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VARIABLES AND ATTITUDES

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Abstract. The phenomenon of quantification into attitude ascriptions has haunted broadly Fregean views, according to which co-referential proper names are not always substitutable *salva veritate* in attitude ascriptions. Opponents of Fregeanism argue that a belief ascription containing a proper name such as 'Michael believes that Lindsay is charitable' is equivalent to a quantified sentence such as 'there is someone such that Michael believes that *she* is charitable, and that person is Lindsay'. They conclude that the semantic contribution of a name such as 'Lindsay' is the same as the semantic contribution of a variable under an assignment, which these opponents suggest is merely the object assigned to that variable. However, renewed interest in variables suggests that they make a more complicated contribution to the semantic processing of sentences that contain them. In particular, a variable contributes both an assignment-unsaturated and an assignment-saturated semantic value. I use this dual role of the semantics of variables to develop a response to the argument from quantifying in. I take as my point of departure Cumming's (2008) view that an attitude ascription relates the subject of an attitude to the assignment-unsaturated semantic value of an open sentence. I argue that this approach fails. I propose an alternative, according to which the truth of a belief ascription depends on both the assignment-saturated and the assignment-unsaturated semantic value of an open sentence. This approach reverses standard assumptions concerning the relation between quantification and substitution.

Millianism is the view that names and other singular terms such as indexicals and demonstratives are directly referential, contributing nothing more than their referents as inputs to the semantic processing of sentences that contain them. This entails that sentences differing only by the substitution of co-referential proper names are synonymous. Many take this to be a strike against Millianism, since there are contexts in which the substitution *salva veritate* of co-referntial proper names looks implausible. The worry is most pressing in the case of attitude ascriptions. For instance, even if 'Lindsay' and 'Nellie' refer to the same individual in the relevant context, it is tempting to suppose that (1) and (2) may differ in truth-value at a time when Michael is ignorant of the fact that Lindsay has two names.

(1) Michael believes that Lindsay is charitable.
(2) Michael believes that Nellie is charitable.

Yet, Millianism entails that (1) and (2) have the same semantic content and thus agree in truth-value.[1]

Anyone who rejects this Millian conclusion must respond to an argument developed by Quine (1976a) and Kaplan (1989) and more recently articulated by Salmon (1986: 1-9, §9.2) and Soames (2010: §1.16) for the claim that a referential interpretation of names best explains the phenomenon of quantification into modal and epistemic contexts. I will focus on epistemic contexts, since the rigidity of proper names under modal operators is largely a settled matter. Millians suggest that (1) entails (3) and its typical regimentation (3a).

(3) There is someone such that Michael believes that *she* is charitable, and that person is Lindsay.
(3a) There is an x such that x=Lindsay and Michael believes that x is charitable.

In these sentences, the name 'Lindsay' occurs outside of the scope of the belief ascription. Inside the scope of the belief ascription is an anaphoric pronoun 'she' in (3) and a variable x in (3a).[2] The semantics of anaphoric pronouns are often assimilated to the semantics of variables, so I will focus on (3a). Sentence (3a) is true just in case there is an assignment function σ that assigns the variable x to Lindsay and the embedded sentence 'Michael believes that x is charitable' is satisfied by σ. According to the Millian, this embedded sentence is satisfied by σ provided that Michael stands in the believing relation to the semantic content of the sentence 'x is charitable' relative to σ. Millians argue that – relative to assignment σ – the variable x contributes nothing more than its value, Lindsay, to the semantic

[1] Of course, some Millians posit additional structure in the attitude ascription to block these results. See Crimmins and Perry (1989).
[2] I omit quotes for expressions containing only formalism. I also will frequently use simple quotation for corner quotes.

processing of sentences that contain it. As Kaplan (1989: 484) says, "free variables under an assignment of values are the paradigms of [...] *directly referential* terms. In determining a semantic value for a formula containing a free variable we may be given a *value* for the variable [...] but nothing more." Since (1) entails (3a), the singular term, 'Lindsay', must contribute at least its referent to the semantic processing of (1). Millians suggest that it overly complicates the semantics if the name contributes more than the variable. They conclude that the name contributes its referent and nothing more to the semantic processing of the belief ascription.

The orthodox reaction to this argument proposes that a name contributes more than an anaphoric pronoun or variable does to the semantic value of sentences that contain it. Attitude ascriptions whose embedded clauses contain variables or anaphoric pronouns in place of names are then construed as incomplete specifications of the attitudes they ascribe using a trick for "quantifying in" learned from Kaplan (1968).

However, renewed interest in the semantics of variables holds the promise of an alternative reaction to the Millian's argument. In particular, it has been observed that in addition to its semantic value relative to an assignment a variable also has an *assignment-unsaturated* semantic value. A variable x designates different individuals relative to different assignment functions. This assignment-unsaturated semantic value enters into the compositional processing of sentences containing *bound* variables. For instance, the truth or falsity of $\forall x \Phi(x)$ relative to an assignment σ depends on whether $\Phi(x)$ is true or false relative to every x-variant of σ. In a sense, variables contribute two semantic values to sentences that contain them: (i) the value of the variable relative to whatever assignment function the sentence takes as input and (ii) the potential to designate different individuals relative to different assignment functions. It has recently been popular to appeal to these two aspects of a variable to try to account for the semantics of proper names and other singular terms.[3]

In this paper, I develop a response to the problem of quantifying in that makes no use of Kaplan's machinery. I take as my point of departure Sam Cumming's (2008) attempt to use these two aspects of the semantics of variables in order to develop a semantics for attitude ascriptions. Very roughly, Cumming's proposal says that an attitude ascription relates the subject of the attitude to the assignment-unsaturated semantic value of an open sentence, rather than its assignment-saturated semantic value. The semantic content of a proper name such as 'Lindsay' or 'Nellie' can then be identified with the content of a variable without identifying the contents of (1) and (2). But – as I show – Cumming's approach fails: whether 'S believes that Φ' is satisfied by assignment σ depends, in part, on the interpretation of Φ under σ. Otherwise, the truth of the belief ascription loses its connection to the truth conditions of the belief ascribed. Instead, I argue that the truth-value of an attitude ascription depends on both the assignment-saturated and the assignment-unsaturated semantic value of the sentence embedded in the that-clause. It will turn out that belief is a relation to what I will call a *quasi-open* proposition. This approach reverses standard assumptions concerning the relation between quantification and substitution. If I am correct, then the fact that Michael believes of Lindsay that she is charitable does not entail that he believes of someone who happens to be identical to her (namely, Nellie) that she is charitable.

1. The Semantics of Variables

According to the standard semantics for first-order logic since Tarski, sentences are not true or false *simpliciter*. Rather, they are satisfied or not by functions that assign values to all free variables.[4] This assignment sensitivity also plays a role in

[3]Most contemporary philosophers who identify the contribution of some class of singular terms with that of free variables trace elements of their view to Heim (1982) and Kamp (2002). Sometimes appeal is made to dynamic semantics developed in Groenendijk and Stokhof (1991). Dever (1998) defends a different version of the thesis that names are free variables.

[4]Tarski (1983a: 189-191) reasoned from the fact that the quantifiers $\exists x$ and $\forall x$ attach to syntactic objects of the same kind as standard truth-functional sentential operators such as negation $\neg$ and disjunction $\vee$. For this reason, Tarski argued that – even for the purposes of extensional semantics

natural language semantics.[5] However, I focus on the narrow language of first-order modal logic with a belief operator. The language includes the following expressions and sentences.

Terms: A countable set of variables, x_1, x_2,...
Predicates: A countable set of n-ary predicates, R^1_n, R^2_n,...
Sentences: If t_1,...,t_n are n terms and R_n is an n-ary predicate, then $R_n t_1 \ldots t_n$ and $t_1 = t_n$ are sentences. If Φ and Ψ are sentences and t is a term, then $\Phi \vee \Psi$, $\neg\Phi$, $\forall x\Phi$, $\exists x\Phi$, $\Box\Phi$, $\Diamond\Phi$, and $B_t\Phi$ are sentences.

I will briefly sketch a standard semantics to fix ideas. The semantics of the belief operator $B_t\Phi$ will be left for later.

Let model $M =< D, W, I >$ have domain D, set of worlds W, and interpretation function I. In the following semantic clauses, a variable contributes its assignment-saturated value to the semantic value of sentences that contain it.[6]

Terms: $||t||_{\sigma,w,M} = \sigma(t)$
Predicates: $||R_n||_M = I(R_n)$, which is a function from worlds drawn from W onto n-tuples of individuals drawn from the domain D.
Formulas: $\vDash_{\sigma,w,M} R_n t_1, ..., t_n$ iff $< ||t_1||_{\sigma,w,M}, ..., ||t_n||_{\sigma,w,M} > \in ||R_n||_M(w)$
=: $\vDash_{\sigma,w,M} t_1 = t_n$ iff $||t_1||_{\sigma,w,M} = ||t_n||_{\sigma,w,M}$
$\vee$: $\vDash_{\sigma,w,M} \Phi \vee \Psi$ iff $\vDash_{\sigma,w,M} \Phi$ or $\vDash_{\sigma,w,M} \Psi$
$\neg$: $\vDash_{\sigma,w,M} \neg\Phi$ iff $\nvDash_{\sigma,w,M} \Phi$
$\Box$: $\vDash_{\sigma,w,M} \Box\Phi$ iff for every $w* \in W$, $\vDash_{\sigma,w*,M} \Phi$
$\Diamond$: $\vDash_{\sigma,w,M} \Diamond\Phi$ iff for some $w* \in W$, $\vDash_{\sigma,w*,M} \Phi$

Other operators such as conjunction & and the conditional $\Rightarrow$ can be defined in the standard way.[7]

– the semantic value of a sentence cannot be its truth-value. In particular, the truth-value of a quantified claim $\forall x\Phi(x)$ depends on the truth-value of $\Phi(x)$ relative to every possible value of x. Since quantifiers may attach to sentences containing an arbitrary number of variables, Tarski concluded that a sentence can only be true or false relative to an assignment (or a sequence) that assigns a value to every variable.

[5]This role is sometimes disguised by the fact that each quantifier's dual roles of variable binding and generalization are divided between quantifiers proper and λ-abstraction operators. See Heim and Kratzer (1998: §§5.2.2-5.2.3). A nice discussion can be found in Rabern (2012).

[6]Since this paper does not concern modality, I am using rather unsophisticated characterizations of the modal operators.

[7]If one prefers a semantic theory that assigns a single semantic value to all entities rather than semantic values relative to an assignment function, one could also characterize the fixed meaning of a variable by the axiom $||t||_{M,w} = \lambda\sigma(\sigma(t))$. The details don't matter for my discussion. See Lewis (1998a).

The fact that a variable has an assignment-unsaturated meaning emerges in the case of variable binding operators, specifically quantifiers. Let $\sigma[x/d]$ be the assignment function that agrees with σ for all values, except possibly x, which is mapped to object d. Then the quantifiers can be characterized as follows.

∃: $\vDash_{\sigma,w,M} \exists x\Phi$ iff for some $d \in D$, $\vDash_{\sigma[x/d],w,M} \Phi$
∀: $\vDash_{\sigma,w,M} \forall x\Phi$ iff for every $d \in D$, $\vDash_{\sigma[x/d],w,M} \Phi$

A universal claim such as $\forall x\Phi(x)$ is satisfied by a sequence σ just in case the embedded sentence $\Phi(x)$ is satisfied by every x-variant of σ. On the other hand, an existential claim such as $\exists x\Phi(x)$ is satisfied by a sequence σ just in case the embedded sentence $\Phi(x)$ is satisfied by some x-variant of σ. In this sense, quantifiers have the effect of shifting the assignment functions relative to which an embedded sentence is to be evaluated. The truth-value of a quantified claim depends on the assignment-unsaturated semantic value, its ability to designate different individuals relative to different assignments.

2. Variables and Other Singular Terms

Renewed appreciation for this dual life of variables has led some to regiment names and other singular terms as variables and yet deny that their position has the unwelcome consequences of Millianism. I focus on names. On this view, (4) can be regimented as (4a).

(4) Nellie was adopted.
(4a) Ax_n

There are a number of complications that arise from such a regimentation. The most pressing challenge arises from the fact that (4a) is an open sentence and unlike a closed sentence whose truth-value is fixed by the world of evaluation, has different truth-values relative to different assignment functions. It is therefore unclear how to assess the utterance of a sentence like (4a) for correctness. What supplies the missing parameter?[8]

One approach appeals to the fact that utterances such as (4) always occur as parts of larger discourses such as (5) and its regimentation (5a).

(5) Nellie was Michael's sister. Nellie was adopted.
(5a) Sx_n, Ax_n

On this approach, the multiple sentences are collected into a single representation which has existential truth conditions such as $\exists x_n(Sx_n \& Ax_n)$. This seems to be Cumming's (2008: §2.3) approach to cases of what he calls bound names. Discourse Representation Theory (DRT) deploys a similar treatment for all names using a construction algorithm to generate a representation that has existential truth conditions.[9]

Another approach – preferred by Cumming (2008: §3.1) for "free occurrences" of names – says that the assignment function is provided by context.[10] On this view, the assignment function is an index supplied by the context as part of the semantic processing of the sentence. A more radical view along these lines is developed by Dever (1998: §2.3.2.2, §2.3.2.4) who suggests that the relevant assignment function is not required for semantic processing, but results from the genuinely post-semantic process of determining speaker meaning.[11]

[8] I note, following Dever (1998: §2.3.2.4.3.1) and Cumming (2008: §3.3), that variables are rigid designators on this semantics, in the sense that the interpretation of a variable varies only with the assignment function and not with the possible world. Given a fixed assignment, the interpretation of a variable is fixed. Thus, $\forall x, y(x = y \Rightarrow \Box x = y)$ is true in any model and world. Though variables have assignment-saturated semantic values, they do not have world relative semantic values. As a consequence, treating names and variables as making semantic contributions of the same kind poses no obvious challenge to the thesis that names are rigid designators. Of course, this result is an artifact of two decisions. First, we have chosen not to take modal operators as variable binding operators. Second, we have chosen not to relativize assignment functions to worlds as might be suggested by Carnap (1947/1988: §§10, 40).

[9] Kamp (2002).

[10] See also Rabern (2012).

[11] A final approach is employed by dynamic semanticists. They modify the semantic role of sentences so that they function as transitions between sets of assignment functions within a discourse. The discourse will be regarded as correct provided that there is some assignment function such that it yields an output when it is entered as an input to the conjunction of the sentences of the discourse taken in order. Though my sympathies lie with the dynamic view, I will set it aside for the purposes of this paper. Rather, I will follow Cumming in assuming that context supplies an assignment function fixing the assignments of the free variables.

3. Attitudes as Relations to Assignment-Unsaturated Meanings

According to the view that names function as variables, sentences (1) and (2) are to be regimented as (1a) and (2a), respectively.

(1) Michael believes that Lindsay is charitable.
(1a) $B_m Cx_l$
(2) Michael believes that Nellie is charitable.
(2a) $B_m Cx_n$

I assume that the contextually supplied assignment function, σ, assigns both x_l and x_n to Lindsay. To avoid complications, I represent the subject of an ascription by subscripting a closed singular term such as m regimenting 'Michael'.

The important question is whether this regimentation and the accompanying new appreciation for the semantic role of a variable can be used to differentiate the truth conditions of (1) and (2). Millians would argue that no progress has been made. It follows from their view that (1) and (2) are true just in case (1a) and (2a) are true an under assignment σ that assigns both x_l and x_n to Lindsay. Oversimplifying considerably, one may model an agent's doxastic state as a set of worlds compatible with how she takes reality to be.[12] A belief ascription such as $B_m Cx_l$ is true under assignment σ just in case Cx_l is true under σ for every world w in the doxastic state of $\sigma(m)$. But, Cx_l and Cx_n determine the same set of worlds under σ. So, given any assignment function that assigns x_l and x_n to the same individual, (1a) and (2a) will be true in the same possible worlds.

The crucial assumption in the Millian's reasoning is that in assessing whether a belief ascription, $B_m Cx_l$, is satisfied by assignment function σ, one must determine whether the agent stands in a relation to the content of the embedded sentence, Cx_l, *under the very same assignment function,* σ. However, the dual semantic features of variables reveal that sometimes we must assess the satisfaction of a

[12] The semantics constructed above assigns truth conditions to sentences relative to various parameters such as a world, an assignment function, and a context (though I will suppress the last of these), rather than assigning structured meanings to a sentence relative to these parameters. I do so purely for convenience, and everything I say could, with some effort, be translated into a framework invoking structured meanings. One version of this framework is developed in Soames (2010: chapter 5).

whole sentence under an assignment σ in terms of the satisfaction of its components under a different assignment. In particular, $\exists x Fx$ is satisfied by σ just in case the embedded sentence Fx is true under $\sigma[x/d]$ for some $d \in D$.

Cumming (2008: 545-6) uses this fact to challenge the crucial assumption. According to Cumming, a belief ascription relates a subject not to a proposition – modeled as a set of worlds – but to an *open proposition*, a set of world-assignment pairs. A belief ascription 'S believes that Φ' is true just in case Φ is true in every world-assignment pair in the agent's belief box. The doxastic state of an agent x in a world w is modeled as a set of world-assignment pairs, $DOX(x, w)$. Cumming's (2008: 550-1) semantic clause for belief ascriptions – presented with only minor notational changes – is as follows:

BELIEF: $\vDash_{\sigma,w,M} B_i\Phi$ if and only if for every $< w*, \tau > \in DOX(||i||_{\sigma,w,M}, w)$, $\vDash_{\tau,w*,M} \Phi$

BELIEF allows us to differentiate the satisfaction conditions of (1a) and (2a). (1a) $B_m Cx_l$ is satisfied by assignment σ just in case every world-assignment pair in Michael's belief box makes true Cx_l. Similarly, (2a) $B_m Cx_n$ is satisfied by assignment σ just in case every world-assignment pair in Michael's belief box makes true Cx_n. But there are assignments relative to which the variables x_l and x_n have different values. Therefore, it is possible that there is a pair $< w, \tau >$ in Michael's doxastic state such that x_l is assigned to an individual by τ who is in the extension of C in w, but x_n is assigned to an individual who is not in the extension of C. Thus, Cumming's view makes it possible that Michael believes that Lindsay is charitable without believing that Nellie is charitable, even if Lindsay is Nellie.

Of course, this brief characterization leaves open a number of questions, including: what does it mean to believe an open proposition? In his informal discussion, Cumming explains the possibility of believing open proposition in terms of an individual's joint attitudes towards the world and towards the salient function that assigns referents to the terms available in the context. Each world-assignment pair in an agent's doxastic state represents a way the world could be that is compatible with the agent's belief set together with an assignment of referents to her terms compatible with her beliefs.

Though I do not intend to make an issue of this, I am uncomfortable with the meta-linguistic explanation of the agent's beliefs. I think it would be better to understand the fact that variable x_l associated with the name 'Lindsay' may designate different individuals relative to different assignment functions as representing the fact that Lindsay *may* be each of these individuals, where this '*may*' reflects epistemic possibility. Thus, the fact there is an assignment in Michael's belief box that maps x_l and x_n to different individuals merely reflects the fact that Michael is unsure of whether *Linsday* and *Nellie* are the same person. However, the issues that I will raise can be separated from the informal understanding of the formal tools used to model an agent's belief set.

3.1. **A Problem with Quantifying In.** On Cumming's semantics, whether an attitude ascription such as (1a) $B_m Cx_l$ is satisfied by a sequence σ depends only on whether for every world-assignment pair $< w, \tau >$ in Michael's belief box, Cx_l is true in world w and assignment τ. The value of the variable x_l on the input assignment, σ, is irrelevant to assessing the satisfaction conditions – and thus the truth – of the attitude ascription. So whether Michael believes Cx_l depends only on whether he stands in a relation to the assignment-unsaturated satisfaction semantic value of the open sentence Cx_l. It does not depend on who x_l is.

In other words, 'believes' is a variable binding operator. The Millian held that the variable x_l is free in the sentence (1a) $B_m Cx_l$. By way of contrast, x_l is a bound variable according to Cumming's semantics, even though it is not bound by an explicit quantifier. The variable is bound by the belief operator itself, as are all free variables in a sentence embedded under a belief ascription.

As a result, a variable within the scope of a belief ascription cannot be not bound by any quantifiers outside of the scope of the belief ascription. Recall (3a):

(3a) There is an x such that x=Lindsay and Michael believes that x is charitable.

After the initial quantifier, 'there is an x such that', variable x occurs two more times in this sentence. In the first occurrence, it is bound by the quantifier 'there is'. Thus, the sentence is true just in case there is an assignment function σ that maps x to Lindsay and σ satisfies 'Michael believes that x is charitable'. This, in turn, depends on whether for every $< w, \tau >$ in Michael's belief box, $< w, \tau >$ makes true 'x is charitable'. The value of x according to σ plays no role in assessing whether σ satisfies 'Michael believes that x is charitable'. Suppose, for instance, that 'Michael believes that x is charitable' is true under the contextually supplied assignment function σ, assigning the variable x to, say, Tobias. It will then also be true under an assignment function that assigns x to Lindsay. Thus, it will follow that (3a) is true. This means that (3a) is not a good regimentation of (3) 'there is someone such that Michael believes that *she* is charitable, and that person is Lindsay'.

A comparison might be helpful. In the sentence (6), the occurrences of x in Fx and in Gx are bound by different quantifiers.

(6) $\exists x(Fx \& \exists x Gx)$

According to the standard semantics for first-order logic, (6) is true relative to assignment σ just in case there is an individual who is F and there is an individual (not necessarily identical to the first) who is G. It does not require the existence of an individual who is both F and G. The second occurrence of the existential quantifier "resets" the assignment function breaking any semantic connection between the two occurrences of x. Analogously, in (3a) the occurrence of x under the scope of the belief ascription is bound by a different operator than the occurrences of x that precede it.

Cumming (2008: 548-9, footnote 48) is admirably clear about this problem. He considers the sentence 'every man thinks he is wise'. Regimenting this sentence using the material provided so far results in the following.

(7) $\forall x(Mx \Rightarrow B_x Wx)$

On Cumming's semantics, this will be true on an assignment σ just in case every man believes the same open proposition: that x is wise. In particular, if there is a single individual$_x$ – let's again say Tobias – such that each man$_y$ believes that this individual$_x$ is wise, then it will follow on Cumming's semantics that each man$_x$ believes that he$_x$ is wise. There is no coordination between the occurrences of x outside of the belief ascription and the x occurring within the belief ascription.

Cumming tries to solve this and analogous problems by distinguishing *de dicto** and *de re** belief ascriptions. Sentences such as (1a) and (1b) are *de dicto** belief ascriptions insofar as they require Michael to have a belief relating to a specific open proposition: the proposition that x is charitable. By way of contrast, we want to be able to say that Michael believes of a specific individual that she is charitable. Relatedly, we want to be able to say that each man believes of himself that he is wise, where each man's belief is true under different conditions (world-assignment pairs) from the others' beliefs.

*De re** ascriptions are meant to regiment English belief ascriptions that contain pronouns anaphoric on noun phrases outside of the scope of the ascription, such as 'Michael believes of Lindsay that she is charitable'. Cumming understands the statement that Michael believes of Lindsay that she is charitable as requiring that Michael believes an open proposition expressed by a sentence of the form 'α is charitable' for some variable α that designates Lindsay. He introduces Greek letters such as α as metalinguistic variables that range over object-level variables. Very roughly, the substitutional quantifier $\exists\alpha\Phi$ is true if $\Phi[\alpha/x]$ is true for some variable x.[13] Cumming regiments a sentence such as 'Michael believes of Lindsay that she is charitable' as follows.

(8) $\exists\alpha(\alpha = x_l$ and Michael believes that α is charitable).

This sentence is true on an interpretation σ just in case there is a variable α such that the interpretation of that variable is co-referential with x_l and Michael believes

[13]Following Cumming, I use $\Phi[\alpha/x]$ to designate the result of substituting x for α in Φ.

the open proposition expressed by 'α is charitable'. Similarly, 'every man believes of himself that he is wise' is regimented as:

(9) $\forall x(Mx \Rightarrow \exists\alpha(\alpha = x \& B_x W\alpha))$

This sentence is true relative to an assignment σ just in case for every x-variant of σ assigning x to a man, there is a variable α co-referential with x such that the referent of x believes the open proposition expressed by 'α is wise'. Cumming (2008: 548-9, footnote 48) points to some problem cases for this strategy, but nonetheless seems to take his treatment of the core cases to be adequate.[14]

3.2. **The Problem Re-emerges.** *De re** belief ascriptions only partially specify the beliefs they ascribe. They assert that an individual believes an open proposition expressed by an open sentence containing some variable α, but they do not require the agent to believe the open proposition expressed by a sentence containing any particular variable. Thus, Cumming's approach is a version of the analysis of quantification into attitude constructions developed in Kaplan (1968). This appeal to quantification over variables in order to capture *de re** readings is disappointing. On any version of the Kaplanian strategy, quantification into attitude contexts functions differently from quantification in other contexts. At very least, the bound pronoun inside scope of the attitude ascription must somehow be moved outside and linked with a new variable that occupies its place within the attitude context. One might have hoped that the assignment-saturated and assignment-unsaturated semantic values of variables could adequately resolve the issues concerning quantification into attitude contexts on their own. The addition of quantification over variables dashes this hope.

But one might think think: at least this mechanism allows him to avoid some objections to his account. However, I will now argue that it serves to exacerbate

[14]One might wonder whether Cumming's regimentation (9) even predicts the correct truth conditions of 'every man believes that he is wise'. Suppose, once again, that the variable x designates Tobias on the default assignment and that – as a matter of fact – every man believes that Tobias (as represented under variable x) is wise. Then it will follow that so long as x designates a man under assignment function τ, the sentence $B_x Wx$ will be true. But then, $x = x \& B_x Wx$ will also be true under τ, as will $\exists\alpha(\alpha = x \& B_x W\alpha)$. It is a short step from this to (9). Similar arguments are given in the next section, where I attempt to diagnose the source of the problem.

existing tensions. I have shown that, for Cumming, a belief ascription binds any free variables in the embedded sentence. Thus, a belief report $B_t\Phi(x)$ is a closed sentence (provided that t is closed). Assessing this belief ascription relative to any two assignment functions delivers the same result. But, if a belief ascription relates a subject to an open proposition, then the truth-value of the proposition believed has two parameters: a world and an assignment function. The world is – as usual – the world in which the agent has the belief. But where does the assignment function come from?

Cumming seems to propose that the truth-value of the belief ascribed in a context is determined by the assignment function supplied by the sentential context. This emerges, in particular, when he suggests that a *de re** belief report asserts a relation between an agent and a "closed proposition":

> [D]e re* belief connects up in a nice way to what I have been calling "closed" propositions. The truth condition for ['Biron thinks *de re** that Hesperus is visible'] establishes a relation between Biron and the coarse-grained, closed proposition that Venus is visible. ['Biron thinks *de re** that Hesperus is visible'] says that Biron stands in the belief relation to an open proposition (of a particular form) whose projection at the *contextual assignment* is the set of worlds at which Venus is visible.[15]

Thus, the assignment function given by the immediate sentential context of a belief ascription provides the assignment function parameter for assessing the truth of the belief ascribed by that ascription. As a result, the alethic features of the belief ascribed – including what the belief is about – depend on the context in which the belief is ascribed.

So, *what an agent believes* is fixed for any assignment function, but the truth-value of the belief depends on which assignment function is operative. There are, of course, forms of relativism which accept a structure of this sort. But unlike more benign forms of relativism, Cumming's view entails that whether an individual has a given belief is fixed for any input assignment function, while the individuals believed to be thus-and-so depend on the input assignment function.

[15]Cumming (2008: 547-8). Emphasis added.

To see the problem, suppose that Michael believes of an individual that she is charitable and that his belief is false. That is, assume Michael believes of an uncharitable individual that she is charitable. This can be regimented by (10).

(10) $\exists x(\neg Cx \& \exists\alpha(\alpha = x \& B_m C\alpha))$

(10) requires the existence of an uncharitable individual such that for some variable y that designates this uncharitable individual, Michael believes the open proposition expressed by 'y is charitable'. So in order for (10) to be true under σ, there must be some *de dicto** belief ascription involving y such as (11) that is also true under σ.

(11) $B_m Cy$

Recall that $B_m Cy$ is closed and is therefore true under any assignment whatsoever, given that it is true under σ.

Suppose now that there is at least one charitable person in the universe. We can represent this as:

(12) $\exists z Cz$

I claim that it follows from (11) and (12) that *Michael believes of the charitable individual that she is charitable.* One regimentation of this claim is:

(13) $\exists y(Cy \& \exists\alpha(\alpha = y \& B_m C\alpha))$

In particular, I will now argue that this disastrous result is true on any assignment function whatsoever. (I present a more rigorous version of this argument in the appendix.)

Let τ be an arbitrary assignment function. (13) is true according to τ just in case (14) is true for some y-variant of τ.

(14) $Cy \& \exists\alpha(\alpha = y \& B_m C\alpha)$

Because (12) is true, we know that there is some individual $d \in D$ who is charitable. Thus, we know that Cy is true according to $\tau[y/d]$.

But we can also show that $\exists\alpha(\alpha = y\&B_mC\alpha)$ is true on $\tau[y/d]$. Recall that B_mCy is true on any assignment function whatsoever, including $\tau[y/d]$. The formula $y = y$ is also true on any assignment function whatsoever, including $\tau[y/d]$. Thus, their conjunction $y = y\&B_mCy$ is true on $\tau[y/d]$. According to Cumming (2008: 551), $\exists\alpha(\alpha = y\&B_mC\alpha)$ is true relative to $\tau[y/d]$ just in case the result of replacing α in $\alpha = y\&B_mC\alpha$ with some variable is true relative to $\tau[y/d]$. But $y = y\&B_mCy$ – which I've just established to be true relative to $\tau[y/d]$ – is the result of replacing α in $\alpha = y\&B_mC\alpha$ with y. Thus, it follows that $\exists\alpha(\alpha = y\&B_mC\alpha)$ is true relative to $\tau[y/d]$. So, I have established that both Cy and $\exists\alpha(\alpha = y\&B_mC\alpha)$ are true under $\tau[y/d]$. We can infer that (14) $Cy\&\exists\alpha(\alpha = y\&B_mC\alpha)$ is true under $\tau[y/d]$. And from this, it follows that (13) is true under arbitrary τ.

This result is utterly disastrous. We have inferred from Michael's falsely believing of someone that she is charitable, that he also truly believes of someone that she is charitable. So just by having a false belief that one individual is charitable, Michael thereby has the true belief of a different charitable individual that this individual is charitable. We should not be able to infer that Michael has this very specific true belief from the fact that he has a false belief that someone is charitable. So, the account should be rejected.[16]

Here is another example that shows the extent of the problem. Suppose that George believes that Lucille is his wife, or B_GWx_L. I will now show that we can infer that for every individual identical to, say, Vladimir Putin, George believes of that individual that he is George's wife. We know that the sentence B_GWx_L is closed, so it is true under every assignment function. But that means it is true under any assignment function that assigns x_L to Putin. From this, we can infer that for every individual identical to Putin, there is a variable α designating that individual such that George believes the open proposition expressed by the sentence 'α is George's wife'. Thus, $\forall x_L(x_L = x_p \Rightarrow \exists\alpha(\alpha = x_L\&B_GW\alpha))$. So, for every individual identical to Putin, George believes of that individual that he is George's

[16] The argument bears some similarities to the "operator arguments" against two-dimensionalism in Soames (2007), and critically discussed in Dever (2007).

wife. Needless to say, George does not have any such belief. Thus, the account should be rejected.

By making belief a relation to an open proposition but leaving the alethic features of the belief subject to the vicissitudes of the assignment function, the account does not allow us to assess a belief for correctness in a natural way. Indeed, it makes it impossible for us to really say what the belief is about, since that too depends on the shiftable assignment function. This leads me to reject Cumming's view that the truth-value of a belief ascription is constant under all assignment functions. The truth-value of the belief ascribed and the individuals the belief concerns vary with the assignment function. But this means that the truth-value of a belief ascription must depend on the assignment-saturated semantic value of its embedded that-clause in some way.

4. A Comparison

My argument against Cumming closely mirrors Kaplan's argument against the corresponding view that belief is a relation to a character. Kaplan (1989) claims that – like variables – indexicals and demonstratives have two layers of semantic content: a context-unsaturated semantic value and a context-saturated semantic value. Consider separate utterances of (15) made by the twins George and Oscar.

(15) I've made a huge mistake.

There is a sense in which these two utterances mean the same thing. Each utterance is true just in case concerning the agent in the context, *he* has made a huge mistake. Thus, the two uses of the indexical 'I' have a common semantic feature which Kaplan calls *character*: in any context, 'I' designates the agent of the context.

However, there is a sense in which the two uses of 'I' differ in meaning since the two uses of 'I' designate different individuals. Kaplan calls this the *content* of the indexical. When George utters (15), his utterance is true just in case George has made a huge mistake. By way of contrast, when Oscar utters (15), his utterance is true just in case Oscar has made a huge mistake.

Of these two semantic features of indexicals – the context-unsaturated character and the context-saturated content, only the content plays a role in semantic processing, according to Kaplan (1989: §8). Specifically, Kaplan argues that there are no *monsters*: function terms that operate on the character rather than the content of an embedded expression. There is no operator such as 'In some contexts...' such that 'In some contexts I've made a huge mistake' is true just in case there are contexts in which the agent in those contexts has made a huge mistake.[17]

One manifestation of – and motivation for – Kaplan's disbelief in monsters concerns the behavior of indexical expressions under attitude ascriptions. Kaplan found himself attracted by the view that characters play a role in our cognitive lives. He (1989: 531) imagines that two twins like George and Oscar each utter a sentence containing 'I' such as (15) 'I've made a huge mistake'. Kaplan suggests that, in some sense, the twins are in the same cognitive state. For instance, George and Oscar may feel the same regret at having made a huge mistake or the same anxiety over the mistake's consequences.

A theorist tempted by the similarity in their mental states might conclude that characters – functions from contexts into sets of possible worlds – are the objects of attitudes. On this view, when George and Oscar think to themselves 'I've made a huge mistake', each thought is a relation to the character of this sentence, the features of this sentence that are independent of context. The twins have thought *the same thing*.

This view is precisely analogous to Cumming's proposal. Just as, for Cumming, belief is a relation to the assignment-unsaturated semantic value of an open sentence, for the theorist under consideration, belief is a relation context-unsaturated semantic value of a context sensitive sentence. I showed how Cumming's view has the result that the conditions under which an agent has a belief come apart from whether she stands in any interesting relation to the belief's alethic features. This

[17]Discussion in Schlenker (2003), Dever (2004, 2007, Unpublished), and Rabern (2012).

issued from the fact that the alethic features we ascribe to a belief depend on the assignment function operative in the context of ascribing it.

Kaplan pointed out that a parallel problem arises for the view that characters are the objects of belief. Suppose that George has made a huge mistake, but Oscar has not. Then, when each repeats to himself the sentence 'I've made a huge mistake', George has a true belief and Oscar has a false one. But if belief is a relation to a character, then the two of them believe the same thing, so their beliefs should have the same truth-value. For this reason, Kaplan proposes that the object of thought is a content, in this case the content of the utterance of (15). So just as alethic considerations suggest that we cannot hold that belief is a relation to an assignment-unsaturated semantic value of an open sentence, analogous considerations suggest that we cannot hold that belief is a relation to the context-unsaturated semantic value, character, of a context sensitive sentence.

Kaplan responds to these alethic considerations in the same way in each case. In the case of variables, he proposes that belief is a relation to the assignment-saturated semantic value of an open sentence. In the case of indexicals, he proposes that belief is a relation to the context-saturated value of a sentence, the Kaplanian content. I will argue that – in the case of variables – this response constitutes a slight overreaction to the alethic considerations. In particular, the alethic considerations alone show only that the truth conditions of a belief ascription depend on the assignment-saturated meaning of the sentence embedded in the that-clause of the ascription. Absent further argument, they leave open the possibility that the belief ascription *also* depends on the assignment-unsaturated meanings of a sentence.[18]

5. A Two Factor Theory

As we have seen, Cumming treats belief as a relation to an assignment-unsaturated open sentence meaning, what he calls an open proposition. In contrast, the Millian construes belief as a relation to an assignment-saturated sentence meaning, which

[18]One might view two-dimensionalists such as Chalmers (2011) as making the same move in the case of indexicals.

I am modeling as a set of worlds. Cumming (2008: 545) suggests that on his treatment belief is a relation to an object "more fine grained than a set of worlds[,]" just as treating belief as a relation to a structured proposition means that belief relates an agent to something more fine grained than a set of worlds.

We have seen, however, that this is misleading. An open proposition determines a set of worlds only relative to an input assignment function. So in making belief a relation to an open proposition, we lose information about which set of worlds are compatible with the agent's belief. That is, the same belief – the belief that Fx – may be true in one set of worlds relative to assignment function σ and a different set of worlds relative assignment function τ and yet the truth conditions of 'S believes that Fx' will be the same under σ and τ.

What is wanted is a theory that is sensitive to both the state of the world that the agent believes to obtain, modeled as a set of worlds, and the peculiar take she has concerning "who is who" in this state of the world, modeled as an open proposition. So I propose to do just this. An agent's doxastic state depends on both the set of world-assignment pairs compatible with her take on the world, compatible with who she takes to be who, and also the set of worlds that are actually compatible with her belief given her circumstances. In other words, the truth conditions of a belief ascription 'S believes that Φ' depend on both the assignment-saturated meaning of Φ, which is a set of possible worlds, and the assignment-unsaturated meaning of Φ, which is a set of world-assignment pairs.[19]

[19]The view that I develop is intended to make room for the possibility that sentences differing only by the substitution of co-referential proper names such as 'Michael believes that Lindsay is charitable' and 'Michael believes that Nellie is charitable' differ in truth-value. Cumming shares this goal. But he also wants to solving a puzzle concerning necessarily false propositions arising from possible-worlds accounts of belief. According to Cumming, the belief ascription 'Michael believes that Lindsay is Tobias' may be true, even though the embedded that-clause is a necessarily false identity claim. Moreover, this should not entail that Michael believes anything whatsoever. But if to believe that p requires that the worlds that are epistemically open for an agent include only p-worlds, then an agent cannot believe a necessarily false proposition without believing everything. Cumming avoids this problem by denying that one's epistemic possibilities are given by a set of worlds, but rather by a set of world-assignment pairs. The fact that Cumming's view has this additional flexibility over my own might be seen as an explanatory advantage. However, Cumming provides only a partial solution to a general problem. His account makes it possible to believe necessarily false propositions without believing everything, only for certain necessarily false propositions which can be expressed using proper names. In other cases, the account does not make room for the possibility that an agent believes other necessarily false propositions without

5.1. **A Naive Regimentation.** The satisfaction conditions for belief ascriptions in our model $M = < D, W, I, DOX >$ must depend both on the input assignment function and on the alternative assignment functions compatible with who the agent takes to be whom. The natural semantic characterization of belief ascriptions satisfying these two requirements is as follows.

BELIEF*: $\vDash_{\sigma,w,M} B_t\Phi$ iff for every $< w*, \tau > \in DOX(||t||_{\sigma,w,M}, w)$, both of the following conditions hold:
(i) $\vDash_{\tau,w*,M} \Phi$
(ii) $\vDash_{\sigma,w*,M} \Phi$

In order for 'S believes Φ' to be true relative to assignment function σ, Φ must be true on every world-assignment pair $< w*, \tau >$ in the agent's doxastic state and Φ must also be true on the pair consisting of the world $w*$ and the operative assignment function, σ. Thus, the truth conditions of a belief ascription depend on both the assignment-unsaturated and the assignment-saturated meanings of the sentence embedded in its that-clause.

The phenomenon of "quantifying in" can be handled naturally on this account. Unlike Cumming, I have no need for Kaplan's analysis. I regiment sentence (3) 'there is someone such that Michael believes that *she* is charitable, and that person is Lindsay' in the standard way as (3a): 'there is an x such that x=Lindsay and Michael believes that x is charitable'. I regiment 'Michael believes of an uncharitable individual that she is charitable' as $\exists x(\neg Cx \& B_m Cx)$. Similarly, the sentence 'every man believes that he is wise', which forced Cumming to introduce *de re** beliefs, can be handled straightforwardly as (7).

(7) $\forall x(Mx \Rightarrow B_x Wx)$

(7) is true just in case for every assignment function σ that assigns x to a man, that man believes of himself (under that variable) that he is wise. We get the desired coordination of beliefs with believers.

believing every proposition. Thus, if one believes that there is water containing no hydrogen, then one believes everything. In my view, the problems associated with believing necessary falsehoods should be solved together, likely by appeal to the semantic structure of the embedded clause. So, I would dispute that Cumming's view has a genuine explanatory advantage. Thanks to Brian Rabern and Seth Yalcin for discussion.

Thus, one advantage of the account is that it delivers a straightforward treatment of quantification into attitude ascriptions without relying on quantification over senses, discourse referents, or variables as in other accounts which respect the Fregean *data*. Aside from an argument that I will reject in the next section, we simply have no need for the device introduced by Kaplan (1968). Everything is done with the independently motivated dual semantic roles of variables.

The problematic step in the argument against Cumming was that if $B_m Cx$ is true on one assignment, then it is true on every assignment function. But this is not true on the semantics I am suggesting. $B_m Cx$ is what we might call a *quasi-open sentence*. Its truth-value relative to an assignment σ depends in part on the value of x relative to σ. Thus, if τ assigns x to a different individual than σ, the sentence $B_m Cx$ may be true for σ but not for τ. I call the sentence quasi-open, because it does not allow the free substitution *salva veritate* of one variable for another. To put the view in a slogan: belief is a relation between an agent and a *quasi*-open proposition.

5.2. **Quantifying In, Again.** One consequence of any account on which 'believes' operates on open propositions – even if it also operates on closed propositions – is that certain familiar variable interchange theorems will fail. In particular, the following theorem will fail:

(16) $\vDash \forall x(B_t\Phi(x) \rightarrow \forall y(y = x \rightarrow B_t\Phi(y)))$.

That is, just because one believes of, say, Lindsay that she is charitable, it doesn't follow that one believes of everyone who happens to be identical to Lindsay that she's charitable.

To see that the inference is invalid suppose that assignment σ makes true $B_t\Phi(x)$ and that x and y co-refer on σ. In order for (16) to be a theorem, it must follow that σ makes true $B_t\Phi(y)$. Does this follow? No, because whether $B_t\Phi(y)$ is true on assignment σ depends not just on the value of x relative to σ, but also on whether every world-assignment pair in the agent's belief set makes true $\Phi(y)$. The assignments in her belief set may assign different individuals to x and y, even though x and y co-refer on the input assignment. So we may not infer that σ makes true $\Phi(y)$. Thus, (16) is not a theorem.

These considerations have consequences for the validity of natural language arguments containing sentences in which a pronoun within the scope of an attitude verb is anaphorically dependent on a quantifier outside with wider scope than the verb. Consider the following argument.

(17) Michael believes of his sister that she is charitable.
(18) The woman at the counter is Michael's sister.
(19) Therefore, Michael believes of the woman at the counter that she is charitable.

Millians might suggest that this argument is straightforwardly valid. But on my account, the validity of this argument depends on how it is regimented. The following regimentation is valid. (Treating 'his sister' as a quantifier.)

(17a) Michael believes of his sister$_x$ that she$_x$ is charitable.
(18a) The woman at the counter is Michael's sister.
(19a) Therefore, Michael believes of the woman$_x$ at the counter that she$_x$ is charitable.

But the following alternative regimentation is not valid.

(17b) Michael believes of his sister$_x$ that she$_x$ is charitable.
(18b) The woman at the counter is Michael's sister.
(19b) Therefore, Michael believes of the woman$_y$ at the counter that she$_y$ is charitable.

This argument is not valid because in order for (19b) to be true, Michael must believe of the woman at the counter that she is charitable under the mode of presentation associated with y.

The case of 'every man believes that he is wise' showed that Cumming's account cannot straightforwardly handle quantification into belief contexts. For that reason, he introduced the notion of belief *de re**. As we saw, Cumming regiments this sentence as $\forall x(Mx \Rightarrow \exists\alpha(\alpha = x \& B_x W\alpha))$. Cumming would therefore offer the following regimentation of the above argument.

(17c) Michael's sister$_x$ is such that for some variable α designating her$_x$, $\ulcorner$Michael believes that α is charitable$\urcorner$ is true.
(18c) The woman at the counter is Michael's sister.
(19c) Therefore, the woman$_y$ at the counter is such that for some variable α designating her$_y$, $\ulcorner$Michael believes that α is charitable$\urcorner$ is true.[20]

This alternative regimentation puts the variables x and y outside of the scope of the belief ascription. Because of this, all occurrences of the variable x in *de re** belief ascription can be substituted *salva veritate* for a distinct variable y, if y does not already occur in the ascription. Thus, Cumming's *de re** ascriptions allow substitution of distinct variables assigned to the same individual.

I have tried to do without the Kaplanian mechanism for quantifying in. As we have seen, my suggestion allows for distributive attributions of belief without complications. I do not endorse Cumming's use of *de re** propositions even for cases in which a belief ascription contains a pronoun anaphorically dependent on a quantifier phrase. Thus, I have made no provision for the substitution of distinct anaphoric pronouns under the scope of belief ascriptions, even if these pronouns are satisfied by the same individual. On my view, this is as it should be, if we want to respect the Fregean *data*. If one takes Frege's puzzle cases seriously for proper names, then one should also take them seriously in cases of quantifying in. Consider (20).

(20) Looking at the girl soliciting for charity, Michael doesn't for a moment believe that she's lazy. In fact, the girl is his niece, Maeby. And if there's one thing that he believes concerning his niece, it's that she's lazy.

This string of sentences strikes me as consistent. The sentences seem no more problematic than any other Frege's puzzle case. I suggest that the consistency of this report be accounted for by a distinction in the variables used to quantify into the belief reports as follows.

[20]Recall that the quantification over variables was understood substitutionally. I have instead used explicit metalinguistic discourse and replaced the identities with designation relations in order to smooth over discussion.

(20a) Looking at the girl$_x$ soliciting for charity Michael doesn't for a moment believe that she$_x$ is lazy. In fact, the girl is his niece, Maeby. And if there's one thing that he believes concerning his niece$_y$, it's that she$_y$ is lazy.

If we introduce a new variable y – even if this variable tracks the same individual in the local sentential context – there is no guarantee that the variable tracks the individual in the same way. The truth conditions of a belief ascription depend not only on what is required of the world for her belief to be true, but also on her subjective take on this requirement. Thus, we cannot infer from the fact that two variables are co-referential on assignment σ, that belief ascriptions which differ only be the substitution of one variable for the other are equivalent under σ.[21]

Indeed, there are a host of similar examples suggesting that Frege's cases are just as puzzling when names are replaced by bound variables. Many of these cases are similar to the hooded man examples discussed in Priest (2002). Consider another example:

(21) After glancing at a man in the jail cell, Michael thought that he deserved that punishment. On the other hand, Michael never thought of any of his uncles that they deserved that punishment. In fact, the man in the cell was an uncle, Oscar.

(21a) After glancing at a man$_x$ in the jail cell, Michael thought that he$_x$ deserved that punishment. On the other hand, Michael never thought of any$_y$ of his uncles that they$_y$ deserved that punishment. In fact, the only man in the cell was Michael's uncle, Oscar

(21) seems to be a consistent way of assigning beliefs to Michael. That is, just like any other case, it doesn't seem inconsistent upon first reading. Those who want to do justice to Fregean intuitions generally would do well to offer a consistent interpretation. The most plausible strategy, once again, that distinct, but co-referential variables cannot be substituted *salva veritate* in attitude ascriptions. As regimented by (21a), the first sentence requires Michael to believe the proposition that Oscar – on the mode associated with x – deserves that punishment. The

[21]If one is suspicious of my use of definite descriptions, consider a case containing less controversial quantifier phrases: Looking at each man$_x$ soliciting for charity, Michael doesn't for a moment believe that he$_x$ is lazy. In fact, though, each man soliciting for charity is one of Michael's brothers. And if there's one thing that he believes concerning each$_y$ of his brothers, it's that he$_y$ is lazy.

second sentence requires that he fail to believe of each of his uncles (including Oscar) that he – under the mode of presentation associated with y – deserves that punishment. For this reason, the regimentation correctly predicts that the discourse has a consistent reading.

I should mention that it's hardly a new thought that in natural language belief reports, one cannot substitute *salva veritate* one anaphoric pronoun for another, even if the two anaphoric pronouns designate the same individuals relative to the contextually supplied assignment function. One example arises in the context of Kit Fine's semantic relationism, which blocks the substitution of distinct variables *salva veritate* in belief ascriptions (2009: 115-117), even if these variables have value-ranges including the same single individual. Fine's position is only briefly sketched and is developed outside to the Tarskian framework. Nonetheless, he endorses substitution failures related to those that I am considering.

DRT accounts of attitude ascriptions also fail to validate the substitution of pronouns anaphoric on different quantifier phrases within the context of belief ascriptions, even if the pronouns are assigned to the same individual on the relevant assignments. According to Asher (1993: §5.1), a belief ascription in natural language containing a pronoun anaphoric on an external quantifier is modeled using two variables. One variable is indeed bound by the quantifier and contributes its value on the relevant assignment function. This variable, which Asher says provides the *external anchor*, grounds the alethic features of the belief ascribed. The other variable is potentially co-indexed with other variables in the agent's belief set. It ranges over internal representations the agent might have. Asher says that this second variable provides an *internal anchor*. Thus, Asher's strategy looks like a variant of Kaplan's mechanism for quantifying in. But using the resources of DRT allows Asher (1993: §2.8) to block the argument from (17) and (18) to (19), as I think is appropriate. Moreover, the appeal to the distinction between internal and external anchors is independently motivated within the DRT framework.[22] However, the

[22] Asher (1993: §2.8).

off-the-shelf nature of the approach does not necessarily carry over outside of this framework. For this reason, I have emphasized my development which achieves a similar effect solely in terms of the semantic values of a single variable.

5.3. **Equivalent Variables.** One might object to my proposal on the grounds that for any syntactically distinct variables, there is some assignment function that distinguishes them. This means that belief ascriptions containing distinct free variables in their embedded clauses are *never* equivalent, even when used in the null context. I concede that this result would be problematic.

However, the result is an artifact of the use of assignment functions rather than an alternative semantic treatment of variables. Specifically, I would prefer a semantics, modeled on Tarski's own proposal, that assigns each variable in a context to a position in a sequence.[23] A sentence will then be true or false relative to a sequence s which assigns to every variable v the object in the contextually determined v position of the sequence. As the conversation progresses, different variables will be assigned to different positions in sequences. This doesn't foreclose the possibility that two variables are merely linguistically different from each other, but are semantically the same, since they are assigned to the same position of a sequence. I believe that such an account is independently motivated to resist recent arguments against the Tarskian semantics for variables in Fine (2009: 9-11).[24] On my account as long as uses of variables are "coordinated" so as to pick out the same position in sequences, attitude ascriptions involving these variables will be interchangeable.[25]

Finally, I would like to leave open the possibility of a more Carnapian treatment of quantification according to which a quantification varies not only the object assigned to the variable by the sequence, but also the position in the sequence

[23]Tarski (1983a: 190-191, including footnote 1).
[24]See Dever (Unpublished) for a related view.
[25]For example, Tarski toys with the idea that the nth variable in a sentence is assigned to the nth position in the sequence. Thus, the nth variable in any two sentences will always have the same interpretation. The function from contexts to variables needn't be so crude, however.

associated with the variable. Such a view would still invalidate (16), but would validate some other variable interchange theorems.[26]

6. Summing Up

I endorse the view that if Michael believes of Lindsay that she is charitable, he need not believe of someone who happens to be identical to Lindsay that she is charitable. If I am right, then I have accounted for the phenomena of quantifying in without appeal to any variant of the machinery of Kaplan (1968). I take this to be a great strength of my modification over Cumming's original account. If my view about the substitution of variables should prove to be untenable, then I could appeal to the same version of this machinery that Cumming introduces. My modification would still be required to block the argument from (11) and (12) to (13), which his account fails to do.

I have explored responses to the Millian argument from quantifying into attitude constructions which preserve the Fregean view that sentences differing only by the substitution of co-referential proper names may differ in truth-value. I have argued that Cumming's view that belief is a relation to an open proposition fails to achieve this goal, since it unmoors the conditions under which an agent has a belief from the alethic features of that belief. In its place, I have proposed an account on which the truth conditions of a belief ascription depend on both the assignment-saturated and the assignment-unsaturated meaning of an open sentence. As I have said, belief is a relation to a quasi-open proposition. This view offers a more elegant response because it accounts for the phenomenon of quantification into attitude constructions wholly in terms of the "off the shelf" semantics of variables, making no appeal to Kaplan's treatment of quantifying in.

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[26]The difference between the two approaches is that – as stated – my approach differentiates variables in different sentences syntactically bound by different quantifiers. The Carnapian (1947/1988: §43) approach – at the level of modal operators – only holds *intra*sententially, as in $\exists x \exists y(x = y \& \neg\Box(x = y))$. I want to leave open the possibility that a correlate of this result holds at the level of attitude ascriptions as in (16), but does not extend to variables occurring in different sentences was in the argument (17)-(19) above. On the more Carnapian semantics, this argument would be valid, but an intrasentential version of this argument would be invalid.

Throughout, I have taken the Fregean judgments about substitution of co-referential proper names at face value. I still take it to be an open and significant challenge for the anti-Millian to show that these judgments are robust enough to be incorporated into a semantic theory. I must admit that the arguments in Kripke (1979) and Salmon (1986) give me pause. However, if the judgments are to be taken seriously as a basis for semantic theory, then I think that the position I have developed will be a component of the full story.[27]

Appendix

In this appendix, I give a more explicit derivation of the result presented in §3.2. Suppose (A) and (B):

A: $\vDash_{\sigma,w,M} \exists xCx$ and

B: $\vDash_{\sigma,w,M} BCy$.

I will use these to derive the disastrous conclusion, that for any assignment function τ:

C: $\vDash_{\tau,w,M} \exists y(Cy \& \exists\alpha(\alpha = y \& BC\alpha))$.

In order to derive this result, I assume the semantic clauses for substitutional and objectual quantification which I transcribe directly into my symbolism from Cumming (2008: 551) as well as the usual semantic clauses given in §2.

Substitutional Quantification: $\vDash_{\gamma,w,M} \exists\alpha\Phi(\alpha)$ if and only if $\vDash_{\gamma,w,M} \Phi(\alpha)[\alpha/x]$ is true for some $x \in Var$, where Var is the set of variables.

Objectual Quantification: $\vDash_{\gamma,w,M} \exists x\Phi(x)$ if and only if $\exists d \in D$ and $\vDash_{\gamma[x/d],w,M} \Phi(x)$

From (A), it follows that for some $d \in D$

[27]This work received helpful comments from audiences at St Andrews, The University of Lisbon, The Italian Society for Analytic Philosophy, and the University of Barcelona. I have also benefited from discussion with Derek Ball, Adrian Briciu, Sam Cumming, Josh Dever, Manuel García-Carpintero, Max Kölbel, Genoveva Martí, David Rey, Jason Stanley, and Seth Yalcin. Particular thanks are due to Ray Buchanan, Dilip Ninan, Brian Rabern, Stephan Torre, and an anonymous referee at Noûs. The research leading to these results benefited from partial funds from projects CSD2009–00056 and FFI2012-37658 (Spanish Government) and from the LOGOS Research Group 2009SGR-1077 (Catalan Government).

D: $d \in ||C||_M$.

Let $\tau* = \tau[y/d]$. It is then sufficient for (C) to show that:

E: $\vDash_{\tau*,w,M} (Cy \& \exists\alpha(\alpha = y \& BC\alpha))$.

From (D), we get:

F: $\vDash_{\tau*,w,M} Cy$.

Recall that (B) is a closed sentence. Therefore, if it is true under assignment σ it is also true under assignment $\tau*$:

G: $\vDash_{\tau*,w,M} BCy$

It trivially follows that:

H: $\vDash_{\tau*,w,M} y = y \& BCy$

By the semantic clause for Substitutional Quantification, we have:

I: If $\vDash_{\tau*,w,M} (\alpha = y \& BC\alpha)[\alpha/y]$, then $\vDash_{\tau*,w,M} \exists\alpha(\alpha = y \& BC\alpha)$.

Claim: $(\alpha = y \& BC\alpha)[\alpha/y] = (y = y \& BCy)$.

Thus by Claim and (I), we get:

J: $\vDash_{\tau*,w,M} \exists\alpha(\alpha = y \& BC\alpha)$.

By (F) and (J) we get immediately to the desired (E):

E: $\vDash_{\tau*,w,M} (Cy \& \exists\alpha(\alpha = y \& BC\alpha))$.

Since $\tau* = \tau[y/d]$ and $d \in D$, (C) follows immediately.

C: $\vDash_{\tau,w,M} \exists y(Cy \& \exists\alpha(\alpha = y \& BC\alpha))$.

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