A causal approach to analogy

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Abstract: Analogical reasoning addresses the question how evidence from various phenomena can be amalgamated and made relevant for theory development and prediction. In the first part of my contribution, I review some influential accounts of analogical reasoning, both historical and contemporary, focusing in particular on Keynes, Carnap, Hesse, and more recently Bartha. In the second part, I sketch a general framework. To this purpose, a distinction between a predictive and a conceptual type of analogical reasoning is introduced. I then take up a common intuition according to which (predictive) analogical inferences hold if the differences between source and target concern only irrelevant circumstances. I attempt to make this idea more precise by addressing possible objections and in particular by specifying a notion of causal irrelevance based on difference making in homogeneous contexts.

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1. Introduction

When evidence regarding different phenomena is amalgamated in the sciences in order to predict, to explain or to develop a conceptual framework, this can often be understood in terms of analogical reasoning. After all, analogical inferences, according to a typical explication, are inferences based on similarity: If two phenomena, source A and target A*, are similar and A has a characteristic C, then under certain circumstances it is plausible or probable to assume that A* has characteristic C as well.

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At all times in history, scientists have stressed the epistemological significance of analogy, including such luminaries as William Gilbert, Johannes Kepler, Joseph Priestley, or James Clerk Mawell. Johannes Kepler, for example, wrote in his *Opticks*: "I cherish more than anything else the Analogies, my most trustworthy masters. They know all the secrets of nature." (Kepler 1604; cited in Polya 1954, p. 12) And indeed, analogical reasoning was a major source of creativity in Kepler's scientific method. In his analysis of the solar system, he crucially relied on the analogy between the emission of light and the propagation of what he called the *anima motrix*, i.e. the spirit that moves the planets around the sun:

> "Let us suppose, then, as is highly probable, that motion is dispensed by the Sun in the same proportion as light. Now the ratio in which light spreading out from a center is weakened is stated by the opticians. For the amount of light in a small circle is the same as the amount of light or of the solar rays in the great one. Hence, as it is more concentrated in the small circle, and more thinly spread in the great one, the measure of this thinning out must be sought in the actual ratio of the circles, both for light and for the moving power [motrice virtute]." (Kepler, 1596/1981, p. 201, cited in Gentner et al. 1997, p. 414-415)

Thus, the analogy suggests that the anima motrix, just as light, constitutes a conserved quantity acting according to an inverse square law. The example demonstrates well, how evidence from two different sources, i.e. the theory of optical phenomena and phenomenological knowledge about the solar system, can be combined in order to develop a model concerning the interaction of material bodies in the solar system.

As another example of analogical reasoning in the sciences, consider animal models that are used in medicine and pharmacology to determine the efficacy of a treatment in human beings. Again, evidence from disparate phenomena, here mice and human beings, is amalgamated to further the knowledge about these phenomena. I will argue later that this example is different from the previous one in important respects. Most importantly, it aims at prediction, while Kepler was primarily concerned with theory or model development.

In view of these examples of successful scientific practice, it is remarkable that many methodologists doubt, whether analogical inferences can at all be reliable, stressing the allegedly only heuristic nature of this type of reasoning. In particular, influential authors have questioned, whether there are any universal rules governing analogical inferences. For example, Paul Bartha, who has written the most extensive modern-day treatise on analogical reasoning (2010), states: "Despite the confidence with which particular analogical arguments are advanced, nobody has ever formulated an acceptable rule, or set of rules, for valid analogical inferences. There is not even a plausible candidate." (Bartha 2013, Sec. 2.4) In a similar vein, Patrick Maher writes: „Argument by analogy is a generally accepted form of inductive reasoning and many think that inductive reasoning can be represented using the probability calculus. From these facts one might expect that there would be accepted probability models that can represent inference by analogy, but no such model exists." (Maher 2001, p. 183) As an example from the statistics and computer science literature, Henri Prade and Gilles Richard write in their recent overview of the field: "analogical reasoning is not amenable to a formal framework in a straightforward manner due to the brittleness of its conclusions." (2014, 5)

The most pressing and interesting epistemological problem with respect to reasoning by analogy therefore is how to bring these two aspects together, the ubiquitous use of analogy in scientific practice on the one hand and the widespread doubt about the reliability of analogical inferences on the other hand. Since analogical reasoning aims at combining evidence from different, but related phenomena, any general framework of evidence amalgamation has to find an answer to this conundrum as well.

In the next section, relying on short case studies from the history of scientific method, I argue for three interrelated points. First, I briefly present Carnap's framework of induction, building mainly on enumerative induction. While he tries to implement analogical reasoning in his approach, he fails to find a convincing manner to do so. This situation leads me to argue for a general failure of enumerative approaches to implement analogical reasoning. Instead, eliminative approaches, focusing on the variation of circumstances rather than the repetition of instances as in enumerative induction, are much more amenable to analogical reasoning, as the second case study on Keynes' approach to induction shows. Third, I introduce two influential contemporary frameworks by Mary Hesse and Paul Bartha, which address one of the major problems of Keynes' approach, the so-called counting problem. To this purpose, they develop a two-dimensional framework, which takes into account the 'horizontal' similarities between different phenomena, but also the 'vertical' nature of the relations between similarities and differences.

In the third section, a distinction between two types of analogical reasoning is introduced, namely conceptual and predictive analogies. These differ in their epistemic aim, the nature of the vertical relations, the criteria of evaluation, and the methodological framework. I argue that the widespread skepticism concerning analogical inferences partly results from a failure to recognize this distinction. While conceptual analogies indeed serve mostly a heuristic function, this is not true for predictive analogies.

In section four, I then sketch a framework for predictive analogies building on the intuition that 'a predictive analogical inference holds, if the differences between source and target are irrelevant to the prediction'. I discuss some preliminary objections and argue that irrelevance must be understood in causal terms. Examining different explications of the notion of causal irrelevance from the literature, I find none of them suitable for the context of analogical reasoning. My own proposal construes causal irrelevance in terms of difference making in a given background context.

Since the framework that was developed so far is intended for deterministic situations, I briefly address in section five, how it can be extended to include probabilistic analogical inferences. While there are straightforward ways to implement probability, a crucial problem remains regarding the interpretation of probability in this context.

2. Three historical perspectives

The history of methodological thinking about analogy is quite rich. In the following, I concentrate on three more recent episodes or case studies that will provide the groundwork for the approach to be outlined later in the article.

2a. Carnap and the inadequacy of enumerative approaches

Rudolf Carnap developed one of the most extensive and detailed inductive frameworks in the 20^{th} century, in which he explicitly aimed to include considerations of analogy. Carnap's approach is based on a confirmation function c(h|e), which designates the confidence in a hypothesis h based on some evidence e. As is well-known, Carnap was a dualist about probability, distinguishing an empirical and a logical role of probability—the former regarding relative frequencies while the latter is usually identified with rational degree of belief in a hypothesis based on some evidence.

Carnap construes analogical inference as an inference from one individual to another based on their known similarity, much in line with the general understanding that was presented in the introduction: "The evidence known to us is the fact that individuals b and c agree in certain properties and, in addition, that b has a further property; thereupon we consider the hypothesis that c too has this property." (1950, p. 569)

Carnap's general approach to induction is based on what is often called the 'straight rule' of induction: Given a family of predicates P, i.e. a mutually exclusive but exhaustive group of predicates that applies to a number of individuals, the degree of confirmation corresponds to the relative frequency s_j/s of a property P_j in the first s individuals. In other words, the straight rule of induction is just ordinary enumerative induction. Carnap recognizes the deficiencies of this simple rule and consequently extends it to 'a continuum of inductive methods' which is determined by a number of additional parameters. There are several versions in his writing over the course of his life, the best known being the so-called λ-γ system developed in his mature, posthumously published *Basic System of Inductive Logic* (1971, 1980) with a confirmation function

$$c_j(s_j, \ldots, s_k) = \frac{s_j + \lambda\gamma_j}{s + \lambda}.$$

Here, s can be interpreted as the number of real individuals and λ the number of virtual individuals. Among the former s_j have the property P_j, among the latter $\lambda\gamma_j$. This confirmation function can be rewritten in terms of an empirical and a logical part:

$$c_j(s_j, \ldots, s_k) = \frac{s}{s + \lambda}\frac{s_j}{s} + \frac{\lambda}{s + \lambda}\gamma_j$$

For large s, the empirical part dominates, for small s, the logical part. Thus, the logical part can be interpreted as an *a priori* contribution to the confirmation function.

In general, analogical influence is considered to belong to this logical part. Carnap specifies several kinds of analogical influence. First, he draws a distinction between similarity influence, which takes into account the distance between properties, and proximity influence referring to the distance between individuals—presupposing in both cases that an adequate metric exists. With respect to the former, Carnap further distinguishes between analogical influence within one predicate family and that between different predicate families. While he acknowledges that the latter is much more common than the former, he mainly addresses in the *Basic System* analogical influence within one predicate family, presumably because it is the simpler problem (for a very brief discussion of analogical influence between different predicate families, see Carnap 1950, §110 D). Furthermore, Carnap's analysis of analogy is restricted to individuals which have certain properties in common, while in typical analogical inferences individuals are also known to differ in certain other properties—a critique spelled out in some detail by Mary Hesse (1964).

Carnap suggests treating analogical inferences in terms of the mentioned γ corresponding to the width (or weight) of properties and an additional η corresponding to the distance between properties. If two properties P_1 and P_2 are sufficiently similar, i.e. are close in terms of the distance measure, then the relative frequency of P_1 will influence the confirmation function for P_2 and vice versa. Naturally, the width also has to be taken into account: basically, the more weight a property has, the greater its influence. According to Carnap, such analogy influence "is usually very small", it "decreases with increasing [evidence in terms of number of individuals] s", and therefore can "be practically neglected" if s is large (1980, p. 41). To repeat, this is because analogy influence belongs to the logical and a priori part of the confirmation function, which can be neglected for $s >> \lambda$. As an example, Carnap uses the color space to illustrate the concepts of width, essentially the range or variation subsumed under a specific color, and distance, i.e. the perceived similarity between different colors. Both are determined by the chosen metric of the color space (1980, Sec. 14.A).

Carnap's treatment of analogy remains brief and fragmentary—in contrast to his very detailed treatment of induction in general—and this situation may already cast doubt over the suitability of enumerative approaches to analogy, i.e. essentially those approaches that are based on some version of the straight rule. There have since been a number of attempts to integrate analogical reasoning within an essentially Carnapian approach to inductive logic (e.g. Hesse 1964, Kuipers 1984, Romeijn 2006, Maher 2001). It seems fair to say that no agreement has been reached (for a helpful overview, see Huttegger forthcoming). Many decades after Carnap published his approach to inductive logic, it continues to be doubtful whether his framework is capable to cover analogical reasoning in a sensible manner.

One strain of criticism attacks the use of additional parameters such as γ or η which must be derived from a metric over properties, which rarely is explicitly available. These parameters seem considerably ad hoc as is well illustrated by the example of the color space for which a wide variety of representations are possible (Reibe & Steinle 2002). In fact, this situation has led Wolfgang Stegmüller, a close collaborator of Carnap, to suggest that Carnap is really talking about subjective rather than logical probability (Stegmüller 1973, 514)—which would further undermine any attempt to justify reliable predictions based on analogical reasoning, even though these seem ubiquitous in the sciences, as the examples from the introduction suggest.

In the end, what seems the most problematic aspect about Carnap's approach is its focus on the straight rule and on relative frequencies as the core concepts for confirmation—automatically confining analogy to prior considerations, which wash out as increasing evidence in terms of instances is gathered.[2] After all, scientific practice suggests otherwise: relative frequencies are generally a bad indicator for confirmation, while analogies can often provide highly reliable evidence. The lesson from the case study on Carnap's treatment of analogy thus seems to be that just as enumerative approaches to induction in general, enumerative approaches to analogy, confining analogy to prior considerations, run into deep and presumably unsolvable problems.

2b. Keynes and the ubiquity of analogical reasoning

It is often thought that the essence of inductive reasoning lies in the multiplication of instances and Carnap's approach with its reliance on the straight rule and on relative frequencies attempts to formalize this intuition. However, there has been for many centuries an alternative tradition of inductive reasoning which focuses on the variation of circumstances rather than on the number of instances. Proponents of this later tradition, which is sometimes referred to as eliminative induction, are among others Francis Bacon, John Stuart Mill and more recently John Maynard Keynes. It turns out that its basic inductive framework is much more amenable to analogical reasoning. After all, an analogical inference concludes from one instance with certain circumstances to another with different circumstances. Indeed, proponents of eliminative induction have often considered analogical inference as the core of inductive reasoning. The best example in this regard is John Maynard Keynes, who in his *Treatise on Probability* lays out a general framework for induction based on analogy:

> „In an inductive argument, therefore, we start with a number of instances similar in some respects AB, dissimilar in others C. We pick out one or more respects A in which the instances are similar, and argue that some of the other respects B in which they are also similar are likely to be associated with the characteristics A in other unexamined cases. The more comprehensive the essential characteristics A, the greater the variety amongst the non-essential characteristics C, and the less comprehensive the characteristics B which we seek to associate with A, the stronger is the likelihood or probability of the generalisation we seek to establish." (Keynes 1921, 219-220)

Note again that Keynes's description closely resembles what we had defined as an analogical argument in the introduction, while he considers it the fundamental form of an inductive argument. Keynes introduces some terminology that has since become standard in literature on analogical reasoning. The *positive analogy* concerns those properties which source and target have in common, the *negative analogy* those properties in which source and target differ, and the *unknown analogy* those properties of which it is yet unknown whether they belong to the positive or negative analogy. Finally, the *hypothetical analogy* concerns those

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[2] Note that Bayesian approaches to confirmation often assume a similar role for analogy as being confined to prior considerations (e.g. Salmon 1990): "I suspect that the use of arguments by analogy in science is almost always aimed at establishing prior probabilities. […] The moral I would draw concerning prior probabilities is that they can be understood as our best estimates of the frequencies with which certain kinds of hypotheses succeed. These estimates are rough and inexact..." (186-187).

properties which are known of the source phenomenon and predicted of the target phenomenon (see also Bartha 2013).

Keynes' approach to induction turns the Carnapian view upside down. While for Carnap enumerative induction in the form of the straight rule is central and analogy is confined to prior considerations that wash out with increasing evidence, for Keynes, analogical inferences are fundamental and enumerative induction only plays a subordinate role by controlling for circumstances whose influence thus far has not been explicitly considered:

> "The object of increasing the number of instances arises out of the fact that we are nearly always aware of *some* difference between the instances, and that even where the known difference is insignificant we may suspect, especially when our knowledge of the instances is very incomplete, that there may be more. Every new instance *may* diminish the unessential resemblances between the instances and by introducing a new difference increase the Negative Analogy. For this reason, and for this reason only, new instances are valuable." (Keynes 1921, 233)

Relatedly, Keynes denies that relative frequencies can be used to determine probabilities along the lines of the straight rule. The reason is that instances vary in different ways regarding their circumstances and thus there is usually no reason to count them with equal weight as the straight rule presupposes:

> „I do not myself believe that there is any direct and simple method by which we can make the transition from an observed numerical frequency to a numerical measure of probability." (Keynes 1921, 367)

In summary, Carnap's system implements a clear distinction between enumerative induction and analogy, it confines analogical influence to a priori considerations, and it endorses a principle of instantial relevance ("one of the basic characteristics of customary inductive reasoning", Carnap 1971, 161), according to which any positive instance strictly increases the confirmation function that the next instance is positive as well.[3] All this is incompatible with Keynes's approach, who argues that all induction basically relies on analogy, even seeming applications of enumerative induction actually aim at increasing the negative analogy. He rejects any simple frequentist approach to confirmation, which quantifies confirmation based on some variant of the straight rule. Relatedly, he rejects the principle of instantial relevance: in particular, if two instances are fully identical in all their relevant circumstances, then the additional instance does not confirm at all (1921, 233).

Unfortunately, the shift away from enumerative induction to an inductive framework based on analogy, while conceptually sensible, eliminates the most obvious candidate for a measure of confirmation, namely relative frequencies. Instead, a quantitative measure could consist in a weighted comparison between positive and negative analogy. Bartha suggests the following characterization of this widespread intuition:

> "Suppose S and T are the source and target domains. Suppose $P_1, \ldots, P_n$ (with $n \geq 1$) represents the positive analogy, $A_1, \ldots, A_r$ and $\neg B_1, \ldots, \neg B_s$ represent the (possibly

[3] Carnap qualifies that the strict inequality only holds if the original confirmation function is not zero or one.

> vacuous) negative analogy, and Q represents the hypothetical analogy. In the absence of reasons for thinking otherwise, infer that Q^* holds in the target domain with degree of support $p > 0$, where p is an increasing function of n and a decreasing function of r and s." (2013, Sec. 2.4)

But, as many authors including Bartha have stressed, this approach leads to the notorious *counting problem*. While counting instances in enumerative induction seems straight-forward, counting properties in analogical reasoning is not. If two instances have property 'color' in common, but differ in property 'size', how possibly should one compare color and size? It appears impossible to formulate general rules for this task, which has led many to conclude that analogical reasoning is necessarily contextual. As a result, Keynes' approach remains almost entirely qualitative, which may have contributed to the fact that it is barely used in contemporary science.

Still, Keynes does derive some general guidelines for analogical reasoning. Inductive arguments which conclude from a number of examined instances to a generalization can be strengthened by the following means:

- "by reducing the resemblances known to be common to all the instances, but ignored as unessential by the generalization,
- by increasing the differences known to exist between the instances,
- by diminishing the sub-analogies or unessential resemblances known to be common to some of the instances and not known to be false of any." (Keynes 1921, 231-232)

For this, either new instances have to be examined or the knowledge of familiar instances has to be extended. Most standard treatments of analogical reasoning propose similar qualitative guidelines (see Bartha 2013, Sect. 3.1 for a comprehensive list of commonsense guidelines).

In summary, Keynes' framework bases inductive reasoning on analogical inferences, i.e. every inductive inference is conceived as an inference based on similarity. While this is conceptually plausible, proponents have largely failed to come up with a quantitative confirmation measure for such an approach.

2c. Hesse, Bartha and the two-dimensional approach

No solution to the counting problem seems to be forthcoming. Apparently, how properties are counted very much depends on the specific context. There is, however, one crucial insight that has occasionally been pointed out in discussions of analogical reasoning, but that was most forcefully stressed by Mary Hesse and more recently by John Norton and Paul Bartha. For analogical reasoning it is important to not only consider the similarity and differences in properties between source and target, but also the nature of the relation between these properties:

> „Under what circumstances can we argue from, for example, the presence of human beings on the earth to their presence on the moon? The validity of such an argument will depend, first, on the extent of the positive analogy compared with the negative (for example, it is stronger for Venus than for the moon, since Venus is more similar to the earth) and, second, on the relation between the new property and the properties

> already known to be parts of the positive or negative analogy, respectively. If we have reason to think that the properties in the positive analogy are causally related, in a favorable sense, to the presence of humans on the earth, the argument will be strong. If, on the other hand, the properties of the moon which are parts of the negative analogy tend causally to prevent the presence of humans on the moon the argument will be weak or invalid." (Hesse 1966, 58-59; cited in Norton 2011, 8)

In other words, Hesse proposes a two-dimensional model, where the horizontal relations concern the similarity between source and target, i.e. the identity or difference in properties, and the vertical relations concern the relations between properties, which Hesse believes to be causal in most cases. Simply comparing the negative and the positive analogy thus will not do, but rather the nature of the relationship between the properties in the positive and the negative analogy with the properties in the hypothetical analogy have to be taken into account.

In his recent influential work on analogical reasoning, Paul Bartha very much builds on Hesse's two-dimensional account (Bartha 2010, briefly summarized in 2013, Section 3.5.2). He classifies different types of analogical reasoning in terms of different vertical relations, e.g. logical, causal, or statistical. Bartha's *principle of prior association* then demands that some kind of connection between the positive analogy and the hypothetical analogy has to be established, taking into account the negative analogy as well. Bartha's second principle, the *principle of potential for generalization*, requires that there should be reason to expect that the relationship between positive and hypothetical analogy in the source obtains for the target as well. In particular, there should be no "critical disanalogy" between source and target.

Let me emphasize again that these modern authors have established that any reasonable approach to analogy has to take into account both similarity in properties between source and target as well as the relations between these properties and the hypothetical analogy.

3. Predictive and conceptual analogies

In the following, I introduce a distinction between predictive and conceptual analogies, which differ in various respects: concerning the epistemic aim, the nature of the vertical relations, the criteria of evaluation, and the methodological framework. Arguably, the failure to clearly hold these types of analogical reasoning apart has led to considerable confusion in the debate on analogical reasoning. Maybe most importantly, only for conceptual analogies the role of analogical reasoning is primarily heuristic, while predictive analogies aim at true or at least probable inferences. As argued in section 2b when discussing Keynes' approach these latter constitute the core of inductive and causal reasoning.

An example of a predictive analogy is the use of animal models such as the mouse model in pharmacology to determine the effectiveness of certain medication to cure diseases in human beings. Predictive analogies aim to establish reliable prediction or effective intervention. Consequently, the relevant vertical relationships must be of causal nature. This follows from a view of causation in the sciences as the crucial concept to distinguish between effective and ineffective strategies—as developed by Nancy Cartwright and others (especially Cartwright 1979). Only if there is a causal link between administering the medication and recovery both in the mouse and in the human being, the analogical inference is reliable.

More exactly, a strategy how to effectively intervene in a phenomenon has to be based on a direct causal relationship between some circumstances in the positive analogy and the hypothetical analogy. Similarly, a reliable prediction must be based on some causal connection, which however need not consist in a direct causal link, but can also result from a common cause structure. In particular, an analogical inference aiming at prediction may infer from a correlation between two variables with a common cause in the source phenomenon to a similar correlation in the target phenomenon. By contrast, a merely accidental correlation that does not result from some causal connection cannot be used either for prediction or for intervention. In summary, no matter if they aim at effective intervention or at reliable prediction, predictive analogies always have to establish a causal relationship in the target phenomenon based on some knowledge about a corresponding causal relationship in the source phenomenon.

Predictive analogies are evaluated by verifying whether an intervention suggested by the analogy works or whether a prediction turns out to be true. After all, there is a matter of fact, whether a medication that cures a disease in a mouse will also lead to recovery in a human being afflicted by a similar disease. Of course, as this example demonstrates, such predictive analogies will in general not be deterministic, but statistical, i.e. they will only hold with a certain probability. Thus, methodological frameworks for predictive analogies try to determine the truth or at least probability for analogical inferences. Both Carnap's and Keynes' approaches to analogy, as delineated in the previous sections, are examples of such probabilistic frameworks for analogical reasoning—covering chiefly predictive analogies.

An example for a conceptual analogy is the analogy between the transfer of heat and interaction in electromagnetic phenomena as it was elaborated in great detail by William Thomson and James Maxwell towards the end of the 19th century—resulting in the modern particle-field theory of classical electrodynamics:

> "The laws of the conduction of heat in uniform media appear at first sight among the most different in their physical relations from those relating to attractions. The quantities which enter into them are *temperature*, *flow of heat*, *conductivity*. The word *force* is foreign to the subject. Yet we find that the mathematical laws of the uniform motion of heat in homogeneous media are identical in form with those of attractions varying inversely as the square of the distances. We have only to substitute *source of heat* for *centre of attraction*, *flow of heat* for *accelerating effect of attraction* at any point, and *temperature* for *potential*, and the solution of a problem in attractions is transformed into that of a problem in heat. […]
>
> It is by the use of analogies of this kind that I have attempted to bring before the mind, in a convenient and manageable form, those mathematical ideas which are necessary to the study of the phenomena of electricity." (Maxwell 1855/56, 157)

As is clear from this quote, Maxwell's aim in developing the analogy between heat and electricity is not primarily prediction or intervention. Rather, Maxwell wants to develop a conceptual framework for electromagnetic phenomena based on another framework that was more familiar and much better developed at the time, namely the theory of heat. Such reasoning facilitates transferring certain results and solutions from one field to the other.

Since the primary aim is neither prediction nor intervention, the relevant vertical relationships in such conceptual analogies are in general not causal—arguing again with a Cartwrightian concept of causation as sketched above. In the example of classical electrodynamics, there are good reasons to assume that the considered relationships are to considerable extent definitional or conventional. In particular, this perspective is in accordance with a standard view on the nature of axioms and laws of fundamental scientific theories—interpreting these as implicit definitions of basic theoretical terms. Certainly, it cannot be the place here to defend this view, but typical arguments range from underdetermination of abstract theory to the observation that the laws in fundamental theories are too abstract to have themselves considerable empirical content. Only when supplemented by further assumptions, e.g. bridge principles according to the classic syntactic view of scientific theories, do these laws acquire empirical meaning. This observation alone might suffice to establish the non-causal nature of the fundamental laws of abstract scientific theories.

Relatedly, conceptual analogies are evaluated by whether they play a fruitful role in transferring established solutions and results from one field to another rather than in terms of truth and probability. While in predictive analogies, one can verify whether an analogical inference corresponds to a matter of fact, e.g. whether a prediction turns out true or not, this is in general not possible for conceptual analogies. To verify, whether a Poisson equation for the electric potential holds, when postulated in analogy to the Poisson equation for temperature in the theory of heat, is certainly not as simple as verifying predictive analogies. One reason lies in the considerable underdetermination of abstract conceptual frameworks. Indeed, Maxwell stressed the underdetermination of classical electrodynamics insisting that there exists considerable flexibility how to formulate the fundamental laws. For example, a choice between action at a distance and field theory in electrodynamics remains possible (Pietsch 2012).

Thus, conceptual analogies are a creative endeavor. Whether they hold, is not so much a matter of truth and probability but to considerable extent depends on the ingenuity of the scientists—whether they are successful in mapping (part of) the fundamental structure from one phenomenon to the other. Consequently, such analogies cannot be treated in terms of probabilistic frameworks like those of Carnap or Keynes. Approaches to analogical reasoning based on structure mapping, such as from the work of Dedre Gentner (1983), seem much more adequate. Gentner's framework relies on a classification of various entities, attributes and relations as well as a quite sophisticated set of inference rules. Analogies are evaluated according to a *systematicity principle*, essentially that those analogies are more plausible that result from a mapping of mutually connected higher order relations compared with those mapping only isolated properties. Note that this main criterion of the structure mapping theory can hardly be translated into probabilities and consequently, Gentner's theory, while well suited for conceptual analogies, seems unable to serve as a framework for predictive analogies.

4. A deterministic framework for predictive analogies

4a. A first suggestion

There exists a core intuition about valid analogical reasoning that can be found across the literature and that is in line with the two-dimensional model sketched in Section 2c. This intuition is for example encapsulated in Bartha's second principle that for valid analogical inferences no essential disanalogy between source and target should exist. The basic idea is the following (PA):

> A (predictive) analogical inference holds, i.e. the hypothetical analogy is true for the target, if and only if the negative analogy concerns only *causally irrelevant* circumstances.

Note that in line with the distinction introduced in Section 3, the vertical relations of interest are causal in nature since the focus lies on predictive inferences. To repeat, this insight stems from a Cartwrightian understanding of causation, the core feature of which is to draw a distinction between effective and ineffective strategies, including between reliable and unreliable prediction.

I will in the following suggest a methodology for predictive analogical inferences that builds on this core intuition (PA). Before discussing the crucial notion of causal irrelevance, let me briefly point out some possible objections against the proposed approach which are then mostly addressed later on. A first issue concerns situations, where an analogical inference is valid even though some circumstances in the negative analogy are causally relevant—i.e. (PA) is not a necessary condition for predictive analogical inferences. Notably a factor may be causally relevant, but may play no role in the considered analogy, because other contributing factors are not instantiated, e.g. the burning match does not cause a fire since there is no combustible material present. Also, the influences of some causally relevant circumstances could exactly cancel each other. For example, one might infer from the acceleration that a stone receives on the earth to the acceleration that a stone of the same mass receives on the moon. The acceleration is indeed the same, if the difference in gravitational field is exactly compensated by an acceleration of the system of reference on the moon. Similarly, the same effect can be due to alternative causes, e.g. the acceleration of a body may be caused by gravitational or by electromagnetic fields. An analogical inference is still valid even if in various instances different alternative causes are active.

Secondly, certain cases suggest that (PA) is not a sufficient condition for predictive analogical inferences. In particular, predictive analogical inferences may sometimes be based on relationships other than causal relevance, e.g. on mere correlations. More exactly, even if the negative analogy is causally irrelevant, the analogical inference could nevertheless fail to hold due to mere correlations between some circumstances in the negative analogy and the hypothetical analogy. For example, given a correlation between gravitational and magnetic fields due to the common cause of the planet earth, one might be tempted to conclude from different gravitational forces to different magnetic forces. Even though the gravitational field is causally irrelevant to the magnetic field, a change in the former may suggest a change in the latter due to the mentioned non-causal correlation—contradicting the claim that only causally relevant conditions are important for analogical inferences.

Relatedly, predictions are sometimes based on definitional relations. This can again result in situations where analogical inferences fail to hold even though the causal structure has not changed. For example, an analogical inference from the gravitational field in one location to another at the same distance from the earth could fail just because the concept of a gravitational field is understood differently in both situations.

A third point concerns the distinction between properties (which are ‘one-place’) and relations (which are ‘many-place’). While the Keynesian terminology of positive and negative analogy suggests a focus on properties rather than relations, many scholars insist that analogy is less about a supposedly superficial similarity in terms of common properties of source and target, but rather about similarity in terms of relations. For example, in the analogy between heat and electricity, the essential similarity is not between corresponding terms such as temperature and electric potential or source of heat and charge. Rather it concerns relations between these terms, e.g. that they obey a Poisson equation.

To resolve this issue, note first that relations always link properties with each other. Thus, it would be wrong to think that one could exclusively focus on relations neglecting properties altogether. The Poisson equation, for instance, relates temperature and sources of heat as well as charges and electric potential. Furthermore, the proposed approach (PA) obviously takes into account relationships as well, by examining the causal relevance or irrelevance of certain properties for others.

It might still be questionable, whether complex analogies can be formulated in terms of positive and negative analogies. After all, it does not appear straightforward how to compare concepts like temperature and electric potential in terms of differences and similarities? In response, it should be stressed that if shared relations exist one can always formulate shared properties corresponding to these relations. For example, both temperature and electric potential share the abstract property that they serve as potentials which by means of corresponding forces lead to the distribution of certain quantities. By contrast, electric potential and temperature differ in terms of the nature of the potential, in particular regarding the quantity on which it acts, namely either charged matter or heat. In this manner, positive and negative analogy can be distinguished. With sufficient ingenuity, this is always possible. Finally, let me stress that these ‘linguistic’ difficulties how to formulate positive and negative analogy are usually much more prominent for conceptual analogies than for predictive analogies.

Fourth and last, there are substantial worries concerning the notion of causal irrelevance. For example, it is far from certain, whether causal irrelevance can ever be established at all. After all, a circumstance that is normally considered irrelevant may suddenly become causally relevant in some obscure situation. The constellation of the stars at birth is usually not considered relevant to the fate of a person, but in some contrived story it might have an impact. For example, the person may be superstitious and the astrological prediction of a psychic may be so scaring that it becomes a self-fulfilling prophecy. The ultimate lesson to draw from such counterexamples is that causal irrelevance is context-dependent and that in an explication of analogical reasoning this must be taken into account. Such context-dependence is of course not surprising to anyone familiar with the philosophical debate on causation. It was stressed in particular by John Mackie, who in his approach to causation introduced the crucial notion of a causal field, to which all causal statements are relative (1980).

Whether the basic intuition (PA) has merits or not, crucially depends on the construal of the notion of causal irrelevance. To this issue we will turn now.

4b. The notion of causal irrelevance

In the following, I discuss several suggestions from the literature how to define causal irrelevance and based on these will later lay out my own proposal. All in all, it seems fair to say that the notion of causal irrelevance has not played a central role especially in philosophical accounts of causation, which are almost exclusively focused on the notion of cause in a positive sense. Therefore, the following overview can be rather brief.

First, one might try to define causal irrelevance based on statistical independence. The most straightforward connection between these notions originates within a probabilistic approach to causation (see e.g. Hitchcock 2016 for a useful overview). If, broadly speaking, causal relevance of an event C to another event E is identified with the increase or decrease of the conditional probability $P(E|C) \lesseqgtr P(E|\neg C)$, it seems natural to define causal irrelevance of C to E in terms of an unchanged probability $P(E|C) = P(E|\neg C)$. As mentioned, most accounts of probabilistic causation do not explicitly address the notion of causal irrelevance in much detail. A notable exception in this regard is John Eels who distinguishes positive, negative, mixed, and neutral causal relevance—the latter corresponding to causal irrelevance (Eels 1991).

One important problem for a definition of causal relevance and irrelevance along these lines are common cause structures, where a correlation between two variables F and G does not result from a direct causal link between them, but rather from a common cause H that is causally relevant to both variables. Let us assume in the following for reasons of simplicity that all variables are binary. Even though no direct causal relevance between the variables exists, the conditional probability changes, e.g. $P(G|F) \neq P(G|\neg F)$. However, it is well known that common causes shield off such correlations—i.e. while $P(G|F) \neq P(G|\neg F)$, we have $P(G|F\&H) = P(G|\neg F\&H)$ when conditionalising on H. Thus, one needs to control for common causes in order to identify the true relations of causal relevance or irrelevance.

For his definition of causal irrelevance, Eels suggests considering the probabilistic impact of a potential cause X on a potential effect Y in various causal background contexts. In each causal background context, all factors $F_1, \ldots, F_n$ that are causally relevant to Y, independently of X[4], are held fixed. Only, if the probability of Y is not changed by X in *all possible* contexts, should one speak of causal irrelevance (Eels 1991, 86). This condition is often called *contextual unanimity*. Note that Eels' definition of causal irrelevance is circular to some extent since the definiens itself employs the notion of causal relevance. However, he argues that the circularity is not vicious, since the definiens refers to the causal relevance of factors other than X, of which the irrelevance is examined (87). Eels further relativizes causal relevance and irrelevance to "a particular population, as well as to a kind that the token population exemplifies" (87). In part, this is required in order for a probability distribution to exist at all. Certainly, causal and probabilistic functions will differ between populations and kinds of populations.

If Eels' definition of causal irrelevance is employed for predictive analogies, the approach falls prey to the first objection of Section 4a. It does not identify analogical inferences as valid where a causally relevant factor is present but contributing causes are not instantiated or where causal influences exactly cancel. Consider again the example of a valid analogical inference where in the two instances different alternative causes are active. According to Eels' definition, these alternative causes would not be causally irrelevant and therefore the analogy must incorrectly be assumed not to hold, according to (PA). Furthermore, an approach to analogy based on Eels' definition of causal irrelevance would have problems dealing with analogies based on correlations, i.e. the second issue addressed in Section 4a. Since two variables with a common cause would be identified as causally irrelevant to each other, analogical inferences based on correlations between such variables could not be accounted for. In addition, there are general problems for probabilistic approaches, to which I will turn now.

In recent years, a link between causality and probabilistic independence has been elaborated in the context of causal modeling on the basis of directed acyclic graphs satisfying the causal Markov condition—which are often referred to as causal Bayes nets. The causal Markov condition implies a range of probabilistic independency relations. In particular, the probabilities for all nodes must be probabilistically independent when conditionalising on all parents PA of the nodes in the graph:

$$P(X_1, X_2, \ldots, X_n) = \Pi_i \, P(X_i \mid PA(X_i))$$

Conditions like faithfulness or minimality further restrict the range of possible causal models. Faithfulness, for example, states that both conditional and unconditional probabilistic independencies in a graph must follow from the causal Markov condition. In particular, if two variables are probabilistically independent there should be no causal link between them.

The faithfulness condition illustrates well the difficulties that arise when building causal models merely from statistical relationships. On the one hand, premises like faithfulness are indispensable to reduce the number of possible models to a manageable amount. On the other hand, a range of counterexamples shows that faithfulness and related conditions can be little

[4] i.e. factors that are causally relevant to Y but to which X is not causally relevant—excluding in particular factors that lie on a causal chain from X to Y.

more than pragmatic and fallible tools to develop causal models. As an example, the faithfulness condition cannot account for causal relationships that exactly cancel each other.

Generally speaking, statistical independence is neither a sufficient nor a necessary criterion for causal irrelevance. As mentioned, when causal influences between two variables exactly cancel each other, there is presumably a causal link between these variables even though they are probabilistically independent. Also, two variables may be probabilistically independent, but in a number of instances of measure zero, there may nevertheless be causal relevance. Such cases show that probabilistic independence is not sufficient for causal irrelevance. But probabilistic independence is not necessary either. After all, as is elaborated in the following, there are plausible candidates for methods that determine causal irrelevance in the absence of evidence in terms of probabilistic independence. Furthermore, probabilistic independence can never be fully established empirically as fluctuations will always lead to some small dependence.

As a second approach, let us take a look at deterministic definitions of causal relevance, i.e. definitions that do not refer to probability distributions. A typical definition is given by Christopher Hitchcock:

> "X is *causally relevant* to Y, if and only if there is some set of variables, and some set of values of those variables, such that when we intervene to set all those variables to those values, at least some interventions on the value of X will lead to different values of Y." (2009, 305)

In a similar vein, Michael Baumgartner and Gerd Grasshoff, who advocate a sophisticated regularity view of causation, suggest:

> „Factor A is causally relevant for the occurrence of an effect B, if and only if there exists at least one causal process, in which an event of type A (partly) causes the occurrence of an event of type B." (Baumgartner & Grasshoff 2004, 49; my translation)

While most authors discussing causal relevance do not bother to explicitly define causal irrelevance, it can easily be construed as complementary to causal relevance. Essentially, this would require that *no* intervention can lead to a change in values of the effect variable or that *no* process exists where an event of type A at least partly causes the occurrence of an event of type B.

Such definitions of causal irrelevance generally are not adequate for an analysis of analogical inferences based on the intuition (PA). After all, it may well happen that a circumstance is causally relevant in some situation, while for the considered analogical inference it plays no role, for example because other contributing causal factors are missing or because there is a counteracting cause (cf. again the first objection in Section 4a). Thus, circumstances that are causally relevant according to the above definitions may change from source to target, while the analogical inference may still be valid—contradicting (PA).

Indeed, very few circumstances will turn out causally irrelevant according to the above definitions, because in some obscure situation these might all be causally relevant (cp. the fourth objection in Section 4a). For this very reason, Baumgartner and Grasshoff largely reject the notion of causal irrelevance (2004, 212). The main lesson to draw from these attempts to define causal irrelevance is that context-dependence is not taken into account in an adequate manner. Exceptions to causal irrelevance in more or less obscure situations should be discounted on the basis that they occur within a different context than the one that is employed in the analogical inference.

David Galles and Judea Pearl belong to the small number of authors, who in an influential article (1997) explicitly define deterministic causal irrelevance and carefully implement context dependence:

> "A variable X is causally irrelevant to Y, given Z […] if, for every set W disjoint of X ∪ Y ∪ Z, we have
>
> $$\forall(u, z, x, x', w), \quad Y_{xzw}(u) = Y_{x'zw}(u)$$
>
> where x and x' are two distinct values of X." (reproduced in Pearl 2000, 235-6)

Here, u are the values of the background or exogenous variables of the model. According to Pearl, this definition captures the intuition that "if X is causally irrelevant to Y, then X cannot affect Y under any circumstance u or under any modification of the model that includes do(Z=z)." (Pearl, 236)

It may be possible to use this definition for an approach to predictive analogies based on (PA). However, this would turn out unnecessarily complicated. The first reason concerns model dependence. Galles and Pearl relativize their definition to a specific causal model that is determined by a number of exogenous or background variables U, a number of endogenous variables, and functions that determine each endogenous variable based on the other variables. It may not be necessary to introduce such rather sophisticated model dependence for the notion of causal irrelevance. Secondly, by relying on an interventionist account of causation, Galles and Pearl subscribe to a distinction between interventions and observations, which leads them to introduce the do-calculus for formally handling interventions. However, the notion of intervention plays no major role in analogical reasoning, neither in predictive nor in conceptual analogies, which suggests that an interventionist framework might not be the first choice for explicating analogy.

Note finally that while the basic intuition (PA), which constitutes a deterministic framework for predictive analogies, naturally suggests looking for a deterministic explication of causal irrelevance, this raises the question how to deal with indeterministic contexts and with situations, in which the evidence allows to formulate only probabilistic dependencies—an issue that will be briefly addressed in Section 5.

4c. A suggestion

Instead of an interventionist and model-theoretic approach to causal relevance and irrelevance, let me in the following sketch an account based on difference-making in context. While we cannot ultimately defend the following definitions here, they should get some initial plausibility from their close resemblance to the method of difference, which is arguably the most successful rule in scientific method to determine causal dependence. One crucial advantage with respect to the definitions of Galles and Pearl is that no sharp distinction between interventions and mere observations must be presupposed and consequently no extra do-calculus for interventions has to be introduced. Another advantage is that while background dependence will be emphasized, the definitions are not model dependent in the stronger sense intended by Galles and Pearl, i.e. the definitions are not relativized to specific causal models.

Causal relevance shall be defined in the following manner (CR):

> A is *causally relevant* to C in a context B, if and only if (i) an instance exists, where A and C occur in the context B, (ii) a second instance exists, where neither A nor C occur in the same context B, and (iii) B guarantees homogeneity.

Note again that this definition largely corresponds to the method of difference as given in particular by John Stuart Mill. Note further that causal relevance of A to C with respect to B implies that a change in A within a context B *always* leads to a change of C—in contrast to all definitions given in the previous section. Causal irrelevance can then be defined as the complementary notion (CI):

> A is *causally irrelevant* to C in a context B, if and only if (i) an instance exists, where A and C occur in context B, (ii) a second instance exists, where A does not but C still occurs in the same context B, and (iii) B guarantees homogeneity.

Causal irrelevance of A to C with respect to B implies that a change in A within context B never leads to a change in C. For example, a switch is causally irrelevant to a light given two instances, one, in which both switch and light are on, and another, in which the switch is off but the light still on, while nothing else that could be relevant to the light has changed—the last premise essentially corresponding to homogeneity. By contrast, the switch is causally relevant, if in the second instance, the light is off.

The context or background B is constituted on the one hand by a set of circumstances that are allowed to change and on the other hand by a set of circumstances that must remain constant. Homogeneity, which was already invoked by Mill in his formulation of the method of difference, essentially captures the intuition that factors in the background that are causally relevant to the examined phenomenon may not change. It is a concept that is used both in counterfactual approaches such as by Rubin and Holland (e.g. Holland 1986) and also in sophisticated regularity approaches such as Baumgartner and Grasshoff (2004). The latter provide an extensive discussion (2004, 208).

While homogeneity is usually defined that causally relevant factors must remain constant, I prefer the complementary perspective that only causally irrelevant circumstances are allowed to change. In combination with the definitions discussed in the previous Section 4b, this has some subtle implications. In particular, the explication of homogeneity given in the following is less demanding in that more circumstances are allowed to change. Notably, some circumstances, e.g. causal factors in certain contexts, that are causally relevant for example according to the definitions of Baumgartner and Grasshoff are now identified as causally irrelevant based on (IR). Thus, these would have to remain constant for homogeneity according to Baumgartner and Grasshoff, while they are allowed to change according to the following explication of homogeneity (H):

> Context B guarantees homogeneity with respect to the relationship between A and C, if and only if only circumstances that are causally irrelevant to C can change, (i) except for A and (ii) circumstances that are causally relevant to C in virtue of A being causally relevant to C.

The second exception allows for circumstances to change that lie on a causal chain through A to C or that are effects of circumstances that lie on this causal chain.[5]

Let me now briefly address how to deal with the problems that were raised in Section 4a. Concerning the first objection, consider for example cases where two influences A and B exactly cancel each other and therefore the analogical prediction remains valid even though causally relevant circumstances change. In reaction, let us specify that not every property in the negative analogy taken by itself must be causally irrelevant, but strictly speaking only all properties in the negative analogy *taken in conjunction*. If A and B compensate each other, then a change from A and B to ¬A and ¬B is irrelevant. Similarly, if A and B are alternative causes for a phenomenon C, then a change from A and ¬B to ¬A and B is irrelevant for the phenomenon C.

Note further that if a factor which is commonly considered a cause fails to be relevant for the considered analogy because other contributing factors are not instantiated, e.g. the burning match does not cause a fire since there is no combustible material present, such a factor is actually identified as causally *irrelevant* with respect to the considered context according to the proposed definition (IR)—in contrast to all other definitions of causal irrelevance discussed in the previous section. Therefore, according to the proposed definition of causal irrelevance and the intuition (PA), such analogical inferences are correctly identified as valid.

The second problem raised in Section 4a concerned analogies based on correlations. Consider again the example that a circumstance F changes from source to target, which is causally irrelevant to the hypothetical analogy G, but which is correlated with it and therefore leads to the failure of the analogical inference. It turns out that such situations are precluded in the sketched approach. Indeed, according to the view of causation introduced in Section 3, any meaningful correlation between variables *must* result from a common cause. Therefore, in order for an analogical inference to fail in the described manner, the corresponding common cause variable must change. However, such a change in common cause variables is precluded by (PA), since these are not causally irrelevant to the hypothetical analogy.

In response to the problem that analogies may be based on definitional, instead of causal relevance, one might be tempted to restrict predictive analogies to causal vertical relationships only. However, this runs into problems with familiar epistemological issues such as

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[5] The notion of "causal relevance in virtue of" cannot be discussed here in further detail due to lack of space. An exact explication is: "A condition X is causally relevant to C in virtue of A being causally relevant to C with respect to a background B, iff in all contexts within B, in which X is causally relevant to C, A is causally relevant to C as well (but not necessarily vice versa)."

confirmational holism and relatedly the lack of a clear distinction between empirical and definitional statements. Instead, I broadly suggest to integrate definitional relevance in the framework which should be rather easily done since definitional relevance can be defined in much the same manner as causal relevance—given that the main difference merely lies in the nature of the necessity between antecedent and consequent.[6]

With respect to the two other issues that were raised in Section 4a, the distinction between properties and relations was already discussed. Concerning the notion of irrelevance, (CI) in combination with (H) is supposed to yield an adequate explication. In particular, by introducing strict context-dependence, (CI) aims to avoid the problem that was pointed out in Section 4a, namely that causal irrelevance is practically inexistent, since any circumstance can be relevant in some obscure situation.

But a crucial question remains, namely what exactly should be chosen as an adequate context for the statement of irrelevance in the basic intuition (PA). Remember that a context consists of circumstances that are allowed to change and others that must remain constant. Since the impact of all circumstances that change (i.e. the negative analogy) is explicitly considered as antecedent, these cannot be ascribed to the context. What is left to account for are thus all circumstances that remain constant, i.e. the positive analogy. Apparently, these then constitute the context.

In summary, the proposed deterministic approach to predictive analogical inferences is given by the following explication (PA') in combination with the definitions of causal irrelevance (CI) and homogeneity (H):

> Predictive analogical inferences are valid, if and only if the negative analogy (taken in conjunction) is causally (and definitionally) irrelevant to the hypothetical analogy with respect to a context constituted by the constancy of the positive analogy.

5. Analogy and probability

Thus far, we have only addressed deterministic analogical inferences that hold with certainty. Of course, analogical inferences are typically only probabilistic: Given a certain known positive, known negative and unknown analogy, what is the probability that the hypothetical analogy is the case for the target phenomenon?

An extension of the approach delineated in Section 4c to cover such probabilistic inferences is straightforward. Let me briefly discuss the most important cases: (i) first, there may be an unknown analogy, which is causally relevant; (ii) second, there may be a negative analogy, which is causally relevant only with a certain probability; (iii) there may be situations of indeterminism.

Regarding the first case, we have thus far only considered ideal situations, in which every circumstance is known to belong either to the positive or to the negative analogy. Now, as

[6] Basically, one needs to replace in (PA), (CI), (CR), and (H) "causally irrelevant" with "causally and definitionally irrelevant" as well as "causally relevant" with "causally or definitionally relevant".

Keynes has rightly pointed out, in actual situations it is usually unknown of a number of circumstances whether they belong to the positive or negative analogy. Assume for the sake of simplicity that the unknown analogy consists of only a single circumstance which is causally relevant in the respective context. Then, the analogical inference is valid with the probability that this circumstance belongs to the positive rather than the negative analogy, if otherwise (PA’) holds. Equally, if there is more than one factor in the unknown analogy, one has to determine the combinations which are causally relevant and then add up the respective probabilities belonging to those combinations.

In the second case, one may be uncertain whether circumstances that belong to the negative analogy are causally irrelevant. Thus, the analogical inference is valid with the probability that these circumstances taken in conjunction are causally (and definitionally) irrelevant. Finally, in cases of indeterminism, i.e. in cases where the circumstances determine the hypothetical analogy only up to a certain probability, analogical inferences are valid with that probability. Of course, in such situations, causal relevance has to be interpreted in a probabilistic manner determining a probability distribution over states and not the state itself.

These are the principal cases, how probabilities may enter the assessment of analogical inferences. Of course, various combinations are possible, for example there may be an unknown analogy of which it is unknown whether it is causally relevant. While one has to carefully keep track of the corresponding probabilities, these complications do not add any substantial conceptual problems. One crucial question, however, which goes far beyond the present article, concerns the interpretation of probability in such probabilistic analogical inferences. On the one hand, the interpretation presumably needs to be objective as prediction and intervention concern matters of facts rather than subjective credence. On the other hand, the most common objective interpretation, the frequentist approach, is not an adequate interpretation since it belongs to the tradition of what was called above enumerative approaches to induction, which are generally hostile to analogical reasoning.

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