Logos, logic and maximal infinity

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Abstract: Recent developments in the philosophy of logic suggest that the correct foundational logic is like God in that both are maximally infinite and only partially graspable by finite beings. This opens the door to a new argument for the existence of God, exploiting the link between God and logic through the intermediary of the *Logos*. This article explores the argument from the nature of God to the nature of logic, and sketches the converse argument from the nature of logic to the existence of God.

Introduction

Arguments for the existence of God typically make use of logic.[1] Yet there have been few, if any, arguments linking the specific nature of logic to the existence of God.[2] Recent developments in the philosophy of logic, however, strongly suggest that the correct foundational logic shares some of the attributes traditionally ascribed to God. This opens the door to a new argument for the existence of God.

Briefly put, there is a striking analogy between the infinity of God and the infinity of logic. In Christianity, God the Son is the *Logos* or Word: both reason and the creative force that structures the universe. The infinite *Logos* transcends human understanding and is revealed to us in part. Strikingly, recent advances tend to show the same is true of logic. Logic structures all truths, is highly infinitary, and can only be partially grasped by finite beings. The connection between the *Logos* and *logic* is in part verbal, but it is also much more than that. The business of this article is to articulate this connection further, and to consider whether it supports theism, in particular Christian theism.

To set out the God-logic analogy and the subsequent arguments in each direction, we need to have a clear view of both sides of the analogy. The next section summarises recent work in the philosophy of logic for non-specialists (I shall assume little more than a grasp of the predicate calculus). The third section sets out and assesses an argument from God's maximal infinity to logic's maximal infinity. The fourth section sketches the converse argument from logic to the existence and nature of God. The third section is thus concerned with the argument *to* logic and the fourth with the argument *from* logic. Following the conclusion, the Appendix contains a few further technicalities.[3]

Logic on the invariantist picture

All logics may be thought of as formal systems, worthy of mathematical investigation. Yet only one of them correctly captures the logical features of our language and its extensions.[4] Some of the properties of this logic have been known for a long time. Virtually all philosophers and logicians agree, for example, that it contains principles such as the law of non-contradiction and, more generally, all tautologies of propositional logic. To go beyond this, the best place to start is the work of the Polish-born logician Alfred Tarski. In 1936, Tarski published his paper 'On the Concept of Logical Consequence', which introduced the now-standard model-theoretic definition of logical consequence.[5] On the model-theoretic definition, a sentence *A* implies another sentence *B* just when any model in which *A*'s formalisation in the correct logic is true is also one in which *B*'s formalisation in the same logic is true. For instance, assuming logic includes the predicate as well as the propositional calculus, 'Fido is a dog and Bruin is a bear' implies 'There is a dog and there is a bear' because any model in which the former's formalisation Df & Bb is true is also a model in which the latter's formalisation $\exists xDx$ & $\exists xBx$ is true. It is crucial for this verdict that so-called logical terms such as & and $\exists$ are always interpreted the same way in any model; in contrast, the interpretation of non-logical ones such as D, B, f and b is allowed to vary.[6]

As the example suggests, Tarski's account of logical consequence needs to be supplemented with a demarcation of the logical constants. Tarski tried to fill the gap in a lecture first delivered in London in 1966, posthumously published as 'What are Logical Notions?' twenty years later. In it, he characterised logical terms—his examples include 'not', 'and', 'or', 'is', 'every' and 'some'—as those of an entirely general character, which turn up in every domain of discourse and inquiry. His thought was that the logical constants do not concern any particular subject matter (such as numbers, neutrons or nervous systems), but rather apply generally to anything. Very roughly, the logical constants have the same semantic value whatever exists. Tarski thereby inaugurated the *invariantist* approach to logical constants, which sees logical constants as invariant under transformations of the domain. This is now the dominant approach to the logical constants, *inferentialism* being another, less promising and less popular one. Invariantism embodies a semantic approach to the logical constants, which fits well with the dominant model-theoretic understanding of logical consequence, whereas inferentialism embodies a syntactic approach.

More precisely, Tarski offered a *permutation-invariance* criterion for logical constanthood. A permutation, recall, is a function from a set S to S with two properties: no two distinct elements are mapped to the same thing (the function is one-to-one); and everything in S is mapped to by some element or other (the function is onto). Informally, a permutation 'shuffles' the objects it acts upon. For example, ACB and BCA can be thought of as permutations of ABC, and the function $f(x) = x + 1$ on the integers is a permutation since it is both one-to-one and onto.[7]

Tarski's criterion for logical constanthood was refined and extended by Gila Sher in a 1991 book and subsequent publications,[8] and in joint work by the present author,[9] among others. Though invariantists disagree over details, almost all of them agree on some key points, notably that logic is maximally infinitary.

To explain what this means, recall from basic set theory that there are different types of infinity, known as alephs, indexed by ordinals, the first size of infinity being $\aleph_0$ ('aleph-zero'), the second $\aleph_1$ ('aleph-one'), and so on. Each of the alephs represents a particular infinity, smaller than the next one in the aleph sequence. The collection of all cardinals is often said to be absolutely infinite because it is not of aleph size but transcends all the alephs. The Greek letter κ is standardly used for cardinals, and in this use encompasses both finite cardinals (0, 1, 2, ...) as well as infinite ones ($\aleph_0$, $\aleph_1$, ...).

Logic's maximal infinity can for our purposes be understood as comprising the following claims: any number (finite or infinite) of logical formulas can be conjoined or disjoined; these formulas can contain quantifiers over any number (finite or infinite) of variables; and as a consequence they can express statements such as 'there are κ many' not just for finite cardinals κ (0, 1, 2, ...) but infinite ones as well. In particular, notions such as 'is infinite' or 'is of size $\aleph_1$' are logical on the invariantist picture. For however you permute the elements of a domain, you cannot change its size: a permutation of a set shuffles its elements but does not reduce or increase their number. Any domain that is infinite, for example, will remain so under permutation; hence 'is infinite' expresses an invariant notion. (The Appendix fleshes out these ideas in more technical detail.)

The main ground for the conclusion that logic is maximally infinitary is the 'top-down' invariantist argument just sketched. This argument, as we saw, proceeds from the nature of the logical constants. It may also be combined with a host of more concrete or 'bottom-up' arguments. Unlike top-down arguments, the latter eschew philosophical assumptions about the nature of the logical constants and proceed more directly. (The Appendix sketches one of these bottom-up arguments.)[10] Remarkably, top-down and bottom-up arguments converge to the same conclusion: the correct logic is maximally infinitary.

If the correct logic is maximally infinitary then it must be incomplete; that is to say, no deductive system can capture its consequence relation. In more formal terms, if, as usual, '$\vDash$' represents logical implication, there is no deductive system, represented by '$\vdash$', such that $\Gamma \vDash \varphi$ if and only if $\Gamma \vdash \varphi$ for any set of sentences Γ and sentence φ. This follows from logic's infinitary nature. Indeed, unlike some incomplete logics,[11] the correct logic's set of logical truths cannot be generated by an idealised being whose powers are bounded by some cardinal (finite or infinite). For example, no being capable of running through and determining the logical status of an at-most-countably-infinite set of statements can generate the set of logical truths. (A countably infinite set is of the same size as the set of natural numbers.) Any idealised being that can generate all the theorems of the correct logic must have absolutely infinite powers: they must be able to determine the logicality of sentences from a collection whose size transcends all the alephs.

The current state of play in the philosophy of logic may therefore be summarised as follows. The most promising and most popular approach to the logical constants is invariantism. Although there are several forms of invariantism, most of them share a core set of commitments and implications. The most striking of these is that logic is as infinitary as possible: it is maximally infinitary. As a consequence, it defies full human comprehension. Moreover, we can come to appreciate this fact through the kinds of arguments just summarised.

From God to Logic

We have covered the logic side of the God-logic analogy. On the most popular approach to the logical constants, logic turns out to be maximally infinitary, transcendent and only partially graspable. We abbreviate this by saying that logic is maximally infinite; by implication, this covers the epistemic dimension—e.g. our inability to fully grasp the set of logical truths—since our abilities are finite. Let us turn now to the other side of the analogy. Since we are interested in how a theist is likely to conceive of logic, or argue from the nature of God to the nature of logic, the perspective in the present section will be a theistic one and will help itself to premises available from that perspective. More specifically, our perspective will be that of Christian theism. In contrast, when we consider the argument from logic for the existence of God in the next section, no such premise can be assumed. But as we shall see, it will be very relevant for that section's argument to have first established how much support theism lends the idea that logic is maximally infinite.

It is worth noting that Western philosophy and theology have had little systematic to say about the relation of God to logic. St Anselm asserted that God is the source of all necessities,[12] and therefore presumably of their logical structure as well. Descartes believed that 'eternal truths' such as those of logic had been laid down by God and depend upon

Him.[13] Leibniz likewise maintained that eternal truths depend on God's understanding (rather than His will).[14] These isolated remarks are typical fare: we find similar ideas scattered in the philosophical canon, but nothing worked-out on the relation between God and a *specific* logic. Philosophers have been more voluble on related topics,[15] but have had little if anything specifically to say about God and logic. We might speculate that part of the reason for this neglect is the common conception of logic from Aristotle to the present day as finitary.[16]

The argument we shall focus on is what I will call the *Logos* Argument. It consists of three premises and two sub-arguments.

P1. God the Son is maximally infinite
P2. God the Son is the *Logos*
Therefore: C1. The *Logos* is maximally infinite (from P1 and P2)
P3. Logic is a manifestation of the *Logos*
Therefore: C2. Logic is maximally infinite (from C1 and P3)

The first sub-argument has premises P1 and P2 and conclusion C1; the second sub-argument's premises are C1 and P3, and its conclusion C2. Both sub-arguments (and thus the overall *Logos* Argument) are put forward from the perspective of Christian theism. We now gauge the strength of each sub-argument and thereby that of the overall argument.

The first sub-argument

The great monotheistic traditions understand God in different ways; moreover, each tradition encompasses many different perspectives. Despite that, all these faiths share a core set of commitments, the relevant ones here being that God is infinite, transcendent and opaquely discernible. (These properties are related to but distinct from other divine qualities such as, say, omniscience and omnipotence.) Naturally, there are different ways of interpreting each of these claims, but in this basic formulation, the majority of theists would explicitly avow them or implicitly accept them.

Let us briefly go over some of the evidence, starting with scriptural evidence, and limiting ourselves to Christianity mainly for reasons of space. In the Old Testament, God's 'measure' is said to be as deep and as high as can be, and not discoverable by us (Job 11:7–9). God's understanding is characterised as infinite (Psalm 147:5). Moses, upon asking the Lord to reveal Himself, is permitted only to see His back (Exodus 33:12–23); likewise, Isaiah avers that the God of Israel hides Himself (Isaiah 45:15); and even His name is an unfathomable mystery (Genesis 3:14). God's wisdom and knowledge run so deep that His judgements and ways defy human comprehension (Romans 11:33–34), and our true life is hidden (Colossians 3:3).

Following its development in Philo and Plotinus, the idea of God's infinity and transcendence was taken up by Christian theologians, including prominently Gregory of Nyssa, Augustine, Thomas Aquinas and Nicholas of Cusa.[17] God's infinity is avowed in several doctrinal definitions, including the Athanasian creed in which God is described as immeasurable or incomprehensible,[18] and many confessions of faith, e.g. the Westminster Confession of Faith or the Thirty-Nine Articles.

Now modern set theory distinguishes transfinite infinities (of size measurable by one of the alephs) and absolute infinity (of size greater than any aleph). A natural question is then whether God, being infinite, is transfinitely infinite or absolutely infinite. As one would expect, the Bible, creeds, confessions of faith, and theological writings prior to the late 19th century are silent on this point; this is hardly surprising, since no such distinction was known to anyone before Cantor introduced it in the 1870s. This silence notwithstanding, if the question is well-posed (something to which we will return) then its answer should be clear: God is *absolutely* infinite. This is in agreement with Cantor's own view, who called absolute infinity 'the true infinite or absolute, which is in God, [and] admits no kind of determination' (Cantor 1883, p. 175). That God must be absolutely infinite rather than (merely) transfinitely infinite follows straightforwardly from the fact that any transfinite infinity is bounded. (See the Appendix for details.)

In sum, P1 is sanctioned by Christian theism: God the Son is maximally infinite. For on the traditional Christian view, God the Father is maximally infinite and God the Father and God the Son are consubstantial. The same conclusion could (but need not) be reached more directly by equating God with God the Son, if each person of the triune Godhead is equatable with God Himself.[19]

The second premise P2 is an identity statement rather than a predication. It encapsulates the important Christian idea, foreshadowed earlier, that God the Son, i.e. Jesus Christ, is the *Logos*. (Though no English word answers to the Greek *λόγος*, its usual English translation is the 'Word', in line with the Vulgate's *Verbum*.) The Christian tradition equates divine reason with the second person of the Trinity. This is most striking in the first few verses of John's gospel, the so-called Hymn to the Word, which starts with 'In the beginning was the Word, and the Word was with God, and the Word was God.' (John 1:1). Then a few verses later: 'And the Word became flesh and dwelt among us, full of grace and truth; we have beheld his glory, glory as of the only Son from the Father' (John 1:14). First Epistle of John (1:1) reprises the theme in its opening verse, and Revelation (19:13) also explicitly identifies Christ with the *Logos*.[20]

The identification enshrined in P2 has been part of Christian apologetics since the second century. Justin Martyr (d. 165) took the *Logos* to be the source of all human knowledge, to have established the heavens (Psalms 33:6), and in its divine form to have sowed its seeds throughout history (before the birth of Christ as well as after). Justin also maintained that 'the Logos Himself … assumed a human form and became man, and was called Jesus Christ'.[21] Other Christian thinkers of the pre-conciliar era such as Ignatius of Antioch,[22] Theophilus, Athenagoras, Irenaeus, Tertullian and Origen likewise explored the identity of the *Logos* with Christ. The Chalcedonian creed (AD 451) enshrined the identity by equating all of the following: 'One and the Self-same Son and Only-begotten God, Word, Lord, Jesus Christ'. Cyril of Alexandria (d. 444), whose view became recognised as orthodoxy at Chalcedon, put it unequivocally: 'If anyone distributes between two characters or persons the expressions used about Christ in the Gospels, etc. … applying some to the man, conceived of separately, apart from the Word … others exclusively to the Word … let him be anathema'.[23]

P2 is thus orthodox Christian doctrine. Of course, there are Christological traditions that start 'from below' and focus on Jesus' historical ministry, paying little heed to the Hellenistic metaphysical doctrine of the *Logos*.[24] Still, unless they renounce it, they arguably remain committed to it. In any case, the point is that any creedally orthodox form of Christianity is committed to both P1 and P2. Naturally, one may be heretical in all sorts of other ways (e.g. accept virtually any manner of heresy about the Trinity), and still remain committed to P1 and P2. Assenting to P1–P2 is necessary but not sufficient for being an orthodox Christian.

The first sub-argument's inference step is an instance of the identity of indiscernibles. As such, it ought to be uncontroversial: if entity *A* is identical to entity *B* then anything true of *A* must be true of *B*. When it comes to the Trinity, however, identity has to be handled with care.[25] 'Jesus is God, God is the Father, so Jesus is the Father' is not, for Christians, a valid argument, even though it uses the otherwise indisputable transitivity of identity. However, as the first sub-argument involves only one person of the Trinity, it is hard to see how it could be vulnerable to this sort of challenge.

The conclusion C1 may be reached in other ways than via the first sub-argument. Some Christians—for instance, Unitarian Christians—construe all talk of the *Logos* as non-literal personification of the unipersonal God's wisdom. Even if they do not get to C1 via P1 and P2, they too might find C1 acceptable, because they take God's wisdom to be maximally infinite. And on this non-hypostasised conception of the *Logos*, many non-Christian theists will also subscribe to C1 (or better: are committed to it). Islam, for example, accepts the identification of Jesus as God's Word (*kalimat Allāh*, Qur'an 3:39, 3:45, 4:171), which it sees as maximally infinite, without, of course, deifying Jesus.[26] Personifications of God's Wisdom (Proverbs 8) or the Word of God (Book of Wisdom 7:26) are found in the Hebrew Bible too, and some Hellenistic Jewish scholars went as far as to hypostasise the Word. Theists of many stripes may thus agree that the *Logos*, conceived either literally as God's Reason and Wisdom or non-literally as a personification of the same, is maximally infinite. C1 is thus compatible with: orthodox Christianity; other, less orthodox, forms of Christianity; and many other forms of theism, such as Islam and Judaism—so long as it is interpreted appropriately in each case.

The second sub-argument
Premise P3 avers that logic is a manifestation of the *Logos*. To someone who understands the *Logos* as the supreme Reason that underlies and fashions all things, as in the Hellenistic metaphysical tradition early Christianity co-opted, this premise verges on the tautological. The logical structure of propositions (and/or facts) is by definition part of the rational structure of things. The premise is not strictly tautological, of course, since there is room for rational structure to diverge from logical structure. But there is at least a close affinity here. Note that there is no equivocation between C1 and P3, as the intended reading of *Logos* in each is the same.

Clearly, the inference from C1 and P3 to C2 is not formally valid. It is intended rather as a plausible inference that trades on the nature of the *Logos* and the meaning of 'manifestation'. Logic, being the manifestation of the *Logos*, must share its properties. Specifically, then, it must be maximally infinite. That logic is a *manifestation* of the *Logos* is supposed to establish a tight connection between their respective properties; manifestation is like incarnation, only stripped of bodily connotations. The alleged connection is thus tighter than if logic were merely *part* of the *Logos*, or its *effect*, or its *creation*. The analogous argument that if x has property P then any part of x has P is clearly fallacious: I may be dark-haired but my little finger is not. Similarly for the argument that if x has P and x causes y then y has P, or that if x has P and x created y then y has P. *Manifestation* is supposed to link the *Logos* and logic more tightly than the relation of parthood, effect, or creation.[27]

In his *Soliloquium* (Burnett 1984), Peter Abelard (d. 1142) commented that it is Christians who, through following the *Logos*, are the true logicians; and, by the same token, that Christian teaching is the true logic.[28] The *Logos* Argument aims to do better than Abelard's mere play on words. This is not to deny that the argument exploits Christianity's identification of elements within the wide semantic range of the single Greek word *λόγος*; but the point—the key point—is that this identification is enshrined in Christian doctrine. Evidently, though, there remains work to be done, from a Christian perspective, to make P3 more plausible.

To help bridge the gap between the *Logos* and logic, theists might try invoking an idea central to the Christian faith: that of *kenōsis* (*κένωσις* in Greek). This is the idea that in the Incarnation, Christ emptied Himself of His divine qualities (Philippians 2: 6–7). One theological reading of this idea is that God or the divine *Logos* became emptied of some attributes and retained others in Christ. The former include metaphysical ones—omniscience, omnipotence, etc.—and the latter moral ones (in particular, omnibenevolence and holiness).[29] There is also a sense in which the creation was an act of divine *kenōsis*.[30] Now logic is static and lacks God's dynamic attributes. So the thought might be that in logic God empties Himself of some attributes (e.g. the moral ones) and retains others, such as absolute infinity and transcendence. That understanding of logic as a manifestation of the *Logos* would lead directly to the conclusion that logic and the *Logos* have (some) common attributes. Clearly, though, the idea of a *kenōsis* of the *Logos* into logic parallel to the Incarnation needs more spelling out.[31]

To summarise, P1, P2 and the inference from them to C1 are reasonably secure. But even here, room for scepticism remains. For instance, one might object that God the Son's maximal infinity should not be taken literally, or at least not entirely literally, if a literal construal just means absolute infinity in the set-theoretic sense. Or to put it a different way, one might object that the argument equivocates: God the Son and the *Logos* are infinite, logic is infinite, but there is no real sense in which they are infinite in the same way. Are we really talking about something like set-theoretic infinity—the infinity of a multitude of things—when we talk about divine infinity? That said, the idea that the *Logos* is infinite *prima facie* has more literal content than the same attribution to God the Son—so the argument for C2 could simply start from C1. As far as the second sub-argument is concerned, we saw that P3 has some plausibility, though less so than P1 and P2. We also saw that the move from P3 and C1 to C2 is something of a promissory note, as it is unclear whether the 'logic of manifestation' permits such an inference.

The *Logos* Argument is therefore promising but must clear several hurdles before it may be unreservedly regarded as sound. Although there is no space to pursue the dialectic further here, a note of optimism may nonetheless be sounded. To show that theism meshes with a maximally infinite conception of logic does not require showing that the former directly entails the latter. It suffices to show that theism favours an account of logical constants—invariantism—which in turn virtually guarantees logic's maximal infinity. As we saw, Christian theists link God and logic, via the *Logos*. Precisely how to understand this connection remains a source of difficulty, as highlighted. And even if the connection is robust, we are still far from a precise account of logic, such as that delivered by the previous section's invariantist arguments.[32] What we can say, though, is that theism favours a broadly invariantist account of the logical constants over an inferentialist one. For according to (the most plausible and accepted) forms of invariantism, logic is maximally infinitary, transcendent and incomplete.[33] In contrast, according to pretty much all forms of inferentialism, logic is finitary and complete (see the Appendix for a brief explanation as to why). So although Christian doctrine by itself does not directly underwrite the idea that logic is infinite, it meshes with an invariantist conception of logical constanthood in the indirect fashion suggested here. Christian theism's 'meshing with' invariantism is a matter of epistemic coherence rather than deductive implication.

That, then, is the support a maximally infinite conception of logic receives from (Christian) theism.[34] The *Logos* Argument points to an important connection between Christian theism and infinitary logic, albeit one it may have only imperfectly succeeded in capturing. As we shall see shortly, that theism meshes with the maximally infinite conception of logic will also be a crucial component of the argument *from* logic.[35]

From Logic to God

The second section set out the invariantist conception of logic and the third the argument from theism to logic's maximal infinity. The moral was that (Christian) theism meshes with an invariantist as opposed to an inferentialist conception of the logical constants, even if there is no exact argumentative route from God's maximal infinity to logic's. What about the other direction? Can we argue from the nature of logic to God's existence?

The short answer is yes, for a simple epistemological reason. If learning that *A* should increase your confidence in *B* then, conversely, learning that *B* should likewise increase your confidence in *A*.[36] Here *A* is (Christian) theism and *B* is logic's maximal infinity. Thus an argument *to* logic can always be turned round to underwrite an argument *from* logic. The question is how *much* support logic's nature affords theism. Assessing both the argument to and the argument from logic in detail within a single essay would be overly ambitious, so this section simply makes a start on the latter.

First, though, let us set some parameters. *Naturalism* urges that inquiry should proceed on the basis of broadly scientific evidence. For example, it may proceed on the basis of replicable observational data or the tenets of a well-confirmed scientific theory, or well-established conclusions in logic and mathematics. The discussion in this section, then, unlike that in the previous one, proceeds on a naturalist basis: only broadly scientific evidence will be admissible. Thus, in the previous section, we argued on the basis of (Christian) theistic premises; in the present one, in contrast, we wish to explore the natural-theological argument for God's existence from the nature of logic.

On that understanding, let us consider a Bayesian version of the argument. Such a formulation, though natural given recent trends in the epistemology of religion,[37] is by no means obligatory. But it will help sharpen the argument and clarify some of its features.

We write Φ for Christian theism and φ for its associated probability, i.e. $Pr(\Phi) = \varphi$, and we let *LMI* be the hypothesis that logic is maximally infinite (itself shorthand for the definition given earlier). As usual, we also write $Pr(A/B)$ for the probability of *A* given *B* (when $Pr(B) \neq 0$). We also tacitly assume that all propositions are conditionalised on background evidence.

Consider next the relevant version of Bayes' Theorem:

$$Pr(\Phi/LMI) = \frac{Pr(\Phi \cap LMI)}{Pr(\Phi \cap LMI) + Pr(\sim\Phi \cap LMI)}$$

Now $Pr(\Phi \cap LMI) = Pr(LMI/\Phi).Pr(\Phi) = k_1\varphi$, where k_1 is defined as $Pr(LMI/\Phi)$. Also, $Pr(\sim\Phi \cap LMI) = Pr(LMI/\sim\Phi).Pr(\sim\Phi) = k_2(1 - \varphi)$, where k_2 is defined as $Pr(LMI/\sim\Phi)$. So we may rewrite the above equation as:

$$Pr(\Phi/LMI) = \frac{k_1\varphi}{k_1\varphi + k_2\,(1 - \varphi)}$$

Dividing top and bottom by k_1 yields:

$$Pr(\Phi/LMI) = \frac{\varphi}{\varphi + \frac{k_2}{k_1}(1 - \varphi)}$$

The question is how this compares to the prior probability of theism (i.e. φ). To gauge this, we divide the quantity $Pr(\Phi/LMI)$ by $Pr(\Phi)$ to arrive at the 'multiplier' $\frac{1}{\varphi + \frac{k_2}{k_1}(1 - \varphi)}$. This is the proportional increase in theism's probability given *LMI*: the argument from logic multiplies the probability of theism by precisely this amount. On the assumption that $k_2 < k_1$, the multiplier's denominator is less than 1, since φ and $(k_2/k_1)(1 - \varphi)$ add up to less than $\varphi + (1 - \varphi) = 1$; so the multiplier is greater than 1. Evidence that logic is maximally infinite therefore raises the probability of theism. That, in a nutshell, is the argument from logic.

To put the argument to the test, let us consider three objections to it. The first objection is a circularity worry about our probabilistic framework. Bayesians assume that logical truths have probability 1 and that if *A* entails *B* then *B*'s probability is no less than *A*'s. So what logic is being presupposed here, and is not any such assumption contentious seeing as it is precisely the nature of logic that is *sub iudice*? The answer is that the logic presupposed by Bayesianism is the propositional calculus, which any serious candidate logic will extend. That is to say, if *A* is a propositional tautology then it is a tautology full stop, and likewise if *A* propositionally entails *B* then it entails *B* full stop; any candidate for the correct logic will have to sanction these conditionals. Hence no circularity dogs the use of probabilistic methods to frame the argument from logic. This first objection is easily met.

The second objection is that the negation of Christian theism (~Φ) is a catchall hypothesis. And as several authors have pointed out in other contexts,[38] the probability of evidence on catchall hypotheses is very hard to determine. For example, the negation of theism includes the possibility that God exists yet logic does not reflect His nature; or that the Devil exists and created logic in his own image; or that the Great Clown created logic in someone else's image, for a laugh; and so on. So how do we assign a prior probability to these multifarious hypotheses? And what is *LMI*'s conditional probability on any given one of them? Swinburne (1990) and others have appealed to considerations of simplicity to raise or lower the prominence of various hypotheses. But what this discussion has tended to show is that the issues are vexed and depend on strong prior commitments, for instance on the relative implausibility of a malevolent creator as opposed to a benevolent one. Sober (2018) sees such arguments as irreparably flawed for this reason, and thinks it sensible in such cases only to compare the likelihood of the evidence based on theism and that of the same based on an equally specific hypothesis, such as naturalism. Although abandoning Bayesianism may be too drastic a reaction, Sober's wider point is well-taken. $Pr(LMI/{\sim}\Phi)$ is hard to gauge.[39]

To make some progress, let us assume that naturalism is theism's main rival. The probability of ~Φ is then little more than that of naturalism, and k_2 may be equated with the probability of *LMI* given naturalism. The question then is how k_2 compares to k_1. Given the discussion in the third section, we may assume that $k_1 > k_2$, since theism supplies an additional reason to believe logic is maximally infinite. The multiplier $\frac{1}{\varphi + \frac{k_2}{k_1}(1 - \varphi)}$ depends sensitively on the value of k_2/k_1, and for values of k_2/k_1 close to 1, $\varphi + \frac{k_2}{k_1}(1 - \varphi)$ is approximately equal to φ

+ (1 − φ) = 1. The third objection is that this is indeed the case: k_2/k_1 is close to 1. Theism, according to this objection, adds little support to the idea that logic is maximally infinite; virtually all its support comes from the non-theistic arguments mentioned in the second section.

I take this to be the strongest objection to the argument from logic. Since assessing it in detail is too large a task, I shall merely sketch one promising line of response to it. The force of the objection depends on what exactly the prior and posterior probabilities represent, a question we have studiously avoided so far. Suppose we take probabilities to be something like philosophers' actual credences. So priors here are those of a philosopher before learning that logic is maximally infinite (*LMI*). Now, the most prevalent candidate logics are finitary, and most philosophers are not particularly au fait with arguments for this or that logic being the correct one, still less do they have strong views about the sorts of arguments mentioned in the second section above. Moreover, some philosophers lean more towards inferentialism. So presumably k_2, i.e. the probability of *LMI* given naturalism (recall that we have agreed to equate naturalism with the negation of theism), is not very high.

Now k_1, the probability of *LMI* given theism, includes any non-theistic grounds one might have for *LMI* as well as theistic grounds. As we saw in the third section, Christian theism meshes with *LMI*, though does not entail it. The amount by which k_1 exceeds k_2 thus depends on how optimistically one reads the arguments in the third section. My own view, as expressed there, is that since k_2 is somewhat low, k_2/k_1 is still considerably smaller than 1, even if k_1 is not particularly close to 1. For as we saw, there is a reasonably suggestive—albeit far from watertight—case from Christian theism to logic's maximal infinity. Suggesting exact figures would of course introduce spurious precision, but the broad point stands: k_2/k_1 is not that high, and thus the boost that logic's maximal infinity gives theism is not that low, not because k_1 is particularly high, but because k_2 is on the low side. To develop this response, of course, one would have to do at least three things: pin down the value ranges of k_1 and k_2 further; consider how to apply the arguments to philosophers or others who already have firm views on the nature of the correct foundational logic; and examine other interpretations of the probabilities in question.

To sum up, the circularity objection was a non-starter. The second objection forced a move to a likelihood version of the argument, in which we compared the probability of *E* given theism to that of *E* given a more specific hypothesis, such as naturalism. Since theism and naturalism carve up much of the relevant territory, though, there is a sense in which the Bayesian and likelihood versions of the argument are fairly similar. The third objection was the strongest. How the relative probabilities compare remains unclear, and the success of the overall argument might hinge on how we understand the probabilities involved. Still, on at least one of its construals, the argument from logic does seem promising.

Conclusion

That the correct logic should bear some of the hallmarks of God is a striking fact, demanding explanation. For the theist, especially the Christian theist, these features of logic broadly cohere with her understanding of God. Her beliefs tend to favour a maximally infinite logic. Conversely, as we saw, learning that logic is maximally infinite should boost your degree of belief in Christian theism, though by how much remains to be more precisely determined. There is no irresistible argumentative route either from theism to logic's maximal infinity or from logic's maximal infinity to theism. But if the above is along the right lines, the two are mutually supportive.[40]

Appendix

The appendix fills in some of the technical detail omitted in the main text.

Invariantism

We illustrate the idea of permutation invariance with an example. We may construe the identity relation over an underlying domain as the set of all and only the ordered pairs $\langle a, a\rangle$ for every a in the domain. A permutation on the domain induces a permutation on these ordered pairs: if the permutation maps a to b, the induced permutation maps $\langle a, a\rangle$ to $\langle b, b\rangle$. The extension of the identity relation is invariant under permutation because any permutation maps the set of all ordered pairs of elements of the domain to itself. For example, in the four-membered domain $\{a, b, c, d\}$, the extension of the identity relation is $\{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle d, d\rangle\}$; under the permutation which maps a to b, b to c, c to d and d to a, its image is $\{\langle b, b\rangle, \langle c, c\rangle, \langle d, d\rangle, \langle a, a\rangle\}$, the set itself. This shows that the denotation of the identity predicate is invariant; hence on the invariantist criterion of logicality, identity is logical.

Contrast say the unary relation *being a profound thinker* over the four-membered domain consisting of Aristotle, Hypatia, Donald Trump and Paris Hilton. I trust I will not ruffle too many feathers by taking its extension as the doubleton set {Aristotle, Hypatia}. The image of {Aristotle, Hypatia} under any permutation mapping ancients to moderns is {Donald Trump, Paris Hilton}, a set distinct from {Aristotle, Hypatia}. As the relation's extension in the permuted domain is distinct from its image under the permutation, it fails to be permutation invariant over this domain. So 'being a profound thinker' is not a logical constant, because its denotation, the relation *being a profound thinker*, is not invariant.

The isomorphism-invariance account of logicality generalises the idea behind these examples, replacing permutations on a single domain with isomorphisms between several domains.

Infinitary logic

Mathematical logicians have developed many logics that allow for infinitary operations. An important example is the logic $\mathscr{L}_{\infty\infty}$ (pronounced 'ell infinity infinity'). To get a feel for this logic, observe that in standard logic, finite conjunctions or disjunctions are permitted: if, for example, A, B and C are formulas, so is A & B & C. However, infinite conjunctions such as A_1 & A_2 & … & A_n & … are not permitted (nor are infinite disjunctions). Likewise, in standard logic we may quantify over any finite number of variables: $A(x, y)$ may for instance be prefaced by $\exists x\forall y$ to yield $\exists x\forall yA(x, y)$. But we may not quantify over infinitely many variables.

Infinitary logics relax these constraints. In particular, $\mathscr{L}_{\infty\infty}$ extends the predicate calculus by allowing quantification over any number of variables (κ-many for any cardinal κ, finite or infinite), and allows conjunction or disjunction over κ formulas, again for any cardinal κ. Just as the predicate calculus can express the claim that there are 3 things, or 52 things, or n things for any finite n, so $\mathscr{L}_{\infty\infty}$ can express the claim that there are κ things for any cardinal κ (finite or infinite). For example, it can express the claim that there are infinitely many things by conjoining the infinitely many inequations '$x_i \neq x_j$' (for i and j distinct natural numbers) and existentially quantifying over all the variables (x_0, x_1, x_2, …) in the resulting formula. Intuitively, this infinitary formula is:

$$\exists x_0 \exists x_1 \exists x_2 \ldots (x_0 \neq x_1 \;\&\; x_0 \neq x_2 \;\&\; x_1 \neq x_2 \;\&\; \ldots)$$

'Bottom-up' arguments

Unlike the 'top-down' invariantist argument, 'bottom-up' arguments for the infinity of logic do not rest on theoretical assumptions about the nature of the logical constants. Consider the argument

> There is at least one planet.
> There are at least two planets.
> There are at least three planets.
> ⋮
> There are at least *n* planets.
> ⋮
> Therefore: There are infinitely many planets.

If we accept—as it seems we ought to—that this argument is valid, we are forced to go beyond predicate logic. Predicate logic cannot capture its validity because it cannot capture the validity of arguments whose conclusion follows from infinitely many premises but no finite subset of them. This is a bottom-up argument because it does not presuppose the logicality of 'there are infinitely many'; rather, it infers it from less theoretical commitments. Furthermore, accounting for the 'planets' argument's generalisations takes us all the way to a maximally infinitary logic. (All this is spelt out in much greater detail in ch. 5 of Griffiths & Paseau (forthcoming).)

Natural-language arguments revealing logic's infinitude include not just the 'planets' argument above, but others too, which pack in an infinitary conjunctive or disjunctive content more subtly than the word 'infinite' does. For example, the word 'ancestor' is equivalent to the infinitary disjunction 'is a parent', 'is a grandparent', 'is a great-grandparent', and so on. That is why the following argument is conceptually valid:

> Al is not my parent.
> Al is not my grandparent.
> Al is not my great-grandparent.
> ⋮
> Al is not my great^{n}-grandparent.
> ⋮
> Therefore: Al is not my ancestor.

To turn this argument into a logically valid one we must add the definitional premise that an ancestor is a parent, or a grandparent, or a great-grandparent, etc. In a logic which allows countably infinite conjunction, the validity of the argument thus augmented is easily captured, since, given the definition of 'ancestor', the argument's conclusion is equivalent to the conjunction of its infinitely many premises.

The 'Al' and 'planets' arguments afford us glimpses of logic's infinitary nature. We grasp logic's maximal infinity opaquely, through a glass darkly. To pursue the Pauline metaphor, 'face to face' knowledge of infinitary logics lays bare the true nature of such implications. Seeing them for what they are (reminiscent of First Epistle of John 3:2), we appreciate that their conclusions are simply equivalent to the infinitary conjunction or disjunction of the argument's premises. Likewise, with knowledge of God, afforded us opaquely and indirectly.

Pursuing the analogy even further, to say that logic is maximally infinitary may be likened to an apophatic statement, since it is tantamount to denying that it can be bounded or measured by some cardinal (finite or infinite). We may be able to identify infinitary logic, say equate it with $\mathscr{L}_{\infty\infty}$. But as finite knowers we cannot unfold all its content—e.g. generate its set of logical truths, even in principle—unlike God. Like God's knowledge (2 Corinthians 12:4), it is, for us, inexpressible.[41] The broad idea, then, is that our epistemic limitations vis-à-vis God are roughly parallel to our epistemic limitations vis-à-vis a maximally infinite logic.[42]

Absolute vs transfinite infinity

In the third section we affirmed that God, if He is one of the two, is absolutely rather than transfinitely infinite. For one thing, it is a consequence of the usual theistic understanding of God that He is unbounded as well as infinite. The passage from Job cited earlier and other biblical ones suggest there is no way to bound God's infinity. This conflicts with the idea that God is of particular transfinite size. And it seems ruled out by God's being, in Anselmian terms, that than which nothing greater can be conceived—maximally great.

Second, identifying God's infinity with a particular transfinite size would be arbitrary, even—dare I say—silly. Is God's infinity equal to $\aleph_2$? $\aleph_{707}$? $\aleph_\omega$? There is simply no obvious transfinite candidate for God's infinity. In contrast, there is one and only absolute infinity, transcending all transfinite ones.[43] Faced with a choice between the two, Cantor was surely right to identify God with absolute infinity, the domain of theology, as he put it, rather than any transfinite cardinal, the domain of mathematics and metaphysics. In correspondence with Cantor, Cardinal Franzelin (a papal theologian to the First Vatican Council) approved this very formulation.[44]

Of course, the distinction between transfinite and absolute infinity has no scriptural foundation. But this seems a clear case of accommodation in Calvin's sense: as their message is adapted to human understanding, it stands to reason that the scriptures should draw no such distinction. After all, no-one prior to the 1870s would have been capable of comprehending it; even today, few do.

Inferentialism

As explained, inferentialism offers a different approach to the logical constants from the invariantism described in the main text.[45] Inferentialism about the logical constants was popular a few decades ago, but has suffered a reversal of fortune since. The many reasons include: (a) its generalisation to the rest of language—inferential role semantics—has proved problematic; (b) inferentialism does not mesh with the generally semantic approach taken in linguistics and logic, e.g. as mentioned, the dominant account of logical consequence is model-theoretic rather than proof-theoretic; (c) no clear inferentialist criterion for logicality has emerged; (d) it is generally recognised that one can use a logical constant in a deviant or non-standard way whilst perfectly grasping its sense, through ignorance, error, philosophical cussedness, or for some other reason.[46]

Invariantists for the most part agree that logic is maximally infinite, as outlined above. In contrast, all inferentialists think that logic is finitary, many identify it with the predicate calculus, and virtually all take it to underwrite a complete logic (in the sense of 'complete' spelt out in the main text). This is because they take inferential relations to determine the meaning of an expression; more narrowly, they see the meaning of the logical constants as given by their characterising rules. A typical inferentialist, for example, maintains that the meaning of the sentential connective ‘and’ is given by its introduction and elimination rules. These rules may be depicted as follows:

$$\frac{A \qquad B}{A \text{ and } B} \qquad \text{(introduction rule)}$$

$$\frac{A \text{ and } B}{A} \qquad \text{(first elimination rule)}$$

$$\frac{A \text{ and } B}{B} \qquad \text{(second elimination rule)}$$

The introduction rule gives the circumstances under which it is appropriate to assert the sentence ‘*A* and *B*’, *viz*. just when it is appropriate to assert *A* and appropriate to assert *B*. The two elimination rules specify what follows from an assertion of ‘*A* and *B*’, *viz*. each of the two conjuncts. Taken together, the rules are supposed to capture the use of ‘and’. Similar rules may be given for the other logical constants. Because inferentialism is broadly speaking a syntactic rather than a semantic account of logical constanthood and consequence, it underwrites a complete logic. As logical consequence is ultimately defined by a system of rules, these rules cannot but define a complete logic.

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[1] You might think that all arguments make use of logic, by definition, but I am allowing for non-deductive (e.g. abductive) arguments. Millican (2019) surveys the role of logic in theistic and anti-theistic arguments respectively.
[2] As opposed to arguments, by Descartes for instance (see below), that invoke God to explain logical truths, whatever these are.
[3] Talk of the ‘argument from God’ or of a link between God and logic is a little loose, as on the traditional Christian view, Jesus is both the *Logos* and God the Son, but not God *simpliciter*, as clarified below.
[4] For a defence of ‘logical monism’—arguably the mainstream view among philosophers of logic—see Griffiths & Paseau (forthcoming).
[5] The model-theoretic conception’s full articulation had to wait until Tarski & Vaught (1956).
[6] For simplicity, we assume a standard classical perspective and set aside logical heresies, such as intuitionism, dialetheism, Karl Barth’s views on the law of non-contradiction, etc.
[7] One-to-one because if $f(x) = f(y)$ then $x = y$. Onto because if y is an integer then $y = f(y - 1)$.
[8] Sher replaced permutation invariance with bijection invariance and added some further conditions. For a summary, see Sher (2013, p. 176).
[9] Griffiths & Paseau (forthcoming).
[10] The top-down vs bottom-up terminology is borrowed from Part II of Griffiths & Paseau (forthcoming).

[11] Such as second-order logic with full/standard semantics.
[12] ‘Nothing is necessary or impossible except because He so wills it’ (Anselm 1094–8/1858, bk 2, ch. 18, p. 98).
[13] Letter to Mersenne, 15 April 1630 (Adam & Tannery I, p. 145); quoted on p. 198 of Mawson (2001).
[14] Sections 44–6 of the *Monadology*.
[15] For instance, Brian Leftow’s 2012 book is on God and *broadly* logical possibility, i.e. metaphysical possibility.
[16] The fact that this system, and its modest extensions, was the only one known to religious philosophers prior to the 19th century, and *a fortiori* prior to the Enlightenment’s secularisation of philosophy, might well explain why no ‘argument from logic’ of the type considered here has hitherto been essayed. Aristotle’s rejection of the actual infinite (see especially *Physics* III) also explains Christian theologians’ resistance to this concept, though that is a slightly different story. Hart (2011) and Russell (2011) discuss the transformation of infinity over the centuries from a ‘negative’ to a more ‘positive’ concept. Moore (2019) is a vademecum to infinity’s role in philosophy.
[17] For a review, see Achtner (2011), and for some quotations and citations, see e.g. Hick (2004, p. 238). Philo’s doctrine of the divine *Logos* is a development of the Stoic idea of reason or plan that organises unformed matter; see Krainer (2019) for this and other antecedents in Greek thought. Kelly (2014) is an authoritative account of early Christian doctrines.
[18] Depending on how you translate the Latin ‘immensus’.
[19] Of course, some theists may demur; e.g. process theologians, who see God as finite.
[20] For more on the New Testament identification of Jesus with the Word, see ch. 9 of Cullmann (1963). Cullman also briefly discusses the *Logos* in Judaism, both in its earliest form as the Word of God (*debar Yahweh*) and its later form as *the* Word (*tout court*).
[21] I *Apology* V. For Justin Martyr’s works, see Minns & Parvis (2009).
[22] Whose writings contain the first extant extra-biblical commentary backing P2 (Kynaston 2018, p. 69).
[23] Cited in Morris (1991, p. 29).
[24] Liberation theology is an example that comes to mind.
[25] Branson (2019) and Tuggy (2018) are two recent overviews of the so-called logical problem of the Trinity.
[26] Kynaston (2018) contains a good deal on the Islamic understanding of Jesus as God’s Word.
[27] Incidentally, this is what distinguishes the *Logos* Argument from an argument to mathematics. Although mathematics may in some sense be a part of God or caused by God or created by God, it is not a manifestation of God in the way logic is, assuming God the Son is the *Logos*. Mathematics, though it may be *applied* to anything, is not in any obvious sense a manifestation of the *Logos*. (Logicism aside, of course.) For recent arguments from mathematics along other lines, see for example Menzel (2018) or Goldschmidt (2018), both of whom heeded Alvin Plantinga’s call in a late 20th-century retrospective on Christian philosophy (Plantinga 1998) to develop such arguments.
[28] Compare Gordon Clark’s (1980) view that God is logic, which Clark identifies with Aristotelian logic for no clear reason.
[29] Thomasius (1857).
[30] As argued in Brunner (1952, p. 20).
[31] For a critique of kenotic Christology, see e.g. Morris (1991, pp. 88–102, p. 149).

[32] To take a related example, that Jesus self-identifies with the truth (John 14:6) instructs Christians that God the Son is in a sense truth; but it scarcely reveals the logical structure of truths.
[33] Christian theists not au fait with logical terminology might find it unpalatable that the *Logos* is incomplete, as incompleteness seems to be a deficiency. But 'incomplete' here is a logical term of art; as earlier, it means roughly that there is no finite reasoning procedure that generates all the logical truths. So to say that the *Logos* is incomplete in this technical sense is not to impute a deficiency to it. The deficiency lies not with the *Logos* but with us, who cannot 'complete' the set of logical truths by the finitary means available to us.
[34] There may be others too, which I hope to explore in further work.
[35] A further question concerns the ontology of logic. Platonism about logic or mathematics is seen by some theists as a threat to God's aseity; see Craig (2016) for a recent example and discussion. As I see it, that logic is infinitary is compatible with a host of views about its ontology, though that argument must await another occasion.
[36] The reason: the probability that *B* given *A* divided by the probability of *B* is equal to the probability that *A* given *B* divided by the probability of *A* (assuming neither divisor is zero).
[37] A pioneer being Swinburne (1990).
[38] E.g. Sober (2018).
[39] A similar point could be made about an abductive version of the argument, such as:

> The similarity of logic to God as conceived by Christian theists is a striking coincidence.
> The combination of Christian theism and the invariantist arguments in the second section offers the best explanation for this coincidence.
> Therefore: Christian theism is true.

Casting the argument as an inference to the best explanation does not overcome the difficulty just noted. Part of what makes an explanation good or bad is its antecedent plausibility. So we have to rely on some appreciation of the antecedent plausibilities not just of Christian theism but of various explanatory hypotheses incompatible with it, in order to crown the invariantist-theist combination as the best explanation.
[40] Thanks to Bernhard Salow, Daniel Hill, Owen Griffiths, Tim Mawson and two *Religious Studies* referees for comments on an earlier draft, to Rupert de Borchgrave for stimulating conversations about the *Logos*, and to Brian Leftow and Tim Winter for supplying some references. I wrote this article whilst holding a Research Fellowship awarded by the Leverhulme Trust, whose support I gratefully acknowledge.
[41] There is a tradition within theism that immediate knowledge of God, even union with Him, *is* possible in this sublunary world; see for example Louth (2007), which examines the Christian mystical tradition. Alston (1991) describes experiences of God on the model of sense perception. However, direct apprehension of God is uncommon, or, if common, discloses His identity only very imperfectly.
[42] How exactly to understand our epistemic limitations vis-à-vis God's nature is of course a vexed issue. Oppy (2011) is a recent philosophical analysis.
[43] For the *cognoscenti*: we have skated over a subtlety here. In some second-order systems of set theory, or proper class theories, not all proper classes need be of the same size, e.g. it may be left open whether *On* (the class of ordinals) is the same size as *V* (the class of sets). The theist is likely to see God's infinity as akin to that of *V*, since clearly any proper class injects into it. And in systems with an appropriate Limitation of Size principle, or second-order Choice, all proper classes are provably of the same size: that of *V*.

[44] For much more on Cantor’s theistic understanding of set theory, see Jané (1995) and Dauben (1979), especially ch. 6. In this chapter (based on Dauben 1977), Dauben also summarises Catholic theologians’ reaction to Cantor’s theory of infinity. Hedman (2019) is a more recent account.

[45] Steinberger & Murzi (2017) is an introduction to inferentialism.

[46] See ch. 4 of Williamson (2007).