TADEUSZ BATÓG'S PHONOLOGICAL SYSTEMS

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This paper presents the phonological theories of Tadeusz Batóg. We try to show that there are three stages in the development of Batóg's phonological ideas (i.e. distributional, phonetic and semantic one). All the axioms proposed by Batóg, as well as his construction of phonemic basis of an idiolect and the fundamental hypothesis of phonology are discussed.

1 Introductory Remarks

The phonological works of Tadeusz Batóg concern first of all the logical reconstruction of the concept of the phoneme. It should be stressed at the very beginning that the works in question have established a standard which should be recommended in all attempts at a logical reconstruction of linguistic theories.

The Author uses, along with the standard machinery of the classical predicate calculus, also the extended mereology of Leśniewski — Tarski. Linguistic theories which are taken into account are those of American structuralism (primarily that of Zellig Harris) as well as European structuralism (N.S. Trubetzkoy). References to the works of Bloch, Bloomfield, Jones, Pilch, Jassem, de Saussure can also be found in Batóg's approach.

Up to now, Tadeusz Batóg has devoted ten of his works to the problem of a logical reconstruction of the concept of the phoneme (cf. References). Five of those (viz. Batóg 1961, 1962, 1967, 1971 and 1976) seem to represent the consecutive stages of the development of his phonological theory. The works Batóg (1961a, 1969, 1971a, 1978 and 1978a) propose some subtle corrections of the main ideas or serve as an exposition of Batóg's theory for those readers without a sufficient mathematical background.

The major achievements of Tadeusz Batóg in his attempts at a logical reconstruction of (structuralistic) phonology have been collected in the monographs Batóg (1967 and 1994). The second of these is a collection of

reprints of the most important articles concerning the subject in question (among others, the articles Batóg 1961, 1962, 1969, 1971, 1971a, 1978a).

Besides the logical reconstruction of the concept of phoneme, the phonological works of Tadeusz Batóg also concern some other important problems in theoretical phonology such as algorithms of phonemic-orthographic conversion, distance function in (articulatory) phonetics, and algorithms for establishing the phonemic bases of a given idiolect. These subjects will not be discussed in the present paper.

There are three stages in the development of Batóg's phonological theory (these may be called three theories):

1. The logical reconstruction of the concept of the phoneme, based solely on distributional criteria (Batóg 1961, 1962);

2. The system which uses phonetic features in the characterization of phonemes (Batóg 1967);

3. Extensions of the system given in Batóg (1967) provided by taking into account semantical relations (Batóg 1971, 1976).

Our main reference in the discussion of Batóg's phonological theory is the monograph Batóg (1967) which presents the most elaborate version of it. We will also add remarks concerning the earlier versions as well as extensions of the main system.

The linguistic terminology used in this article is standard. The same concerns mathematical concepts and notation, thus there is no need to recall it here. The only exception is the following proviso: if R is a binary relation, then by $R^\wedge x$ we denote the set of all R-successors of x and, similarly, by $R^\vee x$ we denote the set of all R-predecessors of x (we follow Batóg's original notation in this respect). Furthermore, if $R \subseteq Y \times X$ is a binary relation such that for every $x \in X$ there exists exactly one $y \in Y$ for which yRx, then this unique y will be denoted by R^*x.

For the completeness of exposition we briefly present some fundamental concepts and the system of axioms of *extended mereology*, together with a few intuitive comments. We will follow chapter 3 of Batóg (1967) in this exposition. The system of extended mereology was presented for the first time by Tarski in Appendix E to Woodger (1937). As is well known, mereology is the system of collective set theory created by Stanisław Leśniewski at the beginning of this century (cf. Leśniewski 1916). Tarski extended this system by adding time dependencies to it. Extended mereology is a system from which linguistic science can greatly benefit — remember that utterances (belonging to *parole*) are just individual objects, with a fixed duration in time and extension in space. Distributive and collective set theory can be used together e.g. to reflect the important distinction between individual and abstract objects (tokens and types) in a precise way. The system of extended mereology can be also useful in formal

representations of eventistic semantics.

The primitive (undefinable) terms of extended mereology are:

$\mathbb{P}$ — the relation of being a part of

$\mathbb{T}$ — the relation of precedence in time.

We understand these concepts according to intuitions related to space-time of everyday experience. An expression $x\mathbb{P}y$ is to be read: the thing x is a part of the thing y. Further, $x\mathbb{T}y$ means that either the whole thing x precedes the whole thing y in time, or that the last time slice of x coincides in time with the first time slice of y.

For any set of objects X, by $\mathbb{P}\langle X \rangle$ we will denote the set of all parts of elements of X:

$$\mathbb{P}\langle X \rangle = \{y : y\mathbb{P}x \ \ for \ \ some \ \ x \in X\}.$$

In the distributive set theory we treat sets as abstract objects. A given set is well determined if we explicitly list all of its elements or give a specific feature (property) characteristic of all the elements of this set. On the other hand, in the mereological approach we are able to apprehend a set of individual objects (things) as a separate object (thing) which is again an individual object (thing). In order to realize this goal we use the function of mereological sum, associating a fixed object with any non-empty family of objects. We say that the object y is the *mereological sum* of the set of objects X (in symbols: $y\mathbb{S}X$) if and only if the following conditions are satisfied:

1. $X \subseteq \mathbb{P}^\vee y$
2. for any z such that $z\mathbb{P}y$, $\mathbb{P}\langle X \rangle \cap \mathbb{P}^\vee z \neq \emptyset$.

Thus, y is the mereological sum of the set X if and only if all elements of the set X are parts of y and every part of y has a common part with some element of the set X. The mereological sum of a given set of objects X is therefore a whole obtained by "gluing together", into one individual object, all the elements of the set X. The mereological sum of a set X will be denoted by $\mathbb{S}^*X$, according to the proviso mentioned above.

It is important to notice the differences between the mereological relations $\mathbb{P}$ and $\mathbb{S}$ and the relations $\in$ and $\subseteq$ from the distributive set theory:

— relation $\mathbb{P}$ holds between individual objects only;

— relation $\mathbb{S}$ holds between an individual object and a set (in the distributive sense) of individual objects;

— relation $\subseteq$ holds between two sets of objects;

— relation $\in$ holds between a particular object (individual as well as any set) and a set of objects.

Now, let us introduce several mereological concepts. For any x and y let

$$x \oplus y = \mathbb{S}^*\{x, y\}$$

(the *mereological sum* of x and y). The *mereological product* x and y will be defined in the following way:

$$x \otimes y = \mathbb{S}^*(\mathbb{P}^\vee x \cap \mathbb{P}^\vee y)$$

The mereological product of x and y is thus the biggest (in the sense of the relation $\mathbb{P}$) object which is simultaneously a part of x and y.

Here are the definitions of further mereological concepts:

$$\boldsymbol{mo} = \{x : x\mathbb{T}x\}$$

(*momentary things*). Thus x is a momentary thing if and only if its beginning coincides in time with its end.

$$\boldsymbol{pn} = \{x : \mathbb{P}^\vee x = \{x\}\}$$

(*points*). A thing is a point if and only if it is the only part of itself.

$$x\mathbb{C}y \text{ if and only if } x\mathbb{T}y \text{ and } y\mathbb{T}x.$$

The formula $x\mathbb{C}y$ may be read: the things x and y are *coincident* in time. It may be proved that the field of this relation is the set of all momentary things.

$$\boldsymbol{ms} = \boldsymbol{mo} \cap \{x : \mathbb{C}^\vee x \subseteq \mathbb{P}^\vee x\}$$

(*momentary world-sections* or, simply, *moments*). Elements of the set of moments are maximal (in the sense of the relation $\mathbb{P}$) momentary things ("the whole universe grasped in one fixed moment").

$$x \; \mathbb{T}_{\boldsymbol{c}} \; y \text{ if and only if for any } u, v\text{: if } u\mathbb{P}x \text{ and } v\mathbb{P}y, \text{ then not } v\mathbb{T}u$$

(*complete precedence in time*). It can be seen from this definition that a thing x completely precedes in time a thing y if and only if no part of y precedes any part of x.

$$x \; \mathbb{T}_{\boldsymbol{e}} \; y \text{ if and only if either } x \; \mathbb{T}_{\boldsymbol{c}} \; y \text{ or } x = y$$
$$x \; \mathbb{T}_{\boldsymbol{i}} \; y \text{ if and only if } x \; \mathbb{T}_{\boldsymbol{c}} \; y \text{ and there is no } z \text{ such that } x \; \mathbb{T}_{\boldsymbol{c}} \; z \text{ and } z \; \mathbb{T}_{\boldsymbol{c}} \; y$$

(*immediate precedence in time*). A thing x precedes immediately a thing y in time if and only if x precedes completely y in time and there is no such thing z which simultaneously precedes completely y in time and is completely preceded in time by x. In particular, if $x \; \mathbb{T}_{\boldsymbol{i}} \; y$, then there

is no momentary world-section which completely precedes y in time and, simultaneously, is completely preceded in time by x (i.e. x and y cannot be "separated" by any moment).

The next concept, i.e. that of a set of *linear objects*, has been introduced to the system of extended mereology by Tadeusz Batóg:

$\boldsymbol{ln} = \{x :$ for every $y \in \boldsymbol{ms}$ such that neither $y \ \mathbb{T}_{\boldsymbol{c}} \ x$ nor $x \ \mathbb{T}_{\boldsymbol{c}} \ y$ we have $x \otimes y \in \boldsymbol{pn}\}$.

Intuitively speaking, linear objects are "continuous in time and not extensive in space".

The system of extended mereology is based on the following axioms (to each axiom we add, in square brackets, a few words of intuitive explanation):

1. The relation $\mathbb{P}$ is transitive.

   [Parts of parts of a given thing are again parts of this thing.]
2. For any x, y: if $x\mathbb{S}\{y\}$, then $x = y$.

   [Mereological sum of the set containing one thing only equals this thing.]
3. For any X: if $X \neq \emptyset$, then $\mathbb{S}^{\vee}X \neq \emptyset$.

   [For any non-empty set there exists (at least one) its mereological sum.]
4. For every x, $\boldsymbol{pn} \cap \mathbb{P}^{\vee}x \neq \emptyset$.

   [Each thing has parts which are points.]
5. The relation $\mathbb{T}$ is transitive.

   [If one thing precedes in time a second one and the second the third, then also the first precedes the third.]
6. The relation $\mathbb{T}$ is dense.

   [If one thing precedes in time another one, then there exists a thing which precedes in time the second of the given things and is preceded in time by the first of them.]
7. For any x there are y and z such that neither $y\mathbb{T}x$ nor $x\mathbb{T}z$.

   [For any thing there exist things which neither precede it nor are preceded by it.]
8. For any momentary world-sections x, y: either $x\mathbb{T}y$ or $y\mathbb{T}x$.

   [Any two moments are always comparable in time.]
9. For any x and y: $x\mathbb{T}y$ if and only if for all $u \in \boldsymbol{mo} \cap \mathbb{P}^{\vee}x$ and for all $v \in \boldsymbol{mo} \cap \mathbb{P}^{\vee}y$ we have $u\mathbb{T}v$.

   [One thing precedes in time another one if and only if every momentary part of the first thing precedes in time every momentary part of the second one.]
10. If $x \in \boldsymbol{pn}$, then the set $\boldsymbol{pn} \cap \mathbb{T}^{\vee}x \cap \mathbb{T}^{\wedge}x$ has the power of the continuum.

    [The set of points coincident with a given point has the power of the continuum.]

11. There exists a denumerable set $X \subseteq \boldsymbol{mo}$ such that for any x and y: if $x\mathbb{T}y$ does not hold, then there exists $z \in X$ for which neither $x\mathbb{T}z$ nor $z\mathbb{T}y$.

[There exists a denumerable set of moments such that for any x and y, where the beginning of y precedes in time the end of x one can find in this set a moment z, whose beginning precedes in time the end of x and whose end is preceded in time by the beginning of y.]

Not all of these axioms are necessary for the construction of Tadeusz Batóg's phonological theory. It might be interesting to find a *minimal* fragment of extended mereology sufficient for these purposes. One may add, at this point, that the above axiom system is categorical, that is that it provides — roughly speaking — for a unique (up to isomorphism) interpretation of its primitive terms.

2   Axioms of the System Batóg (1967)

Tadeusz Batóg has employed five sets of primitive terms in his phonological theories. Some of them are common for all of them. Here are the corresponding collections of primitive terms:

Batóg (1961)

ι	idiolect (arbitrary, but fixed)
$\boldsymbol{D}$	the set of all segments of the idiolect ι
$\boldsymbol{O}$	the set of all pauses of the idiolect ι
$\boldsymbol{B}$	the relation of homophony

Batóg (1962)

$\boldsymbol{I}$	the set of all idiolects
$\boldsymbol{D}$	the set of all segments (of all idiolects)
$\boldsymbol{O}$	the set of all pauses
$\boldsymbol{B}$	the relation of homophony

Batóg (1967)

I	the set of all idiolects
O	the set of all pauses
K	the family of kinds of phonetic features

Batóg (1971)

I	the set of all idiolects
O	the set of all pauses
K	the family of kinds of phonetic features
M	the relation of synonymy
Sm	the relation of phonetic similarity

Batóg (1976)

I	the set of all idiolects
K	the family of kinds of phonetic features
O	the set of all pauses
M	the relation of synonymy

Primitive terms in each of those systems are characterized (formally, as mathematical constructs) by a small number of postulates (axioms). We do not agree with Kortlandt that those axioms are trivial from the linguistic point of view (Kortlandt 1972, p. 95). They reflect for example such important features of speech as linearity and temporal ordering. Moreover, conditions imposed on constructs involved by phonologists (e.g. those concerning kinds of phonetic features) have a clear and unambiguous linguistic interpretation.

The main goal of all these axiomatic systems is the characterization of the concept of the phoneme. In Batóg (1961) and (1962) the Author proposes explicit definitions of this concept. However, as we know from linguistic practice, the phonemization of a given idiolect may not be uniquely determined. Thus, beginning from Batóg (1967), the Author characterizes phonemes in a different way, as members of *any* classification of the set of all phones which satisfies suitably chosen conditions.

We are going now to present the axioms of the system from Batóg (1967). It has been already said that this system plays a central role in Batóg's approach. Systems from Batóg (1971) and (1976) are its extensions and the early systems from Batóg (1961) and (1962) are now of historical interest only.

Elements of the set I are called *idiolects* and elements of $\bigcup I$ are called *utterances*. Further, elements of the family K are *kinds of phonetic*

features. By *phonetic features* we understand elements of the set $\bigcup \boldsymbol{K}$. Elements of $\bigcup\bigcup \boldsymbol{K}$ are *proper segments*. Finally, by *elementary segment* we understand any element of the set $\bigcup\bigcup \boldsymbol{K} \cup \boldsymbol{O}$. Notice that phonetic features are treated extensionally here, as sets of segments.

Elements of the set $\boldsymbol{O}$ are called *pauses* or *zero segments.*

The intended linguistic interpretation of these concepts is that any set of linguistically homogeneous spoken texts (individual utterances) is an idiolect (cf. Batóg 1967, pp. 27–28):

> By an idiolect, in the most comprehensive sense, we mean any set of concrete utterances. It is, of course, evident that the majority of such idiolects will be, from the point of view of linguistics, of no interest whatever. For, if any specific idiolect is to have a linguistic value it should fulfil certain indispensable requirements. First of all it must be linguistically uniform, i.e. all utterances which are the elements of a given idiolect should be uttered by members of the same speech community, that is, they should belong to the same dialect. Moreover, an idiolect should be a sufficiently representative sample of a dialect. Therefore, it should be sufficiently ample and internally differentiated. Finally, it is not irrelevant whether the utterances are pronounced carefully, naturally, not too vehemently and if they are representative of the same style of speech etc. To meet all these requirements is not an easy task, especially when the linguist should follow in order to obtain a sufficiently 'good' idiolect since these problems are irrelevant from the point of view of theoretical linguistics. The basic procedures of our phonological system will refer to any idiolect, no matter whether it meets the above requirements. In this respect our attitude does not differ from that of Harris who in his introductory methodological remarks states: 'The procedures discussed below are applied to a corpus of material without regard to the adequacy of the corpus as a sample of the language'.

Providing an interpretation of the concept of elementary segment Batóg quotes Bloch, Pike and Jassem, who use in a similar sense the term "segment":

> A fraction of an utterance between any two immediately successive change-points is a segment (Bloch 1948, p. 12).

> A single sound caused by the movement of a single articulator (or the synchronous movement of several articulators) may be called a sound segment (Pike 1947, p. 11).

Jassem defines segments as "such minimal elements the impressions of which cannot be further divided by the ear" (Jassem 1954, p. 15).

Finally, the concepts of phonetic features and kinds of phonetic features are explained by Batóg as follows (cf. Batóg 1967, p. 31):

> By phonetic features we mean such and only such articulatory and acoustic features which according to phoneticians account for the fact that some two elementary segments are or are not phonetically equivalent (that is 'identical', in phoneticians' wording). We are perfectly aware that the above 'explanation' explains very little. However, we think that the notion of phonetic feature considered here is sufficiently clear for the phoneticians. Therefore for theoretical purposes we assume that every phonetician has at his disposal, so to say, from the start the general set (stock) of phonetic features. Moreover, we also assume that he is also given such a classification of all phonetic features in *kinds*, that two features are of the same kind if and only if they are homogeneous, that means if they are features 'in the same respect'. (Examples of homogeneous features are e.g. voiced, voiceless; discontinuous, continuant. Examples of non-homogeneous features are e.g. nasal, continuant; voiced, discontinuous).

The axiom system from Batóg (1967) is formally elegant because it contains, besides mereological terms, only primitive concepts (the only exception is Axiom 15 which will be discussed in the next section). We list all the axioms below. Their formulation in English should not lead to any confusion — it is easy to find the corresponding symbolic formulation.

Axiom 1

There exists at least one idiolect.

Axiom 2

Every idiolect is a finite non-empty set.

Axiom 3

Every utterance is a linear object.

Axiom 4

Any part of an utterance overlaps at least one elementary segment (proper or not) completely contained in this utterance.

Axiom 5

No utterance consists entirely of pauses (i.e. every utterance contains at least one proper segment).

Axiom 6

For any utterance u of a specific idiolect there exist two points x, y which are parts of u, so that all points which are parts of u and which

precede or coincide in time with x are parts of certain zero segments, and similarly all points which are parts of u and follow or coincide in time with y are parts of certain zero segments.

Axiom 7

Elementary segments of every utterance are linearly ordered by the relation $\mathbb{T}_c$.

Axiom 8

Every non-empty set of elementary segments has the first and the last element (in the sense of the relation $\mathbb{T}_e$).

Axiom 9

Any two utterances which share at least one proper segment are themselves parts of some utterance.

Axiom 10

Elementary segments are non-momentary parts of utterances.

Axiom 11

No pause has any part in common with any proper segment.

Axiom 12

Any distinct kinds of phonetic features are disjoint sets.

Axiom 13

For any kind of phonetic features, every elementary segment is an element of some phonetic feature of this kind.

Axiom 14

Any distinct phonetic features of the same kind are disjoint.

Here are some immediate consequences of the axioms:

1. Every kind of phonetic features is a classification of the set of all proper segments. These classifications do not have any common members.

2. The sets: of all utterances, of all proper segments, of all kinds of phonetic features, of all pauses are non-empty.

3. No distinct proper segments have any parts in common.

4. For any kind of phonetic features, every elementary segment belongs to exactly one phonetic feature of this kind.

5. No proper segment is a pause.

6. Every utterance contains at least three elementary segments. In particular, every utterance starts and ends with a pause.

The formulation of Axiom 15 of the system from Batóg (1967), as well as axioms from Batóg (1971) and (1976) requires a series of definitions (cf. the next section).

It has been shown in Batóg (1969) that the concept of the pause becomes definable (in terms of the set of all idiolects and the family of kinds of phonetic features) if we add to the above axioms an extra postulate

which requires that no utterance contains two pauses occuring immediately one after another. In such a case, axioms 4 and 11 become superfluous and can therefore be omitted.

3 Definitions of Auxiliary Notions

Let $\boldsymbol{esg}$ denote the set of all elementary segments. For any object x, let $\boldsymbol{esg}(x)$ be the set of all elementary segments which are parts of x. We say that x is a *phonetic chain* if:

— x is a part of some utterance;

— x is the mereological union of the set $\boldsymbol{esg}(x)$, i.e. the set of all its elementary segments;

— all elementary segments of any utterance containing x as a part which lie (in the sense of the relation $\mathbb{T}_{\boldsymbol{c}}$) between some elementary segments of x, are also parts of x.

It follows from this definition that:

— phonetic chains are non-momentary linear objects;

— each phonetic chain consists entirely of elementary segments.

One can also prove that any utterance, as well as any elementary segment is a phonetic chain. If two phonetic chains have at least one common elementary segment, then their mereological sum and their mereological product are again phonetic chains.

Let l_x denote the number of elementary segments which are parts of x. All elementary segments of a given phonetic chain x can be enumerated with numbers from 1 to l_x, because the set $\boldsymbol{esg}(x)$ is linearly ordered by the relation $\mathbb{T}_{\boldsymbol{e}}$. For $1 \leqslant n \leqslant l_x$ let $t_n(x)$ denote the n-th elementary segment of x.

In what follows, we will use the term *utterance* only for such phonetic chains u which contain at least one proper segment and whose first and last elementary segment is a pause (i.e. such u, for which $t_1(u) \in \boldsymbol{O}$ and $t_{l_u} \in \boldsymbol{O}$). One can prove that utterances in this sense are non-momentary linear objects.

By a *phrase* we mean any phonetic chain which does not contain any pause and which is limited on both ends by a pause. The precise formal definition of a phrase can be obtained by using the predecessor and successor functions which associate with every phonetic chain the elementary segment immediately preceding this chain in time (respectively, immediatety following this chain in time).

We say that the phonetic chains x and y are *phonetically equivalent*, in symbols $x\boldsymbol{E}y$, if:

— x and y have the same length (the same number of elementary segments);

— for every n such that $1 \leqslant n \leqslant l_x$ the elementary segments $t_n(x)$ and $t_n(y)$ are either both pauses or else have exactly the same phonetic features (i.e. belong to exactly the same elements of the set $\bigcup \boldsymbol{K}$).

The relation $\boldsymbol{E}$ is an equivalence relation. It will be employed in the definition of the concept of a word. However, this last concept should be, for obvious reasons, relativized to an idiolect.

The relation of phonetic equivalence replaces the relation of homophony used in the earlier works of Batóg. Recall that homophony was a primitive concept. By introducing the family of phonetic features the Author is able to *define* homophony. Let us add that pauses are phonetically equivalent with pauses only (thus the duration of a pause plays no role in the present system).

Introducing the concept of word into his system, Batóg recalls some attempts of other linguists at the definition of a word. In a sense, the closest to Batóg's proposals are those of Harris and Palmer (cf. Batóg 1967, p. 62):

> Harris summarizes Bloomfield's conception when discussing his own idea of the notion of word, as follows: 'Every word ... occurs occasionally by itself as a complete utterance. No word is divisible into smaller sections each of which occurs by itself (except, in special circumstances) as a complete utterance. ... Using this property, Bloomfield defined the word in general as a minimum utterance'.

> L.R. Palmer defines a word as 'the smallest speech unit (=constantly recurring sound-pattern) capable of functioning as a complete utterance'.

We say that a set X is a *quasi-phrasal partition* of a phrase x with respect to the idiolect ι, if:

— the mereological sum of X equals x;

— no two distinct elements of X have any parts in common;

— every element of X is a phonetic chain being a part of x;

— every element of X is phonetically equivalent with some phrase from ι.

If x is a phrase in a given idiolect, then its quasi-phrasal partition can be obtained by cutting x into phonetic chains each of which is phonetically equivalent with some phrase of this idiolect.

We say that X is a *word-partition* of a phrase x with respect to the idiolect ι, if:

— X is a quasi-phrasal partition of x (with respect to ι);

— there is no quasi-phrasal partition Y of x such that $X \neq Y$ and every

element of Y is a part of some element of X.

Thus we see that word-partitions of phrases are quasi-prasal partitions with $\mathbb{P}$-minimal phrasal elements.

The set of all *words* of an idiolect ι is defined to be the set of all elements of all word-partitions of all phrases of this idiolect. To be a word in some idiolect means to be an element of some word-partition of some phrase of this idiolect.

One should remember that words of a given idiolect are individual objects. Sometimes one uses the term *word-token* for objects of this sort. Definition of words in abstract sense (*word-types*) requires some additional constructions connected with phonemic representation of word-tokens.

The equivalence classes of the relation of phonetic equivalence on the set of all elementary segments will be called *phones*. The set of all pauses $\boldsymbol{O}$ is a phone. Phones which are different from $\boldsymbol{O}$ will be called *proper*.

By the *phonic structure* of a phonetic chain x we mean the l_x-element sequence consisting of phones associated with the consecutive (with respect to temporal ordering) elementary segments of x. It is clear that such a sequence is uniquely determined. Let $f_n(x)$ denote the n-th phone in the phonic structure of a phonetic chain x $(1 \leqslant n \leqslant l_x)$.

The next concept to be defined, i.e. that of a unit-length segment is a little bit complicated. Unit-length segments are complexes of elementary segments which in some contexts (neighbourhoods) always occur together. Batóg limits himself to the consideration of such complexes with two or three elements only (this limitation is justified on the ground of linguistic practice). He adds, however, that the construction of n-complexes of elementary segments (for any fixed n) can be done as well. The constructions proposed by Batóg correspond to the non-formal ones suggested by Harris (unit-length segment), Pilch (phonematisches Segment) or Jassem (sound).

We say that the elementary segments x, y, z (in this order) are *inseparable* in the idiolect ι, if:

— x, y, z are proper segments of ι;

— x is the predecessor of y;

— y is the predecessor of z;

— no phonic context of the phonetic chain $x \oplus y \oplus z$ is a phonic context either of $x \oplus y$ or $y \oplus z$.

The concept of a phonic context used above needs some explanation. We say that a pair (a, b) of phonic structures is a *phonic context* of a phonetic chain v if the sequence $(a, f_1(v), \ldots f_{l_v}(v), b)$ is a phonic structure. Observe that a and b are treated here as sequences!

The concept of phonic context introduced here was not employed in Batóg (1967). It corresponds, however, to Batóg's understanding of the

inseparability of elementary segments.

The family of all three element sets of elementary segments inseparable in the idiolect ι will be denoted by $\boldsymbol{sgcm}(3, \iota)$. The elements of this family will be called the family of *three-segmental complexes* in ι.

We say that the segment x is *left inseparable* from the segment y (in the idiolect ι), in symbols $x\,\boldsymbol{lisp}_\iota\,y$, if:
— x and y are proper segments of ι;
— x is the predecessor of y;
— neither x nor y is a member of any three-segmental complex of ι;
— no phonic context of $x \oplus y$ is a phonic context of y.

Similarly, we say that the segment x is *right inseparable* from the segment y (in the idiolect ι), in symbols $x\,\boldsymbol{risp}_\iota\,y$, if:
— x and y are proper segments of ι;
— x is the successor of y;
— neither x nor y is a member of any three-segmental complex of ι;
— no phonic context of $y \oplus x$ is a phonic context of y.

If x is left- or right-inseparable from y, then we say that x and y form a *two-segmental complex* in ι. The family of all two-segmental complexes in ι will be denoted by $\boldsymbol{sgcm}(2, \iota)$.

The mereological sum of any three-segmental (respectively two - segmental) complex will be called a *ternary* (respectively a *binary*) *compound segment* in ι. We will use the term *compound segments* for both ternary and binary compound segments.

By a *proper unit-length segment* of the idiolect ι we mean any compound segment of ι as well as any elementary segment which is not a member of any three-segmental or two-segmental complex. *Unit-length segments* of ι are: all its proper unit-length segments and all the pauses of ι. Let us denote the set of all proper unit-length segments by $\boldsymbol{usg}_\iota$ and the set of all unit-length segments by $\boldsymbol{usg}_\iota^0$. We are able now to formulate the last axiom of the system presented in Batóg (1967):
Axiom 15

If X and Y are distinct three- or two-segmental complexes in ι, then X and Y are disjoint.

Phonetic features are associated with particular elementary segments. In order to associate phonetic features with unit-length segments, we should generalize the very concept of a phonetic feature.

Let $\boldsymbol{cgs}_3$ (respectively $\boldsymbol{cgs}_2$) denote the family of all three-segmental (respectively two-segmental) complexes of all idiolects.

Let $\boldsymbol{X}$ be any kind of phonetic features (i.e. a member of the family $\boldsymbol{K}$). By a *compound feature* of the kind $\boldsymbol{X}$ we mean any set X such that:
— every element of X is a two-segmental complex the elementary segments of which belong to some phonetic feature of $\boldsymbol{X}$;

or

— every element of X is a three-segmental complex the elementary segments of which belong to some phonetic feature of $\boldsymbol{X}$.

The set of all compound features of the kind $\boldsymbol{X}$ will be denoted by $\boldsymbol{cf}(\boldsymbol{X})$. We also define the family $\boldsymbol{K}_+$ of *kinds in the generalized sense of phonetic features*:

$$\boldsymbol{K}_+ = \{\boldsymbol{X} : \boldsymbol{X} = \boldsymbol{Y} \cup \boldsymbol{cf}(\boldsymbol{Y}) \ \ for \ some \ \ \boldsymbol{Y} \in \boldsymbol{K}\}$$

The concept of a compound feature is characterized as follows in Batóg (1967 , pp. 80–81):

> We shall illustrate now the notion of a compound feature of a given kind by a concrete example. Let, e.g. x be the first unit-length segment of a phrase due to the uttering of the English sentence *Damn you!* Then x is composed of two elementary segments the first of which is voiceless and the second is voiced. If now $\boldsymbol{X}$ is that kind of phonetic features which contains the features of being voiceless and of being voiced, and X is the set of all binary compound segments in which the first elementary segment is voiceless and the second is voiced, then in accordance with 10.3 [Batóg means here the definition of the set $\boldsymbol{cf}(\boldsymbol{X})$ — J.P.] the set X may be recognized as a compound feature of the kind $\boldsymbol{X}$. This feature might be called the feature of being a voiceless-voiced unit-length segment. It is easily seen that X is not the only compound feature of the kind $\boldsymbol{X}$. There may exist additionally voiced-voiceless, voiced-voiced and voiceless-voiceless unit-length segments, and, moreover, eight sorts of ternary compound segments.

The equivalence classes of the relation of phonetic equivalence on the set of all unit-length segments of a given idiolect are called *sounds* of this idiolect. The set of all sounds of the idiolect ι is denoted by Σ_ι.

For any phonetic chain x let $u_\iota(x)$ denote the number of unit-length segments of x in the idiolect ι.

Every phonetic chain of the idiolect ι which is identical with the mereological union of the set of all its unit-length segments will be called a *complete chain* of this idiolect. One can prove that every utterance of a given idiolect is a complete chain of this idiolect.

By the *phonetic structure* of a phonetic chain x (in ι) we understand $u_\iota(x)$-element sequence of sounds containing the consequtive (with respect to temporal ordering) elementary segments of x. The phonetic structure of a given phonetic chain is obviously uniquely determined.

By a *beginning chain* of ι we understand any complete chain of this idiolect in phonetic structure of which the first element at most is a pause. Similarly, by an *ending chain* of ι we understand any complete chain of this idiolect in the phonetic structure of which the last element at most is a pause. Any pair consisting of a beginning chain and an ending chain will be called an *environmental pair* in ι. If x is an elementary segment and (a, b) is an environmental pair in ι, then we say that (a, b) is an *environ* of x if and only if the mereological union $a \oplus x \oplus b$ is a complete chain of ι. By the *phonetic structure* of an environmental pair (a, b) in ι we mean the pair consisting of the phonetic structure of the chain a and the phonetic structure of the chain b.

The set $D_\iota(x)$, called the *distribution* of an elementary segment x in the idiolect ι is defined to be the set of all phonetic structures of all environs of x in ι.

If X is a sound of the idiolect ι, then the union of all distributions of all elementary segments belonging to X will be called the *distribution* of X and denoted by $\boldsymbol{D}_\iota(X)$.

We say that the sounds X and Y of the idiolect ι are within the relation of *free variation*, in symbols $X \; \boldsymbol{Fv}_\iota \; Y$, if they have the same distribution in ι. If $X \; \boldsymbol{Fv}_\iota \; Y$, then we also say that X and Y are *free variants*.

The relation $\boldsymbol{cm}_\iota$ of *complementary distribution* is the union of the relations $\boldsymbol{1cm}_\iota$ and $\boldsymbol{2cm}_\iota$, defined as follows on the set of all sounds of the idiolect ι:

$X \; \boldsymbol{1cm}_\iota \; Y$ if and only if the distributions of X and Y are disjoint;

$X \; \boldsymbol{2cm}_\iota \; Y$ if and only if:
— the distributions of X and Y have a common element;
— the distributions of X and Y are not identical;
— no maximal environmental pair of ι belongs to the intersection of the distributions of X and Y.

The concept of a maximal environmental pair of ι, used above should be understood in the following way. We say that an environmental pair (a, b) is *maximal* in ι, if the first element of the phonetic structure of a as well as the last element of the phonetic structure of b is a pause.

It should be pointed out here that Batóg's definition of complementary distribution is at the same time more general and more adequate than definitions of this term proposed sometimes in textbooks (cf. Batóg 1967, pp. 93–94):

> Wishing to grasp the proper linguistic meaning of the relation of complementarity we must treat as complementary also any two sounds X and Y which although having some common contexts have at the same time the following property: there exists a con-

stant factor such that every common context of these sounds may
be enlarged so that X will occur in a context including this constant
factor and Y in a context without this factor.

One can prove that the relations of free variation and of complementary distribution exclude each other in a given idiolect. Similarly, the relations $\mathbf{1cm}_\iota$ and $\mathbf{2cm}_\iota$ exclude each other.

4 The Role of Semantics in Axiomatic Phonology

In the former section we have presented all the auxiliary concepts of the system from Batóg (1967) necessary for the construction of phonemes (in the framework of that system). The same goal (i.e. the characterization of phonemes) is pursued in the two extensions of this major system. In Batóg 1971 the Author has considered two additional primitive concepts: those of the relation of *phonetic similarity* $\mathbf{Sm}$ and the relation of *synonymy* $\mathbf{M}$. In turn, in Batóg 1976 one considers the relation of synonymy $\mathbf{M}$ only (in addition to the primitive terms from Batóg 1967). We present the axioms characterizing these concepts below. It should be stressed that by introducing the semantically based relation of synonymy Batóg's theory becomes capable of embracing European structuralism, mainly in its version suggested by Prince N.S. Trubetzkoy. Hence in this version of Batóg's theory we are able to characterize phonemes as fundamental functional units of language responsible for meaning differentiation.

In Batóg (1971) both relations $\mathbf{Sm}$ and $\mathbf{M}$ hold between individual objects.

The relation of phonetic similarity is characterized by the following axioms:

Axiom 16

For any segments x, y, z: if (x $\mathbf{Sm}$ y or y $\mathbf{Sm}$ x) and y $\mathbf{E}$ z, then x $\mathbf{Sm}$ z and z $\mathbf{Sm}$ x.

Axiom 17

Proper segments with exactly the same phonetic features are phonetically similar.

Axiom 18

Pauses are phonetically similar to pauses only.

Axiom 19

For any x, y, z, v, if x $\mathbb{T}_{\boldsymbol{c}}$ y, z $\mathbb{T}_{\boldsymbol{c}}$ v, x $\mathbf{Sm}$ z, y $\mathbf{Sm}$ v, $x \oplus y$ is a phonetic chain in some idiolect and $z \oplus v$ is a phonetic chain in some idiolect, then $x \oplus y$ $\mathbf{Sm}$ $z \oplus v$.

Phonetic similarity is a reflexive and symmetric relation in the set of all phonetic chains. Moreover, any two unit-length segments belonging to a given sound (of some idiolect) are phonetically similar.

The following axioms characterize the relation of synonymy:

Axiom 20

Synonymy is a symmetric and transitive relation; on the set of all words (of any idiolect) it is also reflexive.

Axiom 21

For any x, y, z, v, if $x \ \mathbb{T}_{\boldsymbol{c}} \ y$, $z \ \mathbb{T}_{\boldsymbol{c}} \ v$, $x \ \boldsymbol{M} \ z$, $y \ \boldsymbol{M} \ v$ and $x \oplus y$ as well as $z \oplus v$ are phonetic chains of some idiolect, then $x \oplus y \ \boldsymbol{M} \ z \oplus v$.

The class of all objects synonymous with a given object x will be called the *actual meaning* of x. One can prove that synonymy is an equivalence relation on the set of all words and phrases of any idiolect. This means that if x and y are words or phrases of a given idiolect, then either their actual meanings are identical or else they do not have any elements in common.

By the *potential meaning* of x we mean the union of all actual meanings of all objects which are phonetically equivalent with x. Of course, the actual meaning of any object is a subset of its potential meaning.

In the system presented in Batóg (1976), we find the relation of synonymy but not that of phonetic similarity. Furthermore, in one of the versions of this system Batóg introduces a function associating with any phoneme the set of its phonetic features. It is of secondary importance of whether this is a new primitive concept: one can talk either of the existence of such a function or of special conditions concerning the relation between sounds and phonetic features.

5   Phonemic Bases

We are now in a position to compare all three stages of the development of Batóg's phonological theory. At each of these stages *phonemes* are certain sets of sounds. More exactly, the family of all phonemes of a given idiolect is a certain classification of the set of all sounds of this idiolect.

The systems from Batóg (1961) and (1962) share the following properties:

1. Both systems characterize the concept of the phoneme in purely distributional terms, via the relations of free variation and complementary distribution. In any of those systems the relation of homophony is present — it is simply a primitive term characterized as an equivalence relation. As it has been already said, any more specific characterization of this relation should take into account phonetic features associated with

segments. Let us also add that the enrichment of a purely distributional system with the concept of a phonetic feature makes it possible to *define* the concept of elementary segment (which was a primitive concept in Batóg 1961 and 1962).

2. The primitive terms of both systems are characterized, in fact, by the same set of axioms. The only difference is that in Batóg (1961) one speaks about an arbitrary, fixed idiolect and in Batóg 1962 about the class of all idiolects. Consequently, the meaning of all the remaining primitive concepts should be modified.

3. Both systems provide for an explicit definition of the concept of phoneme. This means that one believes in the existence of a unique partition of sounds into phonemes. The relation of phonological equivalence, understood as the union of the relations of free variation and complementary distribution, plays a central role here. This relation is obviously reflexive and symmetric. In Batóg (1961) the Author makes an additional assumption, saying that if a sound X is within complementary distribution with both the sounds Y and Z, then the sounds Y and Z are either mutual free variants or are in complementary distribution. This assumption assures that phonological equivalence is transitive and hence an equivalence relation. *Phonemes* are simply its equivalence classes. Thus, two sounds belong to the same phoneme if and only if they are either mutual free variants or are in complementary distribution. Such a formulation of this axiom is seemingly too strong from a point of view of linguistic practice. In Batóg (1962) it has been omitted and phonemes have been defined as equivalence classes of the relation associated with phonological equivalence. We recall that if R is an arbitrary binary relation then by the relation *associated* with R we mean the relation R^+ defined in the following way:

xR^+y if and only if: for all z, xR^+z if and only if yR^+z.

The relation associated with any reflexive and symmetric relation is of course an equivalence. According to the above definition, two sounds belong to the same phoneme (in a given idiolect) if and only if they are phonologically equivalent with exactly the same sounds of this idiolect.

Let us compare now the remaining three systems. It will be useful to put all the conditions characterizing phonemes in those systems in one table:

Batóg 1967	Batóg 1971	Batóg 1976
classification	classification	classification
free variation	free variation	free variation
complementary distribution	phonetic similarity	complementary distribution
distinctiveness	—	distinctiveness
—	differentiation	differentiation
economy	economy	economy

Before we discuss particular postulates of the above three systems let us point out to a few general properties of those systems. Each of those systems proposes a characterization of the family of all phonemes (of a given idiolect), called the *phonemic basis* (of the idiolect in question) in an axiomatic way. A phonemic basis is any family of sets of sounds which satisfies certain conditions. Thus, one admits the existence of more than one classification of sounds into phonemes — this solution is closer to linguistic practice. All systems use distributional concepts and kinds of phonetic features. The relation of homophony is replaced by the relation of phonetic equivalence. In Batóg (1971) and (1976) one makes use of semantical concepts. Finally, the relation of phonetic similarity is present only in Batóg (1971).

The first and last postulate (cf. the corresponding rows of the above table) are of a technical character. Thus, the postulate of *classification* requires that the family of all phonemes of a given idiolect is a classification of the set of all sounds of this idiolect. Hence, any phoneme is a non-empty set of sounds and any sound belongs to exactly one phoneme.

The postulate of *economy* is responsible for the minimalization of the number of phonemes. In the monograph Batóg (1967) it has the following form:

> If $\mathfrak{B}$ is a phonemic basis then there is no other classification of the set of all sounds which satisfies all the remaining postulates (of Batóg 1967) and has less members than $\mathfrak{B}$.

In other words, the existence of two phonemic bases with different numbers of phonemes is excluded. This version of the postulate of economy appeared to not be adequate. In Batóg (1969), the Author has suggested a new form of this postulate, involving the concept of summable reducibility. If $\mathfrak{A}$ and $\mathfrak{B}$ are two different classifications of the same set, then we say that $\mathfrak{A}$ is *summably reducible* to $\mathfrak{B}$ if and only if each member of $\mathfrak{B}$ is a set-theoretical union of some members of $\mathfrak{A}$. It is easy to see that if $\mathfrak{A}$ is summably reducible to $\mathfrak{B}$, then $\mathfrak{A}$ is "finer" than $\mathfrak{B}$.

Now, the postulate of economy obtains the following form:

> No two different phonemic bases of a given idiolect are summably
> reducible to each other.

In this sense, there may exist phonemic bases with different number of elements. However, no phoneme in one basis can be a union of some phonemes from a second one (assuming that all other phonemes of those bases are identical).

The postulate of *free variation* has the very clear and intuitive meaning:

> Any free variants belong to the same phoneme.

In other words, any sound belongs to (exactly one) phoneme together with all its free variants. This means that sounds which occur in exactly the same environs should be put in one phoneme. In still another wording, this postulate says that the classification of sounds into (classes of) free variants is summably reducible to any classification of sounds into phonemes.

The postulate of *differentiation* is formulated in those systems which make use of semantic concepts. Before we state it here, one additional concept should be defined. If $(X_1, X_2, \ldots, X_n)$ is the phonetic structure of a phrase x in a given idiolect, and $\mathfrak{B}$ is any classification of all sounds of this idiolect, then by the $\mathfrak{B}$-*structure* of x we mean the sequence of members of $\mathfrak{B}$ to which the consecutive sounds $X_1, X_2, \ldots, X_n$ belong. It is evident that $\mathfrak{B}$-structure of any phrase is determined in a unique way.

Here is the postulate of differentiation:

> If $\mathfrak{B}$ is a phonemic basis, then $\mathfrak{B}$-structures of words with different
> potential meanings are different.

Thus, words with different potential meanings cannot have the same phonemic structure. In a looser formulation, this means that phonemes and not sounds differentiate meanings. It follows from the definitions of the relations of free variation and complementary distribution that the phonetic structure of words with the same potential meanings may differ only with respect to free variants (in the corresponding places in sequences forming these structures).

It is possible to formulate the postulate of differentiation without the use of semantic terms (thus reflecting the spirit of American structuralism):

> The 𝔅-structure of a given phrase uniquely determines (up to free
> variants) its phonetic structure.

The postulates of complementary distribution and of distinctiveness
are strongly correlated. The postulate of *complementary distribution* has
the following form:

> Any two sounds belonging to the same phoneme are either mutual
> free variants or are in complementary distribution.

Hence this postulate does not allow one to put two sounds which are
neither free variants nor within the relation of complementary distribution
into one phoneme. Of course, not every pair of two sounds in complemen-
tary distribution have to belong to the same phoneme. Which sounds
belong to the same phoneme is decided on the basis of other postulates
and especially the postulate of *distinctiveness*:

> For each phoneme X of any phonemic basis there exists a class of
> phonetic features (the so-called *distinctive features*) such that each
> sound that belongs to X has all the features of this class, and each
> sound that does not belong to X lacks at least one of these features.

The distinctive features of a given phoneme are common to all sounds
of this phoneme and only for them. Particular phonetic features may
be common for sounds belonging to different phonemes. Also sounds
belonging to the same phoneme may differ with respect to some phonetic
features. However, all the sounds of a given phoneme have a specific set
of common features which is not, as a whole, associated with any sound
from outside this phoneme.

In Batóg (1971), the postulates of complementary distribution and of
distinctiveness are not present. Instead, we have the postulate of *phonetic
similarity* in the following form:

> If two sounds belong to the same phoneme, then any segment of
> one of these sounds is phonetically similar to some segment of the
> second one.

It follows from the axioms of Batóg (1971), that if two sounds belong
to the same phoneme, then every segment of one of them is phonetically
similar to *every* segment of the second one. The postulate of phonetic
similarity excludes the possibility of grouping into one phoneme sounds
the segments of which are not phonetically similar. One should remember
that the (primitive) concept of phonetic similarity is characterized only
formally here. In order to decide which segments are phonetically simi-
lar, linguists may take into account several parameters (e.g. articulatory,
acoustic, or auditive).

6   The Fundamental Hypothesis of Phonology

We have discussed all the postulates of the last three systems. In each of those systems one can formulate the following claim, which Batóg calls *the fundamental hypothesis of phonology*:

(H)   For every idiolect there exists a phonemic basis.

The sentence (H) is indeed a hypothesis. It cannot be proved in any of the systems discussed. It cannot be refuted, either (i.e. one cannot prove its negation, saying that there exists an idiolect with no phonemic basis). The claim (H) is thus independent of the axioms.

How to justify the metatheorem of the independence of (H) from the axioms is clear. It will suffice to give examples of idiolects for which:

1. there exists at least one phonemic basis;
2. there is no phonemic basis.

Because of the intended interpretation of Batóg's systems this task should be considered in two ways:

a. searching for examples and counterexamples of (H) from among the phonological systems of natural languages;

b. searching for examples and counterexamples of (H) on purely formal grounds, i.e. looking for models of (H) and models for the negation of (H).

ad a.   As far as we know, there are no reports of non-phonemizable languages, i.e. languages for which linguists are unable to propose corresponding sets of phonemes. Of course not all linguists' proposals follow the requirements of Batóg's phonological theory. But even if we accept some version of the axiomatic characterization of the concept of a phonemic basis, it may be (technically) very difficult, if at all possible, to check whether a given classification of sounds meets all the requirements imposed by the postulates. This is caused simply by the incompleteness of our knowledge of existing languages (for the majority of languages we have at our disposal only very imprecise data) as well as by the numerical complexity of the algorithm for establishing phonemic bases of a given idiolect. To summarize: the current state of knowledge of the languages of the world seems to soundly confirm (H).

ad b. This aspect of our goal is not very complicated from a technical point of view. Below, we present two formal constructions which show the independence of (H) from the axioms.

6.1   A model for (H) for the system from Batóg (1976)

We should construct the set of sounds, the set of phonetic features, the set of phonetic structures and the relation of synonymy. We should also

decide which sounds have which features.

Let $\boldsymbol{X} = \{X_1, X_2, \ldots, X_n\}$ be the set of sounds ($n > 1$). The set $\boldsymbol{wrd}$ of all phonetic structures of words is defined as the set of all non-empty sequences (without repetitions) of the elements of $\boldsymbol{X}$ with at most n elements. Let $\boldsymbol{M}$ be the identity relation on $\boldsymbol{wrd}$. Hence the actual meaning of any word w equals $\{w\}$ and is identical with its potential meaning. Let $\boldsymbol{ftr} = \{a_1, a_2, \ldots, a_n\}$ be the set of all phonetic features and assume that the sound X_i has the feature a_i only ($1 \leqslant i \leqslant n$).

Free variation is the relation of identity here and complementary distribution is the complement of free variation. Thus, no distinct sounds have the same distribution (we assume that any combination of elements of $\boldsymbol{X}$ separated by pauses corresponds to a phrase).

It follows from the definition of $\boldsymbol{M}$ that the postulate of differentiation trivially holds. According to the postulates of free variation and complementary distribution, we could put all the sounds from $\boldsymbol{X}$ into one phoneme. However, this is not possible, because no distinct sounds have any common features.

One can check that the only classification of $\boldsymbol{X}$ which satisfies all the postulates from Batóg (1976) is the classification of $\boldsymbol{X}$ into n one-element sets. Hence, each phoneme in this system consists of exactly one sound.

The above construction gives an example of a sound system with exactly one phonemic basis. Let us add that in Batóg 1967 the Author presented a sufficient condition for the existence of a phonemic basis (theorem 13.6 on page 108). Namely, if an idiolect has no non-trivial free variation (i.e. if free variation is simply the identity relation), then there exists at least one phonemic basis for this idiolect. However, this theorem essentially uses the postulate of economy in its weaker form (without the concept of summable reducibility).

6.2   A model for the negation of (H) for the system from Batóg (1971)

An example of a "non-phonemizable" idiolect can be found in Batóg (1971 , p. 36). In order to get such a case it suffices to assume that an idiolect contains two different sounds X and Y which are mutual free variants and such that the following condition holds:

> There exists a segment in X which is not phonetically similar to any segment from Y.

No idiolect containing such sounds could satisfy at the same time the postulates of free variation and of phonetic similarity. Therefore, for such an idiolect there is no phonemic basis.

This example is, of course, purely formal. It is highly improbable that any sane and sober phonetician would suggest that segments of free variants are not phonetically similar.

We think that it would be interesting to look for necessary and sufficient conditions for the existence (and uniqueness) of phonemic bases. It might happen that such conditions would be easier to check in practice than the postulates proposed in the systems discussed above. The machinery of algebraic linguistics and that of the theory of information systems seem to be useful in this respect.

Finally, let us pay some attention to the role of Batóg's proposals in contemporary theoretical phonology. In our opinion, at least three things should be stressed:

1. Of all the formal approaches in phonology which are known to us, the one suggested by Batóg is the most elaborate and magnificent from the logical point of view. The Author has not limited himself to a few of formal definitions — his goal from the very beginning was the construction of a whole system of axiomatic phonology. One important virtue of the construction of the concept of a phonemic basis is that it can be modified in order to embrace several approaches in modern phonology.

2. The application of the apparatus of extended mereology has appeared very fruitful in the description of sound systems. It is Tadeusz Batóg who introduced this machinery into linguistics.

3. Batóg's approach to phonology may be recommended as a pattern to be followed in other domains of linguistic science. In particular, the idea of a *parametrical* description of segments (segments as characterized by features of different kinds — parameters of the description) can be applied to units from several levels of language (e.g. lexical, morphological, syntactic).

References

Batóg, T. (1961). Logiczna rekonstrukcja pojęcia fonemu (A logical reconstruction of the concept of a phoneme). *Studia Logica* **11**, 139–183.

Batóg, T. (1961a). Fragmentaryczny system fonologii aksjomatycznej (A partial system of axiomatic phonology). *Sprawozdania z Prac Naukowych Wydziału Nauk Społecznych PAN*, Rok IV, z. **4** (21), 62–63.

Batóg, T. (1962). A contribution to axiomatic phonology. *Studia Logica* **13**, 67–80.

Batóg, T. (1967). *The Axiomatic Method in Phonology.* London: Routledge and Kegan Paul.

Batóg, T. (1969). A reduction in the number of primitive concepts of phonology. *Studia Logica* **27**, 55–60.

Batóg, T. (1971). A formal approach to the semantic theory of phoneme. *Studia Logica* **29**, 27–42.

Batóg, T. (1971a). On the definition of phonemic basis. *Studia Logica* **27**, 117–122.

Batóg, T. (1976). O klasycznym pojęciu bazy fonematycznej (On the classical concept of a phonemic basis). Poznań: *Komunikaty i Rozprawy*, Institute of Mathematics, Adam Mickiewicz University.

Batóg, T. (1978). Pojęcie systemu fonologicznego (The concept of a phonological system). *Sprawozdania PTPN, Wydział Filologiczno-Filozoficzny*, Nr **94** za 1976r., 45–49.

Batóg, T. (1978a). On the classical concept of phonemic basis. *Lingua Posnaniensis* **21**, 53–64.

Batóg, T. (1994). *Studies in axiomatic foundations of phonology.* Poznań: Wydawnictwo Naukowe UAM.

Bloch, B. (1948). A Set of Postulates for Phonemic Analysis. *Language* **24**, 3–46.

Jassem, W. (1954). Węzłowe zagadnienia fonematyki (The Main Problems of Phonemics). *Bulletin de la Société Polonaise de Linguistique* **XV**, 13–30.

Kortlandt, F.H.H. (1972). *Modelling the Phoneme.* The Hague-Paris: Mouton.

Leśniewski, S. (1916). *Podstawy ogólnej teorii mnogości I (Foundations of the general theory of sets I).* Moscow: Prace Polskiego Koła Naukowego w Moskwie. Sekcya matematyczno-przyrodnicza, no. **2**.

Pike, K.L. (1947). *Phonemics: A Technique for Reducing Languages to Writing.* Ann Arbor.

Woodger, J.H. (1937). *The Axiomatic Method in Biology.* Cambridge.

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