

# Meaning and Inference

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## Introduction

Contemporary theories of meaning can be divided, with a certain amount of oversimplification, into those which see the meaning of an expression as principally a matter of what the expression denotes or stands for, and those which see it as a matter of how the expression is used. A prominent place among the latter ones is assumed by those which identify the semantically relevant aspect of the usage of an expression with an inferential pattern governing it. According to these theories, the meaning of an expression is, principally, its *inferential role*.<sup>1</sup>

In this paper we first propose an exact definition of the concept of inferential role, and then go on to examine the question whether subscribing to inferentialism necessitates throwing away existing theories of formal semantics, as we know them from logic, or whether these could be somehow accommodated within the inferentialist framework. The conclusion we reach is that it is possible to make an inferentialist sense of even those common semantic theories which are usually considered as incompatible with inferentialism, such as the standard semantics of second-order logic.

## Primary inferential roles (of sentences)

Let us use the sign  $>$  for *correct inferrability* (we will also use the term *entailment*). Thus, if  $X$  is a set of sentences and  $S$  a sentence, we will write  $X > S$  to express that  $X$  entails  $S$ . Besides entailment, we may consider one other important inferential property of sets of sentences (which, as we will see later, may or may not be seen as reducible to entailment), namely *incompatibility*. Let us write  $IC(X)$  to express that nobody can be entitled to all elements of  $X$ . Let us, moreover, write  $X \perp S$  as a shorthand for  $IC(X \cup \{S\})$ .<sup>2</sup>

<sup>1</sup> See especially Brandom (1994; 2000) for the ideological background and Lance (1996; 2001) and Kaldernon (2001) for some more technical aspects. See also Peregrin (2001; esp. Chapters 7 and 8) and Peregrin (to appear).

<sup>2</sup> The notation is partly due to Lance (2001).

We will assume that entailment has the following basic properties (we shorten  $\{S_1, \dots, S_n\} > S$  to  $S_1, \dots, S_n > S$ ):

- [R]  $S > S$   
 [T] if  $X > S$  and  $\{S\} \cup Y \Rightarrow S'$ , then  $X \cup Y \Rightarrow S'$   
 [M] if  $X > S$ , then  $X \cup \{S'\} > S$

Moreover, there is the following relationship between entailment and incompatibility:

- [\*] if  $X > S$  and  $IC(Y \cup \{S\})$ , then  $IC(Y \cup X)$

This means that something is incompatible with the consequent of an entailment, it is bound to be incompatible also with its antecedent.

We may also consider reducing incompatibility to entailment by the well known ‘ex falso quodlibet’ axiom:

- [\*\*]  $X \perp S$  iff  $X \cup \{S\} > S'$

Now with this notational apparatus, what we will call the *primary inferential role* of a sentence  $S$  can be exhaustively characterized by the following four sets:

- conditions of  $S$ :  $S^{\leftarrow} = \{X \mid X > S\}$   
 consequences of  $S$ :  $S^{\rightarrow} = \{\langle X, S' \rangle \mid X \cup \{S\} > S'\}$   
 contradicta of  $S$ :  $S^{\times} = \{X \mid X \perp S\}$

Hence as a first approximation, we can set

$$\text{PIR}(S) \equiv_{\text{Def}} \langle S^{\leftarrow}, S^{\rightarrow}, S^{\times} \rangle$$

We define the inclusion of PIR’s as the inclusion of their respective components

$$\text{PIR}(S_1) \subseteq \text{PIR}(S_2) \equiv_{\text{Def}} S_1^{\leftarrow} \subseteq S_2^{\leftarrow} \text{ and } S_1^{\rightarrow} \subseteq S_2^{\rightarrow} \text{ and } S_1^{\times} \subseteq S_2^{\times}$$

and we define the relation *PIR* of sameness of primary inferential roles:

$$S_1 \text{ PIR } S_2 \equiv_{\text{Def}} \text{PIR}(S_1) = \text{PIR}(S_2).$$

We will now prove that two sentences have the same PIR iff they are logically equivalent. First a lemma (where  $\diamond$  is the relation of logical equivalence, i.e. the relation which holds between  $S_1$  and  $S_2$  just in case  $S_1 > S_2$  and  $S_2 > S_1$ ):

**Lemma:** If  $S_1 \diamond S_2$ , then  $\text{PIR}(S_1) \subseteq \text{PIR}(S_2)$

**Proof:** Let  $S_1 \diamond S_2$ . Then

1. If  $X > S_1$ , then  $X > S_2$  (from  $S_1 > S_2$  by [T]).

2. If  $X \cup \{S_1\} > S'$ , then  $X \cup \{S_2\} > S'$  (from  $X \cup \{S_2\} > S_1$  by [T], which from  $S_2 > S_1$  by [M]).
3. If  $X \perp S_1$ , then  $X \perp S_2$  (from  $S_2 > S_1$  by [\*]).

It obviously follows that if  $S_1$  and  $S_2$  are logically equivalent, they share the same PIR; for if  $S_1 \diamond S_2$ , then  $\text{PIR}(S_1) \subseteq \text{PIR}(S_2)$  and  $\text{PIR}(S_2) \subseteq \text{PIR}(S_1)$ , i.e.  $S_1 \text{ PIR } S_2$ . The next simple lemma shows that the converse implication also holds:

**Lemma:** If  $S_1 \text{ PIR } S_2$ , then  $S_1 \diamond S_2$ .

**Proof:**  $S_1 \in S_1^{\leftarrow}$  (by [R]), hence  $S_1 \in S_2^{\leftarrow}$ , hence  $S_1 > S_2$ ; and the same *vice versa*.

Combining the two lemmas we get the anticipated result, namely that  $S_1 \text{ PIR } S_2$  iff  $S_1 \diamond S_2$ .

Admitting some further assumptions about the inferential structure of language, we can further simplify the definition of PIR (for the lack of space, we omit proofs of these facts). Thus, accepting [\*\*] we can reduce contradicta to consequences,

$$S^{\times} = \{X \mid X \cup \{S\} > S' \text{ for every } S'\} = \{X \mid \langle X, S' \rangle \in S^{\rightarrow} \text{ for every } S'\},$$

and  $\text{PIR}(S)$  becomes uniquely determined by  $S^{\rightarrow}$  and  $S^{\leftarrow}$ .

If we assume that for every statement  $S$  there exists a 'minimal incompatible'  $\text{MI}(S)$  such that  $\text{MI}(S) \perp S$  and if  $X \perp S$ , then  $X > \text{MI}(S)$ , then it can be shown that we can reduce contradicta to conditions of the minimal incompatible:

$$S^{\times} = \{X \mid X > \text{MI}(S)\} = \text{MI}(S)^{\leftarrow}$$

Moreover, there is a sense in which we can reduce consequences to conditions (where  $X^{\leftarrow}$  is the set of sets of sentences entailing all elements of  $X$ ):

$$S^{\rightarrow} = \{\langle X, S' \rangle \mid S^{\leftarrow} \subseteq \overline{X^{\leftarrow}} \cup S'^{\leftarrow}\}$$

### Secondary inferential roles

Primary inferential roles are a matter of sentences only. Hence if we want to apply inferentialism also to subsentential expressions, we have to find a concept of an inferential role which would be applicable to them too. Therefore we introduce the concept of *secondary inferential role*, which we will not define explicitly, but only via the relation of sameness. If  $E_1$  and  $E_2$  are expressions, then we will call two sentences  $S$  and  $S'$   $[E_1/E_2]$ -variants iff  $S'$  differs

from  $S$  only in that some the occurrences of  $E_1$  are replaced by  $E_2$ . Then we define the sameness of secondary inferential roles as follows:

$$E_1 \text{ SIR } E_2 \equiv_{\text{Def}} S \text{ PIR } S' \text{ for every } [E_1/E_2]\text{-variants } S \text{ and } S'$$

Thus, SIR's are 'what SIR-equivalent expressions have in common'. In other words, an SIR of an expression is the contribution which the expression brings to the primary inferential roles of the sentences in which it occurs.

Now sentences have *two* kinds of inferential roles, PIR's and SIR's. Are they different? Though in general they are,<sup>3</sup> we will characterize the class of languages for which they may be seen as coinciding. Let us call a language *at most intensional* (i.e.: not 'hyperintensional') iff for every two sentences  $S_1$  and  $S_2$  and every two  $[S_1/S_2]$ -variants  $S$  and  $S'$ ,  $S_1 \diamond S_2$  implies  $S \diamond S'$ .

**Claim:** A language is at most intensional iff for every two sentences  $S_1$  and  $S_2$ ,  $S_1 \text{ SIR } S_2$  iff  $S_1 \text{ PIR } S_2$ .

**Proof:** Let us first consider a language which is at most intensional. The 'only if' part is trivial. Let  $S_1 \text{ PIR } S_2$  and let  $S$  and  $S'$  be  $[S_1/S_2]$ -variants. Then  $S_1 \diamond S_2$ , and as the language is at most intensional,  $S \diamond S'$ . But this means that  $S \text{ PIR } S'$ , and hence  $S_1 \text{ PIR } S_2$ .

Now consider a language which is not at most intensional. Then there are some  $S_1, S_2$  and some  $[S_1/S_2]$ -variants  $S$  and  $S'$  such that  $S_1 \diamond S_2$ , whereas not  $S \diamond S'$ . This means that  $S_1 \text{ PIR } S_2$ , but not  $S_1 \text{ SIR } S_2$ .

This means that if we restrict ourselves to languages which are at most intensional, we can identify the SIR of a sentence with its PIR. The SIR of a subsentential expression can then be seen as the contribution the expression brings to the inferential roles of the sentences in which it occurs.

### Inferential patterns

The crucial claim of inferentialism is that the inferential role of every expression is determined by some *finite inferential pattern*. (The idea is that to grasp the meaning of the expression is just to master the inferential pattern, and so the pattern must be something humanly masterable, i.e. finite.) However, what is an inferential pattern?

As a first approximation, we will identify such a pattern with a finite set of (possibly parametric) instances of inference, such as

<sup>3</sup> Note that the distinction between the PIR of a sentence and its SIR is the inferentialist embodiment of Dummett's (1973) distinction between the "freestanding sense" and the "ingredient sense".

$$\text{Cat}(\text{Tom}), \text{Mouse}(\text{Jerry}) > \text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})$$

or

$$S_1, S_2 > S_1 \wedge S_2.$$

Note that we do not claim that the inferential role of an expression must be specifiable independently of those of other expressions – an inferential pattern may well characterize the role of an expression only relatively to those of some other expressions (which then implies that the language cannot possess the former without possessing the latter).<sup>4</sup> Note also that in no way do we maintain the claim that any inferential pattern is as good as any other in furnishing an expression with a viable meaning (i.e. the claim attacked by Prior, 1960/61).<sup>5</sup>

Let us take an example. We may characterize the inferential role of the sentence  $\text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})$  by the following pattern:

$$\begin{aligned} \text{Cat}(\text{Tom}), \text{Mouse}(\text{Jerry}) &> \mathbf{Cat}(\mathbf{Tom}) \wedge \mathbf{Mouse}(\mathbf{Jerry}) \\ \mathbf{Cat}(\mathbf{Tom}) \wedge \mathbf{Mouse}(\mathbf{Jerry}) &> \text{Cat}(\text{Tom}) \\ \mathbf{Cat}(\mathbf{Tom}) \wedge \mathbf{Mouse}(\mathbf{Jerry}) &> \text{Mouse}(\text{Jerry}) \end{aligned}$$

Expressed in terms of positive conditions, this yields

$$\begin{aligned} \text{Cat}(\text{Tom})^{\leftarrow} \cap \text{Mouse}(\text{Jerry})^{\leftarrow} &\subseteq \text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})^{\leftarrow} \\ \text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})^{\leftarrow} &\subseteq \text{Cat}(\text{Tom})^{\leftarrow} \cap \text{Mouse}(\text{Jerry})^{\leftarrow} \end{aligned}$$

and hence

$$\text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})^{\leftarrow} = \text{Cat}(\text{Tom})^{\leftarrow} \cap \text{Mouse}(\text{Jerry})^{\leftarrow}$$

In fact, in this way we only reduce the inferential role of  $\text{Cat}(\text{Tom}) \wedge \text{Mouse}(\text{Jerry})$  to those of its components, namely  $\text{Cat}(\text{Tom})$  and  $\text{Mouse}(\text{Jerry})$ ; so this definition works only if we already have the inferential roles of the two atomic sentences.<sup>6</sup>

As a further example, let us consider the inferential role of  $\wedge$ , which obviously derives from that of  $S_1 \wedge S_2$  with parametric  $S_1$  and  $S_2$ . The pattern is, of course,

<sup>4</sup> Take arithmetic: the roles of all its constants are inextricably characterized by Peano's axioms.

<sup>5</sup> See Peregrin (2001, §8.5) for more details.

<sup>6</sup> What, then, would be the role of an atomic sentence, such as  $\text{Cat}(\text{Tom})$ ? Well, if we want  $\text{Cat}$  and  $\text{Tom}$  mean approximately what they do in English, then to understand them inferentialistically, we would have to somehow broaden the concept of inference to comprise not only the 'language-language' instances, but also some 'world-language' instances (see Peregrin, 2001, §7.6). Of course the inferentialist does not aspire to reduce meanings of *empirical* expressions to inferences in the usual, 'language-language' sense – such inferences, though vital for any kind of meaning, exhaust only meanings of *non-empirical*, especially *logical*, expressions.

$$\begin{aligned} S_1, S_2 &> S_1 \wedge S_2 \\ S_1 \wedge S_2 &> S_1 \\ S_1 \wedge S_2 &> S_2 \end{aligned}$$

This yields

$$S_1 \wedge S_2^{\leftarrow} = S_1^{\leftarrow} \cap S_2^{\leftarrow}$$

Unlike in the previous case, this is not a reduction, but rather a full specification of the inferential role:  $\wedge$  is characterized as an expression which combines two sentences into a complex sentence with positive conditions equal to the intersection of the positive conditions of its two components.

Now consider  $\vee$ . Part of the inferential pattern characterizing it is clear:

$$\begin{aligned} S_1 &> S_1 \vee S_2 \\ S_2 &> S_1 \vee S_2 \end{aligned}$$

But we want also the disjunction to be true *only if* at least one of its disjuncts is true. If we may make use of negation, then we can complete the pattern by

$$\neg S_1, \neg S_2 > \neg(S_1 \wedge S_2)$$

yielding the desired

$$S_1 \vee S_2^{\leftarrow} = S_1^{\leftarrow} \cup S_2^{\leftarrow},$$

but this presupposes that we have already established the inferential role of negation. However, there is obviously no way of articulating the inferential role of negation, and consequently disjunction, by means of what we have so far called an inferential pattern.<sup>7</sup>

A reaction to this might be to say that hence the classical operators are not accessible for the inferentialist (the worse for them!, from the inferentialist standpoint), but we will indicate that a generalization of the concept of an inferential pattern, which is not too unnatural, will allow us to make inferential sense even of them. (For lack of space we will say nothing about the logical constants specific to *predicate* logic, i.e. quantifiers.)

<sup>7</sup> It might seem that an inferential specification of the meanings of the logical operators is provided already by the standard axiomatics of the propositional calculus. Does it not follow from the soundness and completeness of the calculus that the axioms pin down the denotations of the operators to the usual truth functions? In fact, it does not. As a matter of fact, the axioms are compatible with some non-standard interpretations of the operators – with negations of some falsities being false and with disjunctions of some pairs of falsities being true. What *is* the case is that if the axioms hold *and if the denotations of the operators are truth functions*, then they are bound to be the *standard* truth functions. But the axioms are compatible with the indicated *non-truth-functional* interpretation of the constants. This is a fact noted already by Carnap (1943), but very rarely reflected (see Koslow, 1992, Chapter 19, for a discussion).

### Truth tables and inferential patterns

Our goal, then, will now be to provide a way of reading the rows of the truth table characterizing any of the usual logical operators as generally recapitulating inferential patterns governing the operator. This is straightforward in some cases, but rather more tricky in others. We will consider general types of rows in a possible truth table and try to provide their ‘translation’ into instances of inference (note, however, that from the inferentialist viewpoint this will not be a translation, but rather a case of *archeology*—retrieving the patterns lying *beneath* the tables).<sup>8</sup>

#### Method 1. True arguments $\rightarrow$ true value

First, consider a row with only T’s both in the argument columns and in the value column. Here the inferentialist reading is, of course, quite straightforward: the row states that the complex sentence is entailed by all its arguments:

$S_1$	...	$S_n$	$O(S_1, \dots, S_n)$
T	...	T	T
..	...	...	...

$$S_1, \dots, S_n > O(S_1, \dots, S_n)$$

#### Example

$S_1$	$S_2$	$S_1 \wedge S_2$
T	T	T
..	...	...

$$S_1, S_2 > S_1 \wedge S_2$$

#### Method 1 (generalized). True or arbitrary arguments $\rightarrow$ true value

This can obviously be generalized to clusters of rows with T in the value columns, T’s in some fixed argument columns and all possible combinations of values in the other argument columns. Let us introduce the convention of putting an asterisk into the argument column whose content is irrelevant for the content of the value column:

<sup>8</sup> Note also that what we are after is something quite different from the fact underlying the most usual proof of the completeness of the classical propositional calculus due to Kalmár (1936), namely that using negation and implication, we can express every row of every table as a theorem of the predicate calculus (see, e.g. Mendelson, 1964, §1.4). What we want is to find, for every table, an inferential pattern featuring the single operator whose semantics is given by the table.

$S_1$	...	$S_{i-1}$	$S_i$	$S_{i+1}$	...	$S_n$	$O(S_1, \dots, S_n)$
$v_1$	...	$v_{i-1}$	T	$v_{i+1}$	...	$v_n$	$v$
$v_1$	...	$v_{i-1}$	F	$v_{i+1}$	...	$v_n$	$v$
..	...	..	...	...	...	...	...

→

$S_1$	...	$S_{i-1}$	$S_i$	$S_{i+1}$	...	$S_n$	$O(S_1, \dots, S_n)$
$v_1$	...	$v_{i-1}$	*	$v_{i+1}$	...	$v_n$	$v$
..	...	..	...	...	...	...	...

Then clearly a row containing only T's and \*'s in the argument columns and T in the value column also yields an inferential pattern quite straightforwardly:

$S_1$	...	$S_n$	$O(S_1, \dots, S_n)$
T/*	...	T/*	T
..	...	...	...

$S_{i_1}, \dots, S_{i_n} > O(S_1, \dots, S_n)$ ,

where  $i_1, \dots, i_n$  are the columns containing T (rather than \*)

Example:

$S_1$	$S_2$	$S_1 \vee S_2$
*	T	T
..	...	...

$S_2 > S_1 \vee S_2$

*Method 2. One false argument and others true or arbitrary → false value*

An inferential pattern is also easily seen to be yielded by a row containing precisely one F in the argument columns and F in the value column:

$S_1$	...	$S_i$	...	$S_n$	$O(S_1, \dots, S_n)$
T/*	...	F	...	T/*	F
..	...	...	...	...	...

$S_{i_1}, \dots, S_{i_n}, O(S_1, \dots, S_n) > S_i$ ,

where  $i_1, \dots, i_n$  are the columns containing T

Example:

$S_1$	$S_2$	$S_1 \rightarrow S_2$
T	F	F
..	...	...

$S_1, S_1 \rightarrow S_2 > S_2$



*Method 3. True or arbitrary arguments  $\rightarrow$  false value*

For rows containing no F's in the argument columns, but containing F in the value column we need to invoke either the concept of incompatibility or the 'ex falso quodlibet' axiom:

Example:

$S_1$	...	$S_n$	$O(S_1, \dots, S_n)$
T/*	...	T/*	F
..	...	...	...

$S_1, \dots, S_n \perp O(S_1, \dots, S_n)$ ,  
or  $S_1, \dots, S_n, O(S_1, \dots, S_n) > S$

Example:

$S$	$\neg S$
T	F
..	...

$S \perp \neg S$   
 $S, \neg S > S'$

*Method 4.*

The types of rows for which we have provided 'translations' so far still do not cover all those occurring in the tables of the standard operators: viz. the F F F row within the disjunction truth table or the F T row within the negation one. How are we to make inferential sense of them?

Let us look at an inferential pattern for an operator  $O$  as a means of enumerating all those assignments of truth values to  $S_1, \dots, S_n$  for which  $O(S_1, \dots, S_n)$  yields T (or, alternatively, F). Thus, the pattern

$$S_1 > S_1 \vee S_2$$

$$S_2 > S_1 \vee S_2$$

specifies that  $S_1 \vee S_2$  is true if at least one of  $S_1$  and  $S_2$  is, i.e. for the truth-value pairs  $\langle T, F \rangle$ ,  $\langle F, T \rangle$  and  $\langle T, T \rangle$ . Similarly

$$S_1, S_1 \rightarrow S_2 > S_2$$

says that  $S_1 \rightarrow S_2$  is false if  $S_1$  is true and  $S_2$  false. Now it is not unnatural to assume that if one gives an enumeration of cases, it is meant to be exhaustive. If I say "My children are Tom and Jerry", then what is normally taken for granted is that these are *all* my children. (See McCarthy's, 1980, discussion of

this feature of our enumerative claims).<sup>9</sup> This may suggest that part and parcel of our ‘enumerative’ claims, and especially of our ‘enumerative’ specification of inferential patterns, is what can be called the *exhaustivity assumption (EA)*:

The inferential pattern’s specification of what entails a sentence (or what is entailed by it) is assumed to be exhaustive; i.e. it is assumed that the sentence is entailed by (entails) *only* that which is specified by the pattern.

If this is the case, then it is enough to find a pattern behind all the rows of a table with T in the value column, or all those with F in this column – EA then takes care of the rest.

Consider disjunction. The above inferential pattern can be seen as recapitulated by three of the four rows of its truth table. Now assuming exhaustivity of this implies that  $S_1 \vee S_2$  is true for *no other* assignment of truth values to  $S_1$  and  $S_2$ , hence that for the truth-value pair  $\langle F, F \rangle$  it yields F. In other words, the set of pairs of truth values for which disjunction yields T is the *minimal* set of pairs containing all pairs with at least one component being T:

Example 1 (minimality):

$S_1$	$S_2$	$S_1 \vee S_2$
T	T	T
T	F	T
F	T	T
F	F	F

$$S_1 > S_1 \vee S_2$$

$$S_2 > S_1 \vee S_2$$

$$\text{if } S_1 > S \text{ and } S_2 > S, \text{ then } S_1 \vee S_2 > S$$

Analogously for implication, where the set of pairs of truth values for which the implication yields T is the *maximal* set of pairs not containing  $\langle T, F \rangle$ :

Example 2 (maximality):

$S_1$	$S_2$	$S_1 \rightarrow S_2$
T	T	T
T	F	F
F	T	T
F	F	T

$$S_1, S_1 \rightarrow S_2 > S_2$$

$$\text{if } S_1, S > S_2, \text{ then } S > S_1 \rightarrow S_2$$

<sup>9</sup> McCarty’s considerations resulted, in effect, into the conclusion that an intended model of what we say is always the *minimal* one, which led him to his concept of *circumscription*. See Hintikka (1988) for a further elaboration of this idea.

The conception of logical operators arrived at in this way is in fact that developed by Koslow (1992):

To any logical operator  $\phi$  there corresponds a condition  $\Phi$  such that the operator assigns to each pair (if the operator acts on pairs) the set of all items in the [inferential] structure that are the weakest members of all those that satisfy the condition.

Note that both the truth-functional viewpoint, the reduction of incompatibility to entailing everything, and the exhaustivity assumption lead to ‘desired’ results only if the language considered is rich enough. Consider a language with an explicitly defined truth-functional semantics. In this case, the stipulation  $S, \neg S > S'$  guarantees that the negation of T is F only if the language contains a contradiction. Similarly the stipulation that the disjunction of  $S_1$  and  $S_2$  is the *minimal*  $S$  for which  $S_1 > S$  and  $S_2 > S$  makes the disjunction denote the usual truth function only if there is an expression denoting the function within the language at all; otherwise it yields its ‘closest approximation’.

### From inferential roles to possible worlds

Let  $W$  be the set of all maximal consistent sets of statements (i.e. all such sets which do not entail a contradiction and are not contained in another noncontradictory set). Let us define the *intension* of  $S$ ,  $S'$ , in the following way:

$$S' = \{w \mid w \in W \text{ and there is an } X \in S^{\leftarrow} \text{ so that } X \subseteq w\}$$

In this way the inferential roles can be seen as giving rise to possible world semantics.

An objection to this construction might be that it yields us only a language-dependent notion of possible-world. However, as I argued elsewhere (Peregrin, 1995, §11.5), this is precisely what we should want. The point is that if we want to use possible worlds for the analysis of meanings, then we must exempt the language we analyze out of the possible worlds – the association of the expressions with their meanings must be kept fixed across the space of possible worlds. (Otherwise *every* statement would come out as true only contingently.) Hence the space of the possible worlds is to be limited by what is possible within the framework established by the language – by what the language ‘takes to be possible’.

But there is a deeper objection which concerns incomplete languages such as the languages of second-order logic (with standard semantics). In such a language, there may exist  $S_1, \dots, S_n, S$  such that  $S_1, \dots, S_n$  entail  $S$ , but not

$S_1, \dots, S_n > S$ , and hence not  $S_1^I \cap \dots \cap S_n^I \subseteq S^I$ . In this case we may feel we have ‘too many worlds’. Peano arithmetic, for instance, yields us a world in which, in contrast to the standard model (which is the only model of the *second-order* Peano arithmetic), the Gödel sentence is not true. Thus, inferential roles lead us directly always only to Henkin semantics. If inferentialism were correct, how could standard semantics, and ‘second-order’ consequence come into being?

It might seem that the inferentialist’s response to this must be that this only shows that second-order semantics is simply illusory. But this is not the case: there is a way of making sense even of this kind of semantics within the inferentialist framework; and it is even possible to see it, in cases like the Peano arithmetic, as *the* semantics (thus vindicating the intuition that the standard model is *the* model). We have admitted that inferential patterns may involve the exhaustivity assumption: and reading the Peano axioms as involving the EA is precisely what is needed to exclude the non-standard models. If we say that 0 is a number, the successor of every number is a number, and *nothing else is a number*; then obviously we have the unique specification of the standard natural numbers – it is the EA which is enough to take us from the Henkin to the standard semantics (cf. Hintikka, 1988; 1989).

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