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Orthogonal Time in Euclidean Three-Dimensional Space: Being an Engineer's Attempt to Reveal the Copernican Criticality of Alfred Marshall's Historically-ignored 'Cardboard Model'

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Abstract

This paper begins by asking a simple question: can a farmer own and fully utilise precisely five tractors and precisely six tractors at the same time? Of course not. He can own five or he can own six but he cannot own five and six at the same. The answer to this simple question eventually led this author to Alfred Marshall's historically-ignored, linguistically-depicted 'cardboard model' where my goal was to construct a picture based on his written words. More precisely, in this paper the overall goal is to convert Marshall's ('three-dimensional') words into a three-dimensional picture so that the full import of his insight can be appreciated by all readers.

After a brief digression necessary to introduce Euclidean three-dimensional space, plus a brief digression to illustrate the pictorial problem with extant theory, the paper turns to Marshall's historically-ignored words. Specifically, it slowly constructs a visual depiction of Marshall's 'cardboard model'. Unfortunately (for all purveyors of extant economic theory), this visual depiction suddenly opens the door to all manner of Copernican heresy. For example, it suddenly becomes obvious that we can join the lowest points on a firm's series of SRAC curves and thereby form its LRAC curve; it suddenly becomes obvious that the firm's series of SRAC curves only appear to intersect because mainstream theory has naively forced our three-dimensional economic reality into a two-dimensional economic sketch; and it suddenly becomes obvious that a two-dimensional sketch is analytically useless because the 'short run' (SR) never turns into the 'long run' (LR) no matter how long we wait.

Keywords: completed competition, cardboard model, non-Newtonian economics, orthogonal time

JEL codes: A23, B21, B41, B59

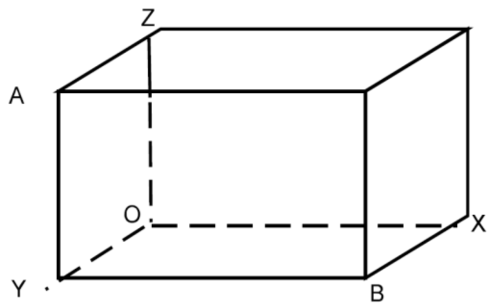
1. The Geometry of Euclidean Three-dimensional Space

We start with Figure 1. It's a simple open-top cardboard box. Notice that we pretend we have X-ray vision so we can see through the cardboard, if required. Several things need to be noted:

1. One corner is labelled 'O' for origin because this will generally be our basic reference point.
2. Angles ZOX and AYB appear as right angles because they are right angles and because they lie 'in' or 'parallel to' the plane of the paper.

3. All other angles (e.g., angle AYO) are also right angles but they do not appear to be right angles when a three-dimensional sketch is forced onto a two-dimensional page.
4. Thus Figure 1 is an orthogonal projection of a simple cardboard box.

Figure 1 A simple open-top cardboard box



We focus our attention on the far lower-left corner. As mentioned above, this shall be the origin of our journey into Euclidean three-dimensional space, so we labelled it as Point O. Next, to aid in the visualisation of what is before us, we imagine that the box has been pushed all the way back against a large piece of white paper thus Point Z, Point O and Point X will be touching the paper. In other words, the side identified as ZOX is a surface lying in the plane of our paper.

Now we look at the side identified by Point A, Point Y and Point B. These three points also create a plane surface but note that, even though AYB is also a plane surface it does not lie in the plane of our paper. It is a flat surface which is parallel to the plane of the paper (and to ZOX). This leaves us with a three-dimensional set of axes on which to place our various musings about reality, Figure 2a.

Figure 2a 3D reality

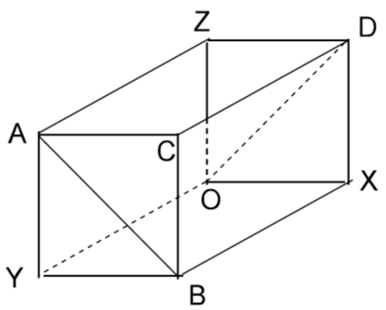
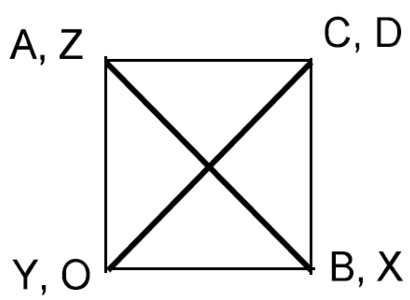


Figure 2b The apparent intersection of two non-intersecting lines when a 3D reality is projected as a 'shadow' onto the (back) plane of the paper (i.e., a misleading 2D sketch of 3D reality).

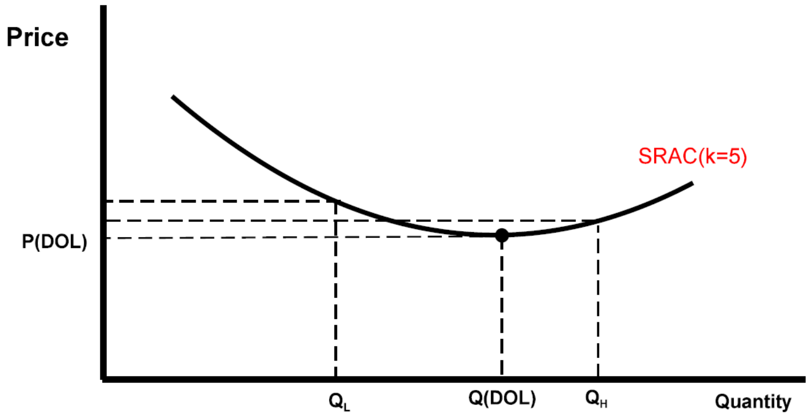


Now we can start to hone in on the crux of the fundamental problem. In order to do this, we re-draw Figure 1 as viewed from a slightly different angle (Figure 2a), we change the proportions to aid in visual clarity and we add lines A-B and O-D. Notice that, in Figure 2a, these two lines do not appear to intersect when drawn in an orthogonal projection (i.e., when drawn in a picture which is closer to our three-dimensional reality) yet, when we naively force the picture back into a two-dimensional sketch (Figure 2b), it now appears as if they do intersect. Here's the reason: we have inadvertently cast a 'shadow' of Figure 2a back onto our paper (Figure 2b). Thus a researcher who was given only Figure 2b on which to base his/her analysis would probably assume that lines A-B and O-D intersect when, in reality, they do not [Appendix, pp. 40-41].

1. Honing In: The Geometry of 'the Short Run' Sketched in Two Dimensions

Let us now use what we have learned by applying it to an examination of the short run average cost (SRAC) curve for our farmer who owns precisely five output-producing tractors, Figure 3.

Figure 3 A typical short-run average cost (SRAC) curve



Note that we have re-labelled the vertical axis as 'Price' and have re-labelled the horizontal axis as 'Quantity' so as to be consistent with conventional economic labelling. Note, also, that we will use either of two standard mathematical expressions to indicate our farmer's capital constraint. Specifically, we will express his ownership of tractors as SRAC(k=5) or even more simply as SRAC(5), depending on our needs at the moment. It is most important that the reader fully understands that, mathematically, the two expressions mean the exact same thing: our farmer – at the time of our initial examination of his 'physical capital' – owns precisely five usable output-producing tractors. As discussed in a just moment, we will let him (if he wishes) add to his physical capital by allowing him the option of purchasing an additional tractor(s) next year (or 'whenever').

In the meantime, as mentioned above, we have recast everything in a format more suitable for an economic analysis of 'the short run'. Also note that we have identified the lowest point on the farmer's SRAC curve as Q(DOL). This is the farmer's Design Output Level when his tractors are being utilised at 100%, no more, no less. This is the production level where the farmer's short run average costs are at a minimum when he owns five tractors [Appendix, p. 41].

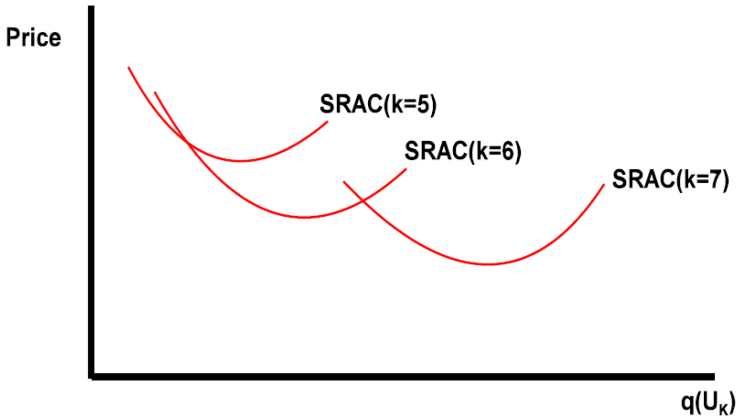
Three additional points need to be mentioned here and we put the crucial point first. Figure 3 is a picture of reality. It is not dependent on any economic theories; neither is it dependent on any (relevant) 'simplifying assumptions'. Second, Figure 3 (for any particular real-world firm) would be constructed from collectable and/or calculable real-world data thus

Figure 3 is a visual presentation of the minimum selling prices (for various levels of output, e.g., Q_L or Q_H, etc.) which would be financially acceptable to the firm for some sustainable future, given its particular and extant arrangement of capital and labour, *ceteris paribus* (here we must translate rather loosely: 'all other things held constant'). Third, we shall not, at this juncture, allow quibbling over the components of 'production costs'; we let the reader make his/her own selection and require only that rigorous consistency be maintained throughout.

Moving on, in Figure 4, we let there be a correctly-anticipated increase in business and therefore allow our farmer to contemplate an increase in his capital; specifically, he contemplates buying one additional tractor (note that we now include SRAC(k=6) in Figure 4.)

Before we proceed further, it's important to understand that, in this paper, our analytic requirements are rather strict. First, the new tractor is not permitted to have any technical improvements, e.g., if the original tractor had a carburettor, this one has a carburettor, not fuel injection).[1]

Figure 4 The farmer buys additional tractors

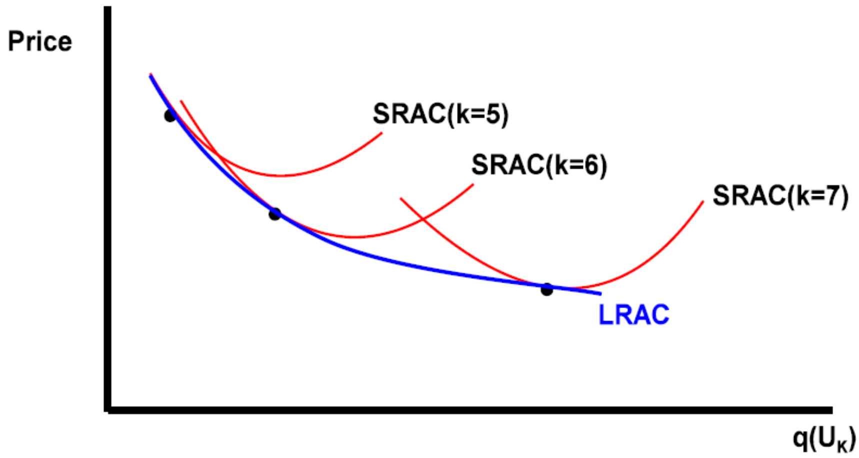


Now we can move on. We let our farmer also contemplate the purchase of two additional tractors (again, Figure 4), thus increasing the number of fully-utilised tractors to seven. When the resulting SRAC curve for the seventh tractor is added to our figure and we force everything into a two-dimensional sketch, we begin to see the problem more clearly, Figure 5. A two-dimensional sketch of our three-dimensional reality gives the viewer the completely erroneous impression that the various SRAC curves intersect in various places and, to the best of this author's knowledge, this is the current state of affairs regarding extant economics theory's current visualisation of a firm's SRAC curves. More importantly, when viewed as in Figure 5, we are forced into the standard 'tangency solution' when we try to construct the firm's LRAC curve because, while a firm can have short-run *economic* losses and long-run *business* profits at the same time, it cannot have short-run *business losses* and long-run *business profits* at the same time[2] [Appendix, pp. 40-41].

[1] In subsequent papers we will be much more lenient because we will want to start moving much closer to reality. Specifically, realistic leniency will allow us to push well beyond Marshall and thus examine our farmer's options in Euclidean five-dimensional space.

[2] It took this author a long time to fully grasp the crucial difference between *economic* profits and *business* profits. An (external), i.e., a real-world lack of adequate competition determines the size of the firm's economic profits whereas a lack of (internal) business acumen determines the size of the firm's business profits. Confusion can arise because both are calculated based on 'left-over' money.

Figure 5 Extant economics' simple but misleading presentation of the relationship between the firm's series of SRAC curves and its LRAC curve



Remember the firm **can** have short-run *economic* losses and long-run *business* profits but it **cannot** have short-run *business* losses and long-run *business* profits. Thus the 'tangency requirement' in this too-simple sketch.

Now can we can turn to Marshall's 'cardboard model' and see how he thought the mis-perception problem should be solved [Appendix, pp. 40-41].

2. We Begin in Ernest: Marshall's Obscure Footnote

We begin by examining the first part of Marshall's footnote. It explains how we could come much closer to our economic reality with regards to this particular economic sketch.

> 'We could get much nearer to nature if we allowed ourselves a more complex illustration. We might take a series of curves, of which the first allowed for the economies likely to be introduced as a result of each increase in the scale of production during one year, a second curve doing the same for two years, a third for three years, and so on' (Marshall, 1990, App. H, Art. 3, footnote 2, p. 667).

Obviously Marshall is not yet describing the precise same picture that we are herein considering but, already, he clearly recognised the need to go beyond the standard two-dimensional schema when trying to visualise the interactions between three economic variables [Appendix, pp. 11, 12]. Now let us turn to the last half of his footnote.

> 'Cutting them out of cardboard and standing them up side by side, we should obtain a *surface*, of which the three dimensions represented amount, price and time, respectively' (Marshall, 1990 App. H, Art. 3, footnote 2, p. 667, italics in original).

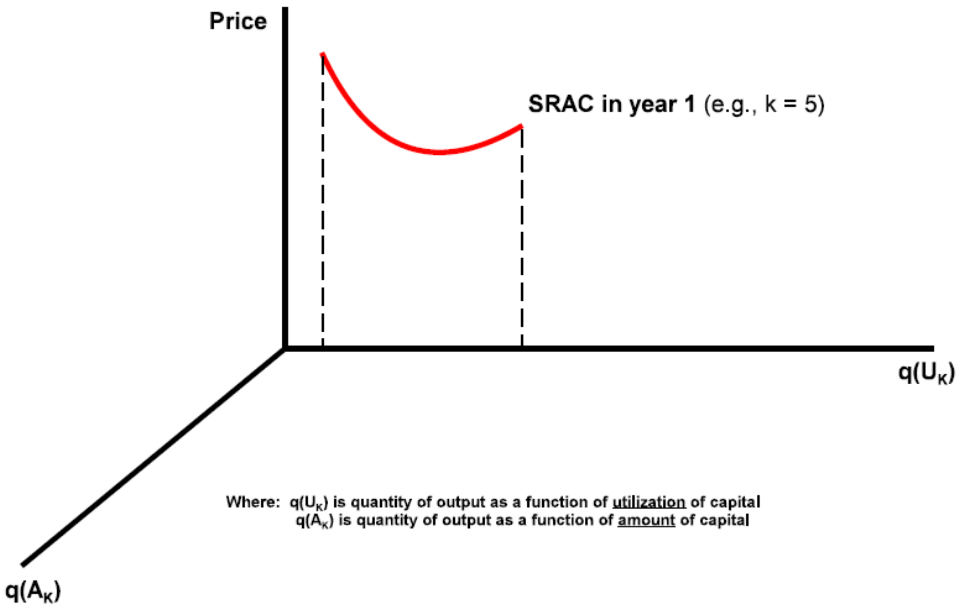
Notice that Marshall used the words '...amount, price and time...' We chose to avoid the actual use of the word 'time' because the pictorial location of any particular SRAC curve does not depend on the passage of time, *per se*; it depends, instead, on the firm's state of production affairs at the end of any 'time interval' during which capital was increased. In other words, our farmer might buy additional tractor(s) at the end of one year or he might buy it/them at the end of two years or at the end of three years. The important point is that our farmer increases his output-producing capital in 'clumps' (he cannot utilize ½ of a tractor).[3] Anyway – usurping some poetic licence regarding Marshall's precise words – we illustrate an unambiguous visual depiction of our farmer's initial situation (i.e., k=5), Figure 6A.

Next, we let him contemplate the purchase of one additional tractor thus he would then own six tractors, Figure 6B. Note that, in figures 6A, 6B and 6C, the axis coming 'out' of the page has now been re-labelled as $q(A_K)$; i.e., output is shown as a function of the amount of capital, not as a function of time.

Finally, we let him consider adding two tractors at the end of the first year, Figure 6C. Certainly, he could have chosen to buy no additional tractors (k=5); he could have chosen to buy one additional tractor (k=6) or he could have chosen to buy two additional tractors (k=7). The wisdom of his decision regarding (a) how many additional tractors to contemplate buying (if any) and (b) when to buy them would, of course, be almost totally dependent on him having reliable real-world cost data and/or cost estimates.

Figure 6a, b and c How to contract Alfred Marshall's historically-ignored 'cardboard model'

Fig. 6a



[3] 'Clumps' might suggest that a 'quantum economics' approach be considered but unfortunately, that terminology is already gaining unwarranted currency.

Fig. 6B

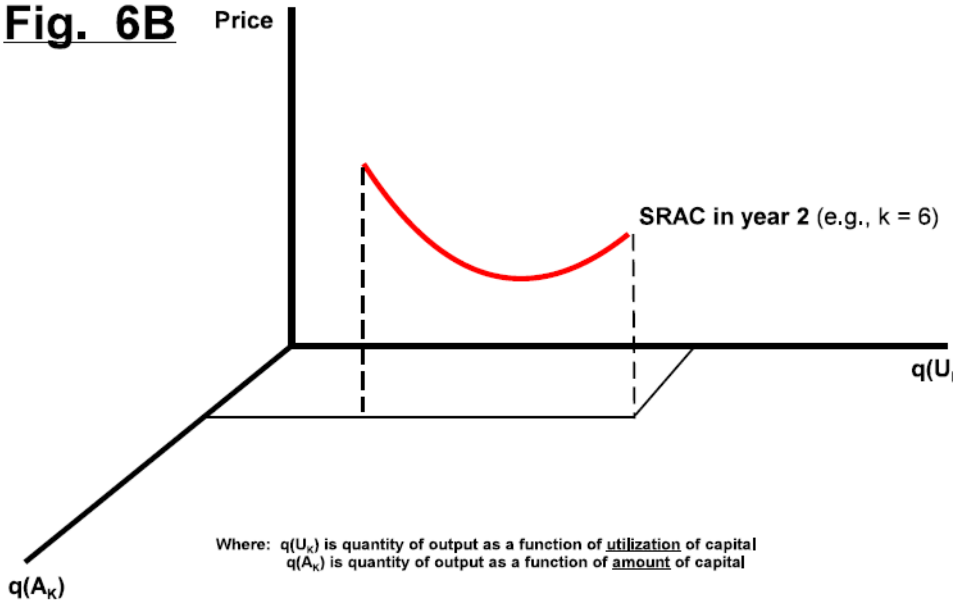
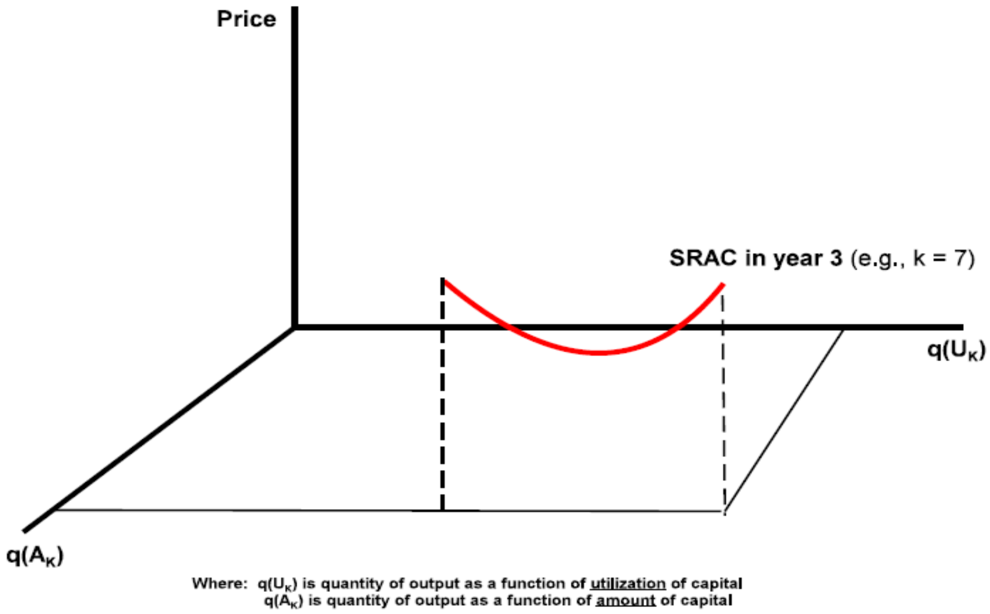
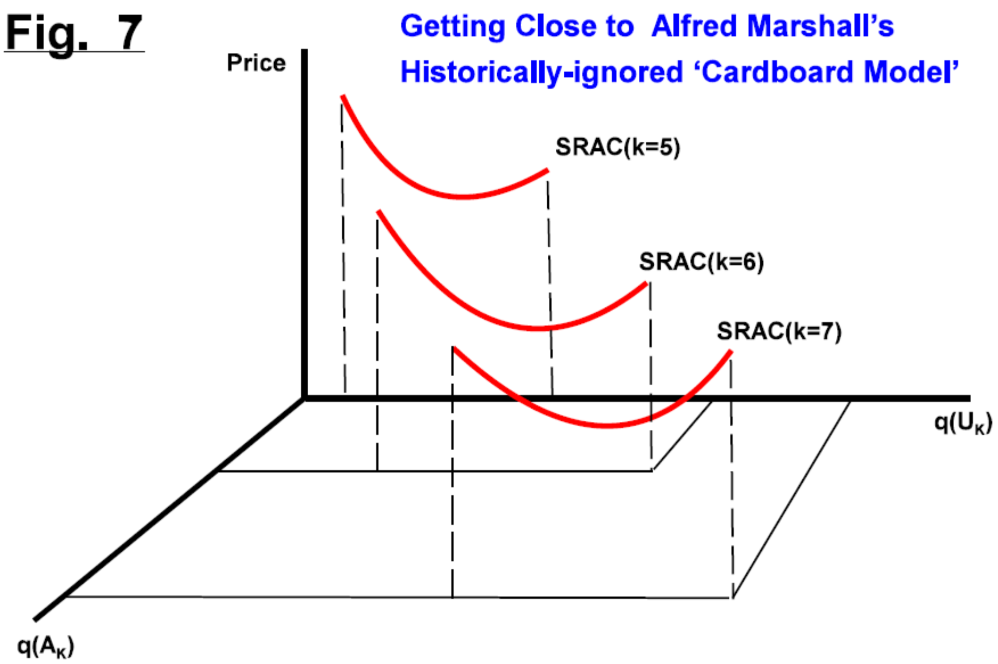


Fig. 6c

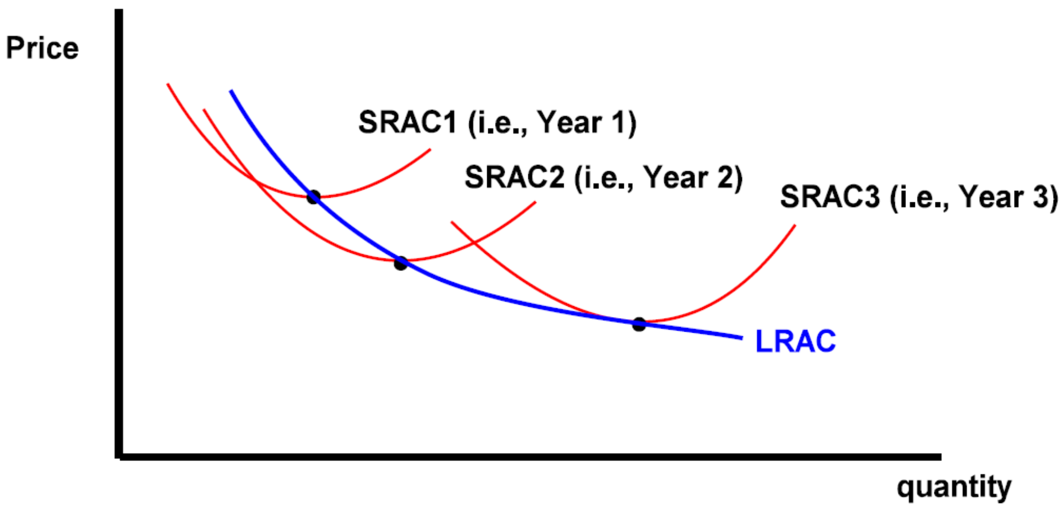


Now we can combine figures 6A, 6B and 6C so as to form Figure 7, thus coming very close to reaching Marshall's cardboard model.

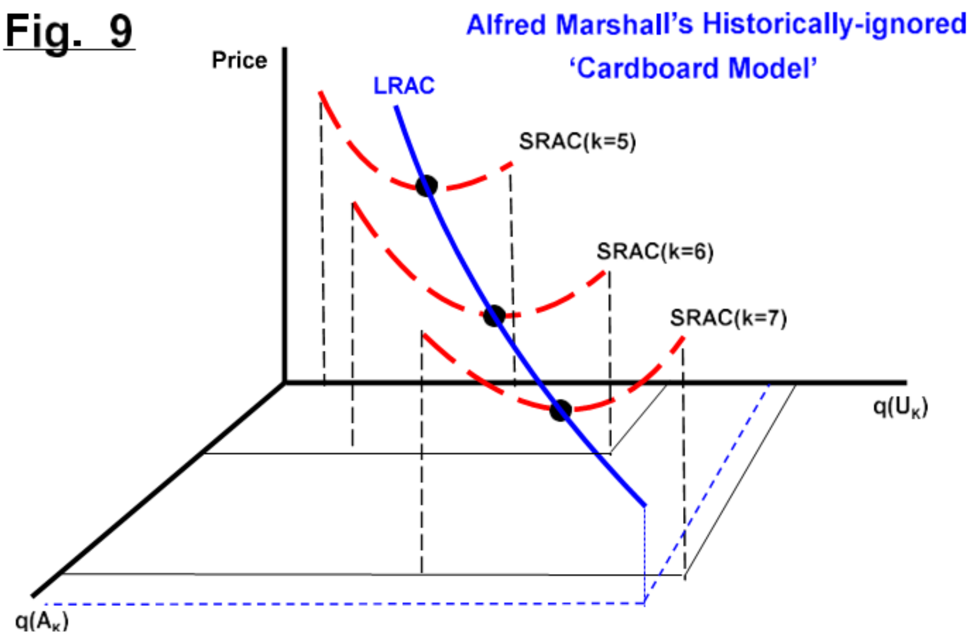


But, before we take the last step, it seems important to show that – if we wanted to – we could (confusingly) force Marshall's 3D model back into a 2D sketch, Figure 8. Note the subtle but crucial difference between Marshall (Figure 8) and extant theory (Figure 5). Specifically, Marshall's depiction *allows for (but does not* ***require****)* a 'low point solution' to the SRAC vs LRAC problem whereas extant theory *requires* the 'tangency solution' [Appendix, pp. 42-43].

Figure 8 A misleading simplification of Marshall's cardboard model



Now let us take the last step. Let us view Alfred Marshall's three-dimensional 'cardboard' model as this engineer believes it was actually meant to be viewed, Figure 9.



In Figure 9, we show the basic 'ribs' which form the skeleton of Marshall's short-run vs long-run 'surface' [Appendix, p. 42]. And, given that a clear appreciation of the SRAC vs LRAC arrangement seems a necessary precursor to more advanced economic theorizing, it would seem that it is time for Marshall's three-dimensional, historically-ignored 'cardboard model' to be given its rightful place as one of the several 'foundations' of modern economic theory.

3. Conclusions

It should now be obvious that my distinction between the 'short run' and the 'long run' has absolutely nothing to do with calendar or clock. Indeed, the distinction must be based solely on the various sizes of the 'clumps' of output-producing capital that a representative firm actually has available at any given instant. Basically, our farmer chooses to own a certain number of tractors (i.e., he chooses a particular short-run curve from a set of long-run options) for his 'course tuning' of output capability then 'fine tunes' his actual output – while 'stuck' on that pre-selected SRAC curve – so as to maximise his profits in accordance with market demand. All things considered, we arrive at the following conclusions:

1. We can join the lowest points on a firm's series of SRAC curves and thereby form its LRAC curve;
2. The firm's series of SRAC curves only appear to intersect because mainstream theory unnecessarily (and misleadingly) forces our three-dimensional economic reality into a two-dimensional economic sketch;
3. A two-dimensional sketch is analytically useless because the 'short run' (SR) never turns into the 'long run' (LR) no matter how long we wait.

In summary - when the words of Alfred Marshall are recognised as being a set of instructions and we then draw a picture based on those words – we begin to understand that (using modern engineering terminology) 'the short run' and 'the long run' are orthogonal functions in Euclidean three-dimensional space.[4]

Appendix

This appendix will utilise the following format. I will quote the reviewer (hopefully, not out of context) and then I will provide my reply. I begin with the comments / suggestions of Professor Duddy because he (appropriately) addressed my (partially successful) attempt to translate 'engineering words' into 'economic words'.

Professor Conal Duddy (CD) wrote: 'The author proposes a new diagram that differs from the original in two ways. Firstly, the new diagram is three-dimensional. Secondly, the author objects to the "tangency solution" that we see in the standard diagram.'

My reply: Professor Duddy is quite correct. My 'new diagram' is, indeed, 'visually different'. In my depiction (based on Marshall's words), I use a three-dimensional sketch for the firm's SRAC curves (plural) because a three-dimensional sketch simply cannot be unconfusingly depicted in a sketch having only two-dimensions. Specifically, the (2D) depictive error creates two separate chimeric problems: (1) the *appearance* of 'intersections' of the SRAC curves and (2) the *appearance* of a 'tangency requirement' regarding the firm's LRAC curve.

A simple real-world example might suffice. Merely hold two wooden dowels up in the air in bright sunlight and let their shadows be cast on the ground. Then arrange them so that their shadows actually do cross. But, obviously, the dowels need not actually be physically touching even as their shadows on the ground create an optical illusion which causes the unwary to (incorrectly) conclude that the dowels are touching.

But the arrangement of our firm's SRAC curves (and their inter-action with the 'longer run' curve) is a bit more complicated than mere shadows of wooden dowels. More to the point, it is my firm contention that, in a proper depiction, any individual SRAC curve lays in its own unique plane and that each of the remaining SRAC curves each lays in its own unique plane and all of the SRAC 'planes' are parallel to each other. Envision the 'first' SRAC curve as being drawn on a piece of semi-transparent graph paper lying on a table. Then place a piece of clear glass over it. On the glass, lay the (semi-transparent) graph of the 'second' SRAC, being sure to align the axes. Repeat the procedure several more times and then look straight down through our 'sandwich'. Voila! But this time, we have a fancier (and perhaps embarrassing) optical illusion: many of the SRAC curves will suddenly appear to intersect. Finally, in my depiction of SRAC curves and the resulting LRAC curve, the LRAC curve is what we get when we 'drill' down through the glass and paper, intersecting each SRAC curve only once. [Note that the LRAC curve may actually be curved or it might be a 'curve' with radius of curvature = 00 (i.e. it might sometimes be a straight line), (Thomas, 1962, p. 588).] Note, therefore, that the (mainstream econ) LRAC cannot be tangent to the series of SRAC curves because it is, in my depiction, somewhat perpendicular to the series of SRAC curves.

[4] Those readers already familiar with orthogonal functions probably realise that, while the axes (price, quantity, capital) *are* orthogonal, a real-world firm's LRAC curve will almost never be fully orthogonal to its collection of SRAC curves because the firm's LRAC curve is actually a 'directional derivative', not a true 'partial derivative' of the overall production function. Our purpose herein was to bring modern attention to Marshall's historically-ignored 'cardboard model' thus we used relatively simple illustrations and/or words and leave gradients and vector calculus to the 'quants'.

Note that I purposely choose the word ‘somewhat’ because (in my depiction of the real world) the ‘longer run’ curve is probably never precisely perpendicular to the series of SRAC curves but is, instead, a ‘directional derivative’ as discussed by Kreyszig (1972, p. 306). Basically, in my (non-Newtonian) depiction of that relationship, the ‘long run’ curve lays in a plane which is *reasonably* perpendicular to the planes of the SRAC curves ***but ‘tangency’ and/or ‘intersection’ requires that <u>all</u> curves under consideration (all SRACs <u>and</u> the LRAC) lay in one single plane***. I hope that this description of my arrangement between a firm’s series of SRAC curves and its LRAC curve adequately explains why I object to the ‘tangency’ depiction and to the ‘intersection’ depiction.

CD: ‘The author argues that the long run curve should instead cross through each short run curve at its *lowest point* (CD’s italics). This can be seen in figures 8 and 9.’

Me: I must apologise for not mentioning that, in the referenced sketches, the ‘crossing’ at the lowest point was chosen only for graphic simplicity. [I confess that I am not very skilled with computer graphics.] In the real world, I would expect that my LRAC curve would probably never intersect the lowest point of any of the firm’s SRAC curves because, based on my perusal of data regarding USA manufacturing output, I concluded that most (established) firms report that they typically operate at roughly 83 +/- % of capacity; they seldom operate at 100% of capacity (what I label as the design output level, DOL). [But do keep in mind that the reported ‘capacity utilisation’ may be influenced by the state of the economy and/or by political motivations.]

In reference to the terminological confusion, Professor Duddy wrote: ‘It may also be appropriate to give a different name to the curve to avoid confusion.’

Me: After I realised that he was quite correct, I searched for a new and different acronym. The best that I can do, for now, is something like ‘non-Newtonian long-run average cost’ curve (nNLRAC). Granted, it’s a mouthful but it should eliminate any future confusion and will be used for clarity when necessary.

CD: ‘...Marshall does not make any reference in this footnote to a long run average cost curve. So, this aspect of the new diagram requires some separate justification.’

Me: I agree that Marshall (Marshall, 1990) does not make any specific reference to a long run average cost curve but he does talk about ‘... the economies likely to be introduced as a result of each increase in the scale of production during one year, a second curve doing the same for two years, a third for three years, and so on’ (Marshall, 1990). But (to me) it seemed apparent that he was talking about some sort of ‘longish’ time frame because (with our tractor example) the farmer might add one more tractor (i.e., to increase his scale of production) during the first year, buy another (additional) tractor after two years, etc. Thus I decided that it was analytically acceptable to express Marshall’s ‘increase in the scale of production’ either in terms of a (non-Newtonian) long run ‘time’ or in terms of an increase in actual physical capital. If my memory is correct, the formal mathematical technique is called ‘conformal mapping’.

Dr Ellerman’s comments are of a rather different nature. He wrote: ‘The question addressed in this paper was already addressed and resolved in the sophisticated discussion by Paul Samuelson in his Foundations of Economic Analysis. See the pages for “Wong” in the index.’

Me: While I can agree that the question 'was already addressed...' in Samuelson's text, I cannot agree that it was 'resolved', regardless of Samuelson's 'sophisticated discussion'. Granted, the importance of using mathematical sophistication was also [well] 'addressed' by Professor Chiang (Chiang, 1984) regarding the need to go beyond geometric models: '...mathematics has the advantage of forcing analysts to make their assumptions explicit at every stage of reasoning' (Chiang, 1984, p. 4).

But I suspect that part of Chiang's preference for using mathematical models is because, on p. 4, he also mentions that the drawing a three-dimensional sketch is 'exceedingly difficult'.

Regardless of the reason for avoiding a three-dimensional sketch, *'what if'* the mathematically-inclined analyst chooses and uses impeccable mathematics but he has chosen the 'wrong' mathematics...? Let us pursue this very intriguing question in greater depth....

The following set of figures illustrate just one of the 'foundational' problems that I have with 'mainstream' economics. Figures 10A through 10C summarise the basic steps in the derivation of the familiar 'cross' which purports to depict equilibrium between supply and demand. Figure 10A illustrates the (perfectly horizontal) demand curve when we assume 'perfect competition' (or Samuelson's 'pure competition'. The demand curve is then integrated to give total revenue, TR_{NE} (Figure 10B, pursuant to profit maximisation), yielding the upward-sloping supply curve in Figure 10C. But from whence came the *downward*-sloping demand curve in Figure 10C? It 'whenced' from 'relaxing' the strictness of perfect competition and thus allowed the demand curve to be depicted with a downward slope (which probably is more in tune with our real world).

Figures 10, A, B and C Newtonian or Mainstream economics

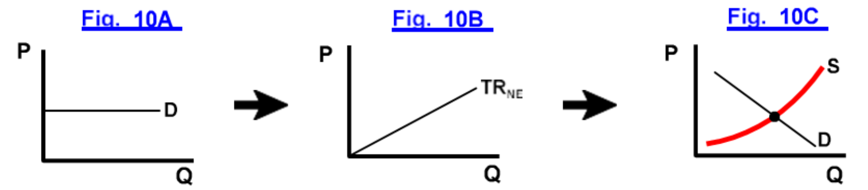
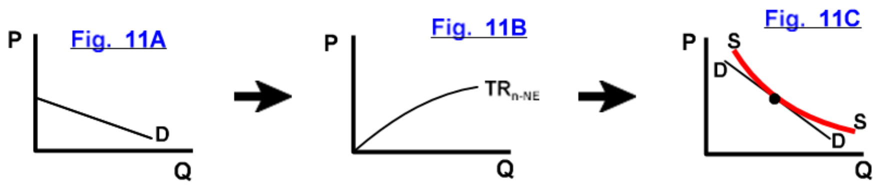


Figure 11, A, B and C Non-Newtonian economics



But...why not start closer to reality (Figure 11A) and follow the precise same mathematical steps as were followed in the 'mainstream' derivation? The result is a ***downward***-sloping supply curve. Thus, instead of our mathematics yielding Marshall's 'scissors' model (Marshall, 1990, p. 290), our mathematics yields a 'wheel and ramp' model of equilibrium between supply and demand, Figure 11C.

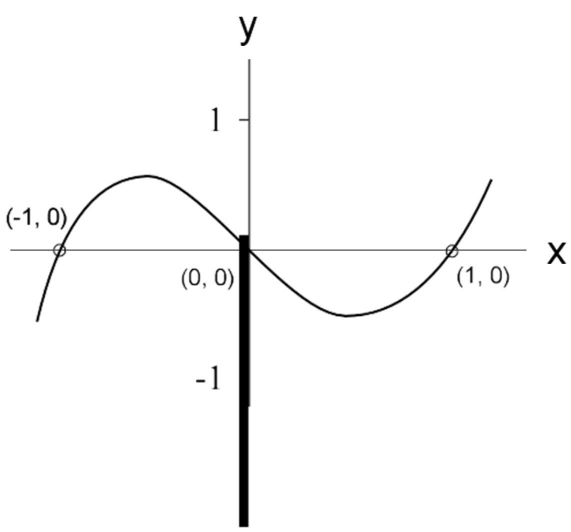
I realise that this is not a general equilibrium model (which would need to employ Samuelson's admittedly sophisticated mathematics) and it is not even a partial equilibrium model. Properly labelled, it might be called a 'single firm equilibrium model'.

In my defense for eschewing sophisticated mathematics, it's been almost 50 years since I studied math at that level and, frankly, I just didn't feel like sweeping off all the cobwebs. Plus, that level of mathematics is not necessary in order to explain my underlying contention: a two-dimensional picture – if it is the correct picture of our economic reality – will often yield results which are much more useful (and, in the case at hand, be rather contrary to) a purely mathematical but naive approach. That's why it was necessary to begin with Marshall's cardboard model; we needed an SRAC curve, totally unencumbered with 'intersections' and/or 'tangency' confusions, before we could tackle the naive use of 'incorrect' mathematics which, in my opinion, resulted in the (incorrect) 'scissors' depiction of equilibrium between supply and demand.

Let me give an example used by Professor Washington (Washington, 1980, p.183) in which he illustrates the potential danger of making a naive decision to employ a specific mathematical technique without first employing a sketch to validate the choice of the mathematics, *per se.* [I came across it when I was reviewing some of my early math texts in preparation for writing this paper. Note that I misplaced my original copy and had to purchase a slightly newer edition, quoted herein.]

Here's the example he used to illustrate the problem, Figure 12:

Figure 12 The graph of $y = x^3 - x$



Find the area between $y = x^3 - x$ and the x-axis.

Note that the area to the left of the origin is above the axis and the area to the right is below. We start with the naive math first... The problem seems simple and straightforward: simply use calculus to determine the area in question.

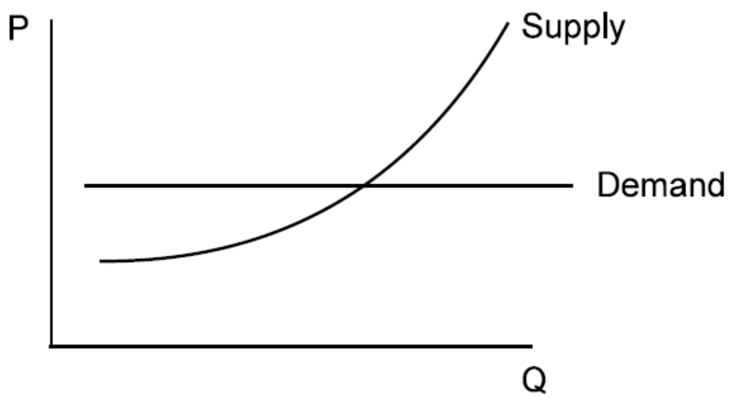
$$A = \int_{-1}^{1} (x^3 - x)dx = x^4/4 - x^2/2 \Big|_{1}^{1} = (1/4 - ½) - (1/4 - ½) = 0$$

But if we recognize that, at $x = 0$, there exists what Kaplan (Kaplan, 1953, p. 554) calls an 'isolated singularity', then we must let the graph override our naive (one step) mathematical approach and use the arithmetic sum of two *separate* integrals to obtain the *correct* answer

$$A = \int_{-1}^{0} (x^3 - x)dx + \int_{0}^{1} (x^3 - x)dx = . . . = 1/2$$

In economics (and in most other disciplines) it seems that the mathematical economist sometimes fails to realise that mathematics is 'dumb', i.e., it is merely a 'tool' that does what it was told to do. And because of the absolute reliability of a mathematical answer, the mathematical economist also sometimes fails to realise that he/she has chosen the wrong mathematical tools (plural). More precisely, in the case at hand, the ('mainstream') mathematical economist starts with one assumption and then chooses the 'appropriate' tool (singular) but, when finished, essentially pretends that he/she started with a different 'tool' based on a different assumption, thereby unknowingly acknowledging that the first tool was the wrong choice. *If consistency of the original assumption had been maintained throughout the complete mathematical 'proof', Marshall's 'scissors' would have looked like Figure 13.*

Figure 13 Equilibrium of supply and demand if maintaining consistency of original assumption



Interesting perhaps, but pragmatically useless. Anyway, this (I believe) is the case with the derivation of the firm's SRAC curve. In my opinion, all economists here-to-fore have chosen the wrong mathematical tool and that's why the firm's SRAC curve is *incorrectly shown as being the* ***upward****-sloping portion of the firm's marginal cost* ***(MC)*** *curve* whereas the correct mathematical tool reveals that it is actually *the* ***downward****-sloping portion of the firm's average cost* ***(AC)*** *curve*.

Acknowledgements

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