The Anderson-Friedman Absolute Objects Program: Several Successes, One Difficulty

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Abstract

The Anderson-Friedman absolute objects project is reviewed. The Jones-Geroch dust 4-velocity counterexample is resolved by eliminating irrelevant structure. Torretti's example involving constant curvature spaces is shown to have an absolute object on Anderson's analysis. The previously neglected threat of an absolute object from an orthonormal tetrad used for coupling spinors to gravity appears resolvable by eliminating irrelevant fields and using a modified spinor formalism. However, given Anderson's definition, GTR itself has an absolute object (as Robert Geroch has observed recently): a change of variables to a conformal metric density and a scalar density shows that the latter is absolute.

1 Introduction

James L. Anderson analyzed the novelty of Einstein's so-called General Theory of Relativity (GTR) as its lacking "absolute objects" (Anderson, 1967; Anderson, 1971). Metaphorically, absolute objects are often described as a fixed stage on which the dynamical actors play their parts. A review of Anderson's definitions will be useful. Absolute objects are to be contrasted with dynamical objects. The values of the absolute objects do not depend on the values of the dynamical objects, but the values of the dynamical objects do depend on the values of the absolute objects (Anderson, 1967, p. 83). Both absolute objects and dynamical objects are, mathematically speaking, geometric objects or parts thereof. Trautman defines geometric objects as follows:

> Let X be an n-dimensional differentiable manifold....
>
> Let $p \in X$ be an arbitrary point of X and let $\{x^a\}, \{x^{a'}\}$ be two systems of local coordinates around p. A <u>geometric object field</u> y is a correspondence
>
> $$y : (p, \{x^a\}) \to (y_1, y_2, \cdots y_N) \in R^N$$
>
> which associates with every point $p \in X$ and every system of local coordinates $\{x^a\}$ around p, a set of N real numbers, together with a rule which determines $(y_{1'}, \cdots y_{N'})$, given by
>
> $$y : (p, \{x^{a'}\}) \to (y_{1'}, \cdots y_{N'}) \in R^N$$
>
> in terms of the $(y_1, y_2, \cdots y_N)$ and the values of [*sic*] p of the functions

> and their partial derivatives which relate the coordinate systems $\{x^a\}$ and $\{x^{a'}\}$.... The N numbers $(y_1, \cdots y_N)$ are called the components of y at p with respect to the coordinates $\{x^a\}$. (Trautman, 1965, pp. 84, 85)

Geometric objects were considered with great thoroughness by Albert Nijenhuis (Nijenhuis, 1952).

Before absolute objects can be defined, the notion of a covariance group must be outlined. Here it will prove helpful to draw upon the unjustly neglected work of Kip Thorne, Alan Lightman, and David Lee (TLL) (Thorne et al., 1973); a useful companion paper (LLN) was written by Lee, Lightman and W.-T. Ni (Lee et al., 1974). According to TLL,

> A group $\mathcal{G}$ is a covariance group of a representation if (i) $\mathcal{G}$ maps [kinematically possible trajectories] of that representation into [kinematically possible trajectories]; (ii) the [kinematically possible trajectories] constitute "the basis of a faithful representation of $\mathcal{G}$" (i.e., no two elements of $\mathcal{G}$ produce identical mappings of the [kinematically possible trajectories]); (iii) $\mathcal{G}$ maps [dynamically possible trajectories] into [dynamically possible trajectories]. (Thorne et al., 1973, p. 3567)

One can now define absolute objects. They are, according to Anderson, objects with components ϕ_α such that

> (1) The ϕ_α constitute the basis of a faithful realization of the covariance

> group of the theory. (2) Any ϕ_α that satisfies the equations of motion of the theory appears, together with all its transforms under the covariance group, in every equivalence class of [dynamically possible trajectories]. (Anderson, 1967, p. 83)

Thus the components of the absolute objects are the same, up to equivalence under the covariance group, in every model of the theory. It is the dynamical objects that distinguish the different equivalence classes of the dynamically possible trajectories (Anderson, 1967, p. 84).

It has been asserted that the novel and nontrivial sense in which GTR is generally covariant is its lack of absolute objects (Anderson, 1967) or "prior geometry" (Misner et al., 1973, pp. 429-431). John Norton discusses this claim with some sympathy (Norton, 1992; Norton, 1993; Norton, 1995), though technical problems such as the Jones-Geroch dust and Torretti constant spatial curvature counterexamples are among his worries (Norton, 1993; Norton, 1995). Anderson and Ronald Gautreau encapsulate the definition of an absolute object as an object that "affects the behavior of other objects but is not affected by these objects in turn." (Anderson and Gautreau, 1969, p. 1657) Anderson claims that absolute objects violate what he calls a "generalized principle of action and reaction" (Anderson, 1967, p. 339) (Anderson, 1971, p. 169). Norton has argued, rightly I think, that such a principle is hopelessly vague and arbitrary and that it should not be invoked to impart a spurious necessity to the contingent truth that our best current physical theory lacks them (Norton, 1993, pp. 848, 849).

In Anderson's framework, an important subgroup of a theory's covariance group is its symmetry group (Anderson, 1967, pp. 84-88). One first defines the symmetry group of a *geometrical object* as those transformations that leave the object unchanged. The symmetry group of a physical system or theory—Anderson makes no distinction between them here—is

> the largest subgroup of the covariance group of this theory, which is simultaneously the symmetry group of its absolute objects. In particular, if the theory has no absolute objects, then the symmetry group of the physical system under consideration is just the covariance group of this theory. (Anderson, 1967, p. 87)

Finding Anderson's definition obscure, Michael Friedman amended it in the interest of clarity (Friedman, 1973; Friedman, 1983). As it turns out, Friedman has made a number of changes to Anderson's definitions, not all for the better. First, Friedman's equivalence relation, which he calls d-equivalence, comprises only diffeomorphism freedom (Friedman, 1983, pp. 58-60), not other kinds of gauge freedom such as local Lorentz freedom or electromagnetic or Yang-Mills gauge freedom, in defining the covariance group. Anderson calls such groups besides diffeomorphisms "internal groups" (Anderson, 1967, pp. 35, 36), though the term does not always fit perfectly for the examples available today. Second, Friedman's mathematical language is less general than Anderson's and fails to accommodate some useful mathematical entities that Anderson's permits. Anderson knows what sorts of mathematical structures physicists need, while Friedman restricts his attention to that narrower collection of entities that all modern coordinate-free treatments of gravitation or (pseudo-)Riemannian geometry presently discuss, namely tensors and connections, but not, for example, tensor densities, which are important for two examples below. Tensor densities, even of fractional or irrational weight, are useful or crucial in a variety of applications, the modern canonical quantum gravity project, the conformal-traceless decomposition in numerical work in general relativity, and massive theories of gravity. Accidentally restricting one's vocabulary also prevents one from using irreducible geometric objects. Friedman's mathematical language also excludes spinors, whether of the usual orthonormal tetrad formalism or the less common formalism of V. I. Ogievetskiǐ and I. V. Polubarinov (Ogievetskiǐ and Polubarinov, 1965), to be discussed below. A third difference pertains to the notion of standard formulations of a theory. Anderson (somewhat confusingly) and TLL require that theories should be coordinate-covariant under arbitrary manifold mappings. Friedman, by contrast, takes as standard a form in which the absolute objects, if possible, have *constant components* (Friedman, 1983, p. 60). Friedman implies that one can always choose coordinates such that the absolute objects (a) have constant components and (b) thus drop out of the theory's differential equations. However, these claims both suffer from counterexamples. Concerning (a), (anti-)de Sitter background metrics of a single value of curvature are absolute but do not have constant components. Concerning (b), absolute objects can appear algebraically in the field equations, not just differently, so their components need not drop out even if constant (Freund et al., 1969). Thus the Thorne-Lee-Lightman fully reduced generally covariant formulation is therefore preferable to Friedman's standard formulation. Friedman's expectation that the components of absolute objects could be reduced to constants in general, though incorrect, usefully calls attention to the role (or lack thereof) of Killing vector fields and the like in analyzing absolute objects. TLL's additional category of "confined" objects is a useful supplement to absolute objects and contains various structures that savor of absoluteness without satisfying a definition of absolute objects.

2 Jones-Geroch counterexample and Friedman's reply

With a clear grasp of absolute objects in hand, one can now consider the Jones-Geroch counterexample that claims that the 4-velocity of cosmic dust counts, absurdly, as an absolute object by Friedman's or Anderson's standards. Friedman concedes some force to this objection made by Robert Geroch and amplified by Roger Jones, here related by Friedman:

> ... [A]s Robert Geroch has observed, since any two timelike, nowhere-vanishing vector fields defined on a relativistic space-time are d-equivalent, it follows that any such vector field counts as an absolute object according to [Friedman's criterion]; and this is surely counter-intuitive. Fortunately, however, this problem does not arise in the context of any of the space-time theories I discuss. It could arise in the general relativistic theory of "dust" if we formulate the theory in terms of a quintuple $\langle M, D, g, \rho, U \rangle$, where ρ is the density of the "dust" and U is its velocity field. U is nonvanishing and thus would count as an absolute object by my definition. But here it seems more natural to formulate the theory as a quadruple $\langle M, D, g, \rho U \rangle$ where ρU is the momentum field of the "dust." Since ρU does vanish in some models, it will not be absolute. (Geroch's observation was conveyed to me by Roger Jones, who also suggested the example of the general relativistic theory of "dust."...) (Friedman, 1983, p. 59)

Here D is the torsion-free covariant derivative compatible with g. Other sources indicate a qualification to *local* diffeomorphic equivalence of nonvanishing timelike vector fields (Jones, 1981, pp. 167, 168) (Trautman, 1965, p. 84), though that qualification does not matter.

Friedman's response is nearly satisfactory, though it has two weaknesses as he expressed it. First, the statement "ρU does vanish in some models" ought to have said

"ρU does vanish in some neighborhoods in some models" to show that he is considering only genuine models of GTR + dust (in which dust vanishes in some neighborhoods in some models), rather than some models with (omnipresent?) dust and some degenerate models which nominally have dust but actually have no dust anywhere. Clearly some models with dust have neighborhoods lacking dust, and it is these models which will prevent the dust 4-velocity from constituting an absolute object. Second, Friedman's unfortunate notation ρU suggests that the mass current density (which I will call J^{μ}) is logically posterior to ρ and an everywhere nonvanishing timelike U^{μ}. If so, then one has not eliminated the absolute object after all. If a timelike nowhere vanishing U^{μ} exists in the theory, then it is absolute even if ρU^{μ} vanishes somewhere and so is not absolute. Thus the significance of Friedman's use of ρU^{μ} is left obscure. Instead one can take J^{μ} to be the fundamental variable, while the timelike U^{μ} is a derived quantity defined wherever $\rho \neq 0$. Alternatively, one can take U^{μ} to be meaningful everywhere (and perhaps primitive), but vanishing where there is no dust. If Friedman had said that J^{μ} or U^{μ} "does vanish in some neighborhoods in some models," then these two infelicities would have been avoided. The Jones-Geroch counterexample fails because there is no physically meaningful everywhere (nonvanishing) timelike vector field in the set of solutions of GTR + dust, because there is none where the dust has holes in some models. Not just globally irrelevant fields, but locally irrelevant portions of fields should be excluded before testing a theory for absolute objects.[1]

3 Hiskes's redefinition of absoluteness, Maidens's worry, and Rosen's answer in advance

To address the Jones-Geroch dust counterexample, Anne Hiskes proposed amending the definition of absolute objects so that no field varied in a theory's action principle would be regarded as absolute (Hiskes, 1984). Such a move makes use of what *prima facie* seems to be a true generalization about absolute and dynamical objects (Anderson, 1967, pp. 88, 89) (Thorne et al., 1973; Lee et al., 1974) and is sometimes presented as a definition (Earman, 2003). Let use call objects "(non)variational" if they are (not) varied in an action principle (Gotay et al., 2004). We have seen that Hiskes's amendment is not necessary to resolve the Jones-Geroch dust counterexample. Anna Maidens has suggested that there might be some way to reformulate special relativistic theories such that the flat metric, which surely ought to count as absolute, is varied in the action principle. If that could be done, then Hiskes's definition of absolute objects

[1]While the virtue of eliminating surplus structure seems generally accepted, discussions of the Anderson-Friedman absolute objects program do not always pay sufficient attention to eliminating "irrelevant" fields. The matter is discussed in detail by Anderson (Anderson, 1967), TLL (Thorne et al., 1973) and me (Pitts, 2006).

would prove to be too strict (the opposite problem from what the Jones-Geroch example suggests about Friedman's), because it fails to count the metric tensor of special relativity as an absolute object in some formulations. Though Maidens's claims are indecisive and not well argued, it is in fact true that one can derive the flatness of a metric from a variational principle with the help of Lagrange multipliers (Rosen, 1966; Rosen, 1973; Sorkin, 2002). So Hiskes's move seems unpromising even for addressing other counterexamples. However, one might argue that the Lagrange multiplier fields are irrelevant variables and so should not be used. Given the qualifications that irrelevant variables should be excluded, Hiskes's proposal might yet have some use in addressing other counterexamples.

Does it follow that Anderson's and others' intuition that fields are absolute iff nonvariational is vindicated? Before accepting such a claim, one must address parameterized theories (Sundermeyer, 1982; Kuchař, 1973; Arkani-Hamed et al., 2003; Norton, 2003; Earman, 2003), in which preferred coordinates are rendered variational. Because the resulting "clock fields" X^A are scalars and their gradients are linearly independent, the Noether-Bianchi identities ensure that $\frac{\delta S}{\delta X^A} = 0$ due to the other fields' Euler-Lagrange equations, even if X^A are nonvariational. If we stipulate that fields should only be varied only there is some benefit to doing so, then preferred coordinates usually should not be varied.

4 Torretti's and Norton's examples have absolute objects

A second long-standing worry concerning the Anderson-Friedman absolute objects project was suggested by Roberto Torretti (Torretti, 1984). He considered a theory of modified Newtonian kinematics in which each model's space has constant curvature, but different models have different values of that curvature. Because every model's space has constant curvature, such a theory surely has something rather like an absolute object in it, Torretti's intuition suggests. Though contrived, this example is relevantly like the cases of de Sitter or anti-de Sitter background metrics of constant curvature that are sometimes discussed in the physics literature, where one often lumps together space-times with different values of constant curvature. The failure of the metrics to be locally diffeomorphically equivalent for distinct curvature values entails that the metric tensor does not satisfy Anderson's or Friedman's definition of an absolute object (or TLL's, for that matter). But it seems intuitively clear to Torretti that his theory has an absolute object, so Friedman's analysis is wrong.

It is not widely appreciated that Anderson's analysis, when applied to Torretti's example, actually does yield a very specific and reasonable conclusion involving an absolute object. Though the spatial metric is not absolute, the conformal spatial metric density, a symmetric $(0,2)$ tensor density of weight $-\frac{2}{3}$ (or its $(2,0)$ weight $\frac{2}{3}$ inverse) is an absolute object. This entity, when its components are expressed as a matrix, has unit determinant. It appears routinely in the conformal-traceless decomposition used in finding initial data in numerical studies of GTR. It defines angles and relative lengths of vectors at a point, but permits no comparison of lengths of vectors at different points. In three dimensions, conformal flatness of a metric is expressed by the vanishing of the Cotton tensor (Aldersley, 1979; Garcia et al., 2004), not the Weyl tensor, which vanishes identically. That the conformal metric density is an absolute object is shown in the following way. Every space with constant curvature is conformally flat (Wolf, 1967). For conformally flat spatial metrics, manifestly the conformal parts are equal in a neighborhood up to diffeomorphisms. The conformal part just is the conformal metric density. Concerning Norton's modification of Torretti's example to Robertson-Walker space-time metrics (Norton, 1993, p. 848), analogous comments could be made: these space-*times* are conformally flat (Infeld and Schild, 1945) and so have as an absolute object the space-time conformal metric density.

5 Tetrad-spinor: Avoiding absolute object by eliminating irrelevant fields

One potential counterexample to the Anderson-Friedman example that seems not to have been noticed arises from the use of an orthonormal tetrad formalism, in which the metric tensor (or its inverse) is built out of four orthonormal vector fields e^{μ}_{A} by the formula $g^{\mu\nu} = e^{\mu}_{A}\eta^{AB}e^{\nu}_{B}$ or the like. Four vector fields have among them 16 components, rather more than the 10 components of the metric, so there is some redundancy that leaves a new local Lorentz gauge freedom to make arbitrary position-dependent boosts and rotations of the tetrad. It is unnecessary to use a tetrad instead of a metric as the fundamental field when gravity (as described by GTR) is coupled to bosonic matter (represented by tensors, tensor densities or perhaps connections). However, it is widely believed to be necessary to use an orthonormal tetrad to couple gravity to the spinor fields that represent electrons, protons, and the like (Weinberg, 1972; Deser and Isham, 1976). The threat of a counterintuitive absolute object then arises. Given both local Lorentz and coordinate freedom, one can certainly bring the timelike leg into the component form $(1, 0, 0, 0)$ at least in a neighborhood about any point. Unlike the dust case, there cannot be any spacetime region in any model such that the timelike leg of the tetrad vanishes. Thus GTR coupled to a spinor field using an orthonormal tetrad gives an example of a Gerochian vector field: nowhere vanishing, everywhere timelike, gauge-equivalent to $(1, 0, 0, 0)$, and (allegedly) required to couple the spinor and gravity and thus not irrelevant. Like clock fields, the timelike tetrad leg also appears to be both variational and absolute. If it is true that coupling spinors to gravity requires an orthonormal tetrad and that an orthonormal formalism for GTR yields an absolute object, then the intuitively absurd conclusion that GTR

+ spinors has an absolute object follows. Though Anderson discusses spinors coupled to a curved metric (Anderson, 1967, pp. 358-360), his treatment is unsatisfactory for various reasons.

The tetrad-spinor example seems rather more serious a problem for definitions of absolute objects than the Jones-Geroch cosmological dust example was, because the spinor field is surely closer to being a fundamental field than is dust or any other perfect fluid. Spinors (actually vector-spinors for spin $\frac{3}{2}$) are also required in supergravity, where internal and external symmetries are combined, not to mention (super)string theory. On another occasion I expect to explain in more detail how to remove irrelevant variables here and thus avoid this unexpected absolute object. This removal is achieved using the alternative spinor formalism of V. I. Ogievetskiĭ and I. V. Polubarinov (Ogievetskiĭ and Polubarinov, 1965) to eliminate "enough" of the orthonormal tetrad as irrelevant that the timelike nowhere vanishing vector field disappears from the theory. A brief summary suffices here. Their formalism's "square root of the metric" resembles an orthonormal tetrad gauge-fixed to form a symmetric matrix by sacrificing the local Lorentz freedom while preserving diffeomorphism freedom. The square root of the metric has only ten components rather than sixteen and can be computed using a binomial series expansion. This work was followed among high energy physicists with further discussion of nonlinear group representations.

6 Geroch's Scalar density example and unimodular GTR: Does GTR lack absolute objects?

Unimodular GTR was invented by Einstein, was discussed by Anderson along with David Finkelstein (Anderson and Finkelstein, 1971), and is rather well known today (Earman, 2003). Still it turns out that consideration of unimodular GTR helps one to reach the startling conclusion that not only it, but GTR itself, has an absolute object on Friedman's definition. (While serving as a referee, Robert Geroch proposed this counterexample, though using different mathematical variables.) Unimodular GTR comes in two flavors: the coordinate-restricted version in which only coordinates that fix the determinant of the metric components matrix to -1, and the weakly generally covariant version that admits any coordinates with the help of a nonvariational scalar density (usually of weight 1 or 2, but any nonzero weight suffices) and a dynamical conformal metric density, which is a $(0,2)$ tensor density of weight $-\frac{2}{n}$ or a $(2,0)$ tensor density of weight $\frac{2}{n}$ in n space-time dimensions. As Anderson and Finkelstein observe, a metric tensor as a geometric object is reducible into a conformal metric density and a scalar density. They have in mind an equation along these lines:

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\sqrt{-g}^{\,\frac{2}{n}} \tag{1}$$

As usual, g is the determinant of the matrix of components $g_{\mu\nu}$ of the metric tensor in a coordinate basis; g is a scalar density of weight 2 and takes negative values because of the signature of the metric tensor. $\hat{g}_{\mu\nu}$ is the conformal metric density. The new variables $\hat{g}_{\mu\nu}$ (or its inverse) and $\sqrt{-g}$ (or any nonzero power thereof) are those of Anderson and Finkelstein or are relevantly similar. They further observe that this scalar density is an absolute object in unimodular GTR. This observation seems unremarkable because that scalar density is not variational. For comparison, recall that one can write the Lagrangian density for GTR in terms of the conformal metric density and a scalar density (Peres, 1963). Surely the result is still GTR and not some other theory. To my knowledge, no one (prior to Geroch, in effect) has ever considered whether the scalar density, even if variational, might still count as an absolute object. Once the question is raised about GTR with the Peres-type variables, a positive answer seems obvious: GTR has an absolute object, on Friedman's definition of local diffeomorphic equivalence. This absolute object is a scalar density of nonzero weight, because every neighborhood in every model space-time admits coordinates (at least locally) in which the component of the scalar density has a value of -1. Thus variationality and absoluteness by Friedman's standards have come apart for GTR. Thus either Anderson's claim that GTR's novelty lay in its lack of absolute objects, or his analysis of absolute objects (as modified by Friedman to require only local diffeomorphic equivalence), is flawed.

7 Conclusion

Reviewing the Anderson-Friedman absolute objects program and various possible counterexamples yields several lessons. First, Anderson's and TLL's demand that irrelevant descriptive fluff be removed needs even more attention that they gave it. This demand as written helps to address the tetrad-spinor case. Irrelevance comes in even more varieties than they imagined, such as local irrelevance for the Jones-Geroch dust case and irrelevant variationality for clock fields. Second, one's mathematical vocabulary should be chosen by the demands of physics, not the accidental fashions of contemporary differential geometry. Thus spinor fields and tensor densities should be considered. Otherwise it is difficult or impossible to discuss the tetrad-spinor and Geroch scalar-density examples, while the Torretti counterexample is greatly overestimated. Third, reducible geometric objects such as metric tensors should be expressed as concomitants of irreducible ones such as certain scalar and tensor densities. Finally, the scalar density counterexample, which arguably is the only serious problem for the Anderson-Friedman framework, shows that either GTR has an absolute object or the Friedman definition of absolute objects is flawed.

References

Aldersley, S. J. (1979). Comments on certain divergence-free tensor densities in a 3-space. *Journal of Mathematical Physics*, 20:1905.

Anderson, J. L. (1967). *Principles of Relativity Physics*. Academic, New York.

Anderson, J. L. (1971). Covariance, invariance, and equivalence: A viewpoint. *General Relativity and Gravitation*, 2:161.

Anderson, J. L. and Finkelstein, D. (1971). Cosmological constant and fundamental length. *American Journal of Physics*, 39:901.

Anderson, J. L. and Gautreau, R. (1969). Operational formulation of the principle of equivalence. *Physical Review*, 185:1656.

Arkani-Hamed, N., Georgi, H., and Schwartz, M. D. (2003). Effective field theory for massive gravitons and gravity in theory space. *Annals of Physics*, 305:96. hep-th/0210184.

Deser, S. and Isham, C. J. (1976). Canonical vierbein form of general relativity. *Physical Review D*, 14:2505.

Earman, J. (2003). The cosmological constant, the fate of the universe, unimodular gravity, and all that. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, 34:559.

Freund, P. G. O., Maheshwari, A., and Schonberg, E. (1969). Finite-range gravitation. *Astrophysical Journal*, 157:857.

Friedman, M. (1973). Relativity principles, absolute objects and symmetry groups. In Suppes, P., editor, *Space, Time, and Geometry*, pages 296–320. D. Reidel, Dordrecht.

Friedman, M. (1983). *Foundations of Space-time Theories: Relativistic Physics and Philosophy of Science*. Princeton University, Princeton.

Garcia, A., Hehl, F., Heinicke, C., and Macias, A. (2004). The Cotton tensor in Riemannian spacetimes. *Classical and Quantum Gravity*, 21:1099. gr-qc/0309008v2.

Gotay, M. J., Isenberg, J., Marsden, J. E., Montgomery, R., Śniatycki, J., and Yasskin, P. B. (2004). Momentum maps and classical fields. Part I: Covariant field theory. *www.arxiv.org*. physics/9801019v2.

Hiskes, A. L. D. (1984). Space-time theories and symmetry groups. *Foundations of Physics*, 14:307.

Infeld, L. and Schild, A. (1945). A new approach to kinematic cosmology. *Physical Review*, 68:250.

Jones, R. (1981). The special and general principles of relativity. In Barker, P. and Shugart, C. G., editors, *After Einstein: Proceedings of the Einstein Centennial*

Celebration at Memphis State University, 14-16 March 1979, pages 159–173. Memphis State University, Memphis.

Kuchař, K. (1973). Canonical quantization of gravity. In Israel, W., editor, *Relativity, Astrophysics, and Cosmology*, pages 237–288. D. Reidel, Dordrecht.

Lee, D. L., Lightman, A. P., and Ni, W.-T. (1974). Conservation laws and variational principles in metric theories of gravity. *Physical Review D*, 10:1685.

Misner, C., Thorne, K., and Wheeler, J. A. (1973). *Gravitation*. Freeman, New York.

Nijenhuis, A. (1952). *Theory of the Geometric Object*. PhD thesis, University of Amsterdam. Supervised by Jan A. Schouten.

Norton, J. (1992). The physical content of general covariance. In Eisenstaedt, J. and Kox, A. J., editors, *Studies in the History of General Relativity - Einstein Studies, Vol. 3*, pages 281–315. Birkhäuser, Boston.

Norton, J. D. (1993). General covariance and the foundations of General Relativity: Eight decades of dispute. *Reports on Progress in Physics*, 56:791.

Norton, J. D. (1995). Did Einstein stumble? The debate over general covariance. *Erkenntnis*, 42:223.

Norton, J. D. (2003). General covariance, gauge theories and the Kretschmann objection. In Brading, K. and Castellani, E., editors, *Symmetries in Physics:*

Philosophical Reflections. Cambridge University Press, Cambridge. http://philsci-archive.pitt.edu/archive/00000380/.

Ogievetskiĭ, V. I. and Polubarinov, I. V. (1965). Spinors in gravitation theory. *Soviet Physics, Journal of Experimental and Theoretical Physics*, 21:1093. Russian volume **48**, page 1625.

Peres, A. (1963). Polynomial expansion of gravitational Lagrangian. *Nuovo Cimento*, 28:865.

Pitts, J. B. (2006). Absolute objects and counterexamples: Jones-Geroch dust, Torretti constant curvature, tetrad-spinor, and scalar density. *Studies in History and Philosophy of Modern Physics*, 37:347. gr-qc/0506102v4.

Rosen, N. (1966). Flat space and variational principle. In Hoffmann, B., editor, *Perspectives in Geometry and Relativity: Essays in Honor of Václav Hlavatý*. Indiana University, Bloomington.

Rosen, N. (1973). A bi-metric theory of gravitation. *General Relativity and Gravitation*, 4:435.

Sorkin, R. D. (2002). An example relevant to the Kretschmann-Einstein debate. *Modern Physics Letters A*, 17:695. http://philsci-archive.pitt.edu/.

Sundermeyer, K. (1982). *Constrained Dynamics*. Springer, Berlin.

Thorne, K. S., Lee, D. L., and Lightman, A. P. (1973). Foundations for a theory of gravitation theories. *Physical Review D*, 7:3563.

Torretti, R. (1984). Space-time physics and the philosophy of science: review of *Foundations of Space-Time Theories: Relativistic Physics and the Philosophy of Science* by Michael Friedman. *The British Journal for the Philosophy of Science*, 35:280.

Trautman, A. (1965). Foundations and current problems of General Relativity. In Deser, S. and Ford, K. W., editors, *Lectures on General Relativity*, pages 1–248. Prentice Hall, Englewood Cliffs, New Jersey. Brandeis Summer Institute in Theoretical Physics.

Weinberg, S. (1972). *Gravitation and Cosmology*. Wiley, New York.

Wolf, J. A. (1967). *Spaces of Constant Curvature*. McGraw-Hill, New York.