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SOLVING JÖRGENSEN'S DILEMMA

ABSTRACT: The dilemma mentioned in the title have long been a puzzle to logic. To solve it, it was necessary to make some philosophical decisions. The article provides these decisions and discusses the way to solve the problem.

KEY WORDS: Jörgensen dilemma, liar's paradox, logic, classical, imperative, deontic, values, logical, true, false, good, evil, philosophy, metaphysics, propositions, sentences, affirmative, evaluative, normative, imperative, evaluations, norms, precepts, state, affairs, reason, intellect, theoretical, practical, will, reasoning, willing

1. A short history of research on the logic of non-declaratives

Traditional logic accepted only these as sentences in the logical sense which can be evaluated as true or false. "True" meant the agreement of the sentence with reality, and "false" – disagreement. As a consequence, the area of logic's reasoning was limited to sentences stating the occurrence of states of affairs and to generally applicable principles and laws.

At the turn of the nineteenth and twentieth centuries, the situation changed thanks to the Polish logician Kazimierz Twardowski, who recognized that the category of sentences[1]

[1] „"Proposition" was replaced by "sentence" because it was a term too loaded with psychologism. Łukasiewicz still used this term in 1912. Only Kotarbiński explicitly abandoned it and replaced it with "sentence". The issue may seem trivial, but propositions, as an expression of the mental activity of judging, could contain some subjective factors, which include values and norms. Sentences, understood in the spirit of neo-positivism, were supposed to describe only facts. This, in turn, could lead to the conclusion that evaluative and normative expressions are obviously not facts, and therefore have no logical value and belong to a completely different area than the object of interest of logic. Such thesis, however, was not put forward by the aforementioned authors, because it would significantly impede the possibility of practicing normative ethics as a science (Pacewicz 2016).

includes not only declarative expressions but also evaluative and normative ones. According to Twardowski, “These expressions are the subject of formal logic and are true and false in a logical sense. This value is not relative to the language or culture in which such expressions occur” (Pacewicz 2016). A similar position was taken by his students – Jan Łukasiewicz (1878-1956), Tadeusz Czeżowski (1889-1981) (Czeżowski 1949), Kazimierz Ajdukiewicz (1890-1963) (Ajdukiewicz 1975) and Tadeusz Marian Kotarbiński (1886-1981) (Kotarbiński 1986). They all considered sentences and norms to be sentences in the logical sense, as long as they could be reduced to a form that allowed them to be considered true or false. Their group was joined in the twentieth century by at least thirty other Polish thinkers who made a significant contribution to the theory of imperatives and norms (Jadacki 2012) initiated by the works of Marian Borowski (1879-1938). The issue of evaluative and normative statements gained the status of a separate section in the textbook *Logika praktyczna* by Zygmunt Ziembiński (1920-1996). Aleksander Peczenik (1937-2005) noted that “At the level of propositional calculus, there are no differences between the logic of norms and the logic of descriptive propositions. Boolean constants such as “or”, “and”, “if... then” have the same meaning both when used as conjunctions that take propositions and when used as conjunctions that take norms” (Peczenik 1964).

Contemporary researchers in this field, Jacek Jadacki (1948-) and his collaborator Anna Brożek (1980-), postulate extending the concept of reasoning so that it includes not only operations on declarative sentences, i.e. declaratives, but also on imperative and interrogative sentences, i.e. imperatives and interrogatives (Jadacki, Brożek 2012). Their opinion on imperatives is shared by the Greek American logician Peter B.M. Vranas, defining inference as “a reasoning process that begins with the recognition of some declarative or imperative sentence (reasoning premises) and ends with the recognition of a declarative or imperative sentence (reasoning conclusion)” (Vranas 2010).

Jadacki and Brożek are aware that their postulate will be met with disapproval concerning the validity of imperative and interrogative reasoning because commands and questions are neither true nor false in the logical sense[2]. Therefore, they propose a solution to the problem

[2] The occurrence of interrogative and imperative reasoning “in practice” and the legitimacy of attempts to create a theory of them have been repeatedly questioned. In our opinion, this is related to the stereotypes about rational mental processes that have been established in the logical and philosophical tradition. The emotional and volitional spheres of the human mind used to be sharply separated from the rational sphere – as if they were governed by separate ‘laws’. Although the differences between these spheres are unquestionable, we are convinced, firstly, that at least higher-order volitional processes (of wanting, of not wanting) are schematized, and secondly, that they are controlled by the rational sphere and therefore with this, it is possible to formulate their normative theory” (Jadacki, Brożek 2012).

by identifying the analogons of truth and falsity that may be “inherited” by the arguments of the logical relation (imperative or interrogative) of consequence and then show that “The nature of the inferential relationships between interrogatives and imperatives is not ... fundamentally different from the relationships between declarations: in explicating these relationships we use the concept of logical truth” (Jadacki, Brożek 2012).

2. The need for philosophical decisions

Opponents of non-declarative logic cite in support of their position the Jörgensen dilemma, stated by the Danish logician Jörg Jörgensen (1899-1969). This dilemma is a simple observation of a strange state of affairs: logically correct reasoning is carried out on norms (practical syllogisms), even though, according to the knowledge we have, they cannot be carried out, because norms are not sentences in the logical sense. Various attempts have been made to solve this dilemma. The Finnish philosopher and logician George Henry von Wright (1916-2003), the founder of deontic logic, recognized that sentences involve something beyond truth, but later concluded that all sentences can be reduced to true and false. Contemporary deontic logic tries two methods: (1) the first consists in making a distinction between a norm and a normative sentence (Hedenius 1941, Von Wright 1963), (2) the second, which grew out of the investigations of conflict-tolerant deontic logic (Van Fraassen 1973, Hansen 2008, Hansen 2013), uses input-output logic (I/O) (Makinson & van der Torre 2000, 2001; Parent & van der Torre 2013), developed to solve the problems related to the philosophy of norms. This logic brings interesting results (SEP 2023); however, Jörgensen's dilemma remains unresolved.

The Polish school of logic shows the way to the solution. As Grzegorz Pacewicz says:

> ...formal logic only provides a formal system without limiting its applicability – after all, what is and what is not a sentence turns out to be a non-logical question in the sense that logical interpretation only assumes that the basic unit in formal logic takes two logical values. Whether evaluative sentences are encompassed by the values of truth and falsity is not decided by formal logic. The awareness of this state of affairs is one of the key achievements of the Polish school of logic, in which it was clearly stated that practicing logic is not possible without certain philosophical decisions (...).(Pacewicz 2016)

Let us, therefore, make the philosophical decisions postulated by the Polish school of logic. As the metaphysical basis of the normative theory of imperative reasoning, let us adopt the realistic theory of cognition.

3. The solution to the Jörgensen's dilemma

Realistic metaphysics distinguishes two types of intellectual cognition – theoretical cognition (Greek θεωρείν – see, look, look at, review) and practical cognition (Greek πρακτικός – active). Depending on the type of cognition, reason[3] is referred to as theoretical or practical. The purpose of cognitive acts of theoretical reason is only to consider the truth, and the purpose of cognitive acts of practical reason is action. As can be seen from this, the reason is one, and it is called theoretical or practical, depending on the purpose of the operations it performs[4].

Theoretical cognition comes from man's natural aspiration to cognition, which is mentioned by Aristotle in the first sentence of the introduction to his *Metaphysics*. Practical knowledge comes from the equally natural drive to act and is divided into two types, which we will call here (1) operational practical knowledge and (2) descriptive practical knowledge. Operational practical cognition concerns the actual implementation of the action, and descriptive practical cognition concerns the description of the action. Each of the listed types of practical cognition is closely related to the corresponding type of volition. Practical operational cognition runs in close connection with practical operational desire. Practical cognition and willing[5] form a dual unity of cooperation of reason and will in moral activity, which Thomas Aquinas described in detail, and the Polish philosopher and theologian, Jacek Woroniecki OP (1878-1949), summarized in the following scheme:

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[3] I use the term "reason" here in the classical sense of the subject of the function of discursive (indirect) cognition. In the Thomistic tradition, it is sometimes used interchangeably with the term "intellect". For a brief description of the relations between intellect, sense, wisdom, and reason see (Kalinowski 1973).

[4] Aquinas T., *Summa Theologiae*, I q. 79, a. 11, c.

[5] "There are certain mental processes in which at least some stages, connected by motivational relationships, consist in taking certain attitudes towards interrogatives or imperatives. These attitudes, however, are, in our opinion, not of a persuasive nature (recognition), but of a volitional nature. (…) When we utter an interrogative seriously, we reveal that we do not know something and at the same time we want to find out. When we express an imperative seriously, we betray that we want a certain state of affairs to occur" (Jadacki, Brożek 2012).

Table 1

	REASON	WILL
Discernment	1. The idea of an object as being good or bad (evaluation)	2. Liking or disliking for the object
Planning	3. Plan – thinking of the object as a goal	4. Intention to achieve the object as a goal
	5. Deliberation – considering means to an end	6. Permission – allowing some means, rejecting others
	7. Intent – judging between means to an end	8. The choice of one of the means
Execution	9. Order – decision or order of an act	10. Active execution

Formulating sentences belongs to reason. The dual reason – theoretical and practical – creates declaratives and interrogatives, normatives and imperatives, suppositives (suppositional sentences) and evaluatives (evaluative sentences). Therefore, if the same reason formulates all kinds of sentences, they should be subject to the same logic. As a consequence, if sentences formulated by theoretical reason assume the logical value of truth or falsehood, then sentences formulated by practical reason should take analogous values, and since the equivalent of the "truth" cognited by the theoretical intellect is the "good" cognited by the practical intellect, practical sentences should take the values "good" and "bad" in a logical sense. Logical "good" and "evil" should concern the realization of desirable or undesirable states of being. The sentence postulating the realization of the desired state of being should be called a logically good sentence, and the sentence postulating the realization of an undesirable state of being – a logically bad sentence, with the proviso that the logical values "refer not to the immutable nature of good or evil contained in things, but to how things relate to the purposes and aspirations of the man who makes knowledge" (Penczek 2012).

It is not difficult to see, however, that calling sentences "logically good" or "logically bad" would always imply a reference to their nature, and not to the postulating of a desirable or undesirable state of being contained in them. A similar situation also takes place concerning the logical values of truth and falsehood, so the question should be asked where these names of values come from and what they mean. Their author was the pioneer of modern logic, Friedrich Ludwig Gottlob Frege (1848-1925). A closer look at the history of his thought

reveals seven phases of searching for answers to our questions. In the first phase, the German logician introduced the categories of recognition (*bejahen*) and negation (*verneinen*) (Frege 1879), in the second – he replaced them with the categories of *correctness* and *incorrectness* (Besler 2010), in the third – he stated that there is an analogy between truth in logic and good in ethics (Frege 1884), and in the fourth he introduced *truth* and *falsehood* as logical values that are "semantic correlates of a sentence" (Frege 1891). This step involved the necessity of rejecting the correspondence theory of truth (as the agreement of the mind with reality) and became the reason for numerous criticisms. The fifth phase brought the thesis on the indefinability of truth (Frege 1897), and the sixth phase – the thesis that one should speak of a significant relationship between logic and truth only for didactic reasons (Frege 1915). "In the seventh and final phase, truth is for Frege a predicate affirmed about thought (Frege 1918), but affirmed on the base of whether the names in the sentence (in which the thought is expressed) have their semantic correlates" (Besler 2010).

As can be seen, the repeated changes in the formulations of what we call logical values testify to Frege's deep dilemma, which accompanied him throughout all the years of his scientific career. His lack of classical philosophical education was clearly to his detriment when, first, he could not distinguish truth as a transcendental from truth as the correspondence of mind with reality, declaring that truth is indefinable, and then stated that he spoke of truth in connection with logic only for didactic reasons. So the question should be asked again: What do the values denoted by zeros and ones in the Classical Propositional Calculus mean? If they were to mean truth and falsehood following the correspondence theory of truth, how could we make such a sentence as "A child in the mother's womb is a human being" as universally true as the sentence "Warsaw is the capital of Poland"? Anyone who has come into contact with an abortionist knows that this is impossible. When an abortionist sees the truth table for the equivalence of the following two statements...

Table 2

p		q		p<=>q
If the child in the mother's womb is a human being,	1	then abortion is murder.	1	1
If the child in the mother's womb is a human being,	1	then abortion is not murder.	0	0
If the child in the mother's womb is not a human being,	0	then abortion is murder.	1	0
If the child in the mother's womb	0	then abortion is not murder.	0	1

is not a human being,				

...he will say – contrary to the obvious – that a child in the mother's womb is not a human being, therefore abortion is not murder. The simple conclusion is that the so-called truth tables can just as well be called falsehood tables. So what we call the truth values of sentences refers to something independent of truth and falsehood. What could it be? The continuation of the discussion with the abortionist will help us find the answer to this question. The logical response to his position is the suggestion: "If you want an abortion, then abort yourself!"

Table 3

p		q		p<=>q
If you want an abortion,	1	then abort yourself!	1	1
If you want an abortion,	1	then don't abort yourself!	0	0
If you don't want an abortion,	0	then abort yourself!	1	0
If you don't want an abortion,	0	then don't abort yourself!	0	1

As can be seen from the table, the logical values of two imperative sentences in the third column, expressing striving for two opposite states – desirable and undesirable, correspond to the logical values of two declarative sentences in the third column of the previous table, expressing the occurrence of two opposite states – existing and non-existing. This confirms Peczenik's thesis that logical constants have the same meaning, regardless of whether they appear as operators of declaratives or normatives. An affirmative sentence in the indicative mood says that there is a certain state of affairs, and in the imperative mood – that the existence of a certain state of affairs is desirable. Thus, both affirmative sentences have in common that they speak of the **existence** of states of affairs. On the other hand, a negative sentence in the indicative mood says that a certain state of affairs does not exist, and in the imperative mood – that it is desirable to a certain state of affairs that it does not exist. Thus, both negative sentences have in common that they speak of the **non-existence** of states of affairs. As a consequence, it can be concluded that the logical values common to the considered sentences are *isness*[6] (Polish *istność,* marked with "I") and *non-isness* (Polish *nieistność,* marked with "N"), and therefore the *isness tables* are common for declarative and imperative sentences. Moreover, if in the first column we put an interrogative instead of the declarative, the isness table will work as well as for the declarative, as can be seen from the well-known slogan "Have you been drinking? Do not drive!"

[6] Compare https://www.merriam-webster.com/dictionary/isness.

Table 4

p		q		p<≠>q
Have you been drinking?	1	Drive!	1	0
Have you been drinking?	1	Don’t drive!	0	1
Haven't you been drinking?	0	Drive!	1	1
Haven't you been drinking?	0	Don’t drive!	0	0

There is, of course, a large group of people who do not consider driving under the influence of alcohol to be wrong. Instead of the exclusive disjunction “Have you been drinking? Don’t drive!” these people will use the biconditional “Have you been drinking? Drive!”. Logic is powerless against such a choice because the choice of good or evil depends on the essential fitness of the mind, not on its logical fitness. For those who live responsibly, however, logic shows that it includes in its values not only declaratives, imperatives, and interrogatives, but also suppositives, i.e. sentences in the subjunctive mood, as shown in the table below:

Table 5

p		q		p<=>q
If I had listened to good advice,	1	I would happily return.	1	1
If I had listened to good advice,	1	I wouldn’t happily return.	0	0
If I hadn’t listened to good advice,	0	I would happily return.	1	0
If I hadn’t listened to good advice,	0	I wouldn’t happily return.	0	1

Thus, the logical values of isness and non-isness can be stated for all kinds of sentences. Moreover, even evaluative exclamation sentences, such as “How beautiful it is here!” or “It's ugly in here!”, reveal such values. Therefore, it can be said that it is difficult to identify any sentences that would not have a logical value. Probably just sentence equivalents like “Guess what?” or “What?” seem devoid of them, but they too contain a connoting, accepting, doubting, questioning, or assuming relationship to logically valuable implicit sentences. We can therefore say that every sentence in the logical sense is either *isnessive* or *non-isnessive* – and only that.

The values of isness and non-isness seem at first glance to encompass even statements that self-declare to be true or false, such as the famous utterance in the liar paradox: “The sentence I am saying is false”. Hearing someone utter such a sentence, one gets the impression that it is a real sentence. Just as true seems to be the opposite sentence: “The sentence I am saying is

true". We are inclined to assume the isness of both of these sentences based on the affirmative form of the copula "is" in them, and not based on the predicate. In the same way, we would probably be inclined to consider the statement "The sentence I am saying is non-isnessive" as an isnessive sentence, but in this case, there is a visible contradiction between the affirmative copula "is" and the logical value "non-isnessive" stated in the predicate. This means that the sentence is self-contradictory. On the other hand, the statement "The sentence I am saying is isnessive" seems to be consistent, as seems the statement "The sentence I am saying is true". The conclusion is that the real cause of the liar paradox is hidden below the verbal layer. Let us, therefore, reach for the metaphysical foundations of the sentences in question and consider the statement "The sentence I am uttering is non-isnessive" as a being *sub ratione entitatis*. This entity has not yet received existence from the person uttering the sentence, and it already contains logical self-evaluation in its content. It has not yet come into being itself, and already it supposedly calls into existence something else. The real existence of such a being is not possible, because to be able to act, one must first exist. This statement requires no proof, just as the statement that a man cannot lift himself by his hair requires no proof. Therefore, meaningful self-asserting sentences (self-indicating, self-assuming) of their isness value are not possible. However, deceptive imitations of such sentences are possible, similar to imitations of names such as "square circle". It remains therefore to say that probably only *self-sentences* are not sentences in the logical sense.

Determining the isness values of logical functions is as simple as determining the isness or non-isness of sentences. Systematic treatment of the logical values of all binary operations using the example of the imperatives "Sing!" and "Dance!" (Vranas 2010) are listed in the table below. "One" means logical isness, and "zero" – logical non-isness.

Table 6

		State 1 singing p	State 1 singing p	Not state 1 not singing ~p	Not state 1 not singing ~p
		State 2 dancing q	Not state 2 not dancing ~q	State 2 dancing q	Not state 2 not dancing ~q
1	Both states coexist (conjunction)	1	0	0	0

	Sing-and-dance! p^q				
2	At least one of the states exists (disjunction) Sing or dance! p∨q	1	1	1	0
3	State 1 exists but state 2 does not (strong and weak inhibition/non-implication) Sing but don't dance! p≠>q	0	1	0	0
4	State 2 exists but state 1 does not (converse of strong and weak inhibition/non-implication) Dance but don't sing! q≠>p	0	0	1	0
5	Both states coexist or co-non-exist (biconditional) If you dance, then sing! p<=>q	1	0	0	1
6	Both states do not coexist or do not co-non-exist (exclusive disjunction) If you dance, then don’t sing! p<≠>q	0	1	1	0
7	Only state 1 does not exist, or both states coexist or do not exist (strong and weak, broad competition) Sing, but dance! p=>q	1	0	1	1
8	Only state 2 does not exist or both states coexist or do not exist (converse of strong and weak, broad competition) Dance, but sing! q=>p	1	1	0	1
9	Both states co-non-exist (non-disjunction) Neither dance nor sing! p ↓ q	0	0	0	1
10	Both states do not coexist (non-conjunction) Either don't sing or don't dance! p ↑ q	0	1	1	1

As one can see, the arrangement of values in the isness table fully corresponds to the arrangement of values in the truth table. Thus, it can be said that thanks to the correct identification of the essence of logical values, the Jörgensen’s dilemma has been solved.

4. Axiomatics of logic

The postulate of replacing the logical values of truth and falsehood with the values of isness and non-isness and the postulate of replacing the implication with the competition, put forward in the article *Solving the Paradox of Material Implication – 2024* (Pociej 2024), raise the question about the need for changes in the system of axioms of logic. The answer is as difficult as Buridan's donkey choosing oats or hay, because there are many such systems (Kisielewicz 2021) and it is not known which one would be subject to possible modification. Therefore, it is rather necessary to refer to at least some axioms of set theory, tested in the fire of over a century of research, and to provide their logical equivalents, introducing possible modifications.

The equivalents of set-theoretic sets in logic are atomic sentences. The equivalents of the elements of sets are terms, and the equivalents of operations on sets are molecular sentences and formulas.

The first axiom of set theory states that there is at least one set. Its logical equivalent should therefore be:

1. There is at least one sentence.

The second axiom of logic, which has no equivalent in set theory but is necessary in logic, should read:

2. Every sentence is isnessive or non-isnessive – and only such. Negation changes an isnessive sentence into a non-isnessive one, and a non-isnessive sentence into an isnessive one.

The next three axioms of set theory define the equality, sum and difference of sets. Their equivalents in logic are logical functions. All propositional calculus functions can be summarized in the following third axiom:

3. There are four types of logical functions: connection, disconnection, entailment and opposition. In each type, there are two kinds of functions: inclusive and exclusive. Exclusive functions are negations of inclusive functions. The inclusive functions are: conjunction, disjunction, equivalence and competition. The exclusive functions are: non-conjunction, non-disjunction, contravalence and inhibition. Competition and inhibition come in two varieties – straight and converse.

The fifth axiom of set theory says that for a given set and a given propositional form therein, there is a set consisting of such and only such elements belonging to the objective set

that satisfy the objective propositional form. The logical equivalent of this axiom should therefore be:

4. Every logical formula includes those and only those sentences whose senses satisfy it.

This axiom has the advantage of once and for all eliminating strange propositions from logic, such as "If the moon is made of cheese, then Paris is the capital of France", and accordingly it should be named, like its set-theoretic pattern, the axiom of excision.

The sixth axiom of set theory states that for any set there is a family of sets consisting of all subsets of that set. It seems that its equivalent in logic should be the following fifth axiom:

5. Sentences belonging to a sentence family that is itself a sentence consist of sentences belonging to that family.

This axiom seems to protect logic against the antinomy of a sentence composed of all sentences, analogous to the antinomy of the set of all sets.

Another axiom of set theory, called the axiom of choice, is considered controversial. It says that for every family of nonempty and disjoint sets, which is itself a set, there is a set that has one and only one element in common with each of the sets of this family. Its logical equivalent should be the sixth axiom:

6. In every family of sentences, which is itself a sentence, there is a sentence that has one and only one sense in common with each of its component sentences.

At first glance, it is difficult to say what such an axiom could be useful for in logic. Logicians admit that "The axiom of choice is sometimes controversial because of some of its surprising consequences" (Urzyczyn 2001-2006, 14). However, there are intuitively obvious facts whose proofs in set theory require the use of this axiom. Therefore, the suitability of its transposed version in logic needs to be further investigated.

This is the proposed system of axioms of logic, based on the axiomatics of set theory. It is of a working nature and requires in-depth study.

5. Summary

To summarize the results of the conducted logical-metaphysical investigations, it should be stated that:

- firstly, the solution of the Jörgensen's dilemma turned out to be possible on the basis of realistic metaphysics,

- secondly, identifying the true nature of logical values allowed to draw the conclusion that all sentences except self-sentences are sentences in the logical sense.

Moreover, a metaphysical solution to the liar's paradox was provided and – due to the discoveries made – a systematics of the axioms of logic was proposed, referring to the axioms of set theory.

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