

Logical Inference and Its Dynamics

Carlotta Pavese¹

*Duke University
Philosophy Department*

Abstract

This essay advances and develops a dynamic conception of inference rules and uses it to reexamine a long-standing problem about logical inference raised by Lewis Carroll's regress.

Keywords: Inference, inference rules, dynamic semantics.

1 Introduction

Inferences are linguistic acts with a certain dynamics. In the process of making an inference, we add premises incrementally, and revise contextual assumptions, often even just provisionally, to make them compatible with the premises. Making an inference is, in this sense, moving from one set of assumptions to another. The goal of an inference is to reach a set of assumptions that supports the conclusion of the inference.

This essay argues from such a dynamic conception of inference to a dynamic conception of *inference rules* (section §2). According to such a dynamic conception, inference rules are special sorts of *dynamic semantic values*. Section §3 develops this general idea into a detailed proposal and section §4 defends it against an outstanding objection. Some of the virtues of the dynamic conception of inference rules developed here are then illustrated by showing how it helps us re-think a long-standing puzzle about logical inference, raised by Lewis Carroll [3]'s regress (section §5).

2 From The Dynamics of Inference to A Dynamic Conception of Inference Rules

Following a long tradition in philosophy, I will take inferences to be *linguistic acts*.² Inferences are *acts* in that they are conscious, at person-level, and

¹ I'd like to thank Guillermo Del Pinal, Simon Goldstein, Diego Marconi, Ram Neta, Jim Pryor, Alex Rosenberg, Daniel Rothschild, David Sanford, Philippe Schlenker, Walter Sinnott-Armstrong, Seth Yalcin, Jack Woods, and three anonymous referees for helpful suggestions on earlier drafts. Special thanks go to Malte Willer and to all the organizers of DEON 2016.

² For example, see [6], [12], [20], [2], [21], [26], and [27], among others.

intentional. They are *linguistic*, in that they consist in the utterance of a list of sentences. These linguistic acts may be private, as when we argue to ourselves, or public, as when we try to convince others that they should endorse a certain conclusion through an argument. Inferences divide into *inductive* and *deductive* inferences, but only deductive inferences are the focus here. In this case, inferences consist in the utterance (mental or public) of a list of premises ϕ_1, \dots, ϕ_n and a conclusion ψ of the form $\lceil \phi_1, \dots, \phi_n; \textit{therefore}, \psi \rceil$.

Now, as linguistic acts, inferences have a *dynamic* aspect. In the process of making an inference, we add premises incrementally, and revise contextual assumptions, often even just provisionally, to make them compatible with the premises. For example, suppose I argue as follows:

- (i) If Marco were in Italy, he would inform me;
- (ii) He has not informed me;
- (iii) Hence, he must not be in Italy.

In making this argument, I add premises to the set of assumptions that I and my listeners already accept, or provisionally revise that set of assumptions in order to make it compatible with the newly introduced premises. If my listeners and I were previously assuming that Marco was in Italy, by uttering the premises (i) and (ii), I am in effect asking to provisionally suspend those assumptions from the initial set and to consider revising them in light of my argument.

Making an inference is, in this sense, moving from one set of assumptions to another — in this case, from a set of assumptions that may or may not be opinionated about Marco’s whereabouts — to a set of assumptions that includes both (i) and (ii) and is adjusted for coherence. The *goal* of an inference is to reach a set of assumptions that *supports* the conclusion of the inference — in this case, to reach a set of assumptions that entails that Marco is not in Italy.

Nothing thus far is particularly surprising. Several have highlighted the dynamic aspect of inferences.³ What is less commonly explored is what this dynamic conception of inference tells us about *inference rules* — such as the rule of *modus ponens* or conjunction introduction.

There seems to be a natural argument from a dynamic conception of inference to a *dynamic conception of inference rules*. The first premise is the dynamic conception of inference:

Premise 1 *An inference is a matter of moving from a set of assumptions to another set of assumptions which is meant to license the conclusion.*

Now, what is the relation between inferences dynamically conceived and inference rules? Inferring is, just like asserting, a linguistic act. And just like assertion, inferring is subject to *rules or norms*. Inference rules codify our inferential practices along certain structural dimensions. The rules of the propositional calculus codify our inferential practices along their truth-functional

³ For example, see [29]’s notion of a reasonable inference and [39]’s notion of informational consequence.

dimensions, whereas the rules for the quantifiers do so along the predicative dimension. That gives us the second premise:

Premise 2 *Inference rules codify our inferences along certain structural dimensions.*

The first and the second premise together lead us to think of the rules that govern inferences *as telling us how to update* a set of assumptions in such a way as to reach another set of assumptions that supports the conclusion:

Premise 3 *Inference rules are rules to move from a set of assumptions to another set of assumptions.*

But note that, according to dynamic semantics, that is exactly what *dynamic semantic values* are supposed to be ([13], [16], [17], [35], [36], and [11]).⁴ Dynamic semantic values are precisely rules to update sets of assumptions and are modeled as *functions* from sets of assumptions to sets of assumptions. This modeling claim is the fourth premise:

Premise 4 *Rules to update sets of assumptions can be modeled as functions from sets of assumptions to sets of assumptions — i.e., as dynamic semantic values.*

Premise 1-Premise 4 yield the dynamic conception of inference rules:

Conclusion *Inference rules can be modeled as functions from sets of assumptions to sets of assumptions — i.e., as dynamic semantic values.* [**Premise 1-Premise 4**]

So, the dynamic conception of inference motivates a dynamic conception of inference rules as *dynamic semantic values*. The next question is: what kinds of dynamic semantic values? Section §3 develops the proposal in some detail.

3 Towards the proposal

3.1 Dynamic Semantics

Sets of assumptions are often referred to as *contexts*. A *context* is a set of assumptions that are mutually shared by the participants of a conversation or that characterize a subject's mental state.⁵ Contexts could be modeled linguistically, as a set of sentences or as a set of linguistically structured propositions. In this section, in order to flesh out my proposal in some detail, it is very convenient to follow Stalnaker and most dynamic semanticists in taking a context to be *a set of possible worlds* — those worlds where *every* proposition in some

⁴ Other advocates of dynamic semantics are, among others, [8], [9], [38], [33], and [34].

⁵ [30], [31], and [32]. On modeling (private) mental states as sets of assumptions, see [40]. This notion of context is to be distinguished from another familiar notion of context, the Kaplanian notion, on which context is whatever is relevant to fix the meaning of context-sensitive expressions of a language. See [25], pp. 3-4 for a brief comparison of these two notions of context.

given set of assumptions is true.⁶ In the last section, I will explain how such a coarse-grained conception of context is not at all required by my proposal, that can be developed also within a more fine-grained notion of context.

The dynamic semantic value of a sentence ϕ is a function from contexts to contexts. More precisely, let $\langle p \rangle$ be the set of possible worlds where p — i.e., *the set of p -worlds*. The dynamic semantic value of a sentence σ — which I will indicate by ‘ $[\sigma]$ ’ — is a function from takes a context as argument and outputs a context as value — i.e., ‘ $c[\sigma]$ ’. The inductive definition is as follows:

Definition 3.1 (Dynamic Semantics)

- (i) If σ has the form p , $c[\sigma] = \{w \in c : w \in \langle p \rangle\}$;
- (ii) If σ has the form $\neg\phi$, $c[\sigma] = c - c[\phi]$;
- (iii) If σ has the form $\phi \ \& \ \psi$, $c[\sigma] = c[\phi][\psi]$;
- (iv) If σ has the form $\phi \ \vee \ \psi$, $c[\sigma] = c[\phi] \cup c[\neg\phi][\psi]$.

The dynamic meaning of an atom p is the function that takes a context c into another context c' that includes all and only the p -worlds from c . The dynamic meaning of a negation $\neg\phi$ is a function that takes a context c into another c' that results from eliminating from c all the ϕ -worlds ($= c - c[\phi]$). The dynamic meaning of a conjunction $\phi \ \& \ \psi$ is a function that takes a context c into the result of updating c first with ϕ and then with ψ ($= c[\phi][\psi]$).⁷ The dynamic meaning of a disjunction $\phi \ \vee \ \psi$ is the dual of the conjunction’s dynamic meaning ($= c[\phi] \cup c[\neg\phi][\psi]$).⁸

Standard presentations of dynamic semantics define a relation of *support* between contexts and sentences of our language.⁹ A context c *supports* ϕ just in case the result of updating c with ϕ is c itself — just in case every world in c is a ϕ -world:

Definition 3.2 (Support) c *supports* ψ ($c \models \phi$) iff $c[\phi] = c$.

Dynamic Entailment (\models_{DE}) is instead a semantic relation holding between sentences:

⁶ This idealization risks narrowing down the scope of my proposal to inferential relations between sentences that are contingently true — and so ruling out mathematical inferences, the sort of inferences that hold between necessarily true (or necessarily false) sentences. The current literature discusses several ways in which one could tweak the current apparatus to make it encompass mathematical inferences. One is to appeal to linguistically structured contexts (see [25] for some discussion). Another is to appeal to metalinguistic propositions, following [30]’s solution to the problem of logical omniscience. Finally, another approach, recently explored by [24] for the specific case of mathematics, consists in adding non-linguistic structure (in particular the structure of *subject matters*), to logical space and to contexts. I will return to this issue in the last section when discussing Lewis Carroll’s regress. A more fine-grained notion of context will turn out to be better suited to apply my proposed conception of inference rules and inferences to Carroll’s paradox.

⁷ [35], p. 18.

⁸ See [18] and [37], p. 10 for discussion of this entry for disjunction.

⁹ for example, [36], p. 221-222

Definition 3.3 (Dynamic Entailment) $\phi_1, \dots, \phi_n \models_{DE} \psi$ iff $\forall c: c[\phi_1] \dots [\phi_n] \models \psi$.

In other words, a set of sentences ϕ_1, \dots, ϕ_n dynamically entails (\models_{DE}) ψ just in case, for every context c , the result of updating c successively with all the premises is a context that supports ψ .

The last dynamic notion that we need is that of *a test*. An expression is a test just in case its dynamic role is *that of checking whether that context satisfies certain conditions*. If the context does satisfy those constraints, the test will return the context itself; otherwise, the test will return the absurd context — i.e., the empty set. So, for example, following Veltman ([36], p. 228), one can think of a sentence containing an epistemic modal such as *must- ϕ* as a test that checks whether ϕ is supported by the current context, in which case it returns the context itself as value; else, it returns the absurd context (— i.e., the empty set):

Definition 3.4 (Example of a Test)

$$\text{If } \sigma \text{ has the form } \textit{must-}\phi, c[\sigma] = \begin{cases} c & \text{if } c \models \phi \\ \emptyset & \text{if } c \not\models \phi \end{cases}$$

3.2 A Dynamic Conception of Inference Rules

We observed that inferences have a *dynamic aspect*. Here is van Benthem ([35], p. 11) explicitly highlighting the dynamic nature of ordinary inferences:

The premises of an argument invite us to update our initial information state, and then, the resulting transition has to be checked to see whether it ‘warrants’ the conclusion (in some suitable sense).

Following van Benthem, we can distinguish between *two* aspects of an inference:

- (i) **update**: updating the initial set of assumptions;
- (ii) **test**: checking whether the update has resulted in a set of assumptions that ‘warrants’, or supports, the conclusion.

For example, consider the inference: $\lceil \phi_1, \phi_2, \phi_3; \textit{therefore}, \psi \rceil$. The first three premises correspond to **update**: uttering them is an invitation to update the current context sequentially with ϕ_1 , ϕ_2 , and ϕ_3 . It is, moreover, plausible that the phrase $\lceil \textit{therefore}, \psi \rceil$ plays the role of the **test** part. As argued by Neta ([21], p. 399), ‘*therefore*’ is a deictic expression in that it refers back to the utterance of certain premises. Because of that, a dynamic interpretation of ‘*therefore*’ — i.e., an interpretation that highlights the role played by the expression within a discourse — seems to be particularly fitting. Moreover, as famously argued by Grice [10], in a sentence such as “John is English and therefore brave,” ‘*therefore*’ does not seem to add anything to the core content of a sentence such as “John is English and brave.” The two sentences may well have the same truth conditions, even though the former also signals (or *conventionally implies*) that John’s being brave follows from his being English. Thinking of ‘*therefore*’ as a test captures the Gricean insight that in some sense,

‘*therefore*’ is informationally empty. A test is in a similar sense informationally empty: its utterance does not alter the context by eliminating assumptions. Rather, it checks that the context satisfies certain constraints. If so, the overall meaning of a one-premise argument of the form $\lceil \phi; \textit{therefore} \psi \rceil$ can be thought of as a function that checks whether the context created by the utterance of the premise ϕ supports the conclusion ψ .

So, an inference of the form $\lceil \phi_1, \phi_2, \phi_3; \textit{therefore}, \psi \rceil$ has an **update** part and a **test** part. My proposal is that we use these two parts of an inference to characterize both inferences and inference rules. As pointed out by Rumfitt ([27], p. 53), the horizontal line in an inference can be thought along the same lines as the English ‘*therefore*’ or ‘*thus*’ — as a function that checks whether the context created by the premises supports the conclusion. This leads us to think of an inference as the *composition* of **update** and **test**:

$$\text{test} \left\{ \frac{\phi_1, \dots, \phi_n}{\psi} \right\} \text{update}$$

This composite function will return the result of updating c successively with the premises ϕ_1, \dots, ϕ_n if the resulting context c' supports the conclusion; else it returns the empty set (or absurd context).

This is a very natural semantic interpretation of an inference (or an argument). Along the same lines, an inference (or an argument) schema corresponding to a certain inference rule can be thought of as a composite function that takes as arguments a context and one or more sentences and returns another context. Let us consider, as a first example, the rule of $\&$ -elimination, represented here as the inference schema that takes a conjunction into either conjunct:

$$\frac{\phi_1 \& \phi_2}{\phi_1, \phi_2} \&\text{-E}$$

In this case, the **update** part is a function that takes the sentence $\phi_1 \& \phi_2$, and a context c , into the result of updating c with that sentence ($= c[\phi_1 \& \phi_2]$). Given **Definition 3.1**(iii), the result of so updating will be a context c' that results from sequentially updating c with ϕ_1 and then ϕ_2 . The second part of the rule is the **test** corresponding to the horizontal line ($/$) and the conclusions ϕ_1 and ϕ_2 — a function that takes the sentences ϕ_1, ϕ_2 and a context c into c having checked that c supports both ϕ_1 and ϕ_2 :

$$\text{Test (for } \&\text{-E)} \ c[\phi_1, \phi_2] = \begin{cases} c & \text{if } c \models \phi_1 \ \& \ c \models \phi_2 \\ \emptyset & \text{if either } c \not\models \phi_1 \ \text{or } c \not\models \phi_2 \end{cases}$$

It then becomes very natural to think of the $\&$ -elimination rule as the *composition* of these two different functions:

Definition 3.5 (&-Elimination)

$$c[\phi_1 \ \& \ \phi_2] \circ [/\phi_1, \phi_2] = \begin{cases} c[\phi_1 \ \& \ \phi_2] & \text{if } c[\phi_1 \ \& \ \phi_2] \models \phi_1, \phi_2 \\ \emptyset & \text{if } c[\phi_1 \ \& \ \phi_2] \not\models \phi_1, \phi_2 \end{cases}$$

The resulting composite dynamic value will take two sentences ϕ_1 and ϕ_2 and a context c and will return the result of updating c successively with ϕ_1 and ϕ_2 if the new context supports both ϕ_1 and ϕ_2 . Otherwise, it will return the absurd context.

Let us consider a second example. Here is &-introduction:

$$\frac{\phi_1, \phi_2}{\phi_1 \ \& \ \phi_2} \ \&\text{-I}$$

Taking the comma above the horizontal line to mean conjunction, we can think of the **update** part of this rule as the function that, for any two sentences ϕ_1 and ϕ_2 , takes a context c into a new context c' that results from updating c successively with ϕ_1 and then ϕ_2 :

Update (for &-I) $c[\phi_1, \phi_2] = c[\phi_1][\phi_2]$

The second part of the rule checks whether the resulting context supports ϕ_1 & ϕ_2 :

$$\textbf{Test (for &-I)} \ c[/\phi_1 \ \& \ \phi_2] = \begin{cases} c & \text{if } c \models \phi_1 \ \& \ \phi_2 \\ \emptyset & \text{if } c \not\models \phi_1 \ \& \ \phi_2 \end{cases}$$

&-introduction corresponds to the composition of these two functions:¹⁰

Definition 3.6 (&-Introduction)

$$c([\phi_1, \phi_2] \circ [/\phi_1 \ \& \ \phi_2]) = \begin{cases} c[\phi_1][\phi_2] & \text{if } c[\phi_1][\phi_2] \models \phi_1 \ \& \ \phi_2 \\ \emptyset & \text{if } c[\phi_1][\phi_2] \not\models \phi_1 \ \& \ \phi_2 \end{cases}$$

In order to specify the dynamic semantic value corresponding to *modus ponens*, I would have to settle the thorny issue of what the dynamic semantic value of the English conditional is. Here, I will set for myself a much less ambitious task, and will only consider the elimination rule for the *material conditional* ‘ \rightarrow ’:¹¹

¹⁰Note that although the function $[\phi_1, \phi_2]$ will return the same value as the function $[\phi_1 \ \& \ \phi_2]$ for any context c , the two functions are *intensionally* different, just like the function ‘ $x + 1$ ’ is different from the function ‘ $x + 1 + 0$ ’, even though they will return the same value for any argument x . By characterizing functions intensionally, rather than extensionally, on my proposal, **&-introduction** is a different function from **&-elimination**.

¹¹That does not mean I endorse a material conditional analysis of the indicative conditional. I do not. Proponents of the material conditional analysis of the English indicative conditional are, notoriously, [10], [14], [15], and [19]. Among its many detractors, see [29], [5], [1], [7], [9], [33], and [34].

$$\frac{\phi \quad \phi \rightarrow \psi}{\psi} \text{MP}$$

The **update** part of this rule is a function that, for any two sentences ϕ and ψ , takes a context c into a context c' that results from successively updating c with ϕ and $\neg\phi \vee \psi$:

Update (for Modus Ponens for the Material Conditional)

$$c[\phi, \phi \rightarrow \psi] = c[\phi][\neg\phi \vee \psi]$$

The composition of this **update** part with the **test** part gives us the following composite dynamic semantic value:

Definition 3.7 (Modus Ponens for the Material Conditional)

$$c([\phi, \phi \rightarrow \psi] \circ [/\psi]) = \begin{cases} c[\phi][\neg\phi \vee \psi] & \text{if } c[\phi][\neg\phi \vee \psi] \models \psi \\ \emptyset & \text{if } c[\phi][\neg\phi \vee \psi] \not\models \psi \end{cases}$$

Generalizing, a rule of the following form:

$$\frac{\phi_1, \dots, \phi_n}{\psi}$$

is a function that takes sentences ϕ_1, \dots, ϕ_n and ψ into the following dynamic semantic value:

Definition 3.8 (Schema)

$$c([\phi_1, \dots, \phi_n] \circ [/\psi]) = \begin{cases} c[\phi_1][\phi_2] \dots [\phi_n] & \text{if } c[\phi_1, \dots, \phi_n] \models \psi \\ \emptyset & \text{if } c[\phi_1, \dots, \phi_n] \not\models \psi \end{cases}$$

So we get that each instance of a rule is a dynamic semantic value. **Schema** works well to characterize simple inference rules, such as the ones just considered, and can be used to capture both the rules of the propositional calculus and those of the predicative calculus. What about meta-rules, such as conditional proof, *reductio ad absurdum*, and argument by cases?

It turns out that we can model meta-rules simply by generalizing **Schema** to cover a new kind of update. To see this, start by considering conditional proof:

$$\frac{\begin{array}{c} [\phi] \\ \vdots \\ \psi \end{array}}{(\phi \rightarrow \psi)} \rightarrow I$$

In this schema, a *sub-proof* seems to play the role of the premise of the argument schema. So the **update** part of this rule must consist in updating a context with a *sub-proof*. But what could updating a context with a *sub-proof* amount to?

Let me make two different suggestions and let me point out one important reason to prefer one of the two. On the first suggestion, updating a context c with a sub-proof, say with a premise ϕ and conclusion ψ , updates the context with the conditional $\phi \rightarrow \psi$:

Definition 3.9 (Update for sub-proofs I)

If σ has the form $\phi_1, \dots, \phi_n/\psi$, $c[\sigma] = c[(\phi_1 \& \dots \& \phi_n) \rightarrow \psi]$.

According to **Update for sub-proofs I**, the dynamic meaning of a sub-proof like ϕ/ψ is the same as the dynamic meaning of the conditional $\phi \rightarrow \psi$ — their difference is merely syntactic. Observe that **Update for sub-proofs I** treats the sub-proof as a *real* premise. For in the dynamic settings explored here, using a sub-proof as a premise means *updating* the current context with it. And note that that is exactly what **Update for sub-proofs I** instructs us to do with subproofs. The main problem with this is that unless we take the dynamic meaning of the conditional $\phi \rightarrow \psi$ itself to be that given by **Schema**, **Update for sub-proofs I** does not make clear how the dynamic meaning of a subproof such as ϕ/ψ arises from the dynamic meaning of an argument from ϕ to ψ , that (as we have just seen) consists in a composite function first updating the context with the premise ϕ and then checking whether the conclusion ψ follows. If we want the meaning of the subproof ϕ/ψ to be compositional on the meaning of the argument from ϕ to ψ , a better solution is to take the dynamic meaning of the subproof ϕ/ψ to result from combining an instruction to first update the context with ϕ and to then check whether ψ follows (just as **Schema** would instruct) with an instruction to *discharge the assumption* ϕ . The upshot of this composite instruction is the same as that of a test — an instruction to return to the context c having checked that $c[\phi]$ supports ψ :¹²

Definition 3.10 (Update for sub-proofs II)

If σ has the form $\phi_1, \dots, \phi_n/\psi$, $c[\sigma] = \{w \in c: c[\phi_1 \& \dots \& \phi_n] \models \psi\}$.

We will see in the last section another conceptual advantage of this second option. For now simply note that both **Update for sub-proofs I** and **Update for sub-proofs II** allow us to think of Conditional Proof along the exact same lines as our previous rules —i.e., as the following composite function:

Definition 3.11 (Conditional Proof)

$$c([\phi/\psi] \circ [/\phi \rightarrow \psi]) = \begin{cases} c[\phi/\psi] & \text{if } c[\phi/\psi] \models \phi \rightarrow \psi \\ \emptyset & \text{if } c[\phi/\psi] \not\models \phi \rightarrow \psi \end{cases}$$

The discussion of **Conditional Proof** also covers also *Reductio ad Absurdum*, for such a rule can be thought as a special instance of **Conditional**

¹² Accordingly, the subproof $\ulcorner \phi/\psi \urcorner$ represents the combination of the argument from ϕ to ψ with the further instruction to discharge the premise ϕ . Plausibly, the latter instruction to discharge premise ϕ is expressed by the ending of the sub-proof line.

Proof, where $\neg A = A \rightarrow \perp$. Finally, consider Argument by Cases:

$$\frac{\begin{array}{cc} [\phi_1] & [\phi_2] \\ \vdots & \vdots \\ \phi_1 \vee \phi_2 & \psi \end{array}}{\psi}$$

Here, we have three different premises — a sentence $\phi_1 \vee \phi_2$ as well as the sub-proofs ϕ_1/ψ and ϕ_2/ψ . Treating each as an individual premise, the update part of this rule will update the context sequentially with $\phi_1 \vee \phi_2$, ϕ_1/ψ , and ϕ_2/ψ . Just like in the previous case of **Conditional Proof**, this update allows us to define the rule of Argument by Cases as one would expect, given our previous discussion (where $c^* = c[\phi_1 \vee \phi_2][\phi_1/\psi][\phi_2/\psi]$):

Definition 3.12 (Argument by Cases)

$$c([\phi_1 \vee \phi_2, \phi_1/\psi, \phi_2/\psi] \circ [/\psi]) = \begin{cases} c^* & \text{if } c^* \models \psi \\ \emptyset & \text{if } c^* \not\models \psi \end{cases}$$

We are now in position to generalize the original **Schema** to encompass the case of meta-rules. The only difference with respect to **Schema** is that the first function can now take as arguments not just sentences ϕ_1, \dots, ϕ_n but any environment above the horizontal line, including sub-proofs. Hence, by letting P (for premise) vary over both sentences and sub-proofs and by letting C vary over conclusions, we arrive at:

Definition 3.13 (Schema-General)

$$c([P_1, \dots, P_n] \circ [C]) = \begin{cases} c[P_1][P_2] \dots [P_n] & \text{if } c[P_1, \dots, P_n] \models C \\ \emptyset & \text{if } c[P_1, \dots, P_n] \not\models C \end{cases}$$

The structure of the premises together with **Schema-General** gives us a general recipe to map any schema into the corresponding context-change potential. Note also that the assignment of dynamic semantic values to schemas is compositional, as it is entirely determined by the meanings of their parts, together with the syntax of the schemas (the order and the syntax of their premises, the horizontal line, and the syntax of their conclusion).

4 Classical versus Dynamic Validity

On the current approach, an inferential rule is valid (\models_0) just in case *the relevant composite function never returns the empty set for any context*:

Definition 4.1 $P_1, \dots, P_n \models_0 C$ iff for every c : $c([P_1, \dots, P_n] \circ [C]) \neq \emptyset$.

It is straightforward to prove that an inference rule is valid (\models_0) just in case it is DE-valid (\models_{DE}):

Theorem 4.2 $P_1, \dots, P_n \models_0 C$ iff $P_1, \dots, P_n \models_{DE} C$.

Proof. Suppose $P_1, \dots, P_n \vDash_0 C$. Then, by **Definition 4.1**, for every $c: c([P_1, \dots, P_n] \circ [C]) \neq \emptyset$. But then, by **Definition 3.13**, for every $c: c[P_1, \dots, P_n] \vDash C$. Hence, by **Definition 3.3**, $P_1, \dots, P_n \vDash_{DE} C$. Now, suppose $P_1, \dots, P_n \vDash_{DE} C$. Then, by **Definition 3.3**, for every $c: c[P_1, \dots, P_n] \vDash C$. Then, by **Definition 3.13**, for every $c: c([P_1, \dots, P_n] \circ [C]) \neq \emptyset$. Then, by **Definition 4.1**, $P_1, \dots, P_n \vDash_0 C$. \square

A possible objection is that this notion of validity (\vDash_0) conflates two notions of validity: *classical validity* and *dynamic validity*. As observed by van Benthem ([35], p. 11 and pp. 18-19), these are indeed different notions of validity. For dynamic validity, the order of the premises and the multiplicity of their occurrence matter. That seems to clash with the basic structural rules of standard classical logic.

Can this important distinction between dynamic validity and classical validity be vindicated on the present approach? As Starr ([33], p. 9) has pointed out, classical entailment can be thought of as a special case of dynamic entailment — i.e., as dynamic entailment in *contexts of perfect information*. Contexts of perfect information only include *the world of the context* — no other world is compatible with a set of propositions that completely distinguishes the actual world from any other possible worlds. So, let the context of perfect information relative to w be $\{w\}$. Classical entailment (\vDash_{CE}) emerges by focusing on perfect information:

Definition 4.3 $P_1, \dots, P_n \vDash_{CE} C$ iff $\forall\{w\}: \{w\}[P_1] \dots [P_n] \vDash C$.

Call a function from contexts of perfect information to other contexts of perfect information a *limiting dynamic semantic value*. The limiting dynamic semantic value of a schema is insensitive to the order of its premises. So when we want to highlight the insensitivity of classical structural rules to the order of their premises, we can then think of them as limiting dynamic semantic values. This move preserves van Benthem's distinction, while clinging to the idea according to which classical inference rules are sorts of dynamic semantic values.

5 The Dynamic Conception of Inference Rule and Lewis Carroll's Regress

What does it mean to follow a rule in an argument? According to a very popular diagnosis of Carroll's regress of the premises, following a rule in an argument is not the same as using, or being guided by, a logical truth. The argument to this conclusion goes as follows. Let us start with premises A and *if A then B*:

- (i) A.
- (ii) *If A then B*.

How does one get from these premises to the conclusion B? Presumably, by appeal to *modus ponens*. But now, suppose the rule of *modus ponens* were identical to the general principle **LT-mp**:

LT-mp *For every X, Y, if X and if X then Y, then Y.*

or to the conditional schema:

LT-mp-schema *If X and if X then Y, then Y.*

Then, presumably, following that rule would be a matter of instantiating **LT-mp** or **LT-mp-schema** for the particular case of A and B — adding an instance of theirs as premise. But by instantiating such a logical truth, one only gets to (iii), still short of the conclusion B:

- (i) A.
- (ii) *If A then B.*
- (iii) *If A and if A then B then B.*

How can one get from (i-iii) to the conclusion? Again, by appeal to *modus ponens*, one would guess. The problem is that if following *modus ponens* is the same thing as using **LT-mp** or **LT-mp-schema**, then arguing by *modus ponens* must amount to instantiating either for the particular cases of the premises. But by so doing, one will only get to the following four-premise argument (by taking the conjunction of A and *If A then B* as the first premise, and (iii) as the second premise), and still short of the conclusion B. And so on. Therefore, if following *modus ponens* were the same as using a general principle such as **LT-mp** or a conditional schema such as **LT-mp-schema** in the course of an argument, following a rule would trigger a regress, making it impossible to reach a conclusion. But we do routinely succeed at reasoning by *modus ponens*. So, the argument concludes that following *modus ponens* cannot be the same as using **LT-mp** or **LT-mp-schema**:

Diagnosis Following an inference rule in the course of an argument is not the same as using a logical truth.

And from **Diagnosis**, it is a shot step to conclude to:

Rules versus logical truths An inference rule is not the same as a logical truth.

This step is motivated by the thought that *the only ways* a truth can be used in an argument are by instantiation (if the logical truth is general, such as **LT-mp**) or by iteration (if the logical truth is singular such as the conditional '*If A, and if A then B, then B*' or an instance of the conditional schema **LT-mp-schema**). This is plausibly a general claim about truths. This claim is not irresistible but to resist it is not an easy task: defending it would require providing a different model of how truths can be used in the course of an argument, one that to my knowledge nobody has ever provided.

It should not come as a surprise, then, that many have underwritten **Rules versus logical truths** as the correct diagnosis of the regress. For example, Dummett [4], p. 303, observes that the main moral of the regress is that an argument of the form:

- (A) *Pietro is Italian; if Pietro is Italian then Andrea is Italian; therefore*

Andrea is Italian.

cannot be identified with the conditional statement:

(C) *If both Pietro is Italian and if Pietro is Italian, then Andrea is Italian, then Andrea is Italian.*

Along similar lines, Ryle ([28], p. 7) argues that knowing a rule is not the same as knowing a truth. Finally, Rumfitt ([26], p. 358) argues that knowledge of a logical truth cannot explain our ability to make deductions and takes that to be the moral of Lewis Carroll's fable.

Although very popular, it should be emphasized that **Rules versus logical truths** is entirely *negative*: it only tells us that rules are not logical truths but it does not tell us *what rules are*. A solution to the puzzle raised by the regress requires replacing a conception of inference rules as logical truths with something else.

The dynamic notion of inference rules I developed in §3 offers a suitable semantic replacement of the notion of rule as logical truth. To see this point, it is now convenient to shift from the coarse-grained notion of context employed so far to a more fine-grained notion of context, as a set of structured propositions. On such a fine-grained notion of context, it makes sense to think of our using a logical truth in the course of an argument as a matter of *adding it to the context as a further premise*.¹³ But now suppose inference rules are not logical truths but dynamic semantic values of the sort that I described in the last section. Following an inference rule in this sense is not a matter of adding a logical truth to the context as a further premise. Rather, it is a matter of *implementing a particular function* and, in particular, one whose implementation does *not* require to go through instantiation of the rule itself as an extra premise. To see this, consider **Modus Ponens for the Material Conditional (Definition 3.7)**. Such rule is a function that, given the premises and a context as arguments, updates the context with the premises, having checked that the result supports the conclusion. So the use of this rule — i.e., the implementation of this function — does not require adding the rule itself to the context nor does it require instantiating the rule itself as an extra-premise.

Relatedly, my proposal, coupled with a suitably fine-grained notion of context, also helps us appreciate Dummett's distinction between an argument such as (A) and a conditional such as (C). Whereas using a conditional such as (C) in the course of an argument plausibly does require to add to the context (C) as an extra premise, on my proposal, using an argument such as (A) does not require to add to the context (A) itself as an extra premise. Here, one might object that arguments do sometimes appear to be treated as premises, as when

¹³ Adopting a more fine-grained notion of context makes it easier to appreciate the difference that my proposal draws between, on one hand, using a logical truth in the course of an argument and, on the other, following a rule in the sense of 'rule' here outlined. By contrast, on a coarse-grained conception of context as set of possible worlds, updating a logical truth (a necessary truth) in the course of an argument is an *ineffectual* update — for it does not eliminate any possible world.

we argue by conditional proof. But recall that, even when arguments are *apparently* used as premises in arguments by conditional proof or by argument by cases, my **Update for Sub-Proofs II** does not take them to *work* as real premises. Rather, according to **Update for Sub-Proofs II**, their role as sub-proofs is equivalent to that of a test — an instruction to first check that the context created by adding their premises supports the conclusion and to then return to the original context after the checking.

Note that my claim is not that the implementation of a context-change potential *never* requires adding an extra premise. To see this, consider the following universal principle:

LT-mp-dynamic *For every X, Y, if X and if X then Y, then must-Y.*

On its dynamic interpretation, **LT-mp-dynamic** *also* expresses a dynamic semantic value. But the relevant context-change potential is different from the one defined in **Definition 3.7**, for **LT-mp-dynamic** is a universal claim. So **LT-mp-dynamic**'s dynamic meaning is function also of the dynamic meaning of the universal quantifier. Assuming a certain dynamic treatment of the universal quantifier and a suitably fine-grained notion of context, its dynamic meaning will consist in an update — in adding it (or an instance of it) as a premise to the context. So while implementing the function corresponding **Definition 3.7** does not require instantiating extra premises, implementing the function corresponding to **LT-mp-dynamic** does (plausibly) require going through instantiation for possible assignments to X and Y.

What should we conclude? The conclusion to draw is that *also* **LT-mp-dynamic** is not the right way to think of the inference rule of *modus ponens*. Of course, it does not follow that my proposal is not correct, for as I just observed, on my proposal *modus ponens* is not the same as the dynamic interpretation of **LT-mp-dynamic** just mentioned. My claim is that *certain* dynamic semantic values (the ones described in section §3) can stop the regress. My claim is *not* that every possible dynamic semantic value will block the regress.

Here is a second potential worry. Could not we imagine a 'dynamic' version of the regress, on which a subject keeps updating the context with more premises, without being able to run a test? If so, how does the dynamic conception of inference rules improve on a conception of rules as logical truths? In response, let me note that the current proposal offers an account of rule-following on which, *if* one can follow a rule at all, one *could not* get stuck in the regress of the premises. It is worth going through this point carefully. On the current proposal, a subject who keeps updating the context with more premises without being able to run a test would prove to be *unable* to follow the relevant rule. That is so because on the current proposal, following an inference rule *requires* being able to run a test. That can be seen from the fact that running a test is a condition for implementing the composite functions with which I have identified inference rules (which are composed both of an **update** part and a **test** part). Hence, if one can follow a rule at all in this sense, one simply cannot get stuck with the regress of the premises. By contrast, if rules are logical

truths, following a rule *will* get us stuck in the regress of the premises. Whereas the notion of following a logical truth is regress-triggering and hence paradoxical, the notion of following a rule developed here is not. That is the important respect under which the dynamic conception of inference rules improves on a conception of rules as logical truths.

6 Conclusions

The dynamic conception of inference rules developed in this essay provides a picture of rule-following which blocks Carroll's regress. As argued in §2, such a dynamic conception is independently motivated by a dynamic conception of inference. As shown in §3, it arises from a plausible and compositional assignment of meanings to argument schemas and from an independently motivated dynamic interpretation of the horizontal line and of the English '*therefore*'. Because of that, it captures the distinction Dummett seemed to be after in the passage mentioned: that between an argument of the form $\lceil P; \text{if } P \text{ then } Q; \text{therefore, } Q \rceil$ and a conditional of the form $\lceil \text{if } P, \text{ and if } P \text{ then } Q, \text{ then } Q \rceil$. All in all, the dynamic conception of inference rules seems to provide a suitably *semantic* replacement of a conception of inference rules as logical truths.

Some outstanding issues are left open by this essay, that I do not have the space to discuss here (though see [22] for discussion). The first is: how does the dynamic conception of inference rules compare to other conceptions of inferential rules, for example, as syntactic mappings or conditional recommendations of sort? Whether or not these conceptions of inference rules can block Carroll's regress, I argue elsewhere that they all fall short in *other* respects ([22]). Either they do not correspond to a plausible interpretation of argument schemas and/or they are not suitably *semantic* conceptions of inference rules, and because of that, they face a version or another of a problem that I call *the problem of understanding*. If I am right, a stronger case on behalf of my proposal is available than I can make here.

The second outstanding issue is this. On the dynamic conception of inference rules, being competent with an inference rule is a matter of *being competent with a function*. As I argue elsewhere ([23], [22]), being competent with a function plausibly requires the function being representable by a subject in terms of operations that the subject can already perform.¹⁴ So if inference rules are dynamically conceived, our being competent with inference rules requires these functions being representable by us in terms of operations that are performable by us — by individuals of average linguistic competence. How plausible is this claim? Although the issue is of great importance, it is not specific to my dynamic conception of inference rules. It arises for any appeal to dynamic semantics as an explanatory theory of our linguistic competence, for on the dynamic picture, knowledge of meanings *is* in general a matter of being competent with functions. Hence, although a thorough defense of the dynamic conception of inference rules does require a detailed defense of the

¹⁴By analogy with the notion of *primitive recursive function* in computability theory.

plausibility of this claim, I consider it to be part of a bigger project that will have to be discussed in further work.

References

- [1] Bennett, J., “A Philosophical Guide to Conditionals,” Oxford University Press, 2003.
- [2] Boghossian, P. A., *What is Inference?*, *Philosophical Studies* **169** (2014), pp. 1–18.
- [3] Carroll, L., *What The Tortoise Said to Achilles*, *Mind* **4** (1895), pp. 278–80.
- [4] Dummett, M., “Frege: Philosophy of Language Vol. 2,” London: Duckworth, 1981.
- [5] Edgington, D., *On Conditionals*, *Mind* **104** (1995), pp. 235–329.
- [6] Frege, G., *Logic. Posthumous Writings* (1979).
- [7] Gibbard, A., *Two Recent Theories of Conditionals*, in: *Ifs*, Springer, 1980 pp. 211–247.
- [8] Gillies, A. S., *A New Solution to Moore’s Paradox*, *Philosophical Studies* **105** (2001), pp. 237–250.
- [9] Gillies, A. S., *Epistemic Conditionals and Conditional Epistemics*, *Noûs* **38** (2004), pp. 585–616.
- [10] Grice, H. P., “Studies in the Way of Words,” Harvard University Press, 1991.
- [11] Groenendijk, J. A. G. and M. Stokhof, *Dynamic Predicate Logic*, *Linguistics and Philosophy* (1991), pp. 39–100.
- [12] Harman, G., “Change in View: Principles of Reasoning,” Cambridge University Press, 2008.
- [13] Heim, I., *The Semantics of Definite and Indefinite Noun Phrases* (1982), PhD Dissertation.
- [14] Jackson, F., *On Assertion and Indicative Conditionals*, *The Philosophical Review* **88** (1979), pp. 565–589.
- [15] Jackson, F., “Conditionals,” Oxford University Press, 1991.
- [16] Kamp, H., *A Theory of Truth and Linguistic Representation*, in: *Formal Methods in the Study of Language (Groenendijk, J. A. G and M. Stokhof (eds.), pp. 277-321)*, Mathematish Centrum, Universiteit van Amsterdam, 1981 .
- [17] Kamp, H. and U. Reyle, “From Discourse to Logic: Introduction to Model-theoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory,” 42, Springer Science & Business Media, 1993.
- [18] Klinedinst, N. and D. Rothschild, *Connectives without truth tables*, *Natural Language Semantics* **20** (2012), pp. 137–175.
- [19] Lewis, D., *Probabilities of Conditional and Conditional Probabilities*, *The Philosophical Review* **8** (1976), pp. 297–315.
- [20] Longino, H. E., *Inferring*, *Philosophy Research Archives* **4** (1978), pp. 17–26.
- [21] Neta, R., *What is an Inference?*, *Philosophical Issues* **23** (2013), pp. 388–407.
- [22] Pavese, C., *Inference rules as dynamic semantic values*, Manuscript https://www.academia.edu/23462009/Inference_Rules_as_Dynamic_Semantic_Values.
- [23] Pavese, C., *Practical Senses*, *Philosophers’ Imprint* **15**, pp. 1–25.
- [24] Pérez Carballo, A., *Structuring Logical Space*, *Philosophy and Phenomenological Research* (2014).
- [25] Rothschild, D. and S. Yalcin, *On the Dynamics of Conversation*, *Noûs* (2015).
- [26] Rumfitt, I., *Inference, Deduction, and Logic*, in: J. Bengson and M. Moffett, editors, *Knowing How: Essays on Knowledge, Mind, and Action*, Oxford University Press, Oxford, 2011 pp. 334–350.
- [27] Rumfitt, I., “The Boundary Stones of Thought: An Essay in the Philosophy of Logic,” Oxford University Press, USA, 2015.
- [28] Ryle, G., *Knowing How and Knowing That: the Presidential Address*, *Proceedings of the Aristotelian Society, New Series* **46** (1945), pp. 1–16.
- [29] Stalnaker, R., *Indicative Conditionals*, in: *Language in Focus: Foundations, Methods and Systems Volume 43 of the series Boston Studies in the Philosophy of Science*, Springer, 1975 pp. 179–196.
- [30] Stalnaker, R., “Inquiry,” MIT Press, Cambridge, Mass, 1987.

- [31] Stalnaker, R., *Context and Content: Essays on Intentionality in Speech and Thought* (1999).
- [32] Stalnaker, R., "Context," OUP Oxford, 2014.
- [33] Starr, W. B., *A Uniform Theory of Conditionals*, *Journal of Philosophical Logic* **43** (2014a), pp. 1019–1064.
- [34] Starr, W. B., *What 'If'?*, *Philosophers' Imprint* **14** (2014b).
- [35] van Benthem, J., "Exploring Logical Dynamics," CSLI Publications and Stanford/Cambridge University Press, 1996.
- [36] Veltman, F., *Defaults in Update Semantics*, *Journal of philosophical logic* **25** (1996), pp. 221–261.
- [37] Willer, M., *An update on epistemic modals*, *Journal of Philosophical Logic* , pp. 1–15.
- [38] Willer, M., *Dynamics of Epistemic Modality*, *Philosophical Review* **122** (2013), pp. 45–92.
- [39] Yalcin, S., *Epistemic modals*, *Mind* **116** (1995), pp. 983–1026.
- [40] Yalcin, S., *Nonfactualism about Epistemic Modality*, in: A. Egan and B. Weatherson, editors, *Epistemic Modality*. *Oxford University Press*, Oxford University Press, Oxford, 2011 pp. 295–332.