Some Thoughts on Relativity and the Flow of Time: Einstein's Equations given Absolute Simultaneity

J. Brian Pitts*
Philosophy of Time Society meeting at APA Central
Chicago, April 2004

July 26, 2004

The A-theory of time has intuitive and metaphysical appeal, but suffers from tension, if not inconsistency, with the special and general theories of relativity (STR and GTR).[1] The A-theory requires a notion of global simultaneity invariant under the symmetries of the world's laws, those ostensible transformations of the state of the world that in fact leave the world as it was before. Relativistic physics, if read in a realistic sense, denies that there exists any notion of global simultaneity that is invariant under the symmetries of the world's laws. If physics is at least a decent guide to metaphysics–as sympathies for scientific realism would suggest–then relativistic physics supports the B-theory. If there were a physically natural way to modify the symmetries of the physical laws so as to remove those that are repugnant to the A-theory, while retaining empirical adequacy, then such an altered physics might be attractive to the A-theorist and would weaken the support given by relativity to the B-theory. I exhibit a way to do so here, displaying a Lagrangian density explicitly containing distant simultaneity, yet implying Einstein's field equations. The modification involves a change in the nature of the lapse function and makes use of the Dirac-Bergmann formalism of constrained dynamics, which recently has been discussed much by John Earman. Here this formalism is adapted slightly to permit both local and global generalized coordinates. A classification of senses in which time might be absolute or not is made along the way. Some suggestions for extending the work by finding a first principles motivation are made. An appendix outlines an argument why many local presents are insufficient and a global present is attractive, while two more appendices review the Dirac-Bergmann apparatus for GTR and then apply it to the theory at hand.

Relativistic physics in its usual form includes the doctrine of the relativity of simultaneity. In the special theory of relativity (STR), simultaneity is a 3-place relation between a pair of space-time points and a reference frame. (For the space-time point arguments, the relation is again reflexive, symmetric, and transitive.) When gravitation is included, one must consider the general theory of relativity (GTR). In GTR, simultaneity is defined not by a choice of three numbers as in STR, but by a choice of a function of four variables. The greater abundance of choices for defining simultaneity in GTR has interesting consequences for many purposes, but the relativity argument only relies on the fact that some merely conventional specification, whether of three numbers or of a function of four variables, or of something else, is required to define distant simultaneity.

*The author thanks Michael Rea, Tom Crisp, and Brad Monton for helpful comments on an earlier version of this paper, and Nick Huggett for commenting on the paper at the PTS meeting.

[1]Quantum nonlocality fits awkwardly with relativity, if one rejects backwards causation (Maudlin, 1994). Thus physics itself is equivocal in support of the B-theory by way of relativity. But my attention will be on classical relativity.

Which metaphysically interesting property the present has depends on the details of the A-theory in question–presentism, "moving spotlight," "growing block" or some other version–but there seems to be no interesting difference as far as the argument from relativity is concerned. Thus the relativity argument against the A-theory depends neither on which A-theory nor which relativistic theory one considers. Expressions of the tension between the A-theory and relativity have been given, for example, by the recent work of Gordon Belot and John Earman on quantum gravity (Belot and Earman, 2001), Hilary Putnam's 1967 argument from orthodox relativistic premises to the conclusion that the whole of space-time is real (Putnam, 1967), and Kurt Gödel's similar argument (Gödel, 1949) from the 1940s.[2]

It seems fairly clear there is no plausible way to modify the A-theory consistent with relativistic physics. At any rate no credible proposal comes to mind: relativized tense (or worse, relativized *existence* for presentists) is difficult, if not impossible, to understand, at least for me (though perhaps not for everyone (McCall and Lowe, 2003)). Relativistically invariant notions such as light cones and worldlines, if regarded as candidates for being the present, entail that there exist preferred spatial locations or some such, which are hardly appealing. If physics and the A-theory of time are to be made consistent, then some sort of preferred frame physical theory is the most credible option. At any rate it is the only option that I will consider. While my concern is with the A-theory in general rather than its presentist version, Ted Sider's consideration of "presentism/Minkowskian hybrids" (Sider, 2001) (pp. 45-52) illustrates the sorts proposals that I do and do not take seriously. Hybrids not to be taken seriously include "here-nowism" or any other hybrid such that the *now* is something other than a spacelike hypersurface. The hybrid to be taken seriously is some version of the "scientifically revisionary" (Sider, 2001) (p. 47) preferred frame "Hybrid 2." Being scientifically revisionary might be a bit embarrassing, but the other presentism/Minkowskian hybrids seem much worse. Some disagree that they are much worse, so an appendix will outline how one might try to argue for a global present, given the local nows of conscious beings.

1 Objective but Unobservable Simultaneity?

One might try to avoid the relativistic disconfirmation of the A-theory by admitting that current physics does not manifest absolute simultaneity, but asserting that an ideal completed physics would indeed detect absolute simultaneity. Thus the true simultaneity is observable, but is too subtle to have been observed yet, at least not unambiguously. In contrast to such bravery, one might instead argue shrewdly for objective simultaneity which happens to be unobservable. The unified four-dimensional world of Minkowski space-time, though simple, empirically adequate, fruitful, and beautiful, might simply fail to exist, because reality might be, for example, a three-dimensional objectively changing realm whose laws of physics just hide the absolute now. Lorentz held views that were not too far from this one. More recently, views somewhat along these lines have been defended by Quentin Smith (Smith, 1993), Nataša Rakić (Rakić, 1997), and William Lane Craig (Craig, 2001).

But it seems unnatural for a metaphysically preferred foliation to exist and yet be unobservable, or even absent from physics altogether. Rakić considers a criticism along these lines: "One possible criticism of our approach is that the new primitive relation R on the events of Minkowski space-time does not have a place in our physical theory." (Rakić, 1997) (p. 276) This tension is an example of Larry Laudan's notion of a conceptual problem (Laudan, 1977). The physicist John Bell, famed

[2]One could also consider his now-famous general relativistic objection to the A-theory of time using a space-time with closed timelike curves (Gödel, 1949). The natural answer to this objection seems to be to deny the physical possibility of such models, perhaps by regarding global hyperbolicity as a law of nature, as will appear below.

for deriving the Bell inequality that made local hidden variable theories refutable, wrote of such a scenario: "As with relativity before Einstein, there is then a preferred frame in the formulation of the theory... but it is experimentally indistinguishable. It seems an eccentric way to make a world." (Bell, 1987) (p. 180, emphasis in the original) Philosopher Tim Maudlin, in attempting to make sense of quantum mechanics and reconcile it to relativity, suggests that backwards causation or a preferred reference frame might be the least unacceptable ways of doing so. Concerning the possibility of a preferred frame, Maudlin, perhaps recalling Einstein's line that God is subtle but not malicious, writes: "One way or another, God has played us a nasty trick. The voice of Nature has always been faint, but in this case it speaks in riddles and mumbles as well. Quantum theory and Relativity seem not to directly contradict each other, but neither can they easily be reconciled. Something has to give: either Relativity or some foundational element of our world-picture must be modified ... the real challenge falls to the theologians of physics, who must justify the ways of a deity who is, if not evil, at least extremely mischievous." (Maudlin, 1994) (p. 242)

It would be interesting to have an adequate explanation of the unobservability of absolute simultaneity. If such an explanation sounds difficult to achieve, one should recall the remarkable but well-known precedent of the "spin 2" derivations of Einstein's equations (Kraichnan, 1955; Feynman et al., 1963; Pitts and Schieve, 2001b) (and references therein), in which plausible physical principles conspire to merge the flat background metric and the gravitational potential into a curved effective metric, so the original flat metric disappears from the field equations. Closer to the present issue is the interesting work of Julian Barbour and collaborators (Barbour et al., 2002; Anderson and Barbour, 2002; Anderson et al., 2003). While Barbour reads his project as the elimination of time (which A-theorists will not like), along the way he provides a sort of first-principles derivation of a theory very similar to GTR empirically, if not identical, and yet containing absolute simultaneity. It seems worth considering whether an A-theorist might adopt part of Barbour's program, while rejecting his claim to eliminate time.[3] One should also mention the work of Brad Monton (Monton, 2001), for which, however, an attractive derivation from a Lagrangian density seems not yet to have been given, so it is difficult to regard it as a complete physical theory.

If there is not (or not yet) any derivation of Einstein's equations from first principles friendly to the A-theory, it would still be interesting to do so from "second principles," so to speak. That is, one could aim simply to postulate (not derive) the key mathematical entity, the Lagrangian density $\mathcal{L}$, from which the theory's field equations are derived. More specifically, one could try to modify the Lagrangian density of GTR to contain absolute simultaneity and yet still yield Einstein's equations, if possible. It turns out that such a Lagrangian density does exist. Furthermore, one can write down a family of examples explicitly. Explaining and demonstrating these facts will require some mathematics, but there are several benefits to using the mathematics. First, the Arnowitt-Deser-Misner (ADM) split of space-time into space over time is a simple bit of algebra that is very

[3] The viability of this suggestion is indicated by comments at a recent lecture, "Absolute and Relative Motion: A Review," which Barbour gave at the European Science Foundation conference on Space-Time at Oxford University, March 27, 2004. Barbour takes configuration space (as opposed to configuration space cross time), to give an adequate description of a system. But what if the curve in configuration space intersects itself, as it does for periodic systems? According to Barbour, a periodic system executes its motion *only once*, the infinitely many extra copies of the motion being mere gauge fluff. This is a remarkable claim, which presumably follows from a strong diachronic application of the identity of indiscernibles. While one might easily reject the claim to eliminate time on this ground alone, one might also reject it because the argument needs a suppressed premise: that there exist no temporal immaterial beings such as the God of Newton and of many contemporary theists, such as William Lane Craig. Such a God would certainly know the difference between one oscillation and infinitely many, unless perchance his activity were also periodic (which seems unlikely). Thus a theistic A-theorist such as Craig might avail himself of Barbour's physical derivation of Einstein's equations with a preferred foliation, while rejecting Barbour's elimination of time. The theistic A-theorist might even thank Barbour for reviving rather than eliminating time. I thank Steve Savitt for discussing parts of this issue.

useful in describing GTR as a theory describing the time evolution of something, as opposed to four-dimensional geometry. The ADM split, which is a standard technique in finding approximate numerical solutions of Einstein's equations by computer, provides a good language for making useful distinctions in the philosophy of time that otherwise might fail to be made. Examples will appear below. Second, the Dirac-Bergmann formalism for constrained dynamics (Sundermeyer, 1982), as John Earman has emphasized repeatedly (including at PSA 2002), is well-nigh indispensable for discussing gauge theories in a precise and sophisticated way (Earman, 2003). In the interest of brevity and accessibility, I relegate the heavy mathematics to the appendices, while merely asserting a few key results in the body. Third, the use of the ADM split and Dirac-Bergmann formalism will make clear how my derivation modifies the usual action principle derivation of Einstein's equations, and yet manages to obtain all of Einstein's equations anyway. In the process, a partial hierarchy of notions of 'absoluteness' for time will be developed. It would be difficult to conceive, describe or justify these various notions without mathematics.

2 ADM Split of Space-time into Space over Time

The ADM split of space-time into space over time (Misner et al., 1973) is a helpful kinematical language for talking about GTR and related theories in a dynamical way. While the ADM split of space-time involves only simple algebra, it is applicable only for causally well behaved space-times. Globally hyperbolic space-times are sufficient, and probably necessary as well. Global hyperbolicity basically is equivalent to the truth of Laplacian determinism for a theory, at least assuming that one takes equivalence classes over the relevant symmetries (as physicists reflexively do) to address the hole argument (Earman and Norton, 1987). Similar requirements on the causal structure exist for Hamilton dynamics to be applicable. One recalls that there exist solutions of Einstein's field equations of general relativity which do not admit any good definition of simultaneity, due to peculiar global properties like nontrivial topology or closed timelike curves. Kurt Gödel famously proposed the latter sort of solution as an objection to the A-theory of time (Gödel, 1949). However, there is nothing like a demonstration that solutions of this sort, *including the causally ill-behaved regions,* are physically possible. The offensive outer reaches of the Gödel spacetime might perhaps be removed while matching the inoffensive interior to some well behaved exterior solution (Bonnor et al., 1998), should the Gödel solution be needed locally. The A-theorist defends a common-sense view of time. At least some portion of the common-sense view of time nowadays, perhaps, is Laplacian determinism, which is tied to global hyperbolicity. Thus the A-theorist might naturally restrict attention to globally hyperbolic spacetimes, which plausible suffice for ordinary needs (Wald, 1984) (p. 202). Such a conjecture is a strong form of the Cosmic Censorship Hypothesis (Earman, 1995).

There being generally no preferred time coordinate for effecting the splitting of space-time into space over time, one picks an arbitrary time coordinate instead, by convention. Thus the four-dimensional coordinate/gauge symmetry is obscured, though the three-dimensional spatial symmetry is still manifest. In an ADM split (Misner et al., 1973) (pp. 506-8), the lapse function $N(x,t)$ describes the relation between physical time and coordinate time. For a worldline orthogonal to a given simultaneity slice, a unit temporal coordinate length corresponds to a proper physical time lapse of N, justifying the name "lapse function" for N. The shift vector field $\beta^i(x,t)$ describes how the spatial coordinate system shifts relative to the timelike vector orthogonal to the simultaneity hypersurfaces. The spatial metric $h_{ij}(x,t)$ is merely the spatial part of the space-time metric $g_{\mu\nu}$. The spatial indices are lowered by h_{ij} and raised by the inverse spatial metric h^{ij}. The space-time metric $g_{\mu\nu}$ and inverse space-time metric $g^{\mu\nu}$ and inverse space-time metric are given by, respectively,

$$\begin{bmatrix} g_{00} & g_{0j} \\ g_{i0} & g_{ij} \end{bmatrix} = \begin{bmatrix} \beta^k \beta^l h_{kl} - N^2 & N_j \\ \beta_i & h_{ij} \end{bmatrix}, \quad \begin{bmatrix} g^{00} & g^{0j} \\ g^{i0} & g^{ij} \end{bmatrix} = \begin{bmatrix} -N^{-2} & \beta^j N^{-2} \\ \beta^i N^{-2} & h^{ik} - \beta^i \beta^j N^{-2} \end{bmatrix} \tag{1}$$

Finally, the determinant g of the components of the space-time metric (a space-time scalar density of weight 2) is related to the lapse and the determinant h of the spatial metric by

$$\sqrt{-g} = N\sqrt{h}.$$

Any repeated pair of repeated indices is summed over, from 0 to 3 for Greek indices or 1 to 3 for Latin indices.

3 Metaphysical Significance of Choice of Lapse Function N

Using an ADM split, one can write GTR in a Hamiltonian form (Wald, 1984; Sundermeyer, 1982; Misner et al., 1973). The most relevant features concern the lapse $N(x,t)$. Apart from weak requirements such as positivity and perhaps some boundary conditions, the lapse is an arbitrary function, whose variation in the action principle to get the field equations implies the "Hamiltonian constraint"

$$\mathcal{H}_0(x) = 0, \tag{2}$$

which is basically the 00 component of the Einstein field equations. Using the Dirac-Bergmann algorithm, one infers that the time evolution of N in GTR is arbitrary. In short, the function N is gauge freedom, a merely conventional choice made in describing the physics. The fact that $N(x,t)$ varies with spatial position and over time in a merely conventional way reflects the lack of absolute simultaneity in the theory, because one can make push simultaneity hypersurfaces forward faster in one place than another at one's whim. (The picture here is perhaps like a growing block or moving spotlight theory, except that one should imagine the front edge of the block to be wiggly and merely conventional in shape. This feature has been called "many-fingered time" (Misner et al., 1973).) The natural reading is that there is no such fact as absolute simultaneity, but merely a variety of choices of slicing of space-time into families of hypersurfaces. One can speak of simultaneity relative to a conventional choice of slicing, but nothing of metaphysical significance should be relative to a conventional choice. The merely conventional status of the lapse N is not a property of the ADM split itself, but rather is a consequence of describing GTR in particular using the ADM split. A different theory might entail that N was less conventional and thus license a different metaphysics from the B-theory suggested the GTR's lack of absolute simultaneity.

One can imagine a variety of notions of time intermediate between that of GTR and Newton's absolute time. STR's relativity of simultaneity to a choice of reference frame, which gives a 3-parameter family of simultaneity definitions, will not make this particular list. Evidently the set of intermediate notions of time between GTR and Newtonian time is only partially ordered. Let us recall that in Newtonian physics (or metaphysics), "absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external" (Earman, 1989) (p. 20). Plausibly Newton did not believe anything about space-time as such, though one can usefully invent a "Newtonian space-time" that reflects many of the properties (with exceptions such as temporal flow) that Newton ascribed to space or time (Earman, 1989). Newtonian space-time has no space-time metric, but one can use Newton's description of time as poetic inspiration for distinguishing an hierarchy of properties of space-time slicings into simultaneity hypersurfaces.

It turns out, using the Dirac-Bergmann formalism of constrained dynamics, that a certain family of lapses with absolute simultaneity gives results *empirically equivalent to GTR*, at least in the field equations. It should be emphasized that such specification of N is imposed *before* performing the variations to get the equations of motion, and so differs in principle from merely imposing a coordinate condition ("gauge fixing") in GTR after the fact to give a convenient conventional slicing of space-time. Building absolute simultaneity into the Lagrangian density and hence the laws ensures that absolute simultaneity exists for every possible model, unlike the "cosmic time" of some cosmological models that some A-theorists have employed, which exists only in the highly idealized case of perfect homogeneity, which excludes planets and human bodies.

Concerning the hierarchy of notions of simultaneity, at one extreme is (with mild abuse of terminology) the generally covariant sort of slicing. This slicing is merely conventional and so generically lacks all three properties of the properties to be considered shortly. At the opposite extreme is the choice $N = 1$, which has all three of them. To be specific, the condition $N = 1$ entails that the space-time slicing is nonconventional, independent of the physical state of the world (briefly, "independent"), and uniform.[4] The property of nonconventionality is just the fact that the $N = 1$ theory's space-time slicing is invariant under those conventional redescriptions that are the symmetries of the theory, which turn out to be spatial diffeomorphisms and rigid time translations. The $N = 1$ slicing is independent just because the number 1 is indifferent to the winds of fortune: however the physical state of the world changes, the number 1 does not change. Finally, the $N = 1$ slicing is uniform because there is a preferred labeling of the absolute simultaneity hypersurfaces (up to the zero point). Between two instants T and $T + 1$ just as much absolute time passes as between $T + 1$ and $T + 2$–hence the majestic capitalization reflecting the metrical significance of the time coordinate T. To have a name, let us call this a fixed foliation. If one set $N = 1$ in the Lagrangian density of GTR, then not all of Einstein's equations would obtain, so this possibility does not fulfill the purpose at hand.

A slightly relaxed view would be setting $N = f(t)$, where $f(t)$ is some function of time, but not a function of space. This choice gives a nonconventional and independent slicing. However, if one substituted this choice into the Lagrangian density$\mathcal{L}$ used here, the functional form of $f(t)$ turns out (using the Dirac-Bergmann formalism) to be merely conventional, so the labeling has no metrical significance. One can transform to another time coordinate t' given by a strictly increasing function $t' = f(t)$ (perhaps requiring that $|f(t)| \to \infty$ as $|t| \to \infty$ as well), and the new description is just as good as the old one. This sort of symmetry is also possessed by "Machian space-time" (Earman, 1989).

Introducing a nondynamical spatial[5] scalar density e of weight 2, one can define an even more relaxed choice of slicing by $N = \sqrt{e/h}^{w} f(t)$ for $w \neq 0$. Given this choice of slicing, it turns out that $f(t)$ is conventional, but depends only on time. Clearly $\sqrt{e}^{w}$ is nonconventional (apart from transforming as a spatial scalar density) and independent. The factor $1/\sqrt{h}^{w}$ is nonconventional but dependent, because it depends on the state of the world, or, to be specific, on the state of the spatial metric. All told, this choice of lapse gives a slicing that is nonconventional, but neither independent nor uniform.

[4]The poetic inspiration came from the respective terms "absolute," "without regard to anything external," and "equably" in Newton's description of absolute time. At least some of the three properties listed were likely never conceived by Newton, however, and "absolute" has too many meanings already to justify using it here. So it seemed best to introduce different terms. For me the modern translation was not so poetically inspiring: "Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly..." (Newton et al., 1999) (p. 408).

[5]It is natural (though not essential if one compensated in the transformation properties of $f(t)$) to take e to be invariant under time reparametrizations.

For these last two cases, one has a notion of simultaneity hypersurfaces that is nonconventional but not metrically significant. Let us call these two cases "reparametrization invariant." This term is borrowed from a version of particle mechanics in which there is no preferred time variable; the relativistic free particle (Sundermeyer, 1982) in manifestly Lorentz-covariant form naturally is reparametrization invariant.[6] As I apply the term here to a theory with field degrees of freedom, the term denotes the possibility of relabeling the simultaneity hypersurfaces, but not of altering their shapes, by convention. Whether the shapes can be altered by physical dynamics differs between the two cases (independent or dependent). Clearly reparametrization invariance is much less slicing freedom than general covariance. This freedom is also less than that of Lorentz covariance in the sense that simultaneity hypersurfaces are fixed, but more than Lorentz covariance in the sense that the hypersurface labels lack metrical significance, in comparison to STR. This straddling shows why simultaneity notions between GTR's and Newton's are only partially ordered. In the interest of generating terminology permitting fine distinctions, I will call the case $N = f(t)$ "independent reparametrization invariance" and the case $N = \sqrt{e/h}^{w} f(t)$ "dependent reparametrization invariance." The independence, as before, refers to the slicing's being independent of the state of the world, and likewise for the term "dependent."

While one can imagine substituting various forms of N into the Lagrangian density for GTR, there is no reason to think that the result will still give Einstein's field equations. A theory that fails to give Einstein's equations will likely not be empirically adequate, whereas one that gives Einstein's equations will be (at least locally). Thus one must consider whether a given altered form of N will in fact give Einstein's equations. Substituting the lapse form $N = \sqrt{e/h}^{w} f(t)$ into the Lagrangian density for GTR gives a theory in which one varies not $N(x,t)$ as an independent variable, but $f(t)$. (e, being an absolute object, is not varied at all.) Having only one number $f(t)$ at each moment, rather than one at each point in space as in GTR, one gets only one constraint, the integrated form

$$\int d^3x \sqrt{e/h}^{w} \mathcal{H}_0(x) = 0, \tag{3}$$

as becomes clear in the appendix B. At this stage the theory looks as if it will not give all of Einstein's equations because only this single integrated constraint has arisen instead of the ∞^3 constraints of GTR,[7] in which case the theory might be empirically inadequate. However, the Dirac-Bergmann algorithm requires ensuring that all constraints, if satisfied initially, remain so over time. Assuming $w \neq 0$, ensuring constraint preservation turns out to entail that the full Hamiltonian constraint $\mathcal{H}_0(x) = 0$ holds: one equation at each of the ∞^3 points in space, not just one equation)! (The derivation is in the appendix. One can show the same thing using the Bianchi identities carefully and patiently.) The simultaneity hypersurfaces are sufficiently bent and wiggly that the spatial constraints and the integrated Hamiltonian constraint entail the full Hamiltonian constraint. It is noteworthy that the quantity $\alpha = N/\sqrt{h}$ has recently been seen as useful in the numerical relativity literature (Anderson and J. W. York, 1998) and has even been given a name, the "slicing density." Choosing $w = -1$, one gets a particular theory reproducing Einstein's equations and having the slicing density given by $\alpha = f(t)/\sqrt{e}$, so that the slicing density depends (nontrivially) only on time. For the case $w = 0$, a theory empirically inequivalent to Einstein's results because the full Hamiltonian constraint does not arise, whereas any choice $w \neq 0$ gives a theory with all of Einstein's equations. Thus any choice $w \neq 0$ gives an alternative theory that

[6] One can also make nonrelativistic particle mechanics reparametrization invariant by introducing a dummy time variable artificially. Sometimes "reparametrization invariant" is used as a synonym for "generally covariant" in field theories, but that is not my usage.

[7] Speaking of ∞^3 constraints is merely a formal shorthand for one constraint at each point in space, not a claim regarding transfinite arithmetic.

is (at least locally) empirically equivalent to Einstein's equations, while being friendly to the A-theory. While the empirical equivalence demonstrated is only local, it seems plausible that these new theories will be empirically adequate globally.

4 Philosophical Import of Derivation

Starting from a Lagrangian density $\mathcal{L}$ that manifestly refers to a preferred foliation of space-time and hence to absolute simultaneity, I have derived (in appendix B) Einstein's field equations. Thus the theory is empirically equivalent to Einstein's GTR, at least locally. To be more accurate, an uncountably infinite family of such theories has been discovered, labelled by w, which differ among themselves by having different slicings of space-time. Given the A-theory of time, these theories will be distinct from each other in ascribing simultaneous presentness to different sets of space-time points. If two firecrackers F_1 and F_2 are ignited at spacelike separation, then some theories might imply that F_1 becomes present before F_2,whereas other theories might imply that F_2 becomes present before F_1. Given an ontology of presentism, the different theories will generally disagree on what exists, even though they yield the same space-times (locally at least). Given the starring role played by h in containing the scale factor $a(t)$ in Big Bang cosmological models, these different theories might show their differences in cosmology, except that the assumed homogeneity and the merely conventional choice of the labelling t will tend to mask these differences. One could find here a case of the underdetermination of scientific theories by data.

More important than the multiplicity of such theories is the fact that *at least one* exists that reproduces the field equations of GTR, while licensing the absolute simultaneity coveted by the A-theory. While it is a mistake to limit a theory's content to its field equations, still they do give the bulk of the content, especially when one considers the rather tame space-times most relevant in applications, or which are consistent with the assumption of global hyperbolicity. One can has a theory of gravity that explains all the empirical success of GTR, yet which is friendly to the A-theory by containing absolute simultaneity. This approach therefore avoids two weaknesses of the approaches of Smith (Smith, 1993), Craig (Craig, 2001), and Rakić (Rakić, 1997). First, in those approaches the absolute simultaneity relation does not appear in the physical theory. Second, those approaches do not readily generalize from STR to GTR. The specification of the set of truly simultaneous events now suffices to define all later (and earlier) such sets in STR. By contrast, in GTR the many-fingered nature of time implies that adding a present true simultaneity hypersurface does not determine earlier and later ones, as the hole argument shows (Earman and Norton, 1987). Moreover, cosmic time is inadequate, because it is approximate and contingent, as Yuri Balashov and Michel Janssen remind us (Balashov and Janssen, 2003) *pace* Craig, whereas true simultaneity should be exact and necessary. The theories presented here address the issue of absolute simultaneity in GTR, from which there solution in STR follows as a limiting case.

What is the philosophical significance of this derivation? Starting from a Lagrangian density that manifestly exhibits absolute simultaneity, one derives as a consequence that Einstein's field equations are satisfied and thus that the absolute simultaneity is unobservable. The many-fingered time of general relativity is reduced to a merely reparametrization-invariant theory, a theory with a preferred ordered set of simultaneity hypersurfaces but no preferred labeling thereon. Such a theory perhaps violates the "flows equably without relation to anything external" clause of Newton's view of time, but still provides the objective absolute simultaneity required by the A-theory of time. What has not been done, however, is to derive the Lagrangian density itself from *a priori* plausible first principles.

5 Conclusion

Relativistic physics causes a difficulty for A-theorist in that the lack of absolute simultaneity in STR and GTR casts doubt on the objectivity of temporal becoming. I have demonstrated the possibility of deriving Einstein's equations using a modified Lagrangian density $\mathcal{L}$ from GTR, where the new $\mathcal{L}$ contained absolute simultaneity. This attempt yielded a family of such theories described by a parameter w for $w \neq 0$. The lack of absolute simultaneity in relativistic physics is thereby made somewhat less worrisome by the provision of (at least locally) empirically equivalent alternative theories with absolute simultaneity. The ADM split used provided a language that made expressible an hierarchy of notions of 'absolute' time, while the Dirac-Bergmann formalism for constrained dynamics was very useful for the derivation. It turns out that there is a notion sufficiently 'absolute' to ground the A-theory, yet 'relational' enough to yield all of Einstein's equations. If the empirical success of relativistic physics still confirms the B-theory to some degree, that degree of confirmation is reduced by this work.

The interest of the theories proposed is primarily as a proof of concept, not as a serious rival for GTR. By contrast, the work of Barbour and collaborators (Barbour et al., 2002; Anderson and Barbour, 2002; Anderson et al., 2003), suitably interpreted gives a different and better motivated theory involving Einstein's equations and a preferred slicing of space-time. The A-theorist would like to have an A-theory friendly derivation along these lines. Some possibilities worth exploring include the following. A generalization of the spin 2 derivations of Einstein's equations making use of the absolute simultaneity hypersurfaces, along the lines of ((Pitts and Schieve, 2001a)), but with a larger gauge group including temporal reparametrization invariance, might prove capable of yielding Einstein's equations in a fashion friendly to the A-theory. Modifying the Poincaré or translational gauge theories of gravity (Utiyama, 1956; Hehl et al., 1976; Blagojevic, 2001) so as to preserve simultaneity, if possible, might be another option. Finally, investigating the set of theories invariant under the simultaneity-preserving coordinate transformations

$$\begin{aligned} t &\rightarrow F(t), \\ x^i &\rightarrow F^j(t, x^i), \end{aligned} \tag{4}$$

might be fruitful. The success of one or more of these approaches might give a satisfactory explanation of the apparent relativistic symmetry of general covariance, and do so using a standard action principle. On the other hand, their failure might strengthen the relativistic argument against the against the A-theory of time.

6 Appendix A: Why a Global Present instead of Many Local Presents?

Above I claimed that a preferred frame theory is less bad for the A-theorist than are alternative presentism/Minkowskian hybrids. Not everyone shares this belief, for some people take the local presents licensed by relativity to suffice. Among these skeptics of a global present are my commentator Nick Huggett (whose comments are in this volume), Don Howard (whose comments were made on another occasion[8]), and James Hartle (Hartle, 2004). It seems appropriate to sketch an outline of an argument for a global present, starting from local presents, so that my physical project will seem well motivated.

[8]Comment in response to my "Reconciling Relativity with Tensed Time," Mephistos 2001 conference, March 30-April 1, History and Philosophy of Science Program, University of Notre Dame.

The relativistic twin paradox is the context for one argument sketch. If one twin stays home while the other travels, the travelling twin will be younger upon his return than the homebody sibling. If the travelling twin merely takes an airplane around the world, this age difference will be negligible and well within the modest precision of common-sense judgments of presentness, as Huggett notes. But if the travelling twin takes a rocket into distant space and then returns, he might be years younger than his homebody sibling. Let us assume that each twin has his own local flow of time and that it is given by his relativistic proper time. Let the traveller leave at age 30, and return at age 35 to meet his 40-year-old twin sister. Though the twins' presents matched adequately before the traveller left, after his return their presents disagree by five years even when they embrace. It would be most peculiar if, upon the traveller's return, the two twins' consciousnesses are years apart when their bodies embrace; then each would encounter not his real twin, but an unconscious zombie (in basically the sense of David Chalmers (Chalmers, 1996)). Such a result is, I take it, a *reductio ad absurdum*. To be sure, the space travelling twin story is merely a thought experiment, unlike the airplane case. But this story (apart from the talk of presents) is the conventional wisdom about the twin paradox in relativity. If this conventional wisdom is accepted, then giving each person his own local flow, and having it correspond to relativistic proper time, leads to the absurdity of zombies. While flowing proper time is adequate for one person, the important condition of intersubjectivity causes trouble. The use of a singular pronoun in Hermann Weyl's famous comment is revealing:

> The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time. (Weyl, 1949) (p. 116)

One might try to avoid a global present by disconnecting the local presents from proper time, to permit the two twins' presents to remain spacelike separated. While this move could remove the zombie threat, it already gives up the strong reading of relativity that presumably was intended to be preserved. One also wonders whether any natural way to keep these presents spacelike separated can be found, other than embedding them in a present that is global (or at least occupies vast regions of space). In addition to the worldlines that the two twins actually occupy, one might further consider counterfactual histories in which one or both of the twins have different world lines, or there are more people in the universe. A global present has a ready-made way to keep all these presents spacelike separated, whereas stories involving only local presents do not. Thus it is far from clear that relativistically respectable resources—local presents defined by relativistic proper time—can yield a credible intersubjective present for cosmic versions of the twin paradox scenario. An obvious solution is a global present. (Issues might be more complicated in some general relativistic spacetimes, so perhaps the present would only be vast rather than global.)

A second argument sketch for a global present involves God, at least if there are good reasons to believe that God exists or one already believes in God. If the community of persons with an intersubjective present involves a person who is spatially omnipresent, then a global present follows rather quickly. Thus Poincaré writes[9]:

> We should first ask ourselves how one could have had the idea of putting into the same frame so many worlds [comprised of different persons' mental lives with psychological times] impenetrable to one another. We should like to represent to ourselves the external universe, and only by so doing could we feel that we understood it. We know we can never attain this representation: our weakness is too great. But at least we desire the

[9] I thank Mark Jensen for help in finding this reference.

ability to conceive an infinite intelligence for which this representation could be possible, a sort of great consciousness which should see all, and which should classify all *in its time,* as we classify, *in our time,* the little we see.

This hypothesis is indeed crude and incomplete, because this supreme intelligence would be only a demigod; infinite in one sense, it would be limited in another, since it would have only an imperfect recollection of the past ... And yet when we speak of time, for all which happens outside of us, do we not unconsciously adopt this hypothesis; do we not put ourselves in the place of this imperfect god; and do not even the atheists put themselves in the place where god would be if he existed? (Poincaré, 1913)

Poincaré proceeds to reject this hypothesis as inadequate, because his purposes are practical rather than metaphysical. But our lack of access to divine simultaneity is no obstacle for the metaphysician. Along similar lines, Lorentz wrote:

A 'world soul' who without being bound to a definite place would penetrate the whole considered system or 'in whom' this system would exist and who could directly 'sense' all phenomena, evidently would immediately prefer a particular one of the systems U, U' *etc.* (Illy, 1989) (p. 275).

(See also (Kox, 1988) (pp. 74, 75) and (Craig, 2001) (pp. 175-177).) Illy explains that "U and U' are synchronized clocks in systems of translational motion relative to each other." (p. 287) God is taken to be omnipresent on both orthodox monotheistic and process conceptions. Thus an omnipresent God who is also temporal would yield a global present. If there are good reasons for an A-theory about finite persons, then divine omniscience will presumably require that God also be temporal (Kretzmann, 1966). Thus theists will have a reason to posit a global present, not merely local presents.

Whether a theist or not, one also presumably needs to ground intersubjective simultaneity in the context of cosmic twin paradox scenarios, and a global present appears to be the one obvious solution. It therefore seems plausible, though not absolutely certain, that the experience of temporal passage, *etc.*, that one should infer a global present, not merely local presents. It seems appropriate, then, to want absolute simultaneity to appear in the laws of physics.

7 Appendix B: GTR in Hamiltonian Form

A review of GTR in Hamiltonian form might be convenient, especially as it facilitates comparing the new theories (to be discussed in the next appendix) with GTR. Using now-standard techniques (Misner et al., 1973; Wald, 1984; Sundermeyer, 1982), one can express the four-dimensionally invariant Hilbert action for GTR $S_H = \int R\sqrt{-g}d^4x$, built using the four-dimensional Ricci curvature scalar $R[g_{\mu\nu}]$, as a boundary term (which does not affect the field equations) and a manifestly three-dimensionally invariant action $S = \int \mathcal{L}d^3xdt$, where (suppressing the dependence on x and t)

$$\mathcal{L} = N\sqrt{h}(K^{ij}K_{ij} - K^i_iK^j_j + R[h_{ij}]). \tag{5}$$

One can try to go over to the Hamiltonian formalism using the recipe above for defining canonical momenta and then the $\mathcal{H}$ out of $\mathcal{L}$. The generalized coordinates will be the fields h_{ij}, β^i, and N, which have time derivatives $\dot{h}_{ij}$, $\dot{\beta}^i$, and $\dot{N}$, respectively. One can define canonical momenta $\Pi^{ij}(x)$, $P_i(x)$, and $P(x)$, respectively as

$$\Pi^{ij}(x) = \frac{\partial\mathcal{L}}{\partial\dot{h}_{ij}}, \quad P_i(x) = \tfrac{\partial\mathcal{L}}{\partial\dot{\beta}^i}, \quad P(x) = \frac{\partial\mathcal{L}}{\partial\dot{N}}.$$

The next step is to define the Hamiltonian density using

$$\mathcal{H} = \Pi^{ij}(x)\dot{h}_{ij}(x) + P_i(x)\dot{\beta}^i + P(x)\dot{N}(x) - \mathcal{L}, \tag{6}$$

and then eliminate the $\dot{h}_{ij}$, $\dot{\beta}^i$, and $\dot{N}$ in favor of the momenta by inverting the equations $\Pi^{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}}$, $P_i = \frac{\partial \mathcal{L}}{\partial \dot{\beta}^i}$, $P = \frac{\partial \mathcal{L}}{\partial \dot{N}}$. However, this last step of inversion is impossible for $\dot{\beta}^i$ and $\dot{N}$. In other words, the Legendre transformation cannot be performed.

Here the fact that we need constrained Hamiltonian dynamics (Sundermeyer, 1982), not ordinary Hamiltonian dynamics, makes itself known. In theories such as Maxwell's electromagnetism or Einstein's gravitation, one uses the formalism of constrained dynamics, which was developed especially by Paul Dirac and Peter Bergmann during the 1950s, to handle "constraints," physical quantities which equal 0 when the equations of motions are satisfied. In the present case $\mathcal{L}$ does not depend on $\dot{\beta}^i$ and $\dot{N}$, so the canonical momenta $P_i(x)$ and $P(x)$ are just 0. These quantities are called "primary constraints" (not to be confused with the orthogonal classification of first-class constraints). When the canonical momenta cannot all be solved for the velocities, the constrained dynamics formalism directs one to calculate $\mathcal{H}$ formally in the same fashion anyway, using the primary constraints $P_i = 0$ and $P = 0$ to reduce the expression to

$$\mathcal{H} = \Pi^{ij}\dot{h}_{ij} - \mathcal{L}. \tag{7}$$

One obtains the Hamiltonian[10] H in the usual fashion $H = \int d^3x\mathcal{H}$.

The hidden time transformation symmetry of GTR manifests itself in some of the constraints of the theory. Constraints are functions of the coordinates, momenta, and spatial derivatives of the same, such that these functions must equal 0. Some of the field equations might just be constraints, while some constraints might be entailed by the field equations less directly. Constraints arise for theories, such as Maxwell's electromagnetism and Einstein's GTR, for which the action S of the theory is invariant under transformations of the field variables depending on one or more arbitrary functions.

For GTR, the Hamiltonian takes the form

$$H = \int d^3x[N(x)\mathcal{H}_0(x) + \beta^i(x)\mathcal{H}_i(x)], \tag{8}$$

where $\mathcal{H}_0$ and $\mathcal{H}_i$ are functions of h_{ij}, Π^{ij}, and their spatial derivatives.

As one notices either by varying $N(x)$ and $\beta^i(x)$ or by demanding that the primary constraints $P(x) = 0$ and $P_i(x) = 0$ have vanishing time derivatives, the quantities $\mathcal{H}_0$ and $\mathcal{H}_i$ are themselves constraints: the theory requires that $\mathcal{H}_0(x) = 0$ and $\mathcal{H}_i(x) = 0$. Given that these constraints have arisen at a later stage than the primary constraints did, they are "secondary constraints." The remarkable feature of the Hamiltonian for GTR is that it is made up entirely of constraints, a fact responsible for most of what is novel and difficult about time in general relativity, including "frozen" aspect of apparent lack of time evolution in "problem of time" in quantum gravity (Earman, 2002). One must also demand that the secondary constraints be dynamically preserved in the sense of having zero time derivatives. (In practice one generally smears out the constraints by integration and multiplication with some arbitrary prescribed functions, in order to handle the

[10] This quantity H is actually the "canonical Hamiltonian" (Sundermeyer, 1982), a nonredundant term in this context, where it is contrasted with the "primary Hamiltonian", which generates the time evolution *via* Poisson brackets. However, this distinction does not matter for our purposes. Furthermore, all boundary terms are ignored, so the Hamiltonian is not equal to the energy (Sundermeyer, 1982). Sometimes greater care pays off, however: the problem of the frozen formalism (Earman, 2002), it has been argued (Salisbury, 2003; Salisbury et al., 2000), can be solved by being sufficiently careful in the canonical formalism.

Dirac delta functions more readily.) To that end one uses the Dirac algebra of constraints, which is the collection of the Poisson brackets of the secondary constraints $\mathcal{H}_0(x)$ and $\mathcal{H}_i(x)$ with each other. The derivation below will require the detailed form of the Dirac algebra (Sundermeyer, 1982), which is

$$\{\mathcal{H}_0(x), \mathcal{H}_0(y)\} = [h^{ij}(x)\mathcal{H}_j(x) + h^{ij}(y)\mathcal{H}_j(y)]\frac{\partial\delta(x,y)}{\partial x^i}, \tag{9}$$

$$\{\mathcal{H}_i(x), \mathcal{H}_0(y)\} = \mathcal{H}_0(x)\frac{\partial\delta(x,y)}{\partial x^i}, \tag{10}$$

$$\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = \mathcal{H}_j(x)\frac{\partial\delta(x,y)}{\partial x^i} + \mathcal{H}_i(y)\frac{\partial\delta(x,y)}{\partial y^j}. \tag{11}$$

The second of these relations is responsible for the surprising success of deriving Einstein's equations in the presence of absolute simultaneity, as will appear below. In requiring that the time derivatives of $\mathcal{H}_0(x)$ and $\mathcal{H}_i(x)$, one notices, using the Dirac algebra, that the time derivatives vanish because of $\mathcal{H}_0(x)$ and $\mathcal{H}_i(x)$ themselves. It follows that the theory is consistent and that no more constraints remain to be found. One gets the remainder of Einstein's equations by varying $h_{ij}(x)$ to get basically the remaining six spatial Einstein equations and $\Pi^{ij}(x)$ to relate the canonical momenta to the corresponding time derivatives of the spatial metric.

8 Appendix C: Deriving Einstein's Equations assuming Absolute Simultaneity

The above material on the Hamiltonian form of GTR was a review of some standard results. I turn now to a novel derivation of Einstein's equations from a Lagrangian density $\mathcal{L}$ and hence a Hamiltonian H that manifestly contains a notion of distant simultaneity invariant under the symmetries of the theory, namely, spatial coordinate transformations and relabeling the simultaneity hypersurfaces. The ADM split language and the Dirac-Bergmann constrained dynamics formalism are used essentially. Using a nondynamical scalar density e of weight 2 under spatial coordinate transformations, one lets a lapse function densitized using the dynamical spatial metric depend nontrivially only on time, not on the spatial coordinates, while preserving invariance under spatial coordinate transformations. (For simplicity, I take e to be independent of the spatial coordinates, though one could admit such dependence as exists for homogeneous spaces. I also take e to be invariant under time reparametrizations $t \rightarrow F(t)$.) Using the detailed form of the Dirac 'algebra' of first-class constraints crucially, one is able to obtain the local Hamiltonian constraint, not merely an integrated version thereof, as a combination of secondary and tertiary constraints. (It is trivial to modify GTR in such a way as to get a modified or incomplete version of Einstein's equations, but such theories are likely to be empirically inadequate.) While my introduction of absolute time into the theory presently lacks the sort of first principles justification of Barbour's work, the mere conceiving of the proposed modification and the fact that it gives Einstein's equations exactly are already interesting. The success of this project indicates that the remarkable empirical success of GTR, which seems to disconfirm strongly the existence of absolute simultaneity, is less embarrassing to the A-theory than has been generally believed.

I will demonstrate this presently using the constrained dynamics formalism sketched above, making small modifications because the theory has both local fields and a global function of time only among its generalized coordinates. The theory therefore is not a local field theory (at least not as formulated here), and so might not fit within the standard framework (*e.g.*, (Earman, 1989)) assumed in analyzing space-time theories. Starting with the Lagrangian density $\mathcal{L}$ from GTR above, one sets

$$N = \sqrt{e/h}^{w} f(t). \tag{12}$$

The absolute simultaneity is manifest in the lack of spatial dependence in f. The generalized coordinates are f, $\beta^i(x)$, and $h_{ij}(x)$, where the spatial dependence is made explicit. In defining the canonical momentum for f, one recalls that particle-like degrees of freedom use L rather than $\mathcal{L}$, as indeed must be the case to avoid a meaningless spatial dependence in the corresponding canonical momentum. One could define the canonical momenta for $h_{ij}(x)$ and $\beta^i(x)$ as *functional* derivatives of L, but this is the same as taking partial derivatives of $\mathcal{L}$. Either way, the canonical momenta are given by

$$P = \frac{\partial L}{\partial \dot{f}}, \quad P_i(x) = \tfrac{\partial \mathcal{L}}{\partial \dot{\beta}^i}, \quad \Pi^{ij}(x) = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}}, \tag{13}$$

giving $9\infty^3 + 1$ canonical coordinates and the same number of momenta, giving a phase space of $18\infty^3 + 2$ dimensions rather than $20\infty^3$ as in GTR.

Given this definition for P, one cannot define a Hamiltonian density $\mathcal{H}$ of which the Hamiltonian H is an integral, at least until the constraint $P = 0$ is used. Instead one defines the Hamiltonian directly as

$$H = P\dot{f} + \int d^3x [\Pi^{ij}\dot{h}_{ij} + P_i(x)\dot{\beta}^i - \mathcal{L}]. \tag{14}$$

Using the primary constraints $P = 0$ and $P_i(x) = 0$, one obtains the canonical Hamiltonian just as before, except with the substitution $N(x) = \sqrt{e/h}^{w} f$. The Poisson bracket takes the mixed field-'particle' form

$$\{A, B\} = \frac{\partial A}{\partial f}\frac{\partial B}{\partial p} - \frac{\partial A}{\partial p}\frac{\partial B}{\partial f} + \int d^3x \left(\frac{\delta A}{\delta \beta^i(x)}\frac{\delta B}{\delta P_i(x)} - \frac{\delta B}{\delta \beta^i(x)}\frac{\delta A}{\delta P_i(x)} + \frac{\delta A}{\delta h_{ij}(x)}\frac{\delta B}{\delta \Pi^{ij}(x)} - \frac{\delta B}{\delta h_{ij}(x)}\frac{\delta A}{\delta \Pi^{ij}(x)} \right). \tag{15}$$

One can show that the two sorts of terms have the same units and that $\{A, H\}$ gives the time evolution as required, so this composite Poisson bracket seems satisfactory.

This Hamiltonian gives the local momentum constraint (actually $3\infty^3$ constraints)

$$\mathcal{H}_i(x) = 0$$

as before, either by varying the shift vector or by demanding the dynamical preservation of $P_i(x) = 0$. However, the only additional secondary constraint is the *integrated* form

$$-\int d^3x \sqrt{e/h}^{w} \mathcal{H}_0(x) \approx 0, \tag{16}$$

which comes from varying f or demanding the preservation of $P = 0$, which is just one constraint, not one at each spatial point. At this stage the theory looks as if it will not give all of Einstein's equations because only this single integrated constraint has arisen instead of the ∞^3 constraints of GTR. However, to ensure consistent dynamical preservation of the constraints, one must search for any tertiary constraints that arise in order to preserve the secondary constraints. In GTR no tertiary constraints arise, but the same need not happen here, as the precedent of unimodular general relativity (Unruh, 1989) reminds us. The details of the Dirac algebra enter essentially here. It is difficult to predict the results without actually doing the calculation, to which I now turn. The time evolution is satisfactorily generated using the canonical Hamiltonian, not just the primary

Hamiltonian. The vector field $\xi^i(x)$ is an arbitrary nondynamical spatial vector field used to smear the constraints, as one usually does with Dirac delta functions present. The time derivative (up to a sign) of the smeared momentum constraint is given by

$$\{\int d^3y\xi^j(y)\mathcal{H}_j(y), H\} = \int d^3y\xi^j(y)\{\mathcal{H}_j(y), H\}$$
$$= \int d^3yd^3x\xi^j(y)\{\mathcal{H}_j(y), f\sqrt{e}^w\sqrt{h}^{-w}(x)\mathcal{H}_0(x) + \beta^i(x)\mathcal{H}_i(x)\}$$
$$= \int d^3x\sqrt{e}^w f\{\int d^3y\xi^j(y)\mathcal{H}_j(y), \sqrt{h}^{-w}(x)\mathcal{H}_0(x)\} + \int d^3xd^3y\xi^j(y)\beta^i(x)\{\mathcal{H}_j(y), \mathcal{H}_i(x)\}.$$

Using $\{\mathcal{H}_i(x), \mathcal{H}_j(y)\} = \mathcal{H}_j(x)\frac{\partial\delta(x,y)}{\partial x^i} + \mathcal{H}_i(y)\frac{\partial\delta(x,y)}{\partial y^j}$ from the Dirac algebra, one sees that the second term in the last line vanishes "weakly," that is, using the constraints themselves (often symbolized by the symbol $\approx$, though it has not been used above). Recalling (Sundermeyer, 1982) (p. 241) that $\int d^3y\xi^j(y)\mathcal{H}_j(y)$ generates spatial coordinate transformations in GTR, in accord with

$$\{\int d^3y\xi^j(y)\mathcal{H}_j(y), h_{ij}(x)\} = -\pounds_\xi h_{ij}(x), \tag{17}$$

it follows that

$$\int d^3x\sqrt{e}^w f\{\int d^3y\xi^j(y)\mathcal{H}_j(y), \sqrt{h}^{-w}(x)\mathcal{H}_0(x)\} = -f\int d^3x\sqrt{e}^w\pounds_\xi(\sqrt{h}^{-w}\mathcal{H}_0)$$
$$= f\int d^3x\sqrt{h}^{-w}\mathcal{H}_0\pounds_\xi(\sqrt{e}^w)$$
$$= f\int d^3x\sqrt{h}^{-w}\mathcal{H}_0[\xi^i(\sqrt{e}^w),_i + w\sqrt{e}^w\xi^i,_i]$$
$$= \int d^3xf[\sqrt{h}^{-w}\mathcal{H}_0\xi^i(\sqrt{e}^w),_i - w\xi^i(\sqrt{e/h}^w\mathcal{H}_0),_i]. \tag{18}$$

Here the Lie derivative of a tensor density of arbitrary weight (Israel, 1979) has been used. Because $\xi^i(x)$ is arbitrary, the vanishing of the integral implies the vanishing of the integrand, which can be rewritten as

$$-wf\sqrt{e}(\sqrt{h}^{-w}\mathcal{H}_0\sqrt{e}^{w-1}),_i \approx 0. \tag{19}$$

This is a tertiary constraint. Discarding some nowhere vanishing factors, one can trivially integrate this partial differential equation to get

$$w\sqrt{h}^{-w}\mathcal{H}_0(x)\sqrt{e}^{w-1} \approx \Lambda(t), \tag{20}$$

where $\Lambda(t)$ is spatially constant but might vary in time. Recalling the secondary constraint $\int d^3x\sqrt{e/h}^w\mathcal{H}_0 \approx 0$, one infers that $\Lambda(t)\int d^3x\sqrt{e} \approx 0$, so $\Lambda(t) \approx 0$ and, most importantly,

$$w\sqrt{h}^{-w}\mathcal{H}_0\sqrt{e}^{w-1} \approx 0. \tag{21}$$

For $w \neq 0$, this last equation is precisely equivalent to the full Hamiltonian constraint

$$\mathcal{H}_0(x) \approx 0. \tag{22}$$

Given the full Hamiltonian and momentum constraints, which preserve themselves in GTR, it is clear the full set of constraints has been found for the $w \neq 0$ theories. Getting the spatial evolution equations and the inverse Legendre transformation equations by varying $h_{ij}(x)$ and $\Pi^{ij}(x)$, respectively in the usual way, one clearly has the *full* set of Einstein's equations, starting from a Lagrangian density that manifestly contains absolute simultaneity.

By contrast, if $w = 0$, then the slicing takes the simple form $N = f(t)$; this case of independent reparametrization invariant slicing is so straight and orderly that invariance under spatial diffeomorphisms can be totally decoupled from the Hamiltonian constraint. The momentum constraint $\mathcal{H}_i(x) \approx 0$ preserves itself, as one sees by setting $w = 0$ in the formulas above. One can show that the remaining secondary constraint

$$\int d^3x \mathcal{H}_0(x) \approx 0$$

gives no tertiary constraints. That is, one gets the integrated Hamiltonian constraint alone (just one, not ∞^3), and even this integrated constraint arises directly from f and P, not indirectly from $\mathcal{H}_i(x) \approx 0$. For the cases $w \neq 0$, the slices tend to be bent and wiggly, mixing in just enough spatial dependence so that the full Hamiltonian constraint is required by the conjunction of the momentum constraint and the integrated Hamiltonian constraint.[11] Thus any choice of $w \neq 0$ gives a theory that yields Einstein's field equations, yet contains absolute simultaneity. While the Lagrangian density given above was for pure gravity, without any matter, nothing much changes when the usual minimally coupled matter is included, because the algebra for the constraints is unchanged by the inclusion of matter (Sundermeyer, 1982). As one would expect, the contracted Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ can also be used to prove that the full set of Einstein's equations are implied by the given Lagrangian density. The derivation, which exchanges the constrained Hamiltonian dynamics apparatus for mere tensor calculus, is too long and messy to present here.

One can also consider the degree of freedom count for this theory. Clearly it must have the same number of degrees of freedom as GTR ($2\infty^3$), given that its equations are like a partially gauge-fixed version of GTR. In this theory all constraints are "first class": their Poisson brackets with each other are zero, either identically or using the constraints. Let m be the number of first class constraints. Some theories also have "second class constraints," constraints such that their Poisson brackets with some constraints are not zero. Let n be the number of second class constraints, and d be the dimension of the phase space. The number of degrees of freedom of a theory is given (Henneaux and Teitelboim, 1992) by $\frac{1}{2}(d - 2m - n)$. Using what seems to be the natural way of counting, the theory at hand has $d = 18\infty^3 + 2$, $m = 7\infty^3 + 1$, and $n = 0$. The count m of first class constraints comes from the $3\infty^3 + 1$ primary constraints P and $P_i(x)$, the $3\infty^3 + 1$ secondary constraints $\mathcal{H}_i(x)$ and $-\int d^3x \sqrt{e/h}^w \mathcal{H}_0(x)$, and the $\infty^3 - 1$ tertiary constraints $-wf\sqrt{e}(\sqrt{h}^{-w}\mathcal{H}_0\sqrt{e}^{w-1})_{,i}$. For comparison, GTR has $d = 20\infty^3$, $m = 8\infty^3$, and $n = 0$. In both cases, the result is $2\infty^3$ degrees of freedom, which is to say, 2 at each point in space.

References

Anderson, A. and J. W. York, J. (1998). Hamiltonian time evolution for general relativity. *Physical Review Letters*, 81:1154. gr-qc/9807041.

Anderson, E. and Barbour, J. (2002). Interacting vector fields in relativity without relativity. *Classical and Quantum Gravity*, 19:3249. gr-qc/0201092, v. 2.

[11] It is noteworthy that the quantity $\alpha = N/\sqrt{h}$ has recently become important in the numerical relativity literature (where the goal is to find approximate solutions using computers) (Anderson and J. W. York, 1998) and has been given a name, the "slicing density." Choosing $w = -1$, one gets a particular theory reproducing Einstein's equations and having the slicing density given by $\alpha = f(t)/\sqrt{e}$, so that the slicing density depends (nontrivially) only on time.

Anderson, E., Barbour, J., Foster, B., and Murchadha, N. O. (2003). Scale-invariant gravity: Geometrodynamics. *Classical and Quantum Gravity*, 20:1571. gr-qc/02112022, v. 2.

Balashov, Y. and Janssen, M. (2003). Presentism and relativity. *The British Journal for the Philosophy of Science*, 54:327.

Barbour, J., Foster, B., and Murchadha, N. O. (2002). Relativity without relativity. *Classical and Quantum Gravity*, 19:3217. gr-qc/0012089, v. 3.

Bell, J. S. (1987). *Speakable and Unspeakable in Quantum Mechanics*. Cambridge University, Cambridge.

Belot, G. and Earman, J. (2001). Pre-Socratic quantum gravity. In Callender, C. and Huggett, N., editors, *Philosophy Meets Physics at the Planck Scale*. Cambridge University, Cambridge.

Blagojevic, M. (2001). *Gravitation & Gauge Symmetries*. Institute of Physics, Bristol.

Bonnor, W. B., Santos, N. O., and MacCallum, M. A. H. (1998). An exterior for the Gödel spacetime. *Classical and Quantum Gravity*, 15:357. gr-qc/9711011.

Chalmers, D. J. (1996). *The Conscious Mind: In Search of a Fundamental Theory*. Oxford University, New York.

Craig, W. L. (2001). *Time and the Metaphysics of Relativity*. Kluwer Academic, Dordrecht.

Earman, J. (1989). *World Enough and Space-Time: Absolute versus Relational Theories of Space and Time*. MIT, Cambridge, Mass.

Earman, J. (1995). *Bangs, Crunches, Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*. Oxford University, New York.

Earman, J. (2002). Thoroughly modern McTaggart: Or, what McTaggart would have said if he had read the general theory of relativity. *Philosophers' Imprint*, 2(3). http://www.philosophersimprint.org/.

Earman, J. (2003). Getting a fix on gauge: An ode to the constrained Hamiltonian formalism. In Brading, K. and Castellani, E., editors, *Symmetries in Physics: Philosophical Reflections*. Cambridge University Press, Cambridge.

Earman, J. and Norton, J. (1987). What price spacetime substantivalism? The hole story. *British Journal for the Philosophy of Science*, 1938:515–525.

Feynman, R. P., Morinigo, F., Wagner, W., and Hatfield, B. (1963). *Feynman Lectures on Gravitation*. California Institute of Technology, reprint Addison-Wesley (1995), Reading, Mass.

Gödel, K. (1949). A remark on the relationship between relativity and idealistic philosophy. In Schilpp, P. A., editor, *Albert Einstein: Philosopher-Scientist*. Open Court, LaSalle, Illinois.

Hartle, J. B. (2004). The physics of 'now'. *www.arxiv.org*, gr-qc/0403001.

Hehl, F. W., von der Heyde, P., Kerlick, G. D., and Nester, J. E. (1976). General relativity with spin and torsion: Foundations and prospects. *Reviews of Modern Physics*, 48:393.

Henneaux, M. and Teitelboim, C. (1992). *Quantization of Gauge Systems*. Princeton University, Princeton.

Illy, J. (1989). Einstein teaches Lorentz, Lorentz teaches Einstein: Their collaboration in general relativity, 1913-1920. *Archive for History of Exact Sciences*, 39:247.

Israel, W. (1979). *Differential Forms in General Relativity*. Dublin Institute for Advanced Studies, Dublin, second edition.

Kox, A. J. (1988). Hendrik Antoon Lorentz, the ether, and the general theory of relativity. *Archive for History of Exact Sciences*, 38:67.

Kraichnan, R. (1955). Special-relativistic derivation of generally covariant gravitation theory. *Physical Review*, 98:1118.

Kretzmann, N. (1966). Omniscience and immutability. *The Journal of Philosophy*, 63:409.

Laudan, L. (1977). *Progress and Its Problems*. University of California, Berkeley.

Maudlin, T. (1994). *Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics*. Blackwell, Oxford.

McCall, S. and Lowe, E. J. (2003). $3D/4D$ equivalence, the twins paradox and absolute time. *Analysis*, 63:114.

Misner, C., Thorne, K., and Wheeler, J. A. (1973). *Gravitation*. Freeman, New York.

Monton, B. (2001). Presentism and quantum gravity. *http://philsci-archive.pitt.edu*.

Newton, I., Cohen, I. B., Whitman, A., and Budenz, J. (1999). *The Principia: Mathematical Principles of Natural Philosophy*. University of California, Berkeley.

Pitts, J. B. and Schieve, W. C. (2001a). Flat spacetime gravitation with a preferred foliation. *Foundations of Physics*, 31:1083. gr-qc/0101099.

Pitts, J. B. and Schieve, W. C. (2001b). Slightly bimetric gravitation. *General Relativity and Gravitation*, 33:1319. gr-qc/0101058.

Poincaré, H. (1913). The measure of time. In *The Foundations of Science: The Value of Science*. Science Press, Lancaster, Pennsylvania. trans. G. B. Halsted; French original 1898.

Putnam, H. (1967). Time and physical geometry. *Journal of Philosophy*, 64:240.

Rakić, N. (1997). Past, present, future, and special relativity. *The British Journal for the Philosophy of Science*, 48:257.

Salisbury, D. C. (2003). Gauge fixing and observables in general relativity. *Modern Physics Letters A*, 18:2475. Proceedings of Spacetime and Fundamental Interactions: Quantum Aspects, May, 2003, honoring the 65th birthday of A. P. Balachandran, Balfest Conference Proceedings, Vietri sul Mare, 26-31 May 2003; gr-qc/0310095.

Salisbury, D. C., Pons, J. M., and Shepley, L. C. (2000). Gauge symmetries in Ashtekar's formulation of general relativity. *Nuclear Physics B (Proceedings Supplement)*, 88:314. gr-qc/0004013.

Sider, T. (2001). *Four Dimensionalism: An Ontology of Persistence and Time*. Oxford University, Oxford.

Smith, Q. (1993). *Language and Time*. Oxford University, New York.

Sundermeyer, K. (1982). *Constrained Dynamics*. Springer, Berlin.

Unruh, W. G. (1989). Unimodular theory of canonical quantum gravity. *Physical Review D*, 40:1048.

Utiyama, R. (1956). Invariant theoretical interpretation of interaction. *Physical Review*, 101:1597.

Wald, R. M. (1984). *General Relativity*. University of Chicago, Chicago.

Weyl, H. (1949). *Philosophy of Mathematics and Natural Science*. Princeton University, Princeton.