

Frege's Unmanageable Thing

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Abstract

Frege famously maintained that concepts are not objects. A key argument of Frege's for this view is, in outline, as follows: if we are to account for the unity of thought, concepts must be deemed *unsaturated*; since objects are, by contrast, saturated entities, concepts cannot be objects. I investigate what can be made of this argument and, in particular, of the unsaturated/saturated distinction it invokes. Systematically exploring a range of reconstructions suggested by Frege's writings, and drawing on contemporary work, I illustrate that no plausible reconstruction is forthcoming. In essence, it is altogether unclear how to simultaneously substantiate, on the one hand, the claim that unsaturated entities must be recognized in order to account for unity and, on the other, the claim that unsaturatedness is incompatible with objecthood.

Keywords: Frege · Concepts · Unsaturatedness · Objecthood · Unity of the Proposition

1 Introduction

Frege famously maintained that concepts are not objects. Since, for Frege, an *object* is anything that can be the referent of a proper name (while a *concept* is anything that can be the referent of a predicate) (Frege 1997f, 186–187), Frege's view amounts to the thesis that concepts cannot be referred to with proper names: concepts are, in this sense, unnameable.¹ This view precipitates a cluster of serious difficulties in Frege's philosophy of logic and language. A concept becomes, in Frege's own words, 'an unmanageable thing' (Frege 1997f, 192). 'In logical discussions', Frege explains (Frege 1997f, 192), 'one quite often needs to say something about a concept, and to express this in the form usual for such predications—viz. to make what is said about

¹I say 'in this sense' because Frege calls expressions referring to functions—among which are predicates, for concepts are a species of function—*names* of functions (*Namen von Funktionen*) or function-names (*Funktionsnamen*) (Frege 1997b, 218–219). This makes for a sense in which concepts are, for Frege, nameable. However, function-names are emphatically not proper names of functions.

the concept into the content of the grammatical predicate' of a sentence and accordingly to make the concept the referent of its grammatical subject. But the grammatical subject must be a proper name or singular term;² hence its referent must be, if anything, an object and not a concept. One's attempt to say something about a concept in this way therefore goes referentially astray: one fails to speak of the concept one intended. The difficult question now arises of how one can even give expression to claims about concepts, such as those that feature in Frege's own doctrines.³

Given that the view causes such trouble—of which Frege was evidently quite aware—why did he accept it? My purpose in this paper is to examine an argument which, it seems clear, formed at least a major part of Frege's grounds for denying that concepts are objects. The nub of the argument is this: concepts cannot be objects because concepts are unsaturated, whereas objects are saturated. My chief concern will be to investigate what can be made of this argument and, in particular, of the unsaturated/saturated distinction it invokes.

2 Unsaturatedness and the Argument from Unity

The *locus classicus* of Frege's view that concepts are not objects is, of course, his 1892 'On Concept and Object', wherein Frege replies to his critic, Benno Kerry, who had contested this view. For much of the essay, however, Frege's reply is apt to seem like mere bullet-biting. Kerry had adduced the likes of 'the concept *horse*' as designating concepts which are nevertheless, by Frege's criterion, objects. In response, Frege insists, against intuition, that 'expressions like "the concept *F*" designate not concepts but objects' (Frege 1997f, 187), whence follows the apparently paradoxical consequence that the concept *horse* is not a concept. Moreover, he seems simply to accept the expressive impediments that are engendered by his thesis that concepts are unnameable, famously counting on his reader to take his pronouncements *cum grano salis* (Frege 1997f, 192):

By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that I was relying on a reader who would be ready to meet me halfway—who does not begrudge a pinch of salt.

What, then, if anything, does the essay offer by way of *reasons* for thinking that concepts are not objects—for thinking that concepts cannot feature as referents of singular terms? Frege

²'Proper name' (*Eigenname*) and 'singular term' (*Einzelname*) are interchangeable in Fregean nomenclature (Frege 1997a, 172).

³These and related difficulties are customarily gathered under the heading of 'the paradox of the concept horse'. (I concur with Ian Proops that the paradox in fact comprises several distinguishable problems (Proops 2013).)

claims that a concept cannot play the part of the referent of a grammatical subject ‘because of its predicative nature [wegen seiner prädikativen Natur]’ (Frege 1967, 171, my emphasis), and explains that ‘[w]hat [he] here call[s] the predicative nature of concepts is just a special case of the need of supplementation, the unsaturatedness, that [he] gave as the essential feature of a function’ (Frege 1997f, 186). It appears, then, that what motivates Frege’s claim ‘that there can be no such thing as a proper name of a concept’, as David Bell summarizes, ‘is that objects are complete in themselves, saturated or self-subsistent, while concepts are, by contrast, incomplete, unsaturated, and unable to stand by themselves’ (Bell 1979, 29).

Among several questions raised by this reasoning is the following: why ought we to think that concepts *are* unsaturated? We receive an answer to this question only once we reach the essay’s penultimate paragraph (Frege 1997f, 192–193), in which Frege’s remarks are most clearly advanced as an *argument* for the thesis that concepts are not objects. The argument, which I shall dub the argument from unity, is addressed to those who would attempt to avoid Frege’s expressive impediments by maintaining that concepts are objects:

Somebody may think that this [the aforementioned expressibility problem resulting from the unnameability of concepts] is an artificially created difficulty; that there is no need at all to take account of such an unmanageable thing as what I call a concept; that one might, like Kerry, regard an object’s falling under a concept as a relation, in which the same thing could occur now as object, now as concept. The words ‘object’ and ‘concept’ would then serve only to indicate the different positions in the relation. This may be done; but anybody who thinks the difficulty is avoided this way is very much mistaken; it is only shifted. For not all the parts of a thought can be complete [*abgeschlossen*]; at least one must be unsaturated [*ungesättigt*] or predicative [*prädikativ*]; otherwise they would not hold together. For example, the sense of the phrase ‘the number 2’ does not hold together with that of the expression ‘the concept *prime number*’ without a link [*Bindemittel*]. We apply such a link in the sentence ‘the number 2 falls under the concept *prime number*’; it is contained in the words ‘falls under’, which need to be completed in two ways—by a subject and an accusative; and only because their sense is thus unsaturated are they capable of serving as a link [*Bindeglied*]. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought. I say that such words or phrases stand for a relation. We now get the same difficulty for the relation that we were trying to avoid for the concept. For the words ‘the relation of an object to the concept it falls under’ designate not a relation but an object; and the three proper names ‘the number 2’, ‘the concept *prime number*’, ‘the relation of an object to a concept it falls under’, hold aloof from one another just as much as the first two do by themselves; however we put them together, we get no sentence. It is thus easy for us to see that the difficulty arising from the unsaturatedness of one part of the thought can indeed be shifted, but not avoided. ‘Complete’ and ‘unsaturated’ are of course only figures of speech; but all that I

wish or am able to do here is to give hints.

One can, if one so wishes, hold that concepts are saturated objects, Frege concedes. But doing so, he claims, will permit one only to shift, not eliminate, the difficulty engendered by the unnameability of concepts. That is, though it is possible to coherently locate concepts, in particular, within the limits of singular reference, doing so will not divest one of a commitment to unsaturated entities transcending those limits. The tacit moral appears to be that the shift, since profitless, should not be made in the first place; rather, since we must countenance unsaturated non-objects somewhere, so to speak, we should, with Frege, recognize concepts as those unsaturated non-objects.

The consideration motivating Frege's claim that unnameability can only be shifted, not banished, is that 'not all the parts of a thought can be complete; at least one must be unsaturated or predicative; otherwise they would not hold together'. Frege seems here to be invoking phenomena usually discussed under the rubric of 'the unity of the proposition': a sentence is no mere list or congeries of names, and the thought expressed by a sentence is no mere list or congeries of the senses of names. Both sentence and thought possess a *unity* lacked, respectively, by a mere congeries of names and a mere congeries of senses of names. (The problem of accounting for that unity, and of explaining in what the difference consists between the complex entities instantiating it and the mere pluralities that do not, is the problem of the unity of the proposition.) For example, the pair of names 'the number 2' and 'the concept *prime number*', hold aloof from one another in failing to form a sentence; and the pair of senses they express correspondingly hold aloof from one another in failing to constitute a thought. Taking account of these phenomena, Frege thinks, requires accepting that for any thought, at least one of its parts is 'unsaturated' or 'predicative'—elsewhere Frege uses 'incomplete' ('*unvollständig*') and 'in need of supplementation' ('*ergänzungsbedürftig*')—and accordingly, that for any sentence, at least one of its parts expresses an unsaturated sense. Moreover, though it is not made entirely explicit in the quoted passage, there can be little doubt that Frege will make the parallel claim with respect to sentences themselves: taking account of the unity the sentence likewise requires recognizing at least one of its constituents as unsaturated.⁴

3 Unities at the Level of Reference

For at least two reasons, it is deeply puzzling that Frege should have thought that these phenomena give us reason to think that concepts are unnameable. Firstly, adverting to these phenomena seems simply to change the subject. Concepts are, for Frege, never constituents of

⁴The idea that linguistic items are (un)saturated will be considered in detail in §5 below.

thoughts and obviously never constituents of sentences. Concepts are referents (*Bedeutungen*) of certain expressions (viz. predicates), never senses (*Sinne*) of expressions, or expressions themselves, out of which thoughts and sentences are respectively constituted. Why then, even if we are compelled to recognise unsaturated constituents of sentences and thoughts, should this reveal anything about the nature of *concepts*? In particular, why should it reveal that *they* must be unsaturated? Secondly, even supposing that considerations of this kind force us to hold that concepts are unsaturated, it is unclear why this should have any bearing upon the possibility of our making reference to them with singular terms. In what does the unsaturatedness of concepts consist, such that their being unsaturated is incompatible with their being objects?

A natural attempt to respond to the first worry is as follows. Just as there are unities at the level of language, the constitution of which by subsentential expressions can only be accounted for by recognizing at least one constituent expression as unsaturated, and just as there are unities at the level of sense, the constitution of which by the senses of subsentential expressions can only be accounted for by recognizing at least one constituent sense as unsaturated, so—Frege means to point out—there are unities at the level of reference, the constitution of which by the referents of subsentential expressions can only be accounted for by recognizing at least one constituent referent as unsaturated. Concepts are just these unsaturated referents, objects their saturated counterparts.

If the consideration that there are such unities at the level of reference were really the operative part of Frege's argument, it would certainly be odd that he neglects to explicitly mention such unities in the quoted passage, but rather treats only of unities at the level of language and sense. However, this response has the virtue of according closely with certain of Frege's remarks elsewhere. The following (Frege 1979b, 178) is particularly strong evidence that Frege would respond in the way just outlined:

[T]he unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. Object and concept are fundamentally made for one another, and in subsumption we have their fundamental union.

Dummett (Dummett 1981a, 174–175) expounds the point thus:

A concept and an object, or a relation and two objects, need no glue to fit them together: they fit together naturally, in a way we can think of as analogous to that in which a predicate and a proper name, or a relational expression and two proper names, fit together to form a sentence.

Without need of any additional mediating element to act as adhesive, concept and object fit together into a fundamental union. The concept's being saturated by the object is precisely this unmediated coming together; and this is possible only because the concept admits of saturation—that is, because the concept is unsaturated. The problem that this conception of the relation between concept and object is clearly intended to obviate is familiar as 'Bradley's regress'.⁵ Suppose we think of an object and a concept under which it falls as united by a third element: the relation of subsumption. We now confront the question of how the subsumption relation unites object and concept; and if we thought that an auxiliary relation was required to unite concept and object, we ought, for parallel reasons, to think that a fourth relation unites object and concept, on the one hand, and subsumption, on the other. These, plainly, are the initial stages of an infinite regress, and one apt to appear vicious. At any stage, n , we can ask, of the uniting relation posited at stage $n - 1$, what unites it with the entities recognised at stage $n - 2$; and a further uniting relation seems requisite at stage n , if one was requisite at stage $n - 1$. Resisting the regress seems to require, at some stage, insisting that the combination of the entities in question is immediate. The only stage at which it seems plausible to halt proceedings in this way is the very first: concept and object engage immediately with one another, without need of an additional mediating relation as cement. Locating the immediate combination at some later stage, $m > 1$, seems arbitrary; indeed, one trades on this fact in setting up the regress: why is an additional uniting relation required at $m - 1$, but not at m ? Although, in the paragraphs from 'On Concept and Object' above, the concern is with the holding together of parts of *thoughts*, something closely resembling the dialectic we have just rehearsed does indeed seem to be present: Frege seems to making the parallel point that, if a sense is genuinely to unite itself and one or more other senses, it must link immediately with those constituents, lest a further sense be required to unite that original link with those other senses. This fact, together with the cited remarks of Frege's elsewhere, constitute a good case that Frege would respond in the manner adumbrated above.

On reflection, however, it emerges that the response is deeply problematic. A (subject-predicate) sentence, s , is composed of those expressions the respective saturatedness and unsaturatedness of which is (allegedly) required for the unity of s . A thought, t , expressed by such a sentence, is composed of those senses the respective saturatedness and unsaturatedness of which is required for the unity of t . But there is nothing composed of objects and concepts

⁵Consider the sentences immediately preceding the quotation just given from (Frege 1979b, 178): 'In the sentence "Two is prime" we find a relation designated: that of subsumption. We may also say the object falls under the concept *prime*, but if we do so, we must not forget the imprecision of the linguistic expression we have just mentioned. This also creates the impression that the relation of subsumption is a third element that occurs as something additional to the object and the concept. This is not the case...' (Frege 1979b, 178, translation altered).

whose unity requires their respective saturatedness and unsaturatedness. Proper names and predicates compose sentences; their senses compose senses of sentences; but, as Frege makes clear, '[t]hings are different in the realm of *Bedeutung*' (Frege 1997e, 365): the referents of proper names and predicates—objects and concepts, respectively—do not compose the referents of sentences. For the referents of sentences are truth-values, and though a concept is a function the value of which for a given object as argument is a truth-value, the concept and the object are not parts of that truth-value. The concept to which 'ξ is a logician' refers, for example, yields the True as value for the referent of 'Frege' as argument; but neither Frege nor that concept are parts of the True.

Or are they? In 'On Sense and Reference', published in the same year as 'On Concept and Object', Frege proposes, remarkably, that judgements be conceived as 'distinctions of parts within truth-values' (Frege 1997g, 159). It is clear, moreover, that the parts which judgements distinguish in truth-values, on the conception Frege is proposing, include objects and concepts; for he indicates that he is here 'calling the *Bedeutung* of a word part of the *Bedeutung* of the sentence, if the word itself is a part of the sentence' (ibid.). So, notwithstanding his later explicit eschewal of the part-whole conception of the realm of *Bedeutung* (the remark just quoted from (Frege 1997e, 365) was written in 1919), Frege does indeed appear, circa 1892, to conceive of the referents of parts of a sentence as parts of the referent of the sentence, such that in particular:

(†) The value, $c(o)$, of any concept, c , for any object, o , as argument has c and o as parts.

Does the above response suffice then after all? Is the crux of the argument from unity that concepts must be deemed unsaturated if we are satisfactorily to account for the unity of *truth-values*?

We should still have serious misgivings about construing the argument from unity in this way, I submit. Firstly, it is unclear how seriously we ought to take Frege's brief and isolated talk of distinguishing parts within truth-values. Dummett has urged that this was merely 'an unfortunate turn of phrase, which Frege later came to regret' (Dummett 1981b, 181). Secondly, construing the argument as presupposing (†) seems assured to render it disastrously weak; for (†) yields a bizarre and incredible metaphysics. It entails that the True has absolutely every object as a part, since every object is such that, for some concept, the value of that concept for that object as argument is the True. By parity of reasoning, it entails that every object is also a part of the False; and since the True and the False are themselves objects, the True must then be a part of the False and the False a part of the True. Thus, a consequence of (†) is that there are distinct objects that are parts of one another. This is in violation of the core mereological principle that parthood is a partial order—in particular, the principle that parthood is anti-

symmetric. A number of other strange and extravagant consequences are derivable from (†),⁶ but I shan't further belabour its implausibility.

For two reasons, though, the foregoing considerations do not altogether put paid to the argument from unity. The first reason is that there is a natural revision of Fregean doctrine that, plausibly, has the consequence that there *are* unities at the level of reference comparable to those at the levels of language and sense. The revision is to this effect: the referent of a sentence is not the truth-value of that sentence; it is rather a state of affairs that obtains just in case that sentence is true and of which (in the case of an atomic subject-predicate sentence) the referents of subject and predicate are parts. This revision of Frege's mature view is arguably also a reprise of his earlier doctrine that sentences stand for judgeable contents. As Mark Textor puts the point, 'Judgeable contents seem to be nothing other than states of affairs' (Textor 2012, §5.2).⁷ (One might, in fact, regard the view that sentences stand for states of affairs as a constant in Frege's philosophy of language, but conceive of the mature Frege as individuating states of affairs in a spectacularly coarse-grained fashion, such that any states of affairs, *S* and *S'*, are identical just in case the proposition that *S* obtains is materially equivalent to the proposition that *S'* obtains (cf. (Williamson 2003, 700)).) The referent of 'Frege is a logician', for example, will be the state of affairs of Frege's being a logician and will be understood to count Frege and the concept of logicianhood as parts. On this revised view, there credibly *is* a problem of unity concerning the referent of a sentence parallel to those concerning the sentence and its sense; for a state of affairs is no mere congeries of referents of subsentential expressions; it is an integrated whole, a unity. This revision eliminates many of the difficulties that arise on Frege's mature view if one supposes that the referent of a sentence is composed of the referents of its subject and predicate. (For example, it rids us of the need to think of the referent of a sentence as a bizarre aggregate of all objects which transgresses the antisymmetry of parthood.) This is not to say that it is unproblematic to maintain that a sentence refers to a state of affairs in which object and concept feature as parts.⁸ Indeed, the view faces difficulties which Frege

⁶See (Dummett 1981b, 180–181) and (Sullivan 1992, 107); Sullivan also urges that Frege's remarks in (Frege 1997g, 159) do not represent a serious commitment to (†).

⁷See also (Beaney 1996, 155): '... "state of affairs" would probably be the best term to use in characterizing what Frege meant [in the *Begriffsschrift* and the *Grundlagen*] by the "content" of a sentence.'

⁸Dummett remarks,

It is generally agreed that, if Frege had to ascribe reference to sentences at all, then truth-values were by far the best thing he could have selected as their referents: at least, he did not go down the dreary path which leads to presenting facts, propositions, states of affairs or similar entities as the referents of sentences. (Dummett 1981a, 182)

The comment is puzzling, however: it is neither clear what underpins Dummett's confidence that general consensus markedly favours truth-values as referents of sentences over states of affairs, nor clear what Dummett has in mind with his 'dreary path' aspersion.

(Frege 1989, 20) himself raises in correspondence with Wittgenstein, concerning the ontology of the *Tractatus*. Nevertheless, I suggest that the view holds the most promise of affording a persuasive reconstruction of Frege's argument from unity, conceived as an argument for the non-objecthood of concepts.⁹

The second reason I think the argument from unity remains of interest is this. Even if the passage were to give us no grounds for thinking that *concepts* are not objects, it may yet give us grounds for thinking that there are parts of *thoughts* or *sentences* that are not objects. It may yet be that accounting for unity at the level of sense or language demands recognition of unsaturated thought-parts or unsaturated sentence-parts and, therewith, recognition of parts that are not objects. That, I take it, would be a conclusion as significant as the thesis that concepts are not objects.

4 Unsaturatedness and Objecthood

We turn now to the second puzzling feature of Frege's argument mentioned above. Even if considerations of unity force us to recognise unsaturated entities, why should this entail recognising entities that are not objects? Why must objects invariably be saturated? Why cannot that which is unsaturated be named? *In what does unsaturatedness consist, such that unsaturatedness is incompatible with objecthood?*

Perhaps these questions already incline the reader to protest on Frege's behalf. After all, Frege repeatedly cautions that 'unsaturated' and its antonym are only figures of speech, that definition is impossible in this territory, and that all he is able to do in this connection is to give hints. He confessedly 'count[s] on the co-operative understanding of the reader' (Frege 1984b, 281), upon her 'agreeing to meet [him] half-way' (Frege 1984c, 292) and not begrudging a pinch of salt (Frege 1997f, 192). Indeed, he claims that the saturated/unsaturated distinction 'must simply be accepted' as a 'logically primitive phenomenon' (Frege 1984b, 281). Are we not, then, already asking too much of Frege?

No. We need not object to granting, for the sake of discussion, Frege's contention that the notion of unsaturatedness is too fundamental to admit of definition. Hints will suffice. But the hints must extend to furnishing us with at least a reasonably firm grip on the attribute towards which Frege is gesturing, on its incompatibility with objecthood, and on why accounting for unity demands recognising entities that possess it. To the extent that expecting hints of that

⁹Gregory Currie (Currie 1984, 342) suggests that Frege's puzzling focus on parts of *thoughts* in the argument from unity may be the result of his tacitly identifying thoughts with the judgeable contents of his earlier ontology. Though exercised by questions somewhat different to mine, Currie offers insightful discussion of many of the themes and passages from Frege on which I focus.

calibre is incompatible with Frege’s insistence that the saturated/unsaturated distinction must simply be accepted, we should reject that insistence. We need not begrudge a pinch of salt; but we are entitled to expect that our co-operative understanding be reciprocated by a co-operative explanation on the part of the author. If *no* light can be cast on the incompatibility of unsaturatedness with objecthood, and upon the need to countenance the unsaturated in order to account for unity, then I suggest that we are being asked, unreasonably, to meet Frege *much* further than half-way. In §5–8, we examine whether that light can be cast.

5 The Unsaturatedness of Linguistic Expressions

It can seem that the notion of unsaturatedness is meant, in the first instance, to be explained by reference to the unsaturatedness of linguistic expressions. It can seem, furthermore, that the unsaturatedness of expressions is simply a matter of their containing gaps or empty places. Let’s take these two points in reverse order.

The second point is readily illustrated with two passages from ‘Function and Concept’:

We can split [$\epsilon(\epsilon^2 - 4\epsilon) = \alpha(\alpha(\alpha - 4))$] into $\epsilon(\epsilon^2 - 4\epsilon)$ and $() = \alpha(\alpha(\alpha - 4))$.

This latter part is in need of supplementation, since on the left of the ‘equals’ sign it contains an empty place.¹⁰

Thus, e.g., we split up the sentence ‘Caesar conquered Gaul’ into ‘Caesar’ and ‘conquered Gaul’. The second part is unsaturated—it contains an empty place;¹¹

The clear impression given is that the unsaturatedness (or need of supplementation) of the predicates in question is just a matter of their containing empty places into which singular terms can be inserted. This hardly seems like a mysterious attribute.

The first point is strongly suggested by the following passage of the posthumously published paper dubbed ‘Comments on *Sinn* and *Bedeutung*’:

[O]ne can always speak of the name of a function as having empty places, since what fills them does not strictly belong to them. *Accordingly*, I call the function itself unsaturated, or in need of supplementation, *because* its name has first to be completed with the sign of an argument if we are to obtain a *Bedeutung* that is complete in itself.¹²

¹⁰Frege 1997c, 140, translation altered.

¹¹Frege 1997c, 139.

¹²Frege 1997a, 174, my emphasis.

Quoting this same passage with the same added emphases, Heck and May conclude: ‘So, in the end, it is the unsaturatedness of the *expression* that is basic’ (Heck and May 2013, 844). Frege’s account here of the unsaturatedness of functions does indeed seem to be based on their being referents of expressions that are themselves unsaturated or incomplete. Now, what is disappointing about the passage is that the explanation *invokes* the notion of completeness at the level of reference with the condition ‘if we are to obtain a *Bedeutung* that is complete in itself’: if the notion of completeness at the level of reference were grasped securely enough for it to be invoked in the explanans of a profitable explanation, we would not need an explanation of *incompleteness* at the level of reference (‘referential unsaturatedness’), which is precisely what we seek; for a referent could be understood to be incomplete just in case it were not complete. Let us ignore, then, this invocation of the complete/incomplete distinction at the level of reference, and instead put our second point above to use. Here is the resultant proposal. The unsaturatedness of concepts (we focus on this particular species of function), and of senses that are modes of presentation of concepts, is to be explained in terms of the possession, by certain expressions, of empty places.

Flesh can be put on the bones of this rather programmatic proposal in two ways, I think. The first involves taking ‘accordingly’ and ‘because’ in the above quotation as seriously as possible: At the level of reference, for something to be unsaturated is for it to be such that any expression of which it is the referent features at least one empty place. This account has the considerable merit of immediately providing an excellent explanation of the incompatibility of referential unsaturatedness with objecthood. To be an object is to be capable of being the referent of a proper name. Proper names lack empty places. To be unsaturated is to be capable of being the referent only of expressions featuring empty places (‘gappy expressions’): this immediately follows from the present account of unsaturatedness if the quantifier phrase ‘any expression’ therein is read—as I suggest it must be in this context—as ranging over all possible expressions. On the present account, the incompatibility of unsaturatedness and objecthood is explicable thus: unsaturatedness *consists*, inter alia, in not being an object.

This merit notwithstanding, the account has several serious shortcomings. Firstly, it does little, as it stands, to cast light on the connection between unsaturatedness and unity: why does accounting for the unity of states of affairs require recognising constituents of states of affairs which can only be referents of gappy expressions? Secondly, the account scuppers the idea, discussed in §2, that what grounds or motivates Frege’s thesis that concepts cannot be named is that they are unsaturated: if the unsaturatedness of concepts partly *consists* in their being unnameable, it would be question-begging to adduce their unsaturatedness as grounds for deeming them unnameable. Thirdly, the account is not satisfactorily extensible to the level of sense. Extending the account to unsaturatedness at the level of sense (‘sential unsaturat-

edness'¹³) must yield the following: at the level of sense, for something to be unsaturated is for it to be such that any expression of which it is the sense features at least one empty place. This account of sential unsaturatedness accords very closely with the view, for which Dummett argues, that the unsaturatedness of a sense 'consist[s] merely in its being the sort of sense appropriate to an incomplete expression' (Dummett 1981a, 291).

The account is unsatisfactory, however. It provides no explanation, as it stands, of the incompatibility of sential unsaturatedness with objecthood, because, for all that has been said so far, it may be that that which can only feature as the *sense* of a gappy expression can nevertheless feature as the *referent* of a gapless expression; in particular, it may be that it can feature as the referent of a proper name.¹⁴ (It might be suggested that Frege could invoke his theory of indirect reference to answer this challenge. According to that theory, in *oratio obliqua*, an expression refers not to that to which it customarily refers, but rather to that which it customarily expresses—what is ordinarily its sense. Thus, an unsaturated sense must also be something that can feature as the *referent* of a gappy expression, since it is the referent of any gappy expression of which it is customarily the sense when that expression appears in indirect speech. These considerations do not suffice to answer the challenge, however. They only establish that, granting Frege's theory of indirect reference, the sentially unsaturated *can* feature as the referent of a gappy expression, not that it *can only* feature as the referent of a gappy expression—and, in particular, not that it is incapable of featuring as the referent of a proper name. That gappy and gapless expressions never co-refer cannot permissibly be assumed in the present context.) The account conflicts, furthermore, with an important thesis of Frege's concerning unsaturatedness (Frege 1984a, 393):

It is really in the realm of sense that unsaturatedness is found, and it is transferred from there to the symbol.

According to Frege, the unsaturatedness of expressions ('linguistic unsaturatedness') is in fact derivative upon sential unsaturatedness. However, the present account of sential unsaturatedness, and the Dummettian view with which it closely accords, precisely invert that order of priority. (The point equally confutes Heck and May's conclusion mentioned above.) Moreover, the following passage (Frege 1997a, 174) strongly suggests that, in Frege's view, the applicability of the saturated/unsaturated distinction at the level of sense is prior, not only to its applicability at the level of language, but also to its applicability at the level of reference:

¹³I owe 'sential' to Michael Bench-Capon (Bench-Capon TS).

¹⁴Though I say 'in particular', this last clause is, for Frege, no more particular than the one that precedes it; for the gapless expressions are, by his lights, just the proper names. We will not follow Frege in this respect, however, but will rather count sentences as gapless expressions that are not proper names.

The words ‘unsaturated’ and ‘predicative’ seem more suited to the sense than the *Bedeutung*; still there must be something on the part of the *Bedeutung* which corresponds to this, and I know of no better words.

This primacy of sential unsaturatedness on Frege’s conception makes all the more dissatisfying the account’s failure to explain the incompatibility of sential unsaturatedness with objecthood. When fleshed out in this first way, then, the proposal that unsaturatedness is to be explained in terms of the gappiness of certain expressions leaves much to be desired.

The second way of fleshing out that proposal is to this effect: unsaturatedness at the levels of sense and reference is to be understood *by analogy with* or *on the model of* gappiness at the level of linguistic expressions. Much that Frege says recommends this account. For example:

I also call the first constituent [‘Two’] saturated; the second [‘is a prime number’], unsaturated. To this difference in the signs there of course *corresponds an analogous one* in the realm of *Bedeutungen*.¹⁵

The suggestion is that we are to think of the unsaturated at the levels of sense and reference as possessed of empty places, in some sense analogous to the sense in which unsaturated expressions possess empty places. A concept, on this picture, contains an empty place which can be occupied by an object to constitute a state of affairs, and the sense of a (monadic) predicate contains an empty place which can be occupied by the sense of a name to constitute a thought, much as a predicate contains an empty place which can be occupied by a name to constitute a sentence.

One immediate concern is whether this picture can be anything more than ‘hopelessly metaphorical’, as Magidor describes it (Magidor 2009, 5). She continues: ‘No semanticist seriously thinks that the semantic values of predicates are literally entities which contain gaps into which we try to fit other entities’ (ibid.). I am not quite so sure. There are entities taken seriously by semanticists and other theorists of intentionality that seem to be literally describable as containing gaps at least to approximately the extent that we speak the literal truth in describing predicates and other incomplete expressions as containing gaps. Perhaps we fall well short of speaking the literal truth in describing predicates as containing gaps; but then, our comfort with describing them thus, and our confidence that we know what we mean in doing so, ought to extend to describing predicate-senses and concepts similarly—making the foregoing picture metaphorical, but certainly not hopelessly so. The entities I have in mind are variously known as *gappy propositions*, *gappy contents* and *unfilled propositions*. Propositions—the things expressed by declarative sentences—are commonly conceived as complex entities

¹⁵(Frege 1984b, 281, my emphasis). Similar remarks are to be found in (Frege 1979b, 177), (Frege 1980, 142) and (Frege 1984c, 292).

that possess a structure comprising a number of positions or places in relation to one another. These positions are taken to be occupiable by propositional constituents—be they, as Russell maintained, the very objects, properties and relations the proposition is about, or, as a Fregean would maintain, only modes of presentation thereof.¹⁶ Gappy propositions differ from propositions so conceived just in having at least one position that is unoccupied or unfilled. They may be posited for various theoretical purposes. The gappy cousins of Russellian structured propositions, for example, are accepted by some direct reference theorists as the contents expressed by sentences featuring empty names (see e.g. (Braun 1993), (Braun 2005)). While the proposition expressed by ‘Obama is mortal’ is understood, on this view, to be a structured entity whose two positions are respectively occupied by Obama and mortality, the gappy proposition expressed by ‘Vulcan is a planet’ is an entity with the same structure whose two positions are respectively unoccupied—since ‘Vulcan’ is empty—and occupied by planethood. Gappy propositions have also been proposed as the contents of certain perceptual experiences (Siegel 2010, §5.1).

If gappy propositions are accepted, it is natural similarly to accept gappy sentences: We conceive of sentences as complex entities that possess a syntactic structure comprising a number of positions or places in relation to one another. These positions are taken to be occupiable by subsentential expressions. Gappy sentences differ from sentences just in having at least one position in this syntactic structure that is unoccupied. Here, then, is a proposal: predicates are to be identified with gappy sentences; their referents, concepts, are to be identified with gappy Russellian propositions; and their senses are to be identified with gappy Fregean propositions.

That predicates are gappy sentences is an independently attractive interpretation of talk of predicates as possessing empty places; and on the present proposal, the referents and senses of predicates possess empty places in a sense highly akin to that in which predicates possess empty places. It strikes me, therefore, that on this proposal, we speak the literal truth in describing concepts and predicate senses as possessing empty places to at least approximately the degree that we speak the literal truth in describing predicates likewise. The proposal therefore represents a reputable semantic picture which sustains Frege’s claim that the linguistic distinction between the saturated and the unsaturated is paralleled by a corresponding and analogous distinction in the realm of *Bedeutungen*.

What the proposal fails to do, I submit, is to cast any light on the incompatibility between unsaturatedness and objecthood. Why should a structured entity’s possessing an empty place render it unnameable? The gappiness under discussion invites comparison with the possession of vacua of a different kind by certain material objects. We feel no qualms about the possibility

¹⁶For our purposes, we needn’t draw any distinction between Russellian propositions and states of affairs.

of singularly referring to incomplete jigsaw puzzles, pieces of Emmental or molars in need of a filling. How does the gapped nature of concepts differ from the holed nature of these material objects, such that the latter enjoy objecthood, but the former do not? Any air of silliness attending this question attaches to the thought that possession of holes could be an obstacle to being named.¹⁷

The mysteriousness of the incompatibility of unsaturatedness, on the present account, with objecthood is compounded by the fact that, for Frege, linguistic unsaturatedness is *not* incompatible with objecthood. It is clear that Frege regards unsaturated *expressions* as objects. He plainly indicates, for example, in a letter to Russell, that he takes incomplete expressions to be possible referents of proper names. Specifically, they are referents of the proper names that result from simply enclosing them in quotation marks (a fact he exploits in passages previously quoted from ‘Function and Concept’):

While ‘() • 3 + 5’ is a function name, “‘() • 3 + 5’” is a proper name, and its referent is the function name ‘() • 3 + 5’.¹⁸

It might be thought that our conception of referential and sentential unsaturatedness ought to be guided by a looser analogy with the gappiness of certain linguistic expressions. Perhaps the consideration to focus upon is that a monadic predicate, for example, must be supplied with a name in order that a sentence be produced. Martha I. Gibson (Gibson 2004, 4–5) pursues the analogy as follows.

The function itself is incomplete in that it requires supplementation by a certain number of arguments in order to produce a value. ... The incompleteness of the concept is just its need to be supplemented with objects in order to produce the value true or false.

My concern is that this conception of unsaturatedness does not make for a contrast with the nature of objects. It would, after all, be very puzzling to extend Gibson’s remarks by saying

¹⁷While holed material entities have an excellent claim to objecthood, I note that at least one metaphysician—namely, C. B. Martin—has defended the thesis that the holes in them are not objects. The parallel with Frege’s view about concepts is striking in that Martin’s denial of the objecthood of holes (he defends the same position with regard to *absences* or *lacks*) is combined with a robust commitment to their reality. While decrying the ‘deontologizing’ (Martin 1996, 58) of holes or voids, he nonetheless says ‘An absence or lack of something or a hole are not things’ (Martin 1996, 58) and ‘A void is not a *thing* ...’ (Martin 1996, 62, original emphasis).

¹⁸(Frege 1980, 136). Some commentators, including Geach ((Geach 1976b, 59–61), (Geach 1961, 144), (Geach 1976a, 440)) and Hugly (Hugly 1973, 236–242), have maintained that, for Frege, incomplete expressions are themselves functions mapping singular terms or sentences to singular terms or sentences. This is clear evidence to the contrary. It remains open to these commentators to maintain that the most charitable interpretation of Frege’s remarks taken as a whole is one on which incomplete expressions are linguistic functions. However, Alex Oliver (Oliver 2010, 128ff) offers a compelling case that this is not so.

‘On the other hand, by contrast, objects do *not* require supplementation by a certain number of arguments in order to produce a value’. The best that could be made of that remark, I think, would be the claim that if an object is given, then a value of *some* function is thereby given; for the object is the value of some function for some argument. But it is not clear that functions lack this property: Frege himself does not recognise functions whose values are functions, but it is not clear that we should follow him in this respect. As for the consideration that a concept, in particular, needs to be supplied with an object in order to produce a truth-value, it is likewise not clear that objects lack that property. If the concept *horse* alone is given, no truth-value has yet been determined until an argument is also given; but similarly, if the object Earth is given, no truth-value has yet been determined until a concept is also given. I struggle to see why we are not equally entitled, therefore, to say that an object must be ‘supplemented’ with a function in order to produce a truth-value, and hence to characterise objects as unsaturated or incomplete in this sense.

6 Unsaturatedness as Existential Dependence

Another important conception of Fregean unsaturatedness finds textual support in the very earliest explicit appearance in Frege’s corpus of the thesis that concepts are unsaturated. This is in Frege’s letter to Anton Marty of 29/8/1882 (Frege 1997d, 81):

A concept is unsaturated [*ungesättigt*] in that it requires something to fall under it; hence it cannot exist on its own.

The succeeding remarks further confirm that Frege wishes, at this stage, to deny the ‘independent existence of concepts’ (ibid.).

Now, his assertion that a concept requires something to fall under it, and that it is in its requiring this that its unsaturatedness consists, will strike anyone acquainted with Frege’s logicism as very odd. That a concept need *not* have anything falling under it—that there are concepts under which absolutely nothing falls—is exploited by Frege himself in his definition of the natural numbers. In the *Foundations of Arithmetic*, appearing two years after the letter to Marty, zero is defined as the number which belongs to the concept *non-self-identical*, under which precisely nothing falls. Nevertheless, Frege’s conception of concepts, at this stage, as lacking independent existence and as existentially dependent, in particular, upon the objects that fall under them, connects intimately with an important tradition in the metaphysics of properties. It is a conception apt to be echoed, for example, by those metaphysicians who conceive of properties as *ways things are*. (See e.g. (Armstrong 1997b, §3.6) and (Lowe 2006, 90ff). Armstrong explicitly draws the connection between conceiving of properties as ways

things are and considering them unsaturated (he is thinking of properties as universals): ‘the conception of universals as ways... quite naturally goes along with, the important Fregean idea... that universals are “unsaturated” entities’ (ibid.) Ways things are, the thought goes, depend for their existence upon things that are those ways. They cannot exist in isolation, as free-floating denizens of the world, any more than a feline grin could exist without a cat (though a cat could exist without a feline grin). They lack—to use an expression of Frege’s from the *Grundlagen—self-subsistence (Selbständigkeit)*¹⁹. A way something is is always a way *something* is.

Frege’s apparent alignment with this tradition of considering properties to be non-self-subsistent can in fact be reconciled, I suggest, with his subsequent commitment to concepts whose extensions are empty. This can be accomplished by first extending the conception of properties as ways things are to the case of relations: we understand a binary relation, for example, to be a way two things are, one with respect to the other (Lowe 2006, 91); and we consider that relation to depend for its existence upon two things being that way, one with respect to the other. Secondly, we conceive of functions as relations of a particular species, in the customary way—namely, as relations, R , such that, for all x, y, z , if Rxy and Rxz , then $y = z$. Combining these two conceptions with Frege’s conception of concepts as functions from objects to truth-values we secure a position on which concepts, though existentially dependent upon objects, may nevertheless have an empty extension: they are existentially dependent upon objects since they are ways object–truth-value pairs are, one with respect to the other; but they need not have a non-empty extension, since they need not be a way some object and the True are, one with respect to the other. Frege can, then, in this regard consistently maintain that concepts are existentially dependent entities and that it is in this dependence that their unsaturatedness consists.

To underscore an emerging pattern, however, the account of unsaturatedness as existential dependence shows little promise of furnishing a satisfactory explanation of the incompatibility of unsaturatedness with objecthood; for existential dependence appears rife among objects: sets depend for their existence upon their members; trees upon carbon atoms; water waves upon molecules of H_2O ; assassinations upon those assassinated; and, as we have already seen, particular grins, feline or otherwise, upon those who wear them. None of these dependents seem incapable of being the referents of singular terms. Indeed, later in his career, Frege himself explicitly ascribes *Unselbständigkeit* (Frege 1997h, 339) to ideas and sensations, though it seems quite clear that Frege would classify the latter as species of object:

It seems absurd to us that a pain, a mood, a wish, should go around the world

¹⁹Literally: the quality of being self-standing.

without an owner, independently [*selbständig*]. ... ideas need an owner. Things of the outer world are on the contrary independent [*selbständig*].²⁰

The conception of unsaturatedness as existential dependence, coupled with the thesis that objecthood is incompatible with unsaturatedness, seems to yield a wildly un-Fregean classification of entities as objects and non-objects. Further reflection suggests that the alleged existential dependence of properties and relations upon objects would be altogether reciprocated by the objects themselves. Objects must instantiate some properties; they cannot exist alone, thoroughly unpropertied. They must stand in some relations too; they cannot exist alone, thoroughly unrelated. Or at least, these things are to be granted by anyone who countenances properties and relations in the plenitude required by Fregean doctrine.²¹

The kind of dependence we have so far been considering ascribing to properties and relations would be, as it were, a *horizontal* dependence relation among the constituents of unities at the level of reference. One kind of constituent of a state of affairs has been supposed dependent upon another kind of constituent of a state of affairs. The generalized supposition would be that the unsaturated constituents of a unity of a certain kind generically depend upon saturated constituents of unities of that kind. A rather different supposition that also deserves our attention, however, is that there obtains a *vertical* existential dependence on the part of the unsaturated constituents of unities upon the unities themselves. As an interpretation of Frege's doctrine of the unsaturated, this supposition is developed in lucid detail by Peter Simons in his 'Unsaturatedness' (Simons 1981). Concerning unities at the level of sense, Simons (Simons 1981, 80) reads Frege thus:

Thoughts are, for Frege, self-sufficient entities but any thought which contains parts, in particular other self-sufficient parts such as other thoughts, or the senses of proper names, must contain at least one part which is unsaturated or supplement-demanding: in our terms, a dependent part.

Simons defines *dependent part* (Simons 1981, 78–80) by way of a series of definitions inspired by those of Husserl, whom Simons considers to have advanced an account of unity similar to that which he distinguishes in Frege. When Simons' definitions are traced back, it emerges

²⁰(Frege 1997h, 334). The appearance of the notion of (*Un*)*selbständigkeit* in this later context is noted by Ignacio Angelelli (Angelelli 1967, 24–25).

²¹There may still be scope for maintaining that *concepts*—when conceived as ways object–truth-value pairs are, one with respect to the other, as suggested above—are asymmetrically dependent for their existence upon objects. This could be maintained if it were held that the truth-values are contingent existents. In that case, there could not be concepts without objects, but there could be objects without concepts. It's hard to see what significance this asymmetric dependence of concepts on objects could have, though, given that concepts are (at present being conceived as) just a special kind of relation. Just as there are special kinds of relation that depend asymmetrically upon objects, so there are special kinds of object that depend asymmetrically upon relations: tennis balls cannot exist without relations, though relations can exist without tennis balls.

that to say that an unsaturated constituent (say, a monadic predicate-sense), u , of a thought, t , is a dependent part of t is to say this:

1. u is a part of t .
2. For some kinds α, β :
 - (a) u is of kind α ;
 - (b) t is of kind β ;
 - (c) nothing of kind α can exist unless something of kind β exists;
 - (d) t meets u 's need for something of kind β .^{22, 23}

Presumably, the particular kinds, α, β , that are supposed to witness (2) are precisely the kinds *monadic predicate-sense* and *thought*, the claim being that no monadic predicate-sense could exist without some thought existing and that t meets u 's need for a thought. But now it surely becomes clear that we have similar and equally compelling grounds for likewise classifying e.g. name-senses—which Frege considers to be saturated objects—as dependent parts of thoughts: for no name-sense can exist unless some thought exists. Similarly, at the level of reference, while there is a good case for classifying properties and relations as dependent parts of states of affairs, there is an equally good case for classifying objects likewise. Again, objects cannot exist altogether unpropertied, so cannot exist without being part of—and, therefore, without there also existing—some state of affairs. So, although Simons notes early in his exposition that the unsaturated is meant to be distinguished ‘utterly from all objects’ (Simons 1981, 74) by its unsaturatedness, the account he develops of unsaturatedness in terms of dependent parthood fails to preserve this feature in any recognizably Fregean fashion.

For what it is worth, I also doubt that positing horizontal or vertical existential dependence within a complex whole can account for the unity of that whole. Consider horizontal dependence first. We are told that entities of one kind, k , cannot exist without entities of kind k' , where wholes of a certain sort feature constituents from both k and k' . However, this alone does not preclude entities of kinds k and k' merely, as it were, *disparately co-existing*; so it does not alone explain their being united into integrated wholes.

Vertical dependence seems to obviate this problem of disparate co-existence, since it is the integrated wholes themselves whose existence is required by the relevant kind of constituent.

²²In general, a 's meeting b 's need for a β is meant to be consistent with other things' also meeting b 's need for a β .

²³I have ignored one complication in the definitions which, roughly, allows for something to be a dependent part of a whole by depending on a part of that whole rather than the whole itself—i.e. I have ignored the disjunct ‘or by some part of b ’ in definition D3 (Simons 1981, 79). This will not matter for our concerns.

But the generic nature of the vertical dependence relation in question seems to me to prevent it from accounting for unity. Simons (Simons 1981, 79) writes,

It is characteristic of a dependent part that it cannot be detached or isolated from the whole of which it is part,...

However, straightforwardly read, this is wrong. Where w is a whole of which a is a dependent part, a may well be detachable or isolable from w : it may be possible for a to exist and yet not be a part of w (though perhaps the only circumstances in which this is so are circumstances in which w does not exist). Qua dependent part, a cannot exist unless wholes (at least one) of w 's kind exist; but since a 's need for something of w 's kind may have been met, or may in fact be met, by something of that kind *other* than w , it need not be the case that a cannot exist except as a part of w . In this sense, a dependent part is only generically dependent upon the kind of whole of which it is in fact a part. Thus, Simons is closer to the mark when he summarizes (what he takes to be) Frege and Husserl's shared response to the problem of unity:

[W]hat holds the parts of a complex whole together? Their common answer is: such a whole contains at least one part which cannot exist outside that *sort* of whole.²⁴

(In fact, strictly, the definitions do not even entail that a dependent part of a whole cannot exist outside that *sort* of whole. A dependent part requires the *existence* of wholes of that sort, but the definitions do not imply that it cannot exist without being *part* of one of them. But I will assume that this implication is secured by further stipulation.) However, supposing that a whole has a dependent part does not seem to account for its unity; for even that dependent part may be capable of existing detached from that very whole; and thus the question remains: by virtue of what do it and its fellow parts in fact exist attached?

Some, I suspect, will view the considerations of this section simply as indications of the folly of conceiving of ontological dependence according to what Kit Fine calls the *modal/existential account* (Fine 1995). On this account, one (kind of) thing depends on another (kind of) thing just in case the former (kind of) thing cannot exist unless the latter (kind of) thing exists—i.e. iff, necessarily, if the former exists, the latter exists. Inspired by remarks of Frege's, the idea of this section has been to identify unsaturatedness with ontological dependence as conceived by the modal/existential account. The idea is problematic, it has emerged, since, given that identification, many entities which are meant to be saturated turn out to be unsaturated. This might be seen as symptomatic of the implausible weakness of ontological dependence on the modal/existential account: the account makes dependence appear where it intuitively ought

²⁴Simons 1981, 80, my emphasis.

not to (Fine 1995, 270–272). Perhaps, then, the problem lies with the modal/existential account. Where that account characterises dependence in terms of a necessary connection between the existence of the dependent and the dependee, perhaps, as Fine argues, we ought rather to characterise dependence in terms of an *essential* connection between the *essence* of the dependent and the dependee, and refrain from construing essence in modal terms. I don't know whether this is the right course. I am confident, though, that neither the Finean essentialist account, nor any other alternative account of which I am aware, will forfend the difficulty confronting the identification of unsaturatedness with dependence—namely: there are objects exhibiting such dependence, so unsaturatedness emerges as compatible with objecthood. In this sense, the problem does not merely lie with the modal/existential account.

7 The Primacy of the Whole

The claim that some constituents of unities are vertically dependent upon the unities of which they are constituents is closely related to an important current in Frege's philosophy of language. In some significant sense, Frege gives primacy to the whole thought, to the sense of a whole sentence. In his view, it is the thought entire that is primary, basic, fundamental. Its constituents, the senses of subsentential expressions, are secondary, derivative: they are, as it were, abstractions from the complete thought—the products of subjecting the thought to analysis or decomposition. The primacy of the thought goes hand in hand with the semantic primacy of the vehicle of its expression, captured in Frege's *context principle*:

[W]e ought always to keep before our eyes a complete sentence [Satz]. Only in a complete sentence have the words really a meaning.²⁵

Now, passages in which Frege clearly affirms this whole-first order of priority can give the impression that it is characteristic of the *unsaturated* to be the derivative product of decomposition of the whole:

...I start out from judgements and their contents, and not from concepts...And so instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement.²⁶

I do not believe that concept formation can precede judgement, because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content.²⁷

²⁵Frege 1953, 71, translation altered.

²⁶Frege 1979a, 16–17.

²⁷Frege 1997d, 81.

What is distinctive about my conception of logic is that I begin by giving pride of place to the content of the word ‘true’, and then immediately go on to introduce a thought as that to which the question ‘Is it true?’ is in principle applicable. So I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analysing the thought.²⁸

In these passages, which span almost forty years of Frege’s career (the first written in 1880 or 1881, the third in 1919), the recurrent focus is on the status of unsaturated *concepts* as posterior to, and arrived at only by analysis of, the contents of whole sentences.^{29,30} In remarks almost immediately following the first quotation, Frege seems to stress, in particular, the semantic posteriority of expressions designating concepts, as reflected in a perspicuous symbolism:

Hence in the concept-script [concepts’] designations never occur on their own, but always in combinations which express contents of possible judgement. . . A sign for a property never appears without a thing to which it might belong being at least indicated. . .³¹

The idea can suggest itself, therefore, that the unsaturatedness of concepts *consists* in their being derivative abstractions from the contents of whole sentences; or—equivalently, one might propose—that their unsaturatedness consists in their designators’ only being capable of occurring meaningfully in the nexus of an entire sentence.³² The idea certainly animates the image of unsaturatedness as a kind of lack of self-sufficiency.

The idea rapidly founders, though. The context principle applies quite generally to expressions, not merely to those whose *Bedeutungen* are classified as unsaturated. By its reckoning, proper names, as much as predicates, are capable only of occurring meaningfully in the context of a complete sentence. Frege makes quite clear in the *Grundlagen* that the species of self-subsistence he has in mind, and with which we are supposing unsaturatedness to contrast, is not a matter of being designatable independently of the sentential context.³³

The self-subsistence which I am claiming for number is not to be taken to mean

²⁸Frege 1997e, 362.

²⁹The notion of unsaturatedness has not quite appeared in earnest in Frege’s work at the time of the first quotation; but, as Heck and May point out (Heck and May 2013, 840), the germ of that notion is surely present in (Frege 1979a).

³⁰Note the connection made, in the second quotation, with existential dependence.

³¹Frege 1979a, 16–17.

³²The idea appears in Gaskin’s exposition of Fregean unsaturatedness: ‘[T]here is no way of referring to the unsaturated item introduced by a concept-expression other than by use of that very expression (or a synonym) in a sentence; concepts cannot merely be *mentioned*; reference cannot be achieved in a context which falls short of the full act of assertion’ (Gaskin 1995, 165, original emphasis).

³³Wittgenstein (Wittgenstein 2010, §49) similarly stressed that proper names are not exempt from the context principle, as Gaskin (Gaskin 2011, 295–296) discusses. See also (Conant 2002, 433).

that a number word signifies something when removed from the context of a sentence...³⁴

For Frege, it is, accordingly, *all* parts of a thought falling short of *being* a thought—not just those classified as incomplete—which are posterior to the thought in toto (as the final clause in the passage from (Frege 1997e, 362) quoted earlier in this section confirms). It would, therefore, be natural for Frege to consider states of affairs—now thought of as the referents of sentences—as prior to all of their constituents that fall short of being a state of affairs: objects, as much as concepts, would naturally be considered abstractions from the whole. Something very close to this view of states of affairs and their constituents seems to be defended by Armstrong. ‘[S]tates of affairs come first’ (Armstrong 1997b, 118), he claims; their constituents, particulars and properties, are ‘vicious abstractions (in the non-Quinean sense of “abstraction”, of course!)’ (Armstrong 1997a, 109–110) therefrom.³⁵

The prospects look poor, then, for explicating unsaturatedness by reference to the primacy of the whole. Conceiving of the unsaturatedness of a constituent as a matter of its being posterior to the whole to which it belongs does not seem to vindicate Frege’s sharp segregation of the unsaturated from (entities he wishes to classify as) objects.

An alternative view would be that the relevant unities are prior only to some of their constituents. Perhaps it would be viable to hold, for instance, that states of affairs and the particulars occurring in them are, so to speak, ontological contemporaries, whereas properties and relations are secondary—as Armstrong puts it, just ‘everything that is left in the state of affairs after the particular particulars involved in the state of affairs have been abstracted away in thought’ (Armstrong 1997b, 29). Combining this view with the conception of unsaturatedness as posteriority to the whole fares better in classifying entities as (un)saturated in a way acceptable to Frege. Even when combined with this view, however, this conception of unsaturatedness appears to offer no explanation of its incompatibility with objecthood. For one thing, the notions in which this conception trades are, as they stand, rather murky: one thing’s being prior or posterior to another; its being more or less basic or fundamental than another; its being a mere abstraction from another; its being only arrived at by the analysis or decomposition or splitting up of another. It is far from being immediately clear how to understand these (perhaps equivalent) notions. I don’t wish to deny that they admit of elucidation. (It may be that they are to be clarified in terms of some notion of dependence, returning us to the discussion of §6.) What I cannot see, though, is how these notions might be suitably connected with nameability in order to provide the explanation we seek. Armstrong’s mention above of ‘the non-Quinean

³⁴Frege 1953, 72e, translation altered.

³⁵Armstrong in fact explicitly proposes that, in accordance with this view, particulars, as well as properties, should be thought of as unsaturated (Armstrong 1997a, 110).

sense of “abstraction”’ is apposite: if that which is an abstraction (posterior, non-basic, arrived at only by analysis, etc.) is nevertheless *real* (which it is in Armstrong’s non-Quinean sense of ‘abstraction’), why should its being an abstraction constitute an essential obstacle to its being referred to with a singular term?

It might be suggested that unsaturatedness ought not to be identified simply with being a result of splitting up a previously given whole, but rather with being a particular *kind* of result of such decomposition. Certainly, there are remarks of Frege’s with which this suggestion closely accords—e.g. in ‘Function and Concept’ (Frege 1997c, 134). The challenge, however, when faced with the inevitable follow-up, ‘Which kind of result?’, is not to resort merely to the reply ‘The unsaturated kind’. One might try to appeal here to an illustrative geometric comparison Frege offers in the aforementioned passage of ‘Function and Concept’. The comparison is with the clean division of a line by a point. The dividing point must be counted as belonging to one of the resulting line segments or the other, but not both, lest the dividing point be counted twice over. The segment to which that point is taken to belong becomes, Frege says, ‘fully complete in itself (*völlig in sich abgeschlossen*)’ (Frege 1997c, 134); by contrast, the other segment ‘is lacking in something (*etwas fehlt*)’ (ibid.)—namely, the dividing point—and only becomes something complete in itself if supplemented with an endpoint or a line with two endpoints. It is hard to see how the comparison might help, however. After all, an open ray, no less than a closed ray, is perfectly available to singular reference: its openness is perfectly compatible with its objecthood.³⁶

A suggestion related to the one introduced in the previous paragraph is that the unsaturatedness of an entity—its referential unsaturatedness, though the suggestion could be duly extended to cover sentential and linguistic unsaturatedness—is a matter, not merely of its designation’s being semantically posterior to the whole sentence, but of the particular *kind* of semantic posteriority to the sentence its designation exemplifies. As Dummett explains, the semantic primacy Frege attributes to the sentence does not require that ‘every explanation of a word must make an explicit allusion to its occurrence in sentences’ (Dummett 1981a, 6). Frege would not deny, Dummett continues, that the referent of a *proper name* can be specified (and therewith, its sense shown) without such allusion. One can, for example, proffer the likes of,

- (i) ‘London’ refers to London.

One might claim that predicates differ from proper names in just this regard. They are semantically posterior to sentences not only in that they lack meaning outside of the context of a sentence—an attribute they share with proper names—but in that their semantics is to be given

³⁶See also (Gaskin 2008, 148).

in a manner that explicitly alludes to their occurrence in sentences. One might, furthermore, identify the unsaturatedness or incompleteness of a concept with its designation's exhibiting precisely this kind of posteriority to the sentence. A version of this account of unsaturatedness is, in fact, proposed by Heck and May (Heck and May 2013, 848):

[T]he claim that concepts are 'incomplete' is far more adequately expressed by the semantic thesis that the meaning of a predicate should be given by stating the meaning of an arbitrary atomic sentence in which it occurs...

The kind of semantic clause for predicates they intend is as follows:

(ii) $\lceil \Delta \text{ swims} \rceil$ denotes the True iff for some x , Δ denotes x and x swims.

For our concerns, the crucial question concerning Heck and May's account is this: what is the import of 'should' in the claim 'the meaning of a predicate should be given by stating the meaning of an arbitrary atomic sentence in which it occurs'? On its most natural reading, the claim that the meaning of a predicate should be given thus is consistent with its being *possible*, in a relevant alethic sense, that the semantics of a predicate be given otherwise. In particular, it is consistent with the possibility of specifying the referent of a predicate in the manner of (i), as follows:

(iii) ' ξ swims' refers to the concept *swimming*.

Something may significantly speak against treating predicate semantics in this way. Perhaps it fosters a false impression of the logical structure of atomic sentences—an impression that the atomic sentence asserts the obtaining of a *relation* between the referent of its subject and the referent of its predicate (Heck and May 2013, 847). This is consistent, nevertheless, with its being possible to specify the referents of predicates in this way; and the claim that it is possible, I submit, is roughly equivalent to the claim that concepts are objects. Thus, on the most natural reading of Heck and May's interpretation of the Fregean claim that concepts are unsaturated, that claim need not clash with the thesis that concepts are objects. Again, the alleged incompatibility of unsaturatedness with objecthood remains unexplained.

On the other hand, if the import of 'should' is really that of an alethic 'must', such that the claim that predicate semantics should be given in the manner of (ii) amounts to a denial of the possibility of treating predicate semantics in the manner of (iii), then we are granted an explanation of the incompatibility of unsaturatedness with objecthood; since, to reiterate, the denial of that possibility is roughly equivalent to the denial of the objecthood of concepts. However, on this reading, Heck and May's account confronts some of the same difficulties discussed in §5

in connection with an earlier account. As yet, it leaves unclear why we must accept that concepts are unsaturated if we are adequately to account for the unity of the proposition (although we will return to this issue in the following section). Moreover, because of the aforementioned rough equivalence, it renders question-begging any adduction of the unsaturatedness of concepts as *grounds* for denying that concepts are objects. Someone who maintains that concepts are objects will, qua proponent of that view, hold that the meaning of a predicate can be given in the manner of (iii), and so will, qua proponent of that view, reject the claim that concepts are unsaturated, on its present interpretation.

One final point deserves to be made before we leave the topic of the primacy of the whole. Frege's view that the thought in its entirety is primary or basic does not seem to do his argument from the unity of the thought any favours; since, as several authors have argued,³⁷ that view seems to *obviate* the problem of the unity of the proposition, whereas Frege's argument from unity seems to be to the effect that the problem requires solution by the invocation of unsaturated entities. If the parts of a thought only arrive on the scene, so to speak, as a result of the decomposition of the complete and antecedently given thought, then there is, one might argue, no problem about how those parts hold together in the whole.³⁸

8 The Unsaturated as *Bindemittel*

Notwithstanding this *prima facie* tension between the whole-first current in Frege's thought, on the one hand, and his argument from unity on the other, it is natural to propose a conception of unsaturatedness drawn straightforwardly from the role assigned to the unsaturated in that argument. The unsaturated is allegedly required to discharge the role of *Bindemittel* or *Bindeglied*. In Geach's translation of the argument from unity quoted in §2, both German words are rendered as 'link'. However, similar uses of 'Bindemittel' elsewhere in Frege's writings have alternatively been translated as 'means of connection' and 'cement' (see below), and the expression might equally be rendered as 'bonding agent' or 'binding material'. The unsat-

³⁷See (Textor 2009, 63), (Hale and Wright 2012, 112) and (Travis TS, §1.1).

³⁸Mark Textor, for example, writes: '[T]he problem of the unity of the proposition simply does not arise for [Frege]' (Textor 2009, 63). As I shall explain in §9, I suspect that this idea—that Frege's doctrines obviate, rather than solve the problem of the unity of the proposition—offers the most promising answer to the question of what, if any, support Frege's theory of concepts gleans from considerations of unity. However, *pace* the authors just cited, I suggest that it is not the primacy of the whole thought that obviates the problem of unity for Frege: after all, even if the parts of the thought are secondary to or derivative upon the thought in its entirety, there is still the mere congeries of those parts; and the problem remains of explaining in what the difference consists between that mere congeries and the unified thought (I thank Richard Gaskin for impressing this upon me). I shall go on to suggest (§9) that it is instead the very non-objecthood of concepts that strictly prevents the problem from even arising for Frege.

urated sense of the relational predicate ‘falls under’, for example, serves, Frege tells us, to bind together the saturated senses of the singular terms ‘the number 2’ and ‘the concept *prime number*’ into a unitary thought, where these senses would otherwise hold aloof from one another. This picture of the unification of senses is reflected, Frege elsewhere (Frege 1984b, 281) seems to indicate, at the level of reference:

An object, e.g. the number 2, cannot logically adhere to another object, e.g. Julius Caesar, without some means of connection [*Bindemittel*]. This, in turn, cannot be an object but rather must be unsaturated.

Where the *monadic* unsaturated is concerned—e.g. the sense or referent of a monadic predicate—Frege stresses the lack of necessity for any additional bonding agent. To return to a passage we’ve already met (Frege 1979b, 178):

[T]he unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement [*eines besonderen Bindemittels*]. Object and concept are fundamentally made for one another, and in subsumption we have their fundamental union.

The concept is such as to allow unmediated, limpet-like union with the object: it is itself a limiting case of a *Bindemittel*, binding not a plurality of other entities, but binding itself to one other or—equivalently—one other to itself. (What are we to make of Frege’s claim that it is in *subsumption* that one finds the fundamental union of concept and object? If Frege means that it is in the object’s being subsumed under the concept that their union consists, the claim is surely amiss. For if we acknowledge compound unities—viz. states of affairs—as the referents of sentences, such that the union of the parts of these unities needs to be accounted for, then we must account, in particular, for the unity of the referents of *false* sentences; and where false sentences are concerned, subsumption will not obtain. For example, in the referent of ‘Obama is immortal’—viz. the state of affairs of Obama’s being immortal—the union of Obama with the concept of immortality cannot be found in the former’s being subsumed under the latter, since the former is not subsumed under the latter. It seems that Frege must allow that an object is united with—saturates—a concept regardless of whether that object is subsumed under that concept.³⁹ To this end, he might claim that the referents of false sentences, as much as those of true sentences, contain a subsumption relation, and that in both cases that relation is the locus of the union of object and concept, even though in the case of the referent of a false sentence the subsumption relation is not in fact instantiated by object and concept. The problem is that this appears quite at odds with the putatively immediate engagement of object with concept;

³⁹This view is ascribed to Frege by Gibson (Gibson 2004, 11–12).

for it seems to place the subsumption relation in the role of mediating cement, the necessity of which Frege is precisely concerned to deny.)

Frege claims in the argument from unity that it is ‘only because’ of a sense’s unsaturatedness that it is capable of serving as a bonding agent uniting other senses into a complete thought (Frege 1997f, 193). On the conception of unsaturatedness I now want to discuss, this consideration has a simple explanation: unsaturatedness just *is* that very capacity to serve as bonding agent; it is precisely the kind of adhesive or copulative potency that permits an entity to consolidate elements—of language, thought, or extra-representational reality—that would, left to their own devices, remain aloof.

Now, this conception immediately invites the complaint that Frege has really failed to give any substantial account of the unity of the proposition. Jeffrey C. King objects, for example, that Frege’s position ‘seems to essentially amount to saying that the parts hold together because some of them are “sticky”’ (King 2007, 18), without the provision of a ‘substantial theory of *stickiness*’ (ibid.) to give the account genuine content. Similarly, Donald Davidson charges that Frege’s classification of certain meanings as unsaturated, towards preventing the disintegration of the meaning of a sentence into a mere congeries of disparate elements, ‘seems to label a difficulty rather than solve it’ (Davidson 1967, 304). There is no doubt a good measure of justice in this kind of objection, as I suspect Frege would be prepared to concede. But one respect in which the charge does not stick—no pun intended—is this: Frege insists that that which serves as bonding agent *cannot be an object* (Frege 1984b, 281). This *is* a substantial thesis and, in at least this respect, his account of unity does not merely label the phenomenon of which explanation was sought; and it is precisely this respect which here concerns us.

Why, then, cannot that which serves as *Bindemittel* be an object? I turn now to two contemporary authors who might be thought to have answered this question: Graham Priest and Richard Gaskin.

8.1 Priest on Gluons

According to Priest, Frege was quite right to insist that ‘if something is to perform the role of explaining how it is that a unity of objects is achieved, it cannot just be another object’ (Priest 2014, 9). Priest offers two at least presentationally distinct arguments for Frege’s view. Concerning the posited bonding agent holding together parts in a unity, Priest (Priest 2014, 9) argues that

it cannot be an object. If it were, the collection of parts *plus* the [bonding agent] constitute a plurality, just as much as the original. So the problem of binding would not be solved.

In somewhat more detail, Priest (Priest 2014, 9) reasons as follows:

Take any thing, object, entity, with parts, $p_1, \dots p_n$ A thing is not merely a plurality of parts: it is a unity. There must, therefore, be something which constitutes them as a single thing, a unity. Let us call it, neutrally (and with a nod in the direction of particle physics), the *gluon* of the object, g . Now what of this gluon? Ask whether it itself is a thing, object, entity? ... [I]t is not, since, if it is, $p_1, \dots p_n, g$, would appear to form a congeries, a plurality, just as much as the original one. If its behaviour is to provide an explanation of unity, it cannot simply be an object.

This is, in my judgement, a non sequitur. To be sure, if a complex unity's *Bindemittel*—its *gluon*, in Priestian terminology—is an object, then it and the (other) parts of the unity form a plurality (or congeries). Of course they do; for Priest so intends 'plurality' that to say of some things that they form a plurality is just to say that they are *some things* (and does not imply that they form some *one* thing) (Priest 2014, 9). But that they form a mere plurality does not entail that they *merely* form a plurality. They may also form a unity, and may do so, for all that Priest has said, by virtue of the unifying action of the gluon. That is, the fact that $p_1, \dots p_n, g$ form a plurality is perfectly compatible with the circumstance that $p_1, \dots p_n, g$ also constitute a unity and are bound together in that unity by g . Equally, it is consistent with the circumstance that $p_1, \dots p_n$ constitute a unity and are bound together in that unity by g . Indeed, for Frege, these circumstances will be one and the same, for the plurality $p_1, \dots p_n, g$ will be none other than the plurality $p_1, \dots p_n$: the gluon of a unity is, on Frege's account, itself a part of the unity. (For example: 'This unsaturatedness of *one of the components* is necessary, since otherwise the parts do not hold together' (Frege 1979b, 177, my emphasis). Recall also: '[N]ot all the parts of a thought can be complete; *at least one* must be "unsaturated"' (Frege 1997f, 193, my emphasis).)

Priest's second argument (Priest 2014, 9-11) proceeds as follows:

It will pay to become clearer about why a gluon cannot be an object. A vicious regress stands behind this. ... Suppose that we have a unity comprising the parts, a, b, c, d , for example. There must be something which, metaphysically speaking, binds them together. This is the object's gluon, g . But then there must be something which binds g and a, b, c, d together, a hyper-gluon, g' . There must, then, be something which binds g', g and a, b, c, d together, a hyper-hyper-gluon, g'' . Obviously we are off on an infinite regress. Moreover, it is a vicious one.

Is it Priest's assumption here that Frege was mistaken in supposing that the gluon of a unity is itself a part of that unity? This is the strong implicature of, e.g., his use of the (otherwise redundant) plural term ' g and a, b, c, d '. At any rate, let us grant this assumption for the moment: g is not one of a, b, c, d . But now, what justifies Priest's contention that 'then there must be

something which binds g and a, b, c, d together'? Suppose, by way of illustration, we accept an account of unity according to which what binds a, b, c, d together as a unity is the fact of their being related in a certain manner—the fact that $Rabcd$, where ' $R\alpha\beta\gamma\delta$ ' is some quaternary predicate. Moreover, this fact is, *pace* the Wittgenstein of the *Tractatus*, itself considered an object. If Priest now attempts to ignite a vicious regress by insisting that we must recognize some additional binding agent unifying a, b, c, d and the fact that $Rabcd$, the proper response on our part would be to return a puzzled stare and remind Priest that we would only need to recognize such an additional binding agent if we thought that a, b, c, d and the fact that $Rabcd$ were bound together into a unity: but we need not think that. We have only agreed to recognizing a, b, c, d —a distinct plurality on the present supposition—as constituting a unity. Indeed, if a, b, c, d are not themselves facts, common sense favours refusing to recognise any one thing of which a, b, c, d and the fact that $Rabcd$ are parts. On the present account of unity, therefore, the regress seems to be forestalled at this early stage. I do not claim that such an account is ultimately tenable. I only wish to illustrate that on the un-Fregean supposition that the gluon of a unity is not a part of that unity, it is far from obvious that one must recognize a second gluon binding the first together with the parts of the original unity. The ignition of the regress depends on that claim, however.

On the other hand, let us now instead grant Frege's view that the gluon of a unity is itself a part of that unity: g is one of a, b, c, d . If we are asked what binds a, b, c, d together, we reply that g does. If we are then asked what binds g and a, b, c, d together, we should simply reply that our questioner has repeated himself, for the plurality of g and a, b, c, d is none other than the plurality of a, b, c, d . We simply refer him back to our first answer. If we really wish, we can nominally proceed with the regress, answering this second question by saying that some gluon g' effects the binding, answering the follow-up by saying that some gluon g'' effects the binding, and so on. But as soon as it is alleged that the regress on which we have embarked is vicious, we must reply that there is no question of its being anything other than benign, since $g \overbrace{t \dots t}^n = g \overbrace{t \dots t}^{n+1}$, for all n . Equally, therefore, Priest does not seem to have given reason to think that a vicious regress ensues when the Fregean view is granted.

We might alternatively suppose that a, b, c, d are only the *non-gluon* parts of the unity in question. By Fregean lights, it is appropriate to describe the gluon of that unity, g —also a part of that unity, we now suppose—as binding together a, b, c, d . (Recall, for example, Frege's claim that the unsaturated sense of 'falls under' binds together the saturated senses of 'the number 2' and 'the concept *prime number*' into a thought: the latter two senses are the non-gluon parts, so to speak, of the thought in question.) But perhaps Priest might now attempt to ignite the regress as follows. 'If some additional part g must be acknowledged to account for the

holding together of the parts *a, b, c, d*, some yet additional part *g'* must, by the same token, be acknowledged to bind the parts *a, b, c, d* and *g* together.' Of course, it is exactly this conditional (or rather, the instances of this conditional schema) that Frege denies: he maintains that the gluon of a unity copulates immediately—precisely without the help of additional glue—with the non-gluon parts of that unity. The immediacy of this copulation is reflected in its being appropriate not only to describe that gluon as binding together the non-gluon parts of a unity but also to describe that gluon as binding itself together with those parts. Crucially for our purposes, it remains unclear that only by classifying the gluon of a unity as a non-object is one able to affirm its immediate copulation with the remaining parts. The conditional in quotes above seems to amount not to an assertion of the specific incompatibility of objecthood with the capacity for immediate copulation, but rather to a general rejection of that capacity.

It strikes me that the foregoing arguments of Priest's do not, therefore, satisfactorily answer the question of why that which serves as *Bindemittel* or gluon cannot be an object.⁴⁰ Now, what I have so far suppressed in my quotations from (Priest 2014) on the issue of the (non)objecthood of gluons is Priest's equal and opposite conviction that a gluon *must* be an object. '[W]e can refer to it, quantify over it, talk about *it*', he stresses. 'If this does not make something an object, I am at a loss to know what could' (Priest 2014, 15). (This leaves no doubt that if an entity lacks objecthood by Priest's lights, then it lacks objecthood by Frege's. This suffices for present purposes; I shan't pursue the question whether the converse holds, for our principal concern here is with the putative *non*-objecthood of gluons.) Remarkably—though not altogether surprisingly, for if anyone is known for his willingness to contain multitudes it is Priest—he proposes simply to bite this bullet and accept that gluons both are and are not objects. Gluons have contradictory properties. Thus, the truth about gluons witnesses dialetheism: the thesis that some contradictions are true.

Priest goes on (Priest 2014, 16–37) to develop a detailed account of *how* it is that gluons succeed in binding together the parts of a complex unity. (Frege's own failure to provide such an account met with criticism above.) This account, it emerges, corroborates both Priest's judgement that gluons are possessed of contradictory properties and (in consequence, as I shall shortly explain) his conviction that gluons are not objects. Indeed, in a sense, this account of how gluons glue constitutes the most important argument in Priest's book for the non-objecthood of gluons. The essence of the account can be introduced by again pressing the question whether the gluon of a unity is itself a part of that unity. Priest's answer is that not only is the gluon one of the parts, it is *each* of the parts. The gluon binds together the parts into a unity by being numerically *identical* to each of them. It is this, Priest thinks, that prevents the

⁴⁰I have offered a similar response to Priest in (Price 2017).

precipitation of the regress described above. No further gluon, g' , is required to unite g with a, b, c, d ; for g is identical to each of a, b, c, d . There is no ‘metaphysical space’ (Priest 2014, 17) between, for example, a and g , across which a further join must be made; likewise for b, c and d .

The immediate objection to this account is this: The phenomenon of which we sought an explanation is the holding together of *distinct* parts as a unity—the case in which, e.g., $a \neq b \neq c \neq d$. Thus, the phenomenon cannot be explained by the supposition that some one part is identical to each of the parts, since if that were so, the parts would not be distinct: e.g., if $g = a$ and $g = b$, then also, by the symmetry and transitivity of identity, $a = b$, contrary to the hypothesis. Priest’s bold response to this objection is to deny the transitivity of identity. So in the present case, he maintains that while $g = a$ (hence $a = g$) and $g = b$, it is not the case that $a = b$. Though the gluon is identical to each of the parts, the parts may nevertheless be distinct.

This is, of course, a radical departure from the orthodoxy concerning the identity relation. The definition of identity Priest adopts, though, is a standard Leibnizian one, also adopted by Frege:

$$(L) \quad \alpha = \beta \equiv_{df} \forall X (X\alpha \equiv X\beta)$$

However, as a dialetheist, Priest is working with a paraconsistent logic—a logic that does not validate the inference from a contradiction to an arbitrary proposition—in which the relation of material equivalence (sharing the same truth-value) expressed by ‘ \equiv ’ is not transitive. (Let P be just true, let Q be a *dialetheia* (both true and false), and let R be just false; then, $P \equiv Q$ and $Q \equiv R$ but $\neg(P \equiv R)$.) Consequently, the relation asserted between α and β by ‘ $\forall X (X\alpha \equiv X\beta)$ ’ is also not transitive, and neither is that asserted between α and β by the definitionally equivalent ‘ $\alpha = \beta$ ’. It can quickly be seen that, granted (L), Priest’s account of how gluons glue entails that gluons do indeed possess contradictory properties.⁴¹ Let a and b be distinct parts of some unity, u , of which g is the gluon. So $\neg a = b$ and thus, by (L) and the substitution of definitional equivalents, $\neg \forall X (Xa \equiv Xb)$. Therefore, $\exists X \neg (Xa \equiv Xb)$. Hence, either $\exists X (\neg Xa \wedge Xb)$ or $\exists X (Xa \wedge \neg Xb)$. Assume the latter (the former case is alike). Let ‘ P ’ denote an arbitrary witness of this latter second-order existential generalisation, so that $Pa \wedge \neg Pb$. Now, according to Priest’s account, $g = a$ and $g = b$. Hence, $\forall X (Xg \equiv Xa)$ and $\forall X (Xg \equiv Xb)$, and in particular, $Pg \equiv Pa$ and $Pg \equiv Pb$. Since $Pa \wedge \neg Pb$, it follows that $Pg \wedge \neg Pg$.⁴² Thus, g has contradictory properties.

⁴¹The following draws particularly on (Priest 2014, §2.8).

⁴²Strictly: this follows granted the assumption that a and b are not themselves contradictory with respect to P .

From here, there is a very strong case that g is not an object. Since $Pg \wedge \neg Pg$, we have that $\neg(Pg \equiv Pg)$ and thus that $\exists X\neg(Xg \equiv Xg)$. Hence, by (L), $\neg g = g$. But if g is not identical to itself, it is not identical to anything! That is, $\neg\exists x(g = x)$. To see this, consider an arbitrary thing, e . Granted the Law of Excluded Middle (which Priest accepts), $Pe \vee \neg Pe$. If Pe , then, since $\neg Pg$, $\neg(Pe \equiv Pg)$. Hence $\exists X\neg(Xe \equiv Xg)$ and therefore $\neg e = g$. On the other hand, if $\neg Pe$, then since Pg , $\neg(Pe \equiv Pg)$. Hence $\exists X\neg(Xe \equiv Xg)$ and therefore $\neg e = g$. So in either case $\neg e = g$. But e was arbitrary. Thus, quite generally, $\neg\exists x(g = x)$. But this latter proposition very plausibly amounts to a denial of the objecthood of g . Singular reference and first-order quantification are constitutively connected, such that to be a possible referent of a singular term is to be ranged over by—to be a value of a variable bound by—the unrestricted first-order quantifiers. Since in the present context ‘ \exists ’⁴³ is to be understood as absolutely unrestricted, ‘ $\neg\exists x(g = x)$ ’ implies that g is not ranged over by the unrestricted first-order quantifiers. Hence, it is not a possible referent of a singular term, which is to say, not an object.

Thus, Priest can, in summary, be understood to offer the following answer to our question of why that which serves as *Bindemittel* must not be an object: something can only bind together distinct parts into a unity—can only copulate immediately with them in the manner Frege envisaged, such as to forestall a vicious infinite regress of auxiliary *Bindemittel*—if it is numerically *identical* to each of those distinct parts; that requires that it have contradictory properties; and that, in turn, requires that it not be an object.

We will find this answer satisfying only if we are prepared to accept Priest’s account of the unification effected by the *Bindemittel*; and there is simply no getting away from the fact that this entails acquiescing in the dual heresies of dialetheism and the denial of the transitivity of identity. (The shock of the latter is perhaps mitigated in some measure by the former, since, as we have seen, the failure of the transitivity of material equivalence in paraconsistent logic, coupled with the definition of identity (L), does indeed have the consequence that identity is not transitive.) There are those for whom the essential role of dialetheism in Priest’s answer is alone sufficient reason to reject it. ‘[D]ialetheism’, Williamson avers, ‘is a fate worse than death’ (Williamson 2007, 387). For my part, I counsel a weaker form of logical conservatism. In my judgement, it would be a grave decision to relinquish consistency. Though we should not rule out doing so, it should take a formidable case indeed to induce us to do so—the kind of case which cannot, I submit, be made (entirely) in one small neighbourhood of metaphysics, such as the problem of the unity of the proposition, where consistent theories appear still to be

⁴³Priest in fact uses a different symbol for the particular quantifier, for reasons connected with his acceptance of *noneism*: the thesis that some objects do not exist. (Priest 2014, xxii)

in the running. The extremely high price (in my estimation) of tolerating true contradictions should buy us more philosophical amenity than an explanation of unity. Of course, Priest’s wider body of work makes precisely the case that dialetheism buys us much more—*inter alia*: a unified response to the semantical, set-theoretical and soritical antinomies.⁴⁴ It evidently lies well beyond the scope of this essay to evaluate this wider case for dialetheism. Thus, I cannot accept Priest’s answer to our question concerning the objecthood of *Bindemittel*, though nor can I reject it: I must place it in abeyance pending the outcome of debates elsewhere.

8.2 Gaskin on the Copula

A rather different, but similarly innovative treatment of the unity of the proposition is developed by Gaskin (Gaskin 1995).⁴⁵ Gaskin takes an approach reminiscent of that which Frege says ‘may be done’ in the argument from unity quoted in §2. Taking his cue from (Wiggins 1984), he essentially proposes to identify the *copula* of a sentence—paradigmatically (though not invariably), the finite form of ‘to be’ in its predicative sense, or the finite ending of the relevant verb—rather than the whole predicate, as that which, by virtue of its unsaturatedness, accounts for propositional unity. Unsaturatedness is shifted to the copula away from the concept expression, which is now understood to be simply a species of proper name—its referent, thus, simply a species of object. This shift is not profitless, Gaskin claims *contra* Frege, since it permits us to talk about concepts—to singularly refer to and quantify over them. *Pace* (Wiggins 1984), Gaskin maintains that the copula is itself referential. He proposes to capture the referentiality of the copula with the following semantic clause:⁴⁶

(C) If $R^n(x_1 \dots x_n)$ then $x_1 \dots x_n$ instantiate ^{$n+1$} R^n .

Thus, for example, if Shergar is a horse, then Shergar instantiates² horsehood. (One immediately worries that (C) isn’t grammatically well-formed, since ‘ R^n ’ features in incompatible syntactic positions therein: predicate position in the antecedent and term position in consequent. In the example just given, I have nominalized the concept expression to resolve this tension (thus, ‘horsehood’), and though the issue goes unremarked in Gaskin’s paper, I shall assume

⁴⁴See particularly (Priest 2006); also, e.g. (Priest 2010).

⁴⁵I shall focus on the account developed in (Gaskin 1995), rather than Gaskin’s later treatise, (Gaskin 2008). The account developed in the paper is more germane to our investigation of Frege’s views, in that Gaskin there seeks to explain unity by proposing, with Frege, a distinction between saturated and unsaturated features of the sentence, and by ascribing a special kind of unifying referent to the unsaturated feature of a sentence. By contrast, in (Gaskin 2008), Gaskin departs from Frege in maintaining that the significant components of a sentence are *all* unsaturated if any are (Gaskin 2008, Ch. 3).

⁴⁶I take it that the copula is here represented by sheer concatenation of the concept expression ‘ R^n ’ with its argument expressions ‘ x_1 ’ . . . ‘ x_n ’. I indicate adicity with a numerical superscript, whereas Gaskin uses a subscript.

that in the consequent of (C) ‘ R^n ’ is simply elliptical for some nominalization of the predicate in the antecedent.) Accordingly, a first pass at specifying the referent of the copula would be to say that the copula refers to the (dyadic) relation of instantiation. But the consequent of our Shergar example can equally be plugged into (C) as the antecedent, yielding as consequent that Shergar and horsehood instantiate³ instantiation². Putting these two instances of (C) together, we have that if Shergar is a horse, then Shergar and horsehood instantiate³ instantiation². Thus, by the same token, a more considered specification of the referent of the copula would be that it refers to the (triadic) relation of the instantiation of (dyadic) instantiation. By similar reasoning, a third and in Gaskin’s estimation even better effort to specify the copula’s referent would be that it refers to the (quaternary) relation of the instantiation of the (triadic) instantiation of (dyadic) instantiation. We can continue this process *ad infinitum*, obtaining ever more complex referent-specifications for the copula. The parallel with Bradley’s regress is unmistakable: indeed the possibility of repeating this process, Gaskin says, ‘is just the possibility of generating Bradley’s regress’ (Gaskin 1995, 174). Repeating the process yields referent-specifications of ever greater fidelity to the real situation; but each such specification, Gaskin claims, is ‘inevitably inchoate’ (Gaskin 1995, 176), a yet more complete specification always lurking over the horizon. The final statement of what the copula refers to irremediably ‘keeps eluding our grasp’ (ibid.). And it is precisely in the inevitable inchoateness of any specification of its referent that the unsaturatedness of the copula consists. Since this inchoateness is a matter of its being possible to generate Bradley’s regress, the unsaturatedness of the copula is in fact underwritten by that regress. Moreover, since it is, in Gaskin’s view, by virtue of the unsaturatedness of the copula that the proposition has unity, the regress similarly underwrites that unity: so far from being vicious, ‘Bradley’s regress is [actually] the metaphysical ground of the unity of the proposition’ (ibid.). It is the infinitism introduced into the proposition by the unsaturated copula, in the form of the regress, that constitutes the proposition as a unity. In a slogan (with a confessedly paradoxical air): ‘what stops a proposition from being a “mere list” is that it is an *infinite* list (of the specified kind)’ (ibid.).

Though Gaskin does not explicitly say as much, it is very natural to surmise that his account of propositional unity is, like Frege’s, one on which the unsaturated component of a sentence, though it refers, does not refer to an object. For I suggest that if the copula did refer to something to which, possibly, some singular term, t , also refers, then we could not take fully seriously Gaskin’s claim that a final specification of the referent of the copula always eludes our grasp—that specification of its referent is ‘inevitably inchoate’: for the claim that the copula refers to t would then, on the contrary, seem finally to specify the copula’s referent.⁴⁷ Thus

⁴⁷That Gaskin would accept that the copula does not refer to an object is also strongly suggested by his remarks on

Gaskin’s account of propositional unity does appear to furnish an answer to our question of why that which serves as *Bindemittel* is to be deemed a non-object—at least in the case of the referent of the unsaturated component of the sentence.

I suggest that answer shouldn’t satisfy us, however; for Gaskin overestimates the inevitability of inchoateness in specifying the copula’s referent. The n -adic instantiation relations in terms of which Gaskin proposes to explain the semantics of the copula can be well-ordered by making dyadic instantiation first, triadic instantiation second and, generally, $n+1$ -adic instantiation n^{th} . Let ‘ I_i ’ name the i^{th} such relation. (Gaskin accepts that each such relation is a ‘perfectly good object’ (Gaskin 1995, 173).) Now define an $n+1$ -adic relational expression ‘copulate’ as follows:

(C*) $x_1 \dots x_n$ copulate $^{n+1} R^n$ iff $_{df}$

1. $x_1 \dots x_n$ instantiate $^{n+1} R^n$, and
2. if $x_1 \dots x_n, R^n, I_1 \dots I_m$ instantiate $^{n+1+m} I_{m+1}$,
then $x_1 \dots x_n, R^n, I_1 \dots I_m, I_{m+1}$ instantiate $^{n+1+m+1} I_{m+2}$.

Roughly, $x_1 \dots x_n$ copulate $^{n+1} R^n$ iff $x_1 \dots x_n$ instantiate R^n , and $x_1 \dots x_n$ and R^n instantiate instantiation, and $x_1 \dots x_n, R^n$ and instantiation instantiate instantiation, ... *and so on*. But the ellipsis—the inchoateness—of this rough explanation is eliminated in (C*) by the second, inductive clause. We can now nominalize ‘copulate’, using whatever device of nominalization we accept—as, say, ‘copulation’. As far as I can see, the singular term ‘copulation’ then designates that to which, on Gaskin’s account, the copula refers. The referent of the copula is, therefore, finally specifiable: it refers to copulation. And copulation, too, is a perfectly good object.

On the conception we’ve been examining, unsaturatedness is the capacity to serve as a bonding agent securing the unity of a sentence, thought or state of affairs. That conception presents a now familiar problem for Frege’s argument from unity: we have found no acceptable answer to the question of why unsaturatedness, so conceived, must be incompatible with objecthood.

the concept *horse* paradox: ‘What Frege’s paradox shows is that one cannot both talk about concepts... and continue to insist on their essentially unsaturated... nature.’ (Gaskin 1995, 166) On the other hand, Gaskin’s exposition does feature *unapologetic* use of singular terms purporting to co-refer with the copula: e.g. ‘what the copula refers to’ [175], ‘the reference of the copula’ [176] (he also uses ‘referent’). Indeed, he even adduces, as a reason for deeming the copula referential, that ‘we want, as philosophers, to talk about predicative being’ [177]. I fear this just represents a tension in Gaskin’s paper.

9 Conclusion

Frege's argument from the unity of thought for the non-objecthood of concepts is, I hope to have shown, deeply problematic. In essence, it is altogether unclear how to simultaneously substantiate, on the one hand, the claim that unsaturated entities must be recognized in order to account for unity and, on the other, the claim that unsaturatedness is incompatible with objecthood.

Is it the case, then, that nothing can be made of Frege's argument? Two considerations might somewhat temper one's confidence in an affirmative answer to this question.

The first is simply that we have proceeded by trying to find a satisfactory reconstruction of the argument and failing. Obviously, we have no full assurance that some persuasive reconstruction has not been neglected: our investigation of Fregean incompleteness may be incomplete.

The second is that there is, I suggest, a sense in which Frege's doctrine that concepts are not objects might after all glean some support by reference to the problem of the unity of the proposition. As Gaskin puts it, by 'the unity of the proposition' is traditionally meant 'the ability of a proposition (or, as we would more naturally say now, a sentence) to say something ... rather than merely list its referents' (Gaskin 1995, 162). The problem of the unity of the proposition is then to explain a sentence's differing from such a list in being capable of saying something. But there is a sense in which, granted Frege's view, this problem simply does not arise, because there *can be no such list* as the kind of list a proposition's difference from which allegedly needs to be explained. Some of a sentence's referents are, Frege holds, not objects. They are unnameable. On an utterly standard conception of *list*, the unnameable is *unlistable*. Thus, the particular instances of the problem cannot strictly even be posed. If we ask Frege how it is that 'Shergar is a horse' manages to say something, while 'Shergar, the concept *horse*' does not, we have in fact, by Frege's lights, not inquired about the list we'd intended: 'Shergar, the concept *horse*' is, strictly, not a list of the referents of 'Shergar is a horse'. The problem of the unity of the proposition is one of accounting for the difference between two kinds of linguistic production. The Fregean is relieved of this problem, since s/he simply does not recognize one of these kinds of linguistic production. Thus, an argument for Frege's view: to accept the view is to be unburdened by this vexing philosophical problem.

I suspect that this, if anything, is what can be made of the argument from unity. I conclude the discussion by offering three reasons for considering even this last take on the argument from unity to be unpersuasive.

Firstly, the argument ought to pull us towards Frege's view only to the extent that we are pessimistic about the prospects for a satisfactory solution to the problem of the unity of

the proposition. For, I propose, if such a solution is to be had, it wins out over the Fregean *dissolution* of the problem, simply because it grants an intuitive datum that the Fregean response denies—viz. that it is possible to list the referents of a sentence’s constituent expressions. However, it is not at all clear that such pessimism is warranted. Certainly, a number of impressive attempts at a solution have been made.⁴⁸

Secondly, evading the problem of propositional unity by appeal to the unlistability of (the totality of) the referents of a sentence’s sub-expressions is a strategy equally available to one who accepts that those referents are all objects: indeed they can, to this end, appeal to Frege’s own context principle. As formulated in the *Grundgesetze*: ‘We can inquire about reference only if the signs are constituent parts of sentences expressing thoughts’.⁴⁹ The kind of list that is supposed to contrast with a sentence would appear to involve just what the context principle, straightforwardly understood, rules out: reference in isolation from the complete sentential context. A stringent insistence upon the context principle forestalls the problem of unity in essentially the same way as the thesis that concepts are not objects, but leaves open the possibility of singular reference (within a complete sentence) to concepts. Thus, the argument lends no clear advantage to Frege’s view.

Thirdly, the argument would, it seems, involve Frege in a kind of dialectical foul play. Recall Frege’s plea for a pinch of salt over his own use of singular terms purporting to co-refer with predicates and other incomplete expressions: taken literally, he acknowledged, his expressions go referentially astray. Well, if we attempt to pose an instance of the problem of propositional unity, we are, by Frege’s lights, merely in the very same bind; and if we did not begrudge the said pinch of salt, and consented to meet Frege halfway, we can surely ask for the same in return. That same pinch will allow us to pose the unity problem precisely in so far as it allows Frege to communicate his own philosophical semantics. The only option for Frege would be, when confronted with an attempt to pose the problem of unity, to begrudge the very pinch of salt for which he had himself pleaded. That would surely be foul play.

Acknowledgements

Many thanks to Paul Egré, Richard Gaskin, Adrian Moore, James Openshaw, Robert Trueman, Timothy Williamson and an audience at the University of Oxford for valuable comments on earlier versions of this paper. Much of the work in this paper was undertaken while I was receiving a doctoral studentship from the Arts and Humanities Research Council and then a

⁴⁸See, for example, the works I have cited by Gaskin, Gibson and King.

⁴⁹Quoted in (Heck and May 2013, 849).

post-doctoral award from the Philosophical Fellowship Fund at the University of Oxford. I thank both funding bodies for their generous support.

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