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Abstract: The place of Heinrich Hertz's The Principles of Mechanics in the history of the philosophy of science is disputed. Here I critically assess positivist interpretations, concluding that they are inadequate.

There is a group of commentators who seek to align Hertz with positivism, or with specific positivists such as Ernst Mach, who were enormously influential at the time. Max Jammer is prominent among this group, the most recent member of which is Joseph Kockelmans. I begin by discussing what Hertz and Mach had to say about one another, and I specify certain respects in which their views are indeed similar. I then go on to detail their differences, looking at Hertz's attitude to the atomic theory, to the mechanical world-view, to simplicity, to unobservables and metaphysics, and his objections to Newtonian forces. I conclude that the positivist interpretation of Hertz's mechanics significantly overplays its similarities to Mach's views.

Keywords: Mach, Hertz, mechanics, positivism, atomism, force, metaphysics

Positivist Readings of Hertz's Principles of Mechanics

One standard way of portraying the position of Heinrich Hertz's last work, The Principles of Mechanics, in the history of the philosophy of science has been to assimilate his views to those of Ernst Mach, casting Hertz as a positivist. Max Jammer, Peter Alexander, and John Blackmore took this line in the 1950s, 1960s, and 1970s, respectively, but it has recently been reasserted by Joseph Kockelmans.[1]

Jammer regards Hertz, along with Mach and Gustav Kirchoff, as having an 'empirical, antimetaphysical attitude', and as putting forward 'more or less

[1] More recent work on Hertz, such as the papers in Baird, Hughes & Nordmann (1998), have tended not to take, but have also failed to critique, the line I am interested in here.

positivistic doctrines in mechanics and physics' (Jammer 1957, p.211). Their common program was to eliminate the concept of force from physics, since they considered it to be (like Hume's 'necessary connection' in causation) 'an idea illegitimate in science' (ibid.), and this program was *successful*, since in their work 'the logical development of the process of eliminating the concept of force from mechanics was completed' (ibid., p.229, also p.241).

Peter Alexander had a very similar reading of The Principles of Mechanics as 'an attempt to rewrite classical mechanics in such a way as to exhibit its systematic nature, increasing its rigour, reducing its assumptions to a minimum, and keeping it as empirical and nonmetaphysical as possible' (Alexander 1967, p.492). He interpreted Hertz's aims, in that book, as being 'firmly in the spirit of his teachers [Helmholtz and Kirchoff] and of Ernst Mach' (ibid.).

John Blackmore, the author of a well-known book on Mach's work, life, and influence, also interprets Hertz's mechanics as being 'couched within…Mach's philosophy of science' (Blackmore 1972, p.119)).[2] Hertz, Blackmore claims, based his own '"modern physics" largely on Mach's philosophy of science' (ibid.).

It is somewhat ironic that the latest representative of this reading should be Joseph Kockelmans, a philosopher who has pronounced hermeneutic-phenomenological leanings. He follows Jammer to the letter in his exposition of Hertz, arguing that 'the concept of force gradually developed, from being a basic concept with a clearly defined content in Newton's Principia, into a merely auxiliary, or perhaps intermediate, concept of primarily methodological importance under Kirchoff and von Helmholtz' (Kockelmans 2002, p.99). 'Jammer correctly observes', says Kockelmans, 'that with the works of Mach, Kirchoff, and Hertz the logical development of the process of eliminating the

[2] Blackmore says, in full, 'couched within Kant's epistemology and Mach's philosophy of science'.

concept of force from mechanics altogether was finally completed' (ibid., p.103).

This positivist interpretation of Hertz insists on something very important which is often missed or denied by other sorts of commentators: Hertz is trying to *replace* the Newtonian concept of force. Most of those interested in Hertz because of his influence on Wittgenstein, in particular, fail to recognise that Hertz is trying to 'clarify' the Newtonian concept of force only by *eliminating* it as a basic concept. Those who assimilate Hertz to positivism are absolutely right about this, and avoid the trap of interpreting Hertz as some kind of proto-Wittgensteinian or analytic philosopher.[3]

In what follows, I shall address the general question of whether Hertz's views are rightly characterised as positivistic, as well as the specific issue of the relations between his views on mechanics and those of Mach, who was very influential at the time, and is now generally thought of as the leading representative of positivism at the turn of the twentieth century.[4] Hertz and Mach were concerned with similar issues, and there are certain similarities between their views. Hertz's program of eliminating forces from mechanics (endorsed by Jammer and Kockelmans, among others), certainly seems positivistic. Mach seems to have approved of the idea. Nevertheless, I want to enter some reservations about characterising Hertz as an ally of Mach, or as a positivist.

Hertz and Mach: Mutual Admiration

Hertz and Mach read one another's work, referred to one another, and certainly admired one another. Hertz first read Mach's The Science of

[3] This temptation I have tried to combat in my Preston (forthcoming).

[4] I shall not be concerned with whether it is right to characterise Mach as a positivist, partly because the commentators discussed here agree in this characterisation, and partly because Mach himself apparently became resigned to the designation.

Mechanics, the first edition of which appeared in 1883, in March 1884.[5] In his own book he pays compliment to Mach, numbering him among 'distinguished physicists' (p.8), and remarking in his Preface that 'In a general way I owe very much to Mach's splendid book on the development of mechanics'.[6]

Naturally, Mach, for his part, was as aware as any contemporary physicist of Hertz's achievements in the field of electromagnetism. But he also expressed his admiration for Hertz's work in mechanics. He read Hertz's book some time between its publication in 1894 and the appearance of the third German edition of The Science of Mechanics in 1897, to which he added a section in which he says some very complimentary things about Hertz's book. Hertz's Mechanics, he says, 'marks a distinct advance' in the right direction (p.317), 'his own novel views must be regarded as a great step in advance' (p.320); the book 'must be read by everyone interested in mechanical problems' (p.317). Mach even seems to have thought that, despite a difference in their general *perspectives*, Hertz's actual *ideas* in mechanics were very similar to his own. In the preface to the second edition of his History and Root of the Principle of the Conservation of Energy, composed significantly later (in 1909), he says '[Hertz's ideas] coincide as exactly as is possible with my own, considering that Hertz was a supporter of the mechanical and atomic physics and a follower of Kant' (Mach 1909, p.11).

The Aim of Science

On the issue of what science is *for*, there does seem to be a genuine point of contact between Hertz and Mach. Blackmore has argued that a complex,

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[5] Blackmore 1972, p.119. References to Hertz, unless otherwise noted, are to the English translation of Hertz 1894 (Hertz 1956), and those to Mach, unless otherwise noted, are to the English translation of Mach 1897 (Mach 1960). Because the pages of Hertz's Preface are unnumbered in the English edition, references to the Preface do not have page numbers here.

[6] The exact way this compliment is phrased might suggest to modern readers that Hertz could have meant almost anything by it, maybe even that he *disagreed* with Mach's book! But there is no reason to read back our sceptical attitudes back into Hertz, or to think that he did not admire and learn from Mach.

three-part specification of the aim of science gradually emerges in Mach's work. The 'internal' aim of science, for Mach, is the description and relation of sensations. Its 'intermediate' aim is 'scientific economy' – 'the most complete, consistent, cohesive… world picture, a world picture of the greatest possible *stability*' (Mach 1900, p.366). And its 'external' purpose is to provide humans with as good a means of orienting themselves as possible.

Hertz, unfortunately, has much less to say on the issue, which he mentions explicitly only in the very first passage of his Introduction. There he suggests that the main task of science is simple, and singular:

> The most direct, and in a sense the most important, problem which our conscious knowledge of nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. As a basis for the solution of this problem we always make use of our knowledge of events which have already occurred, obtained by chance observation or by rearranged experiment (p.1).

This specification of the aim of science (which may, of course, have been only *part* of a fuller specification which Hertz could have given) has two aspects congenial to positivists. First, it presents scientific discovery in an inductivist way. Second, in referring only to the anticipation of future events, it is surprisingly thin and instrumentalist. Blackmore (ibid., p.169) suggests that Hertz actually influenced Mach in his conception of the 'intermediate' aim of science. But what Blackmore has in mind is not the instrumentalistic aspect of Hertz's conception, but his use of the concept of a 'world-picture' (*Weltbild* (p.25), *Bild des Weltganzen* (p.26)), which Mach (as well as Ludwig

Boltzmann) seems to have taken on.[7] It is worth noting, though, that Mach's first reaction was to *complain* about Hertz's conception:

> People require of science that it should *prophesy*, and Hertz uses that expression in his posthumous Mechanics. But, natural as it is, the expression is too narrow. The geologist and the palaeontologist, at times the astronomer, and always the historian and the philologist, prophesy, so to speak, *backwards*. The descriptive sciences, like geometry and mathematics, prophesy neither forwards nor backwards, but seek from given conditions the conditioned. Let us say rather: *Science completes in thought facts that are only partly given*. (Mach 1894, p.253).

Hertz does indeed use the terms '*vorauszusehen'* and '*Voraussicht'* (to foresee, foresight) in the German original of the passage quoted above. Mach's objection shows (perhaps surprisingly to those who think of him as a paradigm instrumentalist) that his own conception of the aim of science is broader than that to which Hertz gives expression. Hertz's specification of the aim of science seems at least as positivistic as Mach's.

Criteria for Evaluating Scientific Theories

For Hertz it is important that our scientific 'images' are not predetermined by experience, but consciously constructed under the guidance of three requirements, one logical, one empirical, and one pragmatic. He proposes, famously, that a scientific 'image' [*Bild*] must be *logically permissible* [*logische zulässig*] in that it must not contradict 'the laws of our thought' (p.2). It must be *correct* [*richtig*], in that its 'essential relations' must not contradict the 'relations of external things' (ibid.). And finally it must be *appropriate* [*zweckmässig*] in

[7] See Blackmore 1972, p.160.

that it must represent as many of the essential relations of its object as possible, while minimising the number of 'superfluous or empty relations' (ibid.) it contains.

If, in emphasising the freedom and conscious activity involved in our construction of scientific images, Hertz had Mach himself in mind as a target, then his remarks are understandable, but misguided. Mach does occasionally say things which might lead one to interpret him as the kind of empiricist who thinks our theories are straightforwardly determined by experience. But these moments are outweighed by those in-depth discussions of particular scientific laws and principles, in which he clearly recognises that the relation between experience and theory is not direct. One of the major claims of The Science of Mechanics, after all, is that the 'main result' of considering the development of dynamics is that 'one great fact' or perception slowly emerged (p.306, cf. pp.478-9, 573). That fact, that bodies mutually determine in each other accelerations dependent on definite spatial and material circumstances was 'embodied in' a great number of mechanical principles, and its perception was 'not reached at once, but slowly and by degrees' (p.306).

Nevertheless, Mach evidently considered that Hertz's insistence on freedom in theory-construction involved a misplaced emphasis. He countered that although our concepts (his preferred term) 'are formed consciously and purposely by us', they 'are nevertheless not formed altogether arbitrarily, but are the outcome of an endeavour on our part to adapt our ideas to our sensuous environment' (p.318). This biological, Darwinian aspect of Mach's methodology finds no echo in Hertz, but may be the root of the disagreement between them over the extent to which our theory-construction is constrained.

Mach also finds other things to complain of in Hertz's discussion of logical permissibility. When Hertz argues that the images we form should be 'such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured' (p.1),

Mach objects to the phrase ‘necessary consequents in nature’ [*die denknotwendigen Folgen der Bilder*] because it appears to him to invoke a kind of natural or non-logical necessity. Against this, Mach wants to insist that

> logical necessity…is the only necessity that *we* have knowledge of. The belief in a necessity obtaining in nature arises only in case where our concepts are closely enough adapted to nature to ensure a correspondence between the logical inference and the fact. But the assumption of an adequate adaptation of our ideas can be refuted at any moment by experience. (p.318).

Finally, although Mach values clearness, he doesn’t have the same concern with *logical* clearness that Hertz has, perhaps because he only really recognises two classes of statement: those expressing matters of experience, and those expressing matters of ‘arbitrary convention’ (p.304), i.e. definitions. The only one of Hertz’s three requirements to which Mach does not object is that of ‘appropriateness’ but, as we shall see, Mach’s assimilation of this feature to his own idea of ‘economy’ is problematic.

Criticisms of Newtonian Mechanics

Hertz and Mach are both well-known as critics of Newtonian mechanics. Both distinguish between its *content* and its *form* (Hertz pp.8-9, Mach pp.245, 555), and both are convinced that its existing defects are defects of form or presentation only, not of content.

They do seem to disagree, though, about how serious those defects are. Hertz indicted the customary (Newtonian-Lagrangian) image of mechanics in two respects. By the end of his Introduction, he has decided that despite its practical utility, the current form of the customary image of mechanics is neither fully logically permissible, nor fully appropriate.

Mach, however, insisted that 'Hertz's criticisms of existing systems of mechanics cannot be accepted in all their severity' (pp.319-20). He felt that the objections Hertz levelled on the score of logical permissibility applied only to 'logically defective expositions, such as Hertz doubtless had in mind from his student days' (p.319). When, for example, Hertz suggested that the customary representation of mechanics counts an object's inertia twice (once as inertia, and once as centrifugal force) (p.6), Mach protested that, even as far back as Huygens and Newton, mechanics had not fallen into this confusion.

Mach also objected to Hertz's characterization of forces as 'idle wheels', and thus to Hertz's objection on the score of appropriateness. To characterize forces as being frequently 'empty-running wheels', Mach says, 'as being frequently not demonstrable to the senses, can scarcely be permissible' (p.319). Here, though, Mach seems to have misunderstood Hertz's objection. Hertz objects to Newtonian forces *not* because they are unobservable, but because they are *inoperative*. He calls them '*leehrgehende Nebenräder*' (idling side-wheels)[8] not because one cannot observe them, but because most of them don't actually produce motion.

Further disagreement between Hertz and Mach appears at the more detailed level of their *reasons* for objecting to the current form of mechanics. For Mach, 'the present form of our science of mechanics rests on an historical accident' (p.316), in that the Newtonian development of Galilean mechanics has been preferred to others (such as the Huygenian) which could just as well have been pursued. He also considers that the current form of the principles of mechanics still reflect their origins in theology (p.554). Mach's well-known objections and amendments to the Newtonian form of mechanics mean that although he reveres and admires Newton, he feels that 'we can only comprehend with difficulty [Lord Kelvin's] opinion that the Newtonian

[8] The existing English translation of Hertz disappointingly renders this key phrase 'sleeping partners' (pp.11-12).

doctrines still remain the best and most philosophical foundation that can be given' (p.305). Mach judges that although Hertz's criticisms of previous representations of mechanics 'contain some very noteworthy epistemological considerations', they nevertheless 'from our point of view… stand in need of certain modifications' (p.318).

Economy and Appropriateness

According to Mach, 'Hertz's criterion of appropriateness coincides with our criterion of economy' (p.318).[9] This is by no means clearly so.

Economy is the central, if not the single, desideratum in Mach's account of science; his 'fundamental conception of the nature of science' is of 'Economy of thought' (p.xxiii). 'Physics', he says elsewhere, 'is experience, arranged in economical order' (Mach 1882, p.197). 'All physical ideas and principles are succinct directions, frequently involving subordinate directions, for the employment of economically classified experiences, ready for use' (ibid., p.204). Because it is his central desideratum, Mach certainly applied the concept of economy in different ways, but his core idea was to construe it as a matter of minimising the *effort* that one expends in getting to grips with a given subject-matter. Science is communicated 'in order that one man may profit by the experience of another and be spared the trouble of accumulating it for himself' (p.577).

> The communication of scientific knowledge always involves description, that is, a mimetic reproduction of facts in thought, the object of which is to replace and save the trouble of new experience. Again, to save the labor of instruction and of acquisition, concise, abridged description is sought. This is really all that natural laws are. (Mach 1882, pp.192-3).

[9] Alexander, we saw above, endorses this thought.

Economy, for Mach, is a matter of labour-saving.

Hertz's 'appropriateness' is only like this to a certain extent, being more limited in scope. Appropriateness, for him, contains two features, lexically ordered. The first (but less clear), *distinctness*, refers to the ability of an image to picture as many as possible of its object's 'essential relations'. The second and more important feature, *simplicity*, is a matter of the image's containing as few 'superfluous or empty relations' as possible. This second feature concerns labour-saving in that an image is simple, for Hertz, to the extent that it avoids postulating entities or properties which perform no *function*. However, while, for Mach, science itself is always ultimately in the service of our practical ends (its 'external purpose', detailed above), Hertz makes it absolutely clear that the appropriateness he is concerned with '*has no reference to practical applications or the needs of mankind*' (p.40, emphasis added). He concedes completely that with respect to such purposes, 'it is scarcely possible that the usual representation of mechanics, which has been devised expressly for them, can ever be replaced by a more appropriate system' (ibid.). This is an anathema to Mach. Because, for him, 'economy' is a portmanteau category, an ideally 'economic' theory would be one that met every demand we could make of it. Noting Hertz's concession, Mach rightly observes that Hertz himself, 'with his characteristic candour, admits [that for practical purposes the usual representation of mechanics is preferable to his own]' (p.324). Hertz, he also claims, 'partly retracts his criticism…; or at any rate, he qualifies it' (p.319). But although the dialectic of Hertz's Introduction means that he does not end up pressing home all of the possible objections which he considers, still, by the end of that Introduction he has complaints about the customary representation on two of the three possible scores (logical permissibility and appropriateness), and he leaves the further question of that representation's correctness to be decided by our future experience.

Observability, Experience, Perception

For Hertz, of course, just as for Mach, observability, accessibility to perceptual experience, has the central role in the specification of what makes an image 'correct': 'What enters into the images for the sake of correctness is contained in the results of experience, from which the images are built up' (p.3). When discussing the merits of the customary image of mechanics, he says: 'No one will deny that within the whole range of our experience up to the present the correctness is perfect; that all those characteristics of our image, which claim to represent observable relations of things, do really and correctly correspond to them' (p.9). Hertz's second requirement on images is thus undoubtedly one that an empiricist can endorse.

The most pregnant passage in which Hertz uses such concepts pertaining to perceptual experience is the one in which he deploys his objection to the customary image on the score of appropriateness:

> The weight of a stone and the force exerted by the arm seem to be as real and as readily and directly perceptible as the motions which they produce. But it is otherwise when we turn to the motions of the stars. Here the forces have never been the objects of direct perception; all our previous experiences relate only to the apparent position of the stars. Nor do we expect in future to perceive the forces. The future experiences which we anticipate again relate only to the position of these luminous points in the heavens. It is only in the deduction of future experiences from the past that the forces of gravitation enter as transitory aids in the calculation, and then disappear from consideration. Precisely the same is true of the discussion of molecular forces, of chemical actions, and of many electric and magnetic actions. And if after more mature experience we return to the simple forces, whose existence we never doubted, we learn that these

> forces which we had perceived with convincing certainty, were after all not real. More mature mechanics tells us that what we believed to be simply the tendency of a body towards the earth is not really such: it is the result, imagined only as a single force, of an inconceivable number of actual forces which attract the atoms of the body towards all the atoms of the universe. Here again the actual forces have never been the objects of previous experience; nor do we expect to come across them in future experiences. Only during the process of deducing future experiences from the past do they glide quietly in and out (p.12).

Jammer and Alexander take this passage to be evidence that Hertz endorsed an instrumentalistic view of explanation. Jammer says that for Hertz, in advanced problems such as those of celestial mechanics, force is 'a transitory aid in the calculation and disappears finally from our considerations' (ibid., p.226). For Alexander, Hertz 'looks askance at forces that "cancel out in the calculations" as robbing an explanation of its simplicity, or what Mach calls its economy' (p.493).[10]

It is not so clear, though, that Hertz is really endorsing this instrumentalistic view. The very next thing he says suggests otherwise:

> But even if the forces have only been introduced by ourselves into nature, we should not on that account regard their introduction as inappropriate. We have felt sure from the beginning that unessential relations could not be altogether avoided in our images. All that we can ask is that these relations should, as far as possible, be restricted, and that a wise discretion should be observed in their use. But has physics always been sparing in the use of such relations? Has it not rather been compelled to

[10] Perhaps this passage, together with Hertz's specification of the aim of science, is part of the reason why Karl Popper lists him among those he considers 'instrumentalists in various ways' ((Popper 1963), p.99, note 5).

> fill the world to overflowing with forces of the most various kinds – with forces which never appeared in the phenomena, even with forces which only came into action in exceptional cases? (pp.12-13).

He goes on immediately to advert to the sheer number of forces (all the gravitational, magnetic, electric, inter-atomic ones) which the customary image postulates in the case of a piece of iron resting on a table, saying

> Some of these forces are not small: if only a part of these forces were effective, this part would suffice to tear the iron to pieces. But, in fact, all the forces are so adjusted amongst each other that the effect of the whole lot is zero; that in spite of a thousand existing causes of motion, no motion takes place; that the iron remains at rest. Now if we place these conceptions before unprejudiced persons, who will believe us? Whom shall we convince that we are speaking of actual things, not images of a riotous imagination? (p.13).

This strongly suggests that the question, for Hertz, is not whether we have used forces only as transitory aids to calculation, but whether we have simply postulated *unbelievably many* forces. His objection to the current form of the customary representation is more a matter of ontological profligacy than epistemological inaccessibility. Thus it is not the objection that Mach thinks it is, and neither is it the objection that advocates of the positivist reading suppose. While this kind of objection to forces is one that a positivist might nevertheless accept, it isn't as *characteristic* of positivism as the objections that are often attributed to Hertz.

Nevertheless, when discussing the 'energeticist' representation of mechanics, whose central idea is that physical phenomena should be explained in terms of laws governing energy-transformations, Hertz imposes a strongly

empiricist requirement. The energeticist, he says, must 'indicate clearly by what concrete experiences we ultimately establish the presence of mass and energy' (p.15).[11] This requirement does not seem to be imposed merely because energeticists *themselves* accepted it, rather than because Hertz himself did so. In this same discussion, he admits that energeticism's advantage over the customary representation of mechanics lies in the fact that 'in the hypotheses of the problems there only enter characteristics which are directly accessible to experience, parameters, or arbitrary co-ordinates of the bodies under consideration; that the examination proceeds with the aid of these characteristics in a finite and complete form; and that the final result can again be directly translated into tangible experience' (p.18). This certainly seems to constitute an advantage *to Hertz*, as well as to the energeticist. Having discussed the appropriateness of energeticism he says 'These are the merits which have endeared this method to present-day physics' (p.18), and he concludes that the energeticist representation of mechanics is indeed 'appropriate'.

'Concealed Masses' and their Operation

There can be little doubt, then, that Hertz really did accept this empiricist requirement to indicate by what experiences we establish the presence of the phenomena answering to all the basic concepts of our physical theories. This, however, sheds a curious light on his own representation of mechanics. Consider his procedure in introducing the 'concealed masses' [*verborgene Massen*] and 'concealed motions' [*verborgene Bewegungen*] which his own system postulates (p.26). It is somewhat unusual, and noteworthy. He begins by issuing a warning:

[11] Likewise, he later insists that 'we have to specify by what simple, direct experiences we propose to define the presence of a store of energy and the determination of its amount' (p.21).

> If we try to understand the motions of bodies around us, and to refer them to simple and clear rules, paying attention only to what can be directly observed, our attempt will in general fail. We soon become aware that the totality of things visible and tangible do not form a universe conformable to law, in which the same results always follows from the same conditions. We become convinced that the manifold of the actual universe must be greater than the manifold of the universe which is directly revealed to us by our senses (p.25).

This warning that one can never eliminate unobservable entities from mechanics is something neither Mach nor positivists could (in the final analysis) endorse. Neither is the lesson Hertz draws from it:

> If we wish to obtain an image of the universe which shall be well-rounded, complete, and conformable to law, we have to presuppose, behind the things which we see, other, invisible things – to imagine confederates concealed beyond the limits of our senses (ibid.).

In the customary (Newtonian-Lagrangian) and energeticist images, though, Hertz complains, the confederates introduced (viz., force and energy) are '*entities of a special and peculiar kind*' (ibid., emphasis added). There is, however, an alternative:

> [A]nother way lies open to us. We may admit that there is a hidden something at work, and yet deny that this something belongs to a special category. We are free to assume that this hidden something is naught else than motion and mass again, - motion and mass which differ from the visible ones not in themselves but in relation to us and to our usual means of perception. Now this mode of conception is just our hypothesis (ibid.).

In a subsequent discussion of the issue Hertz starts by claiming that in his own representation he is indeed complying with the empiricist requirement: 'We first introduce the three independent fundamental ideas of time, space, and mass as objects of experience; and we specify the concrete sensible experiences by which time, mass, and space are to be determined' (p.26). But then he immediately says: 'With regard to the masses we stipulate that, in addition to the masses recognisable by the senses, concealed masses can by hypothesis be introduced' (pp.26-7).

Hertz, then, seems to have thought that he *was* respecting this empiricist requirement. He says of his own system that 'all our statements represent possible experiences; if necessary, they could be confirmed by direct experiments, viz. by measurements made with models' (p.30). But his introduction of 'concealed masses' and 'concealed motions' seems in this respect like a trick. Having introduced the concept of mass in an empiricist way, he then feels free to postulate unobservable masses whose *only* difference from observable ones is supposed to be their unobservability. They are not supposed to be 'entities of a special and peculiar kind', because despite being in-principle unobservable, they are *more like* observable masses than are forces, for example. This move certainly raises some important conceptual problems: can we, as it requires, deny that whether or not an object is observable supervenes on other, more basic properties? Could there really be objects which are exactly like one another, except that one is observable and the other not? Alexander, as we saw, claims that Hertz's book is 'an attempt to rewrite classical mechanics… keeping it as empirical and nonmetaphysical as possible' (Alexander 1967, p.492). Hertz's concealed masses, though, may be 'nonmetaphysical' in *some* sense, but this does not make them unproblematic. His system eliminates forces in favour not of observables but in favour of concealed masses and motions which are by definition and *in principle* unobservable, and which are not

supposed to be mere temporary expedients, later to be discarded. Unsurprisingly, therefore, Hertz's replacements for forces are less acceptable to Mach than forces themselves. Mach complained that the latter 'are decidedly in the advantage on this score, as compared with "hidden masses" and "hidden motions"' (p.319), which he considered 'occult' (p.323). Neither did Mach think Hertz's system was as empirical as possible. In fact, he complained that it was *just the opposite*. The physical side of mechanical problems, he says, 'is not only not disposed of, but is not even so much as touched, by the elaboration of such a formal mathematical system of mechanics' (p.322). Although Hertz's system gave a 'beautiful ideal form' to mechanics, 'its physical contents have shrunk to an apparently almost imperceptible residue' (pp.322-3).

Notoriously, Hertz gave no examples at all of how his concealed masses might actually operate to produce observable motions. As Ludwig Boltzmann aptly commented, it was unfortunate that, in the same moment that Hertz created this 'strikingly simple system of mechanics', 'his lips became forever sealed to the thousand requests for clarification that are certainly not on the tip of my tongue alone' (Boltzmann 1974, pp.89-90). In the wake of his book's publication, a very few of Hertz's peers, students and followers did try to supply the needed illustrations. Mach was perhaps the first. In the case of a mass m, moving uniformly in a circle of radius r and with a velocity v, he suggests, instead of referring its movement to a postulated centripetal force 'we might instead of this conceive the mass to be rigidly connected at the distance $2r$ with one of the same size having a contrary velocity' (p.324). But although he makes this suggestion, it is notable that Mach does so in order to support his view that to take Hertz's mechanics seriously in this way would be 'to resort, even in the simplest cases, to fantastic and frequently questionable fictions, to which the *given* accelerations would be far preferable' (ibid.). Mach, in other words, illustrates how the concealed masses of Hertz's mechanics might operate only in order to suggest that the application of that mechanics is preposterous. For

him, it is the equations governing the ways in which the motions of masses depend on one another that are 'the essential thing, - the thing established by experience' (p.321), rather than the forces or the connections between masses.

Finally, it is true that Hertz insists on (what we would now call) operational definitions for the basic concepts of mechanics (§§298-300). But these are *in addition to* his prior definitions of space, time, and mass (§§2-4), which are supposed to be '*a priori* judgements in Kant's sense' (§1). Instead of *constituting* those concepts, as they would for a positivist, and thus being necessary conditions on their intelligibility, they merely connect those (already intelligible) concepts with experience.

In sum, I suggest that despite broad agreement over the roles of observability and perceptual experience the interpretation and application of such features is rather different in Hertz and Mach. This comes out most clearly in their respective attitudes towards unobservable entities. It is not only empiricists, though, who may well feel that a trick is being executed in the introduction of 'concealed masses'. Mach was right to challenge the idea that they are an improvement on forces. He was right, however, not for his own empiricist reason, but because Hertz's concealed masses face worse conceptual problems than forces do.

Attitudes to Atoms

Hertz's attitude to the atomic theory certainly bears comparison with Mach's. Hertz was a pupil of Kirchoff, who exhibited a positivistic streak in his thought on this subject:

> To an investigator like Gustav Kirchoff, who was accustomed to rigid reasoning, it almost gave pain to see atoms and their vibrations wilfully stuck in the middle of a theoretical deduction. The arbitrarily assumed properties of the atoms may not affect the final result. The result may be

> correct. Nevertheless the details of the deduction are in great part presumably false; the deduction is only in appearance a proof. The earlier mode of thought in physics scarcely allowed any choice or any way of escape (Hertz, p.18).[12]

Mach's exact attitude to atomism is a vexed issue but, in general, he took an equally hard line.[13] In his earliest work (1862) it is true, he seems not only to have accepted the atomic-molecular theory 'as a useful working model and hypothesis' (Hiebert 1970, p.79, cf. Brush 1968, pp.197-9), but also to have used it. By 1872, however, he had come to reject it with vehemence and, despite some later mellowing in his view, seems never to have been reconciled to it. Although he continued to admit the utility of atomism as an hypothesis, for him 'the unreality of atoms…was a basic axiom' (Brush ibid., p.210).

Hertz's attitude to atomism is not comparable to that of Mach, or Kirchoff. He does discuss atoms and atomism in several places, but none of those discussions should lead us to think that he shared a Machian or a positivist attitude. (Mach recognised this, as we saw, in identifying Hertz as 'a supporter of the mechanical and atomic physics').

In the famous passages (pp.12-13, quoted above) in which Hertz levels the 'idle wheels' objection at the customary representation of mechanics, he is evidently sceptical about whether we should suppose that each atom in the universe is related to every other such atom by gravitational and electromagnetic forces. But it is the inoperative forces *themselves* he is sceptical about, not the atoms.

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[12] It may be, however, that Hertz is here exaggerating Kirchoff's resistance to atomism. See (Rosenfeld 1973), p.382.

[13] Larry Laudan at one point suggests that perhaps Mach merely has 'reservations' about atomic hypotheses, rather than rejecting them (Laudan 1981, p.210), but this seems to me significantly to downplay the force of his complaints.

When discussing the 'energeticist' representation, Hertz mentions that physicists find it attractive 'Because in this way [physics] best avoids talking about things of which it knows very little, and which do not at all affect the essential statements under consideration' (p.17). The context makes it clear that this is a reference to atoms. But the view expressed should not be assimilated to Mach's. To say we know very little of atoms, or even that their existence does not (at the moment) affect our theories is not to *deny* their existence or to count assertions of their existence as meaningless. In fact it is clear from other places that Hertz *doesn't* oppose atomism. His considered view is best expressed in this passage:

> We have already had occasion to remark that in tracing back phenomena to force we are compelled to turn our attention continually to atoms and molecules. It is true that we are now convinced that ponderable matter consists of atoms; and we have definite notions of the magnitude of these atoms and of their motions in certain cases. But the form of the atoms, their connection, their motion in most cases – all these are entirely hidden from us; their number is in all cases immeasurably great. So that although our conception of atoms is in itself an important and interesting object for further investigation, it is in no wise specially fit to serve as a known and secure foundation for mathematical theories (pp.17-18).

For Hertz, although we are now convinced that matter consists of atoms, our conception of them is *not yet* a suitable basis for physical theory.[14] They are 'unknown systems' (p.21). Any properties we currently ascribe to them (such as shape, attractive or repulsive forces, etc.) are 'arbitrarily assumed properties' (p.18). But this does not mean that Hertz considers atoms illegitimate

[14] This is well argued in Lützen 2005, pp.46-7, 49.

postulates. For one thing, he twice clearly acknowledges that Kelvin's theory of vortex atoms is one of the inspirations for his work (pp.26, 37). For another, when faced with the objection that there are in reality none of the 'rigid connections' between material points which his mechanics requires, Hertz concedes that '[i]n seeking the actual rigid connections we shall perhaps have to descend to the world of atoms' (p.34, cf. §330). From an anti-atomist, such a concession would be unintelligible. The most that can be said about Hertz's mechanics, in this respect, is that it does not make reference to atoms. But this is because Hertz thinks of his mechanics as an intermediate level theory, a theory that will, with luck, remain valid *whatever* is discovered about the microphysical entities whose existence underpins mechanics.[15] That this is no evidence that Hertz was an anti-atomist is clear from the fact that exactly the same can be said of the ether: his mechanics makes no mention of it, and yet we know that he not only believed in it, but considered its investigation to be 'the all-important problem in physics' (Hertz 1889, p.286). There is no trace, in Hertz's mechanics, of Mach's attitudes to atoms as (at best) convenient fictions, temporary and transitional expedients (p.589). For Mach, atomic theories had *already* outlived their usefulness. Hertz, by contrast, seems to think they had yet to come into their own.

The Mechanistic World-View

At the very beginning of Hertz's Preface he explains that mechanics has a foundational status within physics: 'all physicists agree that the problem of physics consists in tracing the phenomena of nature back to the simple laws of mechanics'. Some have taken this to be a commitment to 'the prevalent view of German physicists at the time that the prime task of physics was to reduce *all* phenomena to mechanics' (Mulligan 1994, pp.65-6, emphasis added, Lützen

[15] I am grateful to Jesper Lützen for some informal comments on this issue. (He, nevertheless, is not to be blamed for any use I make of the comments in question).

2005, ch.3, cf. McCormach 1972, p.347). This would make Hertz an advocate of radical *mechanism*, the 'mechanical *world-view'*, which had been of such importance in Germany from the 1830s. But although Hertz's project concerns the foundations of mechanics, there is in his text very little suggestion that *other* sciences should be reduced to physics, let alone to mechanics. In the book's Introduction he expressly limits the range of his new mechanics to inanimate nature, saying 'how far its laws extend beyond this we leave as quite an open question' (p.38, see also §§318-22, §342). He takes it to be an advantage of his fundamental law that it 'shows us what are the limits of this domain [viz., mechanics]' (p.38). Hertz is certainly a *moderate* mechanist, but the evidence that he is a radical one is inconclusive.

Opposition to mechanism, though, is one of the central and unmistakable aspects of Mach's philosophy. After an initial phase as a mechanist, he came to consider the 'mechanical world-view' a *mere* prejudice (pp.596ff.), associating it with the materialism and atomism which he eschewed.[16] The physico-mechanical view of the world, for him, was a result of an unacceptable belief in the magical power of science (Mach 1882, pp.187-9), and he looked forward to a cooperation between natural science and psychology whose results would 'far outstrip those of modern mechanical physics' (ibid., p.212). Mechanical hypotheses, he insisted, effect no genuine economy (p.599). For Mach, mechanics is just one branch of physics among others, and neither its historical priority nor anything else suggests that it is the foundation of physics, let alone of all science (p.612).

If I am right that Hertz is merely a moderate mechanist, it may seem ironic that Mach later rails against Hertz's restriction of his own mechanics to inanimate nature (Mach 1905, p.183, note 6). He does so, however, because for Mach only when physics is conceived to have a narrow base (e.g. mechanics)

[16] Einstein, of course, credited Mach with having woken physicists from their dogmatic slumbers in this respect ((Einstein 1949), p.21).

can it be assured of *not* being applicable to biological processes. Such processes need not be a mystery to physics, if the latter is properly (i.e. widely) conceived.

Attitudes to Metaphysics

Hertz's attitude to metaphysics is *very* different from that of Mach. He only talks of metaphysics once, when discussing the logical permissibility and appropriateness of the energeticist system. It can be objected to this system, he points out, that Hamilton's principle (or whatever other integral principle is selected) attributes intentions to inanimate nature. The usual answer to this objection, which physics nowadays keeps ready is, Hertz says, that

> these considerations are based upon metaphysical assumptions; that physics has renounced these, and no longer recognises it as a duty to meet the demands of metaphysics. It no longer attaches weight to the reasons which used to be urged from the metaphysical side in favour of principles which indicate design in nature, and thus it cannot lend ear to objections of a metaphysical character against these same principles (p.23).

But this 'usual answer', as Mach makes very clear in his comments on Hertz's book, *is* Mach's answer. Mach does not think 'the case with energic mechanics so bad as Hertz would have it' (p.319). Whereas Hertz objects that the employment of minimum principles 'involves the assumption of purpose and presupposes tendencies directed toward the future', Mach claims that his own book 'shows…quite distinctly that the simple import of minimum principles is contained in an entirely different property from that of purpose' (ibid.). Hertz, however, not only fails to endorse this 'usual answer', he objects to it:

> If we had to decide upon such a matter we should not think it unfair to place ourselves rather on the side of the attack rather than of the defence. A doubt which makes an impression on our mind cannot be removed by calling it metaphysical; every thoughtful mind as such has needs which scientific men are accustomed to denote as metaphysical (p.23).

This remarkably conciliatory comment fits neither the classical positivist attitude, according to which metaphysics forms only an early phase in the development of science, nor Mach's attitude, which is relentlessly critical of metaphysics. Indeed, if one had to identify anyone as the most likely target of Hertz's comment, it would have to be Mach.

Eliminating Forces?

Finally, what of the suggestion that Hertz and Mach are involved in a common programme, for the elimination of force from physics? Mach, it is true, does suggest the possibility of a future physics which would not contain the concept of force, and interprets Hertz's mechanics as a move in this direction (p.317). He also remarks that 'modern physical science shows traces of fetishism… in its "forces"' (p.558). In general, though, he just isn't as concerned with the concept of force as Hertz was. The Science of Mechanics contains nothing one could call a critique of the concept of force. Mach is far more worried about the concept of *mass* (e.g. pp.237ff, 264ff), mainly because he thought that Newton had made a mess of it. Mach gives his own definition of mass (p.266), advertising it as the only possible rational definition of that concept (p.341). He even suggests that this definition would take 'a more organic and more natural place in Hertz's mechanics than his own, for it contains implicitly the germ of his "fundamental law"' (p.341, note). By contrast, though, Mach seems entirely happy with Newton's definition of force,

remarking that ‘It is, we may say, a matter of taste whether we shall embody the explication of the idea of force in one or several definitions. In point of principle the Newtonian definitions are open to no objections’ (p.301). Of course he does not approve of the metaphysical idea that forces are ‘unknown causes’ (p.307), but as long as force is understood (à la Kirchoff, for example) as the product of mass and acceleration, Mach raises no objections to it. In fact at one point he appears to suggest (contrary to his own idea that there could be a forceless physics) that attempts to set aside the concept of force are fruitless:

> Force is any circumstance of which the consequence is motion. Several circumstances of this kind, however, each single one of which determines motion, may be so conjoined that in the result there shall be no motion. … The circumstances determinative of motion that are best known to us, are our own volitional acts – our innervations. In the motions which we ourselves determine, as well as in those to which we are forced by external circumstances, we are always sensible of a pressure. Thence arises our habit or representing all circumstances determinative of motion as something akin to volitional acts – as *pressures*. *The attempts we make to set aside this conception, as subjective, animistic, and unscientific, fail invariably* (p.95, emphasis added).

It is hard to square this remark with the way Jammer and Kockelmans read Mach, as relentlessly striving to eliminate the concept of force from mechanics. Mach may have had that idea, but it figures only peripherally in his critique of existing mechanics. For him, the concept of pressure, at least, which is the phenomenal ground of the concept of force, has a secure place in our ontology.

The Fate of Hertz’s Mechanics

Earlier I commended those who take the positivist reading of Hertz for recognising something which some other commentators (those interpreting him as a proto-analytic philosopher) often get wrong – the fact that he is trying to replace the concept of force. However, both these kinds of commentators are regularly wrong about something else of significance: they far too readily assume the *success* of Hertz's project. Kockelmans, for example, declares that 'In the space of some 250 pages, Hertz effectively shows that it is indeed possible to give a mechanical account of all known phenomena of nature' on the basis of his own assumptions (ibid., p.108). For this to be true, Hertz's mechanics would have to meet some reasonable set of requirements on scientific theories, e.g. that it be logically permissible, empirically adequate, and as 'appropriate' as any other system of mechanics. Hertz, it is true, *claimed* that his mechanics had these virtues. But the verdicts of those who came after him ought to cast serious doubt on this. All the leading physicists and scientifically-oriented philosophers of Hertz's generation, and of the next one, (Helmholtz, Mach, Boltzmann, Lorentz, FitzGerald, Einstein, Russell, Poincaré, Duhem) reacted to his book. Almost all of them found its new mechanics interesting and beautiful, but either baffling or unsuccessful, or both. This reaction cannot be written off by the reminder that these people are scientists, whereas Hertz's aim is philosophical. These are the great *philosopher*-scientists of two generations. Hertz certainly intended his work to be intelligible to them and to make a difference to their activities, since his project is supposed to allow us to make *progress* in physics (Preface). But despite a few attempts to apply Hertz's mechanics, it cannot be said to have been a success. The concept of force might be thought to have eventually succumbed to elimination in Einstein's general theory of relativity, but not by Hertz's method.

Conclusion

In sum, Hertz and Mach admired one another's work, and were in broad agreement over the aim of science and the deficiencies of the Newtonian representation of mechanics. Their detailed critiques of this representation, however, differed in significant respects, some of which can be traced at least in part to divergences in their criteria for evaluating scientific theories. Hertz and Mach differed in other, more general philosophical respects, too. They had different attitudes towards the programme of mechanical explanation, towards metaphysics, towards unobservables in general and towards atoms and 'concealed masses' in particular. Mach, it transpires, was right in identifying Hertz as 'a supporter of the mechanical and atomic physics and a follower of Kant' (Mach 1909, p.11). His judgement that, *modulo* these differences, Hertz's ideas 'coincide as exactly as is possible with my own' (ibid.), overplays their similarities, but does so less than some of Mach's sympathisers who have tried to enlist Hertz in his cause.

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