A Critical Review of Wigner's Work on the Conceptual Foundations of Quantum Theory [1]

Hans Primas [2] *and Michael Esfeld* [3]

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2 Laboratory of Physical Chemistry, ETH-Zentrum, CH-8092 Zürich (Switzerland),

3 Fachgruppe Philosophie, Fach D24, Universität Konstanz, D–78457 Konstanz (Germany),

A Critical Review of Wigner's Work on the Conceptual Foundations of Quantum Theory[4]

Hans Primas[5] *and Michael Esfeld*[6]

Jagdish Mehra (ed.), *The Collected Works of Eugene Paul Wigner. Part B. Historical, Philosophical, and Socio-Political Papers. Volume VI. Philosophical Reflections and Syntheses.* (Berlin: Springer-Verlag, 1995), 631 pp.

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Arthur Wightman (ed.), *The Collected Works of Eugene Paul Wigner. Part A. The Scientific Papers. Volume I. Part I: Eugene Paul Wigner – A Biographical Sketch. Part II: Applied Group Theory 1926–1935. Part III: The Mathematical Papers.* (Berlin: Springer-Verlag, 1993), 717 pp.

Arthur Wightman (ed.), *The Collected Works of Eugene Paul Wigner. Part A. The Scientific Papers. Volume III. Part I: Particles and Fields. Part II: Foundations of Quantum Mechanics.* (Berlin: Springer-Verlag, 1997), 576 pp.

Introduction

Background of Eugene Paul Wigner

Jenö Pál Wigner was born in 1902 in the Jewish community of Budapest. For four years he went to the same high school in Budapest as his life long associate Johann von Neumann. Under his influence he developed a deep affinity to pure mathematics and mathematical physics. But following his fathers advice, he decided to study chemistry. After a first year at the technical University in Budapest, he switched to Berlin in 1922. At the same time, several other Hungarians such as Michael Polanyi, Leo Szillard and Johann von Neumann were also in Berlin, and they had close contact with each other. Under the supervision of Michael Polanyi, Wigner got the Dr. Ing. degree in chemical engineering from the Technische Hochschule in Berlin in 1925. In the same year, he studied the first paper by Born and Jordan on matrix mechanics. By 1929 he had already made seminal contributions to the new quantum theory, and two years later his classic textbook appeared on his powerful group-theoretical methods for the discussion of the symmetry of atoms and molecules.[7] In 1929 Princeton University offered John von Neumann a professorship. Since von Neumann wished to retain his contact with Hilbert's mathematical community in Göttingen, he suggested to share this professorship with Wigner on a half-time basis. Upon arrival in Princeton in 1930, Wigner was offered a continuation of this arrangement for five years. In 1933, the Nazi regime cancelled his German appointment. This event started Wigner's lifelong career in the United States during which he made impressive contributions to physical chemistry, solid state physics and nuclear physics. His contribution to the foundations of quantum theory culminated in his seminal 1939 paper on relativistic invariance.[8] He became a citizen of the United States in 1937. From 1938 to his retirement in 1971 he was Thomas D. Jones Professor of Mathematical Physics at Princeton University. In 1939, the discovery of nuclear fission initiated a collaboration with Leo Szillard and Enrico Fermi on nuclear chain reactions. From 1942 to 1945 Wigner acted as director of the theoretical section at the Chicago Metallurgical Laboratory. Here he was actually working on reactor engineering, leading to the detailed engineering plans for the first nuclear reactors that where ultimately built at

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5 Laboratory of Physical Chemistry, ETH-Zentrum, CH-8092 Zürich (Switzerland),

6 Fachgruppe Philosophie, Fach D24, Universität Konstanz, D–78457 Konstanz (Germany),

7 E. P. Wigner, Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren, Braunschweig, Vieweg (1931).

8 E. P. Wigner, *On unitary representations of the inhomogeneous Lorentz group,* Annals of Mathematics **40,** 149–204 (1939). Reprinted in the *Collected Works,* volume 1, pp. 334–389.

Hanford. In 1960 he received the *Atoms for Peace Award*, and in 1963 he shared the *Nobel Price in Physics* for the "discovery and application of fundamental symmetry principles" and his pioneering work in nuclear physics with Marie Goeppert-Mayer and Hans Daniel Jensen. He died 1995 at the age of 92 in Princeton.

The Collected Works of Eugene Paul Wigner

All of Wigner's mathematical, physical, engineering, historical, philosophical and political papers as well as hitherto unpublished materials are reprinted in the Collected Works in eight volumes. The material is divided into part A (edited by Arthur Wightman), covering the scientific papers, and part B (edited by Jagdish Mehra), dealing with historical, philosophical, and socio-political papers. The five volumes of part A are already published. Volume 1 covers the mathematical papers; volume 2 is on nuclear physics; volume 3 deals with particles, fields, and the foundations of quantum mechanics; volume 4 collects the papers on physical chemistry and solid state physics; volume 5 contains the work on nuclear energy. From the three volumes of part B, only volume 6 with the title "Philosophical Reflections and Syntheses" is published so far. Volume 7 ("Historical and Biographical Reflections and Syntheses", and volume 8 ("Socio-Political Reflections and Civil Defence") are in preparation.

Wigner's collected works are beautifully produced and competently commented. It is, however, regrettable that no summarizing list of his publications is included. Unfortunately, some of the papers are not reprinted from the original papers but from earlier reprint volumes. In the reprinted lecture *Interpretations of Quantum Mechanics*[9], the references are missing. Moreover, the collaboration between the editors of parts A and B leaves much to be desired. The division of the papers on the measuring process between volume 3 and volume 6 is capricious and very inconvenient for the reader. Furthermore, it makes no sense to reprint six papers (some even from different sources) both in volume 3 and in volume 6.[10]

The topics to be discussed

As a starting point for our review, we discuss Wigner's philosophical reflections collected in volume 6. We use this volume as a springboard for a short review of some modern views on the conceptual foundation of quantum theory which is indispensable to acknowledge Wigner's contributions properly. In order to appreciate Wigner's fundamental contributions to quantum theory and the problem of measurement, we also discuss important papers from part III ("The Mathematical Papers") of volume 1, and of part I ("Particles and Fields") and part II ("Foundations of Quantum Mechanics") of volume 3. But we do not consider his important contributions to physical chemistry, applied group theory, solid state physics, nuclear physics, nuclear engineering and civil defence.

We shall review Wigner's contributions on conceptual problems of quantum theory in the light of the recent developments, but we neglect the historical point of view which alone could do justice to the achievements of this distinguished engineer and scientist. We shall place our emphasis on a critical evaluation of the differences between the orthodox view as represented by

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9 *Collected Works,* volume 6, pp. 78–132.

10 The paper in volume 3, pp. 415–422 is also contained in vol.6, pp.147–154; the paper in volume 3, pp. 432–439 is also contained in vol.6, pp.155–162; the paper in volume 3, pp. 440–441 is also contained in vol.6, pp.31–32; the paper in volume 3, pp. 442–451 (reprinted from the original paper) is also contained in vol.6, pp.163–180 (reprinted from a reprint volume); the paper in volume 3, pp. 475–482 (reprinted from the original paper) is also contained in vol.6, pp.181–188 (reset by Springer-Verlag for this volume), the paper in volume 3, pp. 567–571 is also contained in vol.6, pp.139–143.

von Neumann and Wigner and the recent developments in theoretical as well as in experimental science. In particular, we shall consider the following topics:

- In *chapter 1* we compare Wigner's view of physical reality with some recent developments in philosophy. In particular, we discuss Wigner's statement that quantum mechanics on its own makes it necessary to refer to consciousness in physics. Moreover, we contrast Wigner's instrumentalistic view with realistic approaches which are not committed to hidden variables.
- In *chapter 2* we discuss the relations between quantum theory and experimental science. Wigner's adoption of von Neumann's measurement model and the assertion that a measurement of an observable gives as a result one of the eigenvalues of that observable is critically evaluated and compared with the modern developments in theoretical and experimental physics. We use Scheibe's distinction of individual and statistical descriptions, and of ontic and epistemic interpretations to clarify the questions connected therewith.
- In *chapter 3* we deal with Wigner's presumption that quantum mechanics applies only to strictly closed systems. This limitation is not only unnecessary but also unrealistic. It has been overcome by the modern theory of open quantum systems which is indispensable for an understanding of many present-day experiments. The referent of modern quantum theory is not an isolated system but always an open system which interacts with its environment and which may be entangled with its environment.
- In *chapter 4* we discuss Wigner's speculation that in some contexts the appropriate Schrödinger equation may be nonlinear. In order to appreciate this idea one has to distinguish carefully between individual and statistical descriptions. Moreover, one has to differentiate between the superposition principle and the linearity of the Schrödinger equation.
- In *chapter 5* we elaborate on the modern formalism of quantum theory which owes much to Wigner's pioneering work. The more recent developments replace von Neumann's irreducibility postulate by a general representation theory of the Heisenberg–Weyl commutation relations. The existence of inequivalent representations sheds a new light on many conceptual problems which troubled Wigner.
- In *chapter 6* we consider the existence of classical features of quantum systems. Wigner's tacit assumption that quantum systems cannot exhibit a classical behaviour is not valid. The existence of classical quantum states makes the following clear: Bohr's requirement that measuring instruments have to admit of a classical description is not in contradiction with von Neumann's postulate that even instruments must be described by quantum mechanics.
- In *chapter 7* we discuss Wigner's insolubility theorem (and its generalizations) for von Neumann's model of the measurements process. These theorems merely show that von Neumann's model for measurements of the first kind is inappropriate but not that quantum mechanics cannot describe experimental facts. We outline the requirements for a valid theory of experiments.
- In *chapter 8* we argue that Wigner's positivistic position is inappropriate for appreciating the fact that a stochastic behaviour of physical systems is compatible with deterministic equations of motion. The modern developments in the theory of deterministic chaos are essential to understand laboratory measurements and quantum processes which lead to the emergence of irreversibility and objective facts.

1. Wigner's view of the physical reality

1.1 Von Neumann's model for the measuring process

All of Wigner's contributions to the interpretation of quantum theory are based on von Neumann's problematic model of the measuring process. Since Wigner draws far-reaching philosophical conclusions from the associated so-called measurement problem, we first sketch in this section Neumann's model in its simplest form. In the following chapters we shall more critically review the problem of laboratory measurements. For the moment we just warn the reader that von-Neumann measurements never describe measurements which can be actually performed in the laboratory. This misuse of a well-established engineering terminology has led to many confusions in the discussion of the interpretation of quantum theory.

As early as 1926, Paul Adrien Maurice Dirac proposed that in special cases one can attribute numerical values to observables:

Dirac's assignment of numerical values to observables (1)

> "We may regard an eigenfunction ψ_n as being associated with definite numerical values for some of the constants of integration of the system. Thus, if we find constants of integration $a, b, \cdots$ such that $a\psi_n = a_n\psi_n$, $b\psi_n = b_n\psi_n$, $\cdots$, where $a_n, b_n, \cdots$ are numerical constants, we can say that ψ_n represents a state of the system in which $a, b, \cdots$ have the numerical values $a_n, b_n, \cdots$." [11]

In his early textbook of 1930, Dirac *postulated* that an measurement of an observable always yields one of its eigenvalues:[12]

Dirac's jump postulate (2)

> "A measurement always causes the system to jump into an eigenstate of the dynamical variable that is being measured, the eigenvalue this eigenstate belongs to being equal to the result of the measurement." [13]

From a slightly different point of view, based on a repeatability axiom (requiring that two measurements of the same observable made in rapid succession give equal results), Johann von Neumann[14] came to an equivalent mathematical formulation of an idealized measurement operation. Unfortunately, in much of the literature such fictive operations are simply called "measurements". In order to avoid misunderstandings, we shall adopt a terminology proposed by Wolfgang Pauli[15], and refer to the (theoretically meaningful) von-Neumann operation as a *"measurement of the first kind"*, never simply as a "measurement"

In the framework of the irreducible Hilbert-space representation of traditional quantum mechanics, a pure state of an object system can be represented by a normalized vector in a separable complex Hilbert space $\mathscr{H}_{\text{obj}}$, called *state vector*. Von Neumann offered a schematic model for a

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11 P. A. M. Dirac, *On the theory of quantum mechanics,* Proceedings of the Royal Society (London) **A 112,** 661–677 (1926). The quotation is on p. 666.

12 Compare also the equivalent formulation in M. Born & P. Jordan, *Elementare Quantenmechanik,* Berlin, Springer (1930), p. 291.

13 P. A. M. Dirac, *The Principles of Quantum Mechanics,* Oxford, Clarendon Press (1930), p. 36.

14 J. von Neumann, *Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik,* Nachrichten von der Gesellschaft der Wissenschaften, Göttingen, Mathematisch Physikalische Klasse **1927,** 245–272 (1927). Compare p. 271.

15 W. Pauli, *Die allgemeinen Prinzipien der Wellenmechanik,* in: H. Geiger & K. Scheel (Hgg.), *Handbuch der Physik,* 2. Auflage, Band 24, 1. Teil. Berlin, Springer (1933), pp. 83–272. Compare Ziff. 9, p. 152.

description of a measurement of the first kind in which the apparatus is included.[16] It is assumed that the joint system consisting of the object system and the measuring apparatus is a strictly closed quantum system which can be described by the rules of traditional quantum mechanics. In this approach, the measuring apparatus is treated as a quantum mechanical system, so that the Hilbert space $\mathscr{H}_{\text{obj}}$ of the object system is enlarged to $\mathscr{H} = \mathscr{H}_{\text{obj}} \otimes \mathscr{H}_{\text{app}}$, where $\mathscr{H}_{\text{app}}$ is the Hilbert space of the apparatus. Von Neumann considers an observable A of the object system and assumes that A has the simple discrete spectrum $a_1, a_2, \cdots$ and the orthonormalized eigenvectors $\alpha_1, \alpha_2, \cdots$, $A\alpha_k = a_k \alpha_k$, $\alpha_k \in \mathscr{H}_{\text{obj}}$.[17] As a first step, he defines a unitary map $U : \mathscr{H}_{\text{obj}} \otimes \mathscr{H}_{\text{app}} \to \mathscr{H}_{\text{obj}} \otimes \mathscr{H}_{\text{app}}$ with the property

$$U\{\alpha_k \otimes \beta\} = \alpha_k \otimes \beta_k \tag{3}$$

for every initial state vector $\beta \in \mathscr{H}_{\text{app}}$ of the apparatus. Von Neumann requires a one-to-one correspondence between the eigenvalue a_k and the data indicated by a pointer of the apparatus. Therefore he chooses the unitary map U in such a way that the vectors $\beta_1, \beta_2, \cdots$ form an orthonormalized system. He showed that such a choice is always possible. The linearity of the map U implies that for an arbitrary initial state vector $\alpha = \sum_k c_k \alpha_k \in \mathscr{H}_{\text{obj}}$ of the object system the von-Neumann map is given by

$$U\{\alpha \otimes \beta\} = \sum\nolimits_k c_k \alpha_k \otimes \beta_k \;\;,\;\; \alpha, \alpha_k \in \mathscr{H}_{\text{obj}} \;\;,\;\; \beta, \beta_k \in \mathscr{H}_{\text{app}} \;\;,\;\; |c_k|^2 = |\langle \alpha_k | \alpha \rangle|^2 \;\;. \tag{4}$$

Since this operation gives the same result when immediately repeated,

$$U\{U(\alpha \otimes \beta)\} = U\{\textstyle\sum_k c_k \alpha_k \otimes \beta_k\} = \textstyle\sum_k c_k \alpha_k \otimes \beta_k \;\;, \tag{5}$$

von Neumann associates this map with a *repeatable* instantaneous measurement of the first kind. Consider now an apparatus observable $B := \sum_k b_k |\beta_k\rangle\langle\beta_k|$ with $b_k \neq b_\ell$ for $k \neq \ell$. If we know that the apparatus observable has the value b_j, then we know that the transformed vector $\alpha \otimes \beta$ equals $\alpha_j \otimes \beta_j$, so that after this operation the observable A has the value a_j. In general, the resulting joint state vector after a measurement of the first kind is not an eigenvector of the observable $A \otimes B$, but a superposition $\sum_k c_k \alpha_k \otimes \beta_k$ of eigenvectors $\alpha_k \otimes \beta_k$. In order to know the value of the apparatus observable B, we have to iterate this procedure. That is, we introduce an additional apparatus with an associated observable $C := \sum_k c_k |\gamma_k\rangle\langle\gamma_k|$. Of course, the resulting joint state vectors is still a coherent superposition $\sum_k c_k \alpha_k \otimes \beta_k \otimes \gamma_k$ of eigenvectors $\alpha_k \otimes \beta_k \otimes \gamma_k$ of the observable $A \otimes B \otimes C$. Evidently this does not help since in this way we simply get a chain of apparatuses which observe apparatuses. As underscored by Wigner many times,[18] this procedure does not lead to a measurement. Nonetheless, von Neumann obtained the important mathematical result that any cut between the observed system and the observing tools is consistent with the quantum-theoretical formalism. Von Neumann adds that the principle of psycho-physical parallelism implies that this iteration can be pushed arbitrarily deeply into the interior of a human observer. In order to avoid an infinite regress of this chain of measurements,

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16 J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Berlin, Springer (1932), chapter VI.

17 The generalization for observables with degenerate eigenvalues is not essential for our discussion. It has been discussed by G. Lüders, *Über die Zustandsänderung durch den Messprozess,* Annalen der Physik **8**, 322–328 (1951).

18 Compare p. 12 in E. P. Wigner, *The problem of measurement,* American Journal of Physics **31**, 6–15 (1963), (in the *Collected Works,* volume 6, on p. 173); and p. 5 in E. P. Wigner, *The non-relativistic nature of the present quantum-mechanical measurement theory,* Annals of the New York Academy of Science **480**, 1–5 (1986), (in the *Collected Works,* volume 6, on p. 143).

von Neumann makes use of Dirac's jump postulate and terminates the chain by an ad hoc *reduction postulate*

$$\sum_k c_k\,\alpha_k \otimes \beta_k \otimes \gamma_k \;\rightarrow\; \alpha_j \otimes \beta_j \otimes \gamma_j \quad \text{with probability } |c_k|^2 = |\langle \alpha_k | \alpha \rangle|^2 \quad , \qquad (6a)$$

or in abridged reduced formulations as

$$\sum_k c_k\,\alpha_k \otimes \beta_k \;\rightarrow\; \alpha_j \otimes \beta_j \quad \text{with probability } |c_k|^2 = |\langle \alpha_k | \alpha \rangle|^2 \quad , \qquad (6b)$$

$$\sum_k c_k\,\alpha_k \;\rightarrow\; \alpha_j \quad \text{with probability } |c_k|^2 = |\langle \alpha_k | \alpha \rangle|^2 \quad . \qquad (6c)$$

The reduction postulate describes an acausal transition of a pure initial state to a pure final state. In the literature it is also called the "collapse postulate". The nonlinear *state reduction map* $\sum_k c_k\,\alpha_k \rightarrow \alpha_j$ is also referred to as "collapse of the wave packet" or the "reduction of the wave function". It corresponds to the "quantum jumps" of Bohr's old quantum theory.[19]

Von Neumann those arrives at two radically different types of evolution: the deterministic and linear Schrödinger equation which describes the continuous change of the state vector when the object is not subject to a measurement, and the discontinuous, stochastic and nonlinear state change when a measurement of the first kind is carried out on the system.[20] According to Wigner, "the assumption of two types of change of the state vector is a strange dualism." [21] It is one of the main topics of the so-called measurement problem of quantum mechanics.

The fact that "the 'reduction of the wave packet' enters quantum mechanics as a *deus ex machina*, without any relation to the other laws of this theory" [22], has led to a long controversy, lasting for over half a century. The main questions are:

- Is quantum mechanics a complete theory or has it to be completed by introducing hidden variables?
- Is quantum mechanics valid for macroscopic systems?
- Is a discontinuous state reduction $\sum_k c_k\,\alpha_k \rightarrow \alpha_j$ necessary at all for the discussion of laboratory measurements?
- If the state reduction $\sum_k c_k\,\alpha_k \rightarrow \alpha_j$ is necessary, is it an irreducible law of nature, or is it just a convenient working rule?
- If the state reduction $\sum_k c_k\,\alpha_k \rightarrow \alpha_j$ is irreducible, is it a physical process, a mathematical process[23], or an act of consciousness?

In all his work, Wigner defends the necessity of state reductions. In his last paper on conceptual questions, he says that the reduction postulate $\alpha_k \otimes \beta \rightarrow \alpha_k \otimes \beta_k$ "is both natural and necessary".[24] Considering the many worlds interpretation, he takes the notion of a state function of the

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19 In 1913, Niels Bohr suggested that light and matter interact in such a way that an atom jumps instantaneously between stationary states and thereby absorbs or emits a light quantum. Compare N. Bohr, *On the constitution of atoms and molecules. Part I,* Philosophical Magazine **26**, 1–25 (1913),

20 Compare section VI.1, pp. 222–225, in: J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Berlin, Springer (1932).

21 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31**, 6–15 (1963). The quotation is on p. 7; in the *Collected Works,* volume 6, on p. 165.

22 E. P. Wigner, *Two kinds of reality,* The Monist **48**, 248–264 (1964). The quotation is on p. 188; in the *Collected Works,* volume 6, on p. 36.

23 Defended by Heisenberg in a letter of February 2, 1960, to Renninger. Quoted on p. 496 in M. Jammer, *The Philosophy of Quantum Mechanics,* New York, Wiley (1974).

24 E. P. Wigner, *Some problems of our natural sciences,* International Journal of Theoretical Physics **25**, 467–476 (1986). The quotation is on p. 469; in the *Collected Works,* volume 6, on p. 618.

universe to be meaningless.[25] Consequently, he refuses to dissolve the problem by assuming a splitting of worlds instead of a state reduction. Furthermore, Wigner renounces all attempts to change quantum mechanics by introducing hidden variables.[26]

In his early papers, Wigner rejects the idea that the measuring instrument does not admit of a description in terms of quantum mechanics of isolated systems.[27] In 1978 he mentions for the first time doubts on the validity of this view.[28] On the basis of the obvious fact that a macroscopic system can never be considered as isolated from its environment, he later concludes "that quantum mechanics' validity has narrower limitations, that it is not applicable to the description of the detailed behavior of macroscopic bodies".[29]

1.2 Does consciousness modify the usual laws of physics?

Von Neumann follows the chain of measurements via physiological processes up to brain processes; but he is not very explicit on the termination of the chain of observing systems. In reviewing von Neumann's theory, Fritz London and Edmond Bauer unmistakably attribute the capacity to select a product state out of the superposition to the human consciousness.[30] Wigner's initial proposal for a solution to the state reduction problem consists also in postponing the state reduction to the very end of the von Neumann chain. In spite of his assertion that "we know far too little of the properties and the working of the consciousness to propose a philosophy of consciousness" [31], he adopted for many years the thesis that the reduction of the wave packet takes place only in the consciousness of the observer.[32] Typical for Wigner's view in this period is the following statement:

> "[The reduction of the wave packet] takes place whenever the result of an observation enters the consciousness of the observer – or, to be even more painfully precise, my consciousness, since I am the only observer, all other people being only subjects of

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25 Compare p. 382 in E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385 (in the *Collected Works,* volume 6, on p. 68); on p. 294 in E. P. Wigner, *Interpretation of quantum mechanics,* in: J. A. Wheeler & W. H. Zureck (eds.), *Quantum Theory and Measurement,* Princeton, Princeton University Press (1983), pp. 260–314 (in the *Collected Works,* volume 6, on p. 112).

26 Compare pp. 377–378 in E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385 (in the *Collected Works,* volume 6, on pp. 63–64); pp. 289–297 in E. P. Wigner, *Interpretation of quantum mechanics,* in: J. A. Wheeler & W. H. Zureck (eds.), *Quantum Theory and Measurement,* Princeton, Princeton University Press (1983), pp. 260–314 (in the *Collected Works,* volume 6, on pp. 107–115); on pp. 73–75 in E. P. Wigner, *Review of the quantum mechanical measurement process,* in: P. Meystre & M. O. Scully (eds.), *Quantum Optics, Experimental Gravity, and Measurement Theory,* New York, Plenum Press (1983), pp. 43–63 (in the *Collected Works,* volume 6, on pp. 235–237).

27 Compare for example pp. 433–434 in E. P. Wigner, *Epistemology of quantum mechanics,* In: *Contemporary Physics. Volume II,* Vienna, Atomic Energy Agency (1969), pp. 431–437 (in the *Collected Works,* volume 6, on pp. 50–51).

28 E. P. Wigner, *New dimensions of consciousness,* Mimeographed notes (1978). Reprinted in the *Collected Works*, Volume 6, pp. 268–273

29 E. P. Wigner, *Review of the quantum mechanical measurement process,* in: P. Meystre & M. O. Scully (eds.), *Quantum Optics, Experimental Gravity, and Measurement Theory,* New York, Plenum Press (1983), pp. 43–63. The quotation is on p. 78; in the *Collected Works,* volume 6, on p. 240.

30 Compare §11 in F. London & E. Bauer, *La théorie de l'observation en mécanique quantique,* Paris, Hermann (1939).

31 E. P. Wigner, *Two kinds of reality,* The Monist **48,** 248–264 (1964). The quotation is on p. 251; in the *Collected Works,* volume 6, on p. 36.

32 His first paper on this topic is of 1962 (E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301. Reprinted in the *Collected Works*, Volume 6, pp. 247–260).

> my observations. … The measurement is not completed until its result enters our consciousness. This last step occurs when a correlation is established between the state of the last measuring apparatus and something which directly affects our consciousness. This last step is, at the present state of our knowledge, shrouded in mystery and no explanation has been given for it so far in terms of quantum mechanics, or in terms of any other theory." [33]

Although Wigner thus concedes that we do not have a description at our disposition of how a state reduction is effected in consciousness, he suggests that the dynamics of quantum theory has to be modified in such a way that state reductions by consciousness are taken into account. In particular, Wigner describes his proposal as amounting to "the postulate that the equations of motion of quantum mechanics cease to be linear, in fact that they are grossly non-linear if conscious beings enter the picture." [34]

Wigner's main argument for his view is that it is "difficult to accept the possibility that a person's mind is in a superposition of two states … We ourselves never have felt we were in such superpositions." [35] In this context, Wigner discusses the famous "paradox of Wigner's friend": If one observer describes another observer who observes something like a von Neumann chain and if the first observer describes this whole process in the terminology of quantum mechanics, he will end up with ascribing a superposition of different states of consciousness to the second observer. Wigner avoids this paradox by maintaining that quantum theory does not apply to consciousness so that there are no superpositions of different states of consciousness. A state reduction occurs at the level of the friend's consciousness.[36]

1.3 The content of consciousness as the primary reality

The initial stance which Wigner takes on the state-reduction problem can be explained as a consequence of his view that the contents of consciousness are the primary reality. Wigner distinguishes two kinds of reality, the contents of the consciousness of each person on the one hand and physical objects on the other hand.[37] By the contents of consciousness, Wigner means sense impressions in particular. Wigner claims that the content of consciousness of an observer is the primary reality: it cannot be denied, it is absolute. He holds that the reality of physical objects is relative to the contents of consciousness. According to Wigner, physical objects are constructed on the basis of the contents of consciousness: to assume the existence of physical objects is useful in order to account for the connections among the contents of consciousness.

The notion of the content of consciousness with which Wigner works is such that the content of consciousness is immediately accessible only to the respective person. Wigner therefore maintains that the existence of other persons with contents of consciousness of their own is on the

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33 E. P. Wigner, *Two kinds of reality,* The Monist **48,** 248–264 (1964). The quotation is on pp. 249–250; in the *Collected Works,* volume 6, on pp. 34–35.

34 E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301. The quotation is on p. 279; in the *Collected Works,* volume 6, on p. 259.

35 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 381; in the *Collected Works,* volume 6, on p. 67.

36 E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301. Compare pp. 293–294, in the *Collected Works,* volume 6, pp. 255–257.

37 E. P. Wigner, *Two kinds of reality,* The Monist **48,** 248–264 (1964). Reprinted in the *Collected Works*, Volume 6, pp. 33–47.

same footing as the existence of physical objects.[38] That Wigner regards the contents of consciousness as the primary reality and considers physical objects to be constructs in order to account for the connections among the contents of consciousness explains why he invokes the consciousness of the observer in order to solve the measurement problem: Given this philosophical presupposition, there is no reason to seek for an interpretation or a modification of quantum theory which yields a description of a state reduction at some link within the von Neumann chain. The only thing which counts for Wigner as an absolute reality is the content of the consciousness of an observer. Wigner thus adopts an instrumentalistic attitude towards physical theories. Consequently, on the question whether life and consciousness can be explained in terms of the laws of physics, he holds that

> "the regularities which modern physical theory – that is quantum mechanics – furnishes are probability connections between subsequent observations, i.e. contents of the consciousness of the observer. The primitive facts in terms of which the laws are formulated are not positions of atoms but the result of observations. It seems inconsistent, therefore, to explain the state of mind of the observer, his apperception of the result of an observation, in terms of concepts, such as positions of atoms, which have to be explained, then, in terms of the content of consciousness." [39]

In his later papers, Wigner expressed his "desire for a less solipsistic theory"[40] but only in last papers he considers the consequence of solipsism as a sufficient reason to repudiate his earlier views:

> "This means, particularly if we want to avoid the solipsistic attitude, that we should not consider an observation to be the basic concept but should try to describe it also, not only ours but also those of others, even including those of animals, using the ideas of quantum mechanics. This is a natural requirement – but runs into difficulties." [41]

At the end of a paper published in 1977, Wigner expresses the hope

> "that quantum mechanics will also turn out to be a limiting case, limiting in more than one regard, and that the philosophy which an even deeper theory of physics will support will give a more concrete meaning to the word 'reality', will not embrace solipsism, much truth as this may contain, and will let us admit that the world really exists." [42]

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38 Compare E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301, pp. 285–291 (*Collected Works*, Volume 6, pp. 248–254); E. P. Wigner, *Two kinds of reality,* The Monist **48**, 248–264 (1964), , pp. 189–194 and p. 199 (*Collected Works*, Volume 6, pp. 37–42 and p. 47); E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385, p. 377 and p. 382 (*Collected Works*, Volume 6, p. 63 and p. 68).

39 E. P. Wigner, *Are we machines?,* Proceedings of the American Philosophical Society **113**, 95–101 (1969). The quotation is on p. 96; in the *Collected Works,* volume 3, on p. 484.

40 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 382; in the *Collected Works,* volume 6, on p. 68.

41 E. P. Wigner, *Review of the quantum mechanical measurement process,* in: P. Meystre & M. O. Scully (eds.), *Quantum Optics, Experimental Gravity, and Measurement Theory,* New York, Plenum Press (1983), pp. 43–63. The quotation is on p. 68; in the *Collected Works,* volume 6, on p. 230.

42 E. P. Wigner, *Physics and its relation to human knowledge,* Hellenike Anthropistike Hetaireia, Athens, 283-294 (1977). In the *Collected Works*, Volume 6, on p. 593.

1.4 Critical evaluation of Wigner's view of physical reality

Wigner frequently claims that his view according to which the content of consciousness is the primary reality and the reality of physical objects is relative to consciousness is just obvious.[43] Nonetheless, this view shows a strong influence of a specific stream in philosophy on Wigner's thinking – namely the logical positivism of the twenties in particular[44] and the Cartesian tradition in modern Western philosophy as a whole. Logical positivism holds that science is to be built upon a firm foundation of basic statements such as statements describing sense data. As a result, scientific theories are considered as constructions upon such basic statements or upon sense data, and their objects are posits to explain regularities among sense data. According to Wigner, "positivistic philosophy means that we attribute reality only to what can be observed." [45]

Wigner claims that "on the part of most physical scientists" there is a "return to the spirit of Descartes's *'Cogito ergo sum'*, which recognizes the thought, that is, the mind, as primary."[46] Descartes seeks an indubitable foundation of knowledge which is reached by self-reflection. In Descartes, claims about the existence of physical objects are deduced on the basis of clear and distinct ideas in consciousness. Wigner does not, like Descartes, think of body and mind as two distinct entities.[47] Nonetheless, his position that the content of consciousness is the primary reality shares at least two premises with the Cartesian tradition:

- The object of knowledge is primarily the content of consciousness.
- The content of consciousness is independent of the external world in the sense that it could be the same if the physical world were totally different from the way it actually is or even if there were no physical world at all.

Only on the basis of the second premise is it possible to maintain that physical objects are constructed upon the contents of consciousness and that the reality of physical objects is relative to the contents of consciousness. Both these premises are strongly challenged in today's philosophy of mind:

- Instead of maintaining that the content of consciousness is the primary object of knowledge, many contemporary philosophers favour a direct realism: things in the physical world are the direct object of knowledge including perceptual knowledge. The main argument is that if there are basic contents of consciousness (such as sense impressions, for instance), they cannot be both something simply given prior to conceptualization and a foundation which serves to justify knowledge.[48] Consequently, when it comes to

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43 Compare p. 192 in E. P. Wigner, *Two kinds of reality,* The Monist **48,** 248–264 (1964). In the *Collected Works*, Volume 6, on p. 40.

44 Compare Wigner's bold statement "for a positivist (as most of us are) …", in E. P. Wigner, *The philosophical problem,* in: B. d'Espagnat (ed.), *Foundations of Quantum Mechanics. International School of Physics "Enrico Fermi", 1970,* New York, Academic Press (1971), pp. 122–124. In the *Collected Works*, Volume 6, on p. 219.

45 E. P. Wigner, *The limitations of determinism,* in: *Absolute Values and the Creation of the New World. Proceedings of the 11th International Conference on the Unity of Science, 1982,* New York, International Cultural Foundation Press (1983), pp. 1365–1370. The quotation is on p. 1370; in the *Collected Works,* volume 6, on p. 138.

46 E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301. The quotation is on p. 98; in the *Collected Works,* volume 6, on p. 248.

47 E. P. Wigner, *The extension of the area of science,* in: R. G. Jahn (ed.), *The Role of Consciousness in the Physical World,* AAAS Symposium No.57, Boulder, Westview Press (1981), pp. 7–16. In the *Collected Works,* volume 6, on p. 609.

48 This argument goes back to W. Sellars, *Empiricism and the philosophy of mind,* in: W. Sellars (ed.), *Science, Perception and Reality,* London, Routledge (1963), pp. 127–196. For a contemporary elaboration see, for

knowledge, it is maintained that it is physical things which are the primary and direct objects of our beliefs.

- It is argued that we can have beliefs only if we are embedded in a physical world which makes it that most of our beliefs about that world are true. This dependence is not thought of as a causal one. The main argument is that the individuation of beliefs and thus the conditions for the identity of beliefs depend on what qualitative character the physical world has in which the subject of these beliefs lives[49]. One can thus not take the contents of beliefs or the contents of consciousness to be absolute and regard the reality of physical objects as being relative to beliefs.

The point of these two moves in contemporary philosophy of mind is to preempt the separation of mind and world which haunts modern thought since Descartes. These developments in contemporary philosophy of mind are usually not taken into consideration when claims about the observer and his mind or consciousness are made in the interpretation of quantum theory.[50]

Considering these recent developments in the philosophy of mind, the premises which Wigner takes to be obvious and which constitute the framework on which his early interpretation of quantum mechanics is based turn out to be questionable, to say the least. The second of Wigner's above mentioned premises is intimately connected with the specifically modern philosophical problem of scepticism with respect to the existence of an external world: If the content of consciousness is the primary reality, it makes sense to doubt whether our knowledge is about an external world and whether there is an external world at all. Wigner's refusal to commit himself to quantum mechanics being about a physical reality, i.e. his instrumentalistic attitude towards quantum mechanics, is a manifestation of this scepticism consequent upon regarding contents of consciousness as the primary reality.

1.5 Realism and quantum mechanics

In all his papers Wigner favours an instrumentalistic approach:

> "We also recognize that the laws of quantum mechanics only furnish probability connections between results of subsequent observations carried out on a system." [51]

According to this instrumentalistic view, quantum theory is an instrument to calculate connections among measurement results and to predict measurement results, given a description of the experimental arrangement. By contrast, a realistic approach holds that quantum theory says something about nature independently of measurements. In the last decade, a realistic approach to quantum mechanics has gathered momentum in philosophical circles. This realistic approach is not committed to hidden variables. It amounts to a non-classical physical realism based upon nonseparability.[52]

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instance, J. McDowell, *Mind and World,* Cambridge (Massachusetts), Harvard University Press (1994), in particular lecture 1.

49 For an introduction to this argument, see G. McCulloch, *The Mind and its World,* London, Routledge (1995).

50 A notable exception is the book by M. Lockwood, *Mind, Brain and the Quantum,* Oxford, Blackwell (1989), who examines and argues against central points of this contemporary, anti-Cartesian stream in the philosophy of mind. See in particular chapters 9 and 16.

51 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963). The quotation is on p. 13; in the *Collected Works,* volume 6, on p. 176.

52 Compare for example M. Redhead, *Incompleteness, Nonlocality and Realism. A Prolegomenon to the Philosophy of Quantum Mechanics.,* Oxford, Clarendon Press (1987); M. Redhead, *From Physics to Metaphysics,* Cambridge, Cambridge University Press (1995); T. Y. Cao, *Conceptual Developments of 20th Century Field Theories,* Cambridge, Cambridge University Press (1996).

By and large, working scientists are unabashed realists, they stubbornly believe that there is a real external world. For many theoreticians, this belief is the only raison d'être of physical theories. They would like to have a description of how, fundamentally, the world *is*. For many working scientists, the task of science is to explain what really happens in nature. This view is well summarized by Albert Einstein:

> "Es gibt so etwas wie den 'realen Zustand' eines physikalischen Systems, was unabhängig von jeder Beobachtung oder Messung objektiv existiert und mit den Ausdrucksmitteln der Physik im Prinzip beschrieben werden kann. … Diese These der Realität hat nicht den Sinn einer an sich klaren Aussage, wegen ihrer 'metaphysischen' Natur; sie hat eigentlich nur programmatischen Charakter. Alle Menschen, inklusive die Quanten-Theoretiker halten aber an dieser These der Realität fest, solange sie nicht über die Grundlage der Quantentheorie diskutieren." [53]

Every working scientist considers carefully all details of his measuring instruments. Nevertheless, he never worries about the so called measurement problem. In particular, the London–Bauer–Wigner idea that consciousness reduces the state vector is in conflict with engineering practice. For example, much of our knowledge about the absolute age of prehistoric events is drawn from measurements of the relative content of an radioactive nucleus. According to the generally accepted judgement of the scientific community, the disintegration of the radioactive substance is due to the spontaneous decay of certain nuclei, an objective quantum process which happened long ago. Moreover, there are good reasons to assume that in the cosmological and biological evolution there are objective events, encodings and registrations which are independent of the existence of beings to which consciousness can be attributed. Over and above that, in most modern experiments the data of measurement are not directly observed by a human observer but collected automatically and stored in the memory of a computer. Often the subsequent data processing is performed before anybody takes note of the measuring results. According to the well-established engineering wisdom, the measurement is completed before the data are stored in the local computer memory. It is difficult to see how Wigner's earlier claim that a "measurement is not completed until its result enters our consciousness" [54] can be squared with the experimental facts. However, in an earlier paper he adopts von Neumann's view that a "measurement is not completed until its result is recorded by some macroscopic object." [55]

2. Quantum theory and experimental science

2.1 Wigner's concept of a pure quantum state

The relation between theory and experiment is much more complicated than von Neumann's measurement model presupposes. *No single measurement gives a numerical value which can be attributed to an observable.* Since even for the simplest case of a two-level quantum system, the experimentally relevant sample space for a measurement is uncountable, no experimentalist can realize a measurement of the first kind. A statement like "a measurement gave the result that the

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53 A. Einstein, *Einleitende Bemerkung über Grundbegriffe, Louis de Broglie und die Physiker,* Hamburg, Claassen Verlag (1955), pp. 13–17. The quotation is on p. 14.

54 E. P. Wigner, *Two kinds of reality,* The Monist **48**, 248–264 (1964). The quotation is on p. 187; in the *Collected Works,* volume 6, on p. 35.

55 H. Salecker & E. P. Wigner, *Quantum limitations of the measurement of space–time distances,* Physical Review **109**, 571–577 (1959). The quotation is on p. 571; in the *Collected Works,* volume 3, on p. 148.

observable A has the value $a \in \mathbb{R}$" is operationally untenable since no experiment can distinguish between a rational and an irrational number.

Example: No single measurement gives a numerical value which can be attributed to an observable.

It may be well to illustrate this point by the simple example of a measurement of a two-level quantum system whose algebra of observables is generated by the three Pauli matrices σ_1, σ_2 and σ_3. A measurement of the first kind of the observable σ_3 should result in one of the eigenvectors Ψ_+, Ψ_- of σ_3, $\sigma_3 \Psi_\pm = (\pm 1)\Psi_\pm$. A more realistic (but still crudely idealized) description of a laboratory measurement of the observable σ_3 has to consider the full set of state vectors in the Hilbert space $\mathbb{C}^2$. It is convenient to parametrize the possible state vectors by the Cayley angles ϑ and φ, $0 \le \varphi < 2\pi$, $0 \le \vartheta \le \pi$,

$$|\vartheta,\varphi\rangle := e^{i\varphi/2}\cos(\vartheta/2)\Psi_+ + e^{-i\varphi/2}\sin(\vartheta/2)\Psi_- \in \mathbb{C}^2 \quad .$$

These pure states form an overcomplete set and generate a normalized positive operator-valued measure $\mathcal{B} \mapsto F(\mathcal{B})$ on the σ-field $\boldsymbol{\Sigma}$ of Borel sets of the unit sphere in three dimensions

$$F(\mathcal{B}) := \int_{\mathcal{B}} |\vartheta,\varphi\rangle\langle\vartheta,\varphi| \sin(\vartheta)\, d\vartheta\, d\varphi \quad , \quad \mathcal{B} \in \boldsymbol{\Sigma} \quad .$$

All experimental facts depend in one way or the other on statistical decision tests which imply a partitioning of the sample space. If the measuring apparatus is axial symmetric, the relevant normalized positive operator-valued measure for a σ_3-measurement is given by

$$F([\vartheta_1,\vartheta_2]) = \int_0^{2\pi}\int_{\vartheta_1}^{\vartheta_2} |\vartheta,\varphi\rangle\langle\vartheta,\varphi| \sin(\vartheta)\, d\vartheta\, d\varphi \quad , \quad 0 \le \vartheta_1, \vartheta_2 \le \pi \quad .$$

For example, we may consider the following three hypotheses:

(i) the observation comes from the population $0 \le \vartheta \le \varepsilon$,
(ii) the observation comes from the population $\pi - \varepsilon \le \vartheta \le \pi$,
(iii) the observation comes from the population $\varepsilon < \vartheta < \pi - \varepsilon$.

In order to be able to make statistical tests, the experimentalist must have sufficient information about the error probability of his measuring instrument. For an appropriate choice of ε (for a well-constructed instrument we expect that $0 < \varepsilon \ll 1$) he may, for example, adopt the following decision rule: if the measured value in a single experiment is in the interval $[0,\varepsilon]$, then he decides for "spin up"; if it is in the interval $[\pi-\varepsilon,\pi]$, then he decides for "spin down"; if it is in the interval $(\varepsilon,\pi-\varepsilon)$, then he is forced to assume that an error has occurred. Every experimenter will check whether his experimental arrangement leads to statistically reproducible results. He will repeat the experiment very often, and use statistical tests for the data by its experiments. So he may decide that his experiments are reproducible if the frequency of events in the interval $(\varepsilon,\pi-\varepsilon)$ is inconsequential. He may also improve his experimental arrangement so that he can lower the threshold value ε, but he can never reach the value $\varepsilon = 0$. That is to say, *even in this simple example, measurements of the first kind are experimentally impossible.*

In our age of digital devices, practically all modern measuring instruments create well-defined numbers on the readout display with the aid of *built-in threshold devices.* But this exact number is generated by the measuring instrument and cannot be attributed to a property of the object system. The requirement that experiments have to be reproducible does not refer to individual experiments but to equivalence classes of experimental data. Therefore, *von Neumann's repeatability axiom refers to the Boolean classification of final statistical decision methods necessary to arrive at "experimental facts", and not to the Boolean structure of the projection-valued measures associated with the observables of the object system.* The knowledge one can get about an object can be expressed by a *nonpure* quantum state (for example represented by a non-idempotent density operator), never by a pure quantum state (for example represented by a idempotent density operator or a state vector). That is, *it is impossible in principle to determine a pure state experimentally.*

The main difficulty in Wigner's presentation of quantum theory is that his conceptual characterization of the state vector is contradictory. He says:

"The state of the object, if one has the maximum possible knowledge about it, can be described by a state vector. … [The] state vector [is] a description of the best possible knowledge available to the observer." [56]

This statement is operationally untenable since there is no such thing as "the best possible knowledge available to the observer". For every experiment on a quantum system there exist a more informative experiment, but the sequence of more and more informative experiments does not converge to a realizable experiment giving maximal possible knowledge. Wigner's attempts to formulate natural laws "directly in terms of observations independently of any specific theory" [57] leads to severe difficulties. In order to put his instrumentalistic approach into a mathematically formulated theory, Wigner needs a postulate "demanding the possibility of defining a state with final accuracy on which no further improvement is possible." He remarks that this postulate is not at all obvious, but he does not see the operational *impossibility* of this requirement: "It is not yet clear to what extent the reservations concerning the possibility of preparing pure states are valid, and at any rate, the theory of invariance in quantum mechanics has not yet been worked out without the concept of pure states. We shall therefore continue to use [this] postulate." [58] The most striking difference between pure and nonpure states concerns their operational accessibility and their time evolution. The deterministic laws refer to *pure* states which are, however, operationally inaccessible. Every epistemic state is nonpure; its time evolution is in general *not* given by the deterministic Schrödinger equation. Therefore it is in general not possible to embed a nondeterministic instrumentalistic description into a non-instrumentalistic deterministic structure. Deterministic equations of motion can show an extremely sensitive behaviour on the pure-state initial conditions. As we will discuss in more detail in chapter 8, there exist deterministic dynamical quantum systems which in every operationally accessible description show an irreducibly chaotic behaviour. This phenomenon can play a crucial role for epistemically acausal nature of the observations process. It is indeed considered as an "attractive possibility" by Wigner:

"It is possible, and in fact natural, … to assume that the initial state of the apparatus is not known … . The probabilistic nature of the out come of the observation then could be blamed on our ignorance of the initial state of the apparatus." [59]

Wigner rejects this explanation since it "is conflict with the linearity of the equations of motion." This is true for the von-Neumann model of measurements but not for more realistic models.[60]

The necessity to introduce experimentally inaccessible pure state corroborates Einstein's dictum: "We now realize, with special clarity, how much in error are those theorist who believe that theory comes inductively from experience." [61] In Wigner's instrumentalistic approach, the epis-

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56 H. Margenau & E. P. Wigner, *Discussion: Comments on Professor Putnam's Comments,* Philosophy of Science **29**, 292–293 (1962). The quotations are on p. 292 and on p. 293; in the *Collected Works,* volume 6, on p. 31 and on p. 32.

57 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37**, 595–632 (1965). The quotation is on p.603; in the *Collected Works,* volume 3, on p. 214.

58 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37**, 595–632 (1965). The quotations are on p.611; in the *Collected Works,* volume 3, on p. 222.

59 J. M. Jauch, E. P. Wigner & M. M. Yanase, *Some comments concerning measurements in quantum mechanics,* Nuovo Cimento **48 B**, 144–151 (1967). The quotation is on p.147; in the *Collected Works,* volume 6, on p. 184.

60 We shall come back to this important question in chapter 8.

61 A. Einstein, *Physics and reality,* Journal of the Franklin Institute **221**, 349–382 (1936). The quotation is on p. 360.

temically hidden pure states are fictive idealizations, used as a convenient tool for the construction of precise probabilistic predictions. Nevertheless Wigner speaks of "the state of the object"[62], and says that if a first-kind measurement of an observable A with the eigenvalues $a_1, a_2, \ldots$ and the eigenvectors $\alpha_1, \alpha_2, \ldots$ gives the result a_j, then the object system *is* after the measurement in a pure state described by the state vector α_j[63]. Yet, he tends to reject the "point of view to elevate the state vector into a description of 'physical reality' (whatever that may mean)" [64]. A scientific realist would take the opposite position: a pure state is supposed to describe the quantum object independently of measurements. From a *formal* point of view it makes little difference whether we interpret pure states of an individual isolated quantum system instrumentally as a fictive extension of genuine epistemic states, or as states of a realistic interpretation. But the motivation is different. While "both realism and instrumentalism agree that science has an overriding aim, … they disagree as to what that aim is. For realism, science aims at truth (about the *World*). For instrumentalism, science aims at instrumental (including observational) reliability." [65]

2.2 Individual and statistical descriptions, ontic and epistemic interpretations

As emphasized by Erhard Scheibe, "the statements in the relevant original papers, including those of von Neumann, relating to the concepts of probability, state, measurement, and reduction of states are *not* sufficient to compile a single formulation of quantum mechanics that might be regarded as *the* orthodox formulation." [66] This characterization applies also for all papers by Wigner. Scheibe distinguishes *four* interpretations of traditional quantum mechanics. First of all, he discriminates

> "between formulations which do not include the concept of the *state of an individual object* and those which do. In the first category there is a further distinction between the theories in which the propositions regarding probabilities are taken to express the *state of knowledge* concerning an individual object (epistemic interpretation) and those in which they are taken to express essentially, relative frequencies in *statistical ensembles* of individual objects (statistical interpretation)." [67]

States which refer to an epistemic interpretation are called *epistemic states*, while states which refer to a statistical interpretation are called *statistical states.* An ontic interpretation "refers primarily to individual states but *not* to states of knowledge or statistical states." [68] States which refer to an ontic interpretation are called *ontic states.* Ontic states are assumed to give a complete description of a system. In an ontic interpretation, one has again to distinguish

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62 E. P. Wigner & M. M. Yanase, *Analysis of the quantum mechanical measurement process,* Annals of the Japan Association for the Philosophy of Science **4**, 171–186 (1973). The quotation is on p. 171; in the *Collected Works,* volume 6, on p. 549.

63 Compare p. 101 in E. P. Wigner, *Die Messung quantenmechanischer Operatoren,* Zeitschrift für Physik **133**, 101–108 (1952). In the *Collected Works,* volume 6, on p. 147.

64 J. M. Jauch, E. P. Wigner & M. M. Yanase, *Some comments concerning measurements in quantum mechanics,* Nuovo Cimento **48 B**, 144–151 (1967). The quotation is on p.145; in the *Collected Works,* volume 6, on p. 182.

65 A. Fine, *Unnatural attitudes: Realist and instrumentalist attachments to science,* Mind **95**, 149–179 (1986), p. 157.

66 E. Scheibe, *The Logical Analysis of Quantum Mechanics,* Oxford, Pergamon Press (1973), p. 5.

67 E. Scheibe, *The Logical Analysis of Quantum Mechanics,* Oxford, Pergamon Press (1973), pp. 50–51.

68 E. Scheibe, *The Logical Analysis of Quantum Mechanics,* Oxford, Pergamon Press (1973), p. 82.

> "between theories which express the idea that the state of the object concerned is incompletely known, and theories of statistical ensembles of individual objects in definite states (epistemic or statistical extension of the ontic formulation)." [69]

In his basic formulations, Wigner adopts an *individual* interpretation. He is forced to give up his favoured operationalistic approach, and he has to introduce epistemically hidden pure states. These pure states are not meaningless, but they play a crucial role in the formulation of first principles. In spite of the fact that Wigner hesitates to interpret pure states realistically, they have exactly the function of ontic states. In this sense we can consider the epistemic states in Wigner's formulation as extensions of ontic states in the sense of Scheibe.

In classical theories, there is a transparent relationship between statistical and individual descriptions. In the classical case, the convex set of all statistical states is a simplex so that a *unique* decomposition of every nonpure state into pure states is warranted. In quantum theories, however, a nonpure state can be decomposed in infinitely many different ways into pure states. The idea that nonpure states represent ignorance of the system's real individual state is referred to as the *ignorance interpretation.* In classical theories a statistical state specifies a unique ensemble, so that an ignorance interpretation of classical theories is always logically feasible. However, since many decades it is well known that *a simple ignorance interpretation is not defensible in quantum mechanics.* This fact leads to grave problems for a purely statistical interpretation of quantum mechanics. We conclude that *a coherent statistical interpretation in terms of an ensemble of individual systems requires an individual interpretation as a backing.*

2.3 Statistical description of state reductions: The projection postulate

Let us again consider an observable A with a simple discrete spectrum $a_1, a_2, \cdots$ and the spectral decomposition $A = \sum_k a_k P_k$. Then the individual description of a measurement of the first kind of A is given by the stochastic nonlinear map

$$\Psi \to P_j \Psi / \|P_j \Psi\| \quad \text{with probability} \quad p_j := \langle \Psi | P_j \Psi \rangle \quad , \quad \sum_j p_j = 1 \quad , \qquad (7)$$

for every initial state vector $\Psi \in \mathscr{H}^{\text{obj}}$ where $\mathscr{H}^{\text{obj}}$ is the Hilbert space of the object system. In a statistical description, a statistical state is represented by a positive operator of trace one on a separable complex Hilbert space $\mathscr{H}^{\text{obj}}$, called *density operator*. If $D_{\text{obj}} := |\Psi\rangle\langle\Psi|$ is the density operator representing the initial state of the object system immediately prior to the measurement, then the statistical average of the final density operators $P_1, P_2, \cdots$ is given by the density operator $D^{\#}_{\text{obj}} = \sum_k p_k P_k$. That is, a *statistical* description of a measurement of the first kind is given by the mapping

$$D_{\text{obj}} \to D^{\#}_{\text{obj}} = \sum_k p_k P_k = \sum_k P_k D P_k \text{ for every initial density operator } D_{\text{obj}} \in \mathscr{B}(\mathscr{H}_{\text{obj}}) \ . \qquad (8)$$

This *linear* map is referred to as the *projection postulate*.[70] In the eigenbasis of the observable A, the mapping $D_{\text{obj}} \to D^{\#}_{\text{obj}}$ annihilates the off-diagonal elements of the density matrix $(\langle \alpha_j | D_{\text{obj}} \alpha_k \rangle)$, $\langle \alpha_j | D_{\text{obj}} \alpha_k \rangle \to \langle \alpha_j | D^{\#}_{\text{obj}} \alpha_k \rangle = \delta_{jk} \langle \alpha_j | D_{\text{obj}} \alpha_j \rangle$. It should be emphasized that the statistical transition map (8) is a consequence of the individual state-reduction postulate (7). However, the linear map $D_{\text{obj}} \to \sum_k P_k D_{\text{obj}} P_k$ does not imply the individual stochastic and

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69 E. Scheibe, *The Logical Analysis of Quantum Mechanics,* Oxford, Pergamon Press (1973), p. 51.

70 For observables with simple eigenvalues this postulate has been introduced in chapt. V.1 in: J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Berlin, Springer (1932). The generalization for observables with degenerate eigenvalues is due to G. Lüders, *Über die Zustandsänderung durch den Messprozess,* Annalen der Physik **8**, 322–328 (1951).

nonlinear state reduction map $\Psi \to P_k \Psi / \|P_k \Psi\|$. The reason is that the convex set of all statistical quantum states is not a simplex, so that every nonpure density operator allows infinitely many different decompositions into pure density operators. Consequently, the same statistical map $D_{\text{obj}} \to D^{\#}_{\text{obj}}$ can be obtained from many different individual realizations (linear and nonlinear ones). That is, *the individual state-reduction postulate (the "collapse of the wave packet") is not the same as the statistical projection postulate.* From an exhaustive individual description, the statistical description can be derived, *but not the other way round.*

The density operator $D^{\#}_{\text{obj}}$ does admit an ignorance interpretation since it represents *by construction* a proper mixture, or in the terminology of Hermann Weyl a *Gemenge*.[71] The construction *presupposes* the existence of the nonlinear state-reduction map $\Psi \to P_k \Psi / \|P_k \Psi\|$. A formally identical density operator can be obtained by restricting the *pure* state of the combined object and apparatus system before the state reduction to the object system. Let $\Xi \in \mathcal{H}_{\text{obj}} \otimes \mathcal{H}_{\text{app}}$ be the state vector of the combined system after the interaction but before the state reduction,

$$\Xi := \sum_k c_k \, \alpha_k \otimes \beta_k \quad , \quad \alpha_k \in \mathcal{H}_{\text{obj}} \quad , \quad \beta_k \in \mathcal{H}_{\text{app}} \quad , \quad |c_k|^2 = |\langle \alpha_k | \alpha \rangle|^2 \quad . \qquad (9)$$

The restriction $\mathcal{D}_{\text{obj}}$ of pure-state density operator $D := |\Xi\rangle\langle\Xi|$ to the object system is called a *reduced density operator*,

$$\mathcal{D}_{\text{obj}} := \operatorname{tr}_{\mathcal{H}^{\text{app}}} \{D\} = \sum_k p_k \, |\alpha_k\rangle\langle\alpha_k| = \sum_k p_k P_k \quad . \qquad (10)$$

Reduced density operators were first introduced by Lev Davidovich Landau in 1927.[72] The reduced state of an entangled pure state is nonpure and cannot be interpreted as a proper mixture. In particular, if the state vector $\Xi \in \mathcal{H}_{\text{obj}} \otimes \mathcal{H}_{\text{app}}$ refers to a *single* system, then the reduced density operator $\mathcal{D}_{\text{obj}}$ also refers to a single system, and not to an ensemble of single systems. Therefore $\mathcal{D}_{\text{obj}}$ cannot be interpreted in terms of an ensemble in which the object observable have a definite value. The nonpurity of such a state described by Landau's reduced density operator is exclusively due to Einstein–Podolsky–Rosen correlations between the object system and the apparatus system.

The fact that the density operator $D^{\#}_{\text{obj}}$ for the mixture generated by a state reduction and the reduced density operator $\mathcal{D}_{\text{obj}}$ of a single system before the state reduction are represented by the same mathematical objects is just an example for the mentioned fact that nonpure quantum states do not fully specify the system under discussion. For a proper understanding of the state reduction process, it is important not to confuse individual nonpure states described by Landau's reduced density operator with statistical density operators. Unfortunately, Wigner never distinguishes explicitly between Landau's concept of reduced nonpure states of an individual object and the nonpure states of proper statistical mixtures.[73]

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71 See p. 12 in H. Weyl, *Quantenmechanik und Gruppentheorie,* Zeitschrift für Physik **46,** 1–46 (1927).

72 L. Landau, *Das Dämpfungsproblem in der Quantenmechanik,* Zeitschrift für Physik **45,** 430–441 (1927).

73 Compare for example E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301, footnote 13 on p. 301 (in the *Collected Works,* volume 6, on p. 259); E. P. Wigner, *Theorie der quantenmechanischen Messung, Physikertagung Wien 1961,* Mosbach, Physik Verlag (1962), pp. 1–8, p. 2 (in the *Collected Works,* volume 3, on p. 433); E. P. Wigner & M. M. Yanase, *Information contents of distributions,* Proceedings of the National Academy of Sciences USA **49,** 910–918 (1963), p. 911 and reference 4 (in the *Collected Works,* volume 3, on p. 453).

2.4 Wigner on the relation between quantum theory and experimental science

In most discussions of conceptual problems of quantum theory (in particular in all papers by Wigner), von Neumann's model is uncritically adopted.[74] In a joint paper, Jauch, Wigner and Yanase say that they "continue to believe that von Neumann's description of the measurement process is basically correct."[75] Therefore Wigner uses the reduction postulate to discuss the statistical correlations between the outcomes of successive measurements of the first kind on a strictly isolated object system whose automorphic dynamics is governed by the Hamiltonian H. Let $A_1, \cdots, A_n$ be observables with simple discrete spectra, and let $A_k(t) = \sum_\nu a_k^{(\nu)}(t)\, P_k^{(\nu)}(t)$ be the spectral decomposition of the operator $A_k(t) := \exp(itH/\hbar)\, A \exp(-itH/\hbar)$. If the initial state is given by the state vector Ψ, the celebrated "Wigner formula" gives the joint probability

$$\langle P_n^{(\nu_n)}(t_n) \cdots P_1^{(\nu_1)}(t_1)\, \Psi | P_1^{(\nu_1)}(t_1) \cdots P_n^{(\nu_n)}(t_n)\, \Psi \rangle \qquad (11)$$

that a sequence of first-kind measurements of $A_1,\ A_2,\ \ldots,\ A_n$, carried out at times $t_1 \le t_2 \le \cdots \le t_n$, yields the outcomes $a_1^{(\nu_1)}(t_1),\ a_2^{(\nu_2)}(t_2),\ \ldots,\ a_n^{(\nu_n)}(t_n)$, respectively.[76] In spite of the unwarranted idealization necessary to derive this relation, Wigner concludes "that *the laws of quantum mechanics only furnish probability connections between results of subsequent observations carried out on a system.*"[77] Moreover, he claims that the expressions of the type of eq. (11) for the correlations between results of measurements of the first kind "constitute the content of the laws of nature, or of a theory."[78] That is, Wigner assumes that it is possible to formulate the laws of nature entirely in terms of the von-Neumann model of measurements.

Furthermore, Wigner shows that observables which do not commute with the additive conserved quantities cannot be measured precisely, where tacitly a "measurement" is conceived as a measurement of the first kind.[79] Since most observables do not commute with the additive conserved quantities, Wigner claims that a measurement of such observables requires "a very large measuring apparatus".[80] If this statement were true, then quantum mechanics would already fail on the molecular level. For example, enzymes act as microscopic measurement mechanisms, and the biological molecular code realizes, at a molecular level, a highly reliable classical, irreversible

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74 A hardly convincing defense of measurements of the first kind and its trivial generalizations can be found on pp. 18–19 of E. P. Wigner & M. M. Yanase, *Analysis of the quantum mechanical measurement process,* Annals of the Japan Association for the Philosophy of Science **4,** 171–186 (1973). In the *Collected Works,* volume 3, on pp. 552–553.

75 J. M. Jauch, E. P. Wigner & M. M. Yanase, *Some comments concerning measurements in quantum mechanics,* Nuovo Cimento **48 B,** 144–151 (1967). The quotation is on p. 145; in the *Collected Works,* volume 6, on p. 182.

76 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37,** 595–632 (1965), (in the *Collected Works,* volume 3, pp. 206–243); E. P. Wigner, *The subject of our discussion,* in: B. d'Espagnat (ed.), *Foundations of Quantum Mechanics. International School of Physics "Enrico Fermi", 1970,* New York, Academic Press (1971), pp. 1–19, (in the *Collected Works,* volume 6, pp. 199–217).

77 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963), The quotation is on p. 13; in the *Collected Works,* volume 6, on p. 176.

78 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37,** 595–632 (1965). The quotation is on p. 599; in the *Collected Works,* volume 3, on p. 210.

79 E. P. Wigner, *Die Messung quantenmechanischer Operatoren,* Zeitschrift für Physik **133,** 101–108 (1952). Reprinted in *Collected Works,* volume 3, pp. 415–422, and in *Collected Works,* volume 6, pp. 147–154. For a generalized approach, compare E. P. Wigner & M. M. Yanase, *Analysis of the quantum mechanical measurement process,* Annals of the Japan Association for the Philosophy of Science **4,** 171–186 (1973), in the Collected Works, volume 3, pp. 549–564.

80 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963), The quotation is on p. 14; in the *Collected Works,* volume 6, on p. 177.

and nonanticipating memory which in fact acts as a measuring instrument of molecular size. In modern nanotechnology other examples of extremely small measuring devices can be constructed. We conclude that the macroscopic nature of most measuring instruments of experimental science is irrelevant to the quantum mechanical measurement problem.

Wigner's result is based on the well-known fact that a simultaneous first-kind measurement of noncommuting observables with a simple spectrum is impossible. But this mathematical theorem has little to do with laboratory measurements. In fact, it is well known for long in the engineering community that incompatible observables can easily be measured together,[81] and that such measurements also have a proper quantum theoretical description.[82]

According to Wigner, we use quantum mechanics, "as a rule, either to determine material constants or the possible values of essentially only one observable, the energy."[83] These statements are not in accordance with modern experimental science. For example, relation (11) allows not even the discussion of the simplest nuclear magnetic resonance experiment with a spin-$\frac{1}{2}$ system as described by Bloch's equation.[84] In this experiment, the relevant observable is the magnetization which is proportional to the spin angular momentum $\frac{1}{2}\hbar\boldsymbol{\sigma}$. The measured quantity is not an eigenvalue of an observable, but the induced voltage in a receiver coil. According to Faraday's induction law, this voltage is proportional to the corresponding component of the time derivative of the *expectation values* (in contradistinction to the eigenvalues) of the observables σ_x, σ_y and σ_z. By using three mutually orthogonal receiver coils one can measure simultaneously the expectation values of these three mutually incompatible observables as a continuous function of an externally applied magnetic field. The resulting information about the local molecular magnetic field gives very detailed information about the molecular structure and the way molecules move, and it can even lead to precise determinations of the structure of complex proteins.[85]

Referring to spectroscopic experiments, Wigner states that "energy is the most important microscopic quantity that we know how to measure" [86]. However, *no spectroscopic measurement is a first-kind measurement of energy*. The appropriate theory for spectroscopic (and other experiments) is *response theory* which gives the expectation values of observables of an object system in response to an external stimulus, say an electromagnetic field or a mechanical force.[87] In spec-

81 E. Arthurs & J. L. Kelly, *On the simultaneous measurement of a pair of conjugate observables,* Bell System Technical Journal **44,** 725–729 (1965); J. P. Gordon & W. H. Louisell, *Simultaneous measurement of noncommuting observables,* in: B. L. Kelley, B. Lax & P. E. Tannenwald (eds.), *Physics of Quantum Electronics,* New York, McGraw-Hill (1966), pp. 833–840; C. Y. She & H. Heffner, *Simultaneous measurement of noncommuting observables,* Physical Review **152,** 1103–1110 (1966).

82 Compare in particular the engineeringly sound and mathematically rigorous approach by A. S. Holevo, *Investigations in the general theory of statistical decisions,* Proceedings of the Steklov Institute of Mathematics **Issue 3,** 1–140 (Russian number 124, 1976, pp.1–135) (1978); A. S. Holevo, *Probabilistic and Statistical Aspects of Quantum Theory,* Amsterdam, North-Holland (1982).

83 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 374; in the *Collected Works,* volume 6, on p. 60.

84 F. Bloch, *Nuclear induction,* Physical Review **70,** 460–474 (1946).

85 Compare for example R. R. Ernst, *Nuclear magnetic resonance Fourier transform spectroscopy,* in: B. G. Malmström (ed.), *Nobel Lectures in Chemistry,* Singapore, World Scientific (1997), pp. 12–57.

86 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 374; in the *Collected Works,* volume 6, on p. 60.

87 Modern response theory started with the so-called Bloch equation (F. Bloch, *Nuclear induction,* Physical Review **70,** 460–474 (1946)) for the magnetization of a spin system under the influence of a external time-dependent magnetic field. This approach has be generalized to arbitrary systems, for example by R. Kubo & K. Tomita, *A general theory of magnetic resonance,* Journal of the Physical Society of Japan **9,** 888–919

troscopy the measured observable is a molecular electric or magnetic multipole moment, never the energy. Only by using the intricate theory of line broadening and level shifts, information about energy levels can be *deduced*. Wigner's objection that quantum theory gives no rules how a measurement of an observable can be performed[88] is not in accordance with engineering quantum mechanics. It is true that there is no simple relationship between observables and measuring instruments but in simple case (like in magnetic resonance experiments), quantum mechanics gives the *full* information for the construction of an appropriate measurement instrument. Over and above that, *all these experiments can be understood fully in terms of the expectation value postulate, without any recourse to the projection postulate.*

2.5 The expectation-value postulate provides the link between theory and experiment

Wigner admits that von Neumann's model of measurement of the first kind is a "highly idealized description of the measurement." [89] He objects in particular to its instantaneous character: "The instantaneous very strong interaction is an unrealistic idealization even in nonrelativistic theory."[90] The concept of an instantaneous measurement of the first kind precludes a dynamical explanation of the irreversibility connected with every measurement. Furthermore, it leads to difficulties in a relativistic quantum theory since an instantaneous state change is not Lorentz covariant. Nonetheless, Wigner believes that "the paradoxes which are often found alarming, such as Schrödinger's cat, have little to do with this problem. It is important to realize this point, nevertheless, because it also detracts from the simplicity and hence beauty of the mathematical formulation of the theory." [91] We find it difficult to appreciate this attitude.

Wigner proposes to "scrutinize the orthodox view very carefully and to look for loopholes which would make it possible to avoid the conclusions to which the orthodox view leads" [92]. It is astonishing that in spite of his critical attitude and his deep insight, Wigner never questioned the basic assumption underlying von Neumann's model of measurements of the first kind. A theory which deals with the connection between theoretical descriptions and laboratory experiments should at least fulfill the following desiderata:

(1954); R. Kubo, *Statistical mechanical theory of irreversible processes. I,* Journal of the Physical Society of Japan **12,** 570–586 (1957); J. M. Blatt & T. Matsubara, *The electric and magnetic response of a thermodynamic system,* Progress of Theoretical Physics **21,** 696–712 (1959). For a text-book version, compare for example E. Fick & G. Sauermann, *Quantenstatistik dynamischer Prozesse. Band IIa. Antwort- und Relaxationstheorie,* Leipzig, Akademische Verlagsgesellschaft (1985).

88 Compare p. 102 in E. P. Wigner, *Die Messung quantenmechanischer Operatoren,* Zeitschrift für Physik **133,** 101–108 (1952), (in the *Collected Works,* volume 6, on p. 148).

89 E. P. Wigner, *Interpretation of quantum mechanics,* in: J. A. Wheeler & W. H. Zureck (eds.), *Quantum Theory and Measurement,* Princeton, Princeton University Press (1983), pp. 260–314. The quotation is on p. 284, in the *Collected Works,* volume 6, p. 102.

90 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37,** 595–632 (1965). The quotation is on p.625; in the *Collected Works,* volume 3, on p. 236.

91 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The two quotations are on p. 371; in the *Collected Works,* volume 6, on p.57. Compare also p.284 in: E. P. Wigner, *Interpretation of quantum mechanics,* in: J. A. Wheeler & W. H. Zureck (eds.), *Quantum Theory and Measurement,* Princeton, Princeton University Press (1983), pp. 260–314, (in the *Collected Works,* volume 6,on p. 102)..

92 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963), The quotation is on p. 7; in the *Collected Works,* volume 6, on p. 164.

- In the *engineering domain*, quantum theory should in principle be able to provide a description of measuring instruments and of our general experimental laboratory equipment. Therefore, a full-fledged codification of quantum mechanics must include the successful engineering theories like classical point mechanics, chaotic nonlinear dynamical systems, continuum mechanics, hydrodynamics, classical stochastic processes, thermostatics including phase transitions, and Maxwell's electrodynamics.
- Since not every physical system is a measuring instrument, the apparatus and its interaction with the objects to be measured must be characterized by special properties. This is not done in von Neumann's model. From experimental science we know that *every* measuring instrument can register facts in a macroscopically irreversible way and that the relevant degrees of freedom of an observing system can be described classically.
- A realistic model for measurements has to take into account that every measurement takes time. The instantaneous character of first-kind measurements is unphysical and leads to deep difficulties.
- Experimenters have no difficulties to measure incompatible physical quantities simultaneously. A realistic measuring theory should conform to this empirical fact.
- Recent experiments on single atoms and molecules require a careful distinction between individual and statistical descriptions, and between ontic and epistemic interpretations.

The first step towards a realistic theory of measurements is to acknowledge that Dirac's valuation (1) of observables by point functions, which is taken over from classical point mechanics, is inappropriate for quantum mechanics. Dirac's rule does not value all properties which can be measured. In particular, observables with a continuous spectrum and noncommutative observables are not valuated at all. Furthermore, Dirac's valuation is not continuous. For example, for every elementary observable represented by a projection operator F, there exists in quantum mechanics an elementary observable F_ε which is incompatible with F, $F F_\varepsilon \neq F_\varepsilon F$, and which for any arbitrarily small positive number ε satisfies the relation $\| F - F_\varepsilon \| = \varepsilon$. If the state vector ψ is an eigenvector of F with the eigenvalue 1, then Dirac proposes to say that in this state the system *has* the value 1. According to Dirac's rule, the incompatible but physically indistinguishable observable F_ε cannot be valued. Moreover, no observable of an individual *open* quantum system ever has a dispersion-free value. Fortunately, it is straightforward to extend Dirac's valuation by point functions to a continuous valuation of any observable of an arbitrary (isolated or open) classical or quantum system by set functions:

Continuous valuations of observables by set functions (12)

> Any physical system is characterized by an appropriate C*-algebra $\mathcal{A}$. There is a one-to-one correspondence between the properties of the physical system and the commutative C*-subalgebras of $\mathcal{A}$. Every commutative C*-subalgebra $\mathcal{A}_A$ can be generated by a selfadjoint element $A \in \mathcal{A}$. For historical reasons, such a generating selfadjoint element A is called an *observable*. Every observable A generates a unique commutative C*-subalgebra $\mathcal{A}_A \subseteq \mathcal{A}$; it is the smallest C*-algebra which contains all powers of A. The state (pure or nonpure) of the system at time t is represented by a normalized linear functional, called the state functional ρ_t. The valuation of every property of the system is given (independently of any measurement) by the continuous *set function* $\mathcal{B} \mapsto \rho_t\{E_A(\mathcal{B})\}$, where $E_A : \Sigma_A \to \mathcal{A}_A$ is the spectral measure of the observable A, and $\mathcal{B}$ is an element of the σ-algebra of Borel sets of the spectrum of A.

The valuation of observables by the state set functions is continuous and interpolates between incompatible observables. If an elementary observable F has a dispersion-free value, $\rho_t(F) = 1$, then for any neighbouring elementary observable with $\|F - F_\varepsilon\| = \varepsilon$, the mean value is almost 1, $\rho_t(F_\varepsilon) \geq 1 - \varepsilon$, and the value distribution is almost dispersionfree, $0 < (\Delta F_\varepsilon)^2 < \varepsilon$. Note that these valuations are *intrinsic* and in no way probabilistic. The valuation of an observable A with respect to a state functional ρ_t can also be characterized by the family $\{\rho_t(A^n) | n = 0, 1, 2, \ldots\}$ of moments $\rho_t\{A^n\}$. Of particular importance is the first moment $\rho_t(A)$, usually called the mean value, and the dispersion $(\Delta A)^2 := \rho_t(A^2) - \rho_t(A)^2$. The dispersion is a characteristic of the set function $\mathcal{B} \mapsto \rho_t\{E_A(\mathcal{B})\}$ and should not be misinterpreted as the spread of a collection of real numbers. If $\Delta A = 0$, the valuation is determined by the single parameter $\rho_t(A)$, and we say then with Dirac that the observable A *has* the dispersionfree-value $\rho_t(A)$. Dispersion-free valuations exist only if ρ_t is a *pure* state functional. In the special case of classical point mechanics, every pure state functional is multiplicative[93], so that in this case all observables have a dispersion-free value with respect to a pure state functional. It is characteristic for quantum systems (that is, if $\mathcal{A}$ is noncommutative), that the valuations with respect to pure states are in general *not* dispersion-free.

The fundamental and highly nontrivial expectation-value postulate relates the intrinsic value distribution with the statistical value distribution obtained experimentally. Consider for example a N-fold repeated experiment, described by a random variable a which can attain the values $a^{(1)}, a^{(2)}, \ldots a^{(N)}$. Provided the limit $N \to \infty$ is well-defined, the empirically based value distribution can be characterized by the family of moments $\mathcal{E}\{a^n\}$,[94]

$$\mathcal{E}_a\{a^n\} := \lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \left(a^{(k)}\right)^n \quad , \quad n = 1, 2, 3, \ldots \quad . \tag{13}$$

The fundamental link between quantum theory and experimental science is not given by Dirac's jump postulate (2) which is based on Dirac's valuation (1) by point functions but by the expectation-value postulate which is based on the valuation (12) by set functions. The expectation-value postulate requires the equality of the moments $\mathcal{E}\{a^n\}$ of the empirical value distribution with the moments $\rho_t\{A^n\}$ of the intrinsic valuation of the observable A with respect to the state functional ρ_t of the quantum system immediately before the measurement, $\mathcal{E}\{a^n\} = \rho_t\{A^n\}$.

The expectation-value postulate[95] (14)

> Let A be a selfadjoint observable with an arbitrary spectrum $\boldsymbol{\Lambda}$, let $\boldsymbol{\Sigma}$ be the σ-algebra of Borel sets of $\boldsymbol{\Lambda}$ and E_A be the spectral measure of A. The probability that a predictive measurement of the observable A gives a value lying in the Borel set $\mathcal{B} \in \boldsymbol{\Sigma}$ is given by the Kolmogorov probability measure $\boldsymbol{\mu} : \boldsymbol{\Sigma} \to \mathbb{R}^+$ defined by $\boldsymbol{\mu}(\mathcal{B}) = \mathrm{tr}\{DE(\mathcal{B})\}$, where D is the density operator describing the initial (pure or nonpure) state of the object system immediately before the measurement.

93 Compare for example R. V. Kadison & J. R. Ringrose, *Fundamentals of the Theory of Operator Algebras. I. Elementary Theory,* New York, Academic Press (1983), proposition 4.4.1, p. 269.

94 In the case that not all moments exist, a proper characterization can be given by the characteristic function $s \mapsto \mathcal{E}\{\exp(isa)\}$, $s \in \mathbb{R}$.

95 The expectation-value postulate has been formulated for the first time in a slightly weaker form in footnote 1 on p. 83 in W. Pauli, *Über Gasentartung und Paramagnetismus,* Zeitschrift für Physik **41**, 81–102 (1927).

Conditioned by a measurement, the expectation-value postulate implies that the observable A is a random observable on the Kolmogorov probability space $(\boldsymbol{\Lambda}, \boldsymbol{\Sigma}, \boldsymbol{\mu})$. The expectation value $\mathcal{E}\{f(A)\}$ of an arbitrary measurable function f of this random variable is given by

$$\mathcal{E}\{f(A)\} = \int_{\boldsymbol{\Lambda}} f(\lambda)\,\boldsymbol{\mu}(d\lambda) \quad . \tag{15}$$

The discussion of measurements of the first kind is based on the dogma that a valuation of physical quantities has to be made by point functions, and not – as both the mathematical formalism and experimental practice strongly suggest – by set functions. Nevertheless, the projection postulate is compatible with the expectation-value postulate, but it is a much too strong requirement. In distinction from the projection postulate, *the expectation-value postulate says nothing about the state after the measurement.* Nevertheless, the expectation-value postulate is sufficient for a comparison of quantum-mechanical predictions with experimental results. In contrast to the projection postulate, the expectation-value postulate is empirically well-confirmed and supposed to hold for any kind of measurements.

3. Quantum mechanics of open systems

3.1 Quantum theory has to deal with open systems

Even in his late papers Wigner claims that "quantum mechanics describes the behavior of isolated systems".[96] In discussing the question of the validity of quantum mechanics for macroscopic systems, he remarks: "The first point that should be considered in this connection is that physics deals with isolated systems, that is systems so far removed from other bodies that these do not exert noticeable influences on the system considered." [97] The fact that even in classical mechanics macroscopic bodies *cannot* be considered as isolated systems, is well known since the beginning of this century.[98] Yet, for the first time in 1971, Wigner tells us that due to the enormous density of the energy levels, a "state isolation is very difficult to maintain" for macroscopic bodies.[99] In his later papers he repeats this old-established point many times: "There are no isolated macroscopic systems." [100] Since Wigner presupposes that quantum mechanics is applicable to isolated systems only, he concludes that "quantum mechanics's validity has narrower limitations, that it is not applicable to the description of the detailed behavior of macroscopic bodies." [101] Since Wigner never acknowledges the fact that since many decades we have a well-established and perfectly functioning statistical theory of open quantum systems, his reason for believing that quantum mechanics is not applicable to macroscopic systems is hardly convincing.

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96 E. P. Wigner, *Physics and its relation to human knowledge,* Hellenike Anthropistike Hetaireia, Athens, 283-294 (1977). In the Collected Works, volume 6, on p. 591.

97 E. P. Wigner, *The extension of the area of science,* in: R. G. Jahn (ed.), *The Role of Consciousness in the Physical World,* AAAS Symposium No.57, Boulder, Westview Press (1981), pp. 7–16. In the Collected Works, volume 6, on p. 606.

98 Compare for example E. Borel, *Introduction géométrique à quelques théories physiques,* Paris, Gauthier–Villars (1914), p. 98.

99 E. P. Wigner, *The subject of our discussion,* in: B. d'Espagnat (ed.), *Foundations of Quantum Mechanics. International School of Physics "Enrico Fermi", 1970,* New York, Academic Press (1971), pp. 1–19. The quotation is on p. 17; in the *Collected Works,* volume 6, p. 215.

100 So for example on p. 380 in E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. In the Collected Works, volume 6, on p. 66, in the *Collected Works*, Volume 6, p. 259.

101 E. P. Wigner, *Review of the quantum mechanical measurement process,* in: P. Meystre & M. O. Scully (eds.), *Quantum Optics, Experimental Gravity, and Measurement Theory,* New York, Plenum Press (1983), pp. 43–63. The quotations are on p. 78; in the *Collected Works,* volume 6, on p. 240.

Wigner never mentions the well-known fact that *due to the omnipresent Einstein–Podolsky–Rosen correlations, a quantum system can as a rule not be considered as isolated.* If his conclusion that "the lack of the possibility of isolating a truly macroscopic body from the environment … limits the validity of our microscopic theory, of quantum mechanics" [102], would be true, it would also apply to microsystems and thus refute quantum theory. All object systems of interest to experimental science are open systems. Since the early years of quantum mechanics it is known that even an atom cannot be isolated from its environment. For example, if one considers hypothetically an atom as an isolated system in an excited energy eigenstate, it would *never* decay into the ground state, in gross contradiction with the experiment. The basic physical mechanism how excited atoms lose energy is the radiative decay due to the interaction with the electromagnetic radiation field. The proper tools to discuss spontaneously decaying atoms as *open* quantum system are the reduced density operators as introduced by Landau in 1927.[103] All this is, of course, well known, in particular to Wigner as a co-author of the famous Weisskopf-Wigner theory of 1930 for radiative decay of unstable quantum systems[104].

3.2 Dynamical semigroups for open quantum systems

In 1982[105] Wigner proposed "to modify the standard quantum mechanical equation for the time-dependence of [the density operator D]" by adding "to the expression for $\partial D/\partial t$ other terms which decrease the off-diagonal elements of D". The equation

$$i\hbar\, \partial D(t)/\partial t \;=\; H D(t) - D(t) H - i\hbar \sum\nolimits_{\ell,m} \varepsilon_\ell \,(\mathscr{L}_{\ell,m} - \mathscr{L}'_{\ell,m})^2\, D(t) \quad , \tag{16}$$

which he proposes, is neither new nor does it imply a modification of traditional quantum mechanics. It is surprising that Wigner apparently was not aware that this equation of motion is nothing else than the simplest example of a completely positive dynamical semigroup. Dynamical semigroups are not ad hoc, but can be *derived* under appropriate conditions by restricting the Hamiltonian dynamics of the joint system consisting of object and environment to the object system. The use of such dynamical semigroups for density operators is a standard tool in molecular spectroscopy at least since 1948[106], and it is well known to every student in physical chemistry.[107] The whole industry of nuclear magnetic resonance spectroscopy depends in an essential way on the use of dynamical semigroups, as introduced by Felix Bloch in 1946, and heuristically derived

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102 E. P. Wigner, *The extension of the area of science,* in: R. G. Jahn (ed.), *The Role of Consciousness in the Physical World,* AAAS Symposium No.57, Boulder, Westview Press (1981), pp. 7–16. In the Collected Works, volume 6, on p. 606.

103 L. Landau, *Das Dämpfungsproblem in der Quantenmechanik,* Zeitschrift für Physik **45**, 430–441 (1927).

104 V. Weisskopf & E. Wigner, *Berechnung der natürlichen Linienbreite auf Grund der Diracschen Lichttheorie,* Zeitschrift für Physik **63**, 54–73 (1930), V. Weisskopf & E. Wigner, *Über die natürliche Linienbreite in der Strahlung des harmonischen Oszillators,* Zeitschrift für Physik **65**, 18–29 (1930). *Collected Works,* volume 3, pp. 30–49 and pp. 50–61. For a review of the improved and generalized improved Weisskopf-Wigner theory of radiation damping compare the text by W. Heitler, *The Quantum Theory of Radiation,* third edition, Oxford, Oxford University Press (1954).

105 First in his hitherto unpublished lecture in Lindau and in Tutzing in 1982, published in the *Collected Works,* volume 6, pp. 72–77.

106 R. Karplus & J. Schwinger, *A note on saturation in microwave spectroscopy,* Physical Review **73**, 1020–1026 (1948).

107 Compare the basic paper by G. Lindblad, *On the generators of quantum dynamical semigroups,* Communications of Mathematical Physics **48**, 119–130 (1976), the review by V. Gorini, A. Kossakowski & E. C. G. Sudarshan, *Completely positive dynamical semigroups of N-level systems,* Journal of Mathematical Physics **17**, 821–825 (1976), and the text by E. B. Davies, *Quantum Theory of Open Systems,* London, Academic Press (1976).

in 1953 from first principles of traditional quantum mechanics.[108] Later, many elegant and mathematically rigorous derivations of general dynamical semigroups have been given. In quantum optics and laser theory such semigroups are also well established.[109]

After an astonishingly uniformed discussion of the validity of the semigroup approach, Wigner remarks: “And its validity, even if only approximate, does answer the question of our quantum mechanics’ validity for macroscopic objects …”,[110] and: “such an equation can also account for the fact that our mind is not in a superposition of different impressions”.[111] These statements are clearly inappropriate.

It is trivial to show that there exist completely positive linear dynamical semigroups implementing the transition map $D_{\text{obj}} \to D_{\text{obj}}^{\#}$ of the projection postulate (8) in the sense that for every initial density operator $D_{\text{obj}} = D_{\text{obj}}(t=0)$ the final density operator is given asymptotically by $D_{\text{obj}}(\infty) := \sup_{t\to\infty} D_{\text{obj}}(t)$ where $D_{\text{obj}}(\infty)$ is the same operator as $D_{\text{obj}}^{\#}$. However, since the set of nonpure quantum states does not form a *simplex*, an ignorance interpretation of the density operator $D_{\text{obj}}(\infty)$ is inadmissible. That is, a statistical interpretation in terms of density operators says nothing about the individual behaviour. Above all, no state reduction is implied. It is true that dynamical semigroups which asymptotically realize the projection postulate eliminate (in the eigenbasis of the observable considered) the off-diagonal elements of the density matrix. But this fact does *not* imply that “the position of the pointer will assume a definite value”, as erroneously claimed by Wigner.[112]

Wigner approves of the popular view that a coherent superposition “resulting from a measurement with a very large apparatus, surely *cannot be distinguished … from a mixture* ”.[113] In the last few years, this old idea[114] of dephasing has attracted much attention under the buzzword *decoherence*[115] The notion of decoherence refers to the “destruction of the off-diagonal terms in the density matrix”[116] – a condition which is evidently *not* sufficient to the explanation of

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108 F. Bloch, *Nuclear induction,* Physical Review **70**, 460–474 (1946); R. K. Wangsness & F. Bloch, *The dynamical theory of nuclear induction,* Physical Review **89**, 728–739 (1953).

109 Compare for example chapter IV of the review by H. Haken, *Laser theory,* in: S. Flügge (ed.), *Encyclopedia of Physics, Volume XXV/2c, Light and Matter Ic,* Berlin, Springer (1970), pp. 1–320.

110 E. P. Wigner, *Review of the quantum mechanical measurement process,* in: P. Meystre & M. O. Scully (eds.), *Quantum Optics, Experimental Gravity, and Measurement Theory,* New York, Plenum Press (1983), pp. 43–63. The quotation is on p. 81; in the *Collected Works,* volume 6, on p. 243.

111 E. P. Wigner, *Some problems of our natural sciences,* International Journal of Theoretical Physics **25**, 467–476 (1986). The quotation is on p. 470; in the *Collected Works,* volume 6, on p. 619.

112 E. P. Wigner, *The limitations of determinism,* in: *Absolute Values and the Creation of the New World. Proceedings of the 11th International Conference on the Unity of Science, 1982,* New York, International Cultural Foundation Press (1983), pp. 1365–1370. The quotation is on p. 1369; in the *Collected Works,* volume 6, on p. 137.

113 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31**, 6–15 (1963). The quotation is on p. 14; in the *Collected Works,* volume 6, p. 177.

114 Compare for example P. Jordan, *On the process of measurement in quantum mechanics,* Philosophy of Science **16**, 269–278 (1949).

115 In particular in quantum cosmology. Compare for example M. Gell-Mann & J. B. Hartle, *Classical equations for quantum systems,* Physical Review **D 47**, 3345–3382 (1993), as well as the popular account in sect. 11 of M. Gell-Mann, *The Quark and the Jaguar,* New York, Freeman (1994).

116 As characterized on p. 461 in: J. J. Halliwell, *Information dissipation in quantum cosmology and the emergence of classical spacetime,* in: W. H. Zurek (ed.), *Complexity, Entropy and the Physics of Information,* Redwood City, California, Addison–Wesley (1990), pp. 459–469.

superselection rules[117], or of the emergence of definite trajectories[118], or of the possibility to interpret nonpure statistical states "in terms of a classical 'mixture', with the exact state of the system unknown to the observer".[119] Again and again – even in the recent literature – a merely statistical averaging (e.g. an epistemic phase averaging) is proposed as a solution for the measurement problem. Therefore we wish to point out that decoherence in the sense of the dynamical disappearance of the off-diagonal elements of the density matrix is just a rather weak necessary but by no means sufficient condition for a proper explanation of the measuring process.[120]

Wigner admits the possibility "that the validity of quantum mechanic's linear laws is limited." [121] But Wigner never proposes explicitly a nonlinear stochastic equation of motion for single systems. It is important to acknowledge that from a statistical Lindblad semigroup for the density operator one cannot derive an equation of motion for individual systems since there are infinitely many individual realizations of a statistical quantum dynamics in terms of nonlinear stochastic Schrödinger equations, but only *one* ontologically relevant individual description.[122] For a genuine individual description of an open quantum system it is necessary to start *directly* from an individual description of the object system and its environment. It is not possible to derive an individual description from a merely statistical one.

An explicit example for the relation between the statistical projection map and the individual state reduction

In order to show that statistical semigroups alone cannot provide a solution of the measurement problem, consider the simplest dynamical semigroup for a two-level system with the Pauli matrix σ_3,

(a) $$\dot{D}(t) = -\tfrac{1}{2}\, i\, \Lambda_3\, [\sigma_3\,, D(t)]_- - \tfrac{1}{4}\, \kappa\, [\sigma_3\,, [\sigma_3\,, D(t)]_-]_- \quad ,$$

where $\Lambda_3 \in \mathbb{R}$ is an angular frequency describing the Hamiltonian dynamics, and $\kappa > 0$ is a relaxation frequency describing the dissipative interaction with the environment. The asymptotic density matrix $D(\infty)$ fulfills the statistical projection postulate for a measurement of the first kind of the observable σ_3 in a statistical ensemble, $D(\infty) := \lim_{t\to\infty} D(t) = \tfrac{1}{2}\,\sigma_3 + \tfrac{1}{2}\,\mathrm{tr}\{D(0)\,\sigma_3\}\,\sigma_3$. The transition map $D \to D(\infty)$ annihilates the off-diagonal elements of the density matrix relative to the eigenbasis of σ_3. The dynamical semigroup (a) allows *many* essentially commutative dilations to larger reversible systems in which the open object system is contained. The simplest *linear* commutative Markov dilation[123] can be realized by a

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117 As claimed by W. H. Zurek, *Environment-induced superselection rules,* Physical Review **D 26**, 1862–1880 (1982).

118 As claimed on p. 462 in: J. J. Halliwell, *Information dissipation in quantum cosmology and the emergence of classical spacetime,* in: W. H. Zurek (ed.), *Complexity, Entropy and the Physics of Information,* Redwood City, California, Addison–Wesley (1990), pp. 459–469.

119 As claimed on p. 47 in: W. H. Zurek, *Quantum measurements and the environment-induced transition from quantum to classical,* in: A. Ashtekar & J. Stachel (eds.), *Conceptual Problems of Quantum Gravity,* Boston, Birkhäuser (1991), pp. 43–62.

120 Compare also the critical letters in response to a paper by W. H. Zurek, *Decoherence and the transition from quantum to classical,* Physics Today **44**, No. 10, 36–44 (1991), published in Physics Today **46 (4)**, April 1993, pp. 13–15 and pp. 81–90.

121 E. P. Wigner, *The subject of our discussion,* in: B. d'Espagnat (ed.), *Foundations of Quantum Mechanics. International School of Physics "Enrico Fermi", 1970,* New York, Academic Press (1971), pp. 1–19. The quotation is on p. 17; in the *Collected Works,* volume 6, on p. 215.

122 Under the name *quantum diffusion model,* an ad hoc individual realization of a statistical Lindblad semigroup for the density operator of open quantum systems by a state vector satisfying a stochastic nonlinear Schrödinger equation has been proposed by N. Gisin & I. C. Percival, *The quantum-state diffusion model applied to open systems,* Journal of Physics A: Mathematical and General **25**, 5677–5691 (1992). Of course, this and similar "unravelings" are highly nonunique. In spite of that, it has been claimed that the quantum diffusion model gives a detailed picture of the behaviour of an open *individu*al system.(L. Diósi, N. Gisin, J. Halliwell & I. A. Percival, *Decoherent histories and quantum state diffusion,* Physical Review Letters **74**, 203–207 (1995)) Such claims has are not justified.

123 B. Kümmerer & H. Maassen, *The essentially commutative dilations of dynamical semigroups on M_n,* Communications in Mathematical Physics **109**, 1–22 (1987).

reversible Hamiltonian dynamics on the tensor product $\mathscr{M}\otimes\mathscr{C}$ where $\mathscr{M}$ is the algebra of (2×2)-matrices and the commutative algebra $\mathscr{C}$ generates a classical white noise force function $t\mapsto \dot{w}(t)\,\boldsymbol{e}_3$ acting on the object system. In this realization, the reduced individual dynamics for the object system is the given by a *linear* time-dependent stochastic Schrödinger equation in the sense of Stratonovich,

(b) $$i\,d\Psi(t) \;=\; \tfrac{1}{2}\Lambda_3\,\sigma_3\,\Psi(t)\,dt \;-\; \tfrac{1}{2}\sqrt{2\kappa}\;\sigma_3\circ dw(t) \quad,\quad t\geq 0 \quad,\quad \Psi(t)\in\mathbb{C}^2 \quad,$$

where $\circ$ denotes the Stratonovich product, and $t\mapsto w(t)$ is a real-valued Wiener process with the covariance $\mathcal{E}\{w(t)w(s)\}=|t-s|$. The mean density operator $D(t):=\mathcal{E}\{|\Psi(t)\rangle\langle\Psi(t)|\}$ realizes the dynamical semigroup (a). Since σ_3 is a constant of motion, this example shows that a simple dephasing process cannot explain a measuring process. The reason is that although a *linear* dilation of a dynamical semigroup incorporates the influence of the environment, it neglects the backreaction of the polarization of the environment on the object system. While in a statistical description the equations of motion for statistical states are necessarily linear, there are no reasons why the time evolution of individual states should be linear. For example, the mean density operator $D(t):=\mathcal{E}\{|\Psi(t)\rangle\langle\Psi(t)|\}$ generated by the solution $t\mapsto\Psi(t)$ of the *nonlinear* stochastic Schrödinger equation

(c) $$\begin{aligned} i\,d\Psi(t) \;=\;& \tfrac{1}{2}\Lambda_3\,\sigma_3\,\Psi(t)\,dt + i\,\kappa\,\langle\Psi(t)\,|\,\sigma_3\Psi(t)\rangle\,\{\sigma_3-\langle\Psi(t)\,|\,\sigma_3\Psi(t)\rangle\}\,\Psi(t)\,dt\\ &+ i\sqrt{\kappa/2}\,\{\sigma_3-\langle\Psi(t)\,|\,\sigma_3\Psi(t)\rangle\}\,\Psi(t)\circ dw(t) \quad,\quad \Psi(t)\in\mathbb{C}^2 \quad,\quad \|\Psi(t)\|=1 \quad,\end{aligned}$$

also satisfies the semigroup equation (a). As shown by Gisin, the individual solution describes asymptotically for $t\to\infty$ a first-kind measurement of the observable σ_3,[124] The Schrödinger equation (c) is purely formal, the corresponding effective Hamiltonian is not selfadjoint, and to the best of our knowledge it has no physical interpretation. Nevertheless, this example is instructive since it shows that among the infinitely many possible individual realizations of the statistical dynamical semigroup (a) there may be one which is relevant for an understanding of some aspects of the measurement process. Of course it not satisfactory just to postulate a nonlinear stochastic Schrödinger equation. Such effective equations of motion should be *derived* from a encompassing system, comprising the essential parts of the measuring apparatus and the dissipative environment. In fact, it can be shown that the asymptotic state reduction process (7) is a solution of a special case of equations of motion which can be derived from a still another nonlinear stochastic effective Schrödinger equation with a selfadjoint Hamiltonian. Such equations can be derived from first principles of quantum mechanics.[125] Neither a stochastic modification nor any kind of extension of quantum mechanics is necessary to describe such stochastic quantum processes. The nonlinearities of the effective equations of motion for the object system arise from the backreaction of the polarized environment on the object.

3.3 Wigner's elementary building blocks are bare systems

The fact that historically the first examples discussed in quantum mechanics were idealized as strictly closed systems should not lead us to the conclusion that quantum theory is only applicable to isolated systems. Wigner asserts just the opposite, namely that "one can, and does, assume that … the object is separated from the rest of the world … during time intervals *between* measurements." [126] The fact that this claim is not correct is of great importance for an understanding of mesoscopic and macroscopic phenomena in terms of quantum theory. Every object system is interacting with the gravitational and the electromagnetic field. In particular, no material object can be screened off against interactions with the low frequency part of the electromagnetic field.

Physics is strongly influenced by atomism, and tries to analyze the world in terms some *elementary* building blocks. It was Wigner who first introduced the modern point of view that elementary systems have to be characterized group theoretically. In 1939 Wigner published his epochal paper on the representations of the Poincaré group (that is, the inhomogeneous Lorentz

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124 N. Gisin, *Quantum measurements and stochastic processes,* Physical Review Letters **52,** 1657–1660 (1984).

125 H. Primas, *Individual description of dynamical state reductions in quantum mechanics,* Unpublished Report (1997).

126 H. Margenau & E. P. Wigner, *Discussion: Comments on Professor Putnam's Comments,* Philosophy of Science **29**, 292–293 (1962). The quotation is on p.292; in the *Collected Works,* volume 6, on p.31.

group).[127] He gave a (from a physical viewpoint complete) classification of all irreducible projective representations of the Poincaré group which led to a comprehensive insight of their physical significance. The physically relevant irreducible projective representations of the Poincaré group are those in which the spectrum of the translation subgroup lies in the future cone. They can be labelled by the eigenvalue m^2 of a Casimir operator, where the parameter m is called *mass*. The further classification is given by the so-called little group of transformations leaving the absolute square m^2 of a vector fixed. When $m^2 > 0$, the little group is isomorphic to SO(3) or SU(2), and is determined by a positive integer or half-odd integer, called the *spin* s. To each pair (m, s) there corresponds only one irreducible representation up to unitary equivalence. For $m = 0$, the little group is isomorphic to the Euclidean group of a two-dimensional plane or to its two-sheeted covering, and is determined by a positive integer or half-odd integer, called the *helicity*. By means of this analysis he provided a *definition* of what we mean by an elementary physical system. According to Wigner, "the concept of an elementary system … is a description of a set of states which forms, in mathematical language an irreducible representation space for the inhomogeneous Lorentz group." [128] Unfortunately, it is still fashionable to call such elementary mathematical structures "elementary particles". Wigner's ideas have been extended to other kinematical groups (in particular also for the Galilei group which appears as a limiting case of the Lorentz group), and to a more general framework which includes classical theories.[129]

Wigner's elementary building blocks allow the construction the Hilbert space of a composite system as well-defined tensor product. However, there is a snag: *Group-theoretically elementary systems have no direct operational meaning.* According to Wigner, an elementary system is nothing else but a conceptual carrier of an elementary representation of an appropriate symmetry group (like the group SU(3), the Poincaré group, or the Galilei group). The state vectors defined by an irreducible projective representation of a kinematical group refer to a hypothetical strictly isolated system without any interactions with other systems. Such systems are called *bare* ("bare particles", "bare fields"). Wigner's analysis made it clear that massive elementary systems are fundamentally different from massless ones (like photons). A massive elementary system which is coupled to an elementary zero-mass field has no longer a definite mass, hence does no longer transform according to an irreducible representation of the Poincaré group. In particular, a bare elementary system carrying an electrical charge (like a bare electron) inevitably interacts with the electromagnetic field it itself generates. The physical states of such systems always contain clouds of low-energy photons and cannot be described by eigenvectors of the mass operator (the so-called infrared problem). The state representing an electron *as actually observed in the laboratory* is called a *dressed electron*, it has a very complicated structure. Heuristically, it can be thought of as consisting of the

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127 E. P. Wigner, *On unitary representations of the inhomogeneous Lorentz group,* Annals of Mathematics **40**, 149–204 (1939). For more than a decade this paper remained unappreciated in the physics community. Wigner reports in the Citation Classic column of *Current Contents* of June 11, 1979, Number 24, p. 20 (not reprinted in the *Collected Works*): "It may be of some interest to recall that when the manuscript was submitted to one of our mathematical journals, it was rejected as 'uninteresting'. … Not all articles originally rejected by a journal prove to be valueless."

128 T. D. Newton & E. P. Wigner, *Localized states for elementary systems,* Reviews of Modern Physics 400–406 (1949). The quotation is in footnote 1 on p. 400; in the *Collected Works,* volume 1, on p. 402.

129 The representation-independent concept of elementarity is not irreducibility but *ergodicity*. Any closed physical system can be realized as an W*-system $(\mathscr{M}, \mathscr{G}, \alpha)$, consisting of a W*-algebra $\mathscr{M}$, a kinematical group $\mathscr{G}$, and a mapping of $\mathscr{G}$ into the automorphism group of $\mathscr{M}$. W*-system is called *elementary* if the action of $\mathscr{G}$ on $\mathscr{M}$ is *ergodic* (that is, if the only operators $M \in \mathscr{M}$ with the property $\alpha_g(M) = M$ for all $g \in \mathscr{G}$ are multiples of the identity). Compare for example A. Amann, *Observables in W*-algebraic quantum mechanics,* Fortschritte der Physik **34**, 167–215 (1986), pp. 172–173.

bare electron, interacting with its own radiation field by emitting and reabsorbing virtual photons. The presence of a virtual cloud not only modifies the properties of the bare elementary system and its bare environment, but also their dynamics.

The relationship between group-theoretically defined elementary systems and directly observable phenomena is notoriously difficult. All actual laboratory experiments refer to dressed objects, never to bare systems. A dressed object is not only adapted to its environment, it is in an essential way created by it. In order to go over to an operational description, one has to find a dressing transformation which gives rise to a new tensor-product decomposition, such that the dressed object and the dressed environment are only weakly entangled by Einstein–Podolsky–Rosen correlations – a well-defined but mathematically difficult problem. While traditional Hilbert-space quantum mechanics has a simple rule how to describe the union of two quantum systems, the inverse problem is much more delicate. Given the Hilbert space $\mathscr{H}$ of a joint quantum system consisting of two elementary systems with the Hilbert spaces $\mathscr{H}_{\mathrm{A}}^{\text{bare}}$ and $\mathscr{H}_{\mathrm{B}}^{\text{bare}}$, $\mathscr{H} = \mathscr{H}_{\mathrm{A}}^{\text{bare}} \otimes \mathscr{H}_{\mathrm{B}}^{\text{bare}}$, how do we find the two physically relevant Hilbert spaces $\mathscr{H}_{\mathrm{A}}^{\text{dressed}}$ and $\mathscr{H}_{\mathrm{B}}^{\text{dressed}}$ of the dressed systems such that $\mathscr{H} = \mathscr{H}_{\mathrm{A}}^{\text{dressed}} \otimes \mathscr{H}_{\mathrm{B}}^{\text{dressed}}$?

4. The superposition principle versus the linearity of the equations of motion

4.1 Nonlinear equation of motion for open quantum systems

Already in 1939, in his basic paper on the unitary representations of the Lorentz group, Wigner adds the cautionary note: "The possibility of a future non linear character of the quantum mechanics must be admitted, of course." [130] Two decades later, Wigner speculates that one way out of the difficulties connected with the reduction of the state vector "amounts to the postulate that the equations of motion of quantum mechanics cease to be linear, in fact that they become grossly non-linear if conscious beings enter the picture." [131] By "non-linear" Wigner means that the equation of motion for the *state vector* is not linear. Wigner speaks of a "non-linearity of equations as an indication of life", but he never mentions the much simpler problems of engineering science: how can the strong nonlinearities and bifurcations of classical physics (celestial mechanics, hydrodynamics, turbulence) or the nonlinear phenomena of laser physics arise from the linear Schrödinger equation? The usefulness of nonlinear Schrödinger equations for the discussion of concrete quantum-mechanical problems is well documented in the literature. Do these enormously successful models actually violate quantum mechanics?

Some examples empirically successful nonlinear Schrödinger equations[132]

a) Interactions with the radiation field
A plausible, though neither rigorous nor fundamental, evaluation of the radiation reaction in quantum theory was proposed by Enrico Fermi already in 1927.[133] He proposed the following nonlinear Schrödinger

130 E. P. Wigner, *On unitary representations of the inhomogeneous Lorentz group,* Annals of Mathematics **40,** 149–204 (1939), footnote 1 on p. 149. In the *Collected Works,* volume 1, p. 334.

131 E. P. Wigner, *Remarks on the mind-body question,* in: I. J. Good (ed.), *The Scientist Speculates,* London, Heinemann (1962), pp. 284–301. The quotation is on p. 297; in the *Collected Works,* volume 6, on p. 259.

132 In the literature there are numerous papers on ad hoc modifications of the Schrödinger equations with the sole purpose of achieving reductions of wave packets, spontaneous localizations, or damping phenomena. These approaches are heuristically interesting but we will not discuss them here, just as little as purely mathematically inspired investigations.

133 E. Fermi, *Sul meccanismo dell' emissione nella meccanica ondulatoria,* Rendiconti dell' Accademia dei Lincei **5,** 795–800 (1927)

equation which includes not only the mutual interaction of the electrically charged particles but also their renormalized self-energy and radiation reaction,

$$i\hbar \frac{d}{dt} \Psi(\boldsymbol{q},t) \;=\; H\Psi(\boldsymbol{q},t) - \tfrac{2}{3}\,(e^2/c^3)\; \boldsymbol{q} \bullet \frac{d^3}{dt^3} \int \Psi(\boldsymbol{r},t)^* \boldsymbol{r}\, \Psi(\boldsymbol{r},t)\, d^3\boldsymbol{r} \quad .$$

The nonlinear term is modelled after the radiative reaction force $\tfrac{2}{3}\,(e^2/c^3)\,\ddot{\boldsymbol{q}}$ of the classical Lorentz electron theory and accounts for the spontaneous emission of radiation. The radiation reaction potential $\int \Psi(\boldsymbol{r},t)^*\, \boldsymbol{r}\, \Psi(\boldsymbol{r},t)\, d^3\boldsymbol{r}$ is the expectation value of the dipole moment of the radiating charge. A related modern approach – using the coupled Dirac–Maxwell equations – has been applied by Asim Barut to discuss the radiative processes in quantum electrodynamics, in particular to compute the Lamb shift, spontaneous emission, and the Casimir–Polder van der Waals forces without field quantization.[134] The results are remarkable: from our present viewpoint they show that *nonlinear feedback effects from a classical environment can account for phenomena usually considered as purely quantal.*

b) Onsager reaction fields

The first conceptually sound discussion of feedback effects via the environment of molecular systems is due to Lars Onsager.[135] He observed that the severe disagreement of Debye's dipole theory with the experiment is due to the neglect of the reaction field in Debye's theory. Rephrasing Onsager's physical ideas in a phenomenological quantum-mechanical language one may write the effective molecular Hamiltonian H in the form $H = H_0 + V$, where H_0 represents the molecular Hamiltonian in the vacuum, and V denotes the interaction energy between the molecular system and the embedding polarizable medium. This effective interaction V can be approximated by the interaction energy between the molecular dipole moment and the reaction field due to the polarization of the surrounding medium, $V = -\boldsymbol{\mu} \bullet \boldsymbol{E}_r$, where $\boldsymbol{\mu}$ denotes the electric dipole moment operator of the molecular system, and $\boldsymbol{E}_r$ is the electric field strength of the reaction field, $\boldsymbol{E}_r = \lambda \langle \Psi | \boldsymbol{\mu} \Psi \rangle$. The parameter λ depends on the dielectric constant of the medium and represents the degree of the polarizability of the environment. In this way one gets the following phenomenological *nonlinear* Schrödinger equation[136]

$$i\hbar\, \dot{\Psi}(t) \;=\; \{ H_0 - \lambda\, \boldsymbol{\mu} \bullet \langle \Psi(t) | \boldsymbol{\mu} \Psi(t) \rangle \}\, \Psi(t) \quad .$$

The nonlinearity is due to the electric dipole interaction of the molecule with the medium which then acts back by the polarization term $\langle \Psi(t) | \boldsymbol{\mu} \Psi(t) \rangle$ on the molecule.

c) The Hartree approximation

We consider an arbitrary quantum system in the traditional irreducible representation on a Hilbert space $\mathscr{H}$. The time-dependent variational principle of quantum mechanics stems from an action functional $\int_{t_1}^{t_2} L(t)\, dt$, where the Lagrangian L is given by $L(t) := \langle \Xi(t) \,|\, i\hbar\, \partial/\partial t - H \,|\, \Xi \rangle$ and H is the Hamiltonian of the system. The Lagrangian L is a functional of a trial state vector $\Xi(t) \in \mathscr{K}$ and its complex conjugate $\Xi^*(t) \in \mathscr{K}$, where $\mathscr{K} \subseteq \mathscr{H}$ has to be a symplectic manifold. The equations of motion are determined by requiring the stationarity of the action, $\delta \int_{t_1}^{t_2} L(t)\, dt = 0$ with respect to fixed end points and variations of $\Psi(t)$ and $\Psi^*(t)$.[137] They are given by $i\hbar\, \partial \Xi(t)/\partial t = \delta \langle \Xi(t) | H | \Xi(t) \rangle / \delta \Xi^*(t)$, $\Xi(t) \in \mathscr{K}$. The choice $\mathscr{K} = \mathscr{H}$ leads to the time-dependent Schrödinger equation $i\hbar\, \dot{\Xi}(t) = H\, \Xi(t)$. In general, the subset $\mathscr{K}$ does not have to be a subspace. But in order that the time-dependent variational principle leads to a well-defined dynamics, the subset $\mathscr{K}$ has to be a symplectic manifold. If the Hilbert space is a tensor product, $\mathscr{H} = \mathscr{H}^{\mathrm{A}} \otimes \mathscr{H}^{\mathrm{B}}$, and if one restricts the trial state vectors to products states,

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134 Compare the review by A. O. Barut, *Foundations of self-field quantumelectrodynamics,* in: A. O. Barut (ed.), *New Frontiers in Quantum Electrodynamics and Quantum Optics,* New York, Plenum Press (1990), pp. 345–370.

135 L. Onsager, *Electric moments of molecules in liquids,* Journal of the American Chemical Society **58,** 1486–1493 (1936).

136 S. Yomosa, *Nonlinear Schrödinger equation on the molecular complex in solution.,* Journal of the Physical Society of Japan **35,** 1738–1746 (1973).

137 Compare P. A. M Dirac, *The Lagrangian in quantum mechanics,* Physikalische Zeitschrift der Sowjetunion **3,** 64–72 (1933); J. Frenkel, *Wave Mechanics. Advanced General Theory,* Oxford, Clarendon (1934), pp. 253–254; J. Schwinger, *The theory of quantized fields. I,* Physical Review **82,** 914–927 (1951); J. Schwinger, *A note on the quantum dynamical principle,* Philosophical Magazine **44,** 1171–1179 (1953); J. R. Klauder, *Restricted variations of the quantum mechanical action functional and their relation to classical dynamics,* Helvetica Physica Acta **35,** 333-335 (1962).

$\Xi(t) := \Psi(t) \otimes \Phi(t)$, $\Psi(t) \in \mathscr{H}^{\mathrm{A}}$, $\Phi(t) \in \mathscr{H}^{\mathrm{B}}$, one gets the so-called Hartree approximation to the Schrödinger equation $i\hbar\dot{\Xi}(t) = H\,\Xi(t)$. With the Hamiltonian $H = H^{\mathrm{A}} \otimes \mathbf{1} + \mathbf{1} \otimes H^{\mathrm{B}} + \sum_k A_k \otimes B_k$ one gets the following state-dependent Schrödinger equations

$$i\hbar\dot{\Psi}(t) = H^{\mathrm{A}}\,\Psi(t) + \sum_k \langle \Phi(t) \,|\, B_k\,\Phi(t) \rangle_{\mathrm{B}}\, A_k \;\;,\;\; i\hbar\dot{\Phi}(t) = H^{\mathrm{B}}\,\Phi(t) + \sum_k \langle \Psi(t) \,|\, A_k\,\Psi(t) \rangle_{\mathrm{B}}\, B_k \;\;.$$

If the solution of the second Schrödinger equation is inserted in the first one, one gets a *nonlinear* effective Schrödinger equation for the state vector $\Psi(t)$.

In the foregoing examples from practical quantum mechanics, the nonlinearities of the effective Schrödinger equations are due to *state-dependent backreaction terms*. Since they can be obtained by the time-dependent variation principle of quantum mechanics with a subsidiary factorization condition, they do not lead to any dynamical inconsistencies. Of course, such nonlinear equations of motion are not universal but depend on the environment of the object.[138]

The optimal factorization $\Xi(t) = \Psi(t) \otimes \Phi(t)$ of an entangled state vector in the sense of the time-dependent variation principle of quantum mechanics is referred to as the *Hartree factorization*. Thereby, the Einstein–Podolsky–Rosen correlations are not simply neglected but taken into account at least partially by the nonlinear reaction terms in the effective Hartree equation of motion. The Hartree factorization can be considered as a replacement of the Hamiltonian $H^{\mathrm{A}} \otimes \mathbf{1} + \mathbf{1} \otimes H^{\mathrm{B}} + \sum_k A_k \otimes B_k$ of a joint system by the two time-dependent Hamiltonians $H^{\mathrm{A}} + \sum_k \langle B_k \rangle_t\, A_k$ and $H^{\mathrm{B}} + \sum_k \langle A_k \rangle_t\, B_k$, where $\langle B_k \rangle_t$ and $\langle A_k \rangle_t$ are the expectation values of the interaction operators at time t. A large part of the success in laser theory and quantum optics is based on such a Hartree factorization of the expectation value $\langle A \otimes F \rangle$ of products an atomic operators A and light-field operator F, $\langle A \otimes F \rangle \approx \langle A \rangle \langle F \rangle$. [139]

The Hartree-factorization is usually regarded as an approximation. Experience, in particular in radiation theory, shows that the success of the factorization approximation is often amazing. However, it is not very easy to explain how the *nonlinear* Hartree-approximation can be a good approximation to the *linear* Schrödinger equation. The main point is that the influence of the environment is not an annoying complication but a conceptually essential feature for the constitution of a quantum object. As the examples show, the Hartree factorization may lead to self-energy and cloud effects which can also be obtained by appropriate dressing transformations.[140] If one performs a unitary dressing transform, the entanglement between the two bare systems can be reduced (or even eliminated) by a new division in terms of a dressed object and a dressed environment, mathematically characterized by a new tensor product decomposition of the Hilbert space of the joint system. That is the environment of a bare object changes it to a dressed object.

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138 Nonlinear effective Schrödinger equations should not be confused with the many unsuccessful proposals to *modify* quantum mechanics by introducing a *fundamental* nonlinearity into the Schrödinger equation. Such ad-hoc modifications have been proposed many times, most recently by S. Weinberg, *Testing quantum mechanics,* Annals of Physics **194,** 336–386 (1989). Weinberg's proposal leads also to severe difficulties, compare for example D. Sahoo, *A deficiency of Weinberg's quantum mechanics,* Letters in Mathematical Physics **31,** 93–100 (1994).

139 Compare for example pp. 50–58 in the review by H. Haken, *Laser theory,* in: S. Flügge (ed.), *Encyclopedia of Physics, Volume XXV/2c, Light and Matter Ic,* Berlin, Springer (1970), pp. 1–320. For a mathematically rigorous justification of such models used in laser theory, compare K. Hepp & E. H. Lieb, *On the superradiant phase transition for molecules in a quantized radiation field: the Dicke maser model,* Annals of Physics **76,** 360–404 (1973); K. Hepp & E. H. Lieb, *Equilibrium statistical mechanics of matter interacting with the quantized radiation field,* Physical Review **A 8,** 2517–2525 (1973); K. Hepp & E. H. Lieb, *Phase transitions in reservoir-driven open systems with applications to lasers and superconductors,* Helvetica Physica Acta **46,** 573–603 (1973).

140 For illuminating simple examples, compare section 7–9 in L. Davidovich & H. M. Nussenzweig, *Theory of natural line shape,* in: A. O. Barut (ed.), *Foundations of Radiation Theory and Quantum Electrodynamics,* New York, Plenum Press (1980), pp. 83–108.

The important point is that in experimental science one never observes naked objects, but always dressed ones. The fact that the empirically relevant tensor-product decomposition of the world becomes time-dependent leads to grave mathematical difficulties. In mathematically much simpler Hartree approximation one retains the tensorization with respect to the bare systems, but describes the dressing effects by nonlinear reaction terms.

Provided the dressed environment admits of a *classical* description[141], hence a contextual description in terms of a commutative algebra, then an pure-state Hartree factorization can easily be shown to be *exact*.[142] There is in fact a nontrivial correspondence between quantum mechanics with nonlinear operators and partially classical theories.[143]

4.2 Nonlinear Schrödinger equations do not violate the superposition principle

In view of the conceptual difficulties of the orthodox view, Wigner examines the possibility that "the superposition principle will have to be abandoned".[144] However, it is not quite clear what he precisely means by this statement. According to Wigner's discussion, the individual state reduction is in conflict with the *linearity* of the Schrödinger equation but not with the superposition principle. Therefore, we have to emphasize that *the concept of a coherent superposition is completely independent of the dynamics.*

A *coherent superposition* of two different pure states gives a *new* pure state which is entirely different from the two generating pure states. Coherent superpositions do not exist in classical theories. The *superposition principle* of quantum mechanics is the postulate that for every two pure states there exist coherent superpositions. If this superposition principle is valid, it easily follows that we cannot only construct one new pure state, but uncountably infinitely many. In the Hilbert-space formalism, a pure state can be represented by a ray, and the ray by a state vector. In this formalism, the superposition principle takes a mathematically most simple (but conceptually perhaps irritating) form, namely: If Ψ_1 and Ψ_2 are two state vectors representing two different pure states, then any linear combination $\Psi = c_1 \Psi_1 + c_2 \Psi_2$, with $|c_1|^2 + |c_2|^2 = 1$, represents a new pure state, called a coherent superposition of the two generating states. A representation-independent formulation is more complicated but very general and conceptually more illuminating.[145]

The superposition principle must not be mixed up with the possible invariance of coherent superpositions under time evolutions. Every automorphic dynamics (like that given by a linear Schrödinger equation) implies the temporal invariance of coherent superpositions. Real-world objects are not strictly isolated so that their dynamics is not automorphic, hence *we have no reason to expect that the dynamics of open systems leaves superpositions invariant.* For example, a dynamical semigroup or a nonlinear Schrödinger equation does in general not preserve superpositions. But it would be mistaken to say that a nonlinear Schrödinger equation violates the quantum mechanical

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141 For further explanations, compare section 6.

142 The relevant theorem can be found in M. Takesaki, *Theory of Operator Algebras I,* New York, Springer (1979), theorem 4.14, p. 211: Let $\mathscr{A}$ and $\mathscr{B}$ be two C*-algebras and $\mathscr{C} = \mathscr{A} \otimes \mathscr{B}$ their minimal tensor product. Every pure state functional γ on $\mathscr{C}$ is of the form $\gamma = \alpha \otimes \beta$ for some pure state functionals α of $\mathscr{A}$ and β of $\mathscr{B}$ if and only if either $\mathscr{A}$ or $\mathscr{B}$ is commutative.

143 Compare for example R. Haag & U. Bannier, *Comments on Mielnik's generalized (nonlinear) quantum mechanics,* Communications in Mathematical Physics **60,** 1–6 (1978); S. Bugajski, *Nonlinear quantum mechanics is a classical theory,* International Journal of Theoretical Physics **30,** 961–971 (1991).

144 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963), The quotation is on p. 13; in the *Collected Works,* volume 6, on p. 177.

145 Compare J. E. Roberts & G. Roepstorff, *Some basic concepts of algebraic quantum theory,* Communications in Mathematical Physics **11,** 321–338 (1969).

superposition principle since at each instant, every coherent superposition of states with arbitrary coefficients still is an admissible initial state. *The quantum mechanical superposition principle does not imply the invariance of coherent superpositions under the time evolution.*

4.3 In every statistical description the equations of motion are inevitably linear

All *statistical* physical theories are based on the concept of convexity, where the basic notion is the formation of mixtures of statistical states.[146] The appropriate mathematical structure for statistical theories is that of an abstract convex structure, which is quite independent of the theory of linear spaces. However, a fundamental theorem by Stone says that every convex structure can be represented by a convex set in a real vector space.[147] If we accept Mackey's axiom IX which postulates the commutativity of the operation of mixing with the time evolution semigroup for a statistical ensemble,[148] then it can be shown that under weak technical conditions the dynamics can be uniquely extended to a one-parameter semigroup of *linear* operators acting in the predual of the algebra of global observables.[149] That is, *the equations of motion for statistical states are necessarily linear.* This result is independent from the quantum-mechanical superposition principle, it is also true for classical statistical theories. Well-known examples are Koopman's Hilbert-space formalism (which rephrases the nonlinear Hamiltonian equations of motion of classical point mechanics in terms of linear equations of motion for classical statistical mechanics),[150] the equivalence of *nonlinear* stochastic differential equations in the sense of Itô (which provide a stochastic individual description) with the *linear* Fokker-Planck equations (which give a statistical ensemble description),[151] or the relation between the nonlinear individual state reduction postulate and the linear statistical projection postulate. The rash argument "that linear equations are usually an approximation to a more adequate theory" [152] certainly does not apply to the linear Fokker-Planck equations, for the same reason it does not apply to the time-dependent Schrödinger equation in the *statistical* ensemble interpretation.[153]

In the literature there is some confusion about the meaning of the concept of *linearity* of the equations of motion. There is always a profound difference between the mathematical representation of the dynamics of a physical system in an individual and in a statistical description. In the context of the nonlinear field equations of general relativity, Albert Einstein characterizes this dissimilitude of individual and statistical descriptions as follows:

> "At the present time the opinion prevails that a field theory must first, by 'quantization', be transformed into a statistical theory of field probabilities according

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146 Compare for example S. P. Gudder, *Convexity and mixtures,* SIAM Review **19**, 221–240 (1977). Erratum: **20**, 837 (1978).

147 M. H. Stone, *Postulates for the barycentric calculus,* Annali di Matematica Pura ed Applicata **29**, 25–30 (1949).

148 G. W. Mackey, *The Mathematical Foundations of Quantum Mechanics,* New York, Benjamin (1963). p. 81.

149 Compare for example W. Guz, *On quantum dynamical semigroups,* Reports on Mathematical Physics **6**, 455–464 (1974). The basic results are due to R. V. Kadison, *Transformations of states in operator theory and dynamics,* Topology **3**, Suppl. **2**, 177–198 (1965).

150 B. O. Koopman, *Hamiltonian systems and transformations in Hilbert space,* Proceedings of the National Academy of Sciences of the United States of America **17**, 315–318 (1931).

151 L. Arnold, *Stochastische Differentialgleichungen. Theorie und Anwendung,* München, Oldenbourg (1973).

152 J. Waniewski, *Mobility and measurements in nonlinear wave mechanics,* Journal of Mathematical Physics **27**, 1796–1799 (1986).

153 As pointed out first by T. Carleman, *Application de la théorie des équations intégrales linéaires aux systèmes d'équations différentielles non linéaires,* Acta Mathematica **59**, 63–87 (1932), any nonlinear differential equation can be converted (in numerous ways) into a linear differential equation of infinite order.

> to more or less established rules. I see in this method only an attempt to describe relationships of an essentially nonlinear character by linear methods."[154]

In contrast to classical mechanics, in the traditional irreducible Hilbert-space representation of quantum mechanics, this situation can easily be overlooked since in this formulation individual states and statistical states are elements of isomorphic Banach space.[155]

In an *individual* description the time evolution transforms individual states into individual states. In the context of algebraic quantum mechanics (which includes classical mechanics), it can be shown that individual states of an isolated system can be represented by extremal linear functionals on the appropriate C*-algebra of observables (i.e. by not necessarily normalized pure states). A priori, there are no reasons why the time evolution of such individual states should be linear (as a rule, the equations of motion of individual states in classical mechanics are in fact nonlinear). In the generic case, we have to expect that the dynamics is nonlinear. If the dynamics of an individual isolated quantum system can be described in the Hilbert-space formalism, the corresponding Schrödinger equation is in general *nonlinear*.

Since a state vector may represent the individual state of a single object as well as a statistical state of a homogeneous ensemble, a failure to distinguish properly between individual and statistical descriptions can lead to serious difficulties. The linearity of the Schrödinger equation for a state vector representing a *statistical* state is a triviality which must not be confused with a possible nonlinearity of an effective Schrödinger equation describing the time evolution of a state vector representing a *single* system.

5. The modern development of quantum theory

5.1 Symmetries and their spontaneous breakdown

In the early years of the development of quantum mechanics Wigner laid the foundations both for the application of group theory to the study of atomic and molecular spectra and for the role of symmetry principles in quantum mechanics. In the preface to the English edition of his seminal text book of 1931, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren*,[156] Wigner remarks that "there was a great reluctance among physicists towards accepting group-theoretical arguments and the group-theoretical view".[157] In the last few decades, this attitude has changed dramatically. One of the reasons is the influence of Wigner's profound work on group representations and on the role of invariances in physical theories. Today, we regard the principles of symmetry as one of the most fundamental parts of our theory of matter. Therefore

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154 A. Einstein, *The Meaning of Relativity,* fifth edition, Princeton, Princeton University Press (1967), p. 157.

155 In the terminology of algebraic quantum mechanics, an individual quantum state can be represented by an extremal positive element in the dual $\mathscr{A}^*$ of the noncommutative C*-algebra $\mathscr{A} = \mathscr{B}^{\infty}(\mathscr{H})$ of all compact operators on some Hilbert space $\mathscr{H}$. A statistical state can be represented by a normalized positive element of the predual $\mathscr{M}_*$ of the W*-algebra $\mathscr{M} = \mathscr{B}(\mathscr{H})$ of all bounded operators on $\mathscr{H}$. Since the dual $\mathscr{A}^*$ of the C*-algebra $\mathscr{A}$ and the predual $\mathscr{M}_*$ of the C*-algebra are both isomorphic to the Banach space $\mathscr{B}^1(\mathscr{H})$ of trace-class operators on $\mathscr{H}$, $\mathscr{A}^* \cong \mathscr{B}^1(\mathscr{H}) \cong \mathscr{M}_*$, there is a one-to-one correspondence between individual states (referring to a single system) and pure statistical states (referring to a homogeneous ensemble). This very exceptional coincidence (which arises only in traditional quantum mechanics) can lead to serious conceptual confusions.

156 E. P. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren,* Braunschweig, Vieweg (1931).

157 E. P. Wigner, *Group Theory,* New York, Academic Press (1959).

the famous violation of the reflection symmetry[158] came as a shock for most physicists. Wigner writes: "The fact that the laws of nature have no pure space-reflection symmetry has one consequence that is unpleasant to admit. It deprives us of the illusion that these laws are – in perhaps a subtle but nonetheless a real sense – the simplest laws that can be conceived and that are compatible with some obvious experience."[159]

In the context of the irreducible Hilbert space representation of traditional quantum mechanics, Wigner defines a symmetry transformation as a mapping of vector rays of one Hilbert space onto the vector rays of another Hilbert space, which admits an inverse and leaves invariant the transition probability (i.e. the absolute square of the inner product) associated with any pair of vector rays. Wigner's celebrated theorem on symmetry operations says that every symmetry is induced by a unitary or antiunitary vector transformation which is uniquely determined up to a phase.[160] This theorem plays a fundamental role in the analysis of the structure of quantum mechanics. For example, if we assume that the dynamics is given by a measurable one-parameter group of Wigner symmetries, then it follows that the time evolution of a state vector is given by a *linear* Schrödinger equation with a self-adjoint Hamiltonian.[161]

The subsequent developments in the theory of symmetries are related to gauge symmetries and broken symmetries. They still are based on Wigner's conviction that symmetries are the basis of physics, but Wigner did not assimilate the new trends. Wigner's sympathy was for *global* symmetries, he disapproved of local gauge symmetries as artificial:[162]

> "I would prefer not to call the gauge invariance a symmetry, because it does not express anything physical. It only tells us something about our mode of description."[163]

However, gauge symmetries assume a central position in the contemporary fundamental theories of matter. All interactions are due to gauge fields.[164] They provide the basis for the standard model which unifies the electromagnetic, the weak and the strong interactions.

Wigner also poses the question why all our direct observations refer to positions. He claims that "quantum mechanics does not give preference in any obvious way to observations of position

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158 Introduced by Wigner in his explanation of Laporte's selection rule as a consequence of parity invariance which allows electric dipole transitions only between even and odd states. Compare E. P. Wigner, *Einige Folgerungen aus der Schrödingerschen Theorie für die Termstrukturen,* Zeitschrift für Physik **43,** 624–652 (1927) (Collected Works, volume 1, pp. 53–819; E. P. Wigner, *Über die Erhaltungssätze in der Quantenmechanik,* Nachrichten von der Gesellschaft der Wissenschaften, Göttingen. Mathematisch Physikalische Klasse **1927,** 375–381 (1927) (Collected Works, volume 1, pp.84–90).

159 E. P. Wigner, *Violation of symmetry in physics,* Scientific American **213,** Number 6, 28–36 (1965). The quotation is on p. 35; in the *Collected Works,* volume 6, on p. 357.

160 E. P. Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren,* Braunschweig, Vieweg (1931), pp. 251–254; E. P. Wigner, *Group Theory,* New York, Academic Press (1959), pp. 233–236 and pp. 325–329.

161 Compare for example B. Simon, *Quantum dynamics: From automorphism to Hamiltonian,* in: E. H. Lieb, B. Simon & A. S. Wightman (eds.), *Studies in Mathematical Physics,* Princeton, Princeton University Press (1976), pp. 327–349, theorem 2.4, p. 331.

162 E. P. Wigner, *Symmetry and conservation laws,* Proceedings of the National Academy of Sciences **51,** 956–965 (1964). The quotation is on p. 961; in the *Collected Works,* volume 6, on p. 305.

163 E. P. Wigner, *The meaning of symmetry,* in: A. Zichichi (ed.), *Gauge Interaction, Theory and Experiment,* New York, Plenum Press (1984), pp. 729–733. The quotation is on p.732; in the *Collected Works,* volume 3, on p. 364.

164 Compare the short historical review by C. N. Yang, *Magnetic monopoles, fibre bundles, and gauge fields,* Annals of the New York Academy of Sciences **294,** 86-97 (1977), pp. 92–97. Gauge theory is based on following principle fundamentally due to Einstein: "If anything is conserved, it must be conserved locally". Quoted from R. Feynman, *The Character of Physical Law,* London, British Broadcasting Corporation (1965), p. 63.

co-ordinates".[165] Yet, the *only* interactions which can provide a coupling between an object system and a laboratory apparatus are due to zero-mass boson gauge fields, namely the electromagnetic and the gravitational field. The forces which originate from these zero-mass boson gauge fields are longe-range interactions which decrease with *spatial* distance. The corresponding interaction operators pick out a function of the *position* operator as an effective observable of the measuring instrument.

If the word were invariant under all transformations of a symmetry group, such a fact could never have been discovered. The conceptual necessity to break symmetries was clearly recognized already a century ago by Pierre Curie: "C'est la dissymétrie qui crée le phénomène."[166] Wigner approves of this view and states that observable consequences "are possible only because our knowledge of the physical world has been divided into two categories: initial conditions and laws of nature."[167] In all his papers dealing with conceptual questions on symmetries and invariance, Wigner emphasizes again and again the importance of the separation between the accidental initial conditions and universal natural laws. But he did not relate the distinction between contingent and necessary truths directly to symmetries. Today, most theoretical physicists believe "that global symmetries, the types studied by Wigner, are unnatural. They smell of action at a distance. We now suspect that all fundamental symmetries are local gauge symmetries. Global symmetries are either all broken or approximate or they are the remnants of spontaneously broken local symmetries."[168]

As an example, we may mention that a description of thermodynamic systems in terms of quantum mechanics, where the interactions are translation and rotation invariant, and given by gauge symmetries. The quantum-theoretical descriptions of the phenomenologically well-known phase transitions of systems in thermal equilibrium involve spontaneous breakdowns of the mentioned symmetry groups. For example, ferromagnetism involves the spontaneous breakdown of the rotation group, crystallization requires the spontaneous breakdown of the translation and rotation group, superfluidity is related to the breakdown of the special Galilei group, while superconductivity is connected with the spontaneously broken gauge group.

Wigner never discussed *spontaneously broken symmetries* which are nowadays on all levels of physics, from elementary particle physics to molecular and solid state physics, of utmost importance. His puzzling remark that "the physicist is not interested in the initial conditions, but leaves their study to the astronomer, geologist, geographer, etc."[169] can perhaps explains his dislike of broken symmetries.[170] On the other hand, Wigner mostly analyzed fictitious isolated systems with a finite number of degrees of freedom whose infinite environment he assumed to have a negligible effect. Yet, the interactions of an object system with its environment can have dramatic

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165 E. P. Wigner, *Epistemology of quantum mechanics,* In: *Contemporary Physics. Volume II,* Vienna, Atomic Energy Agency (1969), pp. 431–437. The quotation is on p. 436; in the *Collected Works,* volume 6, on p. 53.

166 P. Curie, *Sur la symétrie dans les phénomènes physiques,* Journal de Physique **3**, 393–416 (1894)Républié: *Oeuvres de Pierre Curie*. Paris: Gauthier-Villars, 1908. Pp.118–141, p. 402.

167 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37**, 595–632 (1965). The quotation is on p. 596; in the *Collected Works,* volume 3, on p. 207.

168 D. J. Gross, *Symmetry in physics: Wigner's legacy,* Physics Today **48**, No. 12, 46–50 (1995).

169 R. M. F. Houtappel, H. Van Dam & E. P. Wigner, *The conceptual basis and use of the geometric invariance priciples,* Reviews of Modern Physics **37**, 595–632 (1965). The quotation is on p. 596; in the *Collected Works,* volume 3, on p. 207.

170 In his important work on radiation damping or on cross sections of nuclear reactions, Wigner selected of course the proper initial conditions!

qualitative effects such as dressing, symmetry breakdown, and the emergence of qualitatively new properties. It is well known that the coupling of small molecular systems with the environment can lead to symmetry breakings which are of great biological significance such as molecular chirality.

Symmetry breakings arise only in theories of systems with an infinite number of degrees of freedom in which global symmetries can be realized in different ways. The irreducible Hilbert-space representation of traditional quantum mechanics, which Wigner preferred, is simply not rich enough to deal in a rigorous way with these phenomena.

5.2 Superselection rules

Since every physical system interacts with the electromagnetic and the gravitational field, the environment of every physical object is a system with infinitely many degrees of freedom. *In order to understand the influence of the environment on atomic, molecular, mesoscopic, and macroscopic systems, we have to discuss the behaviour of infinite systems*. Yet, a proper discussion of infinite systems (like the electromagnetic field) lies outside of the domain of competence of traditional Hilbert-space quantum mechanics or Fock-space quantum field theory. The first indication in this direction came from the famous paper by Wick, Wightman and Wigner of 1952, in which they showed that the superposition principle is not valid without restrictions.[171] They introduced the concept of a *superselection rule* by specifying subspaces of the Hilbert space of an irreducible representation such that linear combinations of vectors from distinct subspaces (also called *superselection sectors*) do not represent a physical state. In spite of the fact that the concepts of superselection sectors and the associated classical observables have become a most successful tool in molecular physics[172], in solid state physics, and in statistical physics[173], Wigner holds that superselection rules "limit the absolute generality of the rule of superposition … just enough to impair the mathematical beauty of the general, single and uniform Hilbert space as a frame for the description of all quantum mechanical states. They do not seem to alleviate significantly the conceptual question raised." [174] In an earlier paper, Wigner identifies the hypothesis that quantum mechanics is valid for all systems with the hypothesis that the superposition principle is universally valid.[175] From our present point of view, this appraisal is surprising since Wigner often expressed the opinion that the traditional irreducible representation of the basic principles of quantum mechanics does not provide a correct description of macroscopic phenomena.[176]

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171 G. C. Wick, A. S. Wightman & E. P. Wigner, *The intrinsic parity of elementary particles,* Physical Review **88,** 101–105 (1952). Reprinted in Wigner's *Collected Works,* volume 3, p. 102–106.

172 Compare for example the reviews: H. Araki, *On superselection rules,* in: M. Namiki (ed.), *Proceedings of the 2nd International Symposium Foundations of Quantum Mechanics in the Light of New Technology,* Tokyo, Physical Society of Japan (1987), pp. 348–354; A. S. Wightman & N. Glance, *Superselection rules in molecules,* Nuclear Pysics B (Proc. Suppl.) **6,** 202–206 (1989).

173 Compare for example the monographs: F. Strocchi, *Elements of Quantum Mechanics of Infinite Systems,* Singapore, World Scientific (1985); G. L. Sewell, *Quantum Theory of Collective Phenomena,* Oxford, Oxford University Press (1986).

174 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), Pp.369–385. The quotation is on p. 370; in the *Collected Works,* volume 6, on p. 56.

175 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963). Compare p. 15; in the *Collected Works,* volume 6, p. 180.

176 In an earlier paper (E. P. Wigner, *Die Messung quantenmechanischer Operatoren,* Zeitschrift für Physik **133,** 101–108 (1952); in the *Collected Works,* volume 6, p. 159), Wigner concedes that for more complicated systems there may be more essential variables than just rest mass, spin, electrical charge and baryon number . The concept of essential variables has been introduced by J. M. Jauch & B. Misra, *Supersymmetries and*

5.3 The early period of algebraic quantum mechanics

While Wigner adopted von Neumann's Hilbert-space codification almost dogmatically, von Neumann himself did not regard the Hilbert space as indispensable for quantum mechanics. In 1935 he wrote to Garret Birkhoff: "I would like to make a confession which may seem immoral: I do not believe absolutely in Hilbert space any more." [177] Moreover, Wigner was a coauthor of a fundamental paper (essentially written by von Neumann) which attempted to generalize quantum mechanics in a purely algebraic way, by focusing on observables which are not necessarily represented as operators acting on a Hilbert space.[178] Later on, this pioneering work led to a purely algebraic formulation of quantum mechanics in which superselection sectors and classical observables have a natural place, and which contains traditional quantum mechanics and classical mechanics as special cases.[179] But these developments did not change Wigner's stance.

From the modern point of view, which goes back to the fundamental group theoretical contribution by Weyl and Wigner, the basic tool for establishing a fundamental physical theory is the representation of the kinematical group as an automorphism group on an appropriate mathematical structure.[180] Accordingly, the essential relationship between classical and quantum mechanics is not quantization, but rather the common kinematical symmetry. In the Galilei-relativistic case, the rotation subgroup and the Weyl subgroup, describing the space translation and the velocity boosts, are of particular importance. They generate the canonical commutation relations for position and momentum, for the orbital angular momentum, and for the spin angular momentum. The historical codification of traditional quantum mechanics, due to John von Neumann, uses an *irreducible* representation of the canonical commutation relations on a Hilbert space.[181] The idea that the irreducibility has a physical meaning, is a still popular misconception. Irreducibility is a representation-dependent concept, hence physically irrelevant. This can be seen most clearly in a representation-independent algebraic formulation.[182] This formulation of physical theories is called "abstract", since it is defined without any reference to operators acting on some Hilbert space. In the algebraic formulation of physical theories, the basic mathematical object is an abstract C*-algebra. For finitely many degrees of freedom, the uniqueness theorem by Stone and von Neumann implies that the abstract C*-algebraic formulation of quantum theory is physically fully equivalent to von Neumann's irreducible formulation.[183] Consequently, for

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essential observables, Helvetica Physica Acta **34,** 699–709 (1961). Nowadays, essential observables are called classical observables.

177 Quoted on p. 158 in G. Birkhoff, *Lattices in applied mathematics,* Proceedings of Symposia in Pure Mathematics **2,** 155–184 (1961).

178 P. Jordan, J. von Neumann & E. Wigner, *On an algebraic generalization of the quantum mechanical formalism,* Annals of Mathematics **35,** 29–64 (1934).

179 For a review, compare sect. 1.2 *The emergence of the algebraic approach*, in: G. G. Emch, *Algebraic Methods in Statistical Mechanics and Quantum Field Theory,* New York, Wiley (1972). See also chapter 9 in: G. G. Emch, *Mathematical and Conceptual Foundations of 20th-Century Physics,* Amsterdam, North-Holland (1984).

180 This view has been introduced by H. Weyl, *Quantenmechanik und Gruppentheorie,* Zeitschrift für Physik **46,** 1–46 (1927).

181 J. von Neumann, *Mathematische Grundlagen der Quantenmechanik,* Berlin, Springer (1932).

182 Compare for example I. E. Segal, *Mathematical Problems of Relativistic Physics,* Providence, Rhode Island, American Mathematical Society (1963); in particular pp. 16–17.

183 M. H. Stone, *Linear transformations in Hilbert space. III. Operational methods and group theory,* Proceedings of the National Academy of Sciences of the United States of America **16,** 172–175 (1930); J. von Neumann, *Die Eindeutigkeit der Schrödingerschen Operatoren,* Mathematische Annalen **104,** 570–578 (1931).

finitely many degrees of freedom, the choice of a particular representation is just a matter of convenience.

5.4 Inequivalent representations and robust models

If the phase space of the canonical commutation relations is not locally compact (like for example in the case of the electromagnetic field), the uniqueness theorem by Stone and von Neumann is not applicable. In this case, continuously many *physically inequivalent and qualitatively different Hilbert-space representations* exist. The description of a physical system then depends on the choice of the representation space which is characterized by a new *contextual topology*, distinguishing between relevant and irrelevant features.

Not everybody is happy with the introduction of infinite systems, so we may ask: what is the relation between systems with a very large number $N \gg 1$ of degrees of freedom and infinite systems with $N = \infty$? If we start with a finite system and make N larger and larger, some expectation values become extremely sensitive with respect to small changes of the model (for example, sensitive to very small external perturbations). This lack of smoothness leads to instabilities: the model with a very large number of degrees of freedom is *not robust.* Nonrobust models can be investigated with Bogoliubov's method of quasi-averages.[184] One considers an expectation value $\langle A \rangle_{N,\varepsilon V}$ of an observable A of a system with N degrees of freedom and the Hamiltonian $H + \varepsilon V$. Here H is the Hamiltonian of the object under study and εV $(\varepsilon > 0)$ is an arbitrary external perturbation. The physically correct limit for a very large number of degrees of freedom is then taken to be $\lim_{\varepsilon \to 0} \lim_{N \to \infty} \langle A \rangle_{N,\varepsilon V}$. In general this limit depends in a critical way on the choice of the perturbation V, and it is of course different from the unphysical limit $\lim_{N \to \infty} \lim_{\varepsilon \to 0} \langle A \rangle_{N,\varepsilon V}$. All the different limits are equally significant – none of them is a priori distinguished. Depending on the choice of the arbitrarily small perturbation, one gets in general infinitely many physically different results which correspond to the infinitely many physically inequivalent representations of the canonical commutation relations with the associated qualitatively new features like symmetry breakings (as in ferromagnets, superfluids, supraconductors) and classical observables (as in chiral molecules).

Therefore, Wigner's restriction to the idealization of strictly isolated systems with finitely many degrees of freedom is justifiable if and only if these systems shows a robust behaviour with respect to the relevant observables. Very often this is not the case. Not only macroscopic, but already small molecular systems can be nonrobust against small perturbations arising from the environment. A physically correct discussion of such nonrobust systems requires not only the knowledge of the natural laws of motion and the contingent initial state of the object system, but also the contingent situation of the environment.

5.5 Algebraic quantum mechanics

Algebraic quantum mechanics is nothing else than a mathematically correct codification of the original ideas of quantum theory. It is a general representation theory of the basic canonical commutation relations (or the appropriate kinematical group), valid for all classical and quantal physical systems with finitely or infinitely many degrees of freedom. The fundamental laws are expressed in terms of an abstract C*-algebra ✔, and the fundamental symmetries are expressed by groups of automorphism of ✔. For historical reasons, we call the selfadjoint elements of ✔ *intrinsic observables.* They represent intrinsic properties, but have a priori nothing to do with

184 N. N. Bogoliubov, *Lectures on Quantum Statistics, Volume 2,* New York, Gordon and Breach (1970).

observations. The contingent properties are represented by a family of admissible initial states. Given such a family, there exist a general procedure for generating a contextual description adapted to the relevant contingent continuity condition: the so-called GNS-construction (according to Gelfand, Naimark and Segal). The intrinsic topology is given by the norm topology of the abstract C*-algebra $\mathcal{A}$ of intrinsic observables. A new *coarser* contextual topology can be introduced by selecting family of initial states. The GNS-construction enables us to set up a so-called GNS-Hilbert space $\mathcal{H}_\pi$ and a faithful representation π of the C*-algebra $\mathcal{A}$ as an isomorphic concrete C*-algebra $\pi(\mathcal{A})$ of operators acting on the Hilbert space $\mathcal{H}_\pi$. The closure of this C*-algebra $\pi(\mathcal{A})$ in the weak topology of the algebra of all bounded operators acting on $\mathcal{H}_\pi$ is a W*-algebra, called the algebra $\mathcal{M}_\pi$ of *contextual observables*.

The weak operator topology of $\mathcal{M}_\pi$ is coarser than the intrinsic norm topology of $\pi(\mathcal{A})$. It reflects the continuity requirement necessary for a continuous representation of the contingent initial conditions. The context-dependent W*-algebra $\mathcal{M}_\pi$ is always much larger than the generating C*-algebra $\pi(\mathcal{A})$, $\mathcal{M}_\pi \supset \pi(\mathcal{A})$. The observables of the W*-algebra $\mathcal{M}_\pi$ which are not elements of the C*-algebra $\pi(\mathcal{A})$ are called *emergent observables.* They are generated by the algebra of intrinsic observables, but they are not functions of the intrinsic observables.

Even if the center of the basic C*-algebra $\mathcal{A}$ is trivial (that is, if $\mathcal{A}$ does not contain any nontrivial element which commutes with all elements of $\mathcal{A}$), a contextually generated W*-algebra $\mathcal{M}_\pi$ possesses in general a nontrivial center $\mathcal{Z}_\pi$,

$$\mathcal{Z}_\pi := \{ Z | Z \in \mathcal{M}_\pi , ZM = MZ \text{ for every } M \in \mathcal{M}_\pi \} \quad . \tag{17}$$

The nontrivial selfadjoint operators of the center $\mathcal{Z}_\pi$ are called *classical observables.* In general, *most classical observables are emergent,* that is, they are elements $\mathcal{Z}_\pi$ but not elements of $\pi(\mathcal{A})$.

Example: Temperature as an emergent classical observable

The emergence of temperature as a contextual classical quantity of a quantum system without intrinsic classical observables is the result of a contingent auxiliary condition: the KMS-condition which warrants the stability with respect to small local perturbations, characterizing the thermal equilibrium.[185] A GNS-construction with a family of KMS-states as distinguished reference states generates a W*-algebra with a center which contains a classical temperature observable.[186]

If one restricts the dynamics of the W*-system to its center, one gets a *classical dynamical system* whose equations of motion are in general given by *nonlinear* differential equations.[187]

Different inequivalent representations lead to *different* emergent observables, to different symmetries, and to qualitatively new phenomena like symmetry breakings, phase transitions, the emergence of irreversibility and classical observables with their nonlinear dynamics. Note that in the algebraic approach superselection sectors and classical observables are not postulated, but derived from contingent conditions which are necessary in addition to the natural laws to describe physical systems. The existence of many inequivalent representations of the canonical commuta-

185 See R. Haag, D. Kastler & E. B. Trych-Pohlmeyer, *Stability and equilibrium states,* Communications in Mathematical Physics **38**, 173–193 (1974).

186 This result follows from a theorem by M. Takesaki, *Disjointness of the KMS states of different temperatures,* Communications in Mathematical Physics **17**, 33–41 (1970).

187 For an instructive example, compare T. Gerisch, R. Honegger & A. Rieckers, *Limiting dynamics of generalized Bardeen–Cooper–Schrieffer models beyond the pair algebra,* Journal of Mathematical Physics **34**, 943–968 (1993).

tion relations is of utmost physical importance for the explanation of for the variety and richness of the concrete and particular.[188]

5.6 Disjoint states and the measurement problem

A crucial concept for any discussion of the measuring process is the notion of disjointness. Two pure states are called *disjoint* if there exists a classical observable such that the expectation values with respect to these states are different. That is, mutually disjoint states can be distinguished and classified in a classical manner. Classically indecomposable states are called *factor states*. More precisely, a factor state is characterized by the fact that it is dispersion-free with respect to every classical observable. Every quantum state can be decomposed *uniquely* into a sum or integral of mutually disjoint factor states.[189] This so-called *central decomposition* represents the finest unique decomposition of a nonpure state into a *classical mixture.*[190]

A classical mixture allows an *ignorance interpretation* with respect to the central decomposition of a nonpure quantum state into factor states. A factor state is in general nonpure and allows infinitely many physically different pure states. Therefore, an ignorance interpretation of a factor state is not possible. Since in traditional quantum mechanics all states are factor states, an ignorance interpretation is not defensible in traditional quantum mechanics. *The only permissible ignorance interpretation of nonpure quantum states refers to the central decomposition into a classical mixture of mutually disjoint factors states.*

In a statistical description, the measurement problem is not – as often asserted – the problem how a pure state can be transformed into a nonpure state, or how a density operator can become diagonal in a preferred basis. This is a trivial task which can be described by an appropriate dynamical linear semigroup and its Hamiltonian dilation. *A proper statistical description of the measurement process has to show that a physically realistic dynamics can transform an initial factor state into a classical mixture of disjoint factor states.*

6. Contingent classical properties of quantum systems

6.1 On the classical character of the measuring tools

Wigner discusses two different opinions on the way the process of measurement could be considered:[191]

- according to Bohr and Fock, measuring instruments must be described classically,
- according to von Neumann, quantum mechanics is necessary for the description of the measuring process.

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188 Compare for example the reviews: S. Saunders, *The algebraic approach to quantum field theory,* in: H. R. Brown & R. Harré (eds.), *Philosophical Foundations of Quantum Field Theory,* Oxford, Clarendon Press (1988), pp. 149–186, F. Strocchi, *Long range dynamics and spontaneous symmetry breaking in many body systems,* in: A. Amann, L. Cederbaum & W. Gans (eds.), *Fractals, Quasicrystals, Chaos, Knots and Algebraic Quantum Mechanics,* Dordrecht, Kluwer (1988), pp. 265–285, W. Gans, A. Blumen & A. Amann, *Large-Scale Molecular System. Quantum and Stochastic Aspects – Beyond the Simple Molecular Picture,* New York, Plenum Press (1991), N. P. Landsman, *Algebraic theory of superselection sectors and the measurement problem in quantum mechanics,* International Journal of Modern Physics **A 6**, 5349–5371 (1991).

189 For details, compare for example chapter IV.6 in M. Takesaki, *Theory of Operator Algebras I,* New York, Springer (1979).

190 In the terminology of d'Espagnat, a classical mixture is called a proper mixture. Compare B. Espagnat, *An elementary note about »mixtures«,* in: A. De-Shalit, H. Feshbach & L. van Hove (eds.), *Preludes in Theoretical Physics. In Honor of V. F. Weisskopf,* Amsterdam, North Holland (1966), pp. 185–191.

191 E. P. Wigner, *Epistemology of quantum mechanics,* In: *Contemporary Physics. Volume II,* Vienna, Atomic Energy Agency (1969), pp. 431–437. The quotation is on p. 432; in the *Collected Works,* volume 6, on p. 49.

In a nutshell, the apparent contradiction between these two sensible views is the reason for the fruitlessness of so many discussions about the alleged measuring problem of quantum mechanics. Bohr justified his view by the remark that the prerequisite of any communication is the possibility of a description of facts in a classical language:

> "However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. The argument is simply that by the word 'experiment' we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics." [192]

This requirement reflects the actual scientific practice: *every experiment ever performed in physics, chemistry and biology has a classical operational description.*

Wigner attributes Bohr's view[193] that the description of the measuring instrument cannot be included in the realm of quantum mechanics also to Fock: "In fact, the Russian physicist V. Fock declared that the measuring apparata must be described classically, that is not quantum-mechanically, the systems on which the measurement is undertaken are quantum objects." [194] This statement is not correct. Fock emphasizes the necessity of dialectic thinking in quantum physics[195], but he does not exclude a quantum-theoretical description of the measuring instrument. He writes:

> "We call an 'instrument' such an arrangement which on the one hand can be influenced by, and interact with, an atomic object and on the other hand permits a classical description with an accuracy sufficient for the purpose of registering the said influence (consequently, the handling of the instrument so defined does not need further 'means of observation'). It should be noted at once that in this definition of the instrument it is quite immaterial whether the 'instrument' is made by human hands or represents a natural combination of external conditions suitable for the observation of the micro-object. The only essential point is that these conditions, as also the mean of observation in the narrow sense, must be described classically." [196]

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192 N. Bohr, *Discussion with Einstein on epistemological problems in atomic physics,* in: P. A. Schilpp (ed.), *Albert Einstein: Philosopher–Scientist,* Evanston, Illinois, Library of Living Philosophers (1949), pp. 199–241, p. 209.

193 Most clearly state in a letter of October 26, 1935, by Bohr to Schrödinger: "Das Argument ist ja dabei vor allem, dass die Messinstrumente, wenn sie als solche diesen sollen, nicht in den eigentlichen den eigentlichen Anwendungsberich der Quantenmechanik einbezogen werden können." Quoted on p. 510 in: J. Kalckar, *Niels Bohr Collected Works. Volume 7. Foundations of Quantum Physics II (1933–1958),* Amsterdam, Elsevier (1996).

194 E. P. Wigner, *Events, laws of nature, and invariance principles,* Mimeographed notes (1980). Reprinted in the Collected Works, volume 6, pp. 334–342. The quotation is on p. 338.

195 "Ich habe den Eindruck, daß Sie zu viel Wert auf eine vollständige Formalisierung der Theorie legen. Ich glaube, man muß mehr dialektisch denken, und wenn man das tut kann man auch in der gleichzeitigen Benutzung der klasssischen Beschreibung des Meßapparats und der quantenmechanischen Beschreibung des beobachteten Systems keinen Widerspruch erblicken." Letter of October 12, 1971, by V. Fock to H.P.

196 V. A. Fock, *On the interpretation of quantum mechanics,* Czechoslovak Journal of Physics 7, 643–656 (1957), p. 648.

6.2 The existence of classical features does not contradict the first principles of quantum mechanics

Fock's requirement that the relevant degrees of freedom of the means of observation have to admit of a classical description is not in contradiction with the requirement by von Neumann and Wigner that the apparatus has to be described by quantum mechanics. Neither Bohr nor Wigner did realize that quantum systems are capable of developing a contingent classical structure. Wigner's erroneous claim that the requirement to describe measuring instruments classically "is in conflict with the linearity of the quantum theoretical equations of motion" [197] is probably based on his idea that the initial conditions have an inherently random character:

> "There is a distinguishing property of correctly chosen, that is minimal set, of initial conditions which is worth mentioning. The minimal set of initial conditions not only does not permit any exact relation between its elements, on the contrary, there is reason to contend that these are, as a rule, as random as the externally imposed, gross constraints allow." [198]

> "In other words, the atoms in me and the molecules in me are as irregular motion as possible." [199]

Wigner gives no arguments for his belief which contradicts all the facts we know from biochemistry and molecular biology. It is true that the classical aspects of the material world cannot be grasped by using the first principles of quantum mechanics together with randomly chosen initial conditions. But this situation is entirely in accordance with Wigner's general views:

> "Just as legal laws regulate actions and behavior under certain conditions but do not try to regulate all actions and behavior, the laws of physics also determine the behavior of its objects of interest only under certain well-defined conditions but leave much freedom otherwise. The elements of the behavior which are not specified by the laws of nature are called initial conditions." [200]

For special initial conditions, a quantum system can have classical modes which can be described both by classical and by quantum mechanics. Well-known examples are the collective motions of quantum-mechanical many-body systems (like the collective modes describing the classical shape of molecules). The paradigmatic examples for quantum systems allowing a classical description are the so-called quasifree systems, such as the harmonic oscillator or the free electromagnetic field with an arbitrary coherent state as initial state.[201] Such special quantum states are governed by classical equations of motion and are therefore referred to as *classical quantum states.*[202]

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197 E. P. Wigner, *Epistemology of quantum mechanics,* In: *Contemporary Physics. Volume II,* Vienna, Atomic Energy Agency (1969), pp. 431–437. In the *Collected Works,* volume 6, on p. 50.

198 E. P. Wigner, *Events, laws of nature, and invariance principles,* Science **145,** 995–998 (1964). Compare p. 997, in the *Collected Works,* volume 3, p. 187.

199 E. P. Wigner, *The role and value of symmetry principles and Einstein's contributions to their recognition,* in: B. Gruber & R. S. Millman (eds.), *Proceedings of the Einstein Centennial Celebration: Science Symposium on Symmetries in Science,* New York, Plenum Press (1980), pp. 13–21. The quotation is on p. 14; in the *Collected Works,* volume 3, on p. 354.

200 E. P. Wigner, *Events, laws of nature, and invariance principles,* Science **145,** 995–998 (1964). The quotation is on p. 996; in the *Collected Works,* volume 6, on p. 322.

201 For example, H. Beck & A. Thellung, *Representation of external fields by means of coherent states,* Helvetica Physica Acta **42,** 678–684 (1969), proved that the description of a quantum system in an external classical electromagnetic field is equivalent to a fully quantum-mechanical description in terms of a quantum field in a coherent state.

202 Compare for example E. B. Davies, *Quantum Theory of Open Systems,* London, Academic Press (1976), pp. 120–124.

A family of quantum states is called classical if this family does not contain any coherent superposition of its elements. For example, the set of all pure coherent states is a family of classical quantum states. A dynamical quantum system with an initial state from an invariant family of classical quantum states is called a *classical quantum system*. Note that "classical" refers first of all to the *family*. To call a *particular* state classical makes sense only if the time evolution transforms this quantum state again into a state of the same family of classical quantum states. The existence of quantum systems showing a classical behaviour can be explained by the first principles of quantum mechanics. Classical quantum systems depend on Planck's constant $\hbar$, no fictitious limit $\hbar \to 0$ is involved. Whether or not a quantum object has degrees of freedom which behave classically depends on the *initial conditions* of the combined system consisting of the object of investigation *and* its environment. These initial conditions cannot be chosen arbitrarily, since the experimenter cannot manipulate the whole environment of a selected object.

Because the convex hull of a family of pure classical quantum states is a simplex, it generates the state space of a commutative unital C*-algebra which provides a algebraic description of a corresponding classical quantum system.[203] Such an equivalent description of a classical quantum system by a contextual commutative C*-algebra is extremely useful because it allows to apply the statistical decision methods of engineering science to classical quantum states.

A classical quantum state is a possible state for a strictly classical description. By construction, a classical quantum state never contradicts the Heisenberg inequality for noncommuting observables. From this it follows that a *pure* classical quantum state corresponds to a *nonpure* state in a strictly classical description in terms of a commutative C*-algebra. Nevertheless, it is a legitimate state which fulfills exactly the classical equations of motion.

6.3 The existence of classical properties in molecular quantum systems is an empirical fact

Throughout his life, Wigner adheres to the dated idea that a classical description refers necessarily to a macroscopic physical system.[204] In order to pose the measurement problem properly, it is important not to confuse microscopic and quantal, macroscopic and classical. Moreover, the historical distinction between "microscopic" and "macroscopic" has become obsolete since long. On the way from the atomic to the macroscopic level there is no point which marks a qualitative distinction. There is an essentially continuous transition from atoms to small molecules, to macromolecules, to grains, crystals and macroscopic bodies.

There are macroscopic systems showing quantal properties[205]. On the other hand, already rather small molecules can have classical properties, so that *the emergence of a classical behaviour is not a characteristic property of large systems*. Moreover, there are systems which have at the same time quantum and classical properties. For example, there are circular DNA-molecules, which possess at the same time purely quantal properties, like their photochemical characteristics, and strictly classical properties, like their knot-type and knot-number.

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203 Recall that the state space of a unital C*-algebra is a simplex if and only if the algebra is commutative. Compare for example M. Takesaki, *Theory of Operator Algebras I,* New York, Springer (1979), p. 251.

204 Compare for example E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 379; in the *Collected Works,* volume 6, on p. 65.

205 Compare for example the reviews by W. F. Vinen, *Macroscopic quantum effects in superfluids,* Reports on Progress in Physics **31,** 61–121 (1968); and by J. Bardeen, *Superconductivity and other macroscopic quantum phenomena,* Physics Today **43,** No. 12, 25–31 (1990).

Curiously for a scientist with an excellent background in chemistry, Wigner denies the objective reality of molecular chirality:

> "It should be admitted, first that the concept of the positions of atoms continues to be a useful concept but it has turned out to be an approximate one. It is useful under certain condition but it is not difficult to produce situations in which it is meaningless. A particular striking example is an optically active organic molecule in its normal state. This is neither right handed, nor left handed – in fact, the atoms have no clearly defined positions with respect to each other, not even approximately." [206]

This statement is in manifest contradiction to empirical facts. The reality of molecular chirality was dramatically demonstrated by the terrible Contergan tragedy which caused many severe birth defects. Contergan was the trade name of the drug thalidomide (3-phtalimido–2,6–dioxopiperidin, $C_{13}H_{10}N_2O_4$) which exists in two enantiomeric forms. At least from a phenomenological point of view, there exists a superselection rule between the right- and left-handed isomers. That is, the chirality of molecule is a *classical observable* representing an objective property. The left-handed stereoisomer of thalidomide is a powerful and maybe safe tranquilizer, but the right-handed isomer is a disastrous teratogenic agent, i.e. causing physical deformities in the developing embryo and fetus.[207] Irresponsible ignorance of stereochemical effects led to the approval of the racemic mixture of the two enantiomers for marketing as drug preparation in the 1960s.

7. Present-day versions of the statistical von-Neumann model

7.1 Insolubility theorems for the original statistical von-Neumann model

In the framework of von Neumann's model of a measurement of the first kind, Wigner has shown

> "that measurements which leave the system object-plus-apparatus in one of the states with a definite position of the pointer cannot be described by the linear laws of quantum mechanics."[208]

This result, as well as the various generalizations which remove some idealizations of Wigner's setup,[209] is undoubtedly correct, but the reference to the linearity of the equations of motion is irrelevant and misleading. The crucial point is that if something qualifies as a "pointer" of a measuring apparatus, it has necessarily to admit of a classical description (which does not exclude the possibility that it also has a quantum-theoretical description). Moreover, different pointer positions have to be described by asymptotically disjoint quantum states.

The algebraic version of Wigner's and related insolubility theorems is much more general and conceptually illuminating. It says that *the dual of an automorphic time evolution on any C*-algebra*

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206 E. P. Wigner, *Are we machines?,* Proceedings of the American Philosophical Society **113**, 95–101 (1969). The quotation is on p. 97; in the *Collected Works,* volume 3, on p. 485.

207 W. H. DeCamp, *The FDA perspective on the development of stereoisomers,* Chirality **1**, 2–6 (1989).

208 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31**, 6–15 (1963), The quotation is on p. 12; in the *Collected Works,* volume 6, on p. 174.

209 Compare for example B. Espagnat, *Two remarks on the theory of measurement,* Supplemento al Nuovo Cimento **4**, 828–838 (1966),,M. H. Fehrs & A. Shimony, *Approximate measurement in quantum mechanics. I,* Physical Review **D 9**, 2317–2320 (1974), A. Shimony, *Approximate measurement in quantum mechanics. II,* Physical Review **D 9**, 2321–2323 (1974), P. Busch & A. Shimony, *Insolubility of the quantum measurement problem for unsharp observables,* Studies in History and Philosophy of Modern Physics **27**, 397–404 (1996).

cannot generate disjoint states.[210] Accordingly, even in the very general framework of modern algebraic quantum theory, an automorphic dynamics cannot describe a measurement process which reaches in finite time classically different final states. In this modern formulation, Wigner's no-go theorem looses much of its force. It is known that there exist *linear* but nonautomorphic time evolutions which in a statistical description map an initial factor state into a classical mixture of disjoint factors states. Moreover, there exist automorphic time evolutions which produce asymptotically disjoint final states.

7.2 Realistic time evolutions are nonautomorphic

In the papers where Wigner considers quantum mechanics to be valid also for measuring instruments, he always assumed a unitary dynamics. He never gave a reason for this choice. In the formalism adopted by Wigner, a unitary transformation induces an automorphism of the underlying mathematical structure.

There are severe objections against the view that the most fundamental dynamics is given by a one-parameter group of automorphisms. First of all, we have to recall that every description in terms of a well-defined mathematical structure has to use irrelevant elements. Two descriptions which changes only these irrelevant elements but leave the relevant structure invariant, are called *isomorphic*. An isomorphism of the mathematical structure to itself is called an automorphism. If the time evolution is an automorphism, then *from an internal point of view nothing happens*. That is, automorphic time evolutions just change the point of view but do not lead to events. Therefore it seems to be inappropriate to postulate an automorphic dynamics – which is nothing but a logical symmetry – for a system which includes the observing tools.

Secondly, a time evolution cannot be simply postulated, but it has to be *derived* from the interactions between the systems considered. Group-theoretical arguments indeed suggest that the dynamics of the fictitious elementary systems (bare "particles", bare fields) is automorphic. But the known interactions between the bare elementary systems (described by zero-mass boson gauge fields) most probably destroy the automorphic character of the time evolution of the interacting (and self-interacting) joint system. Due to self-interaction and renormalization problems this question is mathematically very difficult and not yet solved in a fully satisfactory way. From models rigorously discussed in the framework of algebraic quantum mechanics we know that for systems with long-range interactions the dynamics is *not* an automorphism of the basic C*-algebra.[211]

7.3 Automorphic dynamics with asymptotically disjoint final states

The possibility that a time evolution which for all finite times is automorphic can lead to asymptotically disjoint states has been proved by Klaus Hepp.[212] Moreover, there is a dilation theorem saying that for every observable with a simple discrete spectrum there exists an apparatus quantum system and an automorphic dynamics for the joint system which transforms an arbitrary initial factor state asymptotically into the appropriate classical mixture of disjoint factor states.[213] In this

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210 K. Hepp, *Quantum theory of measurement and macroscopic observables,* Helvetica Physica Acta **45**, 237–248 (1972), lemma 2, p.246.

211 Compare the short review by F. Strocchi, *Long range dynamics and spontaneous symmetry breaking in many body systems,* in: A. Amann, L. Cederbaum & W. Gans (eds.), *Fractals, Quasicrystals, Chaos, Knots and Algebraic Quantum Mechanics,* Dordrecht, Kluwer (1988), pp. 265–285.

212 K. Hepp, *Quantum theory of measurement and macroscopic observables,* Helvetica Physica Acta **45**, 237–248 (1972).

213 K. C. Hannabuss, *Dilations of a quantum measurement,* Helvetica Physica Acta **57**, 610–620 (1984).

construction, the constants in the dynamics can be chosen in such a way that the asymptotic states can be approached arbitrarily closely in an arbitrarily short time interval.

While Hepp's contribution was crucial, it did not solve the statistical measurement problem since it is still possible to undue the measurement at any finite time.[214] The key to fix this problem lies in recognizing the measuring apparatus as an irreversible dynamical system which breaks the time-inversion symmetry in such a way that it acts as a causal system. A model which combines asymptotic disjointness with genuine irreversibility has been worked out by C.M. Lockhart and Baidyanath Misra.[215]

It has been objected that a measurement process with asymptotically disjoint final states implies an infinite measurement time.[216] This is a gross misunderstanding: every measurement in engineering physics is of this type. In order to explain this assertion, we consider a customary statistical decision procedure. In experimental science, all observations have to be considered to be subject to random variations. The result of a statistical experiment is characterized by a probability measure on some measurable space $(\boldsymbol{\Omega}, \boldsymbol{\Sigma})$. Consider a statistical test for deciding whether the state of the apparatus is given either by the probability measure $\boldsymbol{\mu}'$ or by the probability measure $\boldsymbol{\mu}''$. Independently how we perform this test, the *minimal error probability* is given by[217]

$$e(\boldsymbol{\mu}', \boldsymbol{\mu}'') := \inf_{\mathcal{B} \in \boldsymbol{\Sigma}} \{\boldsymbol{\mu}'(\mathcal{B}) - \boldsymbol{\mu}''(\boldsymbol{\Omega} - \mathcal{B})\} = 1 - \|\boldsymbol{\mu}' - \boldsymbol{\mu}''\| \quad , \tag{18}$$

where $\|\boldsymbol{\mu}' - \boldsymbol{\mu}''\|$ is Kolmogorov's variation distance between $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}''$, $0 \leq \|\boldsymbol{\mu}' - \boldsymbol{\mu}''\| \leq 1$. If the minimal error probability vanishes, a perfect decision can be made with probability one. Such statistical tests are called *singular. In engineering science, models which lead to singular decisions problems are considered as ill-posed and unacceptable*.[218] If the essential degrees of freedom of the measuring instrument are realized by a commutative C*-algebra, a classical quantum state induces a GNS-representation as a Lebesgue space $L^{\infty}(\boldsymbol{\Omega}, \boldsymbol{\Sigma}, \boldsymbol{\mu})$ where the probability measure $\boldsymbol{\mu}$ represents the classical quantum state. If $\boldsymbol{\mu}'$ and $\boldsymbol{\mu}''$ represent disjoint classical quantum states, then the two probability measures are singular with respect to each other, $\|\boldsymbol{\mu}' - \boldsymbol{\mu}''\| = 1$.[219] That is, disjoint quantum states lead to singular decision problems which engineers reject as unrealistic idealizations.

The fact that the quantum states of the apparatus allow a classical description in terms of probability measures, opens the possibility to use the well-established distance measure between probability distributions to introduce a physically meaningful *measure of the approximate disjointness of two classical quantum states.* It is given by the minimal error probability of the associated statistical decision problem. In the models describing the measurement process

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214 J. S. Bell, *On wave packet reduction in the Coleman–Hepp model,* Helvetica Physica Acta **48**, 93–98 (1975).

215 C. M. Lockhart & B Misra, *Irreversibility and measurement in quantum mechanics,* Physica **136A**, 47–76 (1986). Compare also Mathematical Reviews ***87k***, November 1987, Review #87k:81006.

216 For example by N. P. Landsman, *Observation and superselection in quantum mechanics,* Studies in History and Philosophy of Modern Physics **26**, 45–73 (1995), p. 55.

217 A. Rényi, *On the amount of missing information and the Neyman-Pearson lemma,* in: F. N. David (ed.), *Research Papers in Statistics. Festschrift for J. Neyman,* London, Wiley (1966), pp. 281–288, A. Rényi, *Statistics and information theory,* Studia Scientiarum Mathematicarum Hungarica **2**, 249–256 (1967).

218 W. L. Root, *Singular Gaussian measures in detection theory,* in: M. Rosenblatt (ed.), *Proceedings of the Symposium on Time Series Analysis,* New York, Wiley (1963), pp. 292–315; W. L. Root, *Stability in signal detection problems,* in: R. Bellman (ed.), *Stochastic Processes in Mathematical Physics and Engineering,* Providence, Rhode Island, American Mathematical Society (1964), pp. 247–263; W. L. Root, *The detection of signals in Gaussian noise,* in: A. V. Balakrishnan (ed.), *Communication Theory,* New York, McGraw-Hill (1968), pp. 160–191.

219 Compare for example theorem 2 on p. 82 and theorem 4 on p. 86 in E. Nelson, *Topics in Dynamics. I: Flows,* Princeton, Princeton University Press (1969).

statistically with asymptotically disjoint final states, the effective measuring time can be arbitrarily short. For example, if $\boldsymbol{\mu}'(t)$ and $\boldsymbol{\mu}''(t)$ describe two classical quantum states which become asymptotically disjoint, $\lim_{t\to\infty}\|\boldsymbol{\mu}'(t)-\boldsymbol{\mu}''(t)\|=1$, then the error probability for a decision at time t which of the two possible final states $\boldsymbol{\mu}'(\infty)$ and $\boldsymbol{\mu}''(\infty)$ will asymptotically realized, is given by $1-\|\boldsymbol{\mu}'(t)-\boldsymbol{\mu}''(t)\|$. Given a threshold level $\varepsilon \ll 1$ for the error probability, the effective measuring time τ_ε is given by $\varepsilon = 1-\|\boldsymbol{\mu}'(\tau_\varepsilon)-\boldsymbol{\mu}''(\tau_\varepsilon)\|$. In the mathematical model for this process, the parameters in the Hamiltonian can be chosen in such a way that for any fixed threshold level $\varepsilon > 0$ the effective measuring time τ_ε can be made arbitrarily small.

8. Individual description of state reduction processes

8.1 Stochasticity within determinism

In this chapter, we examine the following important question Wigner poses:

> "The equations of motion of both quantum mechanics and of classical theory are deterministic; why, then, are the predictions not uniquely given by the inputs?" [220]

For classical systems, Wigner's question has a well-known answer. An individual *ontic state* in classical point mechanics can be represented by a point $\boldsymbol{\omega}$ of a phase space $\boldsymbol{\Omega}$. The knowledge available to an observer is never given by a point of the phase space, but by a Borel set of nonvanishing Lebesgue measure.[221] To every Borel set representing some knowledge, there exist a smaller Borel set representing a more detailed knowledge. But there is no smallest Borel set of nonvanishing Lebesgue measure representing maximal knowledge. Since a point (representing an ontic state) is a Borel set of Lebessgue measure zero, it can never represent an epistemic state. That is, in classical mechanics the ontic states are epistemically hidden. In the best case, an epistemic state specifies a measure $\boldsymbol{\mu}$ which gives the probability $\boldsymbol{\mu}(\mathscr{B})$ that the actual ontic state lies in the Borel set $\mathscr{B}$.

Classical point mechanics is deterministic in the sense that given an initial point $\boldsymbol{\omega}$ of the phase space $\boldsymbol{\Omega}$, the individual ontic state $\boldsymbol{\omega}_t \in \boldsymbol{\Omega}$ at all other times $t \in \mathbb{R}$ is uniquely determined by the Hamiltonian equations of motion. One of the most important results of contemporary classical dynamics is the proof that the deterministic differential equations of some smooth classical Hamiltonian systems have solutions exhibiting a genuine stochastic behaviour.[222] That is, the distance between any two trajectories having an arbitrarily small nonzero difference in initial conditions increases exponentially as a function of time.

The paradigmatic example for a deterministic mechanical system which behaves stochastically is Boltzmann's model of a hard sphere gas (consisting of hard spheres in a strictly isolated finite rectangular box and colliding elastically which each other and the walls). In a famous paper, Ya. G. Sinai gave a rigorous proof that the deterministic Boltzmann model is a K-system, hence

220 E. P. Wigner, *The philosophical problem,* in: B. d'Espagnat (ed.), *Foundations of Quantum Mechanics. International School of Physics "Enrico Fermi", 1970,* New York, Academic Press (1971), pp. 122–124. The quotation is on p. 122; in the *Collected Works,* volume 6, on p. 218.

221 The only reasonable choice for the finest Boolean algebra of events is the σ-algebra $\boldsymbol{\Sigma}/\boldsymbol{\Delta}$, where $\boldsymbol{\Sigma}$ is the σ-algebra of Borel sets of $\boldsymbol{\Omega}$, and $\boldsymbol{\Delta}$ is the σ-ideal of Borel sets of Lebesgue measure zero. Compare J. von Neumann, *Zur Operatorenmethode in der klassischen Mechanik,* Annals of Mathematics **33**, 587–642, 789-791 (1932), in particular pp. 595–598.

222 Compare for example A. J. Lichtenberg & M. A. Lieberman, *Regular and Stochastic Motion,* New York, Springer (1983).

showing complete instability and perfect epistemic stochasticity.[223] For such chaotic systems the time evolution of every point in phase space is strictly deterministic but neighboring initial points exhibit a *qualitatively* different evolution. In order to describe this nonrobust behaviour, the *individual trajectories* of the deterministic topological dynamical system have to be replaced by *bundles of trajectories*, corresponding to arbitrarily small open subsets of the phase space as initial states. Mathematically this means that we replace the topological dynamical system (corresponding to a C*-system) by a measure-theoretical dynamical system (corresponding to a W*-system). If the deterministic topological system is not robust with respect to slight variations of the initial state, then every bundle of trajectories spreads in course of time exponentially so that long-term predictions are impossible. In this sense, the measure-theoretical epistemic description is no longer deterministic.

In order to understand the chaotic behaviour of some deterministic systems it is of crucial importance to distinguish between individual and statistical descriptions, and between ontic and epistemic states. Determinism is a concept which refers exclusively to ontic states while predictability refers to epistemic states. Ontologically deterministic systems which do not allow long-term predictions have been called *cryptodeterministic* by Edmund Taylor Whittaker.[224] The deterministically evolving ontic state of a cryptodeterministic individual system is fundamentally not accessible by any kind of experiment. The probability concept associated with an epistemic description of a cryptodeterministic system is determined by the underlying deterministic dynamics of the ontic states, hence objective. Long ago, Marian von Smoluchowsky pointed out that the concept of probability can be defined and the laws of probability can be derived from the theory of strictly deterministic but nonrobust classical dynamical systems.[225] The modern theory of mixing and K-systems elucidate clearly the origin of the laws of probability: the probability distributions arising in these systems can be traced to a causal origin.

Wigner asserts:

> "But surely, we must realize that the quantum mechanical measurement process, the outcome of which is probabilistic, is not describable by the present quantum mechanical equations which are deterministic. It is natural to try to attribute the probabilistic outcome of the measurement to the uncertainty of the initial state of the measuring apparatus but it is easy to prove that this is impossible …" [226]

Wigner refers here to his insolubility theorem which in its most general form says that an automorphic dynamics cannot transform an initial factor state into a classical mixture of disjoint factor states. But this theorem does not imply that a deterministic dynamics (relative to an ontic description) cannot lead to outcomes which are (from an epistemic point of view) irreducibly probabilistic.

A major difficulty in any attempt to grasp Wigner's view is that he never distinguishes between the epistemically hidden pure states (which we refer to as ontic states) and genuine epistemic states. When he says

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223 Ya. Sinai, *On the foundations of the ergodic hypothesis for a dynamical system of statistical mechanics,* Soviet Mathematics Doklady **4,** 1818–1822 (1963).

224 E. T. Whittaker, *Chance, freewill and necessity in the scientific conception of the universe,* Proceedings of the Physical Society (London) **55,** 459–471 (1943).

225 M. von Smoluchowsky, *Über den Begriff des Zufalls und den Ursprung der Wahrscheinlichkeitsgesetze in der Physik,* Naturwissenschaften **6,** 253–263 (1918).

226 E. P. Wigner, *Events, laws of nature, and invariance principles,* Mimeographed notes (1980). Reprinted in the Collected Works, volume 6, pp. 334–342. The quotation is on p. 338.

"that the permanency of the validity of our deterministic law of nature became questionable as a result of the realization … that the states of macroscopic bodies are always under the influence of their environment; in our world they can not be kept separated from it," [227]

he seems to confuse ontic determinism with epistemic predictability. Clearly, the deterministic behaviour of the system refers to the Hamiltonian dynamics which governs the time evolution of ontic states, not the states representing our knowledge.

8.2 Von Neumann's model does not describe individual first-kind measurements

In the individual description, the von-Neumann model assumes that the interaction between the object system and the apparatus transforms an initial product state vector $\varPsi \otimes \beta$ ($\varPsi \in \mathscr{H}_{\text{obj}}$, $\beta \in \mathscr{H}_{\text{app}}$) into an entangled state vector

$$\varPsi \otimes \beta \;\to\; \sum_k c_k\, \alpha_k \otimes \beta_k \quad , \quad |c_k|^2 = |\langle \alpha_k | \varPsi \rangle|^2 \quad , \tag{19}$$

where $\alpha_k \in \mathscr{H}_{\text{obj}}$ is an eigenvector of the object observable to be measured. In this setting, the alleged state reduction problem is the question how the entangled state vector $\sum_k c_k\, \alpha_k \otimes \beta_k$ can be disentangled to some product vector $\alpha_j \otimes \beta_j$.

Wigner never asks whether von Neumann's model is appropriate for the description of first-kind measurements. Neither von Neumann nor Wigner attempt to discuss measurements in physical terms. In particular, von Neumann's model does not make any allowance of the fact that every experiment whatsoever can be described also in terms of classical engineering physics. Von Neumann and Wigner never make any special assumption about the nature of a measuring system. Of course, a measuring system has to fulfill very stringent additional conditions. If we fulfill Fock's requirement that the essential degrees of freedom of the measuring instrument must *also* admit of a classical description, then von Neumann's model does not apply. The reason is that a classical system cannot be entangled with an object system by Einstein–Podolsky–Rosen correlations.[228] In this case the time evolution (be it automorphic or not) maps a pure product state into pure product states. Accordingly, the interaction between the object system and the apparatus system should not to produce the map (19), but the map

$$\varPsi \otimes \beta \;\to\; \varPsi_t \otimes \beta_t \quad , \quad t \geq 0 \quad , \quad \varPsi, \varPsi_t \in \mathscr{H}_{\text{obj}} \quad , \quad \beta, \beta_t \in \mathscr{H}_{\text{app}} \quad . \tag{20}$$

Due to the interaction, the time evolutions $t \to \varPsi_t$ and $t \to \beta_t$ are not autonomous: the vector $\varPsi_t$ of the object system depends both on the initial state vector $\varPsi$ of the object system and on the initial state vector β of the apparatus. Similarly, the vector β_t depends both on the initial vector β and on the initial vector $\varPsi$. For all times, the factorization

$$\langle \varPsi_t \otimes \beta_t | (A \otimes B)\, \varPsi_t \otimes \beta_t \rangle \;=\; \langle \varPsi_t | A\, \varPsi_t \rangle_{\text{obj}}\, \langle \beta_t | B\, \beta_t \rangle_{\text{app}} \quad , \tag{21}$$

is valid, so that the dynamics is given by a Hartree-type evolution equation which leads in general to a nonlinear equation of motion for the state vector $\varPsi_t$ of the object system.

227 E. P. Wigner, *Events, laws of nature, and invariance principles,* Mimeographed notes (1980). Reprinted in the Collected Works, volume 6, pp. 334–342. The quotation is on p. 334.

228 Compare for example M. Takesaki, *Theory of Operator Algebras I,* New York, Springer (1979), theorem 4.14, p. 211.

8.3 Chaotic quantum processes

The map (20) refers to an individual description in terms of ontic states. As Wigner points out many times[229], a description is operationally meaningful only if the initial conditions can be reproduced experimentally. Since it is impossible *in principle* to prepare an initial pure state, we have to consider a family $\{\Psi(\boldsymbol{\omega})\otimes\beta(\boldsymbol{\omega})|\boldsymbol{\omega}\in\boldsymbol{\Omega}\}$ of initial state vectors $\Psi(\boldsymbol{\omega})\otimes\beta(\boldsymbol{\omega})$, together with a probability measure $\boldsymbol{\mu}$ which describes the distribution of the pure initial states. The individual time evolution maps pure states into pure states

$$\Psi(\boldsymbol{\omega})\otimes\beta(\boldsymbol{\omega}) \;\to\; \Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega}) \quad , \quad t\geq 0 \quad . \tag{22}$$

The statistics of the collection $\{\Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega})|\boldsymbol{\omega}\in\boldsymbol{\Omega}\}$ of state vectors refers to an individual system in a stochastic description over a *classical* probability space $(\boldsymbol{\Omega},\boldsymbol{\Sigma},\boldsymbol{\mu})$. This statistical description deals with a probability measure on the space of individual pure states. It should not be confused with the entirely different Gibbsian ensemble description in terms of density operators.[230] The σ-algebra $\boldsymbol{\Sigma}$ of Borel sets of $\boldsymbol{\Omega}$ corresponds to the experimentally accessible epistemic states, and $\boldsymbol{\mu}(\mathcal{B})$ denotes the probability that the parameter $\boldsymbol{\omega}\in\boldsymbol{\Omega}$ characterizing the ontic state vector $\Psi(\boldsymbol{\omega})\otimes\beta(\boldsymbol{\omega})$ lies in the Borel set $\mathcal{B}\in\boldsymbol{\Sigma}$.

In a situation corresponding to a laboratory experiment, the family $\{\beta_t(\boldsymbol{\omega})|\boldsymbol{\omega}\in\boldsymbol{\Omega}, t\geq 0\}$ is a family of classical quantum states. If we turn to a purely classical description, the sample space $\boldsymbol{\Omega}$ plays the role of a classical phase space. Moreover, the phase space can be chosen in such a way that the points $\boldsymbol{\omega}$ of $\boldsymbol{\Omega}$ are in a one-to-one correspondence with the state vector $\Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega})$, so that $\Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega})=\Psi(\boldsymbol{\omega}_t)\otimes\beta(\boldsymbol{\omega}_t)$, $\boldsymbol{\omega}_t\in\boldsymbol{\Omega}$. In this case, the individual time evolution $t\mapsto\Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega})$ is completely determined by the deterministic classical motion $t\mapsto\boldsymbol{\omega}_t$. Since ontic states are experimentally inaccessible, one has to investigate the dependence of the motion $t\mapsto\boldsymbol{\omega}_t$ on small changes of the initial condition.

If the classical motion $t\mapsto\boldsymbol{\omega}_t$ shows a sensitive dependence on the initial conditions, then the motion $t\mapsto\Psi_t(\boldsymbol{\omega})\otimes\beta_t(\boldsymbol{\omega})$ is cryptodeterministic so that it is not true

> “that the state vector is only a shorthand expression of that part of our information concerning the past of the system which is relevant for predicting (as far as possible) the future behavior thereof.”[231]

Since the electromagnetic field – which always belongs to the environment – acts like a K-system[232], we have to expect that every realistic model for the measuring process shows a sensitive dependence on the initial conditions. If in addition the corresponding statistical dynamics maps every initial factor state into a classical mixture of disjoint factors states, then the process $t\mapsto\Psi_t\otimes\beta_t$ can create new facts. As a closer discussion shows, the reduced individual description of the object system is in this case given by a stochastic and nonlinear equation for the state vector $\Psi_t(\boldsymbol{\omega})$. Thereby the dissipative environmental effects are represented by classical fluctuating forces and damping terms, while the polarizations and reaction fields lead to feedback effects

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229 See for example E. P. Wigner, *The extension of the area of science,* in: R. G. Jahn (ed.), *The Role of Consciousness in the Physical World,* AAAS Symposium No.57, Boulder, Westview Press (1981), pp.7–16. I the *Collected Works,* volume 6, p.608.

230 Compare also A. Amann, *Modeling the quantum mechanical measurement process,* International Journal of Theoretical Physics **34,** 1187–1196 (1995); A. Amann, *Structure, dynamics and spectroscopy of single molecules: A challenge to quantum mechanics,* Journal of Mathematical Chemistry **18,** 247–308 (1995).

231 E. P. Wigner, *The problem of measurement,* American Journal of Physics **31,** 6–15 (1963). The quotation is on p.13; in the *Collected Works,* volume 6, p.176.

232 See L. C. Thomas, *A note on quantising Kolmogorov systems,* Annales de l’Institut Henri Poincaré, Physique théorique **A 21,** 77–79 (1974).

which give rise to the nonlinearity of the equations of motion. Wigner would like such a possibility as "the most natural explanation of the indeterminate outcome of the measurement", but he rejects it "though only reluctantly" since it contradicts his favored von-Neumann model.[233] But his additional remark that "this explanation … cannot give the probabilities for the various outcomes of the measurement" hits the mark. There exist reasonable Hamiltonian quantum-theoretical models which generate purely nondeterminisitic (K-flow type) stochastic quantum processes with asymptotically final states producing facts. There also exist processes which in addition realize the expectation-value postulate but this is the exception. Therefore the problem is not the state reduction but the validity of the expectation-value postulate, a question never addressed by Wigner. *How can an experimentalist decide whether his apparatus gives results in agreement with the expectation-value postulate?*

9. Conclusions

Many of Wigner's papers on mathematical physics are great classics. Most famous is his work on group representations which is of lasting value for a proper mathematical foundation of quantum theory. The modern development of quantum theory (which is not reflected in Wigner's work) is in an essential way a *representation theory* (e.g. representations of kinematical groups, or representations of C*-algebras). This view owes very much to Wigner's seminal papers on the unitary representations of compact and noncompact groups (assembled and reviewed in volume 1 of the *Collected Works*).

Wigner showed much courage in relating the then unresolved questions of the measurement problem to the much deeper problem of consciousness. In view of this very unorthodox proposal it is astonishing that Wigner was very reactionary with respect of the dogmas of orthodox quantum mechanics. In contrast to von Neumann himself, he took the old von-Neumann codification of quantum mechanics as authoritative and not to be questioned. Much of the efforts to interpret the meaning of this codification and to prove no-go theorems, such as the insolubility of the measurement problem or the impossibility of a quantum theory of individual objects, are physically irrelevant since they are based on a codification of quantum mechanics that is valid only for strictly closed systems with finitely many degrees of freedom. However, in nature there are no such systems. Every material system is coupled to the gravitational and to the electromagnetic field – systems which require in a Hamiltonian description infinitely many degrees of freedom. A deeper insight into the conceptual problems of quantum theory is possible only if the modern development of a quantum theory of infinite systems is taken into account.

The last three decades have witnessed striking progress in understanding the interface between quantum-theoretical and classical descriptions. This is based on our deeper understanding of the consequences of Einstein–Podolsky–Rosen correlations for a proper concept of a subsystem, the role of inequivalent representations, superselection sectors, the phenomenon of the spontaneous breakdown of symmetries, and the emergence of classical properties. In spite of their outstanding significance for a satisfactory solution to the interpretative problems raised by the founders of quantum theory, these advances in mathematical physics are hardly reflected in the contemporary philosophical literature. They owe much to Wigner's work in mathematical physics, but they did not decisively influence his own thinking about conceptual questions.

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233 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotations are on p. 375; in the *Collected Works,* volume 6, on p. 61.

It is difficult to understand why Wigner never used his deep knowledge of the underlying mathematical structure of quantum theory to overcome the serious limitation of the Hilbert-space formalism of the traditional codification of quantum mechanics. It is hard to believe that Wigner did not recognize the importance of the many possible physically inequivalent representations of the canonical commutation relations and the associated symmetry breakings for our understanding of molecular, mesoscopic, and macroscopic phenomena. His main hesitation seems to be related to "the blemishes on the beauty of the conceptual structure of quantum mechanics."[234] Wigner refers to the irreducible Hilbert-space representation with the unrestricted validity of the superposition principle. He does not seem to appreciate the beauty of the more abstract modern mathematical codifications of the fundamental ideas of quantum theory which permit more flexibility by the possibility of introducing contextually adopted topologies.

Volume 1, 3 and 6 of Wigner's *Collected Works* will be indispensable both to philosophers working in the foundation of quantum mechanics and to scientists interested in epistemological questions. Wigner makes it plain that the traditional proposals for the interpretation of quantum theory within the traditional framework of von Neumann's codifications are not, and cannot be satisfactory. But the reader should acknowledge that Wigner's discussions on conceptual questions are all based on an outdated mathematical codification of quantum theory and do therefore not reflect the present state of the art.

From our present point of view, most of the so-called paradoxes which Wigner discusses are not deep conceptual problems, but simply technical challenges for the mathematical physicists. The "measurement problem" as formulated by Wigner is ill-posed. Von Neumann and Wigner did not discuss measurements in physical terms. All experiments can be explained by the expectation-value postulate without ever using the state reduction or the projection postulate. The processes which lead to actual measurements have no special place in quantum theory, they are nothing but a subclass of naturally occurring objective physical processes. From our present point of view, we can conclude that much of the philosophical discussions on the so-called measurement process was premature, it was simply caused by a slightly inadequate mathematical codification of the historical first principles of quantum mechanics. However, the excellent experimental confirmation of quantum mechanics is not matched by sufficient understanding. From a fundamental point of view, the open question is not the state reduction but the *derivation* of the statistical expectation-value postulate from an individually formulated fundamental quantum mechanics in an ontic interpretation – a problem never mentioned by Wigner.

Wigner's confusion of conceptual and mathematical problems is due to misplaced physical idealizations (like the idealization of instantaneous observations, the neglect of the effects of the environment, a narrow positivistic view) and to inadequate mathematical codifications (like the inadmissible use of the uniqueness theorem by Stone and von Neumann). Nevertheless, Wigner's discussions of the measurement problem was historically important. He could not give a generally acceptable solution. Yet to see a problem is much more difficult and important than to give a solution. Wigner was the leading scientist who always insisted that there is a problem, and that it is important to see the problem.

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234 E. P. Wigner, *Epistemological perspective on quantum theory,* in: C. A. Hooker (ed.), *Contemporary Research in the Foundations and Philosophy of Quantum Theory,* Dordrecht, Reidel (1973), pp. 369–385. The quotation is on p. 373; in the *Collected Works,* volume 6, on p. 59.