by unitary operators of the family $\hat{U}_t = e^{-i\hat{H}t}$, generated by the Hamiltonian $\hat{H}$. But this is not a relation that the states of the regions at which the system is exactly located satisfy, as I now show. A relevant fact about the family of unitaries $\hat{U}_t$ is that its members jointly determine a unique (self-adjoint) Hamiltonian operator $\hat{H}$ only if the operator-valued function $t \mapsto \hat{U}_t$ is continuous in time.[52] But Proposition 1 says that this restricted family can't possibly be continuous in time, and so this set of states necessarily fails to determine a dynamical evolution of the system.[53]

[51]See Balashov (2007) for a similar principle applied to stage theory, i.e., the diachronic composition of the singly located instantaneous stages that constitute a persisting object on that account.

[52]Stone's Theorem says that every strongly continuous one-parameter family of unitary operators uniquely determines a self-adjoint operator, and conversely. See (e.g.) (Prugovečki 1971, 331–338).

[53]Another way to put this is to say that a set of states $\{\psi_{t_k} | \psi \in \mathcal{H}, \hat{P}_\Delta \hat{U}_{t_k} \psi = \hat{U}_{t_k} \psi\}$ cannot

So, considered alone, the states at those regions are not related in a way that can support the identity of the system through time. They are thus poor candidates to ground the relation of immanent causation that MURIC says must hold between them if the system is to endure. The alternative is to let in every state in the evolution of the system, even those where the system is not located at any bounded region. However, Proposition 1 says here that the vast majority of those states will be such that the system cannot be said to be located at any bounded region since the set of times at which it is so located has measure zero.[54] This leaves the endurantist who wishes to make use of immanent causation facing the following dilemma:

Either (almost) every region that lies outside the path of a persisting object is related by immanent causation to the regions it is located at,[55] *or* if the states related by immanent causation are just those of the regions it is located at then they don't together compose a single object persisting through time (according to the dynamical law of quantum mechanics).

Thus it seems that if immanent causation is required to knit the individual three-dimensional stages into a single persisting thing then it must hold between regions (almost) everywhere in spacetime, or it fails to do its job of grounding the identity of the various three-dimensional things. In this way, the discontinuous nature of the spatiotemporal path of a persisting object implied by Proposition 1 proves problematic for the locational endurantist, who claims a persisting thing is mutiply-located at its path, as it also does for the *B*-perdurantist, who holds that persisting things exactly occupy temporally extended spatiotemporal regions.

4.2 Objection: Quantum Mechanics is Open to Interpretation

Interpreting quantum theory is a tricky business. Of especial concern for the project undertaken here—which involves taking results from a particular quantum formalism and interpreting them metaphysically—is the fact that distinct interpretations of quantum mechanics represent divergent ontologies, and so implications that follow from one interpretation may be strictly false according to another. For example, according to the so-called

be inter-related by a corresponding set of unitaries unless $\hat{P}_\Delta = \mathbb{I}$, since otherwise $\hat{U}_{t'_k}\hat{P}_\Delta\hat{U}_{t_k}$ is not a unitary operator.

[54] That is, by Proposition 1, regions $I \setminus \Delta_t$ at which the system is definitively not located must also be confined to a set of times of measure zero.

[55] The proviso '(almost) every region' is meant in the usual sense that regions with measure zero (with respect to spacetime volume) are excluded. (It could be objected that the relation of immanent causation is intended to be temporally asymmetric, in which case modify this disjunct accordingly.)

Bohmian interpretation,[56] each particle of an n-particle (non-relativistic) quantum system is located at a single point at every time, just as in classical mechanics. So, for the Bohmian, any uncertainty about the position of a particle is a strictly epistemic affair, and she would have little reason to care about my result.

However, the Hilbert space formalism I have made use of here has a good claim to be regarded as the canonical form of quantum mechanics, in the sense that it is generally the predictions of the Hilbert space formalism that a viable interpretation is required to replicate, rather than vice versa.[57] But even in interpreting the result of the previous section in terms of a 'no-go' result for a temporally extended spatiotemporal path and a 'go' result for a discontinuous spatiotemporal path, it may be objected that I am assuming too much: what has been established is that a discontinuous path is not *ruled out*; the existence of a discontinuous path is consistent with these results but not implied by them.

Indeed, the import of such results is described by Wallace (2014) as follows:

> Wave-packets with compact support cannot be created (they require infinite potential wells); if they were to be created, they would spread out instantaneously. (1)

However, the idea that these *instantaneously* localized states require an infinite potential is itself interpretation dependent. For example, in the standard 'Copenhagen' interpretation, which endorses both directions of the eigenstate-eigenvalue link, when we are in a position to infer that a system has an eigenvalue for an observable we may infer that it is in the corresponding eigenstate. In that case, when a position measurement at time t reveals that the system is located in Δ at time t (i.e., the projector $\hat{P}_\Delta$ has eigenvalue 1) we infer that the system is in the corresponding eigenstate, updating the state ψ by applying the projection $\hat{P}_\Delta$, i.e. $\psi_t \rightarrow \hat{P}_\Delta \psi_t$. On this interpretation, instantaneous localization requires no infinite potential

[56]Named after David Bohm, but with an important precedent in the work of Louis de Broglie.

[57]In part, the reason for this is historical: Hilbert space was the mathematical arena in which von Neumann was able to unify Schrödinger's wave mechanics and Heisenberg's matrix mechanics, showing them to be different ways of representing the *same* theory. But there is also a sense in which this formalism directly reflects essential posits of the theory, such as Dirac's Superposition Principle, and rests on deep mathematical results that knit together (physically interpreted) symmetry groups and their representation by families of unitary operators (such as $\hat{U}_t$) on a Hilbert space Mackey (1963).

well.[58]

Of course, such considerations merely establish the possibility that a quantum system will have a series of exact locations and so possess a spatiotemporal path (as defined above). What can we say if this is not the case? First, if a system has no exact locations then it could persist by being entirely located at *all* of space.[59] In that case, one may regard it as having spatial parts given by the localization scheme, and the perdurantist faces my earlier argument by analogy against temporal parts (Pashby 2013).

However, one could also explore the use of different location relations. For example, inspired by Albert and Loewer (1996), we could use ϵ-location rather than entire location, where a quantum system is ϵ-located at a bounded region Δ just in case $\langle\psi|\hat{P}_\Delta\psi\rangle > 1-\epsilon$, where $0.5 \geq \epsilon > 0$. This would lead to a family of temporally extended 'ϵ-paths,' nested according to the value of ϵ.[60] This move could be motivated by reasons internal to quantum mechanics but would do violence to the metaphysics of location (based on the relations of entire and exact location) that formed the foundation of this paper.

5 Conclusion

This paper began with an attempt to bring into contact the metaphysics of spatial location with the location relations enjoyed by physical systems, as described by physics. I argued that both classical (namely, continuum) mechanics and quantum mechanics provide explications of the relation of 'exact location' entertained by metaphysicians. Interestingly, it turned out these two physical theories maintain that a physical object occupies a region of space in different ways: according to classical mechanics, a body pertends the region at which it is exactly located; according to quantum mechanics, a quantum system spans the region at which it is exactly located.

Having extended this analysis to metaphysical theories of persistence

[58]Note that there are also interpretations of quantum mechanics that introduce a *spontaneous* localization of the wavefunction (Ghirardi et al. 1986).

[59]Recall that if ψ_t is normalized then $\langle\psi_t|\psi_t\rangle = \langle\psi_t|\hat{P}_S\psi_t\rangle = 1$ for all t. If the system is not located in any bounded spatial region at a time, then it is located at their union: all of space.

[60]Although for an isolated quantum system the phenomenon of wavepacket spreading would prove a problem for permanence (Blank et al. 2008, 236–238), consideration of decoherence effects should serve to secure permanence of macroscopic objects (Butterfield 2006c, §5) (albeit at the cost of their multiplicity).

that make use of the relation of 'exact occupation', I presented a result from quantum mechanics that implies that the spatiotemporal path of a persisting quantum system must be discontinuous in time. This discontinuity presents severe difficulties for Butterfield's (2006a; 2006b; 2006c) 'anti-pointilliste' perdurantist, and for an endurantist who relies on immanent causation to secure identity through time (Gilmore 2008; Zimmerman 1997). This suggests that such theories may not be well suited to describing the persistence of microscopic objects: if the ordinary objects of everyday experience are considered fundamentally to be quantum systems, then presumably these accounts of persistence fail there as well. More generally, metaphysicians who use location in spacetime to express their theories of persistence owe us an account of whether persisting objects are to be understood at temporally extended objects, or objects with mere temporal extent. This paper, then, has represented something of a negative contribution to the persistence debate, corresponding to Gilmore's fear that quantum theories "may show that the current range of options is incomplete or somehow ill-formulated" (Gilmore 2008, 1247).

However, I would like to conclude by attempting to expand the set of commonly considered metaphysical options for persistence by suggesting a candidate that seems especially apt for describing the way that something with a discontinuous path might persist. In particular, in the event-based ontologies of Whitehead (1925) and Russell (1927) the persistence of material bodies is given a distinctive interpretation in terms of recurring patterns of discrete spatiotemporally located events.[61] Moreover, these metaphysicians were explicitly concerned with taking into account the morals of modern physics: relativity and the burgeoning quantum theory.[62] I will save my attention for Whitehead. Although the process theory that Whitehead was to develop would involve a richer ontology, discussing the persistence of electrons in 1925 he proceeds as follows:

> An electron for us is merely the pattern of its aspects in its environment [...] I want to suggest that *reiteration* where it differs from *endurance* is more nearly what the organic theory requires. [...] in the organic theory, a pattern need not endure in undifferentiated sameness through time. The pattern may be essen-

[61]Since the events are intended to be ontologically prior to spatiotemporal points, which are defined notions in this framework, this way of speaking is more than a little forced. Nonetheless, it will suffice for present purposes.

[62]Russell (1927) acknowledges the assistance of Cantabrigian physicist R. H. Fowler, the premier quantum theorist in Britain at that time (save for his brilliant protege P. A. M. Dirac).

> tially one of aesthetic constants requiring a lapse of time for its unfolding. A tune is an example of such a pattern. Thus the endurance of the pattern now means the reiteration of its succession of constants. (Whitehead 1925, 133–134, original emphasis)

I will not attempt an exegesis of Whitehead's philosophy here, but this idea that persistence through time can be more like a tune—a melody composed of a series of discrete notes—seems especially apt for consideration in this context. In particular, a melody sustains its identity through time even if a note is not being played at all times. Indeed, typically there are gaps (or rests) left between notes. And thinking of a tune may even impart more qualitative difference to the 'parts' of a melody than strictly necessary: think not of a melody but of a *rhythm*.[63] A rhythm may be composed of a series of qualitatively indistinguishable sounds (think here of a synthesized drum hit) which nonetheless extends through time in virtue of the distinctive, repeating pattern of time intervals left *between* sounds.[64]

While there may be no sound at a particular time, a rhythm persists through time while it continues to be played. What constitutes the temporal continuity here? Literally, the *reiteration* of the repeating pattern of temporal intervals between events. In this way, the times that are *not* occupied are as vital to the persistence of the rhythm through time as the times that are, but there are no 'temporal parts' of the rhythm at the times where no sound is present. And neither is there a way in which the preceding sound 'brings about' the next in a causal sense. But note how the persistence of the rhythm as a whole emerges at the level of an extended temporal period, i.e., the rhythm requires "a lapse of time for its unfolding." Although this account of reiteration requires further elaboration, it does suggest the possibility of persistence through means other than the occupation of a spatiotemporally continuous path.

Appendix

The proof of the proposition makes use of the following lemma, due to Hegerfeldt (1998).

Lemma 1. *(Hegerfeldt) For any positive operator P, any vector $\psi \in \mathcal{H}$, and*

[63] This is, in fact, the terminology adopted by Russell (1927).

[64] For example, a simple clave rhythm in 4/4 time may be represented as X o o X o o X o o o X o X o o o, where 'X' is a crotchet (quarter note), and 'o' a crotchet rest (during which no sound is made).

any unitary group $\hat{U}_t = e^{-i\hat{H}t}$ generated by a self-adjoint Hamiltonian $\hat{H}$ whose spectrum is bounded from below either*:*

1. $\langle\psi|\hat{U}_{-t}P\hat{U}_t\psi\rangle = 0$ *for all t*, or
2. $\langle\psi|\hat{U}_{-t}P\hat{U}_t\psi\rangle \neq 0$ *for (almost) all t.*

Restating the proposition here, we have:

Proposition 1. *Let $\psi \in \mathcal{H}$, a unit vector, be the state of a quantum system. Let $\hat{P}_\Delta$ be the projection associated with a bounded spatial region Δ by the localization scheme. Let $\psi_t = \hat{U}_t\psi$, with $\hat{U}_t = e^{-i\hat{H}t}$ the unitary group of time translations generated by $\hat{H}$, a self-adjoint operator with spectrum bounded from below.*

Then either

1. *the system is always entirely located at Δ, that is, $\hat{P}_\Delta\psi_t = \psi_t$ for* all $t \in \mathbb{R}$, or
2. *the set of times at which $\hat{P}_\Delta\psi_t = \psi_t$ has Lebesgue measure zero, in which case there is no time interval $(t_1, t_2) \subseteq \mathbb{R}$ such that the system is entirely located at Δ at every time $t \in (t_1, t_2)$.*

Furthermore, the Hamiltonian $\hat{H}$ includes an infinite potential on the boundary of Δ if and only if *every state entirely located at Δ at a time is confined to Δ for all t by $\hat{U}_t$, that is, when $\hat{P}_\Delta\hat{U}_t\hat{P}_\Delta\psi = \hat{U}_t\hat{P}_\Delta\psi$ for all $\psi \in \mathcal{H}$, for all $t \in \mathbb{R}$.*

Proof. Consider the projection corresponding to the spatial complement of Δ, $\hat{P}_{\neg\Delta} = \mathbb{I} - \hat{P}_\Delta$. The premises of Hegerfeldt's Lemma are satisfied by ψ, $\hat{P}_{\neg\Delta}$ and $\hat{U}_t$. If Case 1 applies then $\langle\psi|\hat{U}_{-t}\hat{P}_{\neg\Delta}\hat{U}_t\psi\rangle = 0$ for *all t*. Since we have $\langle\psi|\hat{U}_{-t}\hat{P}_\Delta\hat{U}_t\psi\rangle = 1 - \langle\psi|\hat{U}_{-t}\hat{P}_{\neg\Delta}\hat{U}_t\psi\rangle$ it follows that

$$\langle\psi|\hat{U}_{-t}\hat{P}_\Delta\hat{U}_t\psi\rangle = 1 \quad \text{for all } t \in \mathbb{R}. \tag{4}$$

We may ignore the possibility that $\hat{P}_\Delta = \mathbb{I}$ since Δ is by assumption a bounded region. Thus $\psi_t = \hat{U}_t\psi$ describes an evolution of the system such that it is always entirely located at Δ, that is $\psi_t = \hat{P}_\Delta\psi_t$ for all $t \in \mathbb{R}$.

If Case 2 applies then for almost all t the system is not entirely located at Δ. That is,

$$\langle\psi|\hat{U}_{-t}\hat{P}_\Delta\hat{U}_t\psi\rangle = 1 - \langle\psi|\hat{U}_{-t}\hat{P}_{\neg\Delta}\hat{U}_t\psi\rangle \neq 1,$$

except for a set of times of Lebesgue measure zero. Therefore, the set of times at which $\hat{P}_\Delta\psi_t = \psi_t$ has Lebesgue measure zero.

If every state located at Δ is confined to Δ for all t then $\hat{P}_\Delta \hat{U}_t \hat{P}_\Delta \psi = \hat{U}_t \hat{P}_\Delta \psi$ for any $\psi \in \mathcal{H}$, for any $t \in \mathbb{R}$. It then follows from the spectral calculus that $\hat{P}_\Delta \hat{H} \hat{P}_\Delta \psi = \hat{H} \hat{P}_\Delta \psi$, for all $\psi \in \mathcal{H}$. Thus the restriction of the domain of $\hat{H}$ to the subspace defined by $\hat{P}_\Delta$ forms a closed subdomain. This shows that $\hat{H}$ can be written as a direct sum $\hat{H} = \hat{H}_\Delta \oplus \hat{H}_{\neg\Delta}$, where $\hat{H}_\Delta \psi = \hat{P}_\Delta \hat{H} \hat{P}_\Delta \psi$ for all $\psi \in \mathcal{H}$. Blank et al. (2008, 14.6.1) demonstrate that the existence of an infinite potential on the boundary of Δ is sufficient for such a decomposition to exist; it is necessary in the sense that a finite potential will not have this result. □

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