What powers are not

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Abstract This paper analyses and criticizes the idea that powers are representable as vectors. Mumford and Anjum have recently developed a vector model of powers as part of their account of dispositional causation. The purpose of this model is to represent dispositionality, i.e. a sui generis type of modality introduced by their power-based ontology, as well as to explain various features of their account of causation. In this paper, we criticise both the claim that *powers are vectors* and the concomitant claim that the composition of causes can be understood as *vector addition*. We argue that powers cannot be thought of as even analogous to vectors, and that the vector model is simply misleading. We show that the root of the problem is in Mumford and Anjum's thought that powers have *magnitude* and *direction*.

Keywords dispositionality, causation, powers, composition of causes, vectors, vectorial representation

1. Introduction

Power is an ontological category introduced by Aristotle to explain and ground change and activity in nature. It was heavily challenged from the seventeenth century on, but has regained popularity since the last quarter of the twentieth century. 'Power' is not quite defined. But its key feature, at least according to Aristotle and most of his followers, is that a power can exist unmanifested though it can be manifested under certain circumstances. Within the current neo-Aristotelian ontology, causation is widely taken to be the exercising of powers; it is the production of an effect (manifestation) from its dispositional cause(s).

In their recent work (2011a), Stephen Mumford and Rani Lill Anjum (henceforth M&A) have aimed to articulate a power-based theory of causation, putting emphasis on the novel modal category of *dispositionality*. Part of this theory is that powers are represented as *vectors* (2011a, 2011b). They maintain that the vectorial representation captures key features of powers and that it is an indispensable tool for their theory of dispositional causation. Furthermore, they claim that this model plays a heuristic role. Traditionally, powers were taken to operate with metaphysical necessity: causes were conceived as necessarily related to their effects. But M&A take powers to possess an irreducible, *sui generis*, modality: dispositionality. For them, powers dispose towards their manifestation without necessitating it. This view, which is central to their theory, is claimed to be brought out by the heuristic role of the *powers-as-vectors* model; this model is supposed to show how causal production does not occur via necessitation; given that causal production is not a contingent matter either, the powers-as-vectors model is taken to favour dispositionality (2011a, 46).

In this paper we challenge the *very idea* of the vectorial representation of powers. We begin with the strong claim that powers are vectors and we refute it. We then show that even if we were to accept an increasingly looser association of powers with vectors, it would still be the wrong way to go. Either as constitutive of powers or as a metaphor for the composition of causes, vectors fail to represent causes even at a *qualitative* level. Finally, we claim that even the simple idea of *directedness* fails. We contend that, in the end, dispositionality does not emerge from the M&A model as, when understood in terms of directedness, it becomes poorly supported and highly controversial.

2. The representation of powers as vectors

The motivation for choosing vectors as a tool for the representation of powers arises from pandispositionalism: the view that all properties are essentially powers, that is that they are individuated by their causal role. On this view, powers are *the drivers of change* and the grounds of the (new category of) dispositional modality, independently of the level at which they operate.[1] Hence, when a certain effect is produced, this occurs by virtue of the manifestation of some powers. The production of an effect depends on the way various *powers work with each other* (and sometimes against each other). Having said this, causal production does not involve necessitation. Rather, it is readily admitted that a power-based cause is not a sufficient condition for its effect (hence, it does not necessitate its effect), since it is always possible that some counteracting power may interfere in a given causal situation so that the characteristic effect does not follow (2011a, chapter 3). This is a novelty of their view: causation is neither contingent nor necessary. As they put it:

> Causes can be thwarted, for instance, and this should show that they never guarantee their effects, even when they succeed in producing them. Causal production should not, therefore, be conflated with causal necessitation. (2011a, viii)

The view of causation they favour is dispositional. Causes dispose towards their effects:

> Causation involves what we call a dispositional modality. Causes dispose towards their effects, where disposing towards something involves an irreducible *sui generis* modality. The modality is something between pure contingency and pure necessity and is irreducible to neither. (2011a, viii)

The combination of the thoughts that causation involves various interacting powers, and that a cause is something that disposes (*and no more than disposes*) towards its effect, constitute the rationale for the main representational scheme that M&A promote: that powers are vectors. Moreover, powers are thought of, albeit implicitly, as forces. Though this idea is not directly used in the construction of their model, it is implied by their view that a power 'pushes' towards an effect; a view which is meant to replace the traditional idea that powers are active or passive.[2]

Vectors are used by M&A to represent the individual acting powers that operate in a given causal situation that *push* (in a certain sense jointly) towards an effect. They are meant to capture two key features of powers, i.e. *direction* and *intensity*. As M&A put it:

> [A] power will have a direction — that towards which it is disposed — such as fragility being a disposition towards breaking. And it has intensity. A power can be more or less disposed towards an outcome, … [as in] the case of fragility, a wine glass will be more fragile than a car windscreen. (2011a, 24)

To thwart an initial objection, we should stress that M&A take the vector-model utterly seriously as the way to represent power-based causation. Hence it is not a merely useful metaphor. They take it to be licensed by their realist view about powers and, thus, they claim that 'a realist about powers should prefer this way of representing causation' (2011a, 20). *Prima facie*, there is a basis for the model: powers are said to be 'directed' towards their effects and vectors have a direction; powers seem to come in degrees of strength (intensity) and vectors have magnitude.

[1] M&A typically focus on powers and qualities that operate at a macro-level. Though they do acknowledge that the use of certain macro-properties (such as 'being cold' and 'being hot') does not agree with the way science treats the relevant properties, they take it for granted that all properties are clusters of powers and thus that at the macro-level too, insofar as objects are engaged in causal relations, there are powers that ground causal relations among them.
[2] M&A use the notions of power and disposition interchangeably and, although we are in favour of a distinction between the two, it is outside the scope of this paper to discuss this issue.

Though we shall argue that these similarities are, ultimately, misleading and wrong, it should be noted that they are the chief motivations for M&A to introduce the idea that vectors are apt for representing powers. By representing powers as vectors, they claim that not only can they capture the idea of a power disposing *towards* a certain outcome, but also the fact that different powers can have *different dispositional intensities* regarding an outcome. In the example in the quotation above, while both objects are disposed towards breaking, the wine glass is said to instantiate the disposition of breaking in a higher intensity (it is more disposed towards breaking) than the car windscreen. According to M&A, then, were we to represent vectorially the fragility of the wine glass in comparison to the fragility of the car windscreen, the two vectors would have the same direction, as both objects are disposed towards breaking, but they would differ in their length, representing the difference in the intensity of the two powers —with the intensity of the disposition of the wine glass being represented by a longer directed arrow.

How is this idea developed by M&A? How are the vectors plotted? M&A intend to construct models of single causal situations, e.g., the cooling or the heating of a room, or the breaking of a vase. They then define the space that the vectors (the various powers of certain direction and intensity) act upon. Given that they take it that causation involves, most of the times, changes in qualities, they find Lawrence Lombard's quality space to be suitable for playing the role of the basis of the vector space in their model. According to Lombard, the quality space is a set S of simple static properties $\{P_0, P_1, \ldots, P_n\}$ meeting two conditions: first, it consists of mutually exclusive static properties;[3] second, 'quality spaces are kinds of properties that are such that, if any object changes by losing a property belonging to a given quality space, it must come to have another property of the same kind' (1986, 113). Trying to see how this definition can be applied by M&A, let us think of a set whose members are two simple and static properties: being red (R), and being blue (B). Is {R, B} a quality space? Something cannot be both red and blue throughout and at the same time. This satisfies the first condition of Lombard's definition. Note, however, that {R, B} does not satisfy the second condition: although both R and B are properties of the same kind (being both colours), when something ceases to be red, it does not necessarily become blue. In order to conform to both conditions, the quality space S for the kind 'colour' should be expressed in the following way: S = {x: x is a colour}, where each colour is a simple and static property, and when something ceases to be one colour it must become another. In the examples they provide, M&A have not defined the quality space in this strict way, but by using pairs of contradictories, such as 'hot' (the quality of being hot) and 'cold' (the quality of being cold). Every pair of contradictories sets up a one-dimensional quality space for the model, with the two qualities representing the two extreme states that is (metaphysically) possible for a given causal situation. Comparing this to the requirement of Lombard's definition, M&A (2011a, 23) do acknowledge that a one-dimension quality space is actually a *spectrum* of qualities, meaning that every point between the two extreme qualities represents a different quality of the same kind. However, using a pair of contradictories enables M&A to pictorially represent the quality space in a simple way, without the need to specify how fine-grained the quality space is.

To apply the model to a causal situation, one must also determine the initial state and the relevant powers that operate at that time. To fix our ideas, here is an example of a 'causal situation':

> [S]uppose you ingest calcium, which disposes towards bodily health in a number of respects, such as good bones, teeth and muscles. Let us represent this as a vector towards *F*, where *F* stands for bodily health and *G* stands for ill health. At the same time, however, you may be subject to a number of factors

[3] As Lombard (1986, 113) says: 'Static properties that objects can have and then lack [..] divide into kinds in accordance with the following intuitive principle: P and Q belong to the same kind just in case they are contraries and it makes good sense to suppose that a thing having P comes to have Q'.

> that dispose away from health. You could be tired, stressed, have drunk too much coffee, experienced passive smoking, and so on. (2011a, 27)

This is a one-dimensional quality space with two qualities, F (good health) and G (ill health), and various powers in action. In this 'causal situation', the thought is that, given all the operative powers, there is a dominant disposition towards one of the qualities of the quality space. It is assumed that there is a starting point from which causation will operate, which is represented by a vertical line drawn at the point within the quality space which captures the current state of the causal situation (e.g., the *exact* current state of one's health). On the line, the active vectors are plotted having their directions towards the quality (F or G) they dispose. Note, incidentally, that this presupposes that each operative power disposes towards one *and exactly* one quality in the quality-space. This is clearly an unjustified assumption; diet, for instance, can dispose towards both good *and* bad health. But this is an objection that we discuss later; for now, let's take the disposition towards a single quality as an idealization of the model.

Taking a cue from John Stuart Mill's (1882) account of the *total cause*, M&A take the (total) disposing cause to be the 'sum' of the acting powers. Given that they take causes to be complex, consisting in many different powers that can work with or against each other, they claim that the way to understand this 'sum' of powers is via *vector addition*.

The use of vector addition underlines the contrast between pandispositionalism and the traditional view about dispositional causation. Traditionally powers are taken to require a stimulus and some background conditions to manifest themselves. But on pandispositionalism, stimuli must themselves be powerful and this is, at least *prima facie*, in favour of the M&A project: being realist about powers they do not hold a distinction between the single power that becomes manifested and the rest of the conditions that enabled it to do so. As they put it:

> Metaphysically, we should judge them [the powers] on a par to the extent that they all contribute. A distinction between causes and background conditions cannot have any real ontological strength because the effect is not triggered until they are all present, which can in any case be a momentary matter. […] The distinction between causes and background conditions is not an ontologically grounded one, but rather a pragmatic or epistemological one. (2011a, 32-33)

Thus, instead of passive powers being stimulated, C. B. Martin's (2008) idea of mutual manifestation partners looks more conducive to the M&A view.[4] Under this understanding, the total cause consists in multiple powers *working* together. M&A argue that the model has the advantage of presenting the composition of the causes for each individual causal situation: the cause is captured by the resultant vector **R**, as the outcome of *vector addition*.

Vector addition is taken as the *tour de force* of the M&A model. According to them, the vectorial character of powers reveals that they convey only a *tendency* towards an effect; the vector of the resultant power **R** captures the dispositional modality of power-based causation. More specifically, the point within the quality space that coincides with the end-point of the resultant vector represents the disposition that the certain causal situation has the tendency to manifest. It does not represent what *will* or must happen given the specific component powers; it only represents what is disposed to happen (2011a, 175).

On the traditional view of powers, there are two possible states: the state of an unmanifested power, and the manifestation of this power. A power is not (necessarily) preserved after it is manifested. A glass is fragile, and when the fragility of the glass is manifested, the glass breaks and the

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[4] Martin says: 'I have been talking as if a disposition exists unmanifested until a set of background conditions is met, resulting in manifestation. This picture is misleading […] A more accurate view is one of a huge group of disposition entities or properties which, when they come together, *mutually manifest* the property in question; talk of background condition ceases, replaced by talk of power nets' (2008, 50).

power ceases to exist. On the M&A view, things look different. The component powers compose into the (resultant) dispositional cause. However, the ontological commitment to both the component powers *and* the resultant power raises the following question: if every power is associated with a certain manifestation, should the effect be explained in terms of the multiple manifestations of the component powers, or as the outcome of the resultant power? The M&A model seems committed to the latter view since it introduces the resultant power to represent the total disposition of the causal situation. Moreover, were powers to be considered as vectors, Nancy Cartwright's criticism of realism concerning the component forces and the metaphor of the vector addition seems plausible. As Cartwright has argued: 'The vector addition story is […] a nice one. But it is just a metaphor. *We* add forces (or the numbers that represent forces) when we do calculations. Nature does not 'add' forces. For the 'component' forces are not there, in any metaphysical sense, to be added…' (1983, 59). The pandispositionalist account of M&A is meant to deny this. It takes it that component powers are as real as the resultant power. To give an account of their own view, M&A claim that the situation is analogous to the statue and the clay case, so both the component *and* the resultant powers exist as not entirely distinct existences, nor in a part-whole way; rather they stand in a *composition* relation.

Be that as it may, there is another issue in the offing. According to the M&A model, the component vectors represent the exercising powers that act in the specific causal situation; yet, the resultant power is an unmanifested power, since the resultant vector represents nothing more than a tendency (disposition) towards an effect (2001a, 175). This way to view things creates a conundrum. If the component powers are operating, i.e. if they are doing their work, how is it possible for the resultant power to be unmanifested? What more does it need in order that it becomes manifested than the manifestation of the component powers? To resolve this conundrum, we should either assume that the resultant power is a new (emergent) power which needs the addition of another manifestation partner for it to become manifested; or we should assume that the activation of powers does not consist in their manifestation. If we follow the first horn, we are in need of an account of the supposed additional power that is needed, since it seems clear that all the powers necessary for the effect are already present. If the fragility and the striking of a vase (being the component powers) are manifested, then what is the new resultant power? And why isn't it manifested? If we follow the second horn, it seems we are committed to the implausible claim that although a power is operative or activated, it can still exist unmanifested.

Let's leave all this to the one side and focus on the main idea, viz., that powers can be represented vectorially. In the sequel, we pose two questions concerning the vectorial representation of powers, both of which are answered negatively:

- Are powers vectors?
- Can they be usefully viewed as *in some sense* analogous to vectors?

3. Powers are not vectors

In his *Process and Reality*, A.N. Whitehead put forward the claim that *all things are vectors* (1929, 309). At the final stage of the presentation of the vector model, M&A reflected on this claim and noted:

> What exactly Whitehead meant by this, we may never know. His work defies simple and unequivocal interpretation. But we have offered a model for causation in which Whitehead's statement would make some sense. All things have properties and all properties are powers, for the pandispositionalist. If such powers are understood as directed towards a manifestation, with a degree or intensity, then all things become vectors or at least they can be represented as such. (2011a, 45-46)

This may well justify us in attributing to them the strong clam that *powers are vectors*.

3.1 Power-vectors are *not* vectors

Geometrically, a vector can be represented as a directed line segment with an arrow indicating the direction and whose length is the magnitude of the vector. In physics, vectors have been used for representing forces. Forces have magnitude and direction, and they add vectorially. In the M&A model vectors are used to represent the exercising powers that operate upon the quality space (2011a 24). But are powers vectors? Do they, too, add vectorially?

For something to be a vector it is not enough that it has a direction and a magnitude. A moving car has both but it is not a vector. Mathematically, vectors are defined in vector spaces. Let V be a set on which two operations are defined: (1) *vector addition*, which combines two elements of V and is denoted by "+", and (2) *scalar multiplication*, which combines a complex number with an element of V and is denoted by juxtaposition. V, together with the two operations, is a *vector space* over the set of complex numbers $\mathbb{C}$ if the following properties hold.[5]

Additive Closure of Vector Addition
If $\boldsymbol{u}, \boldsymbol{v} \in V$, then $\boldsymbol{u} + \boldsymbol{v} \in V$.

Scalar Closure of Vector Addition
If $\alpha \in \mathbb{C}$ and $\boldsymbol{u} \in V$, then $\alpha\boldsymbol{u} \in V$.

Commutativity of Vector Addition
If $\boldsymbol{u}, \boldsymbol{v} \in V$, then $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$.

Associativity of Vector Addition
If $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$, then $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}$.

Additive Identity
There is a zero vector, $\mathbf{0}$, such that $\boldsymbol{u} + \mathbf{0} = \boldsymbol{u}$, for all $\boldsymbol{u} \in V$.

Additive Inverses
If $\boldsymbol{u} \in V$, then there exists a vector, $-\boldsymbol{u} \in V$, so that $\boldsymbol{u} + (-\boldsymbol{u}) = \mathbf{0}$.

Distributivity across Vector Addition
If $\alpha \in \mathbb{C}$ and $\boldsymbol{u}, \boldsymbol{v} \in V$, then $\alpha(\boldsymbol{u} + \boldsymbol{v}) = \alpha\boldsymbol{u} + \alpha\boldsymbol{v}$.

Scalar Multiplication Associativity
If $\alpha, \beta \in \mathbb{C}$ and $\boldsymbol{u} \in V$, then $\alpha(\beta\boldsymbol{u}) = (\alpha\beta)\boldsymbol{u}$.

Identity Element of Scalar Multiplication
If $\boldsymbol{u} \in V$, then $1\boldsymbol{u} = \boldsymbol{u}$.

Distributivity across Scalar Addition
If $\alpha, \beta \in \mathbb{C}$ and $\boldsymbol{u} \in V$, then $(\alpha + \beta)\boldsymbol{u} = \alpha\boldsymbol{u} + \beta\boldsymbol{u}$.

Something, then, is a vector by being a member of a vector space V.

The properties expressed above implicitly define *vector addition* and *scalar multiplication*.

The *norm* or *length* of a vector is an important type of function that can be defined on a vector space. It is a function satisfying the following:

[5] Bold letters denote vectors and Greek italic letters denote scalars.

1. $\|\boldsymbol{u}\| > 0$, when $\boldsymbol{u} \neq \mathbf{0}$ and $\|\boldsymbol{u}\| = 0$ iff $\boldsymbol{u} = \mathbf{0}$.
2. $\|\alpha\boldsymbol{u}\| = \|\alpha\|\|\boldsymbol{u}\|$, for any scalar a
3. $\|\boldsymbol{u} + \boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$

A basis for a vector space is a linearly independent set of vectors, such that any vector in the space can be written as a linear combination of the elements of this set. In most problems in physics and mathematics, the choice of an appropriate coordinate system provides great computational advantages. In the case of the usual two- and three-dimensional vectors, it is typical to represent an arbitrary vector as a sum of unit vectors, where the unit vector has the same direction as the given (nonzero) vector $\boldsymbol{u}$.

A two-dimensional vector $\boldsymbol{u}$ whose initial point is at the origin $(0, 0)$, can be uniquely represented by the coordinates of its terminal point (u_1, u_2). This is the component form of a vector $\mathbf{u}$, expressed as $\boldsymbol{u} = < u_1, u_2 >$, where u_1 and u_2 are the components of $\mathbf{u}$. The component form of the vector with initial co-ordinates $P = (p_1, p_2)$ and terminal co-ordinates $\mathrm{Q} = (q_1, q_2)$ is $\boldsymbol{u} = < q_1 - p_1, q_2 - p_2 > = < u_1, u_2 >$. Finally, the magnitude of $\boldsymbol{u}$ is given by the following expression:

$$\|\boldsymbol{u}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{{u_1}^2 + {u_2}^2}.$$

From this description of a vector-space it seems obvious that powers *qua* vectors (from now on: *power-vectors*) are nothing like vectors in vector spaces. Quality spaces, where power-vectors live, are not vector spaces. And since vectors can exist only in vector spaces, powers are not vectors.

But could it be that power-vectors exist in a vector space? After presenting the idea of powers having vectorial features, M&A claim that:

> At this point, we are thus able to make use of another notion that comes from vectors as used in physics, namely, vector addition. Powers can combine with additive, and sometimes subtractive, effects […]. When they do so, they may overall dispose in one direction or the other. […] Vector addition, in any case, can proceed in a rough-and-ready way without having to add and subtract the numbers [these that correspond to the intensity of the power-vectors]. We can perform a simple analogical addition by placing the tail of one vector on the head of another. (2011a, 28)

To perform a vector addition as described here, it is required that powers satisfy the associativity and the commutativity of vector addition. Is adding power **a** to power **b** the same as adding **b** to **a**? Or is it the same for power **a** to be added to powers **b** and **c**, as with power **c** to be added to powers **a** and **b**? It seems that powers do not consistently obey the properties of vector addition. The order by means of which two or more powers combine may lead to different manifestations. M&A are silent on this matter. But let us assume that power-vector addition does satisfy the two foregoing laws. How much of the vector theory does the power-vector model take? It's by no means clear. To be sure, the M&A model also includes the zero vector. As they put it:

> There is a special case of vector addition that is very important to distinguish. This is a case where the dispositions towards *F* and the dispositions towards *G* balance out perfectly, such that the resultant vector is neither directed towards *F* nor towards *G*. Where we have perfectly counterbalancing dispositions in this way, we say that we have a zero resultant vector. (2011a, 29)

But no further information is given as to which of the listed properties of vector spaces are satisfied by the power-vector addition. This should be enough to show that power-vector addition is not really vector-addition. And in any case, it should be by now clear that if power-vectors are vectors, they should be characterised by more than 'direction' and 'magnitude'.

To a large extend M&A focus their attention on one-dimensional quality spaces, which makes vector addition easier. This already creates a problem since it requires that *every* kind of quality creates a single dimension. Given a model of a one-dimensional quality space, the power-vectors that operate in it can be seen as being necessarily one-dimensional (see Fig.1). One-dimensional power-vectors are not 'free vectors'; they are supposed to be represented by localized line segments, the origin of which is a point in the quality space. For instance, M&A say:

> If we are trying to represent a causal situation with respect to, for instance, the temperature of someone's hand, then the vertical line represents the temperature of the hand at the starting point such that it could become warmer, if the situation moved towards *F*, or colder if the situation moved towards *G*. (2011a, 24)[6]

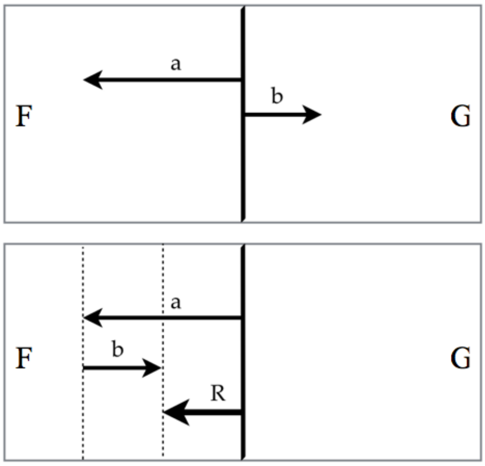

Fig. 1
A one-dimensional vector model of two powers at work and their addition, according to Mumford and Anjum.

Here is a crucial question: what are the identity-conditions of power-vectors? When it comes to vectors, things are straightforward. Two vectors are equal if and only if they have the same direction and magnitude. Besides, any vector can be translated to the single vector that has the same direction and magnitude and its starting point is located at the origin of the coordinate system. Take velocities (or any other vectorial quantities). We can certainly tell when two velocities are the same by saying that they have the same direction and magnitude (that is, they are the same vectors—*in a given vector space*). But that's exactly what we cannot do with powers: we cannot simply tell that two powers are the same iff they have the same direction and magnitude. Hence, the whole aim of using power-vectors qua vectors to identify powers collapses.

To see this, let us suppose that we stipulate that the origin of the 'coordinate system' of the model is the point of the initial state (whichever that is) of the causal situation. Assume that a power-vector **a** (the 'power-vector' representing a power to heat the hand, let's say) has the same direction *and* magnitude with another power-vector **a**′. Assume, also, that **a**′ corresponds to a different power, viz., one disposed to yield a different quality (e.g., the power of the hand to be

[6] Let's forget, for the time being, that temperature is scalar; or that energy—which actually gets transferred between the hand and the environment—is a scalar quantity. Let's play along and assume a heating power (whatever that is!) which is vectorial (*qua* power).

warmed up). Though if **a** and **a′** were vectors they would have to be equal, *qua* power-vectors, they are different since they represent different powers.

Conversely, let us assume that there are more than one powers that are directed towards one and the same quality F (e.g., towards raising the temperature of the hand). Assume also that some of them at least have the same 'intensity' —and hence that they are represented as having the same magnitude. They would all be plotted on the point of origin. But there is no fact of the matter as to what the 'direction' of each power is and how it should be plotted in the quality space. One power-vector **a** might have direction α and another power-vector **a′** might have direction α'. Qua power-vectors all these would be equal, since the represent the *same* power towards F. But they are represented by different vectors, since they have different directions. Should the objection be that powers directed towards the same quality have the same direction, the reply would be that this is not obviously so. It is possible for one and the same power to be represented by different power-vectors of the opposite direction and of different magnitude.[7]

Hence, distinct power-vectors can represent the same power. And conversely, distinct powers can be represented by the same power-vector. So if powers are power-vectors, we cannot tell, in principle, when two powers are the same or not.

There is more to be said against the power-vector model. For instance, there is no stipulation of a coordinate system. Could it be argued that such a system is implied, as it were, on the basis of a given initial point within the quality space, and the postulation of different dimensions as corresponding to different quality kinds? Even if this suggestion could be defended (as we are about to see, there are no unit vectors; nor a basis), it could only work with single-dimensional spaces and power-vectors. Yet, apart from the detailed cases of power-vectors that operate within a one-dimensional model, M&A talk about circumstances that more than one kind of quality can be affected by a collection of acting powers, suggesting that their model can be expanded to having more than one quality-dimensions. This is an example (originated in Geach 1962, 102) that M&A presented as a way to prompt the idea that powers interact with each other in multiple dimensions:

> [A] room contains both a heater and a cooling air conditioner. We will make the case two-dimensional. The heater can, let us say, warm the room to 25^{o}C within an hour with dry air. The air conditioner can cool the room temperature to 10^{o}C within an hour with slightly damp air. Suppose both the heater and the cooler are left on and the powers along the hot-cold and dry-damp dimensions are inseparable. We can still, nevertheless, understand such complex powers along the lines of vectors, and understand their combined effect along the lines of vector addition. (2011a, 44)

This example motivates a version of the model that consists in positioning two pairs of extremes in what seems to be a perpendicular position relative to each other, while the initial point is now a single dot on the two-dimensional vector space, on which the power-vectors of the 'power to warm' and of the 'power to cool' are plotted on a certain angle to each other.

[7] An example we discuss later regards fragility. One can refer to fragility as 'the power of something to break easily when it is knocked', or as 'the power of having low resistance to a knock'. Both descriptions refer to the same quality of a fragile thing; however, were this quality to be represented as a power-vector, this can be done on the basis of either these descriptions, i.e. as a power-vector of large magnitude representing the 'easily-breaking' behaviour, or as a power-vector of small magnitude representing the 'low-resisting' behaviour. These two power-vectors should have different directions.

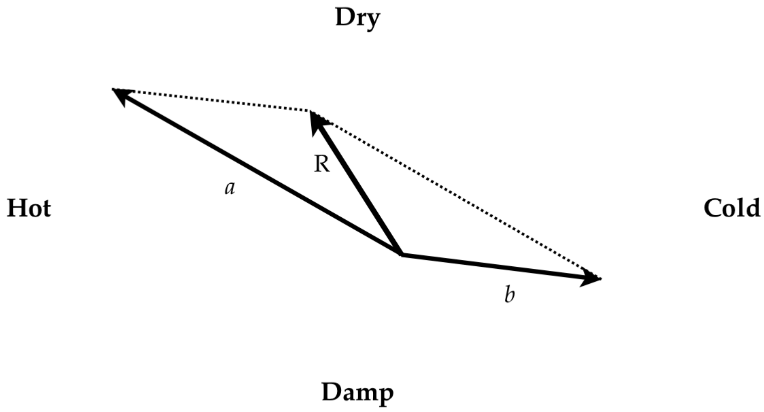

Fig. 2
Vectors within a two-dimensional quality space, according to Mumford and Anjum.

This set up generates many questions about the way it is constructed. For instance, how are the exact positions, directions, and magnitudes of the power-vectors to be specified? But, assuming it for now, it is hard to see how *any* system of co-ordinates can be associated with it. And given that a model such as this, if it is to be realistic at all, should allow for a number of dimensions (pairs of extreme qualities), it makes no sense whatsoever to have a formula for 'calculating' the resultant power-vector. It makes no sense to ask what is the component form of the vector or its norm. In fact, we think there is no fact of the matter. Because, for all we know there is no reason to think that there is a *correct* way to describe the operation of two powers as being additive. Why, for instance, not to think of the interaction between powers as being represented by the *external product* of the vectors?

Without a well-defined coordinate system, it is impossible to locate any power-vector in a multi-dimensional quality space, because no exact angle of a power-vector can be determined. Furthermore, while any vector that forms an angle relative to the axes of its coordinate system can be analysed into component vectors that are parallel to the basis vectors, such a notion is non-existent in the power-vector model. Within a coordinate system, the basis vectors have another role too: they provide the unit upon which a scalar multiplication of the vectors parallel to them can operate. Such a unit is absent from the power-vector model. But even if such a unit were provided, there is another matter to be reckoned with: taking into account that the different dimensions of the M&A model represent kinds of qualities (e.g. temperature, humidity etc.) that are measured in different 'units', how meaningful would it be for a power-vector to be located in the 'unification' of two, or more, incompatible quality-dimensions? What sense does it make for vectors operating in different dimensions to be added?

Hence, a multi-dimensional model for power-vectors is impossible on the basis of i) the impossibility of a well-defined coordinate system, ii) the absence of basis vectors and measuring units, and iii) the incompatibility of the various dimensions regarding measuring units of the qualities involved.

It should, therefore, be obvious that the power-vector model for causal powers cannot be a vector space. Powers are not vectors. To sum up, we don't know how to understand power-vector addition; we have no clue as to whether any of the properties of vector spaces are satisfied; we have no way to define the norm of vectors and no way to specify a unit vector. These are all technical limitations that the power-vector model faces. But the technical limitations are the consequence of a fundamental mismatch: powers are not cut out to be vectors after all. What remains, then, is to look into the weaker claim that such a representation of powers via power-vectors can be grounded on a (weak) *analogy* between powers and vectors.

3.2 Powers are not *analogous* to vectors

It might be that the only elements of vectors that are needed for the M&A model to work —for purposes of philosophical illustration— are the magnitude and the direction (plus the addition law, in *some* sense). But this weaker view cannot be supported either, as it leads to problematic metaphysical implications.

The thought that powers have an intensity is something that a power-theorist could accept. After all a substance can be more or less poisonous, more or less soluble, more or less flammable, or more or less fragile than another. This indicates that powers admit of degrees: x might be more, or less powerful than y to F. Therefore, one might think, something can have a power to F to a certain intensity or strength.

Indeed, this seems to be the case with 'garden-variety' macro-level powers, which are the main examples that M&A use to ground their theory. They do acknowledge that powers at this macro-level are collections of more fundamental (micro-)powers. However, their view takes it that if causation operates at all levels, so do powers. This focus on macro-level causally potent powers commits the power-theorist to properties that are to be found in folk causal explanations, raising issues about the credibility that a power-based theory may have when it comes to physics. Referring, for instance, to *the power to warm* and *the power to cool* would clearly not be accepted as being scientifically accurate. The 'cooling' power and the 'warming' power are processes of energy transfer among objects with different temperatures. And most of the quantities that M&A think as vectorially representable powers are *scalar*. But let's not dwell more on it and just examine the extent to which, if at all, garden-variety powers have 'magnitude' and 'direction'.

In those cases, where we have an intuition that powers are more or less intense, how are we to determine the intensity of a power and represent it by means of length? Accuracy in ascribing the magnitude of each power-vector is important for vector addition. Any difference in the length of the power-vector entails a difference regarding the ending point of the power-vector within the quality space; and hence a difference in intensity. But this, in turn, indicates a different possible state that the causal situation can acquire once the manifestation has occurred and, ultimately, a different resultant disposition. The requirement of accuracy is, therefore, very crucial for the purposes of the model, if it is to offer anything other than vague, arbitrary and purely qualitative representations of the causal situation. But the very idea of a magnitude requires that powers are quantifiable; and more specifically, it requires that there is a unit-power. For only the ascription of a unit can result in the representation of a power in terms of magnitude.

Not all powers are quantifiable. Therefore, there is no way to determine the length of the directed segments in the model. While, for instance, we can claim that given the same amount of pressure exerted on a glass object and on a similar but ceramic object, the former has the tendency to break more (or greater) than the later, there is no certain (or absolute) intensity associated with the manifestation of fragility. Something is characterized as fragile because it is *more disposed to break* in *comparison* to other things, or it is *disposed to break to different degrees* depending on the amount of pressure it will receive. Hence, a power like the fragility of an object is context sensitive, and doubly so. For one thing, it is a comparative power, therefore it needs a contrast-class. For another, it can be manifested in different degrees depending on the force that is exerted on the surface of the object, even if the intrinsic properties of the object are unchanged. The vector model is incapable of capturing this context-sensitivity via the intensity that is ascribed to powers. For instance, there is no way to show that the tendency of a glass to break (consequently, the magnitude of the corresponding power-vector) varies depending on the magnitude of the pressure that is exerted on it. As it has been acknowledged already by D. Manley and R. Wasserman (2007, 70), as long as there are dispositional predicates displaying context sensitivity, degrees, and comparative use of powers, a scale for each power is required, 'along which objects can be compared, and by reference to which the context-dependence of dispositional predicates can be explained'.

The demand for a unit of measuring the intensity of a power might be taken to be too strong by M&A, as they do not require of their model to provide any quantitative results, but to *qualitatively* represent what is disposed to happen in a causal situation. For these purposes, it might seem, depicting *roughly* the intensity of a power would be adequate. After all, they take it that although the power-vector model does not introduce any units for measuring the intensity of a power, it still succeeds in representing it. It is, however, just a lucky accident that the M&A model seems to work for some cases. This happens because the powers being involved in the chosen examples are associated with two kinds of qualities: either they are used to capture a change in some scalar quantity (e.g. a change in the temperature of the room by 10°C), or to represent something that is already a vectorial quantity (e.g. the force that a book exerts on the surface of the table). These are exceptional cases. However, the majority of powers cannot be so represented. Most powers are not even intuitively comparable. Which arrow should be longer in the model for the combustion of a match? Is the power of the flammable material smaller or larger in intensity than the power of the oxygen? There is simply no fact of the matter. And how are arrows to be plotted in the case of the repulsion between two charges? Or the attraction between two masses?

Perhaps all that M&A care about is plotting the resultant disposition. But then it is *totally* irrelevant how we plot the 'intensities' of the various component powers, provided we get right the prevailing intuition about what the dominant disposition is. Furthermore, the power-vector model heavily overdetermines the causal situation. Suppose that in a causal situation, all the powers that are relevant to the breaking of a vase are acting. If there is a dominant disposition towards the manifestation of breaking, the only constraint on picking the 'intensities' of the component powers is that when they are 'added', the 'resultant' is pointing towards the breaking side of the quality-space. The very idea that powers have specific 'intensities', *whatever that means*, is redundant and irrelevant. In the end, it is totally clear that the analysis of the dominant disposition of a causal situation to component powers with certain 'magnitudes' is completely arbitrary.

But if the 'intensity' of a power is a misnomer, can we at least make good sense of the *direction* of powers? The application of the notion of direction to power-vectors is very problematic. It is enough to show that it creates inconsistencies for which any attempt to be resolved is purely *ad hoc*. The core of the problem is in the thought that even if we were to grant that all powers are directed towards their manifestation, this 'directedness' can be seen as analogous to the *direction* of a vector.

One way to see this is by examining the way that the M&A model represents non-acting, that is unmanifested, powers. For they too have a tendency towards their manifestation and therefore they should be represented as power-vectors in the relevant quality space (even though they are not acting). To fix our ideas, let us think of the toy example of an unburned inflammable match. Since flammability is a power, it should be represented as a power-vector within the relevant quality space of the combustion of the specific match. This quality space is one-dimensional and has two boundary qualities: *not burned* (~B) and (say) *totally burned* (B_t). The initial point of this causal situation is on ~B, given that the match is unburned. Suppose that in the environment of the match there is no oxygen. Then the only power-vector to be plotted is the flammability-vector directed towards B_t. Even in this isolated situation, following the norms of the model, what is being represented is the dominant tendency of the current situation; that is, the tendency of the match to become burned just by plotting its (power of) flammability. This example is analogous to other dubious powers that are supposed to be directed towards their manifestations, such as that every living thing being disposed towards its death, or the sunrise being disposed towards the sunset. The possibility of unmanifested powers suggests that an understanding of tendency or dispositionality in terms of directedness is problematic.

M&A themselves have noted the inability of the power-vector model to represent consistently unmanifested or single powers. By emphasizing that powers require manifestation partners in order to manifest themselves, their suggested answer is based on the following idea: as long as there is some other power, or powers, whose existence is required for the manifestation of power *p* to occur, if a power *p* remains unmanifested, this must be because there is an equilibrium that keeps it unmanifested. The motivation of this idea is by the following example:

> Dynamite … has an explosive disposition which seems to manifest only when it is ignited. Without stimulation, it might be thought, it does nothing. On further inspection, however, we find that dynamite's explosive power comes from it being made up in three parts of nitroglycerin, a substance that is so explosive that its disposition has to be counterbalanced by one part diatomaceous earth, which gives it a countervailing power of stability. … [So], nothing happens in respect of the dynamite's power of explosiveness only because it is counteracted by other hidden powers. When the fuse is lit on a stick of dynamite, this is actually the addition of a further power, which then takes the situation out of the equilibrium (2011a, 36-37).

For them, cases like the unmanifested power of explosiveness are being misrepresented. These 'unknown, hidden, or taken-for-granted', powers have to be represented in the model as power-vectors that have opposite direction and equal magnitude to the unmanifested power (2011a, 36). As a result, the powers cancel each other out, their resultant disposition of the causal situation being of zero intensity, thereby allowing—according to M&A—the causal situation to remain unaltered, as if nothing happens. Although they present this as a special case, there is no reason to think that this description does not generalize to *all* possessed but unmanifested powers. After all, according to pandispositionalism, the components of (resultant) powers are powers too. Thus, following the picture that is sketched here, when a power is unmanifested, the component powers are in an equilibrium state. Suppose the fragility (*f*) of a glass is unmanifested, viz., the glass, though fragile, is not broken (~B). The way that M&A describe the situation of the unmanifested power of fragility is the following:

> [F]ragility on its own seems to do nothing, but isn't this only because the fragile object also possesses some countervailing stability and elasticity, which holds that object together enough until some knock is sustained? (2011a, 37).

Is this plausible at all? On the one hand, it inflates unnecessarily the metaphysics of powers, as it leads to the supposition of additional powers to explain (by counteracting to) unmanifested powers. On the other hand, it is inconsistent with the setup of the very M&A model. Figure 3 illustrates the application of the model for the example of unmanifested fragility:

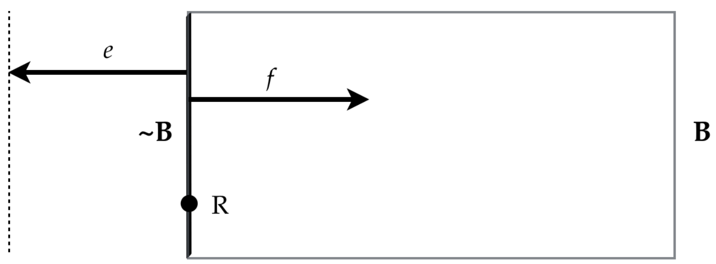

Fig. 3
The equilibrium of unmanifested powers, according to Mumford and Anjum.

The power-vector **e** represents the power of elasticity mentioned in the quotation above, as the one that countervails **f**. The view that powers have directedness even when they are not manifested leads to postulating this countervailing power. However, the restriction in what the direction of **e** should be, so as to explain the equilibrium situation, immediately creates a tension between this setup and the notion of directedness. For, what is **e** directed towards? Towards more elasticity? Or towards more stability? Does it 'push' the glass towards being as much elastic as it is fragile? Then why is the glass characterized as fragile? An object that is characterized as very fragile can be said to be minimally elastic. Then by ‘fragility’ and ‘elasticity’ we seem to denote one and the same power, and it is for purely pragmatic reasons that we opt for using one name over the other. Not only does the M&A solution to the alleged directedness of unmanifested powers not solve the problem with the notion of directedness that it assumes, on the contrary; it actually accentuates it. After all, positing a counteracting power-vector **e** is clearly incompatible with the setup of the power-vector model, as **e** appears to be operating outside the quality space {broken, unbroken} where there is no quality towards which to be directed.

Moreover, if we consider that in every application of the model the resultant power R does not differ ontologically from any other power, in a case of equilibrium where the resultant R has zero ‘intensity’, (it is represented by the ‘zero’ power-vector), this power does not have any direction either. Since R does not have either intensity or direction, it does not differ (and hence cannot be distinguished) from any other zero-resultant power.

When causality occurs, it does by means of what is described as ‘the resultant power’. Thus, given the epistemic inaccessibility to the component powers, one can only intuitively determine the direction of each component power. Maybe in some cases, ‘fixing’ the direction of the contributing powers is more straightforward than others. However, in examples where the directedness is context-dependent, one power can have different directions: while an amount of serum cobalamin (vitamin B12) has the power to treat vitamin B12-deficiency, disposing towards better health, in different circumstances the same amount of vitamin B12 has the power to raise the serum cobalamin levels so high that there are pathological consequences —thereby disposing towards deteriorating health. This example shows that whereas a component power has some directedness towards a certain manifestation, in this case *the power towards raising the levels of serum cobalamin*, in different contexts the very same power has to be represented as having different direction. Consequently, it is wrong to associate the dispositional character of powers with the view that powers have any particular *direction*.

The root of the problem, it seems to us, is that the sense in which the power is ‘directed to its effect’ is, at best, metaphorical. A vector is *directed* quantity; this means it has a direction. It does *not* mean it is directed towards anything. To find the direction of a vector quantity, e.g. velocity, we need to plot a graph in which the direction is fully specified by the angle of the vector relative to the co-ordinates. In fact, a vector can also be regarded as the set of all directed line-segments that are equivalent to a given directed line segment. There is no sense of semblance here between the ‘directedness’ of powers and the direction of vectors. But even as a metaphor, the idea that powers are directed towards their effects is misleading. While in the M&A model the power-vector is ‘directed’ towards an existing point in the quality space, powers have only the tendency towards the change of the state that their manifestation would yield. There is no existing *point* towards which they are directed.

4. **Conclusion**

The idea behind representing powers as vectors is that vectors can capture the key features of dispositional causes: directedness and intensity. As we have shown, the power-vectors in the M&A model are *not* vectors. In fact, we have shown that powers *cannot* be vectors. The idea is physically wrong and metaphysically confusing. If metaphysical views are to be consonant with current physical theories, then powers —whatever their merits— cannot be modeled as vectors. But then we asked: could it be that there is some sense in which powers are *analogous* to vectors, even if strictly speaking they are not vectors? We have shown that the analogy is misconceived. The notions of magnitude and directedness of power-vectors are problematic and incompatible with the idea that vector-powers operate within a quality space via vector addition. Powers cannot even be loosely analogous to vectors by means of their alleged intensity and directedness. It is, then, safe to conclude that the vectorial representation of powers is misleading and cannot cast any light on dispositional causation. Powers are not (and cannot be represented as) vectors.

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