Gentzen's "cut rule" and quantum measurement in terms of Hilbert arithmetic.
Metaphor and understanding modeled formally

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Abstract. Hilbert arithmetic in a wide sense, including Hilbert arithmetic in a narrow sense consisting by two dual and anti-isometric Peano arithmetics, on the one hand, and the qubit Hilbert space (originating for the standard separable complex Hilbert space of quantum mechanics), on the other hand, allows for an arithmetic version of Gentzen's cut elimination and quantum measurement to be described uniformy as two processes occurring accordingly in those two branches. A philosophical reflection also justifying that unity by quantum neo-Pythagoreanism links it to the opposition of propositional logic, to which Gentzen's cut rule refers immediately, on the one hand, and the linguistic and mathematical theory of metaphor therefore sharing the same structure borrowed from Hilbert arithmetic in a wide sense. An example by hermeneutical circle modeled as a dual pair of a syllogism (accomplishable also by a Turing machine) and a relevant metaphor (being a formal and logical mistake and thus fundamentally inaccessible to any Turing machine) visualizes human understanding corresponding also to Gentzen's cut elimination and the Gödel dichotomy about the relation of arithmetic to set theory: either incompleteness or contradiction. The metaphor as the complementing "half" of any understanding of hermeneutical circle is what allows for that Gödel-like incompleteness to be overcome in human thought.

Keywords: cut-elimination, Hilbert arithmetic, metaphor, proposition, propositional logic, quantum measurement

I THE CONCEPTUAL BACKGROUND: HILBERT ARITHMETIC IN BOTH WIDE AND NARROW SENSE AND THEIR MORPHISMS

Cut-elimination, on the one hand, and quantum measurement, on the other hand, belong to two areas fundamentally different (and thus ostensibly unifiable only eclectically) according to the organization of cognition in Modernity and shared by common sense. Indeed, Gentzen (1935) proved his cut rule (or "cut-elimination") as to classical logic and mathematics though thereafter it has been extended to many non-classical domains of logic and mathematics and even metamathematics, however always only within the rigid framework of the Cartesian "mind" where both logic and mathematics are situated in the contemporary "episteme"[1].

On the contrary, quantum measurement refers to an experimental science such as quantum mechanics and consequently, it should be localized on the opposite "shore", i.e. that of the Cartesian "body", thus reliably divided from the former by the "abyss" of dualism. So, what the

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[1] With Michel Foucault's term and concept (1966) and meaning rather the contemporary episteme after Cartesianism including the figure of "human being" as a necessary arbiter about any question touching both "mind" and "body" (i.e. external reality).

article intends to investigate needs a "bridge over it", by the by, invented by quantum mechanics itself forced to do this by its main problem: how to describe uniformly the *discrete* quantum entity and the macroscopic apparatus obeying the *smooth* differential equations of classical mechanics or physics.

At first glance, the objective seems to be different enough from the link between the *mathematical* cut-elimination[2] and the *physical* quantum measurement: a link furthermore necessary for the subject of the article to make sense. However, the theorems about the absence of hidden variables (Kochen, Specker 1967; Neumann 1932) allow for them yo be reinterpreted as the indistinguishability of "model" and "reality" at least in its scope and then, they can be realized also philosophically: as the missing bridge between the Cartesian "body" and "mind" and applied to the narrow problem of the present investigation.

In fact, the corresponding pathway has been passed in other papers (e.g. Penchev 2021 August 24) resulting in a tool "ready-for use" such as Hilbert arithmetic which can be furthermore distinguished in two senses, narrow and wide: a structure able to include two Peano arithmetics, dual, idempotent and anti-isometric to each other, in the former, narrow sense, eventually complemented by a dual structure, both qubit Hilbert spaces, also dual, idempotent, and anti-isometric to each other, in the later, wide sense. So, Hilbert arithmetic in a wide sense is a formal, logical and mathematical framework to investigate the analogue of cut-elimination and quantum measurement by means of a rigorous enough method under the conjecture of their isomorphism or homomorphism adding auxiliary conditions if need be. The structure at issue represents cut-elimination in its branch (i.e. Hilbert arithmetic in a narrow sense), and quantum measurement[3] in the dual branch of the qubit Hilbert space delivering in advance the correspondence between them as classes of equivalence of the latter as the former.

Gentzen's cut elimination is literally formulated to a monotone *infinite* sequence of implications in the framework of classical logic. Nonetheless, it can be thought as an abstract

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[2] A wide enough context of cut-elimination by relative mathematical statements thus assisting the extension of the concept is suggested by Arai (1998). An interesting approach is that of Dowek and Hermant (2012) investigating a necessary condition for cut-elimination: called "super-consistency"; Gil and Rebagliato (2000) research "protoalgebraic Gentzen systems" as an equivalent (necessary and sufficient condition) of cut rule.

[3] The paper of Božić, Marić (1998) considers quantum measurement from a viewpoint very similar to that meant here (the viewpoint of quantum neo-Pythagoreanism), after which "wave function" is not a model, but reality itself: then, quantum measurement maps a branch of reality into its dual counterpart also belonging to reality. Thus, the classical correspondence of model and reality is replaced by their mathematical duality or physical complementarity. As to the "standardly classical" viewpoint to quantum measurement (thus alternative to the cited one), for example the papers (Busch, Lahti 1996; Maki 1989; 1988; Marinkovic, Damnjanovic 1987; Lamb 1986; Machida, Namiki 1980; Lubkin 1979; d'Espagnat 1974; Helstrom 1974; Reece 1973; Fujiwara 1972; Scott 1968; Weidlich 1967; Holze, Scott 1968; Mould 1962) or the book (Busch, Lahti, Mittelstaedt 1996) suggest a detailed outlook (as well as ergodicity conditions, e.g. in: *Daneri, Loinger, Prosperi 1962*) even extendable to the newer approaches to quantum measurement by entanglement and quantum information (Buch 2003) or those following the modal interpretation of quantum mechanics (Cassinelli, Lahti 1993; 1995). An intermediate position can be occupied by the theory of *continuous* quantum measurement (e.g. Diosi 1988; Diosi, Halliwell 1998).

infinite well-ordering only interpreted logically though it can be also not worse seen set-theoretically as an infinite monotone series of subsets as well as successive natural numbers obeying Peano axioms after the fundamental distinction that the set-theoretical interpretation means an *actual* infinite set of well-ordered elements unlike the arithmetic interpretation, for which the axiom of induction implies that all elements of the well-ordering at issue, being natural numbers, are *finite*[4].

What about the proper formulation of Gentzen cut-elimination meaning a monotone sequence of logical conclusions? It is suggested to be *actually infinite* as the case of the set-theoretical interpretation; or on the contrary: *finite*, as in the case of the arithmetic interpretation? One can promptly notice that cut-elimination after the arithmetic interpretation is trivial since it is a direct corollary from the transitivity of implication. So, only the set-theoretical interpretation can make sense as a serious problem and Gentzen proved just that.

Gentzen's proper pathway to prove cut-elimination is well-known and can be traced in his paper in two parts[5]. The present approach by Hilbert arithmetic is rather different mainly due to the following two features: (1) it discusses the generalization about an infinite, monotonically "decreasing" hierarchy furthermore rather arithmetically than properly logically as Gentzen in original; (2) the solution about a hierarchy in Peano arithmetic (i.e. about a sequence of natural numbers, so that the set of its members is infinite) is mediated by Hibert arithmetic inherently doubling it by a dual and anti-isometric counterpart. Those two peculiarities are to be discussed in more detail:

About the former, (1): if an infinite monotone sequence of implications is available, it can be always enumerated by a series of successive natural numbers, so that the set of all them to be infinite. The rule of cut elimination states that it can be reduced to a finite series (i.e. such that the set of its member to be finite), which is logically equivalent to the former one constituting an infinite set of implications. Then, it can be reduced to a single implication in virtue of the transitivity of implication. In other words, cut-elimination generalizes the transitivity of "implication" to an infinite set of implications. So, the statement of cut-elimination is to be "translated into an arithmetic language":

The series of subsequent natural numbers, 1, 2, 3, ..., n, ... enumerates a corresponding sequence of statements belonging to an monotone infinite sequence of implications:

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[4] Indeed, "1" is finite, adding a unit to any natural number "n", "n+1" is again finite. Consequently, the axiom of induction implies for all natural numbers to be finite. (On the contrary, e.g. the axiom of infinity in ZFC implies for the *set* of all natural numbers to be infinite.)

[5] A confusion can arise due to the direct relation of cut-elimination to the Gödel incompleteness in the present paper without referring to Gentzen's consistency proof (1936; 1938). In fact, if the axiom of choice is valid for sets to the ordinals of which the consistency proof is valid, they can be reduced to subsets of a countable set. Then, the consistency proof can be reformulated as the consistency of Peano arithmetic under the condition for the consistency of that countable set which is also meant if the consistency of whether set theory or Hilbert arithmetic has been granted in advance. So, the relation of cut-elimination and Gentzen's consistency proof is meant by virtue of the axiom of choice in the final analysis. The paper of Sanchis (1971) discusses the relation of cut elimination, consistency and completeness in classical logic.

$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \ldots \rightarrow S_n \rightarrow \ldots$ .Obviously their mapping is a bijection and even a homomorphism where any implication is represented by a unambiguously corresponding exemplification of the function successor originating from Peano arithmetic (to which the series of natural numbers at issue belongs). Then, cut-elimination states "arithmetically" that the corresponding infinite sequence of natural numbers can be unambiguously represented by a certain finite series notated e.g. as $1, a_1, a_2, a_3, a_n$. One is to expressly emphasize that a_n is the *last* member of that equivalent finite sequence; and in the final analysis; the initial infinite series of natural numbers is unambiguously determined by its first and last elements, i.e. $1, a_n$.

That arithmetic transcription is trivial in *only* Peano arithmetic in virtue of the axiom of induction due to which all natural numbers are finite. However, it is a serious and very difficult problem in *both* Peano arithmetic and set theory: this can be easily demonstrated by means of the Gödel incompleteness theorems (1931). Indeed, if whatever finite sequence "$1, a_1, a_2, a_3, a_n$" is chosen, it is necessarily incomplete to the initial infinite sequence of natural numbers (keeping the former to be consistent to the latter). So, Gentzen's result overcoming Gödel's restriction is quite not trivial since the rule of cut-elimination states that the former finite sequence is able to determine unambiguously the latter one though remaining always fundamentally incomplete to it.

Even Gentzen's statement seems to be wrong (but of course. only at first glance) once its "Gödel counterpart" (as above) has been granted in advance. Anyway, one can notice that they are distinguishable from each other according to their "directions" in the following intuitive sense. The latter statement means implicitly the direction "from finiteness to infinity" unlike the former referring the opposite direction "from infinity to finiteness". Then, both can be combined in a single consistent one, for example so:

By starting "from finiteness", the finite sequence is constructively or expressly shown therefore necessarily remaining always the same. On the contrary, i.e. "from infinity", one states that finite sequence unambiguously determining the infinite one only exists necessarily, but without explicitly demonstrating it therefore meaning the class of equivalence of all those finite sequences, but not any element within that class. Thus, the consistent reconciliation with the "Gödel counterpart" consists in no constructive method (or any other explicit one) to determine just one element of that class nonetheless that all finite elements of it result in a single implication linking first and last statement thus absolutely unambiguously.

In other words, cut-elimination, arithmetically interpreted, can be distinguished in two successive stages[6]: (1) the elimination of an uncertain, infinite set of elements thus resulting into an also uncertain, but finite set of elements; (2) though the latter, finite set is uncertain (or said otherwise, any of its element only *exists*, but cannot be determined constructively), all elements

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[6] The same two stages can be illustrated also by a relevant proof in intuitionistic predicate logic (e.g. as in: Borisavljević 1999) since that logic postulates a gap between finiteness and infinity due to the suspension of the "excluded middle" as to infinite sets.

can be further eliminated to exactly two ones, both first and last ones, consequently always the same.

Once cut-elimination is reinterpreted arithmetically as above, it turns out to an equivalent to the statement whether that any sequence of natural numbers constituting an infinite set has a certain last element or that the sequence at issue is well-ordered in both directions (being dual and anti-isometric to each other). The latter "paraphrase" of arithmetic cut-elimination[7] suggests immediately that Hilbert arithmetic[8] (even in a narrow sense) is absolutely sufficient to infer cut-elimination (literally, arithmetic cut-elimination and then the proper, logical one in virtue of their homomorphism[9]). Indeed, the second Peano arithmetic postulated in Hilbert arithmetic is both dual and anti-isomorphic to the initial one and therefore guarantees the well-ordering of the infinite sequence of natural numbers in both directions.

If Hilbert arithmetic in a wide sense is involved, all infinite sequences can be not worse represented unambiguously, by means of an nonempty set of "transfinite natural numbers" (equivalent to transfinite natural numbers both greater than any finite ordinal but less than any *countable* ordinal[10]). Furthermore, any of those transfinite natural numbers can be represented as the "uncertainty of the choice" of a corresponding finite sequence, and then, as a probability density distribution of all finite sequences to be chosen: that is, in the final analysis, a certain wave function also equivalently representable in the qubit Hilbert space.

As to the proper cut-elimination both arithmetical and logical, the representation by an ambiguous determined series of transfinite natural numbers is not necessary. However, one needs it (or any relevant equivalent) if the problem refers to the subject of this paper, namely the relation of cut-elimination and quantum measurement researchable only in the framework of Hilbert arithmetic in a wide sense[11].

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[7] As to the relevant counterpart for the arithmetization of the Gödel-like incompleteness in paradoxes of set theory, the paper of Dean (2019) suggests a detailed outlook, and Wiegand (2000), a generalized phenomenological-semantic viewpoint.

[8] The introduction of both induction and coinduction to cut-elimination (e.g. Tiu, Momigliano 2012) implies a viewpoint analogical to that by Hilbert arithmetic.

[9] For example, by the direction for "algebraic proofs of the cut-elimination theorems for classical and intuitionistic logic" (Avigad 2001) or "Gentzen structures" (Belardinelli, Jipsen, Ono 2004) as well as the investigation of Terui (2007) about which structural rules admit cut-elimination. An underlying arithmetic structure is meant if one considers cut-elimination in (e,g.) type theory (Titani 1973). The implicit hypothesis in the present paper is that cut-elimination relies on the fundamental relation of arithmetic and set theory and thus can be found in many interpretations though Gentzen (1935) formulated it to classical logic.

[10] For example, that can be defined recursively as follows. The initial ordinal is the least countable ordinal (notated as a_0). The next ordinal a_1 is defined so that the set of its subsets corresponds to the initial ordinal a_0. Furthermore, this exemplifies the recursive formula defining unambiguously any subsequent ordinal a_{n+1} from the current one, a_n. A paper (Penchev 2005) in Bulgarian discusses them being called there "quantum ordinals".

[11] Cut-elimination can be defined also to quantum logics (e.g. Faggian, Sambin 1998), however the way of linking cut-elimination and quantum measurement is inherently different in the present research as far as classical logic is granted to by absolutely sufficient for any quantum description (thus quantum logics are redundant and cuttable by "Occam's razor") since all quantum phenomena need a different understanding

Indeed, any given quantum measurement among the class of equivalence of all possible measurements of the same quantity of the same quantum entity means the fundamentally random choice of a single value, obeying the probability density distribution at issue. Already in terms only of Hilbert arithmetic in a wide sense, an element of the usual Peano arithmetic and equivalent to a single measured value is bijectively mapped to a transfinite natural number corresponding to a finite natural number, but belonging to the dual counterpart of Peano arithmetic as well as to a probability density distribution of all possible values to be measured.

So, one can notice that quantum measurement and cut-elimination are rather similar if not even and in fact, mathematically homomorphic. Both are absolutely defined by a pair of elements under the condition the one them to belong to the one Peano arithmetic, and the other one, to the dual counterpart of it. Anyway, one can see their essential analogy or identity if one has put off in advance the "spectacles" of our usual organization of cognition and shared by common sense: quantum measurement relates to reality and physics (quantum physics) unlike cut-elimination, which is supposed to be an only mental construction of human mind in mathematics[12].

The sketched scheme identifying cut-elimination and quantum measurement will be considered in more detail in the following sections step by step.

II THE GÖDEL INCOMPLETENESS VERSUS THE GENTZEN COMPLETENESS BY THE "ARROW" OF CUT-ELIMINATION

The viewpoint of Hilbert arithmetic (valid to it in both narrow and wide senses) reduces the relation of any Gödel-like incompleteness and any Gentzen-like completeness[13] to the arrow of any choice, including an elementary choice meant by a bit of information. The states before and after choice (by the by, just as the states before and after quantum measurement) are asymmetric to each other. Accordingly, the Gentzen-like completeness corresponds to the "normal" direction, first "before", then "after", but the Gödel-like incompleteness means the opposite "abnormal"[14] direction: first "after", then "before".

If one repeats that elementary structure of the temporal-like asymmetry similar to time "arrow" as to a bit information, the transition to the decoherent state (i.e after choice or quantum measurement) of whether "1" or "0" can be interpreted to be "Gentzen-likely" complete unlike

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of the relation of finiteness and infinity thoroughly in the framework of classical logic rather than some non-classical logic.

[12] Using the "spectacles" at issue, one would say that cut-elimination can be *only an imperfect mode*l of quantum measurement thus implying an irremovable nonzero finite difference between the. On the contrary, this article states the difference between cut-elimination and quantum measurement is identically zero (i.e. even not an infinitesimally small one).

[13] Both Gentzen-like completeness and Gödel-like [i.e. following the previous Gödel (1930) paper] completeness can be combined very elegantly as the papers of Bowen (1972; 1973) demonstrate. However, this should be rather related to the completeness of quantum mechanics (which is not discussed in the present paper) rather than to quantum measurement. An intuitionistic context of analogical ideas is suggested by DeMarko (2005).

[14] "Normal" and "abnormal" are in quotation marks for being chosen conventionally and idempotent to each other.

the opposite transition into a coherent state (i.e. before choice or quantum measurement) realizable to be "Gödel-likely" incomplete, for example, after erasing of whether "1" or "0" or the case of an "empty" binary cell.

That interpretation of both Gödel-like incompleteness and Gentzen-like completeness can be considered as a conjecture, i.e. as a mathematical statement needing a relevant proof what the nonstandard bijection (discussed in detail in other pares, e.g. *Penchev 2022 June 30; 2022 May 11; 2022 March 11*) can deliver by its two *dual* "directions": $P \to (P^0 \to P^- \otimes P^+)$ & $(P^+ \otimes P^- \to P^0) \to P$.

Then, the former direction, that is: "$P \to (P^0 \to P^- \otimes P^+)$", is meant by the arithmetic counterpart of the Gentzen completeness, and the latter, opposite direction, by the proper Gödel incompleteness. Indeed, "$(P^+ \otimes P^- \to P^0) \to P$" is able to reproduce literally the Gödel dichotomy about the relation of (Peano) arithmetic to (ZFC) set theory (represented in Hilbert arithmetic by "$P^+ \otimes P^-$"): either incompleteness or inconsistency. If just the one "half" (never mind which, whether "P^+" or "P^-") is not "lost" (just as the one "half" of variables in a quantum-mechanical description to the description of the same system in classical mechanics) inherently conditioning incompleteness, the explicit contradiction of their mutual anti-isometry appears immediately.

Then, the two directions meant correspondingly by the Gentzen completeness interpreted arithmetically and the proper Gödel incompleteness are isomorphic (respectively, homomorphic) to those of a bit of information, or symbolically notated:

$$[P \to (P^0 \to P^- \otimes P^+) \, \& \, (P^+ \otimes P^- \to P^0) \to P] \Leftrightarrow$$
$$[B \to (B^0 \to B^- \otimes B^+) \, \& \, (B^+ \otimes B^- \to B^0) \to B]$$

The notations "B", "B^0", "B^-", "B^+" mean correspondingly: a bit of information as a whole; an "empty" binary cell; the two equally probable options relevant to a bit of information. (The Cartesian product "$B^- \otimes B^+$" is reduced to only two admissible states in virtue of the mutual inconsistency of both alternatives to the other one.)

A common and misleading misunderstanding identifies a bit of information with a single opposition, that of the two equally probable alternatives after choice. In fact, it is equivalent to two oppositions, namely: (1) before and after choice; (2) each of both equally probable alternatives (usually notated to be either "0" or "1"). For example, only a single bit (rather than two ones) is lost after quantum teleportation (Penchev 2021 July 8) or the four "letters" of a single bit (rather than those of two ones) are sufficient for the natural alphabet by which the "word" of any entity in the universe (both mental and physical) to be written down correctly (Penchev 2020 July 17).

So, one can speak of a fundamental bit of information underlying the universe (also and particularly relevant to the conjecture that any fact is representable in the final analysis by a classical description established by the Cartesian dualism of "body" and "mind" or "subject" and

"object": *Penchev 2021 August 24*). Obviously, the above equivalence means that bit consisting of two oppositions and realizable as the fundamental bit of information (being necessary and established by scientific transcendentalism: e.g. *Penchev 2020 October 29*): it is meaningless if a bit of information is misinterpreted to be a single opposition.

So, the validity of cut-elimination as well as the asymmetry of Gentzen's completeness and Gödel's incompleteness can be philosophically reflected and traced back to the asymmetry of any bit of information, and shared also by that fundamental bit, or to the postulate of the totality as the rigorous and falsifiable statement of scientific transcendentalism (unlike the traditional philosophical transcendentalism in a proper sense). The relevant asymmetry of the totality can be represent also as the impossibility of it to be inferred from any empirical or experimental data, divided from the correct logical statement by a Gödel-like incompleteness versus the possibility of being postulated and then unlimitedly confirmed, but always by a finite set of confirmations, therefore referable to cut-elimination.

That fundamental and philosophical interpretation of cut-elimination can be complemented by the Humean criticism of causality or induction[15] as well as its rejection by Kantian transcendentalism, on the one hand, and by the main idea of Husserl's phenomenology elucidated also in other papers (Penchev 2021 November 18; 2021 July 26; 2020 June 29), on the other hand. For example, Husserl's "epoché to reality" (by the by, modifiable to a proper mathematical "epoché to infinity" in other papers: *Penchev 2022 March 11*), "phenomenological reduction" (linked to eidetic reduction, directly relevant to mathematics, by Husserl himself) and thus his concept of phenomenon allowing for both psychology and philosophy to be justified as "rigorous sciences" (and therefore connected to mathematics in a proper sense) share an idea analogical to cut-elimination once it is reflected widely enough, i.e. philosophically.

There exists a practical obstacle in front of the approach of the present article (as well as to the analogical method of many previous ones), which can be aphoristically expressed as "the mathematicians are not philosophers, and the philosophers are not mathematicians" (unfortunately as a rule in our age). Though Descaretes, Leibniz, Newton were both more or less, and the philosopher Husserl is a mathematician by education, contemporary philosophy and mathematics are radically opposed to each other just according to the formula suggested by the famous essay of Charles Percy Snow "The two cultures"(1959).

So, philosophy is "enrolled" among "art and humanities" unlike mathematics belonging to the "other culture". No philosophical paper ought to share even loose logical consistency being rather an expression of human nature and freedom, featured by artistry and arbitrary whims. On the contrary, no mathematical paper allows for any philosophical conclusion since that cannot be inferred rigorously and deductively by a formal syllogism. So, any subject of investigation needing both "cultures" is impossible, even inadmissible and "shameful", breaking "good manners" whether in mathematics or in philosophy.

In fact, one can demonstrate that a few great mathematical problems are not real ones, but imposed or appearing because of the requirement to observe or abide by those "good manners"

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[15] For example, in the context of another paper (Penchev 2022 June 30: 20; footnote 21).

(e.g. Penchev 2020 August 25). One can even suggest the "rebellious" conjecture that all the seven great mathematical puzzles (heralded to be those of the Millennium by CMI: e.g. *Carlson, Jaffe, Wiles, eds. 2006*) are rather troubles originating from the necessity for those "good manners" to be followed.

So, if the "unforgivable" behavior of relevant philosophical reflections is admitted therefore breaking the assigned and destined position of mathematics in the contemporary organization of cognition (by the by, visualizable by means of both "The two cultures" and "Le mots et le choses" by Michel Foucault), those puzzle might turn out to be quite elementary (Penchev 2020 August 25) in an alternative mathematics called "Hilbert mathematics" unlike the standard one called "Gödel mathematics"[16].

III THE DEDEKIND-LIKE BIJECTION OF INFINITE SETS INTO FINITE SETS AFTER SKOLEM'S ARGUMENT ("PARADOX") BY PROBABILITY DISTRIBUTIONS

Dedekind (1888) defined a necessary and sufficient condition for a set to be infinite: there has to exist a bijection of it and its true subset. Then if the set at issue does not satisfy that condition (and the framework is not intuitionistic), it is finite. That definition of finiteness (respectively, infinity) is set-theoretical not needing at all Peano arithmetic or natural numbers to be defined in advance.

Then, one can consider the non-empty area between all finite sets and all infinite sets as follows (even without any help of intuitionistic mathematics rejecting the law of excluded middle in relation to infinite sets). One considers a bijection into any finite set under the additional condition for it to be always the same. Obviously, the initial set mapped bijectively is also a finite set sharing the same cardinal number as that one into which is mapped bijectively. One can build the function of the set of all cases of that initial set into the same finite cardinal number shared by both sets after any bijection: it is a constant function. Consequently its converse (reverse) function would be the Dirac δ-function (which is not a proper function in a narrow sense since an element can be mapped into different elements).

Further, one notices that Dedekind's definition determines for a bijection only to *exist* rather than to be the same after any case (interpretable also as a certain experiment in quantum mechanics): the δ-function of Dirac can *smoothly* pass into a "normal" function sharing with the δ-function the same definitive condition its integral on the set of all cases to be a unit; or in other words, the δ-function is smoothly transformable into a usual probability density distribution with some nonzero entropy of its elements, at that increasing more and more.

The same sense and meaning of that probability density distribution once it has been interpreted after a Dedekind-like finiteness consists in implicitly satisfying Dedekind's condition for a set to be infinite: that is a gradual or smooth transition to a limit in the exact mathematical meaning. One can say that the Dedekind-like finiteness is a finiteness as a mathematical limit, i.e. in an infinite transition to it unlike the usual and standard finiteness shared by Peano

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[16] The idea about "Hilbert mathematics" versus "Gödel mathematics" is formulated for the first time in: *Penchev 2010*.

arithmetic being the same limit constantly. Thus, the former generalizes the latter describing the transition from an infinite set to a finite set rather than a finite set as an ultimate result (being "ready-for-use") meant explicitly by Peano arithmetic by its fundamental concept of "natural number". In other words, that Dedekind-like finiteness means the process of how Peano arithmetic appears from any infinite set, or speaking loosely, from infinity just as Hilbert arithmetic in a wide sense means (for example, in relation to that: Fermat's last theorem to be proved: *Penchev 2022 June 30; 2022 May 11; 2022 March 11*).

Indeed, one can trace the mediation of transfinite natural numbers (interpreted to be wave functions in the qubit Hilbert space of quantum information or in the separable complex Hilbert space of quantum mechanics). Indeed, the transition to a proper Dirac δ-function meant by any finite natural number (or speaking loosely, by finiteness) can be quite relevantly and essentially visualized by a "stairs" starting from infinity by adding of new and new qubits as its "steps" therefore narrowing down and down by each "step" (i.e. qubit) the corresponding probability density distribution of which the wave function at issue is its characteristic function so that the Dirac δ-function would appear after an infinite number of "steps" (qubits).

Speaking figuratively, that Dedekind-like finiteness can be considered as an infinite "descend from infinity" by a stairs, the steps of which are qubits and ending to a certain finite natural number being its last "step". Obviously that descend is to be unique for the Dedekind-like finiteness to be determined uniquely by the metaphor of "each last step needs just one corresponding ladder to it". Accordingly said rigorously, what is described figuratively is homomorphic to Gentzen's cut-elimination meant arithmetically as above. Thus, the concept of the Dedekind-like finiteness by the mediation of Hilbert arithmetic in a wide sense allows for cut-elimination to be described in detail as a process of how cut-elimination happens rather than as an ultimate result meant literally by the cut rule:

If cut-elimination is to be represented as a process step by step, qubit by qubit, it would be a probabilistic conclusion admitting in general the infinite set of all relevant or irrelevant conclusions but constituting a probability density distribution, which can be granted to be even a smooth probability density distribution narrowing down and down after each step of the process of cut-elimination. Thus, a certain and single conclusion, where the maximum of the probability density distribution is, becomes more and more probable, but not absolutely true until the result of the current stage of cut-elimination is not reduced to a finite set. Then, the least element of that set, existing necessarily, is the ultimate conclusion of cut-elimination traced as a process step by step as above. Also one may say that cut-elimination ends into the limit of the process demonstrating a certain conclusion to be more and more probable and this limit has been reached once the set of resultative conclusions turns out to be finite in the process of cut-elimination.

One can immediately notice that the description of cut-elimination considered as an abstract and mathematical structure and thus released from the interpretation to an infinite series of implications can be no worse interpreted as the process of the "collapse" of wave function after

measurement. It has been initially only postulated as a physically instant projection[17] of any wave function into any observed readings of the apparatus relevant to it just as cut-elimination can be thought as a "collapse" of the infinite series of implications to a single statement[18] following necessarily as the ultimate conclusion.

Nowadays however, the same ostensibly instant collapse of wave function is considered to be a real process of consecutive decoherence[19] over time by less and less entangled states and resulting as a limit into the absolutely non-entangled state as the ultimate reading of the apparatus (e.g.: Sparenberg, Nour, Manço, Crosta, Gramegna, Ruggiero 2013; Wallace 2012; Schlosshauer 2006; Ford, Lewis, O'Connell 2001; Prosperi 1994).

So, quantum measurement as a consecutive process of decoherence over time and cut-elimination in Hilbert arithmetic share the same structure only interpreted differently: physically in the former case, but logically in the latter case.

The idea that can exist a relevant logic of reality by itself is a fundamental idea of Hegel's philosophy as which he recognized the invented by him dialectical logic, unfortunately inconsistent with classical logic therefore gave impetus to the emergence of non-classical logics by the repairs and improvements of classical logic.

Other papers (e.g. Penchev 2021 April 21) admit that Hegel's dialectics should be repaired and improved in a way to be consistent with classical logic after quantum mechanics and Bohr's complementarity inspired initially by Kierkegaard's non-synthesizing dialectic "either - or" and revising Hegel's[20], and subsequently, by the Chinese philosophy of "Yin and Yang" resulting in "Tao"[21]. In fact, Bohr's conception of complementarity implies a revision of Hegel's dialectic, furthermore absolutely consistent with classical logic, which Hilbert arithmetic is able to demonstrate rigorously and mathematically. However, that approach should not be confused with a series of quantum logics being non-classical and thus more or less inconsistent with classical logic.

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[17] There exist publications demonstrating more or less successful models of quantum measurement without the projection postulate or wave packet collapse (e.g. Cini 1983; Benatti, Ghirardi, Rimini, Weber 1987; Home, Whitaker 1988; Cini, Serva 1992; Belavkin 1994) or linked to the quantum Zeno or anti-Zeno paradox (e.g. Home, Whitaker 1997), or even "without measurement" at all (Kümmel 2006). The paper of Corbet (1988) discusses the measurement of non-Hermitian observables, to which the projection postulate cannot be literally related.

[18] For example, as in automatic proof procedures (Zhung 1991) or investigating the complexity of cut-elimination (Zhang 2006).

[19] That real process is even rather complicated linking coherence and decoherence (e.g. Hou, Liu, Wang, Yang 2017; Luo, Li 2011; Podsędkowska 2007; Roy 1993; Nakazato, Pascazio 1991) and crucial for the theory and implementation of quantum computer (e.g. Leãao 1998; Sun, Zhan, Liu 1998)

[20] The investigation of Bigelow (1982) can assist connecting Kierkegaard's doctrine at issue with the formal and mathematical model of hermeneutical circle (as further in the present paper).

[21] There is a huge volume of scientific publication on complementarity by Niels Bohr himself (e.g. Bohr 1999; Bohr 1948)) as well as considering its very wide scope of interpretations (e.g. Bala 2017; De Gregorio, 2014; Perović 2013; Rabinowitz 2013; Home 2013; Plotnitsky 2012; Katsumori 2011; Camilleri 2007; Sen, Basu, Sengupta 1999; Beller 1992; Mehra 1987).

The Dedekind-like bijection as above can be also interpreted to follow an idea of Skolem in the scope of his conception about the relativity of "set" after the axiom of choice and the Löwenheim - Skolem theorem, suggested by him in 1922 and consisting rather in the relativity of the concept of "set" or "set power" including both finite set power and infinite set power. One meaning Skolem's argument can trace the Dedekind-like bijection even to its initial origin from the axiom of choice and its equivalence to the well-ordering "theorem" inferred from the axiom of choice introduced especially for the proof at issue but very soon proved to be equivalent to the axiom of choice by Whitehead and Russell in *Principia Mathematica*[22].

However, the axiom of choice does not determine unambiguously the corresponding well-ordering, but only that it exists necessarily: even more, it can consist only of the elements of any infinite true subset of the initial set, which is to be ordered, therefore ordering it not absolutely, but "almost thoroughly", i.e. without an arbitrary finite subset remaining unordered after any real case of well-ordering (corresponding to just one measurement if the same abstract structure would be interpreted in terms of quantum mechanics) in virtue of the Dedekind definition for a set to be infinite.

So, meaning a certain "experiment" for the infinite set at issue to be ordered (or respectively, a certain measurement of the same quantum state), it can be unambiguously described by the finite complement (in the exact set-theoretical meaning) of elements remained out of the well-ordering: at that, it can be never determined otherwise, i.e. by the infinite well-ordered subset being equivalent to the initial set or to any other true infinite subset just in virtue of the Dedekind definition at issue.

Then, one can describe all cases of an infinite set to be ordered by just one certain probability eventually, density) distribution (thus implying a corresponding characteristic function interpretable as an equivalent wave function) and granting it to be the infinite set at issue since a bijection can link all wave functions (respectively, all elements of the qubit Hilbert space) and all countable infinite sets, to which all sets can be reduced by virtue of the axiom of choice and what Skolem meant as the relativity of "set" in a narrow sense and then generalized by himself in a wide sense able to include also all finite sets as definitively countable (rather than only all infinite countable sets as in the Skolem relativity in a narrow sense). Indeed, Skolem's justification of that generalization in 1922 is not literally the same as above: the latter refers to the Dedekind definition of "infinite set" and involves "wave function" explicitly.

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[22] In *Principia Mathematica*, *Volume III* (both editions), in the beginning itself (before all enumerated propositions): as the statement that the well-ordering theorem implies the "multiplicative axiom" (equivalent to the axiom of choice). The converse statement (that the axiom of choice implies the well-ordering "theorem") had been proved by Zermelo (1904; 1908) before that, for which he had introduced the axiom of choice being ostensibly obvious according to him or at least more obvious than the well-ordering "theorem".

IV THE INTERPRETATION OF WAVE FUNCTION BY THE PROBABILITY DISTRIBUTION GENERATED BY THE BIJECTION OF AN INFINITE SET INTO AN INFINITE SET OF FINITE SETS

Once "wave function" has been defined properly and thoroughly set-theoretically as in the previous section, this is immediately pioneering a corresponding reflection on quantum mechanics as a physical (and thus experimental) *doctrine of infinity* unlike that by set theory restricting itself only to mathematics, therefore to abstract constructions of human mind divided from physical reality by the Cartesian abyss.

However, once the bridge (as in the present investigation) connecting reliably both shores of the abyss is granted in advance (for example by means of quantum neo-Pyjgaoreanism as here or otherwise), quantum mechanics can be realized (indeed rather in a "seditious" and provocative way challenging the "Beoetians" meant by Gauss not to publish his research founding non-Euclidean geometry) as an experimental approach to infinity (and even eventually much more, to theology, therefore admitting "God" to be the subject of experiments and proper scientific (rather than speculative or metaphysical, e.g. theological) research.

So, quantum mechanics can be interpreted as an experimental study of how infinity is able to transform itself into finiteness obeying quantitative laws that turn out to be probabilistic and challenging or rather generalizing the concept of objectivity in classical mechanics, physics, or science. Speaking loosely, what is really philosophically objective is infinite (and probabilistic because of that) rather than finite, deterministic in a narrow sense, as well as regularly and permanently repeatable thus being always reproducible by anybody in general (though practically by another scientist working in the same scientific discipline).

Properly and set-theoretically, the "transformation of infinity into finiteness" consists in the standard Dedekind bijection of an infinite set into another, also infinite set, however under the additional condition for the elements of the latter to be interpreted as finite sets and then, a certain natural number to correspond as the cardinal number of each of those finite sets. If one considers the mapping of an infinite set into the set of all its finite subsets, that example would not be relevant since the corresponding probability (density) distribution would not possess a maximum at any finite cardinal number. So, the mapping at issue cannot be an automorphism: another and different infinite set consisting of finite sets and able to generate a corresponding distribution similar e.g. to a normal distribution is necessary.

Interpreting the latter consideration physically, one can suggest two alternative mechanisms about the way for an infinite set to be transformed into another where a maximum of the corresponding distribution appears: "classical" versus "quantum". The "classical" way would need an external force or force field to create a relevant deformation and particularly, a maximum of the distribution at issue. The distribution by itself, without external action would remain absolutely even thus not being ever able to transform itself into any finite set or to start that transformation without a relevant external influence.

On the contrary, the "quantum" way can rely on spontaneous quantum fluctuations and a positive feedback amplifying more and more the gradual, more and more discernible

deformation with an increasing maximum accordingly. If the positive feedback by itself manages to reach infinite values, this would result into a fluctuation similar to the Dirac δ-function and corresponding to a finite implication (i.e. consisting of a finite series of consecutive elementary implications) being interpreted as cut-elimination. That necessary quantumness thought set-theoretically rather than physically needs a regular discreteness (for the Planck constant in the physical case) such as that of natural numbers or the function successor of Peano arithmetic as well as the axiom of induction in order to substitute the axiom of infinity relevant to set theory and corresponding to the positive feedback in the physical interpretation.

Indeed, the axiom of induction implies for any natural natural number to be finite and thus all natural numbers to be finite (though set theory states that the *set* of all natural numbers is infinite or actually infinite in virtue of the axiom of infinity). So, thinking both semi-mathematically and semi-physically, one might correspondingly say semi-figuratively that if Peano arithmetic is imported into the "medium" (ZFC) set theory (corresponding to standard or Gödel mathematics and its inherent incompleteness if not inconsistency), it complements itself and by itself to the qubit Hilbert space (respectively, the separable complex Hilbert space of quantum mechanics) in order to *become complete* adding all wave functions able to mediate between the actual infinity of set theory and the inherent and unsurmountable finitess of arithmetic. One may conclude that the physical world appears necessarily "ex nihilo" in virtue of mathematical laws and vizializable by the "immersion" of arithmetic into a "medium" of set theory as far as any entity belonging to the physical world is absolutely and thoroughly defined by its quantum state and corresponding wave function.

The statement that "science has gradually pushed religion out of any explanation not needing the hypothesis of God at least since Newton's age" is common and well-known. Nonetheless, the conjecture of the Big Bang (by the by, suggested by the Catolic priest as well as theoretical physicist and astronomer Georges Lemaître[23] just to reconcile science valid after any finite interval after the Big Bang with religion for God's creation in the Big Bang itself) conserves that hypothesis of God though the scientists atheists do not wish to recognize this granting that a relevant scientific explanation for the Big Bang itself will be discovered sooner or later.

However, their hope cannot ever come true in the framework of the dualistic Cartesian organization of cognition at all because that episteme (by Michel Foucault's term) of Modernity conserves implicitly God (in fact, explicitly by Descartes himself) as the justification of human chauvinism and the unique position of humankind only gifted by "free will" and thus by the capability of making decision and acting according it and linking by that the abyss of "body" and "mind" (featuring the human beings in general and "God"), or respectively that of "object" and "subject" in German classical philosophy as the main problem for resolving[24].

In fact, the conjecture of the Bing Bang (which can be called "theory" only in the sense of "doctrine") implies God's creation in the Big Bang itself though science can be absolutely relevant *after any finite interval* from it when the creation is already thoroughly available and

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[23] A detailed bibliography of Georges Lemaître's papers is suggested by Berger (1984).
[24] This is discussed in much more detail in previous papers (e.g. Penchev 2022 June 30).

"ready-for-use" by humankind and science in particular, and exemplifying the Cartesian abyss just by that finite time interval after the Big Bang. Indeed, God and God's creation in the Big Bang itself, on the one hand, and the creation "ready-for-use" by humankind or science, on the other hand, are reliably divided on the two shores of the abyss by the the finite time interval at issue though it seems to become shorter and shorter according to the development of science.

The class of explanations advocated in the present and other papers is alternative. It does not really need the hypotheses of God or the Bing Bang. The physical world can appear in virtue of mathematical laws (as this is described in detail above) once the Cartesian abyss has been absolutely removed as illusory, misleading and irrelevant, an artifact of humankind's ignorance e.g, as quantum neo-Pythagoreanism suggests. That is: the Big Bang is a myth though what science describes after it is relevant; however, the real cause is quantum decoherence occurring permanently and omnipresently rather than the alleged ostensible Big Bang.

The Big Bang corresponds exactly to all experimental human cognition, especially in the areas of cosmology, astronomy, astrophysics, but under the crucial condition that the total organization of human cognition justified by the Cartesian gapped opposition of "mind" where mathematics is interned not to disturb physics respectively related only to "body". That implicit condition can be visualized by the following tenet. The Big Bang interpreted as the absolute beginning of all being rather than of the physical world would mean that mathematics and physics are indistinguishable from each other at the Big Bang itself and can be divided only after time appears in any finite time interval after it.

Then, just the episteme of Modernity excludes the Big Bang itself to be described relevantly by any science obeying that organization of cognition, particularly by physics, astrophysics, astronomy, cosmology, etc. For example, the Big Bang is a colossal violation of energy conservation without any physical cause though all conservation laws might be valid after any finite time interval after it. So, the proper relevant description of it needs a tool interpretable equally well both physically and mathematically therefore inherently corresponding to the description of the Big Bang itself where physics and mathematics are to be the same.

"Wave function" meets that requirement. The fact that it is quite relevant to all experimental data of quantum mechanics as a physical science is well-known and absolutely corroborated. However, it needs furthermore and not less a successful realization, which the present interpretation suggests (along with other papers, e.g.: *Penchev 2020 August 25*), as placing it simultaneously in the foundations of mathematics, e.g. regulating the Gödel incostency (or dichotomy) of the relation of arithmetic to set theory by the introduction of transfinite natural numbers and their description by wave functions also by the mediation of the Dedekind-like set-theoretical finiteness. Then, the concept of it can be related to the Big Bang to describe it adequately since mathematics and physics (rather than only all the physical world) should be the same particularly transcending the fundamental restrictions and boundaries of any possible scientific representation within the framework of Modernity and its episteme dividing and opposing mathematics and physics.

Particularly, wave function especially visible if being represented as an element of the qubit Hilbert space needs a relevant generalizing quantity able to be both physical and mathematical though in the way of complementarity. Information together with quantum information interpreted as information relevant to infinite series or sets (Penchev 2020 July 10) can be that quantity if physical action by means of the Planck constant is equated to it therefore suggesting the most general law of conservation (Penchev 2020 October 5) so that the mutual transformations of mathematical and physical entities obey it therefore being able to describe the Big Bang absolutely scientifically, i.e. without God's creation in the Big Bang itself rejecting Georges Lemaître's hopes and intentions:

That would be a quantitative description of any "creation *ex nihilo*", particularly the Big Bang, admissible by science and thus rejecting the "hypothesis of God" as redundant cutting it by "Okkam's razor". In other words, an eventual conservation of quantum information would replace the absolute rejection of any "creation ex nihilo" even as anti-scientific or religious (by classical science granting the episteme of Modernity) with the exact mathematical and physical laws, to which any physical phenomena belonging to the "scandalous" class of "creation ex nihilo".

The concept of creation does not need the hypothesis of God any more since it is due to mathematical necessity. The philosophical solution of the fundamental ancient problem "Why anything rather than nothing" consists in the necessary occurrence of the physical world in all variety by virtue of mathematical laws being simultaneously physical in any "creation ex nihilo" such as the Big Bang itself.

V THE "COLLAPSE" OF WAVE FUNCTION AFTER QUANTUM MEASUREMENT

One can interpret quantum measurement as classical in a sense[25]: any physical quantity generates a probability distribution (to which a characteristic function interpretable as a wave function can be assigned) due to the error of the instrument (or respectively, that of the measuring process as a whole). That error theoretically reducible to zero and meant as zero in any smooth (continuous) classical model usually by infinitesimal analysis and differential equations is restricted to be fundamentally finite by the Planck constant involving an irremovable even theoretically "error" being always greater than that constant (or equal to it) therefore needing e.g. Heisenberg's uncertainty.

Analogically, any quantity measured in classical physics collapses in a sense admitting always a relevant projectional operator to describe the measurement at issue, The fundamental distinction from the case of quantum mechanics consists in its interpretation within the episteme of Modernity just as an error belonging to the measurement rather than to "reality by itself". So, classical measurement mediating between the Cartesian "body" and "mind" is conventionally enumerated in the area of the latter, but it can be assigned not worse to "body" by an alternative

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[25] For example, one can follow the approach of Aharonov and Vardi (1981) who "interpret the (formal) postulates of measurement in quantum theory in terms of measurement procedures that can be done in the laboratory (at least in principle)".

convention therefore pioneering an equivalent, however, classical interpretation of quantum mechanics, e.g. such as "Bohmian quantum mechanics".

The same gap between those "body" and "mind" is postulated even within mathematics by the Gödel incompleteness since physical reality by itself described as a relevant smooth manifold by differential equations is accessible to human senses and mind only as a finite result of measurement, i.e. reduced to a rational number, to which a corresponding real number has been "collapsed" in advance, i.e. by its measurement.

If that "collapse" is due to a fundamental restriction such as the Planck constant, one needs the separable complex Hilbert space of quantum mechanics able to represent that natural error in any measurement. So, if one wishes to distinguish classical measurement from quantum one only mathematically, it consists not in the "collapse" itself available in both cases, but *within* the "collapse" possibly infinitesimal in the classical case, however, necessarily finite in the quantum case and possibly generating "quantum measurement chaos" (Martens, Muynck 1991; Muynck 1995). The "measurement problem" in quantum mechanics is closely linked to its interpretation: how much the measurement itself creates or not[26] its result.

The same distinction can be visualized by cut-elimination represented arithmetically and especially in Hilbert arithmetic. The classical "collapse" (as the measurement error) admitting to be infinitesimal or finite (not being restricted by any finite constant at least theoretically) can be related to a natural number as finite (i.e properly, a rational number) as transfinite (i.e. properly, an irrational number). On the contrary, the quantum "collapse" (being due to a fundamental constant of nature) is necessarily restricted to the former case. So, interpreted only mathematically and rather paradoxically, the quantum collapse is a particular case included in the more general classical case.

Hilbert arithmetic (in a wide sense) allows for the description of the quantum collapse to be equated to that of the classical collapse as equally general furthermore corresponding to the contemporary understanding of the "collapse" of wave function in measurement as a gradual process of decoherence, the separate stages of which can be observed experimentally (at least in principle) as less and less entangled states (or said otherwise, less and less coherent states of the measured quantum entity or system). So, the "classical collapse" as an infinitesimal quantity corresponds to the initial stage of "quantum" cut-elimination where the eliminated members (transfinite natural numbers arithmetically or enumerated consecutive implications logically) are a finite set.

Even more, if that "initial stage" is represented dually (or in virtue of "complementarity", in terms of quantum mechanics) would be the final one (though anti-isometric) in the twin counterpart of Peano arithmetic (postulated definitively in Hilbert arithmetic). In other words and speaking physically, the initial stage of the quantum collapse of one conjugate quantity

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[26] That alternative corresponds to the realist interpretation of quantum mechanics and can serve also as a heuristics on how quantum mechanics can be improved to be inherently realistic (as in Hájíček, Tolar 2010; Szamosi 1993; Zeh 1988; 1970) or non-realistic (e.g. Schroeck 1997).

corresponds to the final stage of the other one (though anti-isometrically) corresponding exactly to Heisenberg's uncertainty principle.

As far as Hilbert space is a geometric space (for example, the real Hilbert space can be considered as a generalization of the usual tridimensional Euclidean space of classical physics, and the separable complex one of quantum mechanics, accordingly, as an analogical space generalization of Minkowski space relevant to special relativity: *Penchev 2022 February 4*), the interpretation of the quantum "collapse" as a projection operator is common. Thus, it is not any physical quantity in the sense of the "classical" quantum mechanics since it is postulated to be a Hermitian operator or conserving energy needing an unitary operator. Nonetheless, any projection operator can be considered as a composition of a Hermitean operator and an unitary operator just as any entanglement operator, therefore corresponding exactly to the understanding of quantum mechanics nowadays (i.e. including quantum information), which considers the "collapse" of quantum measurement as the final result of a gradual process over time (e.g. Mizrahi, Moussa, Otero 1993) forced by the preliminary preparation to be measured a certain quantity of both conjugate quantum quantities:

Then, that gradual process can be also visualized geometrically as a series of wave functions (i.e. vectors in the separable complex Hilbert space of quantum mechanics or in the qubit Hilbert space), each of which can be featured by the vector of its angle (rotation) to the ultimate projection result, each component of which decreases after the consecutive members of that series[27] corresponding to an increasing degree of decoherence to the initial coherent state by "itself".

The same process of the "collapse" of wave function after quantum measurement is homomorphically representable as the arithmetic "cut-elimination" by means of Hilbert arithmetic in a wide sense as a series of more and more narrow probability distributions, which results in a Dirac δ-function (corresponding to the absolute decoherence of the ultimate result of

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[27] For example, Ahluwalia (1994) links that series of de-coherent states in the process of quantum measurement and the fundamental Planck constant by the rejection of locality; an analogical viewpoint to measurement by entanglement (Cerf, Adami 1998) as well as the resolution of the measurement problem through decoherence of the quantum state (Dieks 1989) or investigating quantum and classical correlations in quantum measurement (Xi, Li 2013); analogically, by "many Hilbert spaces" (Namiki, Pascazio 1993). An anticipation can be discovered after de Broglie's interpretation of quantum mechanics in relation to measurement as by Andrade e Silva (1982). According to Ballentine (1988) the theory of measurement needs an ensemble interpretation rather than that of a single system, or the necessity of quantum trajectories (Wiseman 1996). Prigogine's model of quantum measurement can be also interpreted as a process of decoherence Baaquie (1994). The process of decoherence in quantum measurement can be equally well interpreted as the transition of the description by quantum mechanics to that by classical mechanics (Bhattacharya, Habib, Jacobs 2003), respectively as the transformation from quantum to classical probabilities (Breuer, Petruccione 1996), as the proliferation of observables measuring in quantum-classical hybrids (Elze 2012) or as the macroscopic limits of quantum measurement (Kobayashi 1992). Wigner (1986), Mensky (e.g. 2001; 1990), Yukalov and Sornette (2008), or Zwirn (2016) suggest far-reaching and philosophical extrapolations of decoherence and measurement. On the contrary to all cited papers, (Zanchini, Barletta 1991) reject the instantaneous transmissions of signals in quantum measurement.

quantum measurement). A certain transfinite "natural number" (meant as a certain transfinite ordinal number less than the least countable ordinal number as to terms of set theory) possessing the same wave function corresponds unambiguously to each of those probability distributions.

If one returns to the initial cut-elimination referring to logical propositions in an infinite sequence of implications (rather than to their numbers as in the arithmetic "cut-elimination"), the process of cut-elimination can be realized as a series of metaphors[28] corresponding to just a single proposition and being able to represent its origin from common language by a metaphor in the course of its use gradually narrowing down and down its meaning and ultimately "collapsing" in its absolutely exact definition (at least theoretically).

Then Gentzen's cut rule would be interpreted linguistically and semantically as the statement that any metaphor implies just one single proposition by an infinite sequence of "narrowing" down and down metaphors therefore becoming less and less "metaphoric". On the contrary, the Gödel incompleteness would mean in an analogical interpretation that any proposition is incomplete to any single "parent" metaphor, or speaking figuratively, it possesses always one more "parent".

Thus, the physical and mathematical interpretation of cut-elimination by its unity with quantum measurement though even alone interesting and instructive enough can be extended further, to language and semantics. This more general consideration is the subject of the next section.

VI THE "COLLAPSE" OF WAVE FUNCTION INTERPRETED BY THE FINITE-INFINITE BIJECTION AND CUT-ELIMINATION AS DERRIDA'S "WHITE METAPHOR"

Another paper (Penchev 2015) suggests both idea and practical method for a probabilistic distribution (and thus a wave function being its characteristic function) of the "veer of all interpretations" according to the real use of the investigated metaphor in language at a certain moment of time to be always buildable as well as the understanding of metaphor at all as the "entanglement" of two or more exact meanings.[29] So, the assignment of a certain probability

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[28] The paper of Alberucci, Jäger (2005) contains an analogical idea developed in the context of "deductive systems for common knowledge above epistemic logics" as well as that Brünnler and Studer (2009) relevant in the same context. Cerrito and Kesner (2004) discuss pattern matching and Kikuchi (2008) the call-by-name reduction as cut-elimination, Baaz and Leitsch (2006; 2014) investigate cut-elimination by a clausal analysis or by the relation of syntax and semantics as well as Ciabattoni and Terui (2006) or Okada (2002); for a calculus with context-dependent rules (Elbl 2001); or the relationship with compact regularity (Moshier 2004): thus all of them also implicitly pioneer the pathway from cut-elimination to metaphor. Other extensions or generalizations of cut-elimination theorems link it with normalization (Pottinger 1977; Zucker 1974), with ϵ-calculus (Yasuhara 1982; Wessels 1977), with other branches of logic (e.g.: McDowell, Miller 2000; Pfenning 2000; Valentini 1983; Mints 1976) or the foundations of mathematics (e.g. Mints 2008).

[29] The same idea being reflected only philosophically is articulated for the first time in an earlier paper (Penchev 1996). A conference presentation entitled "Metaphor as entanglement" (Penchev 2017 August 27) articulates the same idea.

distribution (which can be also extrapolated to a smooth probability density distribution) will be granted in advance by default in the further course of the present section.

Then, cut-elimination meaning consecutively decreasing ("by a unit") transfinite natural numbers and representable as accordingly consecutively narrowing down and down probability distributions (respectively their characteristic functions as qubit "wave functions") in the framework of Hilbert arithmetic in a wide sense can be furthermore interpreted as a series of metaphors narrowing down the scope of their possible interpretations in course and virtue of their repeating use again and again.

Each case of use causes almost imperceptible narrowing in its interpretations therefore restricting their freedom or looseness being integrated over a long enough period of its use many, many times and able to result into an absolute exact meaning in the final analysis in an infinite or "infinite"[30] temporal interval. In fact, science accelerates the natural "crystallization" of any metaphor in the course of its use into a single exact meaning by its proper definition. Then, Gentzen's cut rule meaning literally the reduction to a finite sequence of implications (and to a single one in the final analysis) can be now interpreted as the option for any metaphor to be reduced to a certain proposition by a process analogical to Derrida's "erasure of metaphor"[31].

On the contrary, the Gödel incompleteness in the same context would mean that no metaphor corresponds to a scientific proposition such as an exact definition of a scientific notion. Indeed, scientific practice not admitting metaphors in scientific papers (excluding only art and humanities) answers that statement as a well confirmed fact.

Metaphor is individual and subjective, understandable by somebody, but misunderstood or even absolutely ununderstood by others and thus unacceptable in general as to any proper scientific language. Even more: the symbolic notations in mathematics, physics, chemistry lead to the fundamental removal of any connotations (and thus potential or implicit metaphors), which are abundant in natural language.

However, the omnipresent symbolic notations everywhere it is possible, e.g. and especially in a mathematical proof cause an extremely unwanted side effect. Any Turing machine supplied by a relevant software program can repeat the proof at issue (e.g. to check it) without any understanding, of which it is deprived by definition. Speaking loosely, science tends to cancel any understanding since it is inherently individual and subjective, even non- and anti-scientific in a sense: an approach which is too radical though. Then, one can suggest that metaphor is the main tool for understanding if not the single one.

One can visualize that relation of metaphor (calling for understanding and subjective interpretation) and scientific concept or notation (needing to be absolutely unambiguous) by the idea of hermeneutical circle promoted by philosophical hermeneutics (Heidegger, Gadamer, etc.)

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[30] "Infinite" in quotation marks means a finite interval of an arbitrary length, i.e. extendable *ad lib*.

[31] Derrida's original viewpoint about the "erasure of metaphor" refers rather to the origin of philosophy and philosophical thought from metaphor (Derrida 1974; Morris 2000; Stellardi 2001). However, it can be naturally generalized to an analogical process using any metaphor again and again in language. That wider interpretation is suggested by Derrida himself (e.g. in Butman 2019; Novitz 1985) and shared by authors investigating metaphor, for example by Ricœur (1975).

as the fundamental structure and "atom" of understanding[32]. Indeed or unfortunately, it interpreted formally and logically is a "vicious circle"[33] or redundant tautology as two opposite implications, therefore depending on whether one wishes to prove a statement, starting with another, or considers their logical equivalence. So formal logic contradicts philosophy: a statement familiar at least since Hegel's age and repeated by great philosophers again and again (e.g. by Wittgenstein though in an opposite sense).

Of course, quantum neo-Pythagoreanism advocated here and in other papers (e,g. Penchev 2022 February 4) is inconsistent with that alleged contradiction of philosophy and logic, fortunately being ostensible and seeming. One can visualize further its solution by representing "hermeneutic circle" as a (fundamental) bit of information involving "quantum" complementarity as to the two equally probable alternatives of a bit (notatable e.g. as "0" and "1") as follows (in more detail in: *Penchev 2021 April 21*).

The two opposite implications, to which logic formally and necessarily decomposes any hermeneutic circle, are postulated to be "complementary" to each other philosophically or physically as well as "dual' mathematically. So, they cannot be simultaneously available, by which hermeneutical circle and logic are reconciled as far as the "excluded middle" being a "de Morgan" equivalent of the logically represented "hermeneutical tautology" is avoided,

For example, one can trace how human mind (unlike a Turing machine) can understand by hermeneutical circle realizing the capability of *memory* as to any formal syllogism[34] (always calculable by a Turimg machine): for example, its implication is accomplished step by step, i.e. proved formally and logically, then *memorized*, and the necessary other converse implication is done at a different moment of time but only as a metaphor in general (therefore involving the human "ability of logical mistake", which any metaphor is, and fundamentally inaccessible to any Turing machine), So, human mind is able to "go around" the Gödel incompleteness just by the substitution of the converse implication by a relevant metaphor (though formally and

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[32] The relation between hermeneutical circle, understanding, and interpretation in a context relevant to the formal approach of the present paper is investigated by Colburn (1986) meaning the debate between Gadamer and Habermas, by Debesay, Nåden, Slettebø (2008) as to the interpretation of qualitative studies, or by Rosen (1991) or Shklar (1986) as to the relation of understanding and interpretation. The research of Fischer (1987) interpreting "emergence of Mind from Brain" in terms of hermeneutic circle is able to link it with the critique to the Cartesian dualism of the contemporary organization of cognition (episteme), frequently discussed in the present paper.

[33] It can be formalized and then proved a "vicious circle theorem" (Betz 2005). However, Russell's "vicious circle principle" though a particular exemplification is much more widely discussed (e.g.: Hintikka 2012; 1957; Rouilhan 1992; Fleischhacker 1979; Vardy, Fleischhacker 1979; Tichý 1971; Pap 1954). The mentioned publications are chosen for their relevance to the relation of duality (complementarity) and well-ordering (hierarchy) along with their proper subject of the vicious circle principle. However, the vicious circle can be investigated as inherently logical in a "logic of question and answer" (e.g. Kalman 2011; Kostroman 2006; Hogan 1997; Jones 1988). Furthermore, the availability of vicious circle can be relevantly interpreted in turn as a case of hermeneutical circle as the paper of Ginev (1988) witty demonstrates in relation to the problems in philosophy of science.

[34] The paper of Brown and Vincent (2006) shows how the frequency-dependent selection in the process of evolution can be interpreted as a kind of hermeneutical circle. Their result could be very useful if one undertakes to research the relation of idempotency and hierarchy in terms of hermeneutical circle.

logically wrong and therefore unfeasible by any contemporary computer[35]). Particularly, the same complementing metaphor can be also *relevantly*[36] reducible to an implication generating a logical equivalence, but thus not necessary for the understanding at issue.

So, following the approach sketched by the linguistic and semantic interpretation of cut-elimination and the Gödel incompleteness by metaphor (as to the latter) and its "erasure" (as to the former) one can build a formal and logical model of human understanding, by the by, being absolutely necessary for any release of AI. Hermeneutical circle is logically represented accordingly (i.e. linguistically and semantically by metaphor), however it admits also a proper mathematical formalization by the 'nonstandard bijection" introduced in other papers (cited above) and notable in Hilbert arithmetic in a narrow sense as:

$$[P \rightarrow (P^0 \rightarrow P^- \otimes P^+)] \,\&\, [(P^+ \otimes P^- \rightarrow P^0) \rightarrow P] \,.$$

Then, hermeneutical circle would represent both dual directions of it simultaneously therefore being able to make clear hermeneutical circle as the proper "atom" of a formal fundamental ontology relevant to Heidegger's properly philosophical "fundamental ontology", however furthermore originating from "scientific transcendentalism" also involved in other papers (e.g. Penchev 2020 October 20) as it relies on the nonstandard bijection formally.

Indeed, what fundamental ontology adds to classical transcendentalism by means of both understanding and hermeneutical circle is an interpretation of "phenomenon"[37] *just by them* (and even, not otherwise). That is: what is "Ding an sich" and the same for us (whether human beings or a philosophical subject at all) are able to constitute a transcendental "phenomenon" only by understanding and hermeneutical circle and particularly, by their implementation in the human beings' thinking. The same analogically repeated also to Husserl's "epoché" or "phenomenological reduction" emphasizes the transcendental essence of the humans' psychology, e.g. by its identification with "transcendental reduction", and relative also to "eidetic reduction" inherent for mathematics.

Then, the nonstandard bijection once it has been successfully applied in advance in scientific transcendentalism can be immediately transferred or translated in the "language" of fundamental ontology and philosophical hermeneutics, furthermore continuable to the proper "mathematical edition" of them (and corresponding to Husserl's "eidetic reduction") therefore deepening Heidegger's historical and philosophical destruction in searching the origin from the Presocratics before them: to the Pyhatgoreans.

The nonstandard bijection as to the intended application to hermeneutical circle can be thoroughly represented by the two opposite or "dual" directions implemented in any bit: (1) making a choice between the equally probable alternatives; (2) erasing a made choice in a bit.

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[35] The suggested model admits inherently that a quantum computer (unlike any Turing machine) is able to understand just as a human being. If the "Turing test"' needs understanding necessarily, a quantum computer might overcome it, proving to be a proper artificial intellect.

[36] Cut elimination interpreted linguistically and semantically as above supplies a reverse implication always, however it can be the implication converse to the initial one only in particular thus being irrelevant to it in general.

[37] "Phenomenon" whether after Kant or after Husserl.

Those two directions can be discernibly represented by the visualization of a bit of information as a Turing machine cell, in which either "1" or "0" can be (1) written ; (2) then, each of them (i.e. either "1" or "0") can be erased.

A common, but misleading prejudice interprets a bit of information as a single opposition[38], that of its two equally probable alternatives (e.g. either "1" or "0"). In fact, that opposition is preceded by one more: that visualizable as an "empty" cell and the same cell, in which whether "1" or "0" is recorded. It can be also represented by the two directions of a bit relevant to the model of hermeneutical circle by a bit of information due to the nonstandard bijection.

One can immediately notice that the two directions of a bit (as above) are dual to each other just as the corresponding two directions of the nonstandard bijection. However, one can question whether (and how much) a record (respectively erasure) in a binary cell is a bijection in a rigorous mathematical meaning rather than as a loose metaphor. Obviously, it is a bijection only either to its first opposition (that of its directions) unambiguously mapping its "empty" state into its "recorded" state (e.g. whether "1" or "0") or to its second opposition but not to both simultaneously.

Then and fortunately, one notices that those two oppositions of a bit are in turn dual to each other, which allows for it to be reduced to a single opposition (whether the first one or the second one); that is: the nonstandard bijection to be interpreted as a bijection under the necessary condition the two directions of it to be dual to each other and definitively satisfied by the nonstandard bijection, e.g. in the case of two dual Peano arithmetics in the framework of Hilbert arithmetic in a narrow sense (being in fact borrowed from Hilbert arithmetic in a wide sense).

The same observation is repeated in the case of infinite sets (respectively, qubits if "quantum information" and "qubit" are interpreted as generalizations of classical information, as well as its unit of a bit, to infinite series and sets: *Penchev 2020 July 10*) by the equivalence of the axiom of choice and the well-ordering "theorem". Its "atom" is just a qubit of quantum information thus confirming again that it is to be also considered as a case of the nonstandard bijection just as its finite counterpart of a bit (of classical information).

Indeed, any qubit can be considered to be a bit generalized as follows. Each alternative of a bit (i.e. either "1" or "0") is interpreted to be a Peano arithmetic, i.e two ones both dual to each other, thus constituting a structure equivalent to Hilbert arithmetic in a narrow sense in virtue of the axiom of choice, on the one hand, or an infinite set (even not necessarily countable otherwise than by means of the axiom of choice) satisfying the requirement *ad hoc* of the non-naive set theory (i.e. unlike Cantor's) to be a subset of another set (that of both alternatives together), on the other hand. A finite series of bits of classical information corresponds in the former case, and as to the latter case, it repeats the initial state being to be reduced in the same way.

As a result, hermeneutical circle interpreted by a single bit of information (including the case of the fundamental or "philosophical" bit of information) is also an example of the nonstandard

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[38] That mistake is available in the "law of quantum teleportation", according to which ostensibly *two* bits of classical information are necessarily lost after quantum teleportation. In fact, what is lost are the two *oppositions* of a *single* bit (Penchev 2021 July 8).

bijection whether that of its two directions (the one of which is a rigorous implication then memorized and complemented by a metaphor for understanding) or that of the circle as a whole (i.e. understanding by itself and at all) and either of its directions (i.e. the class of equivalence of the rigorous (thus general and repeatable) implication and loose (thus inherently individual and subjective) metaphor necessary for understanding (and fundamentally inaccessible to any Turing machine being a formal and logical mistake).

One is to discuss the same idea by the representation of hermeneutical circle by interpreting the corresponding bit in the framework of Hilbert arithmetic in a wide sense including a paradox due to the axiom of choice to be elucidated. One can admit that a *maximal* and single metaphor exists in the final analysis so that it is able to generate all the rest metaphors in the process of its erasure therefore being reducible in virtue of cut-elimination to a single implication. In fact, that statement is not quite correct though formally valid as far as an infinite set (due to the Dedekind definition of an infinite set) can be mapped by a bijection not only to its true subset, but to itself being a subset of itself though not a true subset.

The statement is not quite correct because the case of a set itself is satisfied by any finite set and thus it is not able to distinguish unambiguously a finite set from an infinite one. Indeed, the Dedekind definition means only the case of a bijection into a true subset, which obeys the additional condition that its complement is also an infinite set. On the contrary, if its complement is a finite set, it can be called to be an "almost true" subset.

The almost true subset satisfies the Dedekind definition since no finite set possesses any bijection to its almost true subset (in fact, "true subset" and "almost true subset" are identical as to any finite set). However, the natural suggestion about all possible metaphors is to be an infinite set due to which the erasure of any metaphor is a true subset, but not an almost true subset of that of all metaphors. Consequently, it contradicts the alternative conjecture about a single "universal metaphor", ostensibly admissible at least theoretically.

However, if one discusses the hypothesis of a single universal metaphor in terms of the corresponding wave functions, it would be equivalent to the wave function of all the universe (an idea which is common, e.g. in quantum cosmology). The wave function of any entity being a true (rather an almost true) subset of the universe can be also considered to be a partial projection of the wave function of the universe onto a Hilbert true subspace therefore implying a corresponding process of protection, which in turn can be interpreted as a partial erasure of the universal metaphor of the universe able to result in any given metaphor.

So, what one should reject is a universal process of erasure rather than a universal metaphor since it is isomorphic to the wave function of the whole universe, on the one hand. However on the other hand, the problem of whether the axiom of choice (or rather its equivalent of the well-ordering "theorem") is applicable to the case of all erasures of the universal metaphor is essential and closely linked to another fundamental problem discussed in another paper (Penchev 2022 May 11): whether a quantum computer can calculate for a finite time any problem which a Turing machine cannot calculate for any finite time also further relatable to the "P vs NP" problem (Penchev 2020 August 5) and thus, to the Kochen-Specker theorem in the final analysis.

Anyway, one can conclude that the linguistic and semantic extension of the quantum neo-Pythagorean unity of physics and mathematics by the mediation of propositional logic, metaphor (since it connects propositions as logic does, though unlike it probabilistically) and its theory, cut rule and the Gödel incompleteness is quite consistent and fruitful suggesting furthermore a new viewpoint to a series of fundamental problems.

VII A DIRECTIVE CONCLUSION REALIZING QUANTUM MECHANICS AS THE INHERENT LINK OF THE FOUNDATIONS OF PHYSICS, MATHEMATICS, AND SEMIOTICS (LINGUISTICS)

At least a few previous papers (e.g. Penchev 2021 August 24; 2020 October 2020) have emphasized the inherent link of physics and mathematics (in their shared complete foundations unlike the opposition of their higher branches) by Hilbert arithmetic in a wide sense and overcoming the rigorous restriction originating since Descartes's "mind" and "body" (respectively, "subject" and "object" of German classical philosophy) to be gapped even by an abyss surmountable only by God or by the human being as "God's Vicar on earth" being only gifted with free will and thus able to estimate whether "model" (for mathematics) and "reality" (for physics) correspond to each other or not. Other papers (e.g.: Penchev 2021 November 18a) add logic to that unity of physics and mathematics confessed by scientific transcendentalism and quantum neo-Pythagoreanism (also discussed in detail in a few papers: e.g. *Penchev 2021 August 24*; *2020 October 2020*).

Obviously, logic is traditionally grounded in language expressing the human capability of thought though it has been interpreted as a formal mathematical discipline at least since the end of the 19th century. Thus, logic (especially, propositional logic) can be also considered as a "bridge" to semantics and linguistics linked by what a propositions is: whether a notation of a corresponding thought or a sentence very often being metaphorical, eliptic, etc., i.e. containing formally a logical mistake for the viewpoint of propositional logic, but nonetheless being inherent for human understanding and quite inaccessible for the calculation of any Turing machine.

One can question whether the "correct logical mistake" embodied in metaphor and embedded in its theory can be anyway described rigorously and formally, mathematically and logically. Both Gödel incompleteness and cut-elimination, furthermore modeled unambiguously by Hilbert arithmetic in a wide sense able to link the foundations of physics, mathematics, and proposition logic, can interpret metaphor by a relevant mathematical structure, or loosely speaking as a generalization of proposition (inherent for logic and naturally linkable to the "sentence" of linguistics or semiotics) to infinity (in turn inherent for set theory, and to all mathematics by means of it).

So, cut-elimination sharing the same mathematical structure as quantum measurement (also demonstrated in the paper) can be equally well understood as the "erasure of metaphor" in the course of its use again and again and resulting into a single proposition as an ultimate limit of that erasure where "limit" is utilized in its absolutely rigorous mathematical meaning, i.e. as the limit of a well-ordered infinite series of a single metaphor reducing and decreasing its metaphoricity.

Following alternatively the interpretation in terms of quantum measurement, the same process can be also seen as the "collapse" of a wave function attachable to the metaphor at issue

in virtue of the probability distribution of its meaning being inherently uncertain unlike the absolutely exact meaning of a proposition in the framework of propositional logic and rather corresponding to the real practice of any natural language.

Further, the same mathematical model of metaphor assists the "correct logical mistake" necessary for understanding and implicitly meant by the concept of hermeneutical circle not as a tautology or a vicious circle. The fruitful mistake of understanding (being absolutely inaccessible to any Turing machine or contemporary computer) is a relevant metaphor dual and complementing any syllogism so that both constitute a hermeneutical circle. Only it is able to overcome the Gödel incompleteness (conversing the absolutely unambiguous cut-elimination), but being a mistake, always individual and subjective[39], thus formally unique, unrepeatable on general, and thus out of scope of classical science needing, on the contrary, absolutely repeatability of any scientific result. Briefly and paradoxically, classical science cannot understand the understanding being unrepeatable in definition, and particularly, it cannot understand philosophy definitively tending to understanding (for example, turning out to be logically inconsistent from the viewpoint of propositional logic).

A main conclusion can be that scientific areas such as mathematics, logics, physics, linguistics, and semantics, enumerated apparently on the opposite "shores" of the Cartesian "abyss", overcomable "only" by God and human beings in practice and thus furthermore gaping them from nature absolutely deprived of free will (by which God has ostensibly gifted only human beings since they are "God's favorites"), in fact, admit a general uniform description traditionally interpreted as a relevant mathematical structure, but being also inherently philosophical from the viewpoint of quantum neo-Pythagoreanism, however at the price of rejection the prejudice at issue, being, to tell the truth, an anti-scientific and ideological expression of human chauvinism preventing any further cognition.

The specific accent of the paper consists in the extension from logic to linguistics with semantics. They can be considered to be opposed to each other, analogically to the pair of mathematics and physics, i.e. as if on the two "shores" correspondingly also representable mathematically and philosophically as those of finiteness and infinity inherently asymmetric to each other. Indeed, the direction from infinity to finiteness is ruled by cut-elimination, but the opposite direction, from finiteness to infinity, obeys the Gödel-like incompleteness (respectively, contradiction) between them. So, one can say that infinity is "transcendent" to finiteness, on the one hand, however on the other hand, that gap of transcendence needs the human free will's decision each time when it should be surmounted, therefore unlawfully magnifying humankind over all the rest of nature. So, the same Cartesian only seeming abyss bisects the unity of

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[39] The famous first sentence of Tolstoy's "Anna Karenina" ("Happy families are all alike; every unhappy family is unhappy in its own way.") shares an analogical formal structure consisting in the uniqueness of "success" to all possible deviations from it. A paper by Bornmann and Marx (2011 arXiv:1104.0807) even introduces the "Anna Karenina principle" but only as a metaphor (though meaning the same formal structure) to investigate the success / failure of any scientific publication (that is as if a possible paraphrase: published papers are all alike; every unpublished paper is not published in its own way.) However, the "mistake" necessary for understanding is a success though being a logical failure.

language of a "mathematical-like" half of logic (e.g that of propositional logic based just on "proposition") and another\"physical-like" half of linguistics (eventually together with semantics) where "proposition" is replaced by "metaphor" therefore gapping the last two fundamental scientific notions again in virtue of the organization of cognition ostensibly needing that chasm.

On the contrary, the paper relying on a fundamentally non-Cartesian "episteme" suggest the model of the unity of mathematics and physics initially applied to the proper arithmetic version of cut-elimination and quantum measurement by means of Hilbert arithmetic in a wide sense, then, to the derivative gorge dividing logic (proposition) from linguistics (metaphor) to be analogically or isomorphically erased as existing only in the contemporary humankind's imagination, but not in reality by itself.

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