Chapter 1

Intensive and Extensive Quantities

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1 Introduction: Physical Quantities

1.1 The Problem of Quantity

Physical quantities—like mass, charge, volume, and length—are commonly represented in science and in everyday practice by mathematical entities, like numbers and vectors. For instance, we use real number and a unit to represent the determinate magnitudes of mass (like $2kg$, $7.5kg$, πg, etc.). These representations are appropriate because they faithfully represent the physical world as being a certain way, as exhibiting certain structural features. Specifically they represent what we might describe as these physical quantities being structured in a certain way.

There has been a long standing problem in explaining exactly what this physical structure consists in. The difficulty lies in giving an account of quantitative structure without either (1) making ineliminable appeal to abstract Platonic mathematical entities themselves (which seem ill suited to explain their own adequacy as representational tools) or (2) positing primitive, irreducible metric structure at the fundamental level (for instance, a distinct and primitive 'n times as long as' relation for every real n).[1] Call this the *problem of quantity*.

[1]I won't motivate these added constraints here. I take it that the motivations for the latter constraint are transparent. An uncountable infinity of distinct primitive posits is the sort of thing that should be avoided wherever possible. Field (1984) makes the best case for the former constraint. I'll just point out that even the most red-blooded Platonist ought to be suspicious of the idea that the numbers 6 and 10 are somehow involved in the ultimate explanation of, e.g., why this $6kg$ ball ricocheted at this particular speed and angle when it collided with this $10kg$ one.

I examine the ways physical quantities constrain the structure of their worldly instances, specifically the mereology of the physical entities which instantiate them. In this paper, I identify a phenomena which I call "proper extensiveness". Of the physical quantities which do put constraints on mereology, including those one might classify as "additive", only a proper sub-class qualify as properly extensive.

In what follows, I will provide motivations for positing such a phenomena, and argue that proper extensiveness cannot be dependent on dynamics. In the second half of the paper, I make the case for taking proper extensiveness to be metaphysically fundamental (at least relative to most of our other physical ontology), by showing that doing so allows us to construct an elegant and attractive solution to the problem of quantity (though only as it applies to quantities which are properly extensive).

Here's the plan for the paper: The rest of this section contains a primer on quantitative structure and establishes some terminology. The argument that we need to posit proper extensiveness is made in sections 2 and 3. Section 2 introduces a puzzle about explaining the reliable success of paradigm physical measurements. The worry is that no explanation that essentially appeals to dynamics can account for the success of synchronic length measurements, like those involving pairs of aligned rods. The best explanation for this success, I argue, requires a pre-dynamical but modally robust connection between quantitative structure and mereology.

Section 3 outlines two candidate connections, one commonly known as "additivity" and the other a previously unrecognized phenomena which I dub "proper extensiveness". I show that only proper extensiveness is sufficient to underwrite the explanation of the length measurement presented in section 2. Also, taking length to be properly extensive better accords with our modal intuitions involving the quantity.

The final section offers a sketch of an application of the distinction to the problem of quantity. I describe a solution to the problem of quantity only available to properly extensive quantities, as I understand them. I discuss the implications such a result would have on our understanding of the nature and significance of proper extensiveness.

1.2 Quantitative Structure

Every physical quantity is associated with a class of determinate *magnitudes* or *values*, each member of which is a (non-quantitative) property or relation itself. So when a particle possesses mass, charge, or length, it always instantiates one particular *magnitude* of that quantity – like $2.5kg$, $7C$ or $2\pi m$.[2] These magnitudes exhibit, or the objects which instantiate them, exhibit "quantitative structure" just in case they are related to one another by certain "structural relations".

We can represent these relations as between magnitudes and between the instances of magnitudes. Some of them are *metrical*—we say "this pumpkin is over 8.7 times as massive as that gourd" when talking about objects and "$1.5m$ is ten times as much as $15cm$" when talking about magnitudes. Other structure is *sub*-metrical. Let me introduce two relations which handily express the sub-metrical structure we intuitively apply to one-dimensional unsigned scalar quantities,[3] i.e. things like mass, length, and volume (and unlike charge, velocity, and spin).

We say "this pumpkin is less massive than that table" and "$22m^3$ is less than $22.1m^3$", when talking about the *ordering* on (in these cases) massive objects and determinate magnitudes of volume, respectively.

Let '$\prec$' denote a two-place relation symbolizing the intuitive "less than" relation over the magnitudes, Q_i, of some quantity, Q. Intuitively $Q_a \prec Q_b$ when Q_a is "lesser than" Q_b. When an object, x, instantiates a mass magnitude that bears $\prec$ to the magnitude instantiated by another object y, we say that x is *less massive than* y.

We say "this stick is as long as that pencil and this highlighter put together" and "$12kg$ is the sum of $7kg$ and $5kg$", when talking about the *summation* or *concatenation* structure on (in these cases) lengthy objects and determinate magnitudes of

[2]It is sometimes said that quantities are determinables and their magnitudes their determinates, but this is not universally accepted. Certainly the magnitudes of mass, say, are all and only the determinates of the determinable property denoted by the predicate 'has mass' or 'has a mass', but it's not obvious that we should identify the *quantity* mass with this determinable property.

[3]By "one-dimensional scalar" quantity, I mean one which is intuitively gradated along only one axis and which don't involve any notion of direction. By an "unsigned" quantity I mean just those which are not most faithfully divided into categories like "positive" and "negative", where two magnitudes might have the same "degree" but differ in "sign". In what follows, I will drop these descriptors, but my focus, for simplicity's sake, will always be on quantities of this type.

mass, respectively.

Let '$\oplus$' denote a three-place relation over the Q_i's which serves to map two magnitudes to a third magnitude which is their "sum". So when $\oplus(Q_a, Q_b, Q_c)$ we say Q_c is the "sum" of Q_a and Q_b, and we write $Q_a \oplus Q_b = Q_c$. When $\oplus$ obtains between three length magnitudes instantiated by objects x, y, and z respectively, we say that z is as long as x and y *taken together*.

I will say a bit more about metrical structure, since it is our target. We'll say the ratio of Q_a to Q_b is intuitively 4.767 to 1 when $4.767 : 1(Q_a, Q_b)$. Since we are construing these only as relations between magnitudes and not between magnitudes and numbers, every distinct ratio must correspond to a distinct 2-place relation.[4]

2 Quantities and the World

The primary way that we gain epistemic access to facts about quantities is by performing measurements. However, measurements are interesting physical processes/procedures[5] in their own right, even putting aside their crucial epistemic role.

For our purposes, a "Q measurement" is a physical procedure performed on certain objects, a and b, (though there needn't be just two) which instantiate magnitudes of a particular quantity, Q. Measurements have a *ready state*, a specification of the state of the measurement apparatus and of a and b relative to that apparatus, as well as a set of possible (mutually incompatible) *outcomes*. Outcomes can include things like different possible positions of a pointer, the relative positions of plates on a balance scale, or a distribution of illuminated pixels on a readout screen.

A measurement's ready state and the different possible outcomes should be distinguishable without appeal to quantitative features of or relations between a and b

[4]This is why doing away with mathematical entities but still positing irreducible metrical structure is an unacceptable solution to the problem of quantity. It requires making an unweildy (indeed, infinite) number of distinct, primitive posits.

[5]I prefer the term 'procedure' to 'process', and will use the former in what follows. This is for two reasons. First, the same procedure can have different outcomes. Second, processes take time, while some measurement procedures are instantaneous (Section 2.2 gives an example). We could think of procedures as event-types which can be tokened in a few importantly different ways, where these differences amount to the different "possible outcomes" discussed below.

(or their respective parts).[6] That is, the ready state of a mass measurement should not include the condition that a be more massive than b, and two possible *distinct* outcomes of a length measurement cannot be distinguished *only* by whether or not a and b bear the "same length as" relation to each other.

Let's call a particular token measurement procedure, performed on a and b, *successful* if the occurrence or non-occurrence of each possible outcome is *reliably correlated* with the obtaining or non-obtaining of different quantitative relations between a and b (or between the magnitudes of Q they instantiate). A successful such measurement procedure produces a counterfactually robust correlation between its outcomes and the quantitative facts—i.e. it renders true conditionals of the form "If a had stood in R_Q to b (at the time of our measurement), then outcome O_i would have occurred".

Such robust correlations, when they obtain, cry out for explanation. A great many such explanations appeal to the role of Q in the dynamics evolving the ready state into one or another possible outcome (I give an example of a mass measurement with such an explanation in Case 1 below). However, certain paradigmatic length measurements do not admit of explanation by such means, yet they still can be robustly successful. Case 2 describes one such successful length measurement, and offers an intuitive, non-dynamical explanation for its success. The rub is, this explanation depends on a substantive connection – which isn't mediated by dynamics! – between length's quantitative structure and the mereology of lengthy physical entities.

2.1 Case 1: Weights on a scale

In the first case, we want to measure the ordering structure (i.e. to determine which, if either, is *more massive* than the other) of a pair of massive objects, a and b.

[6]Indeed, I require that the outcomes of a given measurement procedure must be distinguished wholly non-quantitatively (i.e. *not* by the obtaining or non-obtaining of any quantitative fact, magnitude, or relation). If I was only concerned with the epistemic role of measurement, this last requirement would be needlessly strong. This requirement screens off measurements whose success is really only revelatory of their relationships with and impact on *other quantities*, and not the non-quantitative world directly.

To do this, we set up a balance scale, with two plates suspended from opposite ends of a bar. The bar is balanced at its center on a rigid, vertical stand. The ready state for the scale is with the bar parallel to the ground and weights a and b positioned on opposing plates. We perform the measurement by releasing the plates and waiting a moment or two. The possible outcomes are: a's plate is lower than b's plate, b's plate is lower than a's plate, or the bar is parallel to the ground.[7]

Suppose we run this measurement and get the first outcome—a's plate is lower. Suppose further that a is more massive than b, and that if a had been *less* massive than (just as massive as) b, the second (third) outcome would have obtained. That is, we have performed a *successful* length measurement on a and b. In this particular case, what explains our measurement's success?

Here the explanation should be clear. Mass's quantitative structure plays a certain role in the dynamical laws of motion and gravitation. Specifically, objects which are more massive experience a greater force pulling them towards the earth. After we set the scale up in its ready state,[8] the weights on the scale are impressed by gravitational forces, as dictated by the physical laws. The downward forces on the plates will unbalance a properly calibrated balance scale just in case the objects differ in mass, with the more massive object being pulled more forcefully. Thus the dynamical laws come together with the quantitative facts and the physical makeup of the scale to bring about one of the three outcomes in a way which is reliably correlated with the "less massive than" relation.

Call a measurement procedure of this sort a *dynamical* measurement. Dynamical measurements are successful in virtue of the dynamics governing the evolution from the ready state to the resulting outcome. While there are other ways the dynamics

[7] One might worry that "lower" is a quantitative notion. However, it is not a matter of any quantitative relations between a and b and, in particular, is not a fact about a and b's masses. Even so, this quantitativeness is easy to get rid of, if we complicate our measuring device a bit. Many balance scales have a needle, perpendicular to the horizontal bar, attached at its center. The point of this needle is exactly above the vertical stand when the bar is parallel to the ground, and can either end up still upright or leaning to the left or the right of the vertical stand after.

[8] It turns out that there's some freedom in which ready state you pick. Even if the scale doesn't start with the bar perfectly parallel to the ground, the dynamics on the system will bring it to the right outcome as long as we wait a sufficiently long time.

can be involved in our general measurement *practices*—e.g. in us perceiving which outcome obtains, or in me building a balance scale—a measurement only counts as *dynamical* when the dynamics play an essential role in the measurement's success.[9]

2.2 Case 2: Aligning Rods

We want to measure the ordering structure for a pair of lengthy objects, in this case straight rigid rods. To do this, we arrange the rods so that they are parallel and lay them side-by-side. We then align them at one endpoint—i.e. while keeping them parallel, positioning one endpoint of rod a such that it is immediately adjacent to the endpoint on the same side of rod b. This is the ready state. There are three possible outcomes, as before: rod a extends past rod b, rod b extends past rod a, or neither rod extends past the other (Where "extending past", for these rods, just means one rod having a part which isn't adjacent[10] to any part of the other rod). We observe which of the rods, if either, extends past the other, and conclude that that rod is longer.

Suppose we perform this measurement and get the second outcome—rod b extends past rod a. Let's also suppose that this measurement is successful. I.e. that b is, in fact, longer than a, and that if b hadn't been longer than a, then b would not have extended past a (etc.). What explains the success of *this* measurement procedure?

In this case, we *cannot* appeal to length's role in the dynamics to explain the success of our length measurement. There are, of course, dynamical laws that *involve* spatial quantities like length, but this measurement *has no temporal component.* The procedure's ready state – a and b laid flush against each other and aligned at one endpoint – is *simultaneous* with the procedure's outcome – b's extending past a. Of

[9]Classical mass, it turns out, *only* admits of dynamical measurement. While there are many mass measurement procedures, including various kinds of scales, as well as "collision tests" where massive objects are knocked against each other to see which resists displacement to a greater degree, they all involve an appeal to the dynamics of the measuring system as it evolves from the ready state to one of the resulting outcomes.

[10]We can make the notion of extending past even clearer by doing away with adjacency. For a and b arranged as described in the text, a extends past b just in case there exists a plane orthogonal to a and b which intersects a part of a and no part of b.

course, the dynamics will play a role in our *observing* the outcome after the measurement, and it will play a role in our *positioning* the rods before the measurement, but the dynamics plays no role in evolving the system *from* the ready state *to* the particular outcome. This means that the success of this measurement, and the reliable correlation between its non-quantitative outcome and the quantitative facts, does *not* depend on the dynamics of length or any other quantity.

Indeed, this length measurement could occur and be successful at a world governed by *no* dynamical laws, which exists only for one moment, as long as, at that moment, the rods a and b are situated in the right way.[11]

If not the dynamics, what can explains the success of a length measurement of this sort? This measurement procedure is so transparently legitimate that it's unclear what mechanism *could* be underlying the correlation. The notion of extending beyond is so close to our conception of being longer (or instantiating a length magnitude bearing $\prec$ to the other) that it's hard to see the gap at all, let alone identify what's bridging it. It's not especially difficult to come up with an intuitively satisfying explanation of this case. The trouble is giving an account of what length must be like such that this explanation applies.

[11] There's a bit of nuance here that we should address. The issue isn't *merely* that the ready state and result state are simultaneous, though this is important. The issue is that the connection between them isn't dynamical. For instance, we could construct a mass measurement which could be performed instantaneously, but it would still depend on the dynamics. In 2.1, I pointed out that, in the case of a balance scale, we have some freedom in where we set the angle of the balance bar suspending the two plates, as long as we wait long enough for the system to enter equilibrium.

What makes the outcome of such a measurement important is that it represents an equilibrium state of the system. The state evolves to equilibrium and then stays in that state. Since there's some freedom as to the angle of the bar, we could start with our bar in *exactly the right position* such that the system is *already in an equilibrium state* when we let it go! In this case, there is a certain sense in which the ready state and the outcome are *simultaneous*.

However, the fact that the two states are simultaneous doesn't mean the success of the measurement isn't dependent on mass's role in the dynamics. What gave that outcome its status was that it's an equilibrium state, but being at equilibrium is a dynamical feature. It's a matter of what the dynamics governing that system *would* do to such a system *if* it were left alone and given a chance to evolve. So even if there *were* a world that existed only for an instant and contained a balance scale in exactly the right position, it would only count as a successful mass measurement if there were dynamics "governing" (or that would govern) that short-lived system, and the system was in equilibrium according to those dynamics. A short-lived world *without* any dynamical laws at all could not support such a measurement.

Here's what I think is going on in this case:
b extends past a. So while there's a part of b that is perfectly aligned with a, but there's also a *remainder*—i.e. another part of b that has no part that's adjacent to any part of a. Call the first part x and the second part, the remainder, y. The existence of such parts doesn't yet establish that b is longer than a. For that we need two bridge principles connecting the mereology and the quantitative facts.

(1) If two rods are laid side by side such that neither extends past either endpoint of the other, then they are as long as each other.

(2) A rod must be longer than any of its proper "rod segments".[12]

(1) establishes that a is as long as x. (2) establishes that b is longer than x. Together they establish that, in situations like our length measurement, above, b is longer than a. While premise (1) is of central importance to the practice of measuring length by laying rods side-by-side, I will not be discussing it much here.[13]

[12]Premise (2) is expressed in terms of rules of thumb for measuring rigid rods, and makes use of the notion of a "rod segment". This is not ideal, but it's important to recognize that the more natural sounding principle: "a rod must be longer than any of its proper parts" has some unfortunate exceptions. In particular, a three meter rod could be cut "lengthwise", so to speak, and thus divide into two three meter parts. Alternatively, it also has proper parts that, intuitively, have no length at all, but are just a few spatially disconnected pieces of rod. The notion of "rod segment" is meant to rule out cases like these.

If the reader is still worried that a rod could be as long as one of its "rod segments", perhaps because the rod segment is just the segment of the rod *minus* some lengthless slice at one endpoint, we can add premise (3):

(3) If a rod can be partitioned into two "rod segments", it is longer than each of them.

What premise (3) relies on is the idea that an infinitely thin slice off the endpoint of a rod is not a "rod segment" (even if its complement is). Once we move beyond this example and do away with talk of rods in favor of talk of spatiotemporal *paths*, we can avoid all the ambiguity involved in the notion of a rod segment.

[13]Premise (1) is likely an approximation of a principle that has its source in Euclid, with his Common Notion 4: "Things which coincide with one another are equal to one another."[Euclid (trans. Heath, 1908)]. Since material bodies can't interpenetrate, the closest to coinciding we can come, practically, is alignment without remainder, i.e. being laid side by side with neither extending beyond the other. There's much more to be said about the spatial structure of the world such that this approximation works, to the extent it does, but that's outside the scope of this paper.

2.3 The puzzle of non-dynamical measurement

This is a patently non-dynamical explanation. The outcome (b extending past a) and the quantitative facts (b being longer than a) are correlated, according to this explanation, not because of length's role in the dynamics but because of certain constraints on the mereology of lengthy objects (i.e. on the possible lengths of objects given their mereological structure and relations, and the possible mereological structure of objects given their lengths and length relations.). This connection between quantitative structure and mereology shows up at two points in the explanation:

The first is obvious. Premise (2) establishes that a rod bears a certain quantitative relation (longer than) to every member of a certain special sub-class of its parts.

The second involves premise (1), though in a more nuanced way: The explanation of the success of a length measurement of a given pair of rods, a and b, such that b extends past a, was presented as *fully general*. That is, for *any* rod shorter than b, which is measured against it in this way, b must have a proper part that's perfectly aligned with that rod. By (1) this implies that b has a proper part that's as long as that rod, for *any* such rod shorter than b! Here this explanation (& specifically its generality) puts substantial constraints on the parts of b and the lengths those parts can have.

Before we go any further, we will have to replace this talk of rods with something more rigorous. (1) and (2) are *approximately* true, as is this assumption about the generality of the explanation. However, though the success of the measurement of the rods a and b *can* be *roughly* explained by appeal to these principles, we don't *need* to tether our explanation to the nature of something as derivative and clunky as the notion of a concrete, straight, macroscopic material rod, and the "rod segments" which make it up. Indeed, if we want to give a truly rigorous and completely general explanation, we will need to give it in terms of the fundamental entities and properties in the vicinity.

Let's say that length is, fundamentally, a property of one-dimensional, open (i.e. non-looped) paths through spacetime. To the extent that a concrete material rod can be said to have length, it has its length derivatively, in virtue of occupying a region containing certain (properly oriented) spatiotemporal paths of such-and-such a length. We should be able to recapture an explanation in terms of rods by appealing to the properties of the regions they occupy. For the remainder of this paper, however, I'll be concerned only with the more general and rigorous principles concerning spatiotemporal paths.

We can capture the significance of premise (2) and of the generality assumption in one principle (By "object of length L_n" I'm only referring to things like substantival paths, and not to anything which has its length derivatively):

(2′) For all objects x of length L_n, and for all lengths $L_m \neq L_n$, x has a proper part of length L_m iff $L_m \prec L_n$.

(2′) puts very strong constraints on the sorts of parts lengthy objects can have, and on the possible lengths those parts can have. Analogously to (2), (2′) implies that a given path is as long or longer than *all* of its lengthy parts. Analogously to the assumption about generality, (2′) implies that a given path of length L_n must have a lengthy proper part corresponding to *every* length property bearing $\prec$ to L_n.

The only explanation for the reliable success of synchronic length measurement on offer requires a principle like (2′). But neither the physical details of the measurement procedure, nor the dynamical laws governing the system, are responsible for conditions like (2′). If this explanation is a good one, then, our theory of quantities like length should be able to account for the truth of (2′) in the relevant situations. To do this, we will have to consider how certain quantities constrain the mereology of their instances. In the next section, I argue that the way quantities are standardly assumed to put constraints on that mereology is insufficient to underwrite this explanation, and I propose an alternative.

3 Constraining the World

In this section, I consider two ways a quantity might put constraints on the mereological structure of its instances. The first is commonly called "additivity", while the second is a hitherto undiscussed phenomena, which I have dubbed "*proper* extensiveness" (though I will argue that it better captures some of our modal intuitions concerning certain physical quantities). I will show that additivity, properly understood, cannot explain the success of instantaneous length measurements, while proper extensiveness can.

3.1 Additivity

An additive quantity, roughly, is one for which composite objects "inherit" their Q-value (what magnitude of Q they instantiate) from the Q-values of their parts (if they have any). For instance, mass and length are both additive quantities. $2kg$ and $3kg$ stand in '$\oplus$' relation to $5kg$ ($2kg \oplus 3kg = 5kg$). Since mass is additive, composites of massive objects "inherit" their masses from their parts; so the mereological sum of a non-overlapping[14] pair of objects weighing $2kg$ and $3kg$ must weigh $5kg$.[15] The inheritance analogy is a powerful one, as it indicates both the strength and – we shall see – the limitations of this connection.

Put more formally, an additive quantity necessarily satisfies the following conditionals. They hold for any magnitudes, Q_i (of the same additive quantity), that satisfy the antecedent. The mereological relations used are these: '$O(x, y)$' for overlap, '$(x, y)C(z)$' for a three-place composition relation, with the third relatum being the fusion of the first two, and '$P(x, y)$' for parthood.

Additive $\prec$: $(Q_m \prec Q_n) \rightarrow \forall x \forall y((Q_n(x) \wedge Q_m(y)) \rightarrow \neg P(x, y))$

Additive $\oplus$: $(Q_m \oplus Q_n = Q_r) \rightarrow \forall x \forall y \forall z((Q_m(x) \wedge \neg O(x, y) \wedge (x, y)C(z)) \rightarrow (Q_r(z) \leftrightarrow Q_n(y)))$

[14] If we're being really strict about it, the parts may either have no overlap or have only "negligible overlap". What counts as negligible overlap depends on the structure and mereology of the quantity in question. For instance, often negligible overlap might just be overlap which instantiates the "zero magnitude", like $0m$ or $0kg$ or $0cm^3$. However, if one's metaphysics of the relevant quantity does not include a zero magnitude ([Balashov, 1999] takes issue with the very idea of a zero magnitude, albeit for reasons I'm not especially sympathetic to) the notion of negligible overlap must be got at in a different way.

[15] For ease of presentation, I assume mereological universalism. Certain complications would arise if we were to drop this assumption. However, none of the substantive points of the paper depends on it. I also assume that none of the massive objects discussed are spinning.

In the case of mass, **Additive** $\prec$ says that no massive object can have a part which is more massive than it. **Additive** $\oplus$ says that the fusion of any two non-overlapping objects has the "sum" of their respective mass magnitudes as its mass, providing they instantiate mass magnitudes at all. These conditionals (on the assumption that $\oplus$ is commutative) fully specify the mereological significance of additivity. These conditionals are modally[16] robust: Suppose *pumpkin* is a $5kg$ object composed out of non-overlapping parts *body* and *stem*. If we consider a counterfactual scenario in which the only difference is that *stem* is $2kg$s heavier (than it actually is), we readily (often automatically) infer that at this world *pumpkin* is $2kg$s heavier as well. Indeed, its difficult to conceive of a world where only *stem*, but neither *body* nor *pumpkin*, changes its mass.

3.2 Additivity and Measurement

The reason additivity cannot explain the success of synchronic length measurement is well illustrated by the "inheritance" analogy. Additivity says that an object's mass is determined by the masses of its parts. However, **Additive** $\oplus$ and **Additive** $\prec$ are entirely silent on *whether* a given massive object has parts (massive or otherwise). This means that length's additivity cannot itself account for the truth of $(2')$.

Since **Additive** $\prec$ and **Additive** $\oplus$ never imply that a given object *must have* parts of some kind, they're consistent with a pair of objects, a and b, instantiating magnitudes, Q_a and Q_b, (of some additive quantity) where $Q_a \prec Q_b$ yet both a and b are *mereological simples*. There's nothing obviously wrong with admitting of such a possibility for *mass*. On the ordinary understanding of most particle theories, elementary particles are assumed to be mereologically simple, and there is no prohibition on different elementary particles ever possessing different masses! How-

[16]The nature of this modal robustness, i.e. the degree of necessity possessed by the conditionals **Additive** $\oplus$ and **Additive** $\prec$, is not entirely clear, and may differ from quantity to quantity. For instance, on some understandings of classical mass, on which it is identical to inertia, the truth of **Additive** $\oplus$ and **Additive** $\prec$ for mass might be grounded in the nature of mass's dynamical role. If so, then these conditionals may well be merely nomologically necessary, when it comes to mass, rather than metaphysically necessary.

ever, the analogous possibility for *lengthy* entities is flatly inconsistent with (2′).[17] Moreover, such a possibility also seems to get the modality of fundamentally lengthy entities (like spacetime paths or trajectories) intuitively wrong (I go more in depth into this issue in particular in the next section).

Mere additivity cannot explain the reliable and general success of synchronic length measurement.

3.3 Proper Extensiveness

I'm going to introduce a phenomena called "proper extensiveness". My contention is that certain physical quantities are properly extensive—length, volume, and temporal duration among them—and that properly extensive quantities, by their nature, put stronger constraints on the mereological structure of the world than merely additive quantities do. Specifically, these constraints are sufficient to entail (2′) for length and thereby support the intuitive explanation offered in the previous section for the success of synchronic length measurement.

Physical quantities can be grouped into the additive and the non-additive (sometimes called "intensive") quantities, and the class of additive quantities can be further divided into the *merely* additive quantities and the *properly* extensive quantities. As such, properly extensive quantities satisfy **Additive** $\prec$ and **Additive** $\oplus$:

Additive $\prec$**:** $(Q_m \prec Q_n) \rightarrow \forall x \forall y((Q_n(x) \wedge Q_m(y)) \rightarrow \neg P(x, y))$

Additive $\oplus$**:** $(Q_m \oplus Q_n = Q_r) \rightarrow \forall x \forall y \forall z((Q_m(x) \wedge \neg O(x, y) \wedge (x, y)C(z)) \rightarrow (Q_r(z) \leftrightarrow Q_n(y)))$

$2m$ and $3m$ stand in $\oplus$ to $5m$ (i.e. $2m \oplus 3m = 5m$). Length is additive, so the fusion of two non-overlapping objects of length $2m$ and $3m$ laid end-to-end (in the right way) will be $5m$ long. If length were *merely* additive, that would be the end of the story. Because length (we are supposing) is also properly extensive, we can say

[17] To see this, realize that it's also consistent with the dictates of additivity that there be two lengthy objects, a and b, of lengths, $2m$ and $5m$ respectively, where b has no proper part as long as a (that is, $2m$ long) because b is a mereological simple.

more—e.g., since $2m \oplus 3m = 5m$, *any* $5m$ path *must* admit of a partition into a $2m$ part and a $3m$ part. We'll understand a partition of o as a class of non-overlapping objects whose fusion is o. That is, properly extensive quantities also necessarily satisfy:[18]

Extensive $\prec$: $(Q_m \prec Q_n) \Rightarrow \forall x(Q_n(x) \to \exists y(y \neq x \wedge Q_m(y) \wedge P(y, x)))$

Extensive $\oplus$: $(Q_m \oplus Q_n = Q_r) \Rightarrow \forall x(Q_r(x) \leftrightarrow \exists y \exists z(Q_m(y) \wedge Q_n(z) \wedge \neg O(y, z) \wedge (y, z)C(x)))$

In the case of length,[19] what **Extensive $\prec$** says is that every spatial path of a given length L_n, such that $L_m \prec L_n$, has an interval (which is to say, a part which is itself a path) of length L_m. **Extensive $\oplus$** says a path can instantiate a length magnitude L_a such that $L_b \oplus L_c = L_a$, if and *only if* it has two non-overlapping parts which respectively instantiate those magnitudes. This is a very powerful condition, because it says that, given the quantitative facts, just instantiating a given length magnitude, L_a, necessarily requires you to have parts with certian length properties standing in certain mereological relations to one another.

Recall that, in order for our explanation of synchronic length measurement in terms of the existence of a remainder to apply, our theory of length must entail that the quantity satisfies:

[18]Additivity and proper extensiveness both involve principles which concern the quantitative features of objects "put together in the right way". For most quantities, like mass or volume, the formula '$\neg O(x, y) \wedge (x, y)C(z)$' will accurately describe this condition. However, since only certain kinds of objects can have length (namely, unbroken non-looping paths), the conditions for putting two paths together "in the right way" are more stringent. It isn't enough for path a and path b to not overlap and to together compose object c. If a is the spatial path from my nose to my upper lip, and b is the shortest path from the surface of the earth to the moon, then their fusion, c, isn't an unbroken path, and so doesn't have length! The conditions for length would be something like this: a and b are both intervals of path c, which is their mereological fusion, and a and b either don't overlap or have a lengthless overlap (either one with $0m$ length or without length, depending on what we want to say about the lengths of unextended points). Since I am more concerned here with the relationship between the second-order $\prec$ and parthood, I will set this issue aside

[19]Technically these conditionals, as stated, only directly apply to properly extensive quantities like volume or surface area. They would need slight tweaking to accurately characterize a quantity like length. How we sort out this wrinkle won't, however, make a difference for our argument concerning measurement.

(2′) For all objects x of length L_n, and for all lengths $L_m \neq L_n$, x has a proper part of length L_m iff $L_m \prec L_n$.

An account of length on which length is properly extensive does entail (2′). By **Extensive** $\prec$, we get that if $L_m \prec L_n$ then x has a part of length L_m, and by **Additive** $\prec$, we get that if x has a proper part of length L_m, then L_m must be either $= L_n$ or $\prec L_n$ (which, given the assumption that $L_m \neq L_n$, implies that $L_m \prec L_n$.

3.4 The significance of Proper Extensiveness

The fact that proper extensiveness is necessary to explain the reliable success of a paradigm measurement procedure is important because it indicates that (1) there is good reason to take length to be properly extensive and (2) that the necessary conditionals characterizing proper extensiveness must be independent of the operation of the dynamical laws. Solving the puzzle of synchronic length measurement is less an end in itself and more a means to introduce and motivate proper extensiveness. In this section I will further examine this phenomena, and in the next outline a very significant application.

Some of our central intuitions regarding physical quantities like length, volume, and temporal duration—specifically those concerning how the mereological structure of the world reflects the quantitative structure of the properties instantiated at it—already suggest a tacit commitment to something like proper extensiveness for these quantities.

One striking consequence of taking length to be properly extensive illustrates this quite well. Suppose we discover a path through space that had a non-zero length, L_u, but no proper sub-paths (i.e. no proper parts which are paths). According to **Extensive** $\prec$, this implies that there are *no* length magnitudes $\prec L_u$ (except the zero-magnitude, $0m$, if there is such a thing)—meaning that the quantity, length, is *discrete* (best represented by the natural numbers plus zero) and that L_u is its *unit length*.

This result very closely accords with our intuitive expectations about what the physical world can tell us about quantities like length. We do not hear metaphysicians raise concerns when physicists run together the possibility that there is a smallest non-zero length (alternatively, that the quantity *length* is discrete) with the possibility that there are shortest possible *paths* (alternatively, that *space* is discrete). Indeed, many discussions of length readily use "shorter than" and "as long as a proper sub-interval of" interchangeably. Similar points can be made for area, volume, and temporal duration. The pervasiveness of this line of thought disguises how significant of a metaphysical commitment it amounts to, once we take it seriously. The notion of proper extensiveness is how we should characterize this commitment.

It is important to stress again how these commitments simply do not hold sway for merely additive quantities. Though mass's status as merely additive is not entirely uncontroversial, treating it that way is in accordance with an extremely common understanding of the quantity.[20] On this understanding, there could very well be two simples (objects without proper parts) with differing, non-zero, masses. When entertaining the epistemic possibility that, e.g., the electron is a point-particle (without spatial extension and, it is presumed, mereologically simple), we don't at all expect every *other* elementary particle to therefore be exactly as massive as the electron! However, that is precisely the sort of conclusion we *should* reach in the analogous scenario for quantities like length and volume!

I've suggested that there exists a distinction in our intuitions about the modal mereology of additive physical quantities. If this is right, it stands as strong evidence in favor of a distinction between the additive quantities into the merely additive and the properly extensive, as I draw it. The lack of acknowledgment or discussion of this phenomena in the philosophical and physical literature means that (as of yet)

[20]The fact that mass is closely associated with a certain dynamical role is good evidence that it's not properly extensive, since we standardly think that the same dynamical role in gravitation or inertia could be played equally well by a mereological complex or a simple. However, for all we know it may turn out that mass more closely aligns with earlier notions of physical mass as the "measure of matter". If that is right, to say that a is less massive than b is to say that a has less matter making it up than b. One way to draw out this understanding would be to treat mass as properly extensive, and to expect its instances to obey the associated mereological constraints (i.e. if b has more matter making it up than a does, then b should have a part which has exactly as much matter making it up as a does).

there are no suggestions on the table as to *why* some quantities are extensive, or *how* this constraining of the mereology is supposed to work. For our purposes, it suffices to say *that* some quantities are extensive and *that* they constrain mereology in a modally robust way that is independent of the dynamical laws.

4 Conclusion: Applying Proper Extensiveness to the Problem of Quantity

In the previous two sections I have argued in favor of positing a distinction amongst the additive quantities into the merely additive and the properly extensive. I have argued that this distintion better captures and explains the data, specifically regarding simultaneous length measurement as well as our modal mereological intuitions about various physical quantities. I'd like to close by gesturing in the direction of a significant potential application of this distinction. Specifically, I will give a few reasons to believe that an elegant and principled solution to the problem of quantity, as it applies to properly extensive quantities, is available if we take proper extensiveness as fundamental

Many metaphysicians of quantity appeal to *measurement theory* in their answer to the problem of quantity. Specifically, they attempt to reduce facts about metric structure to facts about the world satisfying the right measurement-theoretic axioms.[21] Measurement theory is a formal discipline which involves rationalizations, formalizations and defenses of empirical measurement practices. The game of measurement theory is to take a domain of material objects, which instantiate different magnitudes of some quantity, Q, posit some axioms that these objects obey, and then prove theorems which imply that Q can be faithfully represented, up to a point, with a certain mathematical structure, e.g. the real numbers.[22]

Some of the axioms required to prove these theorems impose certain requirements on the *size* and *structure* of the domain itself. They say that domains are well

[21]Field (1980) is the most famous account along these lines.
[22]Cf. [Krantz, et. al. 1971]

populated (existence axiom), and that there's ample variation in which magnitudes of Q are instantiated therein (richness axiom). The satisfaction of such axioms is a contingent matter. If there aren't enough objects, or if they don't instantiate enough different magnitudes, these axioms fail to be satisfied.

But our account of the ground of metric structure ought not to be contingent on the world being well-populated! This contingency problem has been acknowledged in the literature, and various theorists have proposed ad hoc solutions to eliminate this contingency. [Mundy, 1987] gives up on the domain of massive *objects* and instead attempts to apply measurement theory to the domain of mass *magnitudes*, while Arntzenius and Dorr in their (2013) avoid the contingency problem by positing well-populated substantival physical spaces, and identifying the geometry of *this* space with the relevant quantitative structure.

The unique advantage of properly extensive quantities is that any world where such a quantity is instantiated must, by the conditions it places on the mereology of its instances, be well populated and variegated, to a certain degree. Suppose that L_x is a length magnitude, instantiated by a path, p. **Extensive** $\prec$ implies that p will have *at least* as many proper parts as there are length magnitudes which bear $\prec$ to L_x. Similarly, **Extensive** $\oplus$ implies that p will admit of a partition into parts of length L_y and L_z, for every such pair of length magnitudes such that $L^y \oplus L_z = L_x$.

This suggests that a domain where a properly extensive quantity is instantiated, and in which its instances satisfy the necessary constraints its proper extensiveness puts on their mereology, may be of the right form to satisfy the relevant existence and richness axioms. I think this can be shown, but there's no room to do so here. However, if it were true, it would allow for a uniquely elegant and principled solution to the problem of quantity, as it applies to properly extensive quantities.

A result of this kind, if it can be done (and I think it can), is not just important because it moves us closer to a satisfactory solution to the problem of quantity in full generality. It also speaks to the metaphysical depth of the distinction between properly extensive quantities and all other physical quantities, one which manifests not just in the way these quantities relate to mereology, but also in the nature and ground of their metric structure.

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