Modality and Constitution in Distinctively Mathematical Explanations
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Forthcoming in the *European Journal for Philosophy of Science*

Abstract. Lange (2013; 2016) argues that some natural phenomena can be explained by appeal to mathematical, rather than natural, facts. In these "distinctively mathematical" explanations (DMEs), the core explanatory facts are either modally stronger than facts about ordinary causal law or understood to be constitutive of the physical task or arrangement at issue. Craver and Povich (2017) argue that Lange's account of DME fails to exclude certain "reversals". Lange (forthcoming) has replied that his account can avoid these directionality charges. Specifically, Lange argues that in legitimate DMEs, but not in their "reversals," the empirical fact appealed to in the explanation is "understood to be constitutive of the physical task or arrangement at issue" in the explanandum. I argue that Lange's reply is unsatisfactory because it leaves the crucial notion of being "*understood* to be constitutive of the physical task or arrangement" obscure in ways that fail to block "reversals" except by an apparent ad hoc stipulation or by abandoning the reliance on understanding and instead accepting a strong realism about essence.

Keywords: scientific explanation; distinctively mathematical explanation; directionality; explanatory asymmetry; ontic conception; modal conception.

1. Introduction

Lange (2013; 2016) argues that some natural phenomena can be explained by appeal to mathematical, rather than natural, facts. In these "distinctively mathematical" explanations (DMEs), the core explanatory facts are either modally stronger than facts about ordinary causal law or understood to be constitutive of the physical task or arrangement at issue.[1] Craver and Povich (2017) argue that

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[1] Obviously, these are necessary, but not sufficient, conditions. Presumably, there should also be something like a relevance requirement, since if you take any of Lange's examples and add the premise that 2+2=4, you still have an argument whose premises meet these conditions (see Lange 2016: 136). But note also that according to Lange (2013: 507), "there is no criterion that sharply distinguishes the distinctively mathematical explanations from among the non-causal explanations appealing to some mathematical facts. Rather, it is a matter of degree and of context. Insofar as mathematical facts alone are emphasized as doing the explaining, the explanation is properly characterized as distinctively mathematical". In this paper, I

Lange's account of DME fails to exclude certain "reversals". Lange (forthcoming) has replied that his account can avoid these directionality charges. Specifically, Lange argues that in legitimate DMEs, but not in their "reversals," the empirical fact appealed to in the explanation is "understood to be constitutive of the physical task or arrangement at issue" in the explanandum.

I argue that Lange's reply is unsatisfactory because it leaves the crucial notion of being "*understood* to be constitutive of the physical task or arrangement" obscure in ways that fail to block "reversals" except by an apparent ad hoc stipulation or by abandoning the reliance on understanding and instead accepting a strong realism about essence. In Section 2, I briefly review the directionality objection and Lange's (forthcoming) reply. In Section 3, I argue that the notion of "constitution" to which Lange appeals cannot exclude Craver and Povich's "reversals" as DMEs, on pain of abandoning a purely modal conception of DME. In Section 4, I present some non-explanatory "reversals" whose empirical facts are plausibly "understood to be constitutive of the physical task or arrangement," thus meeting Lange's criteria for DME.

2. Directionality and Lange's Reply

Two of Lange's paradigm DMEs are the cases of Königsberg's bridges and Terry's trefoil knot. In the case of Königsberg's bridges, the explanandum is the fact that Marta failed[2] to walk a path across Königsberg's bridges in 1735, crossing each of its bridges exactly once (i.e., an Eulerian walk). The explanantia are the empirical fact that Königsberg's bridges then formed a connected network with four

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am concerned with those explanations that Lange takes to be clearly distinctively mathematical (rather than in between), where the mathematical facts alone are emphasized. See Povich and Craver (2018) for a discussion of some of Lange's other "explanations by constraint" and Craver and Povich (2017: 35 fn. 10) for a brief discussion of "really statistical explanations". I thank an anonymous review for helping me be clear about this.

[2] Or that Marta will always fail or that anyone will always fail; similarly, for the other examples. An anonymous reviewer has the intuition that when the explanandum is characterized this way, the case is not a DME. It might have been that Marta (or everyone on earth) failed to try all the possible paths and that this would make the mathematical fact irrelevant (see also Barrantes forthcoming). I see the force of that intuition, but I ultimately resist it because of the stronger, competing intuition that if something other than the mathematical fact explained a given failure to cross the bridges (or the fact that everyone has failed, or whatever) – not trying all paths, say - that would imply that trying all paths would have led to success, which is not the case. For the purposes of this paper, I will assume that the explanandum as characterized by Lange, Craver, and Povich does have a DME.

nodes (landmasses), three of which had three edges (bridges) and one of which had five, and the mathematical fact that only networks that contain either zero or two nodes with an odd number of edges permit an Eulerian walk (Lange 2013: 489). These facts make it impossible for Marta to succeed, and, so, inevitable that she will fail. In this consists the characteristic modal strength of DMEs, which sets them apart from causal explanations.

In the case of Terry's trefoil knot, the explanandum is the fact that Terry failed to untie his shoelace. The explanantia are the empirical fact that Terry has a trefoil knot in his shoelace and the mathematical fact that the trefoil knot is distinct from the unknot. These facts, according to Lange, make it impossible for Terry to succeed, and, so, inevitable that he will fail. Importantly, in both these cases, the mathematical facts are modally more necessary than causal laws and the empirical facts are understood to be constitutive of the physical task or arrangement at issue (i.e., the bridges and the knot, respectively).

Craver and Povich's (2017) argument is premised on the fact that each of these DMEs (as well as Lange's other examples) can be "reversed" to yield an argument of the same form that appeals to the same mathematical facts but is not an acceptable explanation. In "reversed" Königsberg's bridges, the explanandum is the fact that Königsberg's bridges in 1735 did not form a network with a certain topology, viz., with four nodes, three of which had three edges and one of which had five (see Craver 2016). The explanantia are the empirical fact that Marta completed an Eulerian walk through the bridges and the mathematical fact that only networks that contain either zero or two nodes with an odd number of edges permit Eulerian walks. In "reversed" trefoil knot, the explanandum is the fact that Terry does not have a trefoil knot in his shoelace. The explanantia are the empirical fact that Terry untied his shoelace and the mathematical fact that the trefoil knot is distinct from the unknot. Craver and Povich argue that these "reversals" fit the criteria for DME: the mathematical facts are modally more necessary than causal laws and Craver and Povich stipulate that the empirical facts are

understood to be constitutive of the physical task or arrangement at issue (so that, for example, it becomes constitutive of the knot in Terry's shoelace that he untied it).

In reply, Lange (forthcoming: 3) argues that,

> By making the fact that Terry untied the shoelace a presupposition of the why question, we have not made Terry's having untied the shoelace understood as part of what constitutes the physical task or arrangement at issue. Terry's untying the knot does not help to make the knot trefoil (or nontrefoil). As an analogy, suppose I ask you to bring me some salt. If I add that salt is my favorite spice, this does not change what it takes to bring me some salt. What is understood as constituting salt does not change when I add that it is my favorite spice.

It is plausible that Craver and Povich (2017: 36) are right that presupposing the fact that Terry untied his shoelace makes that fact part of what constitutes *the explanandum* – it helps make the explanandum, the fact to be explained, the fact that it is. It would not be the same explanandum-fact had Terry not untied his shoelace[3]. What is less clear, though, is whether presupposing this fact makes Terry's having untied his shoelace part of *what constitutes the physical task or arrangement at issue*. Lange thinks not, as the quotation makes clear. Let us call the former idea, "explanandum-constitution," and the latter, "task-constitution"; this is the distinction toward which Craver and Povich appear to be gesturing in Section 4 of their paper and toward which Lange appears to be gesturing in the passage above. Lange seems to be arguing that DMEs require task-constitution – only presuppositions that partly constitute the physical task or arrangement at issue, not the explanandum-fact, result in a DME. In other words, task-constitution enforces the required directionality (or accounts for the directionality intuition) of DMEs – the "forward cases" rely on task-constitution, while their "reversals" do not.

Note that, for Lange, whether the empirical facts are understood to be constitutive of the physical task or arrangement at issue must be a relatively objective or non-psychological matter. Lange's use of the word "understood" makes it seem like this is at least partly a psychological matter. If

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[3] Prima facie, the fact that Terry does not have a trefoil knot in his shoelace is different from the fact that Terry does not have a trefoil knot in the shoelace he untied.

it were not partly non-psychological, then what is understood to be constitutive of the physical task or arrangement at issue would be up to those requesting an explanation. There would be nothing to exclude Craver and Povich's "reversals" as DMEs; they could simply stipulate that in some context the person requesting an explanation understands the empirical facts in their "reversals" to be constitutive of the physical task or arrangement at issue. Indeed, Craver and Povich apparently understood them this way themselves (see footnote 8). We return to this point below.

3. Explanandum-constitution

Recall that Lange seemed originally to intend DMEs to conform to a modal conception of explanation that denies any ontic requirements on explanation (Lange 2013; 509–10). Indeed, part of Lange's motivation, at least in his 2013 paper (Ibid.),[4] was arguably to elevate the "modal" conception of explanation from its neglected status in contemporary philosophy relative to the epistemic and ontic conceptions (Salmon 1989).

Craver and Povich's (2017) objections were designed to show that a complete account of DMEs cannot dispense with ontic constraints. In this section, I explain why explanandum-constitution must also result in a DME, if DME is understood in the purely modal sense in which Lange (2013; 509–10) seemed originally to intend it.

According to Lange (2013; 2016), DMEs are distinctive because they reveal their explananda to be mathematically necessary or impossible. The explanantia of a DME make its explanandum *mathematically inevitable* (Lange 2013: 487) or *mathematically necessary* (Lange 2013: 488) or make the non-obtaining of its explanandum *mathematically impossible* (Lange 2013: 496)[5]. This is the source

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[4] Lange's (2016, especially Chapter 3) later account of "explanations by constraint" (of which DMEs are one variety) goes far beyond any traditional modal conception, but I do not have the space here to give his full account the attention it deserves. Here I am only concerned with the extent to which ontic, non-modal elements must be considered in an account of the explanatory power of DMEs. See Section 4 below for an argument that there are still "reversals" even if Lange is right that explanandum-constitution does not result in a DME.

[5] There are similar statements in the book (Lange 2016: 30, 37, 38). I find these claims hard to square with Lange's other claim that the explananda of DMEs need not be necessary (2016: 131).

of the distinctive explanatory power of DMEs, according to Lange. The fact that Terry has a trefoil knot in his shoelace, together with the mathematical fact that the trefoil is distinct from the unknot, *makes mathematically inevitable* or *makes mathematically necessary* that he will fail to untie it or *makes mathematically impossible* that he will untie it. Furthermore, Terry's failure to untie his shoelace, even together with the relevant mathematical fact, does not necessitate that he has a trefoil knot in it, so the exact reversal is not a DME. This might make it seem as if Lange's account does not face directionality problems.

However, all of the purely modal relations that hold between the explanantia and the explanandum of a DME hold in Craver and Povich's (2017) "reversals" as well. On purely modal readings, "x makes y necessary," means "necessarily, if x, then y": every possible world in which x obtains is a world in which y obtains. Likewise, "x makes y impossible" means "necessarily, if x, then ~y": every possible world in which x obtains is a world in which y does not obtain.[6] So, the fact that Terry untied his shoelace, together with the mathematical fact that the trefoil knot is distinct from the unknot, *makes mathematically inevitable* or *makes mathematically necessary* that he did not have a trefoil knot in it or *makes mathematically impossible* that he had a trefoil knot in it. This is because every possible world in which Terry has a trefoil knot in his shoelace is a world where he fails to untie it, and every possible world in which Terry untied the knot in his shoelace is a world where it was not a trefoil knot. Therefore, if Terry's untying his shoelace does *not* help to make necessary that he did not have a trefoil knot in it or to make impossible that he did have a trefoil knot in it, as Lange claims, then Lange must not be working with purely modal readings of "to make necessary" and "to make impossible". Notice, too, that it is precisely the presupposition that Terry untied his shoelace that is what helps to make necessary that he did not have a trefoil knot in it or to make impossible that he had a trefoil knot in it; had Craver and Povich presupposed instead something irrelevant, like that the trefoil

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[6] X makes y impossible if and only if x makes ~y necessary.

knot is his favorite knot (on analogy with Lange's response quoted above), that presupposition would not have helped to make necessary that he did not have a trefoil knot in it or to make impossible that he did have a trefoil knot in it, since it is not true that every possible world in which the trefoil knot is Terry's favorite knot is a world where he did not have a trefoil knot in his shoelace. Therefore, explanandum-constitution also results in a DME, when DMEs are understood in the purely modal sense Lange seemed originally to defend. Lange's (forthcoming) reply does not address this central point.

For many purposes, however, purely modal notions of necessitation and "making impossible" are inadequate (e.g., Schaffer 2010). On the purely modal reading of the former, for example, every contingent fact (and every necessary fact too) necessitates every necessary fact (e.g., the fact that my couch is red necessitates the fact that $2 + 2 = 4$) and, for the latter, every contingent fact (and every necessary fact too) makes impossible every impossible fact (e.g., the fact that my couch is red makes impossible the fact that $2 + 2 = 5$). A possible substitute for the purely modal notion of necessitation is grounding (e.g., the fact that my couch is red does not ground the fact that $2 + 2 = 4$). Perhaps the explanans-facts of a DME, but not its "reversal," ground (at least partially) their explanandum. However, if Lange appeals to a notion that is not purely modal, such as grounding, then he will be conceding Craver and Povich's (2017) main point that some ontologically substantial relation is required to account for the directionality of DMEs.

Lange (forthcoming: 3) might be right that he does not need a general account of what is and what is not constitutive of the physical task or arrangement at issue, and the distinction between task-constitution and explanandum-constitution might effectively sort cases into those that intuitively are DMEs and those that are not, but without an account of *why* only task-constitution results in a DME, the distinction remains ad hoc. Let us follow Lange, though, in taking only task-constitution to result in a DME. Even then, "reversals" can still be constructed.

4. Task-constitution

It turns out that Craver and Povich's (2017) "reversals" do not rely on task-constitution only because their explananda are characterized in a certain way. Each of their "reversals" can easily be rewritten as examples that rely on task-constitution. Before showing how, note that two of Craver and Povich's (2017) "reversals" that Lange does not discuss arguably rely on task-constitution: "reversed" Patty's pendulum and "reversed" chopsticks. In "reversed" Patty's pendulum, the explanandum is the fact that Patty's pendulum is not a double pendulum and the explanantia are the empirical fact that Patty's pendulum does not have at least four equilibrium configurations and the mathematical fact that the double pendulum's configuration space is a torus with at least four stationary points[7]. In this "reversal," presupposing the empirical fact plausibly makes it understood to be constitutive of the arrangement (i.e., the pendulum) in question.

In "reversed" chopsticks, the explanandum is the fact that the chopsticks were tossed non-randomly and the explanantia are the empirical fact that more of the tossed chopsticks were oriented vertically than horizontally and the mathematical fact that there are more ways for a chopstick to be horizontal than to be vertical. In this "reversal," presupposing the empirical fact plausibly makes it understood to be constitutive of the arrangement (i.e., this collection of tossed chopsticks) in question. So, even if the distinction between explanandum-constitution and task-constitution is part of Lange's explanation for why the Königsberg's bridges and trefoil knot "reversals" are not DMEs, this cannot be the reason why the chopsticks and Patty's pendulum "reversals" are not DMEs.

To return to the main point: the reason that some of Craver and Povich's (2017) "reversals" rely on explanandum-constitution and others rely on task-constitution has to do with the nature of their empirical explanantia. Many of the explananda in Lange's examples are event- or action-oriented: they describe the (necessary) non-occurrence of an event or action. In Craver and Povich's (2017)

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[7] If this seems like a definitional explanation, rather than an explanation of a natural fact, consider making the explanandum the fact that Patty's pendulum does not have a flexible joint in it. The same explanation applies. Thanks to Mark Alford for this suggestion.

"reversals," negations of Lange's explananda become the empirical explanantia. Then, when they presuppose those empirical explanantia, they presuppose that someone succeeds in some action (e.g., untying a knot or crossing some bridges). The result of this presupposition is not task-constitution: it is not plausible that any of the relevant actions partly constitute a task or arrangement at issue. However, the explananda in Lange's examples of chopsticks and Patty's pendulum do not consist of the (necessary) non-occurrence of an event or action; they consist of some object's, or collection's, having some property. Thus, when Craver and Povich "reverse" those examples and presuppose their empirical explanantia, the result is task-constitution.

Therefore, we can make "reversals" that rely on task-constitution out of all Craver and Povich's (2017) examples by revising the explananda slightly. Thus, consider a revision to the "reversal" of Terry's trefoil knot. The explanandum is still the fact that Terry does not have a trefoil knot in his shoelace. The explanantia are the now-revised empirical fact that the knot in Terry's shoelace is isotopic to the unknot and the mathematical fact that the trefoil knot is distinct from the unknot. This "reversal" plausibly fits the criteria for DME: its mathematical fact is modally more necessary than ordinary causal law and presupposing the empirical fact makes it understood to be constitutive of the arrangement (i.e., the knot) in question.

Next, consider revising the explanandum in the case of Königsberg's bridges to the fact that Königsberg's bridges did not permit an Eulerian walk. In the "reversal" of this revision, the explanandum is the fact that Königsberg's bridges did not form a connected network with four nodes (landmasses), three of which had three edges (bridges) and one of which had five. The explanantia are the empirical fact that Königsberg's bridges formed a connected network that permitted an Eulerian walk and the mathematical fact that only networks that contain either zero or two nodes with an odd number of edges permit an Eulerian walk. Presupposing the empirical fact plausibly makes it understood to be constitutive of the arrangement (i.e., the bridges) in question.

Lange could respond that a knot's pattern of overs and unders is constitutive of it and that a network's pattern of nodes and edges is constitutive of it. In contrast, the isotopy of a knot to the unknot follows logically from its pattern of overs and unders, but it is not constitutive of the knot (Lange, personal communication),[8] and whether a network permits an Eulerian walk follows logically from its pattern of nodes and edges, but it is not constitutive of it. This might[9] be true when the knot is considered as a topological individual or when the network is considered as a "graphical individual". However, Lange acknowledges that there are contexts where a knot is not considered as a topological individual (forthcoming 3) (and presumably he would say there are contexts where a network is not considered as a graphical individual). For example, when considered as a historical individual, a knot's pattern of overs and unders is not constitutive of it (Ibid.). There seems nothing to prevent Craver and Povich, then, from stipulating contexts in which a knot's isotopy to the unknot is constitutive of it and a network's permitting an Eulerian walk is constitutive of it. As what kinds of individual would the knot and the network be considered in such a context? I know of no name for these kinds of individuals, since contexts are simply stipulated for them – call them an "isotopic individual," the kind of individual such that when a knot is considered as one, its isotopy to the unknot is constitutive of it and an "Eulerian individual," the kind of individual such that when a network is considered as one, its permitting an Eulerian walk is constitutive of it. Practicing knot and graph theorists may not find conceiving of their subject matter this way useful, but that does not show that such a conception is impossible, nor does Lange's account currently prevent such a conception from occurring in an explanatory context.

It would appear, therefore, that Lange's response thus far is only palliative. Even if Lange can

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[8] This is what Lange (forthcoming) has in mind when he says, "Terry's untying the knot does not help to make the knot trefoil (or non-trefoil)" (3). Obviously, it was possible from the beginning to argue that in Craver and Povich's (2017) original "reversals" there was task-constitution as well; that they have stipulated a context in which the knot is considered as an individual such that Terry's untying the knot helps to make it the kind of individual it is. However, this context (and this kind of individual) seems far more artificial than the one stipulated above, thus making the case far less plausible.

[9] This is arguably false on the standard modal account of essential properties. See below.

mount a non-ad hoc argument that only task-constitution results in a DME, he will need to appeal to something else to exclude these revised "reversals" as DMEs, as well as the Patty's pendulum and chopsticks "reversals," which arguably already satisfy his criterion, whatever it should turn out to be.

One route for Lange to exclude these "reversals" is to exclude the kinds of individual to which they appeal (such as the "isotopic individual" and the "Eulerian individual"), because they are not *real* kinds of individual. There are at least two problems with this strategy. First, it presupposes a strong realism about individuals and their essential properties (what Sidelle [1992] calls a "privileged ontology" with objective identity conditions) and an account of essential properties that does not follow the standard modal account. For, on that account, a knot's isotopy to the unknot is essential to it, considered as a topological individual, since there is no possible world where it exists as the topological individual that it is and is not isotopic to the unknot (similarly for the network). Second, this move takes much of the psychological element out of what is *understood* to be constitutive of the task or arrangement at issue, which plays a large role in Lange's account. This understanding determines as what kind of individual a task or arrangement is considered and, thus, whether the explanation proffered is distinctively mathematical at all. At the very least, this move places heavy constraints on what can legitimately be so understood.

Finally, even if Lange can address my previous points, it seems we can simply modify the examples again. In the knot case, let us replace the empirical explanans with the fact that Terry has a Thistlethwaite knot in his shoelace. A Thistlethwaite knot is isotopic to the unknot. The explanandum is still the fact that Terry does not have a trefoil knot in his shoelace. Thus, the explanantia are the empirical fact that Terry has a Thistlethwaite knot in his shoelace and the mathematical fact that the trefoil knot is distinct from the unknot. This case seems to fit Lange's account of DME: it appeals to a mathematical fact and the empirical fact is more clearly constitutive of the physical arrangement or task at issue, since a Thistlethwaite knot is a standard topological individual defined by its pattern of overs

and unders. In the Königsberg's case, let us replace the empirical explanans with the fact Königsberg's bridges formed a $K_{2,2}$ network, a complete bipartite graph that permits an Eulerian walk[10]. The explanandum is still the fact that Königsberg's bridges did not form a connected network with four nodes (landmasses), three of which had three edges (bridges) and one of which had five. Thus, the explanantia are the empirical fact that Königsberg's bridges formed a $K_{2,2}$ network and the mathematical fact that only networks that contain either zero or two nodes with an odd number of edges permit an Eulerian walk. This case also seems to fit Lange's account of DME: it appeals to a mathematical fact and the empirical fact is more clearly constitutive of the physical arrangement or task at issue, since a $K_{2,2}$ network is a standard graphical individual defined by its pattern of nodes and edges. These cases fit Lange's account, and they do not appeal to any contrived, stipulated individuals to which Lange might object.[11]

5. Conclusion

To summarize: for an argument to be a DME, according to Lange (2013; 2016), each of its premises must be either modally more necessary than ordinary causal laws (as the mathematical premises are) or understood to be constitutive of the physical task or arrangement at issue (as the empirical facts are). The mathematical premises of all Craver and Povich's (2017) "reversals" meet the former condition. The debate is thus over (i) whether the empirical facts in their "reversals" meet the second condition or whether they are only understood to be constitutive of their *explananda* and (ii) whether arguments whose empirical facts are only understood to be constitutive of their *explananda* are DMEs.

In this reply, I argued that explanations whose empirical facts are understood to be constitutive

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[10] $K_{2,2}$ consists of two sets of two nodes, where each node of the first set is connected to each node of the second.
[11] Perhaps you think that these arguments require additional premises – that the Thistlethwaite knot is isotopic to the unknot and that $K_{2,2}$ permits an Eulerian walk, respectively. Even if that's true, there is nothing in Lange's account that implies DMEs can only have one mathematical premise. In fact, some of his other examples seem to have more than one mathematical premise, such as the pendulum case (Lange 2013).

of their explananda, but not the task or arrangement at issue, must be DMEs, on pain of abandoning their purely modal character. But I also presented some new (and old) "reversals" and argued that their empirical facts can be understood to be constitutive of the physical tasks or arrangements at issue. I have shown that a slight revision of the explanandum-statement in each case yields an example that sidesteps Lange's response. While Lange's examples explain why I know or should believe that p, they do not explain why p. My hope is that my arguments go some way toward tilting the scales away from a modal conception of DME and toward something like an ontic conception or counterfactual account of DME (e.g., Povich forthcoming, under review; Reutlinger 2016).

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