Syllogistic reasoning with intermediate quantifiers

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Abstract

A system of *intermediate quantifiers* ("*Most S are P*", "$\frac{m}{n}$ *S are P*") is proposed for evaluating the rationality of human syllogistic reasoning. Some relations between intermediate quantifiers and probabilistic interpretations are discussed. The paper concludes by the generalization of the atmosphere, matching and conversion hypothesis to syllogisms with intermediate quantifiers. Since our experiments are currently still running, most of the paper is theoretical and intended to stimulate psychological studies.

Introduction

In 1908, Störring published a psychological study on simple inference processes. The last part of his paper was devoted to syllogistic reasoning (Störring, 1908; Politzer, 2004). He presented syllogism tasks like the following one to his subjects:

All p belong to class a,[1]
All a belong to class d.
Therefore ...

The subjects had to complete the conclusion. In 1935, the well known study of Woodworth & Sells (1935) was published. Most theoretical and empirical research work on syllogistic reasoning, though, was published during the last decades (Johnson-Laird & Byrne, 1994; Bacon et al., 2003; Newstead, 2003; Morley et al., 2004). Syllogisms build a prototypical task to investigate human reasoning in the field of monadic predicate calculus. Comparable prototypical tasks are Wasons's selection task in the field of propositional calculus and, more recently, also in modal logic. Similarly, classical modus-ponens-type tasks investigate parts of propositional calculus. Syllogisms are well-defined in standard logic and were studied in philosophy for more than two millennia. No wonder that they are considered as one of the benchmarks of human rationality.

During the last 50 years many new approaches to old logical problems were developed in philosophy, artificial intelligence, and linguistics. These non-standard approaches are often of special interest to psychology as they try to do more justice to practical reasoning than classical logic. Typical examples are nonmonotonic reasoning or the probabilistic treatment of conditionals.

[1] "Alle p gehören zur Gattung a" (Störring, 1908, p.78)

The present paper tries to exploit a not so well known non-standard development in syllogistic inference, namely intermediate quantifiers, for psychological purposes. The classical universal and the existential quantifiers in syllogisms are either too strict or too weak, respectively. On the one hand, the universal quantifier is too strict because it does not allow for *exceptions*. One simple counterexample falsifies an all-assertion. In everyday contexts exceptions are the rule, we reason with defaults or rules of thumb that hold *normally* or most of the time. Nonmonotonic reasoning formalizes reasoning with exceptions and withdrawing conclusions in the light of new evidence. Experiments in which nonmonotonic reasoning is investigated are reported in Pfeifer & Kleiter (2003, 2005, in press). On the other hand, the existential quantifier is too weak because it quantifies only over at least one individual. Such quantifiers hardly ever occur in everyday life reasoning. Quantifiers that—at least implicitly—actually occur in everyday life reasoning like "*most ...*", "*almost-all ...*", or "*90 percent ...*" are not expressible in classical syllogistics. Such quantifiers that lie "in-between" the existential and the universal quantifier are called *intermediate quantifiers*[2].

Table 1: The universal (A) and particular (I) affirmative, and the universal (E) and particular (O) non-affirmative moods of classical syllogisms and their predicate-logical (PL) form.

Mood	*Read*	*PL-Formula*
A	All S are P	$\forall x(Sx \to Px)$
I	At least one S is P	$\exists x(Sx \land Px)$
E	All S are not P	$\forall x(Sx \to \neg Px)$
O	At least one S is not P	$\exists x(Sx \land \neg Px)$

[2] Also known as "generalized quantifiers". Historically, the first system was proposed by Sir William Hamilton, his dispute with De Morgan is reprinted in (De Morgan, 1847). Generalized quantifiers have been developed in mathematics and logics (Mostowski, 1957; Lindström, 1966; Väänänen, 2004), artificial intelligence and computer science (Schwartz, 1997; Liu & Kerre, 1998; Novák, 2001), linguistics (Barwise & Cooper, 1981; van Bentham & ter Meulen, 1985; Gärdenfors, 1987; van der Does & van Eijck, 1991; Keenan & Westerståhl, 1997). Peterson's work is an improvement of Finch (1957) and is situated in the philosophy of language tradition.

Table 2: The four figures of syllogisms. S, M, and P are the subject, middle, and predicate term, respectively.

	Fig. I	*Fig. II*	*Fig. III*	*Fig. IV*
Major Prem.	MP	PM	MP	PM
Minor Prem.	SM	SM	MS	MS
Concl.	SP	SP	SP	SP

In the present paper we propose intermediate quantifiers as developed by Peterson (1985, 2000) as a promising candidate for investigating and evaluating human syllogistic reasoning. After a short introduction to classical syllogisms, we sketch the formal system of intermediate quantifiers and formulate for this system some (classical) hypothesis of human syllogistic reasoning. The discussion of some relations between intermediate quantifiers and probabilistic interpretations concludes the paper. Since we are presently running experiments, we cannot provide empirical data yet.

Classical Syllogisms

The classical syllogism is a two-premise-one-conclusion argument made by three of four sentence types, or *moods* (Table 1). The order of the predicates involved is regimented by the four *figures* (Table 2). This leads to 256 possible syllogisms,[3] of which 24 are *syllogistically* valid. From a predicate logical point of view, only 15 syllogisms are *predicate-logically valid* (Table 3). All 15 predicate logically valid syllogisms are also syllogistically valid. The reason is that in syllogistics *All S are P* implies *At least one S is P* (Some S are P), because it is implicitly assumed that the subject term Sx is not empty. This assumption is called "existential import". In predicate logic, $\forall x(Sx \to Px)$ does not entail $\exists x(Sx \wedge Px)$. The reason is well known: In predicate logic, formulae like $\forall x(Sx \to Px)$ can be "vacuously true". This is the case when there is no x such that x has the property S. Then, clearly $\exists x(Sx \wedge Px)$ is false (since $\neg\exists x Sx$ is assumed). However, if the existential assumption is made explicit, $\exists x(Sx \wedge Px)$ is a predicate-logically valid conclusion,

$$\forall x(Sx \to Px) \wedge \exists x Sx \quad \vdash \quad \exists x(Sx \wedge Px)\,.$$

The valid syllogisms got names like "Barbara" for mnemotechnic reasons.[4] The vowels in these names indicate the moods of the first and second premise, and the mood of the conclusion of the respective syllogism (in the order just stated) (Hughes & Londey, 1965). A list of valid syllogisms with their traditional names is given in Table 3.

[3] $4^3 = 64$ ways of constituting a two-premise argument (2 for the premises, 1 for the conclusion) by four moods (A, I, E, O). Multiply 64 by the four figures gives $64 \times 4 = 256$ possible syllogisms.

[4] The first mnemotechnic verses of valid syllogisms appeared in William of Sherwood's *Introductiones Logicam (Summulae)*, 13th Century (Kneale & Kneale, 1984, p. 231f.)

Table 3: Classical syllogisms that are predicate-logically (PL) and not predicate-logically valid.

	PL-valid		*Not PL-valid*	
Figure I	AAA	Barbara	AAI	Barbari
	AII	Darii	EAO	Celaront
	EAE	Celarent		
	EIO	Ferio		
Figure II	AEE	Camestres	AEO	Camestrop
	AOO	Baroco	EAO	Cesaro
	EAE	Cesare		
	EIO	Festino		
Figure III	AII	Datisi	AAI	Darapti
	EIO	Ferison	EAO	Felapton
	IAI	Disamis		
	OAO	Bocardo		
Figure IV	AEE	Camenes	AAI	Bramantip
	EIO	Fresison	AEO	Camenop
	IAI	Dimaris	EAO	Fesapo

Syllogisms with intermediate quantifiers

Intermediate quantifiers are quantifiers "between" the all quantifier and the existential quantifier. Examples of intermediate quantifiers are *Almost-all S are P*, *Most S are P*, *Many S are P* or quantifiers with *fractions*, $\frac{m}{n}$ *S are P*. As stated in the introduction, the universal quantifier is too strict and the existential quantifier is too weak and not appropriate to model human reasoning on *a priori* grounds. We therefore suggest to prefer intermediate quantifiers for modeling human syllogistic reasoning.

Intermediate quantifiers have hardly been investigated by psychologists. Exceptions are the logical rule-based approach by Guerts (2003) and Chater & Oaksford's (1999) Probability Heuristics Model. We will not discuss these approaches here. Studies on probability judgment as well can be close to studies on quantifiers with fractions.

Peterson (2000) provides algorithms to evaluate syllogisms with intermediate quantifiers. These algorithms are *correct* and *complete* with respect to arbitrarily many intermediate quantifier syllogisms ($\frac{1}{5}$ *S are P*, $\frac{2}{5}$ *S are P*, $\frac{m}{n}$ *S are P*, ...). For his interpretation of intermediate quantifiers consider the Venn diagram in Figure 1 and Table 4. Figures 2, 3, and Figure 4 list the valid syllogisms with the intermediate quantifiers *Almost-all S are P* (P), *Most S are P* (T), and *Many S are P* (K) and their non-affirmative versions *Almost-all S are $\neg P$* (B), *Most S are $\neg P$* (D), and *Many S are $\neg P$* (G), respectively. Syllogism ATK of Figure I, e.g., is, *All M are P* (major premise), *Most S are M* (minor premise), therefore *Many S are P* (conclusion). The validity of figures I, II, and IV can be directly inspected, since the intermediate quantifiers strengthen the premises or weaken the conclusion (solid or dashed arrows, respectively). Valid syllogisms of Figure III in the shaded boxes of Figure 3 are not derived trivially by strengthening or weakening. Consider, e.g., syllogism TTI of Figure III: *Most M are*

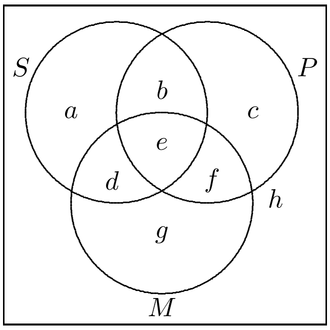

Figure 1: S, M, and P represent the subject, middle, and predicate terms, respectively. Each term represents a class of objects (the S-class, the P-class, and the M-class). $a, \ldots, h$ label the cardinality of the eight possible subclasses of objects.

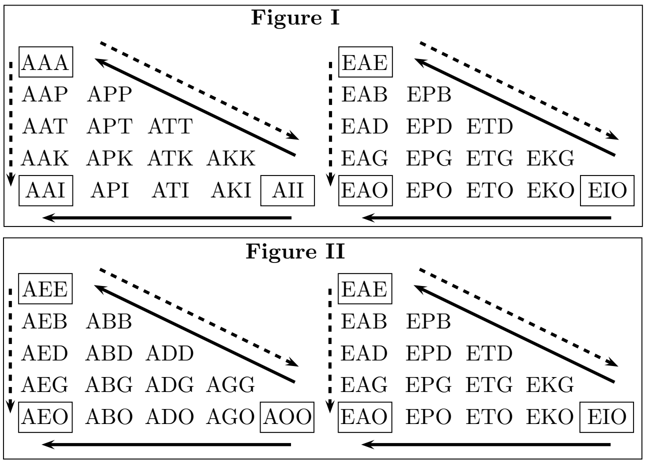

Figure 2: Valid syllogisms with intermediate quantifiers of Figure I and Figure II. Solid arrows indicate strengthening of the premises, and dashed arrows indicate weakening of the conclusions. The classical syllogisms are in boxes (Peterson, 1985, 2000, modified).

P, *Most* M *are* S, therefore, *At least one* S *is* P.

Finally, we note that Peterson's logic of intermediate quantifiers can easily be related to a probability interpretation based on relative frequencies.

Fractionate quantifiers and probability

Let $n = |S|$, $m = |S \cap P|$, and $n - m = |S \setminus P|$ denote the cardinalities of P, the intersection and the set difference, respectively. Peterson (2000) introduced *fractional* quantifiers of the form

More than $\frac{m}{n}$ *the* S *are* P, $(0 \leq m \leq (n-m) \leq n)$.

$\frac{m}{n}$ is the relative frequency (proportion, percentage) of P given S. Syntactically it is straightforward to rewrite the relative frequencies as conditional probabilities. Especially the subjective approach to probability theory (De Finetti, 1974; Coletti & Scozzafava, 2002) stresses the fact, that the formal rules to infer probabilities from a set of given probabilities are determined

Table 4: Semantical interpretation of moods involved in syllogisms with intermediate quantifiers (Peterson, 1985, 2000). The "where …" clause makes the existential import explicit. "$b + e$" denotes the cardinality of the intersection of S and P. "$\gg$" is read as "greatly exceeds". "F*" denotes the quantifiers with fractions. See Figure 1.

Mood	*Semantics*
A	All S are P: $a = 0$ and $d = 0$, where $(b \neq 0$ or $e \neq 0)$
E	No S are P: $b = 0$ and $e = 0$, where $(a \neq 0$ or $d \neq 0)$
P	Almost-all S are P: $b + e \gg a + d$, where $(b \neq 0$ or $e \neq 0)$
B	Almost-all S are $\neg P$: $a + d \gg b + e$, where $(a \neq 0$ or $d \neq 0)$
T	Most S are P: $b + e > a + d$, where $(b \neq 0$ or $e \neq 0)$
D	Most S are $\neg P$: $a + d > b + e$, where $(a \neq 0$ or $d \neq 0)$
K	Many S are P: $\neg(a + d \gg b + e)$, where $(b \neq 0$ or $e \neq 0)$
G	Many S are $\neg P$: $\neg(b + e \gg a + d)$, where $(a \neq 0$ or $d \neq 0)$
I	At least one S is P: $b \neq 0$ or $e \neq 0$
O	At least one S is $\neg P$: $c \neq 0$ or $f \neq 0$
F*	Exactly $\frac{m}{n}$ of the S are P: $(\frac{m}{n}$ of the S are $P)$ and $(\frac{n-m}{n} S$ are $\neg P)$, i.e., $(m(b+e) = (n-m)(a+d))$ iff $[(m(b+e) \geq (n-m)(a+d))$ and $((n-m)(a+d) \geq m(b+e))]$

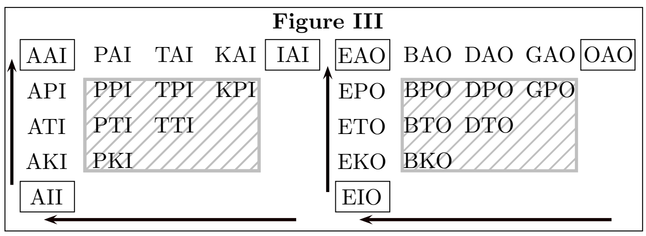

Figure 3: Valid syllogisms with intermediate quantifiers of Figure III. Solid arrows indicate strengthening of the premises. Syllogisms in the shaded boxes are not derived trivially by strengthening of the premises (Peterson, 1985, 2000, modified).

Figure IV		
AAI	AEE	EAO
↑PAI	AEB	↑EPO
TAI	AED	ETO
KAI	↓AEG	EKO
IAI	AEO	EIO

Figure 4: Valid syllogisms with intermediate quantifiers of Figure IV. Solid arrows indicate strengthening of the premises, and dashed arrows indicate weakening of the conclusions. The classical syllogisms are in boxes (Peterson, 1985, 2000, modified).

by *linear constraints*. These linear constraints hold of course also in other approaches to probability theory and correspond (in the finite case) to the Kolmogorov axioms. The probabilities may be given in the form of exact (point) probabilities or as imprecise probabilities (intervals). The method by which Peterson proves theorems is based on analog linear constraints (giving rise to in-equalities, finding upper and lower bounds etc.). Syntactically, Peterson's intermediate quantifiers are special cases of probability theory. His linguistic markers like "most", "many", "almost-all" etc. correspond to intervals of conditional probabilities. Some of the special cases he is struggling with would easily be subsumed into this "linear constraints" probability calculus. Syllogisms such as the TTI (Figure III),

$$.5 < p(P|M) \leq 1, \quad .5 < p(S|M) \leq 1\,,$$
$$\text{therefore:} \quad 0 < p(P|S) \leq 1\,,$$

where $p(M) > 0$, can be solved, for example, by linear programming. In the subjective approach of Coletti & Scozzafava (2002) zero-probabilities in the conditioning events are admissible. That is, if $p(B) = 0$, then $0 \leq p(A|B) \leq 1$, which is the completely uninformative unit interval.

As all syllogisms contain three variables, the complete specification of all possible combinations of binary truth-values by intermediate quantifiers would require seven numbers (the 8$^{\text{th}}$ is obtained by subtracting the sum of the others from 1). The premises of a syllogism gives only two of these values, and these often in the form of intervals only. Thus, the impact of the premises upon the conclusion must be weak. The conclusion often excludes just one value, zero or one, for example. Syllogistic inference is thus an excellent example of reasoning under the condition of partial knowledge.

Although relative frequencies and probabilities are clearly not the same, there are close relationships between both of them. Semantically intermediate quantifiers are a calculus of partially specified relative frequencies. Intermediate quantifiers specify objective properties of the external world. There is no randomness, there are no relative frequencies in the long run, no degrees of belief or similar pointers to the domain of probability, just relative frequency. There are many experimental studies on the processing of frequencies and proportions in animals and humans (Sedlmeier & Betsch, 2002). Moreover, Gigerenzer and his co-workers have stressed the point that human subjects are doing better when judgment under uncertainty tasks are phrased in a frequency format than when they are phrased in a probability format. Frequencies are easier to understand and to process than probabilities. The question arises, when do psychologists investigate human understanding of frequencies and when do they investigate human understanding of probabilities?

Psychological Predictions

The four best known psychological effects in the field of classical syllogisms are the atmosphere, matching, and conversion hypothesis, and the figure effect. They may be generalized in the framework of intermediate quantifiers as follows.

Atmosphere The atmosphere hypothesis (Woodworth & Sells, 1935) consists of two principles:

- *Quality:* If at least one premise contains a negation (O, E), then subjects prefer the conclusion that contains a negation, otherwise a conclusion is preferred that is not negated (I, A).
- *Quantity:* If at least one premise is particular (I, O), then subjects prefer a conclusion that is particular, otherwise it a universal conclusion (A, E) is preferred.

The generalization of the atmosphere hypothesis for the case of syllogisms with intermediate quantifiers is straightforward:

- *Quality*:* If at least one premise contains a negation (O, G, D, B, E), then subjects prefer the conclusion that contains a negation, otherwise a conclusion is preferred that is not negated (I, K, T, P, A).
- *Quantity*:* Let $Q(X)$ denote the quantity of a quantifier X. Then, subjects prefer as the conclusion the smallest quantity, whereas,

$$Q(\text{A}) = Q(\text{E}) > Q(\text{P}) = Q(\text{B}) > Q(\text{T}) =$$
$$= Q(\text{D}) > Q(\text{K}) = Q(\text{G}) > Q(\text{I}) = Q(\text{O})\,.$$

Matching The matching hypothesis (Wetherick, 1993) states that subjects prefer conclusions of the same type as the most *conservative* statement of the premises. The statement "*No S are P*" (E) is the most conservative because it says that *no* object that has the property S has the property P. The statement "*All S are P*" (A) is the least conservative, because it says that *all* objects that have the property S have the property P. Let $C(X)$ be the conservativity of statement X, then,

$$C(\text{E}) > C(\text{I}) = C(\text{O}) > C(\text{A})\,.$$

Since both statements, I and O, speak about at least one object, they are equally conservative. Thus, the generalization of the matching hypothesis to syllogisms with intermediate quantifiers is straightforward:

$$C(\mathrm{E}) > C(\mathrm{B}) > C(\mathrm{D}) > C(\mathrm{G}) > C(\mathrm{I}) =$$
$$= C(\mathrm{O}) > C(\mathrm{K}) > C(\mathrm{T}) > C(\mathrm{P}) > C(\mathrm{A})\,.$$

Conversion The conversion hypothesis (Chapman & Chapman, 1969) states that subjects erroneously the terms in statements that involve the universal quantifier. E.g., *All S are P* is erroneously represented as *All P are S*. The only quantifier where conversion of the terms in not problematic is the existential quantifier:

$$\exists x(Px \wedge Sx) \quad \text{is logically equivalent to} \quad \exists x(Sx \wedge Px)\,.$$

This holds because of the commutativity of the conjunction. The conversion hypothesis can be investigated in the framework syllogisms with intermediate quantifiers, by asking whether subjects misrepresent, e.g., *Almost-all S are P* by *Almost-all P are S*.

Validity and Figure Subjects are performing well at determining validity of classical syllogisms: the 24 valid syllogisms are judged as valid more often (51% of the time on average) than invalid syllogisms (11%; Chater & Oaksford (1999)). We hypothesize that subjects are even better in determining the validity of syllogisms with intermediate quantifiers, since intermediate quantifiers are closer to everyday reasoning than classical quantifiers. An important factor for the difficulty of determining validity is the figure type: syllogisms of Figure I are the easiest, of Figure IV the hardest, and those of figures II and III are in between (Guerts, 2003, p. 229). It is an open question whether subjects in conditions with intermediate quantifiers solve Figure I syllogisms better than Figure IV syllogisms.

Concluding Remarks

Intermediate quantifiers are not restricted to syllogistic reasoning. Insofar the linear constraints are satisfied, intermediate quantifiers can be applied to other inference schemes with more than three variables. Syllogisms are, of course, an interesting case for studying intermediate quantifiers. We showed how the traditional atmosphere, matching and conversion hypothesis are formulated in the framework of intermediate quantifiers. We are currently running experiments on syllogisms with intermediate quantifiers to investigate some of these claims.

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