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Research Article

MOORA under Pythagorean Fuzzy Set for Multiple Criteria Decision Making

Luis Pérez-Domínguez,[1] **Luis Alberto Rodríguez-Picón**,[1] **Alejandro Alvarado-Iniesta**,[1] **David Luviano Cruz**,[1] **and Zeshui Xu**[2]

[1]*Department of Industrial and Manufacturing Engineering, Universidad Autónoma de Ciudad Juárez, Ave. Del Charro 450 Norte, 32315 Ciudad Juárez, CHIH, Mexico*
[2]*Business School, Sichuan University, Chengdu, Sichuan 610064, China*

Correspondence should be addressed to Luis Pérez-Domínguez; luis.dominguez@uacj.mx

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The multiobjective optimization on the basis of ratio analysis (MOORA) method captures diverse features such as the criteria and alternatives of appraising a multiple criteria decision-making (MCDM) problem. At the same time, the multiple criteria problem includes a set of decision makers with diverse expertise and preferences. In fact, the literature lists numerous approaches to aid in this problematic task of choosing the best alternative. Nevertheless, in the MCDM field, there is a challenge regarding intangible information which is commonly involved in multiple criteria decision-making problem; hence, it is substantial in order to advance beyond the research related to this field. Thus, the objective of this paper is to present a fused method between multiobjective optimization on the basis of ratio analysis and Pythagorean fuzzy sets for the choice of an alternative. Besides, multiobjective optimization on the basis of ratio analysis is utilized to choose the best alternatives. Finally, two decision-making problems are applied to illustrate the feasibility and practicality of the proposed method.

1. Introduction

Recently, the area of multiple criteria decision making (MCDM) had suffered a rapid development. MCDM aims to provide methods of ranking alternatives or select the optimal alternatives among a set of possible alternatives regarding several criteria [1]. Due to the commonness of the MCDM problems in modern life, its theories have been widely applied in various domains like military affair, industrial engineering, macroeconomic domain, and management [2]. Likewise, there are numerous multicriteria methodologies to deliver aid in the problematic task of making this decision [3]. In this sense, the most commonly reported methodologies in the literature are elimination and choice translation reality (ELECTRE, 1968) [4], decision support system (DSS, 1971) [5], data envelopment analysis (DEA, 1978) [6], analytic hierarchy process (AHP, 1980) [7], technique for order of preference by similarity to ideal solution (TOPSIS, 1981) [8], dimensional analysis (DA, 1993) [9], multicriteria optimization and compromise solution (vlsekriterijumska optimizacija i kompromisno resenje, VIKOR, 1998) [10], analytic network process (ANP, 1996) [11], multiobjective optimization on the basis of ratio analysis (MOORA, 2006) [12], and preference selection index (PSI, 2010) [13]. In addition, there are conventional MCDM problems that only consider nonfuzzy (crisp) type for appraising the alternatives with respect to each criterion and preferences of the criteria. In this logic, the conventional MOORA method is proficient for establishing the evaluations and rankings of the alternatives without any complexity. Nonetheless, in real-world, there are MCDM problems, where the opinions (feeling, preferences) of the DMs for appraising the alternatives and criteria weights are commonly expressed by means of linguistic terms embracing ambiguity and hesitation [14, 15]. In this manner, the classical MOORA method presents drawback for manipulating the nonfuzzy (crisp) and fuzzy (qualitative) information involved in a problem of MCDM [16, 17]. Then, there exists the panorama to continue developing investigation in decision making to approaches to deal with incomplete and imprecise information involved in MCDM problems.

Moreover, there are frequently reported hybrid methods with fuzzy sets and equally fuzzy set theory has been generalized in order to manipulate vagueness [18]. Nevertheless, these methods by themselves in addition to the hybrids with fuzzy sets still have some drawbacks and there is an imperative demand to present new MCDM methods [3, 19–21].

Additionally, current investigations assert that multicriteria methods are being combined with intuitionistic fuzzy sets (IFS). Principally, the IFS, introduced by Atanassov [22], become of a generality of the conventional fuzzy sets stated by Zadeh [23]. According to literature over the last decade, the academics have paid great attention to the use of IFS in MCDM [3, 24, 25]. The IFS are proficient at imprecise treatment and inexact data [26–28].

On the other hand, the Pythagorean fuzzy set (PFS) [29–32] has arisen as an operational instrument for handling the vagueness of MCDM problems. The PFS is categorized by means of the affiliation degree and the nonaffiliation degree, whose sum of squares is less than or equal to 1. In the circumstance, the PFS can explain the difficulties that the IFS cannot; for example, if a DM gives the membership degree and the nonmembership degree as 0.8 and 0.3, respectively, then it is just operative for the PFS. In this sense, all the IFS degrees are a part of the PFS degrees, which specifies that the PFS is more proficient in handling problems of vagueness. Motivated by the advantages of the MOORA method and PF, this paper proposes two algorithms of MCDM by extending the MOORA to PF environments. Additionally, dealing with the last two challenges mentioned in the paragraph above arises. In this sense, the originality and contribution of this paper can be summarized as follows. First, we propose MOORA under PF environments to overcome the limitation of MOORA for dealing with any other type of arguments rather than crisp data and extend its potential applications to more extensive areas. Second, our approach can simultaneously handle quantitative (tangible) and qualitative (intangible) information, commonly presented in an MCDM problem. Hereafter, the intention of this paper is to extend the MOORA method under the PFS environment for the MCDM field.

The remainder of this paper is organized as follows. Section 2 briefly presents the concepts related to PFS. Section 3 presents the explanation of MOORA. Section 4 pronounces the method proposed in this work. In the Section 5 two numerical cases are presented to describe the proposed methodology and the conclusions are presented in Section 6.

2. Pythagorean Fuzzy Set

Pythagorean fuzzy set (PFS) presented by [29, 33]is explained as follows.

Definition 1 (see [33]). Let Y be an arbitrary nonempty set. A PFS P is a mathematical object of the form

$$P = \{\langle y, P(\mu_A(y), \upsilon_A(y))\rangle \mid y \in Y\}. \qquad (1)$$

Thus, a Pythagorean fuzzy set P in $Y = \{y\}$ is given by $P = \{\langle y, \mu_P(y), \upsilon_P(y)\rangle \mid y \in Y\}$.

Here $\mu_P(y)$ and $\upsilon_P(y) : Y \to [0,1]$ depict the affiliation function and nonaffiliation function of the fuzzy set A; $\mu_A(y) \in [0,1]$ depict the affiliation of $y \in Y$ in A. At the same time, a PFS A in $Y = \{y\}$ is defined as $A = \{\langle y, \mu_A(y), \upsilon_A(y)\rangle \mid y \in Y\}$, μ_A and $\upsilon_A : Y \to [0,1]$. With the condition $0 \le \mu_A(y) + \upsilon_A(y) \le 1, \forall y \in Y$, the numbers $\mu_A(y)$ and $\upsilon_A(y)$ depict the degree of affiliation and degree of nonaffiliation of element y with respect A.

The number $\pi_P(y)$ is named the Pythagorean index degree of the hesitancy of y in P and can be stated as

$$\pi_P(y) = \sqrt{1 - (\mu_P^2(y) + \upsilon_P^2(y))}, \qquad (2)$$

where $\mu_P^2(y) + \upsilon_P^2(y) \le 1$ for each $y \in Y$.

Hence, a PFS P in $Y = \{y\}$ is fully defined with the form $P = \{\langle y, \mu_P(y), \upsilon_P(y), \pi_P(y)\rangle \mid y \in Y\}$. Here $\mu_P : Y \to [0,1]$; $\upsilon_P : X \to [0,1]$ and $\pi_P : X \to [0,1]$. Thus, diverse operations are presented over the PFSs [29]; some of them are revealed in (3), (4), and (5).

Definition 2 (see [32, 34]). Assuming $\alpha_1 = P\{\mu_{\alpha_1}, \upsilon_{\alpha_1}\}$,$\alpha_2 = P\{\mu_{\alpha_2}, \upsilon_{\alpha_2}\}$, and $\alpha = P\{\mu, \upsilon\}$ are three PFNs, then,

$$\alpha^c = (\nu_\alpha, \mu_\alpha) \qquad (3)$$

$$\alpha_1 \oplus \alpha_2 = \left(\sqrt{(\mu_A + \mu_B - \mu_A \cdot \mu_B)}, (\nu_A \cdot \nu_A)\right); \qquad (4)$$

$$\alpha_1 \otimes \alpha_2 = \left(\mu_A \cdot \mu_B, \sqrt{\nu_A + \nu_A - \nu_A \cdot \nu_A}\right) \qquad (5)$$

$$n\alpha = \left(\sqrt{1 - (1 - \mu_\alpha^2)^n}, (\nu_A)^n\right), \quad n > 0. \qquad (6)$$

In fact, to rank the PFNs the next definition is presented.

Definition 3. Let $\alpha = P(\mu_\alpha, \nu_\alpha)$ describe a PFN; then the total function of $\theta(\alpha)$ is presented as

$$\theta(\alpha) = (\mu_\alpha)^2 - (\nu_\alpha)^2. \qquad (7)$$

The large $\theta(\alpha)$ depict the best PFN.

Definition 4. Let $\alpha = P(\mu_\alpha, \nu_\alpha)$ represent a PFN; at that time the precision function of $\Omega(\alpha)$ is introduced as

$$\Omega(\alpha) = (\mu_\alpha)^2 + (\nu_\alpha)^2. \qquad (8)$$

Obviously, $\Omega(\alpha) \in [0,1]$. Thus, $0 \le \Omega(\alpha) = (\mu_\alpha)^2 + (\nu_\alpha)^2 \le 1$. The superior rate of $\Omega(\alpha)$ describes the higher precision of the PFN α.

Thus, per (2) and (8), $\pi_\alpha^2 + \Omega(\alpha) = 1$ can be determined. The inferior hesitant degree makes higher accuracy of the PFN α.

Hence, with the total function and the precision function of PFNs, the ranking method for any two PFNs can be defined as follows.

Definition 5. Let $\alpha_1 = P(\mu_{\alpha 1}, \nu_{\alpha 1})$ and $\alpha_2 = P(\mu_{\alpha 2}, \nu_{\alpha 2})$ depict two PFNs. Here, $S(\alpha_i)$ $(i = 1, 2)$ and $H(\alpha_i)$ $(i = 1, 2)$ describe the rate and the precision of α_1 and α_2. Then,

(i) if $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1 < \alpha_2$;

(ii) if $S(\alpha_1) = S(\alpha_2)$, then,

(1) if $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 < \alpha_2$;

(2) if $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$;

(3) if $H(\alpha_1) > H(\alpha_2)$, then $\alpha_1 > \alpha_2$.

3. MOORA

The MOORA method was introduced by [12], which analyzes the complete throughput for each alternative as the variance between the sums of cost criteria and benefit criteria. Therefore, the MOORA method is definite through steps.

Step 1. Establish the decision-making matrix called β. In this manner β collects n rows that denote the alternatives $A^1 \cdots A^n$ in assessment and $T + Z$ columns that characterize criteria in appraisal (T quantitative criteria and Z qualitative criteria). In this mode, per (9), the decision-making matrix β can be obtained as follows:

$$\beta = [VO, VST]$$
$$= \begin{matrix} A^1 \\ A^2 \\ \cdot \\ \cdot \\ A^n \end{matrix} \begin{bmatrix} x_1^1 & \cdots & x_T^1 & x_{T+1}^1 & \cdots & x_{T+Z}^1 \\ x_1^2 & \cdots & x_T^2 & x_{T+1}^2 & \cdots & x_{T+1}^2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1^n & \cdots & x_T^n & x_{T+1}^n & \cdots & x_{T+Z}^n \end{bmatrix}, \quad (9)$$

where A^i denote the alternatives, aimed at $k = 1 \cdots n$, and x_l^k reflect the inputs of the alternative k with reference to criterion l.

Step 2. Proceed with the normalization of β matrix. Here the Euclidean norm to the criterion x_l is obtained by using

$$|\beta_l| = \sqrt{\sum_{K=1}^{n} x_l^2}. \quad (10)$$

Therefore, the normalization of each entry in the β is calculated by using

$$\overline{\beta} x_{kl} = \frac{x_{kl}}{|\beta_l|}. \quad (11)$$

Step 3. Create the balanced normalized decision-making matrix called $\check{\xi}_{kl}$. Following [12] the different preferences of criteria, the evaluations $\check{\xi}_{kl}$ are computed by using

$$\check{\xi}_{kl} = \omega_i \cdot Nx_{kl}. \quad (12)$$

Step 4. Analyze the global assessments of cost and benefit criteria for each A_i.

In this mode, the global evaluations of benefit criteria Nx_i are estimated as the sum of weights normalized per

$$Nx_i = \check{\xi}_{kl}\Big| \in \delta^{\max}, \quad (13)$$

where $\delta^{\max}$ is related to Nx_i.

Table 1: Scale for evaluation of the preferences of DMs and criteria.

Meaning	PFNs (μ, ν)
Apprentice (Ap)/Very Insignificant (VI)	(0.10, 0.90)
Learner (Lr)/Insignificant (I)	(0.35, 0.60)
Capable (Ct)/Average (A)	(0.50, 0.45)
Skillful (S)/Imperative (Im)	(0.75, 0.40)
Dominant (D)/Very Significative (VS)	(0.90, 0.10)

Likewise, the global assessments of cost criteria Nx_j are calculated by mean of

$$Nx_j = \check{\xi}_{kl}\Big| \in \delta^{\min}, \quad (14)$$

where $\delta^{\min}$ is related to Nx_j.

Step 5. Establish the contribution Ny_i value. Ny_i is obtained via (15) originated by [12].

$$Ny_i = \sum_{l=1}^{g} Nx_i - \sum_{l=g+1}^{m} Nx_j, \quad (15)$$

where Ny_i represents the contribution of each alternative $k = 1 \cdots n$, $i = 1 \cdots g$ are the maximum criteria, and $l = g+1, g+2, \ldots, m$ are the lowest criteria.

4. MOORA under Pythagorean Fuzzy Environment (PF-MOORA)

Let $A = \{A_1, A_2, \ldots, A_i, \ldots, A_n\}$ represent a set of alternatives and $x = \{x_1, x_2, \ldots, x_j, \ldots, x_m\}$ depict a collection of criteria to be appraised. The PF-MOORA method is described in the following steps.

Step 1. Establish a team of DMs and capture the preferences of each one. Here $\text{DM} = \{\text{DM}_1, \text{DM}_2, \ldots, \text{DM}_k, \ldots, \text{DM}_l\}$ denote a group of decision makers (DMs). The preferences of each DM are evaluated via a linguistic term mapping by PFN. The scale and their corresponding PFN used are shown in Table 1.

Let $\text{DM}_k = \{\mu_k, \nu_k, \pi_k\}$ be a Pythagorean fuzzy number for evaluation of DM. Then, the equivalent weight of DM is calculated using the concept fuzzy weighted arithmetic Pythagorean represented by

$$\varepsilon_k = \frac{(\mu_k + \pi_k (\mu_k / (\mu_k + \nu_k)))}{\sum_{k=1}^{l} (\mu_k + \pi_k (\mu_k / (\mu_k + \nu_k)))}, \quad (16)$$

where $\sum_{k=1}^{l} \varepsilon_k = 1$.

Step 2. State the preferences of criteria. Hence, all opinions/preferences need to be considered and fused into one.

Thus, in order to appraise the preference criteria by every DM, the scale linguistic from Table 1 can be used.

Table 2: Scale for assessment alternatives.

Meaning	PFNs (μ, ν)
Enormously Bad (EB)/Extremely Low (EL)	$\{0.10, 0.99\}$
Tall Bad (TB)/Very Little (VL)	$\{0.10, 0.97\}$
Not Good (NG)/Little (L)	$\{0.25, 0.92\}$
Middle Bad (MB)/Middle Little (ML)	$\{0.40, 0.87\}$
Fair-middle (F)/Middle (M)	$\{0.50, 0.80\}$
Middle Good (MG)/Middle High (MH)	$\{0.6, 0.71\}$
Tall (T)/Big (B)	$\{0.70, 0.60\}$
Very Big (VB)/Very Tall (VT)	$\{0.8, 0.44\}$
Exceptional (E)/Tremendously High (TH)	$\{1, 0\}$

Let $\omega_j^{(k)} = \{\mu_j^{(k)}, \nu_j^{(k)}\}$ be a PFN given to criterion x_l by the DM. Then, the weights of the criteria are computed by means of the IPFWA operator proposed by [32]

$$\begin{aligned}\omega_j &= \text{PFWA}\left(\omega_j^{(1)}, \omega_j^{(2)}, \dots, \omega_j^{(k)}, \dots, \omega_j^{(l)}\right) \\ &= \lambda_1\omega_j^{(1)} \oplus \lambda_2\omega_j^{(2)} \oplus \cdots \oplus \lambda_k\omega_j^{(k)} \oplus \cdots \oplus \lambda_l\omega_j^{(l)} \\ &= P\left[\sum_{j=1}^{n}\lambda_j\mu_j, \sum_{j=1}^{n}\lambda_j\nu_j\right],\end{aligned} \tag{17}$$

where $\omega_j = \{\mu_j, \nu_j\}$ and $\omega_j = \{\omega_1, \omega_2, \dots, \omega_j, \dots, \omega_m\}$ and $\sum_{j=1}^{m}\omega_j = 1$.

Step 3. Create the combined Pythagorean fuzzy decision matrix denoting the assessment of A_i according to the preferences of the DMs.

Let $R^{(k)} = (x_{ij}^{(k)})_{nxm}$ be a Pythagorean fuzzy decision matrix (PFDM) of each DM. The scale used to appraise each alternative is presented in Table 2.

All preferences of the DMs need to be involved into a gathered Pythagorean fuzzy decision matrix (PFDM) through PFWA operator: $R = (x_{ij})_{nxm}$.

$$\begin{aligned}x_{kl} &= \text{PFWA}\left(x_{kl}^{(1)}, x_{kl}^{(2)}, \dots, x_{kl}^{(t)}, \dots, x_{kl}^{(z)}\right) \\ &= \omega_1 x_{kl}^{(1)} \oplus \omega_2 x_{kl}^{(2)} \oplus \cdots \oplus \omega_z x_{kl}^{(z)} \oplus \cdots \oplus \omega_j x_{kl}^{(t)} \\ &= \left[\sum_{k=1}^{l}\left(\left(\omega \cdot \mu_{kl}^{(z)}\right), \left(\omega \cdot \nu_{kl}^{(z)}\right)\right)\right],\end{aligned} \tag{18}$$

where $x_{ij} = \{\mu_{A_k(X_l)}, \nu_{A_k(X_l)}, \pi_{A_k(X_l)}\}$ $(k = 1, 2, \dots, n;\ l = 1, 2, \dots, m)$.

Then, the PFDM is defined as

$$\text{PFDM} = R = \begin{bmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{bmatrix}. \tag{19}$$

Explicitly,

$$R = \begin{bmatrix} \{\mu_{A_1(X_1)}, \nu_{A_1(X_1)}, \pi_{A_1(X_1)}\} & \cdots & \{\mu_{A_1(X_m)}, \nu_{A_1(X_m)}, \pi_{A_1(X_m)}\} \\ \vdots & \ddots & \vdots \\ \{\mu_{A_n(X_1)}, \nu_{A_n(X_1)}, \pi_{A_n(X_1)}\} & \cdots & \{\mu_{A_n(X_m)}, \nu_{A_n(X_m)}, \pi_{A_n(X_m)}\} \end{bmatrix}. \tag{20}$$

Step 4. Calculate the combined weighted Pythagorean fuzzy decision matrix called R'. In this step, R' is computed by means of APFDM and the vector ω_j. The elements of R' are calculated via

$$\text{WPFDM} = R' = R \cdot \omega = \left\{\left\langle x, \mu_{A_i}(x) \cdot \mu_\omega(x), \nu_{A_i}(x) + \nu_\omega(x) - \nu_{A_i}(x) \cdot \nu_\omega(x)\right\rangle \mid x \in X\right\} \tag{21}$$

$$R' = \begin{bmatrix} \{\mu_{A_1'(X_1)}, \nu_{A_1'(X_1)}, \pi_{A_1'(X_1)}\} & \cdots & \{\mu_{A_1'(X_m)}, \nu_{A_1'(X_m)}, \pi_{A_1'(X_m)}\} \\ \vdots & \ddots & \vdots \\ \{\mu_{A_n'(X_1)}, \nu_{A_n'(X_1)}, \pi_{A_n'(X_1)}\} & \cdots & \{\mu_{A_n'(X_m)}, \nu_{A_n'(X_m)}, \pi_{A_n'(X_m)}\} \end{bmatrix}. \tag{22}$$

Step 5. Compute the sum of BNx_i and Cx_j.

Consequently, (23) denotes the sum of benefit criteria:

$$BNx_i = \sum_{i=1}^{g}\left(\mu_{A_i'}(x_i), \nu_{A_i'}(x_i), \pi_{A_i'}(x_i)\right), \tag{23}$$

where BNx_i describe the benefit criteria for the alternative $k = 1 \cdots n$. $x_l = 1 \cdots g$ denote the maximum criteria. Then, (24) defines the sum of the cost criteria.

$$Cx_j = \sum_{j=g+1}^{m}\left(\mu_{A_i'}(x_j), \nu_{A_i'}(x_j), \pi_{A_i'}(x_j)\right), \tag{24}$$

where Cx_j denotes the sum of the cost criteria for alternative $k = 1 \cdots n$, and $x_l = g+1, g+2, \dots, m$ are minimum criteria.

Step 6. Defuzzify BNx and Cx_j by mean of

$$\begin{aligned}Nx_i &= \left(\mu_{\alpha_{x_i}}\right)^2 - \left(\nu_{\alpha_{x_i}}\right)^2 \\ Nx_j &= \left(\mu_{\alpha_{x_j}}\right)^2 - \left(\nu_{\alpha_{x_j}}\right)^2.\end{aligned} \tag{25}$$

Step 7. Calculate the value of Ny_i.

It is obtained by means of

$$Ny_i = Nx_i - Nx_j. \tag{26}$$

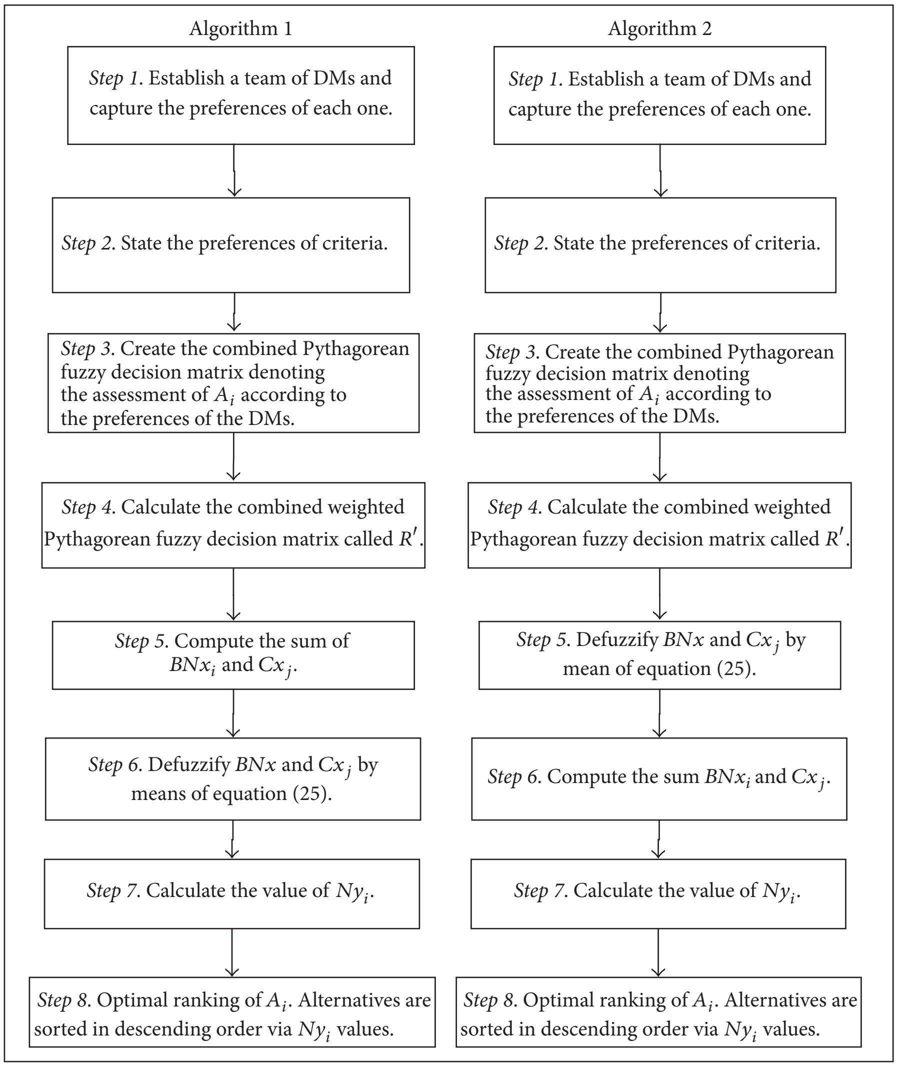

Figure 1: Flowcharts of the algorithms of the PF-MOORA method.

Step 8. Optimal ranking of A_i: alternatives are sorted in descending order via Ny_i values.

In order to explain the proposed method, Figure 1 shows the flowcharts of the different steps used by the PF-MOORA method.

Here, Algorithm 1 is capable of working under completely fuzzy information. Likewise, Algorithm 2 is proficient to work with hybrid information, handling nonfuzzy and fuzzy data, since MCDM problems simultaneously may include both quantitative (nonfuzzy) and qualitative (fuzzy) data.

5. Numerical Case

Example 1 (Algorithm 1). This case belongs to an organization from Maquiladoras of Juárez, México. In fact, the organization is an assembly manufacturing company, in which several components are assembled in its production line. A cost reduction project was implemented and opportunity area belongs to packing item. This company proposes appraising five packing suppliers of electronic components. In this sense, two decision makers are invited for the assessment. In addition, four criteria are raised for depiction of the substantial features of the providers. In this manner, the criteria involved are described as follows:

(i) Cost (x_1): minimum values are ideal.

(ii) Service (x_2): great assessments are preferred.

(iii) Lead time (x_3): high appraisals are preferred.

(iv) Quality (x_4): great appraisals are preferred.

Thus, the collection of suppliers is designated by $P = \{P_1, P_2, P_3, P_4, P_5\}$.

The procedure followed to select the best supplier is shown below.

Step 1. Establish a team of DMs and capture the preferences of each one.

$$\varepsilon_{1,2} = \frac{(0.75 + 0.53\,(0.75/\,(0.75 + 0.4)))}{(0.75 + 0.53\,(0.75/\,(0.75 + 0.4))) + (0.75 + 0.53\,(0.75/\,(0.75 + 0.4)))} = 0.5. \quad (27)$$

The DM preferences are presented in Table 3. Hence, the weight of each DM is obtained by means of (16).

Step 2. State the preferences of the criteria.

The appraisal of each DM is represented in Table 4. The preferences of the DMs are unified via (17).

$$\omega_{\{x_1,x_2,x_3,x_4\}} = \begin{bmatrix} \{0.24\} \\ \{0.24\} \\ \{0.22\} \\ \{0.31\} \end{bmatrix}^T . \quad (28)$$

Step 3. Create the combined Pythagorean fuzzy decision matrix denoting the assessment of A_i by the preferences of the DMs.

The evaluations given for each DM are specified in Table 5. The PFDM is given by (18) and the results are as follows:

$$R = \begin{bmatrix} \{0.10, 0.97, 0.22\} & \{0.25, 0.90, 0.36\} & \{1.00, 0.00, 0.00\} & \{0.80, 0.44, 0.41\} \\ \{0.40, 0.87, 0.29\} & \{0.60, 0.71, 0.37\} & \{0.50, 0.80, 0.33\} & \{0.70, 0.60, 0.39\} \\ \{0.96, 0.04, 0.28\} & \{0.96, 0.04, 0.28\} & \{0.96, 0.04, 0.28\} & \{0.10, 0.97, 0.22\} \\ \{0.71, 0.50, 0.50\} & \{0.10, 0.97, 0.22\} & \{0.10, 0.97, 0.22\} & \{0.25, 0.92, 0.30\} \\ \{0.50, 0.80, 0.33\} & \{0.40, 0.87, 0.29\} & \{0.71, 0.50, 0.50\} & \{0.50, 0.80, 0.33\} \end{bmatrix}. \quad (29)$$

Step 4. Calculate the combined weighted Pythagorean fuzzy decision matrix called R'.

The elements of R' are calculated via (21).

$$R' = \begin{bmatrix} \{0.1, 0.8, 0.1\} & \{0.2, 0.8, 0.1\} & \{0.4, 0.6, 0.1\} & \{0.2, 0.7, 0.0\} \\ \{0.3, 0.6, 0.1\} & \{0.5, 0.4, 0.1\} & \{0.1, 0.8, 0.1\} & \{0.2, 0.8, 0.1\} \\ \{0.5, 0.4, 0.1\} & \{0.6, 0.3, 0.1\} & \{0.3, 0.7, 0.1\} & \{0.0, 0.9, 0.0\} \\ \{0.4, 0.5, 0.1\} & \{0.1, 0.8, 0.1\} & \{0.0, 0.9, 0.0\} & \{0.1, 0.9, 0.0\} \\ \{0.3, 0.6, 0.1\} & \{0.3, 0.6, 0.1\} & \{0.2, 0.8, 0.1\} & \{0.1, 0.9, 0.0\} \end{bmatrix}. \quad (30)$$

Step 5. Compute the sum of BNx_i and Cx_j.

Table 6 shows the results of BNx_i values calculated via (23).

Table 7 describes the Cx_j values calculated using (24).

Step 6. Defuzzify BNx and Cx_j by means of (25).

Tables 8 and 9 describe the results.

Step 7. Calculate the value of Ny_i.

Table 10 shows the ratio for each alternative and its ranking.

Step 8. Rank the alternatives.

The results reveal $P_1 \succ P_2 \succ P_3 \succ P_5 \succ P_4$. Then, alternative P_1 is selected as the top supplier.

Example 2 (Algorithm 2). In fact, this case belongs to a real-life problem; there is a *manufacturing company* that is in charge of manufacturing fixtures, work tables, and holders. The most commonly used materials are plastic, aluminum, and steel. The machines used in CNC are milling machines, lathe, and grinding machines. The current tool selection is based on expertise from operators. Consequently, there is problem reflected in the quality metric from customers due to the poor quality of parts (products). Then, after a six-sigma project and root cause analyses were identified, the major cause is related to the wrong tools assigned to the manufacturing process. In this manner, the MCDM problem is associated with selecting the tools indicated with their respective machining criteria for the types of material used in the manufacturing process through a CNC milling machine.

In this sense, four DMs were invited to the assessment process. Likewise, a group of five tools are involved which can be called alternatives $T = (T_1, T_2, T_3, T_4, T_5)$. At the same time, six criteria are considered and are described as follows:

(i) RPM (c_1): this describes the revolutions per minute (rpm) from the spindle of the lathe. High values are ideal (crisp/nonfuzzy criterion).

(ii) Advance on X/Y (c_2): it describes the advance on axes of X and Y depicted in in/min, respectively. Great assessments are preferred (crisp/nonfuzzy criterion).

(iii) Advance on Z (c_3): it describes the advance on axes of Z depicted in in/min. High appraisals are preferred (crisp/nonfuzzy criterion).

Table 3: The importance of DM.

Decision maker	1	2
Linguistic term	D	D
PF number	{0.75, 0.4, 0.53}	{0.75, 0.4, 0.53}
Weight	0.5	0.5

Table 4: The importance of criteria.

DMs	x_1	x_2	x_3	x_4
DM_1	Im	A	Lr	VI
DM_2	A	Im	VI	Lr

Table 5: The assessments of alternatives A_i.

DMs	Supplier	Criteria			
		X_1	X_2	X_3	X_4
DM1	P_1	VL	L	E	VT
	P_2	ML	T	F	T
	P_3	VB	VB	VB	TB
	P_4	MH	TB	TB	L
	P_5	M	MB	MG	F
DM2	P_1	L	MB	E	TH
	P_2	M	VB	L	VB
	P_3	B	TH	B	L
	P_4	MH	L	EB	MB
	P_5	L	F	MB	MB

Table 6: BNx_i values.

Alternatives	μ	ν	π
P_1	0.609	0.301	0.09
P_2	0.636	0.245	0.119
P_3	0.751	0.18	0.069
P_4	0.212	0.629	0.159
P_5	0.481	0.395	0.125

Table 7: Cx_j values.

Supplier	μ	ν	π
P_1	0.115	0.77	0.115
P_2	0.292	0.613	0.095
P_3	0.488	0.399	0.113
P_4	0.388	0.51	0.102
P_5	0.251	0.643	0.106

(iv) Cutting speed (c_4): it describes the cutting speed depicted in ft/minute. Great appraisals are preferred. Minimum evaluations are desired (crisp/nonfuzzy criterion).

(v) Machining finish (c_5): it is associated with the machining finish of the work piece. Great values are ideal (subjective/fuzzy criterion).

Table 8: Defuzzification of benefits Nx_i.

Alternatives	Crisp
P_1	0.641
P_2	0.675
P_3	0.767
P_4	0.32
P_5	0.538

Table 9: Defuzzification of costs Nx_j.

Alternatives	Crisp
P_1	0.207
P_2	0.354
P_3	0.54
P_4	0.445
P_5	0.323

Table 10: Rank for each alternative.

Alternatives	PF-MOORA method proposed	Rank
P_1	0.576	1
P_2	0.415	2
P_3	0.337	3
P_4	−0.049	5
P_5	0.294	4

(vi) Geometry of the parts (c_6): it is related to how complex the shapes of manufacturing of the parts are. High evaluation are desired (subjective/fuzzy criterion).

Step 1. Establish a team of DMs and capture the preferences of each one.

Table 11 describes the results.

Step 2. State the preferences of the criteria.

Table 12 illustrate the results.

$$\omega_{\{x_1,x_2,x_3,x_4\}} = \begin{bmatrix} \{0.06\} \\ \{0.19\} \\ \{0.26\} \\ \{0.35\} \\ \{0.03\} \\ \{0.10\} \end{bmatrix}^T . \quad (31)$$

Step 3. Create the combined Pythagorean fuzzy decision matrix denoting the assessment of A_i according to the preferences of the DMs.

Table 11: Comparison of rank for each alternative.

Decision maker	1	2	3	4
Linguistic term	Ct	Lr	D	D
PF number	{0.50, 0.45, 0.74}	{0.35, 0.45, 0.82}	{0.90, 0.10, 0.42}	{0.75, 0.4, 0.53}
Weight	0.22	18	0.32	0.28

Table 12: Comparison of rank for each alternative.

DMs	x_1	x_2	x_3	x_4	x_5	x_6
DM_1	VS	I	A	VI	I	Im
DM_2	S	VI	I	I	VS	Im
DM_3	A	D	Im	A	A	A
DM_4	VS	Im	VI	Im	A	VI

Table 13: Defuzzy values of BNx and Cx_j.

Alternatives	BNx	Cx_j
T_1	−0.830	−0.996
T_2	−0.871	−0.978
T_3	−0.747	−0.917
T_4	−0.781	−0.964
T_5	−0.952	−0.991
T_6	−0.753	−0.917

Table 14: BNx and Cx_j values.

Alternatives	BNx	Cx_j
T_1	−0.517	−0.746
T_2	−0.699	−0.728
T_3	−0.613	−0.713
T_4	−0.678	−0.815
T_5	−0.900	−0.914
T_6	−0.701	−0.851

Our MCDM problem involves crisp and fuzzy criteria. The crisp matrix is called T and depicted as

$$T = \begin{bmatrix} 5000 & 50 & 5 & 0.050 \\ 5000 & 50 & 5 & 0.025 \\ 3500 & 30 & 5 & 0.020 \\ 3000 & 30 & 3 & 0.015 \\ 4000 & 10 & 2 & 0.007 \\ 4000 & 5 & 2 & 0.011 \end{bmatrix}, \quad (32)$$

and $\ddot{Y}$ is called Pythagorean matrix depicting the fuzzy criteria:

$$\ddot{Y} = \begin{bmatrix} (0.33, 0.70) & (0.09, 0.98) \\ (0.19, 0.75) & (0.19, 0.91) \\ (0.30, 0.55) & (0.30, 0.68) \\ (0.26, 0.60) & (0.26, 0.87) \\ (0.12, 0.90) & (0.12, 0.96) \\ (0.45, 0.60) & (0.45, 0.73) \end{bmatrix} \quad (33)$$

Finally, the combined Pythagorean decision matrix is called ξ:

$$\xi = \begin{bmatrix} 0.49 & 0.60 & 0.52 & 0.81 & (0.33, 0.70) & (0.09, 0.98) \\ 0.49 & 0.60 & 0.52 & 0.41 & (0.19, 0.75) & (0.19, 0.91) \\ 0.34 & 0.36 & 0.52 & 0.32 & (0.30, 0.55) & (0.30, 0.68) \\ 0.30 & 0.36 & 0.31 & 0.24 & (0.26, 0.60) & (0.26, 0.87) \\ 0.39 & 0.12 & 0.21 & 0.08 & (0.12, 0.90) & (0.12, 0.96) \\ 0.39 & 0.06 & 0.21 & 0.08 & (0.45, 0.60) & (0.45, 0.73) \end{bmatrix}. \quad (34)$$

Step 4. Calculate the combined weighted Pythagorean fuzzy decision matrix (WPFDM).

$$R' = \begin{bmatrix} 0.03 & 0.11 & 0.14 & 0.28 & (0.16, 0.03) & (0.03, 1.01) \\ 0.03 & 0.11 & 0.14 & 0.14 & (0.09, 0.94) & (0.06, 0.99) \\ 0.02 & 0.07 & 0.14 & 0.11 & (0.14, 0.88) & (0.10, 0.96) \\ 0.02 & 0.07 & 0.08 & 0.09 & (0.13, 0.89) & (0.08, 0.99) \\ 0.02 & 0.02 & 0.05 & 0.03 & (0.06, 0.98) & (0.04, 1.0) \\ 0.02 & 0.01 & 0.05 & 0.03 & (0.22, 0.89) & (0.15, 0.97) \end{bmatrix}. \quad (35)$$

Step 5. Defuzzify BNx_i and Cx_j by means of (25).

Table 13 depicts the fuzzification related to BNx_i and Cx_j criteria.

Step 6. Compute the sum of BNx_i and Cx_j.

Table 14 denotes the results.

Step 7. Calculate the value of Ny_i.

Table 15 depicts the results.

Step 8. Rank the alternatives.

Table 15: Rank for each alternative via PF-MOORA.

Alternative	Ny_i	Rank
T_1	0.229	1
T_2	0.028	5
T_3	0.100	4
T_4	0.137	3
T_5	0.014	6
T_6	0.151	2

Table 16: Comparison of rank for each alternative.

Alternatives	IF-MOORA	PF-TOPSIS	PF-MOORA method proposed	Rank
P_1	0.434	−0.219	0.576	1
P_2	0.321	−1.135	0.415	2
P_3	0.227	−1.005	0.337	3
P_4	−0.124	−0.124	−0.049	5
P_5	0.215	−0.760	0.294	4

Table 17: Comparison of rank for each alternative.

PF-TOPSIS	$T_1 \succ T_5 \succ T_2 \succ T_3 \succ T_6 \succ T_4$
PF-MOORA proposed	$T_1 \succ T_5 \succ T_4 \succ T_3 \succ T_6 \succ T_2$
PF-TODIM	$T_3 \succ T_5 \succ T_6 \succ T_4 \succ T_2 \succ T_1$

Table 14 shows the results in which the order of the alternatives reveals that tool number 5 is the best option.

$$T_5 \succ T_2 \succ T_1 \succ T_4 \succ T_3 \succ T_6. \quad (36)$$

After trials run implemented using T_5, good improvements are seen with regard to the quality of the products manufactured by the *company* involved in this study.

5.1. Comparison with Other Methods. In this section, we evaluate the method proposed through comparison with alternative methods. To validate the effectiveness of both PF-MOORA, we conducted a comparative analysis with respect to the Pythagorean fuzzy TOPSIS (PF-TOPSIS) and the intuitionistic fuzzy MOORA (IF-MOORA)

The results reveal $P_1 \succ P_2 \succ P_3 \succ P_5 \succ P_4$ as seen in Table 16. Likewise, the results are just as our method proposed. At the same time, an advantage of PF-MOORA is shown in comparison to alternative methods with regard to contribution value about the ranking of the best alternative. It is clear that the PF-MOORA method is more proficient due to the Pythagorean fuzzy taking into count the membership and the nonmembership degrees to be operative and capture the uncertainty in MCDM problems.

Likewise, a second comparison was performed in Example 2. Table 17 illustrates the results.

The results of comparisons between PF-TOPSIS and our proposed method show coincidence in the selection and determine T_1 to be the best alternative. However, the PF-TODIM reports different order in alternatives as shown in Table 17. In general, the methods of MCDM present a drawback with regard to how to operate nonfuzzy and fuzzy data simultaneously. Similarly, there is weakness in the manipulation of the imprecision and uncertainty involved in measuring the preferences of the decision makers when evaluating criteria and alternatives [35–37]. In this sense, the PF-MOORA method introduces two algorithms in order to manipulate fuzzy and nonfuzzy data. In this mode our method describes a systematic way and computational ratio to choose the best alternative. At the same time, PF-MOORA requires low setup time to carry out the analysis for determining the best alternative.

6. Conclusions

This study introduces a hybrid of MOORA and Pythagorean fuzzy sets (PF-MOORA) for multiple criteria decision making. The PF-MOORA method is defined via eight steps, and two experiments were carried out to illustrate it. The proposed methodology delivers a strong hybrid method that can give decision makers support for choosing the best alternative. In addition to the advantage of this study, intangible information that carries flexibility to handling this kind of data involved in MCDM problems is addressed. At the same time, the comparisons reveal the potential of PF-MOORA to assign the ranking of the best alternative and coincidences with the results from PF-TOPSIS.

In future work, it would be motivating to apply PF-MOORA to diverse MCDM problems, for example, robot selection, risk management, personnel selection, and project selection. Lastly, making comparisons with other methods and appraising the results are recommended, while, at the same time, exploring diverse aggregation operators to evaluate it.

Conflicts of Interest

The authors state that there are no conflicts of interest concerning the publication of this paper.

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