On Two Slights to Noether's First Theorem: Mental Causation and General Relativity*

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Revised, 7 October 2021

Abstract

It is widely held among philosophers that the conservation of energy is true and important, and widely held among philosophers of science that conservation laws and symmetries are tied together by Noether's first theorem (that rigid symmetries yield conservation). However, beneath the surface of such consensus lie two slights to Noether's first theorem.

First, there is a 325+-year controversy about mind-body interaction in relation to the conservation of energy and momentum, with occasional reversals of opinion. The currently popular Leibnizian view, dominant since the late 19th century, claims to find an objection to broadly Cartesian views (and non-epiphenomenalist property dualism) in their implication of energy non-conservation. Here energy conservation is viewed as an oracle, an unchallengeable black box. But Noether's first theorem and its converse show that conservation and symmetry of the laws stand or fall together. Absent some basis for expecting conservation in brains that has a claim on the Cartesian (whose view implies the absence of law symmetries in brains), the objection is circular. An empirically based argument is possible, but is a different argument with little force except insofar as it is rooted in neuroscience.

Second, General Relativity has a 100+-year-long controversy about whether gravitational energy exists and is objectively localized. The usual view is that gravitational energy exists but is not objectively localized, though some deny its existence. Without positive answers to both questions, generally applicable conservation laws do not exist: energy is not conserved. This conclusion is startling in itself and a problem for conserved quantity theories of causation. Yet Noether's first theorem applies to General Relativity, which has uncountably many symmetries of its laws and so has conservation laws, indeed uncountably many of them. Many authors downplay these laws due

*For Springer volume *Rethinking the Concept of Laws of Nature: Natural Order in the Light of Contemporary Science* edited by Yemima Ben-Menahem

to their quirky properties; some authors even attempt to explain the laws' supposed nonexistence in terms of an absence of symmetries of the geometry, which is a distraction. Thus Noether's first theorem is widely ignored, left uninterpreted, or distorted in relation to General Relativity. Taking the theorem seriously seems possible, however, restoring the conservation of energy, or rather, energies.

How do these controversies relate? One sometimes finds claims that General Relativity's supposed lack of conservation laws answers Leibniz on behalf of Descartes. Taking seriously the superabundance of formal conservation laws in General Relativity, however, suggests that General Relativity resists (not facilitates) mind-to-body causation. This conclusion can be proven apart from interpretive controversies. The resistance is, however, finite and tends to be swamped by larger world-view considerations.

Keywords: conservation laws, Noether's first theorem, philosophy of mind, dualism, Cartesianism, interactionism, gravitational energy

1 Introduction

This paper considers two areas where Noether's first theorem tends to be neglected, one in the philosophy of mind, one in physics and the philosophy of physics, and finds the two debates mutually illuminating. In the philosophy of mind, there is a tendency to believe that physics has revealed that energy is conserved, full stop, whereas Noether's first theorem implies (*inter alia*) that energy is conserved if and only if the laws are the same at all times—a claim that sometimes is and ought to be denied by interactionist dualists, against whom energy conservation is often invoked. In General Relativity, on the basis of traditional arguments there is a tendency to downplay the conservation laws that at least formally exist according to Noether's first theorem, but these traditional arguments have become less convincing in the last decade or two. My goal in discussing interactionist dualism is not to make a defense of the position, but simply to critique a very widely received claim that the view has been refuted by physics, in particular by conservation laws.

The energy conservation objection to nonphysical mental causation has been made from the 1690s (Leibniz, 1997) to the 2010s (Churchland, 2011). According to Leibniz's *Theodicy*,

> ...two important truths on this subject have been discovered since M. Descartes' day. The first is that the quantity of absolute force which is in fact conserved is different from the quantity of movement, as I have demonstrated elsewhere. The second discovery is that the same direction is still conserved in all bodies together that are assumed as interacting, in whatever way they come into collision. If this rule had been known to M. Descartes, he would have taken the direction of bodies to be as independent of the soul as their force; and I believe that that would have led direct [*sic*] to the

> Hypothesis of Pre-established Harmony, whither these same rules have led me. For apart from the fact that the physical influence of one of these substances on the other is inexplicable, I recognized that without a complete derangement of the laws of Nature the soul could not act physically upon the body. (Leibniz, 1985, p. 156)

To paraphrase, Descartes's volume $\times$ speed quantity is not conserved, but momentum (mass times velocity, including direction) is conserved, as is 2$\times$kinetic energy (*vis viva*), so the soul's deflecting matter while leaving its speed unchanged violates the newer notion of conservation. Being a rationalist, Leibniz had high aesthetic standards for physics and metaphysics. While co-inventing calculus, Leibniz did not inherit (as we do) an expectation that physical laws would come as differential equations that generically cannot be solved exactly. Hence what he counted as a "complete derangement" might not be nearly so disturbing nowadays.

While philosophy of mind texts often leap from Leibniz to the 20th century as though nothing happened in between except perhaps more of the same, this objection has waxed, waned and waxed again over time. After much influence for a season in the 18th century German world, this objection, resisted explicitly by Crusius and Knutzen (Watkins, 1995; Watkins, 1998) and implicitly by Newton and Euler (the best physicists in the world) (Pitts, 2020d), was defeated in the 18th century, restoring interactionism as the default position. It probably helped that the conservation of *vis viva* (proto-energy) in that era was a rejected Leibnizian opinion rather than a generally accepted law. In the 19th century, as the conservation of *vis viva* was rehabilitated as the conservation of energy, the first law of thermodynamics, the Leibnizian objection reappeared. This reappearance seems to owe as much to cultural opportunism and the *Zeitgeist* (including the carefully cultivated Baconian inductive naivité) as anything else; the question whether one should make exceptions for spiritual influence is clearly untouched by Joule's paddle wheel experiment. Newtonians had always been willing to accept mental forces, which would have falsified momentum conservation, so doing the same for energy conservation would not have been at all difficult. And yet the conservation of energy has been held to have been more damaging to world-views involving spirits (including but not only Christianity) than was Darwin (Turner, 1974, p. 27). Some good physicists, including Boussinesq and Maxwell, questioned or rejected the argument (van Strien, 2015), but without the intensity of its scientific naturalist proponents. Much of the story has been told for the German-speaking world (Heidelberger, 2004; Wegener, 2009), though often without the normative coloration that I would give.

While common in the philosophy of mind, the traditional (not general relativistic) argument from conservation against dualism is rarely made by experts in physics even when they discuss the mind. Thus noted philosopher of physics Jeremy Butterfield writes:

> ... [A] traditional argument against interactionism is flawed, because of this false picture of physics.... The idea is that any causal interaction between mind and matter would violate the principle of the conservation of en-

> ergy.... But, says the argument, physics tells us that energy is conserved in the sense that the energy of an isolated system is constant, neither increasing nor decreasing.... And there is no evidence of such energy gains or losses in brains. So much the worse, it seems, for interactionism. (Though traditional, the argument is still current; for example, Dennett endorses it (1991, pp. 34-35).) This argument is flawed, for two reasons. The first reason is obvious: who knows how small, or in some other way hard to measure, these energy gains or losses in brains might be? Agreed, this reason is weak: clearly, the onus is on the interactionist to argue that they could be small, and indeed are likely to be small. But the second reason is more interesting, and returns us to the danger of assuming that physics is cumulative. Namely: the principle of the conservation of energy is not sacrosanct. ... [A]lthough no violations have been established hitherto, it has been seriously questioned on several occasions. It was questioned twice at the inception of quantum theory.... And furthermore, it is not obeyed by a current proposal ... for solving quantum theory's measurement problem. In short: physicalists need to be wary of bad reasons to think physicalism is true, arising from naivety about physics. (Butterfield, 1997)

The next section will elaborate on Butterfield's remark.

To be brief, the main problem is circularity or question-begging: when one understands where conservation laws come from in terms of Noether's first theorem, what seemed to be a scientific fact, categorical energy conservation, is replaced by a biconditional relation between symmetries and conservation laws, with no dialectically useful basis for assuming symmetries. If there are souls that act on bodies (and not in the same way at all times and places), then the symmetries of the laws fail: my willing to raise my arm involves my soul's doing something to my brain during my lifetime, but does not affect William the Conqueror or the Moon (at least not directly in the case of the Moon). So one needs to assume the falsehood of interactionism to justify the symmetries of the laws, from which conservation follows by Noether's first theorem. But then one has begged the question. It is of course possible that neuroscience provides good reasons to reject interactionist dualism, but that is a quite different argument, one based on evidence about the brain.

One might wonder why this paper pays no attention to quantum mechanics and quantum field theory. As explained in more detail elsewhere (Pitts, 2020a), one reason is that it is not very clear what to say about conservation in quantum mechanics and quantum field theory, in light of the measurement problem, (on some views) the collapse of the wave function, and the like, as well as the uncertainty relations. Some authors have invoked quantum mechanics' arguable non-conservation to make room for mental causation; I do not need to take a stand on the issue. I think that I have something interesting to say about energy conservation and mental causation (namely, that the Leibnizian argument fails, regardless of what one says about quantum mechanics and quantum field theory), something interesting to say about General Relativity and energy conservation (namely, that the formal conservation laws that one gets by a

Noether-type recipe can be taken more seriously than most people think), and something interesting to say about General Relativity and mental causation (namely, that General Relativity makes mental causation harder, not easier, a conclusion that coheres with taking general relativistic conservation laws seriously but does not depend on doing so), but I do not have anything interesting to say about quantum mechanics and energy conservation. While General Relativity, being a classical rather than quantum field theory, is not a fundamental feature of the world, it is good practice to understand the theory and to try to make its quantum successor "quantum gravity" (whatever that turns out to be) as much like General Relativity as possible—unless of course General Relativity is wrong even apart from quantum mechanics, a conclusion far more widely entertained in the physics literature now than in, say, 1997.

One might also recall the Steady State cosmological theory as an example of an energy non-conserving theory more in the vicinity of GR.[1] This feature was not particularly satisfactory even to the theory's proponents, however, for later versions posited a "creation field" C with negative energy (Narlikar, 1993, pp. 241-246), so that energy could be conserved, matter could acquire more energy, and the C field's energy could become more negative.

2 Conservation and Symmetry: The Modern View

From the standpoint of the philosophy of physics, a striking weakness in traditional discussions of conservation and the philosophy of mind is the failure to connect with the now-standard (especially 1910s+) relation between symmetries and conservation laws that was put into a recognizably modern form by Herglotz, Mie, Born and Noether (Herglotz, 1911; Mie, 1913; Born, 1914; Noether, 1918; Pitts, 2016a). (Some sources on the history of the relationship between symmetries and conservation laws exist (Houtappel et al., 1965; Kastrup, 1987; Kosmann-Schwarzbach, 2011).) In the 1910s the 19th century mathematical tradition of analytical mechanics (Lagrange, Hamilton, Jacobi) at last became a standard part of physics, and the ideas were widely understood as applied to continua (whether elastic media or fields such as the electromagnetic and gravitational potentials). I have explained this textbook material (Goldstein, 1980, chapter 12) (Davis, 1970) as clearly as I could (at times working with Alin Cucu) in application to the philosophy of mind (Pitts, 2020a; Cucu and Pitts, 2019; Pitts, 2021), so I will be brief here. One infers the laws from a mathematical function of space and time, now called the Lagrangian density $\mathcal{L}$, by taking a curious kind of derivative of the Lagrangian density with respect to the fields/potentials and their temporal and spatial derivatives (rates of change). These are called the Euler-Lagrange equations, which are field equations and/or equations of motion. Given the Lagrangian density, one can discern the conservation (or otherwise) of energy and momentum by inspection. In the early 1910s Max Born expressed how rigid translation symmetries imply conservation

[1] Thanks to Patrick Duerr for helpful remarks here and elsewhere. All conclusions are my own.

laws:

> The assumption of Mie just emphasized, that the function [the Lagrangian density $\mathcal{L}$] is independent of x, y, z, t, is also the real mathematical reason for the validity of the momentum-energy-law. ...We assert that for these differential equations, a law, analogous to the energy law ($3'$) of Lagrangian mechanics, is always valid as soon as one of the 4 coordinates x_α does not appear explicitly in [$\mathcal{L}$]. (Born, 1914)

In this context the "momentum-energy law" is not a global statement that $E = constant$ at each time or the like, but a local statement. In particular, it is a claim (for each time and place) that the temporal rate of change of the density of the conserved quantity in question (energy or a component of momentum) plus the tendency of that quantity to spew out of a place (its "divergence," the sum of the spatial rates of change of the energy or momentum flux) equals 0. A bit of the mathematics will appear later. To give an analogy, if people are walking into a room faster than people are leaving it, then the number of people in the room increases. This seems obvious; what does it exclude? It excludes instantaneously disappearing in one place and reappearing elsewhere, whether at the same time or later; more generally, it forbids disappearance into nothingness and forbids appearance out of nothingness. Instead energy and momentum flow in accord with the laws of motion/field equations derived from the Lagrangian density. (The same can be said of angular momentum, which follows from a rotational symmetry of the Lagrangian density.) So, at any rate, claims the continuity equation. In some simple cases, with spatially isolated matter, one can add up (spatially integrate) the continuity equation over the whole universe and infer that the total energy is constant, a more familiar claim from secondary school chemistry, but one with less content and less assurance of making sense than the continuity equation (Peebles, 1993, p. 139).

The continuity equation, being an equation (or a handful of them for a handful of conserved quantities), says much more than the $E = constant$ darling of many philosophers of mind, so the prospects for a conservation law-based objection to broadly Cartesian mental causation seem brighter. In particular, one strategem that one sometimes sees among dualists, having energy nonconservation due to the mind be compensated by the opposite nonconservation elsewhere, is clearly excluded. This proposal was never worth much anyway. Where does this nonconservation happen—the center of the Earth? How does this other place know to zag when my soul zigs? Matter presumably would not know what my soul is doing; is there a slave-soul that does the opposite of what I do? Is the machinery run by angels? And why find an apparently unobservable compensating violation of conservation laws at all relevant or comforting, given that the conservation of energy is supposed to be empirically motivated by observable events? Surely it was always better to bite the bullet (as a few hardy souls have) even in the late 19th, 20th, and 21st centuries and simply deny conservation in the context of mental causation. The preeminent example of doing so intelligently was the Averill and Keating paper (Averill and Keating, 1981), which made some contact with the physics literature and the relation between symmetries and conservation laws.

While a favorite with some interactionist dualists, a rather beleaguered band until recently, this paper tended to be drowned out by people flaunting their knowledge of secondary-school chemistry. The conservation objection has been very popular, and even interactionists have tended to try to uphold conservation (often unsuccessfully (Pitts, 2020a; Cucu and Pitts, 2019)).

Hence the question of where and when the continuity equation is true can be postponed no longer. Expressing Born's point a bit more clearly, one can say the following. Wherever and whenever the Lagrangian density does not depend explicitly on time t, energy is conserved in the sense of the continuity equation. Wherever and whenever the Lagrangian density does not depend explicitly on the spatial coordinate x, momentum in the x-direction is conserved; likewise for y and z. On the other hand, wherever and whenever the Lagrangian density depends explicitly on t, x, y, or z, energy or the corresponding momentum component is *not* conserved. The claim that symmetries imply conservation laws was later encapsulated as Emmy Noether's first theorem, though Born's special case is sufficiently general for our purposes. Noether also proved the converse, that conservation laws imply symmetries (Noether, 1918; Brading, 2001; Brown and Holland, 2004; Kosmann-Schwarzbach, 2011; Romero-Maltrana, 2015). That sounds more surprising, but this converse was already evident for the cases treated by Born.

So what of the dialectic of the energy conservation objection? The objector is making an argument, something that ought to provide some reason for a person initially sympathetic to interactionist dualism to give up that view. Consequently, it would be awkward to use a premise that is exactly as acceptable as the denial of interactionist dualism in order to motivate the denial of interactionist dualism. An argument like

P. Interactionist dualism is false.
C. Interactionist dualism is false.

would clearly not be impressive, despite its validity. But once one understands the biconditional relationship between symmetries and conservation laws due to Lagrangian field theory and Noether's first theorem, one finds the following popular argument similarly unimpressive.

P1. If interactionist dualism is true, then energy and momentum conservation are false.
P2. Energy and momentum conservation are true.
C. Interactionist dualism is false.

This is a valid argument using *modus tollens*. But once one understands Lagrangian field theory and Noether's first theorem, premise 2 ceases to be the deliverance of an oracular black box, Science, and looks instead like an assumption that interactionist dualism is false—which was supposed to be a conclusion rather than a premise. What went wrong? The dualist has no reason to grant that the laws of nature are the same everywhere and always. Your soul, if interactionist dualism is true, causes the Lagrangian density to vary with time and place within your brain so that something physical behaves in a way that it otherwise wouldn't have, in order to implement your decision to raise your arm. Hence the Leibnizian objection is indeed question-begging, a claim typically made only by the heartiest of theists and metaphysicians in recent centuries (Ducasse, 1960, p. 89) (Larmer, 1986; Plantinga, 2007), but demonstrably true in light of Lagrangian field theory and Noether's first theorem and its converse. The inverse correlation between immersion in physics and commitment to the Leibnizian argument is not accidental, once one distinguishes between domesticated and genuine physics (Ladyman et al., 2007, p. 24). The foundations of physics show conservation laws to be conditional upon symmetries, symmetries in which there is no reason to believe (in the relevant exceptionless form[2]) if one thinks that souls act upon bodies. Hence when one opens the black box of conservation laws, one finds a mirror reflecting one's own opinion about whether there are souls that act on bodies. Little or no material for an objective argument against interactionism remains. If we take Noether's first theorem seriously, we find that the Leibnizian conservation objection dissipates.

Some readers will be rightly disappointed by so vague an expression as that something physical in your brain behaves in a way that it otherwise wouldn't have. Surely one should say which part of the brain does what, and even provide neuroscientific evidence that this actually happens? Indeed, interactionist dualists would be better situated, the more plausibly they could tell a neuroscientific story that fills in the details, especially if empirical evidence for such a story could be found. My aim is not to defend interactionism, but to remove a popular faulty objection to help the discussion be carried out on the proper terrain, which is *a posteriori* and neuroscientific. But that discussion can hardly occur in a balanced way as long as the authority of physics has been wrongly claimed so as to stack the deck against interactionism. Neuroscience poses a potentially serious problem for interactionism, but one in which conservation laws play no essential role.

The issue of conservation laws in General Relativity also bears upon the philosophy of mind. That connection will be explored towards the end of this essay. But first the issue of conservation in General Relativity itself, a matter of longstanding controversy, merits discussion. There one finds a second slight to Noether's first theorem. As it happens, a new philosophy of mind-type example will help to illuminate the (shortage of) content of a well known general relativistic law sometimes offered as a substitute for the conservation of energy and momentum.

[2]The assumption is that a prudent dualist will hold that nature is uniform except in some small spatio-temporal regions, presumably inside our brains, and that such nonuniformity in our brains is sufficiently large to make a difference but sufficiently small not to be obvious. Nothing in the interactionist dualist position as such prohibits thinking that telekinetic powers are ubiquitous, such that anyone could lift a modest spacecraft out of a swamp like Yoda in *The Empire Strikes Back*, or that our souls render the Lagrangian density so strongly dependent on space and time in our heads that we can radiate as much energy as a nuclear reactor. Such views would have the 'virtue' of falsifiability. But everyone has always known from experience that these claims are false.

3 Noether's First Theorem and General Relativity: Another Slight

The story about a Lagrangian density $\mathcal{L}$ and conservation's holding insofar as the space-time coordinates t, x, y, z do not appear in $\mathcal{L}$ applies at least formally in Einstein's General Relativity (GR), our usual (though more often challenged since 2000 than previously) theory of gravitation (Papapetrou, 1948; Schmutzer, 1972). At some level this has been understood since the 1910s, but it is frequently forgotten and its consequences have been kept at arm's length due to the difficulty in interpreting the resulting mathematical expressions. (Some possible leads into the literature are ((Szabados, 2009; Pitts, 2010; Chen et al., 2015).) The natural initial expectation is that one could use the usual machinery of Lagrangian field theory, whether the entry-level "canonical tensor" (Goldstein, 1980, chapter 12) or perhaps an upgraded expression due to Belinfante or Rosenfeld which suffices for other local field theories such as electromagnetism.

It was realized in the 1910s, however, that the canonical tensor, which for GR gives Einstein's "pseudotensor" using Einstein's Lagrangian density,[3] has some rather peculiar properties. In particular, one could apparently make gravitational energy disappear around a heavy round body by choosing coordinates with that aim in mind (Schrödinger, 1918; Cattani and De Maria, 1993). One could also apparently make gravitational energy appear in empty flat space-time by using spherical coordinates (Bauer, 1918; Cattani and De Maria, 1993). Indeed one can make the gravitational energy density $\mathfrak{t}^0_0$ vanish at any point in space-time by a choice of coordinates. Presumably nothing real can be created or destroyed through mere descriptive choice. The idea that all coordinate systems are admissible and equally good in GR seemed to imply that anything worth discussing in the theory should be tensorial (broadly construed), that is, should admit changing its components in one coordinate system into another coordinate system in an algorithmic way—which is to say, should be a "geometric object" in the sense of classical differential geometry (Nijenhuis, 1952; Anderson, 1967; Pitts, 2006; Duerr, 2021; Read, 2022). (A geometric object's transformation law can be affine rather than linear, can be nonlinear, or can involve higher derivatives, for example. The idea arose in the 1930s as a generalization of tensor calculus, which General Relativity made an important branch of mathematics.) A familiar case is the metric tensor, a machine that allows one to infer real spatio-temporal distances (squared) from coordinate displacements, keeping in mind that coordinates might not be distances, but could involve angles, or not be orthogonal, or slosh back and forth in an arbitrary way. The metric tensor components transform like this:

$$g'_{\mu\nu} = g_{\alpha\beta} \frac{\partial x^\alpha}{\partial x^{\mu'}} \frac{\partial x^\beta}{\partial x^{\nu'}}.$$

[3]There is a measure of nonuniqueness, which can sometimes be physically interesting, for the Lagrangian density. That seems true for GR (Pitts, 2022)(Ohanian and Ruffini, 1994, p. 647).

One easily sees that

$$\sum_{\mu'=0}^{3}\sum_{\nu'=0}^{3} g'_{\mu\nu} dx^{\mu'} dx^{\nu'} = \sum_{\mu=0}^{3}\sum_{\nu=0}^{3} g_{\mu\nu} dx^{\mu} dx^{\nu} = ds^2,$$

the invariant line element. It was concluded that gravitational energy is not "localizable," because the total energy could be viewed as behaving reasonably while the local description from Einstein's pseudotensor $\mathfrak{t}^{\mu}_{\nu}$ could not be trusted due to the above-mentioned false negative and false positive and related tendencies. Gravitational energy-momentum pseudotensors not only are not tensors, but also are not even geometric objects (Bergmann, 1958) (Anderson, 1967, p. 428). How, then, one can take their claimed local descriptions of gravitational energy seriously? This nonlocalizability doctrine has remained the majority view to this day.

There later arose a second objection to gravitational energy localization, namely, that the gravitational energy-momentum pseudotensor is not unique (Landau and Lifshitz, 1975; Goldberg, 1958)(Anderson, 1967, pp. 423-430). There are various expressions, indeed infinitely many, it turned out eventually, with arguably comparably good credentials. Which, if any, was the real gravitational energy? With two serious problems, localization seemed hopeless to most.

Some philosophers have gone further, proposing the bold but clear nihilist view that there is no such thing as gravitational energy (Hoefer, 2000; Duerr, 2019a; Duerr, 2019b; Duerr, 2021), because nothing real could have such contradictory properties. Hence one should not say, on this view, that binary black holes or other dense bodies radiate away mass-energy during inspiral, and we detect this energy with LIGO and the like. Rather, one should simply say that binary black holes radiate gravitationally and we can detect this radiation. The bookkeeping device of energy is rejected. One can certainly express the content of Einstein's equations without discussing gravitational energy; nothing is lost except some puzzling and anachronistic bookkeeping, one might think.

Still other authors have introduced a red herring in the discussion by hinting of a sort of rival Noether theorem that supposedly licenses non-conservation in GR. These authors bring up the idea of symmetries (or the lack thereof) of the *geometry*, as though a spatio-temporally varying metric were relevant to the existence of conservation laws (Logunov and Folomeshkin, 1977; Carroll, 2010; Motl, 2010; Hossenfelder, 2016; Physics Stack Exchange, 2017; Siegel, 2018; Maudlin et al., 2020). Because the metric of GR lacks symmetries, one should not expect conservation laws, we are told. But Noether's first theorem does not know or care about geometry; it cares only about symmetries of the action (Noether, 1918) (equivalently, of the laws) or, if one permits nonvariational fields, whether the nonvariational fields have symmetries (Trautman, 1966). If it turns out, *e.g.*, that geometry is conventional—perhaps there is no fact of the matter whether the Einstein 'frame' or the Jordan 'frame' is correct in Brans-Dicke scalar-tensor gravity, partly because there is no fact of the matter whether the variable gravitational 'constant' is geometrical or material—Noether's theorem is perfectly applicable, giving tolerable results either way, because geometry is simply not a relevant category to Noether's theorems. To ask for a field with Euler-Lagrange equations to have symmetries as well in order to have conservation laws, is to require supererogation. At this point one has moved beyond denying that Noether's first theorem has a reasonable interpretation to distorting its content. Hence the nonlocalizability and nihilist views should not be cluttered with such rationalization. There *is* a connection between symmetries of the geometry and conserved quantities *that have uniqueness or coordinate independence properties that most people like*, but that is rather different from having conserved quantities *simpliciter*.

3.1 Giving Up Energy-Momentum Conservation?

One should understand clearly that the cost of either the nonlocalizability view or the nihilist view is simply *renouncing the claim that energy is conserved in general.*[4] Why so? A local conservation law worthy of the name is a law that, when spatially integrated in cases with localized matter, sufficiently rapid falloff of the fields to trivial values, *etc.*, implies secondary school chemistry claims such as $E = constant$, a global conservation law. The kind of law that can yield $E = constant$ is the continuity equation, which implies the absence of sources or sinks. But the only continuity equation that GR implies is one that involves both material stress-energy $T^{\mu\nu}$ (big T), which is usually considered unproblematic, and gravitational (pseudo-?)energy $t^{\mu\nu}$ (little t). Earlier I used the Gothic letter with one index up and one down, $\mathfrak{t}^{\mu}_{\nu}$, which is better for technical reasons (Pitts, 2022; Sorkin, 1977). Hence the best form of the continuity equation involves the Gothic[5] material energy $\mathfrak{T}^{\mu}_{\nu}$ and the Gothic gravitational energy $\mathfrak{t}^{\mu}_{\nu}$; their sum $\mathfrak{T}^{\mu}_{\nu} + \mathfrak{t}^{\mu}_{\nu}$ satisfies the continuity equation:

$$\sum_{\mu=0}^{3} \frac{\partial}{\partial x^{\mu}} (\mathfrak{T}^{\mu}_{\nu} + \mathfrak{t}^{\mu}_{\nu}) = 0.$$

One can already see that no conservation law is generally forthcoming if one tries to write it in terms of the material stress-energy only; moving the gravitational pseudotensor to the right side gives an equation for the *nonconservation* of *material* energy-momentum:

$$\sum_{\mu=0}^{3} \frac{\partial}{\partial x^{\mu}} \mathfrak{T}^{\mu}_{\nu} = -\sum_{\mu=0}^{3} \frac{\partial}{\partial x^{\mu}} \mathfrak{t}^{\mu}_{\nu} \neq 0.$$

Thus material energy-momentum fails to be conserved due to the spatio-temporal variation of the gravitational potential/space-time metric, in general. In short, energy and

[4]Duerr argues, using a notion of inertial frames, that one can be an antirealist about nonconservation where it occurs formally (Duerr, 2019a).

[5]One can (de-)Gothicize and raise and lower indices by matrix manipulations using the metric tensor $g_{\mu\nu}$. An index is moved down or up by multiplication with $g_{\mu\nu}$ or its matrix inverse $g^{\mu\nu}$, respectively. One can make a normal quantity Gothic by multiplication by $\sqrt{-g}$ (g being the determinant of the matrix $g_{\mu\nu}$), thus making it a density of weight 1. Apart from the minus sign due to space-time rather than space, this quantity $\sqrt{-g}$ is familiar in volume integration in spherical coordinates as $r^2 sin\theta$, for example. For the richer world of densities of arbitrary weight, which are often useful and sometimes important, see ((Schouten, 1954; Anderson, 1967)).

momentum just are not conserved. One can perhaps learn to accept such a conclusion, discarding what little many people remember from secondary school chemistry (I imagine), but is it necessary or advisable?

Some authors would comfort us with a material energy-momentum balance law, which can be written in a tensorial way (no pseudotensor t) in terms of the material stress-energy and the space-time metric: $\nabla_\mu T^{\mu\nu} = 0$ or some equivalent expression (recalling the games playable with Gothic letters and powers of the metric tensor to move indices). Such an equation is indeed true in GR, important, and even entailed by Einstein's field equations. But it should not provide us that much comfort. First, though some people call this a conservation law (more often "covariant conservation," which is in most circumstances a special form of nonconservation), it generically does not imply the conservation of anything like $E = constant$ (Weyl, 1922, pp. 236, 269-271) (Landau and Lifshitz, 1975, p. 280) (Misner et al., 1973, p. 465) (Lord, 1976, p. 139) (Stephani, 1990, p. 141). Even playing with Gothic letters optimally, one cannot make this equation turn into the continuity equation.[6] Second, it is not very distinctive of Einstein's equations, following simply from the way that matter and gravity couple without need for Einstein's field equations for gravity (Wald, 1984, p. 456). GR *is* distinctive in that $\nabla_\mu T^{\mu\nu} = 0$ separately follows from Einstein's equations, unlike most other theories of gravity (such as massive spin 2 gravities (Ogievetsky and Polubarinov, 1965; Freund et al., 1969; de Rham et al., 2011; Hassan and Rosen, 2012; Pitts, 2016b)); that is quite interesting. The third reason requires a more detailed discussion.

Third, the content of the energy-momentum balance equation just isn't enough to rule out the kinds of phenomena that many people have expected conservation laws to rule out. For example, it is reported that disembodied spirits make rooms cold; philosopher Stephen Braude, who accepts such phenomena and reports witnessing a table tilting in graduate school, argues that such phenomena are compatible with energy conservation due to cold breezes and changes in the weight of the medium (Braude, 1986; Braude, 1987; Braude et al., 2017). Accomplished psychiatrist Richard Gallagher informs us that in an exorcism, the demon(s) can make the room cold or hot (Gallagher, MD, 2020, p. 65). I have pointed out that local conservation laws, being more demanding than the global conservation presumably envisaged by Braude and being equivalent to the uniformity of nature as formulated in classical field theory, would exclude such phenomena (Pitts, 2020a). (Gallagher, a Catholic, might take the Evil One and his minions to be exempt from conservation laws.) What seems not to have been pointed out, however, is that the material energy-momentum balance law $\nabla_\mu T^{\mu\nu} = 0$ is perfectly compatible with the idea that spirits act on the physical world *at any and all times and places and in arbitrarily strong ways, as long as it is only on*

[6] One sees this point argued invalidly with surprising frequency even in the physics literature, but also in the philosophy literature. For electric charge one *can* turn the conservation equation into the continuity equation by playing with Gothic letters and index position (Anderson, 1967; Duerr, 2019a). One has to use a weight 1 current with index up (contravariant/tangent) in order to remove as many Christoffel symbols as possible, and then check whether any remain. For energy-momentum, one can get rid of one of the two terms but not the other one in this fashion.

space-time/gravity $g_{\mu\nu}$. This is evident from inspection of an action of the form

$$S_{gravity}[g_{\mu\nu}, \Psi_{spirits}] + S_{matter}[u, g_{\mu\nu}],$$

where u standards for all physical fields other than $g_{\mu\nu}$; the equation $\nabla_\mu T^{\mu\nu} = 0$ follows from the second term in the usual way, notwithstanding all the spiritual influence on $g_{\mu\nu}$ in the first term. One could make a similar point in terms of Humean non-uniformities rather than spirit-to-matter causation: the course of nature might change in ways that involve the dynamics of $g_{\mu\nu}$ but not the coupling of $g_{\mu\nu}$ to matter, and $\nabla_\mu T^{\mu\nu} = 0$ would still hold. The point is that $\nabla_\mu T^{\mu\nu} = 0$, though a lovely and important equation, is surprisingly weak in content in some respects, making it unable to ground some of the claims that the conservation of energy and momentum would ground and that many people have wanted conservation laws to ground.

3.2 Why Gravitational Energy Might Be Localized After All

In my view, the case for taking gravitational energy seriously has gotten much stronger in the last few decades, because new replies to the two traditional objections have become available. I start with the nonuniqueness objection, for which I see three or four possible answers. An initial reply, which appeals to the widespread acceptance of material energy $T^{\mu\nu}$, is a *tu quoque* response (Pitts, 2010): gravitational energy is not qualitatively worse off than supposedly unproblematic material energy. Even a scalar field in flat space-time suffers from nonuniqueness due to a multiplicity of comparably plausible candidates, due to the "improved" energy-momentum tensor, which has certain advantages (Callan et al., 1970). While 2 is less than ∞, it is still enough to run a parallel argument against the reality of non-gravitational energy. If that argument is not accepted, then at least the nonuniqueness objection to GR pseudotensors is blunted: nonuniqueness is compatible somehow with reality of the energy distribution in question, even if how has not been specified. A second and more powerful reply comes from the work of Nester *et al.*, according to whom different pseudotensors describe different quasi-localizations with physical meaning tied to boundary conditions (Chang et al., 2000; Nester, 2004). Thus different pseudotensors are right in different contexts. Why should the same one be required in every context, given the close relationship between pseudotensors and boundaries? A third possible reply is that there is a best One True pseudotensor. Perhaps it is the Papapetrou-Belinfante pseudotensor (Papapetrou, 1948) or a higher-tech relative thereof (Petrov and Katz, 2002). Clearly this third reply is incompatible with the second, but one can simply offer their disjunction, or even parts of each: maybe some pseudotensors are always wrong, but others are right in one context or another. A fourth reply is rooted in old work (Bergmann, 1958): nonuniqueness is not a distinct objection, but only a repeat of coordinate dependence. Bergmann noted that "the totality of all conservation laws . . . in one coordinate system is equivalent to one of them, stated in terms of all conceivable coordinate systems." (Bergmann, 1958). The "totality of all conservation laws" refers to different pseudotensors. He shows how to find the Einstein and the Landau-Lifshitz pseudotensors in his expression by choosing $\delta x^\sigma = k^\sigma$ (where k^σ is a set of constants) or $\delta x^\sigma = \mathfrak{g}^{\sigma\alpha} k_\alpha$ (where k_σ is a set of constants). Schutz and Sorkin also find both the Einstein and Landau-Lifshitz pseudotensors as special cases of their Noether operator (Schutz and Sorkin, 1977). Especially if one has a reply to the coordinate dependence objection in terms of infinitely many energies, reducing the nonuniqueness objection to the coordinate dependence objection helps.

Turning to the coordinate dependence objection, that pseudotensoriality is bad—one could phrase this in terms of pseudotensors' not being even geometric objects (Anderson, 1967, p. 428) and hence not physically real—one might consider what service is performed by a coordinate transformation rule (Pitts, 2010). It is clear that a transformation rule does not *create* physical meaning. I could take a number at each point in some coordinate system, $x^1 x^2 x^3 sinh x^0$, postulate a transformation rule (perhaps a scalar density of weight $\frac{43}{89}$, also specifying whether to take absolute values),[7] and have a geometric object: components relative to any coordinate system. (I have assumed that my initial coordinate system covers the whole manifold; otherwise the story is slightly more complicated, but no more illuminating.) A transformation rule, rather, indicates equivalence: a geometric object tells the same story in every coordinate system. A physically meaningful (or meaningless) story in one coordinate system is told equivalently, equally meaningful (or meaningless) in another coordinate system. For a gravitational energy-momentum pseudotensor, the ostensible physical meaning comes not from the transformation rule, but from the algorithmic derivation from the Lagrangian, the realistic interpretation afforded to the results for most or all other field theories, and the *prima facie* expectation of a uniform interpretation for all classical local field theories.[8] But is being a geometric object perhaps a *necessary* condition for physical reality?

It is important to ask how many conserved quantities we should expect. Noether's first theorem tells us to associate with each continuous (*i.e.*, not involving a jump like flipping a right hand into a left hand) rigid (*i.e.*, not depending on the space-time coordinates) symmetry of the Lagrangian density, a current satisfying the continuity equation. The seven most basic such symmetries are time translation invariance, spatial translation invariance (3), and the isotropy of space (3), implying the conservation of energy, of 3 momenta, and 3 angular momenta. Less obviously *a priori*, there is also a principle of relativity, whether due to Galileo or Einstein, Lorentz, Poincaré, and so on, which implies another 3 conserved currents. Thus one has 10, matching the 10 components of the Poincaré group of special relativistic fields. For massless relativistic fields, there can be a handful of additional conserved currents associated

[7]While this choice is made whimsically, sometimes peculiar fractional choices, such as $\frac{5}{9}$ or $-\frac{5}{11}$, are physically motivated (DeWitt, 1967).

[8]Some readers will notice a slight dose of particle physics egalitaritarianism here: General Relativity is presumed to be like other field theories except insofar as it is shown to be otherwise (Feynman et al., 1995; Kaiser, 1998; Pitts, 2017b; Pitts, 2020b). I take this view to be the default view and, in this case, not relevantly defeated. There are respects in which this presumption can be defeated, however, such as regarding observables (Pitts, 2017a; Pitts, 2018; Pitts, 2019).

with the Bateman-Cunningham 15-parameter conformal group. The more symmetries there are, the more conserved currents. Turning to GR, one finds that there are *far more* rigid symmetries, indeed infinitely many, of the Lagrangian density (Bergmann, 1958), implying infinitely many conserved energy-momentum currents. For any vector field, one can find a coordinate system such that its components in some neighborhood take the form $(1, 0, 0, 0)$ (time translation), $(0, 1, 0, 0)$ (spatial translation), or the like. Hence there are, at least formally, uncountably infinitely many energy conservations and momentum conservations. This point was eventually taken seriously by pointing out that if there are infinitely many rigid symmetries and infinitely many conserved currents, then pseudotensoriality makes good sense in that different coordinate systems are related to different symmetries (Pitts, 2010; Pitts, 2009). Why can't they all be real and distinct? On the other hand, the usual complaint that gravitational energy forms a pseudotensor amounts to an anti-Noetherian demand that there be only one conserved energy (4 conserved energy-momenta) even though there are infinitely many symmetries of the Lagrangian density. While Noether's first theorem is a standard part of Lagrangian field theory, the postulate $\infty = 1$ or $\infty = 4$ does not seem to me a good axiom for field theory. Nihilists, recognizing the troubles associated with postulating $\infty = 1$ or $\infty = 4$, have in effect postulated $\infty = 0$ instead. My own view is that one should avoid postulating how many conservation laws there ought to be, and simply ask Noether's first theorem how many there are; the answer is ∞, which just is not subject to evaluation based on *ad hoc* criteria. To give an analogy, one might be puzzled by the inequivalence under translation (analogous to lack of a coordinate transformation rule) between "María es alta" (tall) and "Mary is short"—unless María $\neq$ Mary, in which case there is no reason to expect equivalent heights. Hence most of the complaint about pseudotensoriality is rooted in failing to listen to Noether's first theorem how many conserved quantities to expect in GR when the theorem is volunteering the answer. By ceasing to slight Noether's first theorem, one largely ceases to worry about pseudotensoriality in general, though there might be related more specific issues to consider. Hence Schrödinger's false-negative worry is answered by noticing that there are lots of gravitational energies surrounding the round heavy body, many or most of which are not 0, so the fact that one of them is 0 hardly shows the absence of gravitational energy and so is of no concern. Bauer's false positive objection has some other answers, which can be quite technical (Pitts, 2010; Pitts, 2022).

Given that there are infinitely many conserved currents, there is no reason to expect equivalent stories in each coordinate system. Hence the failure of a pseudotensor to be a geometric object is no problem: there is no reason to expect conserved quantities derived from inequivalent symmetries to be equivalent (related as a geometric object by a transformation rule).

With plausible ideas available for responding to both of the traditional objections to gravitational energy localization $\mathfrak{t}^{\mu}_{\nu}$, one is in a better position to take seriously energy conservation

$$\sum_{\mu=0}^{3} \frac{\partial}{\partial x^{\mu}} (\mathfrak{T}^{\mu}_{\nu} + \mathfrak{t}^{\mu}_{\nu}) = 0.$$

One can also think a bit more broadly about some facets of GR, such as the fact that Einstein's field equations alone (with no separate postulation of material equations of motion/field equations) imply these conservation laws. Sometimes that is viewed as some kind of defect, as was claimed by Felix Klein (Brading and Brown, 2003). On the other hand, in Maxwell's electromagnetism one has the closely analogous result that Maxwell's equations alone (with no separate postulation of the equations for charged matter) imply the conservation of charge; this is an immediate consequence of the antisymmetry of the electromagnetic field strength tensor $F^{\mu\nu}$, the divergence of which equals the charge-current density. But no one thinks that charge conservation is therefore trivial in Maxwell's theory, to my knowledge, though there is an interesting multiplicity of derivations of charge conservation (Deser, 1972; Brading, 2002; Brading and Brown, 2003). Why think that energy-momentum conservation is trivial in GR because Einstein's equations alone imply it? This implication, rather like the presence of infinitely many conserved energy-momentum currents, suggests that GR is *more* conserving of energy-momentum than other theories, not less as one usually hears. Likewise Maxwell's theory is in some sense more conserving of charge than other electromagnetic theories. The obvious competition is Proca's massive electromagnetism, which cheerfully accepts charge conservation or nonconservation, depending on how matter behaves.

4 GR, Energy Conservation, and the Philosophy of Mind

Given the usual views that gravitational energy either is not localized or does not exist, one cannot make sense of the local conservation equation

$$\sum_{\mu=0}^{3} \frac{\partial}{\partial x^{\mu}} (\mathfrak{T}^{\mu}_{\nu} + \mathfrak{t}^{\mu}_{\nu}) = 0$$

and energy is not conserved in general. Some authors, better informed about physics than usual among philosophers, have found in this conclusion an answer to the Leibnizian conservation objection (Mohrhoff, 1997; Mohrhoff, 1999; Collins, 2008; Collins, 2011).[9] If conservation already fails given General Relativity, then there is no conservation remaining for interactionist dualism to spoil, so the usual Leibnizian objection is eliminated. This is an impressive aikido-like move, rhetorically, and shows much better grasp of the physics than usual. Unfortunately the truth is the reverse: General Relativity makes Cartesian mental causation *harder*, not easier. So I claimed some time ago

[9] See also (Penrose, 1994, pp. 334, 344-346) for a related suggestion that this peculiarity of General Relativity might help to address a conservation-related difficulty of spontaneous collapse theories. Penrose invokes gravity to induce collapse of the wave function. This project is also linked to the philosophy of mind.

based on the Noether- and pseudotensor-related considerations recalled above (Pitts, 2010). Many people are not in the habit of trusting such considerations, however, and with some reason. Can one find a more clearly tensorial argument—one that is framed in tensor calculus or at least geometric objects, not in terms of pseudotensors, and hence that clearly has the same meaning in every coordinate system?

In fact one can: a recent paper used the generalized Bianchi identities to show that General Relativity tends to exclude, not facilitate, Cartesian mental causation (Pitts, 2020c). In the simplest case, Cartesian mental influence must be spatio-temporally constant, and hence 0. The difficulty may diminish for more complicated models, however. As it turns out, the more field components worth of mental influence there are, the more readily one can evade the Bianchi identities' strictures by simply sacrificing a handful of fields to satisfy the identities by taking up the slack while the soul's other field components act on the world (such as the brain) in whatever fashion a dualist might expect. Hence GR tends to resist mental causation, but its resistance is finite.

How many field components worth of mental influence should one expect a soul to have? One possible answer is that one hasn't the slightest idea. If that is giving up too easily, then it seems difficult to answer this question without attending to larger world-view considerations. Given naturalism, it would seem that the answer is either 0 because such things could not evolve naturally, or not many because it is difficult (though not impossible) to evolve them, or likely not enough because the number of them is not brought about in a fashion with the foresight to try to circumvent the Bianchi identity restrictions. Hence it seems not very likely (at best) that souls could act on bodies given naturalism. Of course naturalists generally do not believe in souls anyway, so the new argument from GR is perhaps superfluous. On the other hand, given that most dualists are theists, plausibly God has designed (perhaps *via* evolution) souls and bodies to work together in a world that has Einstein's equations for laws (ignoring quantum gravity, which, however, is presumably relevantly similar except where the quantum aspect makes a difference). If so, then presumably souls have sufficiently many field components to act on the world even given GR. So the new argument from GR doesn't work against dualists who are also theists. Thus the new argument from GR against mental causation is unnecessary for naturalists and ineffective for theists. Almost the only targets whom it hits are dualists who are not theists, such as Karl Popper and John Beloff not that long ago. But the argument nonetheless helps one to understand the content of GR in a way that otherwise has not been so evident.

One notes that gravitational energy realism (taking pseudotensor laws seriously) has the correct heuristic force—it leads one to expect stronger restrictions on mental causation, and the Bianchi identities vindicate that expectation—whereas nonlocalization and nihilism lead one to expect looser restrictions on mental causation. Hence the new general relativistic objection provides some support for realism about gravitational energy-momentum in GR. Besides its intrinsic interest, such a conclusion might help to answer on objection to conserved quantity theories of causation (on which see (Fair, 1979; Rueger, 1998; Dowe, 2000; Curiel, 2000)): that energy, the star example of a conserved quantity, is not conserved after all. To the degree that one can take the conservation of energy seriously in GR, such an objection is no longer decisive, though difficulties remain (Pitts, 2020c).

5 Conclusion

In different ways, both philosophers of mind and philosophers of space & time have tended to slight Noether's first theorem, the former by supposing conservation to be categorical when it is really conditional upon symmetries (the failure of which is expected given interactionist dualism), the latter by ignoring or underestimating the conservation laws that GR licenses due to symmetries of its laws. The latter slighting of Noether's first theorem suggests that GR is less conserving of energy than earlier theories, which claim some have invoked as a response to the assumed categorical conservation of the Leibnizian objection. But GR is actually more conserving of energy than earlier theories, a conclusion that is motivated by the infinity of symmetries of the GR laws by Noether's first theorem, but also demonstrable in a tensorial way using the generalized Bianchi identities. Thus GR tends to resist mental causation—though the argument does not make much difference given typical naturalist or theist background beliefs. Surprisingly, thinking about the 325+-year mental causation controversy and the 100+-year gravitational energy controversy sheds considerable light in both directions.

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