5 Laws of Thought and Laws of Logic after Kant

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Road Map

In the mid-1800s, George Boole developed a theory of logic as an instrument for representing the structure of mathematical problems. For Boole, the laws of logic are laws of thought, but they are not (merely) psychological laws, for all that. Boole's approach draws on a dialogue within the post-Kantian "New Analytic" tradition, then current in the United Kingdom.[1]

Some contemporary inferentialists argue "against the view that there are facts of matters of logic that obtain independently of us, our linguistic conventions and inferential practices" (Resnik 1999, 181). Boole, along with those in the post-Kantian tradition who influenced him, the New Analytic, took the position that the laws of logic are laws of thought. Logical laws govern inferences. In that case, there can be facts of the matter about logic, namely facts about the laws of thought that are valid principles of inference in certain domains. One of the key insights of Boole's method is that the epistemic status of such laws can be established by studying logic's application to solving problems in mathematics.

Boole's approach is one origin of the contemporary discipline of model theory, which has branches in philosophy, logic, and mathematics. Model theory analyzes the notion of a given proposition being true under an interpretation (Hodges, forthcoming). Alfred Tarski's "The Concept of Truth in Formal Languages" (1933/1983) is a seminal paper in this tradition. As Hodges notes, Boole's work is a significant precursor to the model-theoretic approach. In particular, Boole pays close attention to the extent to which logic can represent problems in algebra so that, within a given logical interpretation, results in algebra can come out true – that is, desired solutions can be found. Boole's approach takes its cue from critical responses to Kant and from the British reception of

1 Realists about logic take the position that logic is a domain of truths independent of any particular subject matter and of our inferential practices and subjective constitution. Inferentialists need not take any stance about the correspondence of logical propositions to reality or truth. Rather, logical propositions, relationships, and terms acquire their meaning from the use to which they are put in inferences. Peregrin 2014, and Resnik 1999, among many others, define inferentialism in these terms.

the post-Kantian logicians Wilhelm Traugott Krug, Wilhelm Esser, and Jakob Friedrich Fries.

Logic as Art; Logic as Science

In 1826, Richard Whately published the *Elements of Logic*.[2] Before Whately, much of British logic was in the Lockean tradition, seeing logic as the "art" of thinking about the truth, not as a "science" that discovers novel truths. Levi Hedge's 1818 *Elements of Logick* "well illustrates the prevailing view of logic in the Anglophone world before Whately" (Heis 2012, 102). Levi Hedge argues that logic "traces" the development of thought from perception to judgment. Like an artwork, then, logic attempts to give a rendering, tracing, or picture of judgments of truth. But Hedge believes that logic cannot itself prove truths, much less discover them.

Richard Whately responds, to Hedge and to others, that logic is a science as well as an art. For Whately, logic provides "an analysis of the process of the mind in reasoning" and to that extent is strictly a science (Whately 1870, §1, 1). However, logic also concerns itself with "practical rules" for "guarding against erroneous deductions", and, to that extent, logic is an art (§1, 1). Whately stipulates that "a *science* is conversant about speculative *knowledge only*, and *art* is the application of *knowledge* to *practice*" (§1, 1). Whately maintains that the scientific element of logic consists of speculative knowledge about the reasoning process, while an equally significant element of logic consists of applying that speculative knowledge to reasoning in practice.

The Scottish philosopher William Hamilton[3] wrote a substantial review of Whately's *Elements* in which Hamilton took the Anglophone logicians to task for neglecting "contemporary German logics". The debate over whether logic is an art or a science presumes that logic is either screened off from the content of science (art), or is itself an independent tool for the discovery of psychological or metaphysical truths (science). Hamilton defends a different position: that logic can consist of a set of truths, but that they are *formal*, not substantial truths.

According to Hamilton, the Anglophone tradition at the time had no analogue of Kantian formal logic, which is why Hedge, Whately, and others were stuck. As Heis (2012) summarizes Hamilton's account,

> we can more adequately purify logic of intrusions from psychology and metaphysics and more convincingly disabuse ourselves of the conviction that logic is an "instrument of scientific discovery" by

2 For the logical context of Hamilton's work, I draw on Jeremy Heis's excellent essay "Attempts to Rethink Logic" (Heis 2012).

3 Not to be confused with the Irish mathematician William Rowan Hamilton.

> accepting Kant's idea that logic is *formal*.[4] Hamilton's lectures on logic, delivered in 1837–8 using the German Kantian logics written by Krug and Esser (Krug 1806, Esser 1823) thus introduced into Britain the Kantian idea that logic is formal.[5]

If logic is formal, Kant argued, then logic can be a "canon" of rules of inference that have validity over a certain domain. But logic does not, itself, expand the domain of our knowledge: logic is not what Kant calls an "organon" or what Hamilton calls an "instrument of scientific discovery".[6] The laws of thought are normative, formal rules describing "how we *ought* to think", rather than descriptive, psychological laws telling us "how we *do* think".[7]

While Hamilton criticizes Kant's reasoning about 'regulative ideas' and Kant's account of judgment using the categories (*Kategorienlehre*), he adopts Kant's notion of logic as a formal science and Kant's divorce of logic from psychology.[8] Hamilton combines the idea that he had borrowed from common sense philosophy, that thought presupposes principles of thinking, with Kantian formal logic.[9] In the review, Hamilton writes,

> Logic they [the Kantian logicians] all discriminated from psychology, metaphysic, &c. as a *rational*, not a real, – as a *formal*, not a material science. – The few who held the adequate object of logic to

4 Hamilton, "Recent Publications," 139.

5 Heis (2012), 103. Heis cites "Hamilton, *Logic*… from the 1874 3rd ed. (Original edition, 1860)".

6 "General logic for Kant contains the 'absolutely necessary rules of thinking, without which no use of the understanding takes place' (A52/B76). The understanding – which Kant distinguishes from 'sensibility' – is the faculty of 'thinking,' or 'cognition through concepts' (A50/B74; Ak 9:91). Unlike Wolff, Kant claims a *pure* logic 'has no empirical principles, thus it draws nothing from psychology' (A54/B78). The principles of psychology tell how we *do* think; the principles of pure general logic, how we *ought* to think (Ak 9:14). The principles of logic do not of themselves imply metaphysical principles; Kant rejects Wolff and Baumgarten's proof of the principle of sufficient reason from the principle of contradiction (Ak 4:270). Though logic is a canon, a set of rules, it is not an organon, a method for expanding our knowledge (Ak 9:13)" (Heis 2012, 98).

7 Ak 9:14; Heis 2012, 98.

8 See Durand-Richard (2000), §2.3, for more details and historical background on the material in this section.

9 "For [Hamilton], the form of thought is the kind and manner of thinking an object (I 13) or the relation of the subject to the object (I 73). He distinguishes logic from psychology (against Whately) as the science of the *product*, not the *process*, of thinking. Since the forms of thinking studied by logic are *necessary*, there must be *laws* of thought: the principles of identity, contradiction, and excluded middle (I 17, II 246). He distinguishes physical laws from 'formal laws of thought,' which thinkers *ought* to – though they do not always – follow (I 78)" Heis 2012, 103–4.

be *things in general*, held this, however, under the qualification, that things in general were considered by logic only as they stood under the general forms of thought imposed on them by the intellect, – *quatenus secundis intentionibus substabant*. – Those who maintained this object to be the *higher processes of thought*, (three, two, or one,) carefully explained, that the intellectual operations were not, in their own nature, proposed to the logician, – that belonged to the psychologist, – but only in so far as they were *dirigible*, or the subject of laws.[10]

Hamilton identifies the key contribution of "intellectual operations" as not their nature or particular content but their lawlikeness. That is why formal logic can be a kind of a science, as well as an art. It doesn't merely retrace the justification for a particular inference; it also provides laws that are valid for inferences in other domains. Hamilton's initial response to the debate about whether logic is an art or a science is to argue that logic is a formal practice, describing normative laws of reasoning, which can be the basis of inferences beyond the initial domain in which they are analyzed. In this sense, logic is a science, but it is a merely formal one. According to Hamilton,

> Logic is a formal science; it takes no consideration of real existence, or of its relations, but is occupied solely about that existence and those relations which arise through, and are regulated by, the conditions of thought itself. Of the truth or falsehood of propositions, in themselves, it knows nothing, and takes no account: all in logic may be held true that is not conceived as contradictory. In reasoning, logic guarantees neither the premises nor the conclusion, but merely the *consequence* of the latter from the former; for a syllogism is nothing more than the explicit assertion of the truth of one proposition, *on the hypothesis* of other propositions being true in which that one is implicitly contained.[11]

Mill and the New Analytic

As is well known, in 1865, John Stuart Mill published *An Examination of Sir William Hamilton's Philosophy*. In this astonishingly long work – it has two volumes, and volume one is 650 pages – which went through several subsequent editions, Mill subjects Hamilton's work to searching criticism. One of the central points of Mill's criticism is Hamilton's

10 Hamilton 1866/1833, 137. The notion that operations must be *lawlike* will be central to Boole's theory as well.
11 Hamilton 1866/1833, 144.

exclusion of "whatever relates to Belief and Disbelief, or to the pursuit of truth as such" from logic and his restriction of logic to "that very limited portion of its total province, which has reference to the conditions, not of Truth, but of Consistency".[12] Mill disagrees: for him, logic must be a science of truth, or it is not a science at all. Mill objects to Hamilton's theory on practical grounds, as well. He argues that Hamilton does not provide a rigorous way to distinguish, in practice, between the formal and the material elements of logical inferences (Mill 1882, 25 and passim). As a result, Hamilton's attempt to defend logic as a purely formal science fails.

A number of figures within the New Analytic tradition responded to Mill's criticisms, including Francis Bowen (1874), Henry Mansel (1866), James McCosh (1869), and William Thomson, whose response appeared in the many editions of his *Outlines of the Laws of Thought.* Mansel and Thomson, in particular, stress the Kantian notion that thought is a free product of the mind.[13] Mill had objected that Hamilton could not identify in a reliable way what was formal and what was material in a given logical inference. Thomson responds that we can identify logical laws because they are the freely chosen tools we use to investigate the phenomena. Since we choose the logical tools, which Thomson describes as a priori "rules" (see below), we can make a distinction between what is formal and a priori and what is material in any domain we investigate using logical reasoning.

Thomson's *Outlines*[14] begins with an explicit statement that logical reasoning is prior to logical laws, which is why logic is a science. Logical laws do not express psychological laws or metaphysical truths. Instead, logic is a science *of* scientific knowledge, because it encodes the rational process of coming to have scientific knowledge.

It's said, in language reminiscent of a Platonic dialogue,

> Poems must have been written before Horace could compose an 'Art of Poetry,' which required the analysis and judicious criticism of works already in existence. Men poured out burning speeches and kindled their own emotions in the hearer's breast, before an Art of Rhetoric could be constructed.[15]
>
> And wherever our knowledge of the laws of any process has become more complete and accurate; as in astronomy, by the

12 Mill 1882, 25.

13 Mansel (1866) links his argument to a theological one defending the freedom of the will, with which Mill is hardly likely to have been impressed.

14 I cite from the first edition, 1849. While of course only the later editions respond to Mill, Boole's early work responds to the earlier editions of Thomson (see the next section). Citations modernize Thomson's spelling to reduce the irritation of the reader.

15 Thomson 1849, 1.

> substitution of the Copernican for the Ptolemaic system; in history, by a wiser estimate than our fathers had the means of forming, of modern civilization and its tendencies; in chemistry, by such discoveries as the atomic theory and the wonders of electro-magnetism; our progress has been made, not by mere poring in the closet over the rules already known, to revise and correct them by their own light, but by coming back again and again to the process as it went on in nature, to apply our rules to facts, and see how far they contradicted or fell short of explaining them.[16]

To borrow an example from Hans Reichenbach, when we choose to use a meter stick to measure a table, we can establish how many meters long the table is. The measurement yields statements about the properties of the table. One such statement might be "This table is seven meters long". The word "meter" in that statement is a feature not of the table itself but of the standard we used to measure the table. While the statement of the properties of the table mixes formal and material content, we can nonetheless identify in practice what is formal and a priori. For instance, we know that we chose to employ the 'meter' as a standard of measurement. How many meters the table measures is a material property of the table, while the standard of measurement used is an a priori decision.

Logic is a science and not an art, because logic uses an experimental method to uncover the rational justification of scientific knowledge. We use logical "rules" to explain the facts as they emerge and to explain natural processes. By trial and error, we discover to what extent the rules can account for the facts and where we have gone wrong.

Thomson's experimental method evades Hamilton's difficulty of trying to find some principled way of distinguishing between the matter and the form of logical inferences. Moreover, it allows for the possibility that logic can be a formal science: by accounting for all the inferences in natural science. Logic in Thomson's theory is also an art – but it is not a merely aesthetic art of "tracing" inferences, as Levi Hedge had argued. Thomson's logic is the "art" of finding the justification for scientific inferences that result in knowledge. Work in the New Analytic tradition undermines the distinction that others had tried to make between logic as art and logic as science.

George Boole, *An Investigation of the Laws of Thought*

In 1854, five years after the publication of Thomson's *Outlines*, George Boole published *An Investigation of the Laws of Thought*. Boole's *Investigation* responds explicitly to the New Analytic tradition. However, Boole goes well beyond that tradition: in proposing a distinctive method

16 Thomson 1849, 2.

for logic, in linking algebra and logic, and in specifying a particular domain for the justification of the laws of logical inference.

Boole was spurred to take on questions of logic by the priority dispute between De Morgan and Hamilton over the quantification of the predicate. During this dispute, Hamilton argued that logic and mathematics should be separated, because philosophy "answers the question 'Why?', whereas mathematics is credulous in its premises" (Gray 2014, 99). As Boole notes, Hamilton even argues that the study of mathematics is "at once dangerous and useless" (Boole 1847, 11). Boole responds that, while "Of Sir W. Hamilton it is impossible to speak otherwise than with that respect which is due to genius and learning", he disagrees (Boole 1847, 12).

The disagreement is embodied in Boole's fluid employment of the methods of algebra in logical reasoning. Hamilton argues that logic, formal reasoning, must be separated from science as a doctrine of truth and reality. Insofar as Hamiltonian logic is successful, it must *correspond* to truth and reality – but logic itself is not an organon, it is a canon. Boole counters that, in the case of mathematics, logic can play the role Thomson assigns it. Logic can capture the justification for the inferences that result in scientific knowledge. When restricted to the domain of mathematics, logic can depict the reasons why inferences are justified and, to that extent, can be a doctrine of truth that yields real solutions to problems. While this may not amount to full Kantian objectivity, it nonetheless connects logic to mathematical science.

Boole's negative appraisal of Hamilton's position on the relationship between mathematics and logic could be taken, and often is taken, as a negative estimation of Hamilton's work generally. This chapter will encourage a reading on which Boole's critical reading is a step taken *within* the New Analytic tradition, to solve a problem for that approach: how are we to distinguish between the formal and the material content of logical inferences, and how are we to give a foundation for the laws of logic as laws of thought? Boole's project, conceived early on, was to show that applying logical and mathematical (algebraic) reasoning in a restricted domain could yield demonstrations of the validity and scope of logical laws as necessary laws of thought.

Here, we can distinguish two problem structures: first, issues in the foundations of mathematics, including the relationship between arithmetic and algebra, and the study and application of differential equations; second, the derivation of the laws of logic from the laws of the operation of the human mind.

Boole's account of algebra and of logic is intended to solve both problems.[17] Boole's early study of differential equations and complex

17 In working through Boole's contributions in these areas, my work is made much easier by the contributions of a recent volume on Boole (Gasser 2000), especially the essays by Durand-Richard and Panteki, and by recent work by Heis (2012) and Gray (2014).

numbers convinced him that there were holes in the foundations of the study of both.[18] Moreover, Boole's mentors, including Duncan Gregory and his predecessors such as George Peacock, were preoccupied with the question of how to give a foundation for calculation with "impossible" quantities. The question was partly occasioned by John Playfair's introduction of Laplace's *Mécanique celeste* and partly by the difficulties encountered when calculating with complex numbers and differential equations.[19] One response to such difficulties would be to argue that such quantities are merely tools of reasoning. However,

> the mathematicians of the English Algebraic School did not embrace instrumentalism. On the contrary, they were convinced that practices such as those of analytical algebra are fruitful only because they are founded on reason: efficiency results from the laws of symbolical calculus, which they considered it their job to discover... They thus sought to formulate explicitly the principles of a logical and symbolical calculus adequate for founding algebra (Durand-Richard 2000, 153–4).

It had been noted for some time that the operations of algebra and the study of differential equations can lead to the employment of mathematical and logical signs that seemingly have no meaning. Boole argues that they do have meaning if they are interpreted in the context of a demonstration according to the laws of thought that govern a symbolic calculus.

As van Evra (2000) notes, one wing of Boole reception is critical of the presence of "meaningless" symbols in his logical calculations. The presence of meaningless symbols *in the actual practice of mathematics* was precisely the *problem* that Boole was trying to solve in his earliest work. As van Evra observes correctly, Boole's aim was to show that there was a general method for logic that sprang from necessary laws of thought about a given domain of elective symbols. Once that general method and the laws of logic are justified, the laws governing inference could serve as a foundation for calculation even with meaningless symbols, because the symbols would be given a contextual definition within the confines of any given proof.

It is unjust, then, to fault Boole for the presence of meaningless symbols in his work. It is not as if Boole conceived of a general method, and then that method ran aground because it resulted in the presence of meaningless algebraic symbols. Boole was aware that mathematical practice in the English algebraic school had resulted in proofs involving

18 See Panteki 2000 for an excellent analysis, and of course, see Boole 1841 and Boole 1844.

19 Durand-Richard 2000, 152–3 and *passim*.

meaningless or even impossible quantities. His symbol language was intended to provide a secure way to deal with precisely those quantities. The presence of meaningless symbols in his work is a feature, not a bug.

In one of his earliest works of 1847, *The Mathematical Analysis of Logic*, Boole argues for a number of related theses. He splits from Hamilton, going so far as to argue that logic is a doctrine of truth, but he retains the Hamiltonian idea that logic does not deal with the real causes of things (p. 13 and *passim*). Logic is associated not with metaphysics but with mathematics. Election (the choosing of a variable, for instance), selection (selecting among the members of a class), and classification are mental acts or operations which are governed by laws. If those acts were different, the laws – and logic – would be different. Distributive and commutative laws, the syllogistic, and categorical and hypothetical judgments are expressible in elective symbols. The doctrine of elective symbols is independent of quantitative *origin,* though it may be expressed quantitatively. Boole agrees with those who thought formal logic should have a content autonomous of the general doctrine of magnitude, but he thought, explicitly, that that content was expressed mathematically.

By re-expressing mathematical equations in an elective symbol language, Boole provided a way to perform calculations in a distinct system, one that was governed by necessary laws of thought. Between 1847, when he wrote *The Mathematical Analysis of Logic*, and 1854, when he wrote *An Investigation of the Laws of Thought*, Boole became increasingly familiar with the work of the New Analytic.[20]

In the 1854 work, we find a more sophisticated account of how the laws of logic are necessary laws of thought. Boole begins with a method quite close to that defended by Hamilton, by Thomson, and later by Jevons and others:[21] to trace the development of science on the basis of principles taken as axioms. Boole focuses, however, on the role of mathematical thinking in the development of the sciences. Mathematical reasoning may consist in "rearranging" truths to show which are fundamental and which are derived. But such a rearrangement is by no means empty or merely negative.

> All sciences consist of general truths, but of those truths some only are primary and fundamental, others are secondary and derived.

20 In the Preface to *An Investigation*, Boole remarks, "That portion of this work which relates to Logic presupposes in its reader a knowledge of the most important terms of the science, as usually treated, and of its general object. On these points there is no better guide than Archbishop Whately's *Elements of Logic*, or Mr. Thomson's *Outlines of the Laws of Thought*" (Boole 1854, iii).

21 These include, in the German tradition, Adolf Trendelenburg, Hermann Cohen, and Ernst Cassirer, as well as, of course, Ludwig Boltzmann, Heinrich Hertz, David Hilbert, and the axiomatic tradition generally. See Patton 2009 and Patton 2014, including references to further work there.

> The laws of elliptic motion, discovered by Kepler, are general truths in astronomy, but they are not its fundamental truths. And it is so also in the purely mathematical sciences. An almost boundless diversity of theorems, which are known, and an infinite possibility of others, as yet unknown, rest together upon the foundation of a few simple axioms; and yet these are all general truths.
>
> (Boole 1854, 5)

Boole goes on to say that logic allows us to provide "uniform processes" from which we can "deduce" the results of science:

> Let us define as fundamental those laws and principles from which all other general truths of science may be deduced, and into which they may all be again resolved. Shall we then err in regarding that as the true science of Logic which, laying down certain elementary laws, confirmed by the very testimony of the mind, permits us thence to deduce, by uniform processes, the entire chain of its secondary consequences, and furnishes, for its practical applications, methods of perfect generality? Let it be considered whether in any science, *viewed either as a system of truth or as the foundation of a practical art*, there can properly be any other test of the completeness and the fundamental character of its laws, than the completeness of its system of derived truths, and the generality of the methods which it serves to establish.[22]

Thomson's method of testing is here taken as a fundamental method in *logic*, as well as in science. Once we have determined the relationships of interdependence in science, and discovered the basic mathematical statements on which the results depend, we can then re-derive those results using logic. That derivation requires us to find some way to compare logic and mathematics and to show that results in one can be reproduced in the other. Boole does not give an ultimate justification for this method. Instead, he argues, we can prove it by practical demonstration, based on the possibility of science itself.[23]

22 Boole 1854, 5, emphasis added.

23 "Whence it is that the ultimate laws of Logic are mathematical in their form; why they are, except in a single point, identical with the general laws of Number; and why in that particular point they differ; – are questions upon which it might not be very remote from presumption to endeavour to pronounce a positive judgment. Probably they lie beyond the reach of our limited faculties. It may, perhaps, be permitted to the mind to attain a knowledge of the laws to which it is itself subject, without its being also given to it to understand their ground and origin, or even, except in a very limited degree, to comprehend their fitness for their end, as compared with other and conceivable systems of law. Such knowledge is, indeed, unnecessary for the ends of science, which properly concerns itself with what is, and seeks not for grounds of preference

> [This book] is designed, in the first place, to investigate the fundamental laws of those operations of the mind by which reasoning is performed. It is unnecessary to enter here into any argument to prove that the operations of the mind are in a certain real sense subject to laws, and that a science of the mind is therefore possible. If these are questions which admit of doubt, that doubt is not to be met by an endeavour to settle the point of dispute a priori, but by directing the attention of the objector to the evidence of actual laws, by referring him to an actual science. And thus the solution of that doubt would belong not to the introduction to this treatise, but to the treatise itself.[24]

Boole argues that if we restrict the *domain* of the laws of the operations of the mind artificially, to the symbols 0 and 1, we can prove that the laws of logic and of mathematics both are valid in that domain.[25]

> Let us conceive, then, of an Algebra in which the symbols x, y, z, &c. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation alone divide them. Upon this principle the method of the following work is established.[26]

Based on this method, we can show that if a result is derivable in logic, then its counterpart is derivable in algebra. However, the converse is not the case: not every operation in algebra is logical. In particular, algebraic division has no counterpart in logic, as Boole is aware.

In the concluding chapters of *An Investigation*, Chapters V and following, Boole introduces "a fundamentally different topic. It is within these chapters that virtually all of the expressions with which the critics are concerned appear, and it is here that Boole lays out what he calls a 'general method in logic'" (Van Evra 2000, 90; see Boole 1854, 70). As van Evra notes, this method is strikingly innovative. It involves extending the operations of logic to domains other than logic, in order to

or reasons of appointment. These considerations furnish a sufficient answer to all protests against the exhibition of Logic in the form of a Calculus" (Boole 1854, 11).

24 Boole 1854, 3.

25 "Each of the functions serves as an analogue of its arithmetical counterpart, and the laws of logic correspond in like fashion with expressions in mathematics. Boole circumscribes the extent of the similarity by laying particular stress on the law of idempotence, xx =x, which holds universally in the logic, but in standard algebra, only for the values 0 and 1" (Van Evra 2000, 89).

26 Boole 1854, 378.

support results that go beyond logic itself – but also to show that logical operations can illuminate and support extralogical conclusions. Boole argues that

> We may in fact lay aside the logical interpretation of the symbols in [a] given equation; convert them into quantitative symbols, susceptible only of the values 0 and 1; perform upon them as such all the requisite processes of solution; and finally restore to them their logical interpretation.[27]

Logical laws govern sciences that do not belong entirely to logic itself. Extralogical operations, including operations with no logical counterpart like division, can be treated still with the Boolean calculus. But logical reasoning itself also can be expanded by applying it to operations outside the logical domain. The trick is the restriction of the values of a given expression to the "quantitative symbols" 0 and 1.

Concluding Remarks

Boole's work emerged from the difficulties found within the work of the English algebraists, who encountered seemingly impossible or meaningless quantities in their mathematical exploits. Boole borrowed William Thomson's "experimental" approach, arguing that, if the laws of logic are truly the laws of thought, then we should be able to use logic to retrace the demonstration of results within mathematics. Then, we can retranslate those results back into the language of logic and secure not only mathematics but logic itself.

But if logic is considered in its formal aspect, as the doctrine of the laws of thought and their consequences, then what is its content? If it has no content of its own, then we might conclude that logic is not an independent science but only a Lockean art of thinking. We might conclude, as many do, that logic depends on psychological laws and view these laws as contingent.[28]

Boole's approach on this score has much in common with the contemporary inferentialist and model-theoretic approaches in logic. For Boole,

27 Boole 1854, 70, original passage in italics. Van Evra (2000) remarks, "He is suggesting that any logical symbol may be treated as its mathematical counterpart in the manner laid down in Chapter II. Then any available mathematical operation may be used on it, whether that operation is logically interpretable or not. The final (mathematical) expression in the sequence must again be one which corresponds to a logical expression. With the purely mathematical interlude lying between, the sequence may then be treated as the inference of the final (logical) expression from the initial one" (p. 91).

28 See the very illuminating discussion in Kusch 1995, Chapter 1.

establishing the content of logical statements is only a matter of showing how a given symbol works in inference. Boole argues for what he calls the "directive" character of the logical calculus in constructing proofs. Boole's method is to establish a sphere of validity for the laws of logic, which are the laws laid down by the operations of the mind, as expressed in operations on arbitrarily chosen signs. Boolean algebra is based on the idea that, if we assign the values 0 and 1 to algebraic variables, the laws and axioms governing operations on those variables will be identical to the laws and axioms of logic:

> Let us conceive, then, of an Algebra in which the symbols x, y, z, &c. admit indifferently of the values 0 and 1, and of these values alone. The laws, the axioms, and the processes, of such an Algebra will be identical in their whole extent with the laws, the axioms, and the processes of an Algebra of Logic. Difference of interpretation alone divide them. Upon this principle the method of the following work is established (Boole 1854, 37–8).

Boole uses 0 and 1 as values for the arbitrarily chosen symbols to make it clear that there can be no case in which the laws of algebra, under such an interpretation, are distinct from the laws of logic. This, in turn, allows us to argue that algebra can be shown to be governed or interpreted by the laws and operations of logic.

Boole rejected any notion that the symbols of logic are chosen to resemble their objects or their content. Boole is quite clear in Chapters I and II of Boole (1854) that such symbols are arbitrary "signs" and even that classes or sets of objects are chosen by election. For Boole, logic does not track truth because it is a universal language that describes actual thought processes. It is a science because it is a flexible language capable of representing the structure of mathematical problems and because the laws governing logic also govern mathematical inferences.

Boole had a characteristic and innovative method of developing proofs within logic, of relating those proofs to results in mathematics, and he gave a fluid and flexible way to derive the foundations of both sciences. Understanding Boole's achievements requires looking more deeply into the Kantian tradition in logic and epistemology, the German logicians who built on that tradition, and on the reception of both in the English traditions that influenced Boole directly.

The above discussion traces the influence of the "New Analytic", the hidden tradition behind Boole. This tradition, unabashedly Kantian in its origins and motivations, was concerned with the status of the laws of logic. It was also concerned with the Lockean question, popular at the time, of whether logic is an art or a science: whether it has laws and results of its own, or whether it is the art that traces the sources of justification of the true sciences.

The influence of the New Analytic on Boole is deep but also mixed. Boole's approach, of showing the justification of the laws of thought as laws of logic, owes a great deal to the New Analytic. But his approach goes beyond theirs, in drawing an explicit connection between logic and algebra. Boole argues against the idea that logic is purely formal, which was central to the New Analytic approach. He defends the notion that logic has a content, independent of its formal properties as a system of inference. However, that content depends on *using* logic to depict the structure of problems in algebra and differential calculus, a method that Boole develops thoroughly and that became part of the origin story of model theory.

Acknowledgment

I would like to thank Sandra Lapointe for providing the intellectual framework and leadership for this project, for organizing excellent workshops that were the site for substantive collaboration with others working on this project, and for comments on a draft. At the workshops, Scott Edgar, Michael Forster, Gary Ostertag, Consuelo Preti, Nick Stang, Clinton Tolley, and Richard Zach asked questions that provoked reflection on the central questions of the paper and revision to its strategies and composition.

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