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Interval Neutrosophic Multi-Attribute Decision-Making Based on Grey Relational Analysis

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Abstract. The purpose of this paper is to introduce multi-attribute decision making based on the concept of interval neutrosophic sets. While the concept of neutrosophic sets is a powerful tool to deal with indeterminate and inconsistent data, the interval neutrosophic set is also a powerful mathematical tool as well as more flexible to deal with incompleteness. The rating of all alternatives is expressed in terms of interval neutrosophic values characterized by interval truth-membership degree, interval indeterminacy-membership degree, and interval falsity-membership degree. Weight of each attribute is partially known to the decision maker. The authors have extended the single valued neutrosophic grey relational analysis method to interval neutrosophic environment and applied it to multi-attribute decision making problem. Information entropy method is used to obtain the unknown attribute weights. Accumulated arithmetic operator is defined to transform interval neutrosophic set into single value neutrosophic set. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal interval neutrosophic estimates reliability solution and the ideal interval neutrosophic estimates unreliability solution. Then interval neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, an example is provided to illustrate the applicability and effectiveness of the proposed approach.

Keywords: Accumulated arithmetic operator**,** Grey relational analysis, Ideal interval neutrosophic estimates reliability solution, Information entropy, Interval neutrosophic set, Multi-attribute decision making, Neutrosophic set, Single-valued neutrosophic set.

1. Introduction

The concept of neutrosophic sets was introduced by Smarandache [1, 2, 3, 4]. The root of neutrosophic set is the neutrosophy, a new branch of philosophy [1]. The thrust of the neutrosophy creates new field of study such as neutrosophic statistics [5], neutrosophic integral [6], neutrosophic cognitive map [7], etc. The concept of neutrosophic set has been successful in penetrating different branches of sciences [8], social sciences [9, 10, 11], education [12], conflict resoltion [13, 14], philosophy [15], artificial intelligence and control systems [16], etc. Neutrosophic set has drawn the great attention of the researchers for its capability of handling uncertainty, indeterminacy and incomplete information.

Zadeh [17] proposed the degree of membership in 1965 and defined the fuzzy set. Atanassov [18] proposed the degree of non-membership in 1986 and defined the intuitionistic fuzzy set. Smarandache [1] proposed the degree of indeterminacy as independent component and defined the neutrosophic set.

To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[19] restricted the concept of neutrosophic set to single valued neutrosophic set since single value is an instance of set value. Neutrosophic set and its various extensions have been studied and applied in different fields such as medical diagnosis [20, 21, 22, 23, 24], decision making [25, 26, 27, 28, 29, 30, 31], decision making in hybrid system [32, 33, 34, 35, 36], image processing [37, 38, 39, 40, 41, 42], etc.

However, Zhang et al. [43] opinioned that in many real world problems, the decision information may be suitably presented by interval form instead of real numbers. In order to deal with the situation, Wang et al.[44] introduced the concept of interval neutrosophic set (INS) characterized by a membership function, non-membership function and an indeterminacy function, whose values are interval forms.

Broumi and Smarandache [45] studied correlation coefficient of interval neutrosophic sets and applied it in medical diagnosis. Broumi and Smarandache [46] studied cosine similarity measure in interval neutrosophic environment. Zhang et al. [43] studied interval neutrosophic sets and its application in multi attribute decision making. Ye [47] studied similarity measures between interval neutrosophic sets and their applications in multicriteria decision making. Ye [48] proposed improved correlation coefficient of SVNS and studied some of its properties, and then extended it to a correlation coefficient between INS. Chi and Liu [49] developed the order performance technique based on similarity to ideal solution (TOPSIS) method for multiple attribute decision making problems based on interval neutrosophic set.

Grey relational analysis (GRA) studied by Deng [50, 51] is widely used for multi-attribute decision making problems.

Zhang et al. 2005 [52] presented GRA method for multi attribute decision-making with interval numbers. Rao & Singh [53] proposed improved GRA method by integrating analytic hierarchy process and fuzzy logic Wei [54] studied the GRA method for intuitionistic fuzzy multi-criteria decision-making. Pramanik and Mukhopadhyaya [55] presented GRA based intuitionistic fuzzy multi-criteria group decision-making approach for teacher selection in higher education. Biswas et al. [56] proposed entropy based GRA method for multi-attribute decision making under single valued neutrosophic assessments. Biswas et al. [57] also studied GRA based neutrosophic multi-attribute decision-making (MADM) with unknown weight information Mondal and Pramanik [58] applied GRA based neutrosophic decision making model of school choice.

GRA based MADM in interval neutrosophic environment is yet to appear in the literature. In this paper, we present interval neutrosophic multi attribute decision making based on GRA.

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and interval neutrosophic sets. Section 3 is devoted to present GRA method for multi attribute decision-making in interval neutrosophic environment. Section 4 presents a numerical example of the proposed method. Finally section 5 presents concluding remarks.

2 Mathematical preliminaries

2.1 Definitions on neutrosophic Set [1]

Definition 2.1.1: Let E be a space of points (objects) with generic element in E denoted by x. Then a neutrosophic set P in E is characterized by a truth membership function $T_P(x)$, an indeterminacy membership function $I_P(x)$ and a falsity membership function $F_P(x)$. The functions $T_P(x)$, $I_P(x)$ and $F_P(x)$ are real standard or non-standard subsets of $]^-0,1^+[$ that is $T_P(x) : E \to]^-0,1^+[$; $I_P(x): E \to]^-0,1^+[$; $F_P(x)$: $E \to]^-0,1^+[$.

The sum of $T_P(x)$, $I_P(x)$, $F_P(x)$ satisfies the relation

$$^-0 \le \sup T_P(x) + \sup I_P(x) + \sup F_P(x) \le 3^+$$

Definition 2.1.2 (complement) [1]

The complement of a neutrosophic set P is denoted by P^c and is defined as follows:

$T_{P^c}(x) = \{1^+\} - T_P(x)$; $I_{P^c}(x) = \{1^+\} - I_P(x)$, and $F_{P^c}(x) = \{1^+\} - F_P(x)$

Definition 2.1.3 (Containment) [1]

A neutrosophic set P is contained in the other neutrosophic set Q, $P \subseteq Q$ if and only if the following result holds.

$$\inf T_P(x) \le \inf T_Q(x),\ \sup T_P(x) \le \sup T_Q(x)$$
$$\inf I_P(x) \ge \inf I_Q(x),\ \sup I_P(x) \ge \sup I_Q(x)$$
$$\inf F_P(x) \ge \inf F_Q(x),\ \sup F_P(x) \ge \sup F_Q(x)$$

for all x in E.

Definition 2.1.4 (Single-valued neutrosophic set) [19]

Let E be a universal space of points (objects) with a generic element of E denoted by x.

A single valued neutrosophic set [Wang et al. 2010] S is characterized by a truth membership function $T_S(x)$, a falsity membership function $F_S(x)$ and indeterminacy function $I_S(x)$ with $T_S(x)$, $F_S(x)$, $I_S(x) \in [0,1]$ for all x in E. ,

When E is continuous, a SNVS S can be written as follows:

$$S = \int_x \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$$

and when E is discrete, a SVNS S can be written as follows:

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \forall x \in E$$

It should be observed that for a SVNS S,

$$0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3,\ \forall x \in E$$

Definition 2.1.5: The complement of a single valued neutrosophic set S [19] is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x);\ I_S{}^c(x) = 1 - I_S(x);\ F_S{}^c(x) = T_S(x)$$

Definition 2.1.6: A SVNS S_P [19] is contained in the other SVNS S_Q, denoted as $S_P \subseteq S_Q$ iff, $T_{S_P}(x) \le T_{S_Q}(x)$; $I_{S_P}(x) \ge I_{S_Q}(x)$; $F_{S_P}(x) \ge F_{S_Q}(x)$, $\forall x \in E$.

Definition 2.1.7: Two single valued neutrosophic sets S_P and S_Q [19] are equal, i.e. $S_P = S_Q$, iff, $S_P \subseteq S_Q$ and $S_P \supseteq S_Q$.

Definition 2.1.8: (Union) The union of two SVNSs S_P and S_Q [19] is a SVNS S_R, written as $S_R = S_P \cup S_Q$ .Its truth membership, indeterminacy-membership and falsity membership functions are related to S_P and S_Q by the relations as follows:

$$T_{S_R}(x) = \max\left(T_{S_P}(x), T_{S_Q}(x)\right);$$
$$I_{S_R}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right);$$
$$F_{S_R}(x) = \min\left(F_{S_P}(x), F_{S_Q}(x)\right) \text{ for all } x \text{ in } E$$

Definition 2.1.9 (Intersection) [19]

The intersection of two SVNSs P and Q is a SVNS V, written as $V = P \cap Q$. Its truth membership, indeterminacy membership and falsity membership functions are related to P an Q by the relations as follows:

$T_{S_V}(x) = \min\left(T_{S_P}(x), T_{S_Q}(x)\right)$;

$I_{S_V}(x) = \max\left(I_{S_P}(x), I_{S_Q}(x)\right)$;

$F_{S_V}(x) = \max\left(F_{S_P}(x), F_{S_Q}(x)\right), \forall x \in E$

Distance between two neutrosophic sets.

The SVNS can be presented in the following form:

$S = \{(x/(T_S(x), I_S(x), F_S(x))) : x \in E\}$

Finite SVNSs can be represented as follows:

$$S = \begin{Bmatrix} (x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \cdots, \\ (x_m/(T_S(x_m), I_S(x_m), F_S(x_m))) \end{Bmatrix}, \forall x \in E \quad (1)$$

Definition 2.1.10: Let

$$S_P = \begin{Bmatrix} (x_1/(T_{S_P}(x_1), I_{S_P}(x_1), F_{S_P}(x_1))), \cdots, \\ (x_n/(T_{S_P}(x_n), I_{S_P}(x_n), F_{S_P}(x_n))) \end{Bmatrix} \quad (2)$$

$$S_Q = \begin{Bmatrix} (x_1/(T_{S_Q}(x_1), I_{S_Q}(x_1), F_{S_Q}(x_1))), \cdots, \\ (x_n/(T_{S_Q}(x_n), I_{S_Q}(x_n), F_{S_Q}(x_n))) \end{Bmatrix} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance [59] between two SNVS P and Q is defined as follows:

$$d_S(S_P, S_Q) = \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_P}(x) - T_{S_Q}(x)| + \\ |I_{S_P}(x) - I_{S_Q}(x)| + \\ |F_{S_P}(x) - F_{S_Q}(x)| \end{matrix} \right\rangle \quad (4)$$

and normalized Hamming distance [59] between two SNVS S_P and S_Q is defined as follows:

$$^N d_S(S_P, S_Q) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{S_P}(x) - T_{S_Q}(x)| + \\ |I_{S_P}(x) - I_{S_Q}(x)| + \\ |F_{S_P}(x) - F_{S_Q}(x)| \end{matrix} \right\rangle \quad (5)$$

with the following properties

1. $0 \le d_S(S_P, S_Q) \le 3n$ (6)
2. $0 \le {}^N d_S(S_P, S_Q) \le 3n$ (7)

Definition 2.1.11

Let α and β be the collection of benefit attributes and cost attributes, respectively. R_S^+ is the interval relative neutrosophic positive ideal solution (IRNPIS) and R_S^- is the interval relative neutrosophic negative ideal solution (IRNNIS). $R_S^+ = [r_{S_1}^+, r_{S_2}^+, \cdots, r_{S_n}^+]$ is defined as a solution in which every component $r_{S_j}^+ = \langle T_j^+, I_j^+, F_j^+ \rangle$ is characterized by $T_j^+ = \{(\max_i \{T_{ij}\}) \mid j\text{-}th attribute \in \alpha$, $((\min_i \{T_{ij}\}) \mid j\text{-}th attribute \in \beta\}$

Definition 2.1.12

The interval relative neutrosophic negative ideal solution (IRNNIS) $R_S^- = [r_{S_1}^-, r_{S_2}^-, \cdots, r_{S_n}^-]$ is a solution in which every component $r_{S_j}^- = \langle T_j^-, I_j^-, F_j^- \rangle$ is characterized as follows:

$T_j^- = \{(\min_i \{T_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\max_i \{T_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

$I_j^- = \{(\max_i \{I_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\min_i \{I_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

$F_j^- = \{(\max_i \{F_{ij}\}) \mid j\text{-}th attribute \in \alpha, ((\min_i \{F_{ij}\}) \mid j\text{-}th attribute \in \beta\}$,

in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$ (see equation 8) for i = 1, 2, …, n and j = 1, 2, …, m

2.2 Interval Neutrosophic Sets

Definition 2.2 [44]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) M in X is characterized by truth-membership function $T_M(x)$, indeterminacy-membership $I_M(x)$, function and falsity-membership function $F_M(x)$. For each point x in X, we have, $T_M(x), I_M(x), F_M(x) \in [0, 1]$.

For two IVNS,

$M_{INS} = \{< x, ([T_M^L(x), T_M^U(x)], [I_M^L(x), I_M^U(x)], [F_M^L(x), F_M^U(x)]) > | \; x \in X\}$

and $N_{INS} = \{< x, ([T_N^L(x), T_N^U(x)], [I_N^L(x), I_N^U(x)], [F_N^L(x), F_N^U(x)]) > | \; x \in X\}$, the two relations are defined as follows:

(1) $M_{INS} \subseteq N_{INS}$ if and only if $T_M^L \le T_N^L$, $T_M^U \le T_N^U$; $I_M^L \le I_N^L$, $F_M^L \le F_N^L$; $F_M^L \le F_N^L$, $F_M^L \le F_N^L$

(2) $M_{INS} = N_{INS}$ if and only if $T_M^L = T_N^L$, $T_M^U = T_N^U$; $I_M^L = I_N^L$, $F_M^L = F_N^L$; $F_M^L = F_N^L$, $F_M^L = F_N^L \; \forall x \in X$

3. Grey relational analysis method for multi attributes decision-making in interval neutrosophic environment.

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let $A_1, A_2, ..., A_m$ and $C_1, C_2, ..., C_n$ denote the alternatives and attributes respectively. The rating describes the performance of alternative A_i against attribute C_j. Weight vector $W = \{w_1, w_2, ..., w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of attributes C_j (j = 1, 2, ..., m) to the decision maker. The values associated with the alternatives for MADM problems presented in the following table.

Table1: Interval neutrosophic decision matrix

$D_s = \langle d_{ij} \rangle_{m \times n} =$

	C_1	C_2	$\cdots$	C_n
A_1	$\langle d_{11} \rangle$	$\langle d_{12} \rangle$	...	$\langle d_{1n} \rangle$
A_2	$\langle d_{12} \rangle$	$\langle d_{22} \rangle$	...	$\langle d_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle d_{m1} \rangle$	$\langle d_{m2} \rangle$	...	$\langle d_{mn} \rangle$

(8)

Here $\langle d_{ij} \rangle$ is the interval neutrosophic number related to the *i*-th alternative and the *j*-th attribute.

Grey relational analysis (GRA) is one of the adoptive methods for MADM. The steps of GRA under interval neutrosophic environments are described below.

Step1: Determination the criteria

There are many attributes in decision making problems. Some of them are important and others may be less important. So it is necessary to select the proper criteria for decision making situations. The most important criteria may be fixed with help of experts' opinions.

Step 2: Data pre-processing and construction of the decision matrix with interval neutrosophic form

It may be mentioned here that the original GRA method can deal mainly with quantitative attributes. There exists some complexity in the case of qualitative attributes. In the case of a qualitative attribute (quantitative value is not available), an assessment value is taken as interval neutrosophic environment.

For multiple attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed as interval neutrosophic sets. It can be represented with the following forms:

$$A_i = \left[\begin{array}{l} C_1 \big/ \left\langle N_1\left(\left[T_1^L, T_1^U\right], \left[I_1^L, I_1^U\right], \left[F_1^L, F_1^U\right]\right)\right\rangle, \\ C_2 \big/ \left\langle N_2\left(\left[T_2^L, T_2^U\right], \left[I_2^L, I_2^U\right], \left[F_2^L, F_2^U\right]\right)\right\rangle, \cdots, \\ C_n \big/ \left\langle N_n\left(\left[T_n^L, T_n^U\right], \left[I_n^L, I_n^U\right], \left[F_n^L, F_n^U\right]\right)\right\rangle : C_j \in C \end{array} \right]$$

$$= \left[C_j \big/ \left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle : C_j \in C \right]$$

for j = 1, 2,…, n (9)

Here $\left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle$, (j = 1, 2, ..., n) is the interval neutrosophic set with the degrees of interval truth membership $\left[T_j^L, T_j^U\right]$, the degrees of interval indeterminacy membership $\left[I_j^L, I_j^U\right]$ and the degrees of interval falsity membership $\left[F_j^L, F_j^U\right]$ of the alternative A_i satisfying the attribute C_j

The interval neutrosophic decision matrix can be represented in the following form (see the Table 2):

Table2: Interval neutrosophic decision matrix

$d_N = \left\langle \left[T_{ij}^L, T_{ij}^U\right], \left[I_{ij}^L, I_{ij}^U\right], \left[F_{ij}^L, F_{ij}^U\right] \right\rangle_{m \times n} =$

	C_1	C_2	...	C_n
A_1	$\left\langle \left[T_{11}^L, T_{11}^U\right], \left[I_{11}^L, I_{11}^U\right], \left[F_{11}^L, F_{11}^U\right] \right\rangle$	$\left\langle \left[T_{12}^L, T_{12}^U\right], \left[I_{12}^L, I_{12}^U\right], \left[F_{12}^L, F_{12}^U\right] \right\rangle$	...	$\left\langle \left[T_{1n}^L, T_{1n}^U\right], \left[I_{1n}^L, I_{1n}^U\right], \left[F_{1n}^L, F_{1n}^U\right] \right\rangle$
A_2	$\left\langle \left[T_{21}^L, T_{21}^U\right], \left[I_{21}^L, I_{21}^U\right], \left[F_{21}^L, F_{21}^U\right] \right\rangle$	$\left\langle \left[T_{22}^L, T_{22}^U\right], \left[I_{22}^L, I_{22}^U\right], \left[F_{22}^L, F_{22}^U\right] \right\rangle$	...	$\left\langle \left[T_{2n}^L, T_{2n}^U\right], \left[I_{2n}^L, I_{2n}^U\right], \left[F_{2n}^L, F_{2n}^U\right] \right\rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\left\langle \left[T_{m1}^L, T_{m1}^U\right], \left[I_{m1}^L, I_{m1}^U\right], \left[F_{m1}^L, F_{m1}^U\right] \right\rangle$	$\left\langle \left[T_{m2}^L, T_{m2}^U\right], \left[I_{m2}^L, I_{m2}^U\right], \left[F_{m2}^L, F_{m2}^U\right] \right\rangle$	...	$\left\langle \left[T_{mn}^L, T_{mn}^U\right], \left[I_{mn}^L, I_{mn}^U\right], \left[F_{mn}^L, F_{mn}^U\right] \right\rangle$

(10)

Step 3: Determination of the accumulated arithmetic operator

Let us consider an interval neutrosophic set as $\left\langle N_j\left(\left[T_j^L, T_j^U\right], \left[I_j^L, I_j^U\right], \left[F_j^L, F_j^U\right]\right)\right\rangle$.

We transform the interval neutrosophic number to SVNSs by the following operator. The accumulated arithmetic operator (AAO) is defined as follows:

$$N_{ij}\left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle = N_{ij}\left\langle \left[\frac{T_{ij}^L + T_{ij}^U}{2}\right], \left[\frac{I_{ij}^L + I_{ij}^U}{2}\right], \left[\frac{F_{ij}^L + F_{ij}^U}{2}\right] \right\rangle \qquad (11)$$

The decision matrix is transformed in the form of SVNSs as follows:

Table3: Single valued neutrosophic decision matrix in transformed form

$d_S = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{m \times n} =$

	C_1	C_2	...	C_n
A_1	$\langle T_{11}, I_{11}, F_{11} \rangle$	$\langle T_{12}, I_{12}, F_{12} \rangle$	...	$\langle T_{1n}, I_{1n}, F_{1n} \rangle$
A_2	$\langle T_{21}, I_{21}, F_{21} \rangle$	$\langle T_{22}, I_{22}, F_{22} \rangle$	...	$\langle T_{2n}, I_{2n}, F_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle T_{m1}, I_{m1}, F_{m1} \rangle$	$\langle T_{m2}, I_{m2}, F_{m2} \rangle$	...	$\langle T_{mn}, I_{mn}, F_{mn} \rangle$

(12)

Step 4: Determination of the weights of the criteria

During decision-making process, decision maker may often encounter unknown or partial attribute weights. In many cases, the importance of attributes to the decision maker is not equal. So, it is necessary to determine attribute weight for decision making.

3.1 Method of entropy:

Entropy plays an important role for measuring uncertain information. Majumdar and Samanta [59] developed some similarity and entropy measures for SVNSs. The entropy measure can be used to determine the attributes weights when these are unequal and completely unknown to decision maker.

Now, using AAO operator, we transform all interval neutrosophic numbers to single valued neutrosophic numbers. In this paper for entropy measure of an INS, we consider the following notation:

$$T_{SP}(x_i) = \left[\frac{T_{ij}^L + T_{ij}^U}{2}\right], I_{SP}(x_i) = \left[\frac{I_{ij}^L + I_{ij}^U}{2}\right],$$

$$F_{SP}(x_i) = \left[\frac{F_{ij}^L + F_{ij}^U}{2}\right]$$

We write, $S_P = \langle T_{SP}(x_i), I_{SP}(x_i), F_{SP}(x_i)\rangle$. Then, entropy value is defined as follows:

$$E_i(S_P) = 1 - \frac{1}{n}\sum_{i=1}^{m}(T_{SP}(x_i) + F_{SP}(x_i))\left|I_{SP}(x_i) - I^c{}_{SP}(x_i)\right| \quad (13)$$

Entropy has the following properties:

1. $E_i(S_P) = 0 \Rightarrow S_P$ *is a crisp set and* $I_{S_P}(x_i) = 0$ and $F_{S_P}(x_i) = 0 \ \forall x \in E$.
2. $E_i(S_P) = 1 \Rightarrow \langle T_{SP}(x_1), I_{SP}(x_1), F_{SP}(x_1)\rangle = \langle T_{SP}(x_i), 0.5, F_{SP}(x_i)\rangle \ \forall x \in E$.
3. $E_i(S_P) \geq E_i(S_Q) \Rightarrow$ $(T_{SP}(x_i) + F_{SP}(x_i) \leq (T_{SQ}(x_i) + F_{SQ}(x_i)$ *and* $\left|I_{SP}(x_i) - I^c{}_{SP}(x_i)\right| \leq \left|I_{SQ}(x_i) - I^c{}_{SQ}(x_i)\right|$
4. $E_i(S_P) = E_i(S_{p^c}) \ \forall x \in E$.

In order to obtain the entropy value E_j of the j-th attribute C_j (j = 1, 2,…, n), the equation (13) can be written as follows:

$$E_j = 1 - \frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i) + F_{ij}(x_i))\left|I_{ij}(x_i) - I_{ij}^C(x_i)\right|$$

for i = 1, 2, …, m; j = 1, 2, …, n (14)

It is observed that $E_j \in [0,1]$. Due to Hwang and Yoon [60], the entropy weight of the j-th attribute C_j is presented as follows:

$$W_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)} \quad (15)$$

We have weight vector W = (w_1, w_2,…,w_n)^T of attributes C_j (j = 1, 2, …, n) with $w_j \geq 0$ and $\sum_{i=1}^{n} w_j = 1$.

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS) for interval neutrosophic decision matrix

For an interval neutrosophic decision making matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i satisfying the attribute C_j. The interval neutrosophic estimate reliability solution (see definition 2.1.11, and 2.1.12) can be determined from the concept of SVNS cube [61].

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Grey relational coefficient of each alternative from IINERS is:

$$G_{ij}^+ = \frac{\min\limits_i \min\limits_j \Delta_{ij}^+ + \rho \max\limits_i \max\limits_j \Delta_{ij}^+}{\Delta_{ij}^+ + \rho \max\limits_i \max\limits_j \Delta_{ij}^+}, \text{ where}$$

$$\Delta_{ij}^+ = d\left(q_{S_j}^+, q_{S_{ij}}\right), i = 1, 2, …, m \text{ and } j = 1, 2, …, n \quad (16)$$

Grey relational coefficient of each alternative from IINEURS is:

$$G_{ij}^- = \frac{\min\limits_i \min\limits_j \Delta_{ij}^- + \rho \max\limits_i \max\limits_j \Delta_{ij}^-}{\Delta_{ij}^- + \rho \max\limits_i \max\limits_j \Delta_{ij}^-}, \text{ where}$$

$$\Delta_{ij}^- = d\left(q_{S_{ij}}, q_{S_{ij}}^-\right), i = 1, 2, …, m \text{ and } j = 1, 2, …, n \quad (17)$$

$\rho \in [0,1]$ is the distinguishable coefficient or the identification coefficient. It is used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho = 1$, the comparison environment is unchanged. When $\rho = 0$, the comparison environment disappears. Smaller value of distinguishing coefficient will reflect the large range of grey relational coefficient. Generally, $\rho = 0.5$ is fixed for decision making.

Step 7: Calculation of the interval neutrosophic grey relational coefficient

Calculate the degree of interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS using the following two equations respectively:

$$G_i^+ = \sum_{j=1}^{n} w_j G_{ij}^+ \text{ for } i = 1, 2, …, m \quad (18)$$

$$G_i^- = \sum_{j=1}^{n} w_j G_{ij}^- \text{ for } i = 1, 2, …, m \quad (19)$$

Step 8: Calculation of the interval neutrosophic relative relational degree

Calculate the interval neutrosophic relative relational degree of each alternative from ITFPIS (indeterminacy truthfulness falsity positive ideal solution) with the help of following two equations:

$$R_i = \frac{G_i^+}{G_i^- + G_i^+}, \text{ for i = 1, 2, …,m} \quad (20)$$

Step 9: Rank the alternatives

The ranking order of alternatives can be determined based on the interval relative relational degree. The highest value of R_i reflects the most desirable alternative.

Step 10: End

4. Illustrative examples

In this section, interval neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach.

4.1 Example 1

Consider a decision-making problem adapted from [58] studied by Mondal and Pramanik. Suppose a legal guardian wants to get his/her child admitted to a suitable school for proper basic education. There is a panel with three possible alternatives (schools) to get admitted his/her child: (1) A_1 is a Christian missionary school; (2) A_2 is a Basic English medium school; (3) A_3 is a Bengali medium kindergarten. The proposed decision making method can be arranged in the following steps.

Step 1: Determination the most important criteria

The legal guardian must take a decision based on the following four criteria: (1) C_1 is the distance and transport; (2) C_2 is the cost; (3) C_3 is the staff and curriculum; and (4) C_4 is the administration and other facilities.

Step 2: Data pre-processing and Construction of the decision matrix with interval neutrosophic form

We obtain the following interval neutrosophic decision matrix based on the experts' assessment:

Table4. Decision matrix with interval neutrosophic number

$$d_S = \left\langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \right\rangle_{3\times 4} =$$

	C_1	C_2	C_3	C_4
A_1	⟨[0.6, 0.8], [0.2, 0.4], [0.3, 0.5]⟩	⟨[0.6, 0.8], [0.2, 0.4], [0.1, 0.3]⟩	⟨[0.6, 0.8], [0.1, 0.3], [0.3, 0.5]⟩	⟨[0.7, 0.9], [0.2, 0.4], [0.2, 0.4]⟩
A_2	⟨[0.5, 0.7], [0.3, 0.5], [0.1, 0.3]⟩	⟨[0.7, 0.9], [0.4, 0.6], [0.3, 0.5]⟩	⟨[0.6, 0.8], [0.2, 0.4], [0.1, 0.3]⟩	⟨[0.7, 0.9], [0.3, 0.5], [0.4, 0.6]⟩
A_3	⟨[0.5, 0.7], [0.2, 0.4], [0.4, 0.6]⟩	⟨[0.6, 0.8], [0.5, 0.7], [0.1, 0.3]⟩	⟨[0.6, 0.8], [0.4, 0.6], [0.4, 0.6]⟩	⟨[0.7, 0.9], [0.3, 0.5], [0.3, 0.5]⟩

(21)

Step 3: Determination of the accumulated arithmetic operator (AAO)

Using accumulated arithmetic operator (AAO) from equation (11) we have the decision matrix in SVNS form is presented as follows:

Table5: single valued neutrosophic decision matrix in transformed form

	C_1	C_2	C_3	C_4
A_1	⟨0.7, 0.3, 0.4⟩	⟨0.7, 0.3, 0.2⟩	⟨0.7, 0.2, 0.4⟩	⟨0.8, 0.3, 0.3⟩
A_2	⟨0.6, 0.4, 0.2⟩	⟨0.8, 0.5, 0.4⟩	⟨0.7, 0.3, 0.2⟩	⟨0.8, 0.4, 0.5⟩
A_3	⟨0.6, 0.3, 0.5⟩	⟨0.7, 0.6, 0.2⟩	⟨0.7, 0.5, 0.5⟩	⟨0.8, 0.4, 0.4⟩

(22)

Step 4: Determination of the weights of the attributes

Entropy value E_j of the j-th (j = 1, 2, 3, 4) attributes can be determined from the decision matrix d_S (21) and equation (14) as: E_1= 0.6533, E_2 = 0.8200, E_3 = 0.6600, E_4 = 0.6867.

Then the corresponding entropy weights w_j, (j = 1, 2, 3, 4) of the attribute C_j (j = 1, 2, 3, 4) according to equation (15) is obtained as w_1 = 0.2938, w_2 = 0.1568, w_3 = 0.2836, w_4 = 0.2658 such that $\sum_{j=1}^{4} w_j = 1$

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS)

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as it follows:

$$Q_S^+ = \left\langle q_{S1}^+,\ q_{S2}^+,\ q_{S3}^+, q_{S4}^+ \right\rangle =$$

$$\begin{bmatrix} \left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\}\right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\}\right\rangle, \\ \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\}\right\rangle, \left\langle \max_i\{T_{i4}\}, \min_i\{I_{i4}\}, \min_i\{F_{i4}\}\right\rangle \end{bmatrix}$$

$$= \left[\langle 0.7, 0.3, 0.2\rangle, \langle 0.8, 0.3, 0.2\rangle, \langle 0.7, 0.2, 0.2\rangle, \langle 0.8, 0.3, 0.3\rangle\right]$$

The ideal interval neutrosophic estimates un-reliability solution (IINEURS) is presented as follows:

$$Q_S^- = \left\langle q_{S1}^-,\ q_{S2}^-,\ q_{S3}^-, q_{S4}^- \right\rangle =$$

$$\begin{bmatrix} \left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\}\right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\}\right\rangle, \\ \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\}\right\rangle, \left\langle \min_i\{T_{i4}\}, \max_i\{I_{i4}\}, \max_i\{F_{i4}\}\right\rangle \end{bmatrix}$$

$$= \left[\langle 0.6, 0.4, 0.5\rangle, \langle 0.7, 0.6, 0.4\rangle, \langle 0.7, 0.5, 0.5\rangle, \langle 0.8, 0.4, 0.5\rangle\right]$$

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Using the equation (16) the interval neutrosophic grey relational coefficient of each alternative from IINERS can be obtained as the following matrix.

$$[G_{ij}^{+}]_{3\times 4}=\begin{bmatrix}0.8000 & 1.0000 & 0.8000 & 0.6667\\ 0.8000 & 0.5714 & 1.0000 & 0.4444\\ 0.5714 & 0.5714 & 0.4444 & 0.5000\end{bmatrix} \quad (23)$$

Similarly, from the equation (17) the interval neutrosophic grey relational coefficient of each alternative from IINEURS is presented as follows:

$$[G_{ij}^{-}]_{3\times 4}=\begin{bmatrix}0.4545 & 0.3333 & 0.3846 & 0.4545\\ 0.4545 & 0.5556 & 0.3333 & 1.0000\\ 0.7143 & 0.5556 & 1.0000 & 0.7143\end{bmatrix} \quad (24)$$

Step 7: Determine the degree of interval neutrosophic grey relational co-efficient of each alternative from IINERS and IINEURS. The required interval neutrosophic grey relational co-efficient corresponding to IINERS is obtained by using the equation (18) as follows:

G_1^{+} = 0.7961, G_2^{+} = 0.7264, G_3^{+} = 0.5164 (25)

and corresponding to IINEURS is obtained with the help of equation (19) as follows:

$G_1^{-} = 0.4156, G_2^{-} = 0.5810,\ G_3^{-} = 0.7704$ (26)

Step 8: Thus interval neutrosophic relative degree of each alternative from IINERS can be obtained with the help of equation (20) as follows:

$R_1 = 0.6570, R_2 = 0.5556, R_3 = 0.4013$ (27)

Step 9: The ranking order of all alternatives can be determined according to the decreasing order of the value of interval neutrosophic relational degree i.e. $R_1>R_2>R_3$. It is seen that the highest value of interval neutrosophic relational degree is R_1 therefore A_1 (Christ missionary school) is the best alternative (school) for his/her the child for getting admission.

4.2 Example 2

An example about investment alternatives for a multi-attribute decision-making problem studied in [43, 47, 48, 49, 62] is used to demonstrate the applicability of the proposed approach under interval neutrosophic environment.

An investment company wants to invest an amount of money in the best option. There are four possible alternatives to invest the money:

(1) A_1 is a car company;
(2) A_2 is a food company;
(3) A_3 is a computer company;
(4) A_4 is an arms company.

The proposed decision making method can be arranged in the following steps.

Step 1: Determination the most important criteria

The company must take a decision according to the three attributes as follows:

(1) G_1 is the risk;
(2) G_2 is the growth;
(3) G_3 is the environmental impact.

Step 2: Data pre-processing and Construction of the decision matrix with interval neutrosophic form

We obtain the following interval neutrosophic decision matrix based on the experts' assessment:

Table 6. Decision matrix with interval neutrosophic number

$d_S = \langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \rangle_{4\times 3}$ =

	C_1	C_2	C_3
A_1	$\langle [0.4,0.5], [0.2,0.3], [0.3,0.4] \rangle$	$\langle [0.4,0.6], [0.1,0.3], [0.2,0.4] \rangle$	$\langle [0.4,0.5], [0.7,0.8], [0.7,0.9] \rangle$
A_2	$\langle [0.6,0.7], [0.1,0.2], [0.2,0.3] \rangle$	$\langle [0.6,0.7], [0.1,0.2], [0.2,0.3] \rangle$	$\langle [0.8,0.9], [0.5,0.7], [0.3,0.6] \rangle$
A_3	$\langle [0.3,0.6], [0.2,0.3], [0.3,0.4] \rangle$	$\langle [0.5,0.6], [0.2,0.3], [0.3,0.4] \rangle$	$\langle [0.7,0.9], [0.6,0.8], [0.4,0.5] \rangle$
A_4	$\langle [0.7,0.8], [0.0,0.1], [0.1,0.2] \rangle$	$\langle [.6,.7], [0.1,0.2], [0.1,0.3] \rangle$	$\langle [0.8,0.9], [0.6,0.7], [0.6,0.7] \rangle$

(28)

Step 3: Determination of the AAO

Using AAO, the decision matrix (see the table 7) in SVNS form is presented as follows:

Table7: single valued neutrosophic decision matrix

	C_1	C_2	C_3
A_1	$\langle 0.45,0.25,0.35 \rangle$	$\langle 0.50,0.20,0.30 \rangle$	$\langle 0.45,0.75,0.80 \rangle$
A_2	$\langle 0.65,0.15,0.25 \rangle$	$\langle 0.65,0.15,0.25 \rangle$	$\langle 0.85,0.60,0.45 \rangle$
A_3	$\langle 0.45,0.25,0.35 \rangle$	$\langle 0.55,0.25,0.35 \rangle$	$\langle 0.80,0.70,0.45 \rangle$
A_4	$\langle 0.75,0.05,0.15 \rangle$	$\langle 0.65,0.15,0.20 \rangle$	$\langle 0.85,0.65,0.65 \rangle$

(29)

Step 4: Determination of the weights of attribute

Entropy value E_j of the j-th (j = 1, 2, 3) attributes can be determined from the decision matrix d_S (12) and the equation (14). The obtained values are presented as follows: E_1 = 0.4400, E_2 = 0.4613, E_3 = 0.5413.

Then the entropy weights w_1, w_2, w_3 of the attributes are obtained from the eqation (15) and the obtained values are presented as follows: w_1 = 0.3596, w_2 = 0.3459, w_3 = 0.2945 such that $\sum_{j=1}^{4} w_j = 1$

Step 5: Determination of the ideal interval neutrosophic estimates reliability solution (IINERS) and the ideal interval neutrosophic estimates un-reliability solution (IINEURS)

The ideal interval neutrosophic estimates reliability solution (IINERS) is presented as follows.

$$Q_S^+ = \left\langle q_{S1}^+,\ q_{S2}^+,\ q_{S3}^+ \right\rangle =$$

$$\begin{bmatrix} \left\langle \max_i \{T_{i1}\}, \min_i \{I_{i1}\}, \min_i \{F_{i1}\} \right\rangle, \left\langle \max_i \{T_{i2}\}, \min_i \{I_{i2}\}, \min_i \{F_{i2}\} \right\rangle, \\ \left\langle \max_i \{T_{i3}\}, \min_i \{I_{i3}\}, \min_i \{F_{i3}\} \right\rangle \end{bmatrix}$$

$$= \left[\langle 0.75, 0.05, 0.15 \rangle, \langle 0.65, 0.15, 0.20 \rangle, \langle 0.85, 0.60, 0.45 \rangle \right]$$

The ideal interval neutrosophic estimates un-reliability solution (IINEURS) is presented as follows.

$$Q_S^- = \left\langle q_{S1}^-,\ q_{S2}^-,\ q_{S3}^- \right\rangle =$$

$$\begin{bmatrix} \left\langle \min_i \{T_{i1}\}, \max_i \{I_{i1}\}, \max_i \{F_{i1}\} \right\rangle, \left\langle \min_i \{T_{i2}\}, \max_i \{I_{i2}\}, \max_i \{F_{i2}\} \right\rangle, \\ \left\langle \min_i \{T_{i3}\}, \max_i \{I_{i3}\}, \max_i \{F_{i3}\} \right\rangle \end{bmatrix}$$

$$= \left[\langle 0.45, 0.25, 0.35 \rangle, \langle 0.50, 0.25, 0.35 \rangle, \langle 0.45, 0.75, 0.80 \rangle \right]$$

Step 6: Calculation of the interval neutrosophic grey relational coefficient of each alternative from IINERS and IINEURS

Using equation (16), the interval neutrosophic grey relational coefficient of each alternative from IINERS can be obtained as the following matrix.

$$[G_{ij}^+]_{4\times3} = \begin{bmatrix} 0.3913 & 0.6000 & 0.3333 \\ 0.6000 & 0.4737 & 1.0000 \\ 0.3913 & 0.5625 & 0.7500 \\ 1.0000 & 1.0000 & 0.6429 \end{bmatrix} \quad (30)$$

Similarly, from equation (17) the interval neutrosophic grey relational coefficient of each alternative from IINEURS is presented as the following matrix.

$$[G_{ij}^-]_{4\times3} = \begin{bmatrix} 1.0000 & 0.8182 & 0.1.0000 \\ 0.5294 & 0.5625 & 0.3333 \\ 1.0000 & 0.9000 & 0.3750 \\ 0.3913 & 0.5294 & 0.4091 \end{bmatrix} \quad (31)$$

Step 7: Determine the degree of interval neutrosophic grey relational co-efficient of each alternative from IINERS and IINEURS. The required interval neutrosophic grey relational co-efficient corresponding to IINERS is obtained using equation (18) as follows:

$$G_1^+ = 0.4464, G_2^+ = 0.6741, G_3^+ = 0.5562, G_4^+ = 0.8548 \quad (32)$$

and corresponding to IINEURS is obtained with the help of equation (19) as follows:

$$G_1^- = 0.9371, G_2^- = 0.4831,\ G_3^- = 0.7813,\ G_4^- = 0.4443 \quad (33)$$

Step 8: The interval neutrosophic relative degree of each alternative from IINERS can be obtained with the help of equation (20) as follows:

$$R_1 = 0.3227, R_2 = 0.5825, R_3 = 0.4159, R_4 = 0.6580 \quad (34)$$

Step 9: The ranking order of all alternatives can be determined according to the decreasing order of the value of interval neutrosophic relative relational degree i.e. $R_4 > R_2 > R_3 > R_1$. It is seen that the highest value of interval neutrosophic relational degree is R_4. Therefore investment company must invest money in the best option A_4 (Arms company).

4.3 Comparision between the existing methods

The problem was studied by several methods [43, 47, 48, 49, 62]. Ye [47] proposed the similarity measures between INSs based on the relationship between similarity measures and distances and used the similarity measures between each alternative and the ideal alternative to establish a multicriteria decision making method for INSs. have two sets of rankings, $R_4 > R_2 > R_3 > R_1$ and $R_2 > R_4 > R_3 > R_1$ based two different similarity measures. Obviously, the two rankings in [47] conflict with each other. Ye [48] furthet proposed improved correlation coefficient for interval neutrosophic sets and obtained the ranking $R_2 > R_4 > R_3 > R_1$. In contrast, Zhang et al. [43] presented the aggregation operators for interval neutrosophic numbers and obtained the two different rankings $R_4 > R_1 > R_2 > R_3$ and $R_1 > R_4 > R_2 > R_3$. . Şahin, and Karabacak [62] suggested a set of axioms for the inclusion measure in a family of interval neutrosophic sets and proposed a simple and natural inclusion measure based on the normalized Hamming distance between interval neutrosophic sets. Şahin, and Karabacak [62] obtained the ranking $R_2 > R_4 > R_1 > R_3$. Chi and Liu [49] obtained the ranking $R_4 > R_2 > R_3 > R_1$. The above results reflect that the different methods yield different solution or rankings. This ensures that the study of interval neutrosophic decision making is interesting and challenging task. We can observe that our ranking order of the four alternatives and best choice are also in agreement with the results of Chi and Liu's externded Topsis method [49]. In addition, it is simpler in calculation process than of Chi and Liu's method [49].

5. Conclusion

INSs can be applied in dealing with problems having uncertain, imprecise, incomplete, and inconsistent information existing in real scientific and engineering applications. In this paper, we have introduced interval neutrosophic multi-attribute decision-making problem with completely unknown attribute weight information based on modified GRA. Here all the attribute weights information is unknown. Entropy based modified GRA analysis method has been introduced to solve this MADM problem. Interval neutrosophic grey relation coefficient has been proposed for solving multiple attribute decision-making problems. Finally, the effectiveness of the proposed approach is illustrated by solving two numerical examples. However, the authors hope that the concept presented here will open new avenue of research in current neutrosophic decision-making arena. The main applications of this paper will be in the field of practical decision-making, medical diagnosis, pattern recognition, data mining, clustering analysis, etc.

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