Dialogue Structure and Logical Expressivism

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Abstract. This paper aims to develop the implications of logical expressivism for a theory of dialogue coherence. I proceed in three steps. Firstly, certain structural properties of cooperative dialogue are identified. Secondly, I describe a variant of the multi-agent natural deduction calculus that I introduced in Piwek (2007) and demonstrate how it accounts for the aforementioned structures. Thirdly, I examine how the aforementioned system can be used to formalise an expressivist account of logical vocabulary that is inspired by Brandom (1994; 2000). This account conceives of the logical vocabulary as a tool which allows speakers to describe the inferential practices which underlie their language use, i.e., it allows them to make those practices explicit. The rewards of this exercise are twofold: 1) We obtain a more precise account of logical expressivism which can be defended more effectively against the critique that such accounts lead to cultural relativism. 2) The formalised distinction between engaging in a practice and expressing it, opens the way for a revision of the theory of dialogue coherence. This revision eliminates the need for logically complex formulae to account for certain structural properties of cooperative dialogue.

Keywords: Dialogue coherence, logical expressivism, natural deduction, logical vocabulary, inferential practices

1. Introduction

Over the past two decades, the philosopher Robert Brandom (1994; 2000; 2008) has developed a new account of logic, inspired by, among others, the works of Frege (in particular, his early work), Gentzen, Sellars and Dummett. This account, going by the name of 'logical expressivism', conceives of the logical vocabulary as a tool which allows speakers to describe the inferential practices which underlie their language use, i.e., it enables them to make those practices explicit.

Recently, in Piwek (2007), I have proposed a theory of dialogue coherence, henceforth DCT (for Dialogue Coherence Theory), which aims to explain certain structural properties of dialogue as the interplay between (1) the inferential roles of logical connectives, (2) a practice for transferring information between agents, and (3) a practice governing information flow between agents and their environment. The theory is formulated as a Gentzen-style multi-agent natural deduction calculus, but whereas Gentzen's calculus characterises valid inferences, the extended calculus of DCT demarcates a certain type of coherent dialogue. One of the principal aims of DCT is to account for dialogue structure

whilst relying on only minimal assumptions regarding the reasoning and information processing capacities available to dialogue participants.

The insight into logic won by Brandom opens up the possibility of an even more parsimonious version of DCT. The revised version of DCT that results from formalising logical expressivism using DCT's natural deduction-based formal apparatus, resonates with a current trend in work on dialogue modelling that eschews the full complexity of higher-order intention computation to account for dialogue phenomena (see Gregoromichelaki et al., 2011). The current paper thus contributes to dialogue theory by using Brandom's logical expressivism to further streamline DCT. Additionally, by formalising logical expressivism in terms of DCT, the paper aims to illuminate Brandom's logical expressivism.[1]

The paper proceeds as follows. Section 2 introduces Dialogue Coherence Theory. It starts with a specification of the scope of the theory, specifically, as cooperative information-seeking dialogue. A prototypical fragment of this type of dialogue is provided. Next, the aims and assumptions behind the theory are laid out. This is followed by a description of the technical details. The section concludes with an overview of the limitations of the theory as it stands.

Section 3 moves on to the topic of logical expressivism. After a brief introduction to Brandom's expressivism and some of the criticisms it has been subject to, I describe a formalisation of logical expressivism using the machinery of DCT. Having carried out the formalisation, I then return to criticisms of logical expressivism and use the formalisation to illuminate the issues that the objections give rise to.

In Section 4, I return to DCT itself and show how the insights gained from formalising logical expressivism lead to a revised theory of dialogue coherence. In particular, the rigorous distinction between inferential practices and their description, as afforded by logical vocabulary, enables us to streamline the theory of dialogue coherence. It turns out that the recourse in Piwek (2007) to logically complex formulae to account for dialogue structure was superfluous. We can derive structural properties of dialogue directly from the underlying inferential practices for the non-logical vocabulary. The important lesson from this is that logical vocabulary itself is not required to explain dialogue coherence, a finding which sits well with the idea that in language acquisition, engagement in coherent dialogue can precede mastery of logical expressions. It also opens up the possibility of a language community

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[1] Note that this endeavour is very different from the set-theoretic formalisation undertaken by Brandom and Aker in 'Between Saying and Doing', which identifies a consequence relation such that Y follows from X 'just in case everything incompatible with with Y is incompatible X' (Brandom, 2008, p. 137).

that engages in cooperative information-seeking dialogue and yet has no conception of logical vocabulary.

The paper finishes with a conclusion section.

2. A Theory of Dialogue Coherence

In this section, the dialogue coherence theory as proposed in Piwek (2007), henceforth DCT, is introduced. We start by delimiting the scope of the theory and giving an example of a prototypical fragment of the kind of dialogue the theory deals with.

2.1. DIALOGUE STRUCTURE IN COOPERATIVE INFORMATION-SEEKING DIALOGUE

What makes a dialogue coherent? Phrased this way, the question seems to elude an answer. People engage in dialogue for many different purposes; depending on the purpose, what counts as coherent varies. For example, in a dispute participants are permitted, and even expected, to produce assertions and denials that belong to incompatible viewpoints. In contrast, dialogue as encountered at the information desk of train stations, tourist offices, etc. requires participants to jointly achieve a situation in which the information requester has acquired the information they were looking for. In short, there are many different types of dialogue that we can engage in, each with their own notion of coherence.

This paper focuses on the practices underlying one such type of dialogue: *cooperative information-seeking dialogue*. Information-seeking dialogue is a type of dialogue that is identified in the classification of dialogue types proposed by Walton & Krabbe (1995, p. 66). They characterise this type of dialogue as starting from an initial situation of 'personal ignorance' (of one or both interlocutors on one or more topics) with as main aim the 'spreading of information and revealing of positions'. We are particularly concerned with the cooperative variant of this type of dialogue where the participants' aims are to 'gain', 'pass on' and 'show' information. The participants are not seeking to persuade each other to believe or do something – the assumption is that each is an acknowledged expert on the questions that they are asked by the other and are assumed to provide reliable information to each other. As observed by Walton & Krabbe, cooperative information-seeking dialogue can be interleaved with other types of dialogue within the same speech event, e.g., a shift may occur to a persuasion dialogue if the addressee does not regard the speaker as an expert, or has other reasons for not accepting the information the speaker is providing. Such

shifts in dialogue type within a single dialogue are, however, beyond the scope of this paper.

Though cooperative information-seeking dialogue is by no means the only type of dialogue, it does take a central place in linguistic communication.[2] It has been pointed out by, among others, Dewey (1925) that "[The heart of language] is communication; the establishment of cooperation in an activity in which there are partners, and in which the activity of each is modified and regulated by partnership." Similarly, cooperation, through the cooperative principle, is the central notion in the influential work of Grice (1975) on logic and conversation. Additionally, Grice highlights the information-seeking dimension of conversation by tailoring his maxims for conversation specifically to conversation for the purpose of 'maximally effective exchange of information' (Grice, 1975, p. 47).

Let us look at a prototypical example of the kind of dialogue I have in mind. It involves a caller with flu symptoms who rings a health information desk to find out whether she needs to see a doctor. The nurse at the information desk is an expert on this matter. At the same time, the caller is presumed to have privileged access to the information whether she has a temperature. Both dialogue participants are interested in resolving the questions that they pose to each other. In other words, the conditions for a cooperative information-seeking dialogue are met, where each interlocutor is deemed expert on specific propositions that come under discussion.

(1)   1.   *caller*   :   Shall I see a doctor?
      2.   *nurse*   :   Do you have a temperature?
      3.   *caller*   :   Wait a minute [*caller* checks her
                          temperature], yes, I do.
      4.   *nurse*   :   Then see a doctor.

The dialogue illustrates two important structural characteristics of dialogue that have been studied by conversation analysts. Firstly, there is the fact that utterances typically come in pairs, where the occurrence of the second part of the pair is conditional on the occurrence of the first part (Schegloff, 1972; Schegloff and Sacks, 1973). Secondly, pairs can be embedded, with embedded pairs referred to as insertion sequences (Schegloff, 1972). In this example, we have the pairs $\langle 1, 4 \rangle$ and $\langle 2, 3 \rangle$, with the latter embedded in the former. The dialogue also illustrates the fact that linguistic and non-linguistic action are often interleaved in conversation (Levinson, 1979). Utterance 3. involves the interleaving

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[2] Moreover, there are good reasons for modelling non-cooperation in dialogue as supervenient on the rules for cooperative behaviour, see Plüss (2010).

of linguistics actions (words being spoken) with action (an observation by the caller to check their temperature).

2.2. MOTIVATION AND RELATED WORK

For DCT to serve as an account of cooperative information-seeking dialogue, structures such as the ones in dialogue (1) should emerge from the mechanisms for information flow that the theory posits. In DCT, this is accomplished as follows. Cooperative information-seeking dialogue is viewed as being sustained by inferential practices that dialogue participants have mastered. These practices are modelled as inference rules in a natural deduction calculus. The product (i.e., trace) of engaging in such practices are modelled in the theory as proof trees generated by the calculus. We show that these proof trees map to (and thus account for) the structure of certain cooperative information-seeking dialogues.

The approach followed here aims at a parsimonious account of the mechanisms that are required to generate the dialogue structures in Example (1). Our goal is to answer the question *When starting from a natural deduction calculus, which miminal additional machinery is needed to account for these structures?*

We will address the question of why to start with natural deduction in a moment, but first let us elaborate on the minimality requirement. This is interpreted as refraining from introducing complex propositional attitudes and full intention recognition and formation for modelling dialogue, a strategy which recently has been advocated eloquently by Gregoromichelaki et al. (2011). Their argument builds on the foundational work by Millikan (2005) (her teleosemantic approach to language content), Brandom (1994) (the social-inferential account of communication), and a range of findings from psycholinguistic research – including Healey et al. (2003), Horton & Gerrig (2005), Keysar (2007), Pickering & Garrod (2004) and Wellman et al. (2001) – which support a paradigm shift away from classical Gricean and neo-Gricean approaches. In research on computational models of dialogue, the case against using complex propositional attitudes to model cooperative dialogue has also been made by Taylor et al. (1996).

Let us now turn to the choice for (an amended version of) the natural deduction calculus as a starting point. This choice is motivated by the purpose of this calculus, as set out by its inventor Gerhard Gentzen, as a formalism which is as close as possible to that of actual human inferential activity. Since our theory aims to account for actual human dialogue in terms of the underlying inferential practices, a calculus which mirrors as closely as possible human inferential practices is preferable. The natural deduction calculus proceeds from assumptions to a conclusion

via inference rules. Gentzen contrasts this with the systems developed by Russell, Hilbert and others at the time, which all start from a set of logical axioms and, usually, a single inference rule (Modus Ponens), see Gentzen (1934, pp. 176 and 184).

Most research on dialogue adheres to either one of two paradigms. The first approach, as pioneered by Power (1979) proposes to view dialogue as construction and execution of a joint plan, which involves the interlocutors proposing and carrying out planning procedures. These procedures include a fixed stock of conversational procedures (such as ASK, TELL, DISCUSS and ASSESS) that give rise to dialogue that addresses subgoals of the overall plan. Similar to Power's conversational procedures, according to (Levin and Moore, 1988) dialogue is organised around fixed patterns, which they call Dialogue-Games, for achieving specific goals. A second approach, which pervades most current research in dialogue, views dialogue as organised around information states and rules for transitions between information states (update rules) and rules for generation of dialogue act in information states (generation rules). Traum & Larsson (2003) describe a generic framework for information state-based dialogue.[3] Pulman (1999) shows that certain approaches that pre-date the information state-based approach can be formulated in terms of information states, including Ginzburg's (1996) up- and downdating rules for the partially ordered questions under discussion. What all these approaches have in common is that inference is not viewed as central to dialogue activity. Rather, the conversational procedures, games, update and generation rules will, on occasion, call on the inferential capabilities of the dialogue agents. The current paper aims to propose a different order of explanation, where inference takes the centre stage. In this, we follow Brandom who argues that language does have a 'downtown'. Brandom takes aim, in particular, at Wittgenstein's broad notion of a language game ('Sprachspiel') which includes a game involving a builder and his helper, with the builder calling out names of building materials which are then handed over by the helper. According to Wittgenstein this constitutes a 'vollständige primitive Sprache', i.e. a complete primitive language (Wittgenstein, 1984, p. 238). Brandom says the following about this claim:

> Thus the 'Slab' *Sprachspiel* that Wittgenstein introduces in the opening sections of the *Philosophical Investigations* should not [...] count as a genuine *Sprach*spiel. It is a *vocal* but not yet *verbal* practice. By contrast to Wittgenstein, the inferential identification of the

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[3] See also Beun (2001) who introduces the distinction between update and generation rules which operate on the interlocutors' mental states. Beun refers to the system he sets up as a dialogue game, giving the term a different use from that proposed by Levin & Moore (1988).

> conceptual claims that language (discursive practice) has a *center*;
> it is not a motley. Inferential practices of producing and consuming
> *reasons* are *downtown* in the region of linguistic practice. Suburban
> linguistic practices utilize and depend on the conceptual contents
> forged in the game of giving and asking for reasons, are parasitic
> on it. (Brandom, 2000, p. 14)

Following up on Brandom's emphasis on inferential practices as central,
our aim is to show how a minimal extension of a natural deduction
based model inferential practices already gives rise to several of the
key dialogue structures that we find in everyday language.

The order of explanation here is from inferential practices to dialogue
structure. Interestingly, the opposite direction has been worked out by
Lorenzen & Lorenz (1978). They specify the meaning of the logical
constants in terms of their role in rational debates.[4] They show how one
can extract valid patterns of reasoning involving the logical constants
from formal winning strategies for *adversarial* dialogues (see also Barth
& Krabbe 1982). Apart from the difference in the order of explanation,
the current work introduces a new perspective in that it can account
for dialogue structure in terms of inferential practices that are free of
logical vocabulary, whereas in Lorenzen & Lorenz, as in Piwek (2007),
logical vocabulary plays a central role.

2.3. A SYSTEM OF MULTI-AGENT NATURAL DEDUCTION WITH OBSERVATION

2.3.1. *Inference*

To account for the aforementioned structures we set up a system of
natural deduction that is extended with rules for multiple agents and
observation. In this section, we first focus on the core of the system
(without multiple agents and observation).

At the heart of this system is the notion of a judgement. Following
Frege's Begriffschrift – see, in particular, Geach & Black (1952, pp. 1-
10) – inferential relations are taken to hold between judgements, where
a judgement $\vdash A$ expresses the truth of the proposition A. As pointed
out by Göran Sundholm, in a critique on 'ordinary formal theories'
where the derivable objects are propositions:

> It is simply not correct to say that the proposition B follows from
> the proposition $A \rightarrow B$ and A. What is correct is that the *truth*
> of the proposition B follows from the *truth* of $A \rightarrow B$ and the

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[4] Note though that what Lorenzen & Lorenz call rational debates is not based
on empirical studies of the structure of actual debates, but rather on an idealised
version of such debates.

truth of A. Thus *the premises and conclusions of logical inferences are not propositions but judgements as to the truth of propositions.* (Sundholm, 1986, p. 498)

Apart form categorical judgements $\vdash A$, we also have hypothetical judgements of the form:

(2) $H \vdash A$

Here, H is a set of propositions that play the role of temporary assumptions. In a hypothetical judgement, the truth of the proposition A depends on the truth of the assumptions in H.[5] Temporary assumptions hold only for the duration of an inference and need to be discharged before arriving at a categorical judgement (an example follows shortly).

In fact, we can view (2) as the generic form of judgements, provided that we take a categorical judgement to be equal to a hypothetical judgement with $H = \emptyset$.

In addition to temporary assumptions, we also have a set Γ of persistent commitments. These remain valid across inferences and can be updated as a result of learning new information non-inferentially (e.g., through communication and observation). For example, the declaration $\Gamma = \Gamma \cup \{A\}$ results in the addition of A to Γ. Γ is most akin to a global variable in a programming language (whereas the role of H is analogous to that of a local variable).

We restrict the scope of this paper to a language $\mathcal{L}$ consisting of atomic formulae $At \subset \mathcal{L}$, *and* formulae that are constructed from members of $\mathcal{L}$ using the connectives for implication '$\rightarrow$' and conjunction '$\wedge$': if $A, B \in \mathcal{L}$, then $(A \rightarrow B) \in \mathcal{L}$ and $(A \wedge B) \in \mathcal{L}$.

We define practices for this language in the form of the following two types of rule schemas:

(3)   a. $\dfrac{Test}{Judgement}$ b. $\dfrac{Judgement_1 \ldots Judgement_n}{Judgement}$ with $n \geq 1$

For now, we have one instance of schema (3.a):

(4)   (member) $\dfrac{A \in \Gamma \cup H}{H \vdash A}$

In words, if testing that A is a member of the set of temporary (H) or persistent commitments (Γ) succeeds, then this allows us to conclude that A holds. In the event that A is a member of H, but not Γ, A holds provisionally (i.e., dependent on the temporary assumptions in

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[5] These are similar to the hypothetical judgements that can be found in Martin-Löf's (1984) Contructive Type Theory.

H). Note that in the conclusion we have supressed the set of persistent commitments. Strictly speaking we should write $\langle \Gamma, H \rangle \vdash A$ instead of $H \vdash A$.

We have the usual introduction and elimination rules for '$\rightarrow$' and '$\wedge$' as instances of schema (3.b):

$$(5) \quad (\rightarrow \text{intro}) \ \frac{H \cup \{A\} \vdash B}{H \vdash A \rightarrow B} \quad (\rightarrow \text{elim}) \ \frac{H \vdash A \rightarrow B \quad H \vdash A}{H \vdash B}$$

$$(6) \quad (\wedge \text{intro}) \ \frac{H \vdash A \quad H \vdash B}{H \vdash A \wedge B} \quad (\wedge \text{elim}) \ \frac{H \vdash A \wedge B}{H \vdash A} \quad \frac{H \vdash A \wedge B}{H \vdash B}$$

Structural rules, in particular, *contraction* and *exchange* are obviated by our stipulation that H and Γ are sets from the outset (making it not even possible to formulate these rules in our framework). The fact that assumptions come in sets is exploited by the (member) rule – a rule for non-inferential entry into inferential practices. This rule enables us to derive the judgement $H \vdash A$, whenever A is a member of H or Γ; these being sets, there is no concept of number of occurrences (contraction) or ordering of assumptions (exchange) in H and Γ. Of course, in actual human entry practices for inference, the availability of an assumption may be influenced by its salience, number of uses, etc., but we keep such considerations, which would need much more fine-grained notions (e.g., degrees of salience) than possible by switching off contraction and exchange, outside of the calculus. The calculus merely specifies the inferences that are in principle available (see also Section 3.3.2). A further structural rule that we need to consider is *weakening*:

$$(7) \quad (\text{weakening}) \ \frac{H \vdash A}{H \cup \{B\} \vdash A} \ (\text{with } B \text{ fresh in } H \cup \Gamma)$$

In the current calculus *weakening* (or *thinning*) is admissible, i.e., does not affect which judgements are derivable: if we can derive $H \vdash A$, then derivation of $H \cup \{B\} \vdash A$ is also guaranteed: if the (member) rule is able to extract an assumption from a set of assumptions H_1, then it is also able to extract that same assumption from any superset H_2, thus making the entry of A into the inference independent of the members of $H_2 - H_1$. Moreover, the inference rules for the conditional and conjunction always proceed from judgements that depend on sets of assumptions to a conclusion which depends on a set of assumption which is equal to or a subset of the aforementioned assumption sets.

The following example shows the rules in action. We prove that the tautological judgement $\vdash (p \wedge q) \rightarrow p$ holds categorically (i.e., $\emptyset \vdash (p \wedge q) \rightarrow p$):

(8)

$$\dfrac{\dfrac{\dfrac{p \wedge q \in (\emptyset \cup \{p \wedge q\})}{\{p \wedge q\} \vdash p \wedge q} \; (member)}{\{p \wedge q\} \vdash p} \; (\wedge \, elim)}{\emptyset \vdash (p \wedge q) \rightarrow p} \; (\rightarrow \, intro)$$

This is an example of a proof tree. The general definition of proof trees is given below (with T ranging over tests, J over judgements and Π over proof trees) and also defined in Appendix A as the prolog predicate `proof_tree`. In this definition $\mathcal{R}_{entry}$ refers to a set of rules of the form of schema (3.a) and $\mathcal{R}_{infer}$ refers to a set of rules of the form of schema (3.b). The former are rules for entering an inferential practice (e.g., by using an assumption), whereas the latter stand for the inferential practices themselves. A proof (tree) is alway relative to antecedently specified sets $\mathcal{R}_{entry}$ and $\mathcal{R}_{infer}$.

(9) a. $\dfrac{T}{J}$ is a proof tree if it is a member of $\mathcal{R}_{entry}$

  b. $\dfrac{\dfrac{\Pi_1}{J_1} \cdots \dfrac{\Pi_n}{J_n}}{J}$ is a proof tree if $\dfrac{J_1 \ldots J_n}{J} \in \mathcal{R}_{infer}$ and $\dfrac{\Pi_1}{J_1} \ldots \dfrac{\Pi_n}{J_n}$ are proof trees.

  c. nothing else is a proof tree.

2.3.2. *Dialogue: Transfer and Observation*

To extend the framework presented so far to dialogue we introduce two further rules. Firstly, we add a rule to $\mathcal{R}_{infer}$ for transferring information between agents. This requires us to make judgements relative to agents. So instead of $H \vdash A$, we now have $[\alpha] \, H \vdash A$, where α is the name of an agent.

(10) (tr) $\dfrac{[\beta] \, \emptyset \vdash A}{[\alpha] \, H \vdash A}$ pre: $\langle \beta, \alpha \rangle \in \mathcal{C}$ and $\text{expert}(\beta, A)$

         post: $\Gamma_\alpha = \Gamma_\alpha \cup \{A\}$

This transfer rule says that if A follows for β (with no temporary assumptions active), then A also follows for α. Note that this rule, if taken declaratively, means that agent β shares *all* their information with α. However, here we view the rule as specifying an inferential

practice. Such a practice is procedural in nature: for information to be transferred the practice has to be excuted. This makes it possible to distinguish between information that the agents potentially share and information they actually share.

The (tr) rule has a pre- and postcondition. The precondition requires there to be an open communication channel between α and β ($\mathcal{C}$ is the set of communication channels, specified as pairs of agents) and that β is an acknowledged expert on A. The postcondition states that performing this practice results in an update of the persistent context Γ_α of α. Consequently, after the transfer has taken place, α will on subsequent occasions be able to arrive at the conclusion without recourse to β. This seems plausible, given that α has already once (as a result of the application of the (tr) rule) used A in its own proof.[6]

A separate issue is that of whether α should so easily use information provided by β. Currently, the only pre-condition is that there exists a communication channel between the two and β is an expert on A (in what follows this condition is left implicit). One could, however, add further pre-conditions on when information from β should be allowed to enter α's proofs and commitments to account for dialogue types different from the cooperative information-seeking dialogue that we are currently investigating.[7]

Note that whereas for β the transfer rule requires H to be empty, this is not the case for α (though it is permitted that $H = \emptyset$ also for α). We allow H to be non-empty to account for situations where α is in the middle of contructing a proof using the temporary assumptions in H and needs A to complete the proof. The (tr) rule allows α to 'import' A into the proof.

Our second new rule is a rule for entering inferential practices, it belongs to $\mathcal{R}_{entry}$. This rule allows us to proceed from α observing that A to α concluding that A.

(11) (obs.) $\dfrac{observe(\alpha, A)}{[\alpha]H \vdash A}$ pre: $A \in \mathcal{O}_\alpha$ post: $\Gamma_\alpha = \Gamma_\alpha \cup \{A\}$

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[6] For the purpose of this paper, I assume that the global updates are excuted *after* the completion of the proof tree that contains the rules that triggered those updates. This allows us to keep the definition of proof trees relatively simple. If global updates could have effects *inside* the proof in which they were triggered, this would require us to extend our notion of proof trees with constraints on the ordering in which their branches are constructed (to regulate which information from such global updates is available in which branches).

[7] A further consideration is the question how A is precisely recorded in Γ_α. In our current proposal, the information is simply added. There are, however, more fine-grained options. For example, one could add some sort of label to A which records its provenance - *cf.* the labels used in Kibble's (2006) formalisation to record, possibly partial, justifications of Brandomian commitments and entitlements.

Again there are pre- and postconditions. The precondition requires that A is a member of the set of observables for α. The postcondition makes the conclusion that A holds persistently by adding it to α's stock of persistent assumptions.

With all this in place, we are now in a position to model the underlying inferential practices of dialogue (1). We assume that at the outset of the conversation the caller's persistent context Γ_{caller} is empty. The nurse has access to the rule that having a temperature implies that one needs to see a doctor. Thus, $\Gamma_{nurse} = \{ht \rightarrow sd\}$. Given these contexts and the rules described above we can construct the following proof tree:

(12)

$$
\cfrac{
  \cfrac{
    \cfrac{ht \in \mathcal{O}_{caller} \quad obs(caller, ht)}{[caller]\, \emptyset \vdash ht}\ (3)\ (obs.)
  }{[nurse]\, \emptyset \vdash ht}\ (2)\ (tr)
  \qquad
  \cfrac{[nurse]\ ht \rightarrow sd \in \Gamma_{nurse}}{[nurse]\, \emptyset \vdash ht \rightarrow sd}
}{
  \cfrac{[nurse]\, \emptyset \vdash sd}{[caller]\, \emptyset \vdash sd}\ (1)\ (tr)
}
$$

KEY: *ht = have temperature, sd = see a doctor*

POSTCONDITIONS: (1) $\Gamma_{caller} = \Gamma_{caller} \cup \{sd\}$, (2) $\Gamma_{nurse} = \Gamma_{nurse} \cup \{ht\}$, (3) $\Gamma_{caller} = \Gamma_{caller} \cup \{ht\}$.

Construction of the proof tree leads to a situation where $\Gamma_{caller} = \{ht, sd\}$ and $\Gamma_{nurse} = \{ht \rightarrow sd, ht\}$. Note that as a side effect of being engaged in the proof construction/dialogue, both *caller* and *nurse* now *individually* have the information required to, on a later occasion, infer *sd*. In other words, as a side effect of the proof construction the persistent commitment stores Γ_{nurse} and Γ_{caller} have been updated such that *nurse* and *caller* can now infer *sd* from their own commitments without recourse to observation or communication.

From the proof tree, we can extract the moves of dialogue (1). A commented (Prolog) program for carrying out such mappings from proof trees to dialogue structures is provided in Appendix A.[8] The representation of the proof tree (12) which this program takes as input is given in appendix B.2, together with the output that is generated and which corresponds with the dialogue structure of dialogue (1).

Here, we informally describe how the mapping proceeds. We start at the root node of the tree and traverse the tree bottom-up. At the root node, we find a proof goal: $[caller]\emptyset \vdash sd$. This goal is handed over to the nurse via the transfer rule. Transferring a proof goal correspond with

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[8] The code is available in electronic form at `mcs.open.ac.uk/pp2464/resources/`

asking a question, an approximation of the first question of dialogue (1), *Do you have a temperature?*. The approximation of this question in terms of the proof goal can be paraphrased as '*Can you prove that you have a temperature?*' (see Section 2.4 for the limitation of this approximation of the question). The answer has to wait until the entire branch leading to the proof goal has been closed. In this case, as a result of invoking the $\rightarrow$ elimination rule two new branches are created. The lefthand side branch involves a further invocation of the transfer rule, corresponding with the second question of dialogue (1). The branch above this goal is closed through the observation rule. The result is a positive answer to the second question, corresponding to utterance 3. of the dialogue (which includes a trace of the application of the test, i.e., the observation of having a high temperature). The righthand side branch is closed as a result of the nurse holding the information that having a temperature implies needing to see a doctor. With all the branches leading to the root proof goal closed, the answer to the inital question is affirmative ('Then see a doctor'), i.e., utterance 4 in the dialogue.

What we have achieved so far is to show that certain high-level structures of cooperative information-seeking dialogues can be accounted for in terms of inferential reasoning by dialogue participants, without resorting to complex propositional attitudes.

2.4. LIMITATIONS

The main contribution of this paper is the use of the DCT framework to formalise logical expressivism and then repurpose the formalisation to reshape the DCT framework itself. The scope of the project is very much in the spirit of Brandom's account of deontic scorekeeping as 'an artificial idealization [...] Simplified and schematic though the model may be, it should nonetheless be recognizable as a version of what we do.' (Brandom, 1994, p. 158). As far as the simplifications are concerned, the main ones are:

- The theory so far is only concerned with assertion and, consequently, positive proof goals. A proof goal with the content $\vdash A$ corresponds to the question *Can you prove that A is true?* The answer 'no' does not necessarily mean that A is not the case (merely that it cannot be proven by the addressee of the question). This greatly simplifies our use of proof trees, which otherwise, for each polar question A?, would need to incorporate both positive and negative proof goals (for example, both *Can you prove that A?* and *Can you prove that not A?*, respectively). Since this would, however, not subtantially alter the relation between the proof trees

and dialogue structure, for the sake of simplicity, we have restricted our attention to assertion only, and leave the exposition of a calculus which also includes denial and consequently negation for future work. In this respect, our approach is similar to Lance & Kremer (1994), who formalise Brandom's notion of commitment, and focus on a logic restricted to implication and conjunction. Our focus on implication is, apart from simplicity, based on the centrality of the conditional in Brandom's logical expressivism. Lance & Kremer add conjunction only for the purpose of 'simplif[ying] the axiomatizations of our systems' Lance & Kremer (1994, p. 376). In our case, adding conjunction has a more substantive motivation, given that it allows us to make explicit inferential practices with multiple premises, see Section 3.3.2.

– Without denial, contradiction is impossible. This also means that the need for non-monotonic inference does not arise. With most common everyday inference being non-monotonic, we do, however, envisage a future extension of the model with denial that allows for non-monotonic inference.

– In this section, we have focused on mappings from proof trees to dialogue. To deal with a broader range of dialogue, which also permit responses conveying lack of knowledge to questions (e.g., 'I don't know') we need to move from proof trees to proof search trees. This is accomplished in Piwek (2007), but orthogonal to the purpose of this paper.

This concludes our list of limitations which, of course, could be extended substantially given that our 'artificial idealisation' only focuses on a few selected properties of dialogue structure. Further work will show whether the proposed approach can be extended beyond these to include the rich tapestry of different dialogue types, degrees of coherence, cooperation and non-cooperation in dialogue.

3. Logical Expressivism

In his magnum opus 'Making it explicit',[9] Robert Brandom takes up the expressive conception of logic which he traces back to Frege's early work. The starting point for this view of logic is the idea that propositions are characterised in terms of the inferential practices in which they take part, their proprieties of inference. This in contrast with modern

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[9] (Brandom, 1994)

model-theoretic semantics, which conceives of propositions in terms of set-theoretical extensions.

In this section, I aim to do three things. Firstly, I briefly introduce Brandom's expressivism. Next I show how it can be formalised using the natural deduction-based machinery of Discourse Coherence Theory. Finally, I discuss a number of challenges to logical expressivism and show how these can be clarified using the formalisation that I have undertaken.

3.1. Brandom's Expressivism

Brandom characterizes the core tenets of logical expressivism succinctly as follows:

> Logic is not properly understood as the study of a distinctive kind of *formal* inference. It is rather the study of inferential roles of vocabulary playing a distinctive *expressive* role: codifying in explicit form the inferences that are implicit in the use of ordinary, non-logical vocabulary. Making explicit the inferential roles of logical vocabulary then can take the form of presenting patterns of inference involving them that are formally valid in the sense that they are invariant under substitution of nonlogical for nonlogical vocabulary. But that task is subsidiary and instrumental only. The task of logic is in the first instance to help us *say* something about conceptual contents expressed by the use of nonlogical content, not to *prove* something about the conceptual contents expressed by the use of logical vocabulary. (Brandom, 2000, p. 30)

Thus, logical vocabulary is viewed as a *tool* which allows speakers to describe the inferential practices which underlie their language use. The logical vocabulary allows them to make those practices explicit. For instance, the conditional 'if A then B' ('$A \to B$'), involving the non-logical contents A and B, allows us to make explicit the materially substantive practice of concluding B as result of having established that A holds.

This approach to logic, though unconventional, bears great promise, because it suggests an objective requirement on the meaning of logical connectives (such as 'if ... then ...' and 'and', here formalised as '$\to$' and '$\wedge$'). As with the meaning of non-logical vocabulary, the meaning of a logical connective is identified with the inferential practices that govern its use. According to logical expressivism, these inferential practices need to be strictly descriptive: they should describe pre-existing non-logical practices and not change or extend these practices. This idea can be made precise in terms of Belnap's (1962) notion of conservativity, which I formulate here using the terminology introduced in this paper.

The requirement of conservativity holds between two inferential practices (and the languages belonging with each of the practices). Let us assume a language with only non-logical vocabulary $\mathcal{L}_0$ and corresponding inferential practices $\mathcal{IP}_0$. We obtain $\mathcal{L}_1$ by adding logical vocabulary to $\mathcal{L}_0$. $\mathcal{IP}_1$ is obtained by adding inferential practices for the logical vocabulary to $\mathcal{IP}_0$. We speak of a judgement J, of the form $H \vdash A$, as being a member of language $\mathcal{L}$ if: $A \in \mathcal{L}$ and $\{F | F \in H\} \subseteq \mathcal{L}$.

(13) CONSERVATIVITY: $\langle \mathcal{L}_1, \mathcal{IP}_1 \rangle$ is a *conservative extension* of $\langle \mathcal{L}_0, \mathcal{IP}_0 \rangle$ if and only if: $\{J \in \mathcal{L}_0 \mid J \text{ is derivable under } \mathcal{IP}_1\} = \{J \in \mathcal{L}_0 \mid J \text{ is derivable under } \mathcal{IP}_0\}$.

In words, the extended set of inferential practices ($\mathcal{IP}_1$), which includes practices for the logical vocabulary, should make available the same set of judgements in the pre-logical language $\mathcal{L}_0$ as those that were already available under the non-logical inferential practices $\mathcal{IP}_0$.

3.2. EXPRESSIVISM FORMALISED

According to logical expressivism, propositions are characterized in terms of the inferential practices that they take part in. In DCT, we have two schemas for specificying such practices, here given as (3.a) and (3.b). Let us revisit the practice, adopted by the nurse in Section 2.3.2. This practice consists of concluding *See a doctor* (*sd*) from *Has a temperature* (*ht*). It can be captured by adding the following rule, instantiating (3.b), to $\mathcal{R}_{infer}$:

(14) (ht_to_sd) $\dfrac{H \vdash ht}{H \vdash sd}$

This rule stands for the practical ability to derive *sd* when *ht* already follows. Given this rule, *sd* can be concluded if, for example, $ht \in \Gamma$ (or alternatively if *ht* is observed), *without* recourse to $ht \to sd$:

(15)

$$\dfrac{\dfrac{ht \in (\Gamma \cup \emptyset)}{\emptyset \vdash ht}\ (member)}{\emptyset \vdash sd}\ (ht\_to\_sd)$$

Note that on the assumption that $(ht\_to\_sd) \in \mathcal{R}_{infer}$, the conditional is, however, available via the introduction rule. This rule governing the use of the logical connective '$\to$' allows us to make the inferential practice underlying the use of *ht* and *sd* explicit:

(16)

$$\frac{\dfrac{\dfrac{ht \in (\{ht\} \cup \Gamma)}{\{ht\} \ \vdash \ ht} \ (member)}{\{ht\} \ \vdash \ sd} \ (ht\_to\_sd)}{\emptyset \ \vdash \ ht \rightarrow sd} \ (\rightarrow intro)$$

In summary, we read $(ht\_to\_sd) \in \mathcal{R}_{infer}$ as saying that (ht_to_sd) is an inferential practice. This inferential practice can be made explicit through the introduction rule for $\rightarrow$. Note that this does assume that '$\rightarrow$' is available as a logical connective in the first place. In principle, there is nothing that excludes the existence of a language community which can make practical inferences involving materially substantive propositions (such as ht and sd), but which has not mastered any logical vocabulary. The difference between that community and, for example, adult speakers of English is that they can't make their inferential practices explicit.

3.3. Logical Expressivism: Critique and Clarifications

3.3.1. *The charge of relativism*

Despite the clear potential of logical expressivism, existing work on formalising Brandom's notions of commitment and entitlement has shied away from formalising this groundbreaking aspect of Brandom's approach. For example, Kibble (2006, p. 190) notes 'that adopting Brandom's concepts of commitment, entitlement and perspectival commitment stores does not necessarilly involve commitment to "expressivism"'. Lance & Kremer go further and directly attack Brandom's logical expressivism:

> If Brandom is correct in claiming that sentences like 'A→B' purport to "codify" underlying linguistic appropriateness to make inferential relations, previously implicit, explicit then the correct system is the one that most accurately describes these underlying social criteria of inferential appropriateness. However, to adopt such a line is to endorse a form of cultural relativism. It is to say that we ought to enshrine in logical theory whatever inferential moves are endorsed by our linguistic community. (Lance and Kremer, 1994, p. 373)

Having provided a formal version of logical expressivism, we are now in a position to address the critique by Lance & Kremer. They object to logical expressivism because it supposedly leads to cultural relativism. They motivate this claim by proposing that logical expressivism 'enshrine[s] in logical theory whatever inferential moves are endorsed by our linguistic community.'

The main problem with this critique is the notion of logical theory which it assumes. Logical theory seems to be equated with the totality of inferential practices of a linguistic community. There is, however, nothing that forces us to adopt this notion of logical theory. There is a perfectly valid way of keeping alive the usual notion of logical theory. This is already present in the quote above from Brandom, specifically, where he characterises formally valid inferences[10] as the set of inferences that are valid 'under substitution of nonlogical for nonlogical vocabulary' (Brandom, 2000, p. 30). A formally valid inference is for example an inference, using Modus Ponens, from the assumptions (A1) *I have evidence of something* and (A2) *I have evidence of something $\rightarrow$ I think what I like* to the conclusion (C) *I think what I like*.

We can characterise such formally valid inferences in our framework as hypothetical judgements $H \vdash A$[11] that can proved with:

-   $\Gamma = \emptyset$,

-   $\mathcal{R}_{entry} = \{(member)\}$, and

-   $\mathcal{R}_{infer} = \{(\rightarrow intro), (\rightarrow elim), (\wedge intro), (\wedge elim)\}$.

That is, we are allowed to use the full language, but have to restrict the inferential practices to those practices which govern the logical vocabulary.

Thus, logical theory turns out to consist of judgements[12] that can be proved without any assumptions and with only the inferential practices for the logical vocabulary and the member rule. This way no materially substantive inferences can enter into logical theory proper. Even though a materially substantive inferential practice, e.g. from *ht* to *sd*, allows derivation of the judgements $\{ht\} \vdash sd$ and $\vdash ht \rightarrow sd$, these do *not* belong to logical theory, since they rely on a non-logical inferential practice. With logical theory consisting of materially non-substantive inferences, the danger of cultural relativism is averted.

In a similar vein, we can define *tautologies* as those propositions A such that the categorical judgement $\emptyset \vdash A$ belongs to logical theory (as defined above). A tautology is derived without use of any assumptions (both Γ *and* H need to be empty) or non-logical inferences; see for example the derivation in (8) of the tautology $(p \wedge q) \rightarrow p$.

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[10] We take the set of formally valid inferences to be what is meant by 'logical theory'.

[11] This includes categorical judgements, which are treated as a special case of hypothetical judgements with $H = \emptyset$.

[12] Or possibly schemas for judgements, if we allow for parameters to range over the expressions belonging to the non-logical vocabulary.

I suspect that some of the confusion regarding Brandom's notion of logical theory derives from the fact that he does consider materially substantive practices to be *normative* in nature. In other words, these practices are subject to judgements of correctness. We do, however, not offer an analysis of this type of correctness. Such a criterion for correctness is only offered here for the practices associated with logical vocabulary: they are subject to a universal context-independent form of correctness (i.e. conservativity), which arises out of their very nature as a tool for describing the underlying normative inferential practices. The materially substantive practices are judged by other criteria, such as overal coherence and consistency (internally and with respect to observation and action), which in the words of Brandom is 'a messy, retail business' (Brandom, 2000, p. 75).

Take the inferential practice 'Whenever you have evidence of something, think what you like', which, in a nice understatement by one the anonymous reviewers of this paper, '[is] not completely desirable' as a part of logical theory. Let us start by emphasising that we agree that this inferential practice is undesirable. But does that also mean that it can't be part of logical theory? Firstly, note that the conditional that can be derived from this practice is *not* a tautology, since its derivation requires use of the non-logical inferential practice. Secondly, we observe that the conditional does, however, feature in inferences that do belong to logical theory (and in that sense is part of logical theory). In particular, a streamlined version occurs above as (A2) in the formally valid inference from (A1) and (A2) to (C). This is, however, not undesirable. If it were, this would not only be a problem for the current account, but also for classical model-theoretic logic. According to classical logic, formally valid inferences are those inferences which preserve truth from the premises to the conclusion. They do, however, *not* guarantee the truth of the premises themselves. In particular, logic has nothing to say about whether (A2) is true or not – even though (A2) can be used in formally valid inference from (A1) and (A2) to (C). Similarly, our account does not tell us whether 'Whenever you have evidence of something, think what you like' is a correct or incorrect inferential practice. In this respect, we have simply traded the traditional notion of truth of a conditional for that of correctness of an inferential practice. Neither can be decided by logic alone.

Another sense in which the approach could be accused of cultural relativism, which to my knowledge Brandom doesn't discuss, is the very structure of the inferential (and non-inferential) practices themselves. We have captured this structure in terms of the schemas (3.a) and (3.b). These structures are posited as the basic shape of our practices. Since these are at the foundation of this approach, the implicit assumption

is that their shape is universal. For now, however, the support for this claim must remain speculative.[13]

3.3.2. *Multipremise inferences and conservativity*

In Weiss (2010), Brandom's expressivism is subjected to a critical examination. Here, I would like to pick up on two of the points made by Weiss which the current formalisation sheds some light on.

Firstly, Weiss suggests that it is problematic to make explicit a materially good inference with multiple premises. The example he gives is the inference from (f) *Fred is male* and (m) *Fred is Mary's sibling* to (b) *Fred is Mary's brother*. Brandom (2010, p. 354) answers that 'To codify multipremise inferences, one must simultaneously introduce the conditional and conjunction'.

$$(17) \quad (fmb) \quad \frac{H \vdash f \;\; H \vdash m}{H \vdash b}$$

Given the inferential practice in (17), and having introduced the conditional and conjunction in (5) and (6), we can here give the actual derivation of the corresponding multipremise conditional:

(18)

$$\frac{\dfrac{\dfrac{(f \wedge m) \in \{f \wedge m\}}{\{f \wedge m\} \vdash f \wedge m}\,(member)}{\{f \wedge m\} \vdash f}\,(\wedge\,elim) \quad \dfrac{\dfrac{(f \wedge m) \in \{f \wedge m\}}{\{f \wedge m\} \vdash f \wedge m}\,(member)}{\{f \wedge m\} \vdash m}\,(\wedge\,elim)}{\dfrac{\dfrac{\{f \wedge m\} \vdash b}{\emptyset \vdash (f \wedge m) \to b}\,(\to\,intro)}{}}\,(fmb)$$

A second point that Weiss raises concerns the notion of conservativity. Brandom emphasises that logical expressions merely play the

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[13] Brandom (2008, p. 9 fn. 8) says that for the 'purposes of the present project, I will maintain a studied neutrality [...] The apparatus I am introducing can be non-committal as to whether we understand content-conferring *uses* of expressions in terms of social practices or individual abilities, or some more complicated constellation of both'. If we adopt the stance that individual *abilities* do play a role, this opens up the possibility of an account in which the structure of these abilities is constrained by univeral properties of our physiogical make-up, in particular, its capacity for a) taking in sensory stimuli, b) (inductive) learning of conditional relations between event or situation type occurrences, and c) initiating bodily motor actions. Note that this would require at least one further schema, in addition to the entry (3.a) and inference (3.b) schemas. This further schema would accommodate going from judgements to action; in other words, a pattern for exit practices, as opposed to entry and inference.

role of making explicit that which we already tacitly are able to do inferentially. This disregards, however, the epistemic usefulness of logical expressions: they not only allow us to say what we previously could only do, they also enable us to construct extended chains of reasoning, which were previously inaccessible to us. I would like to add to this that it goes even further, in that the logical expressions allow us to outsource the drawing of inferences to computers (i.e., theorem provers). These are already able to construct chains of reasoning well beyond the capabilities of human reasoners.

In short, the problem is that if we take conservativity to mean conservativity relative to the practices in as far as human reasoners have taken these, then logical vocabulary gives these reasoners new powers which allow them to take the practices further than they could do before they acquired the logical vocabulary. In a sense, the logical vocabulary and concomitant inferential practices for using it, extend the set of non-logical propositions that are practically accessible. This suggests that the correct notion of conservativity is relative to the non-logical inferences that we can carry out in principle (given infinite time and memory), rather than in practice. The lesson learned from this is that in (13) the phrase 'J is derivable under $\mathcal{IP}$' needs to be read as 'J is derivable *given infinite resources* under $\mathcal{IP}$'.

3.3.3. *Practices and goal-directed Behaviour*

In the current paper, inferential practices are modelled as natural deduction inference rules. These rules take us from one or more judgements (the premises) to another judgement (the conclusion). They specify *procedural* 'know how', i.e. what can be done when certain premises are encountered. In this respect, they are similar to computer programs that take an input and produce an output. Brandom contrasts this with the traditional account of inference, which assumes that inferences from premises to a conclusion rely on a belief in the truth of a *declarative* conditional that connects the premises with the conclusion.

We employ inferential practices in two distinct modes of reasoning:

— *Forward reasoning from premises*: From an observation or judgement to a conclusion.

— *Goal-directed reasoning towards a goal*: Given a proof goal, find a practice and premises which make the goal succeed.

As one of the reviewers of this paper pointed out, the second of these modes of reasoning is potentially problematic in that 'it seems to re-

quire access to some explicit set of conditional rules which can be read "backwards" from consequent to antecedent'.[14]

Brandom does not address this issue, with his notion of inferential practices and how they are put to use remaining rather opaque. The point he emphasizes is that the practices are tacit, unexpressed, whereas the conditionals that we can construct to describe them are explicit. In this respect, one can argue, without going into the issue of modes of reasoning, that natural deduction provides an adequate account, given that the constructs (horizontal stroke, turnstile and set of assumptions) needed to formulate the deduction rules are not part of the language that our inferential practices allow us to draw inferences with (i.e. in our example a language of propositional constants closed under '$\rightarrow$' and '$\wedge$'), and thus are inexpressible in that language. On the other hand, the logical vocabulary and its associated inferential practices permits for the derivation of conditionals which express these practices in the language.

This, however, does not take away the concern that for inferential practices to be different from conditional rules, whether expressible or not, they need to function as black boxes which, given an input, produce an output. Such a black box cannot be directly run in reverse, suggesting an input for a given target output. This raises the general question, how such black box practices can be deployed in goal-directed activity.

An answer will consist of a solution to the problem of finding a practice p from a set of practices P, given a goal g such that when p is applied to an input i, the goal g is satisfied. Let us assume that there is a finite set of elementary practices P (each corresponding to a natural deduction rule). Given a goal g, we need to hypothesize inputs i, then run $p \in P$ on i and test whether g succeeds. The difficulty here is that with all p being black boxes, they provide us with no information on which input-practice pairs are likely to give us g, making the search computationally expensive. Though I don't have a solution to this problem, problems similar to this one have been addressed in other disciplines. In particular, inspiration can be drawn from work in cognitive psychology which suggests that goal-directed actions are guided by *associations* that people learn between their actions and outcomes and vice versa (see the review by de Wit and Dickinson, 2009). Further suggestions for tackling this problem can be found in Purver and Kempson (2004) who explore the use of context to guide the selection of forward-directed parsing actions (from text to

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[14] This issue has previously been brought to my attention by Rodger Kibble at a presentation in 2010 at soas of this work.

to logical form) for goal-directed language generation. With the goal being a particular logical form, their approach comes down to context-guided hypothesizing of parsing actions (associated with words) that are consistent with the target logical form.

In summary, though computationally expensive, it is, in principle, possible to use forward-directed inferential practices for goal-directed reasoning whilst treating the practices as black boxes (i.e. as opaque with regards to their logical structure). Furthermore, there is work in other disciplines that suggest computationally feasible heuristics for approximating a solution.

4. Dialogue and Logical Expressivism

Having used the framework of DCT to get a better understanding of logical expressivism, we can now turn the insights that have been gained back on DCT itself. One of the crucial insights is that we can have agents engage in conditional reasoning *without* explicit recourse to the conditional ('$\rightarrow$') itself. This means that now we have availed ourselves of an even more parsimonious account of the structure of dialogue (1). Rather than assuming that the nurse in the dialogue has explicit access to the information that $ht \rightarrow sd$, we can rely on an inferential practice (specific to the nurse):

(19) (ht_to_sd) $\dfrac{[nurse]H \vdash ht}{[nurse]H \vdash sd}$

This yields a single branch proof tree for dialogue (1):

(20)

$$\dfrac{\dfrac{\dfrac{\dfrac{obs(caller, ht)(a)}{[caller]\,\emptyset \vdash ht}\ (obs)\ post : \Gamma_{caller} = \Gamma_{caller} \cup \{ht\}}{[nurse]\,\emptyset \vdash ht}\ (tr)\ post : \Gamma_{nurse} = \Gamma_{nurse} \cup \{ht\}}{[nurse]\,\emptyset \vdash sd}\ (ht\_sd)}{[caller]\,\emptyset \vdash sd}\ (tr)\ post : \Gamma_{caller} = \Gamma_{caller} \cup \{sd\}$$

As with proof tree (12), we read off dialogue (1) from proof tree (20) in a bottom up fashion – applying the algorithm of Appendix A to the input representation of proof tree (20) in Appendix B.2.

We start at the root node, where we find the proof goal $[caller]\,\emptyset \vdash sd$. Via the transfer rule, this goal is handed over to the nurse. This correspond to the first move in dialogue (1): *Shall I see a doctor?* The

algorithm uses the approximation 'ask(caller,sd)'. This puts on the nurse the obligation to confirm success of the proof goal as soon as the branch of the proof tree above it is closed (in the algorithm this 'obligation' is recorded in the variable JList). Next the nurse invokes the material rule ht_sd, arriving at the new proof goal $[nurse]\,\emptyset \vdash ht$. This goal is handed back to the caller through the transfer rule (of course, with the agent *nurse* replaced by *caller* in the goal). This corresponds to the second move, *Do you have a temperature?*, in the dialogue. It is now upon the caller to report to the nurse that this goal has succeeded as soon as the branch of the proof tree above it is closed. The branch is indeed closed off at the next step, when the caller observes that she has a temperature. As required, the caller confirms that they have a temperature to the nurse; this is the third move in the dialogue. This signals to the nurse that the branch of the proof tree above the goal $[caller]\emptyset \vdash sd$ is closed off. Confirming this to the caller, the final fourth move in dialogue (1), completes the dialogue.

Note that at each step of the mapping from proof tree to dialogue mirrors backward chaining goal-directed construction of the proof tree itself. Thus, the dialogue can be used to verbalise the proof construction as it takes place.[15]

5. Conclusion

This paper started with the presentation of an elementary system of inference for multiple agents which accounts for certain high-level structures that are exhibited by cooperative information-seeking dialogue. We showed how this system can be used to throw light on a novel conception of logic, logical expressivism, as proposed by Robert Brandom (Brandom, 1994; Brandom, 2000; Brandom, 2008). We found that the formalisation was helpful as a means for clarifying a number or issues, in particular, regarding (1) the charge of cultural relativism, (2) the nature of conservativity and (3) the understanding of practices as goal-directed activities. Finally, we showed how the insights gained from logical expressivism fed back into DCT, a theory of dialogue coherence. It opened up the possibility of an even more parsimonious account of certain dialogue structures. An important consequence of our formalisation of logical expressivism is that it drives a clear wedge between inferential practices on the one hand and their explicit verbalisation on the other. It prepares the ground for modelling both communities

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[15] In this respect, the algorithm of Appendix A is different from that presented in Piwek (2007) which specifies a mapping that does *not* respect the order of goal-directed backward chaining proof construction.

with and without the ability to explicitly talk about their inferential practices. It may also have applications to the modelling of language acquisition, by allowing for a distinction between knowing how (i.e., mastery of the practice itself) and knowing that (i.e., expression of practices).

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Appendix

A. Proof Tree to Dialogue Structure Mapping Code

```
%%% MAPPING PROOF TREES TO DIALOGUE STRUCTURE %%%

% Prolog Source Code tested with
% SWI-Prolog 5.10.1 and SICStus 3.12.8

% VARIABLE NAMING CONVENTIONS (for readability)

% J for Judgement nodes
% JContent and JAgent for content and agent of a judgement
% Entry and Infer for Rule names
% L for Leave nodes which close off a branch of the proof tree
% T for Tree
% D for Dialogue
% R prefix for Remainder (of a list)
% JList records the goals of a tree branch whose success
% needs to be confirmed when they are closed
% (because these goals were introduced through
% the communicative transfer rule and in the dialogue
% match with an 'ask' speech act)

% ENCODING OF PROOF TREES

% This definition checks the global structure of the
% tree, it does not check whether the correct rules
% (corresponding with the rule names) have been used.
```

```
proof_tree((_J,Entry,_L)):-
  entry_rule_name(Entry).

proof_tree((_J,Infer,[T|RT])):-
  infer_rule_name(Infer),
  proof_tree(T),
  proof_trees(RT).

proof_trees([]).

proof_trees([T|RT]):-
  proof_tree(T),
  proof_trees(RT).

entry_rule_name(observe).
entry_rule_name(member).
infer_rule_name(tr).
infer_rule_name(and-intro).
infer_rule_name(and-elim).
infer_rule_name(arrow-intro).
infer_rule_name(arrow-elim).
infer_rule_name(material).

% MAPPING FROM TREES TO DIALOGUE

map_tree2dialogue(T,D):-
  proof_tree(T),
  map(T,[],D).

% A transfer (tr) rule introduces an 'ask' speech
% act into the dialogue

map((J,Infer,RT),JList,[ask(JAgent,JContent)|RD]):-
  Infer = tr,
  !,
  J = (JAgent, JContent),
  RT = [((JAgent2,JContent2),_,_)],
  map_trees(RT,[(JAgent2,JContent2)|JList],RD).

% Inference rules other than transfer are not expressed
% explicitly in the dialogue

map((_J,Infer,RT),JList,D):-
```

```
  infer_rule_name(Infer),
  map_trees(RT,JList,D).

% Entry rules which close of a proof branch, lead
% to expression of all the proof goals that have
% now succeeded (i.e., members of JList)

map((_J,Entry,_L),JList,D):-
  entry_rule_name(Entry),
  express(JList,D).

% The final tree of a sequence of trees
% which all rooted in the same branch
% is associated with the explicitly introduced
% proof goals (via transfer) of that branch, such that
% when the tree is closed off, confirmation of the
% success of the branch's goals is expressed
% (using the 'confirm' speech act).

map_trees([T],JList,D):-
  map(T,JList,D).

% The non-final trees of a sequence of trees
% which are all rooted in the same branch
% are not associated with explicitly introduced
% proof goals (via transfer) of that branch,
% i.e., in map(T1,[],D1) the second argument
% is the empty list. Thus, only when the final
% tree that is rooted in the branch has been
% closed will the succes of the branch's goals be
% expressed (using the 'confirm' speech act).

map_trees([T1|RT],JList,D):-
  map(T1,[],D1),
  map_trees(RT,JList,RD),
  append_lists(D1,RD,D).

% When a branch of the proof is completed, 'express' is
% called to express all the judgements in that branch
% that have now been proven

express([],[]).
```

```
express([(JAgent,JContent)|T],[confirm(JAgent,JContent)|RT]):-
  express(T,RT).

% General purpose predicates

append_lists([],List,List).

append_lists([H|T],List,[H|NList]):-
  append_lists(T,List,NList).
```

B. Input-Output Examples

B.1. INPUT PROOF TREE (12) FOR DIALOGUE (1)

```
  ((caller,sd),tr,
   [
    ((nurse,sd),arrow-elim,
     [
      ((nurse,imply_ht_sd),member,
        successful_test
      ),
      ((nurse,ht),tr,
       [
        ((caller,ht),observe,
          successful_test
        )
       ]
      )
     ]
    )
   ]
  )
```

OUTPUT DIALOGUE STRUCTURE

```
[ask(caller,sd),ask(nurse,ht),confirm(caller,ht),confirm(nurse,sd)]
```

B.2. INPUT PROOF TREE (20) FOR DIALOGUE (1)

```
  ((caller,sd),tr,
```

```
 [
  ((nurse,sd),material,
   [
    ((nurse,ht),tr,
     [
      ((caller,ht),observe,
       successful_test
      )
     ]
    )
   ]
  )
 ]
)
```

OUTPUT DIALOGUE STRUCTURE

`[ask(caller,sd),ask(nurse,ht),confirm(caller,ht),confirm(nurse,sd)]`