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REALISM AND QUANTUM MECHANICS

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A realistic interpretation of quantum mechanics is imperative

By and large, working scientists are unabashed realists, they stubbornly believe that there is a real external world. For many theoreticians, this belief is the only raison d'être of physical theories. They would like to have a description of how, fundamentally, the world *is*. But many popular presentations tell us that quantum mechanics is not compatible with realism. If this view would be true we would be in real trouble. Scientists take no thought of abandoning quantum mechanics since it is probably the empirically best confirmed scientific theory. In spite of many counter-intuitive quantum-theoretical predictions, there is not a single well-performed experiment which contradicts quantum mechanics. Certainly, there are open questions, but no flagrant contradictions between theory and experiment. On the other hand, we cannot abandon realism since the very confirmation of quantum mechanics is based on the acceptance of everyday realism. In the early days, quantum mechanics has been considered as a theory of the microworld, and most scientists did not realize that they cannot consistently adopt different ontologies for the microworld and the everyday world of laboratory instruments. Nowadays we cannot any longer take this position because we know that quantum mechanics is valid also for mesoscopic systems–like DNA-molecules with biochemically important quantum properties *and* genetically important classical properties. Since no scientist is willing to give up some kind of realism in the domain of laboratory experience, we really have to care for a *realistic interpretation of quantum mechanics.*

The philosophical notions about quantum mechanics held by many philosophers and theoretical physicists are incompatible with the actual practice of the working scientist. The lack of a well-founded philosophical discourse on quantum mechanics has harmful consequences in research and in teaching. Nevertheless, quantum theory is by no means in a state of crisis. The problem is only that many scientists and most philosophers are not familiar with the modern technical developments of quantum mechanics, and therefore they still try to solve conceptual problems of quantum theory– like the theory of the measuring process– in terms of old-fashioned Hilbert-space quantum mechanics *which is valid only for finite closed systems.* Strictly speaking, such systems do not exist. Since all material systems are inextricably coupled to the electromagnetic and to the gravitational field, even "reasonably isolated" finite systems do not exist. This does not mean that it is not instructive to study the fiction of closed systems, but one should not confuse tentative investigations and the full-grown theory. In this sense, no exegesis of the writings of Niels Bohr, Werner Heisenberg and other pioneers will lead to a satisfactory solution of the conceptual problems of contemporary quantum mechanics.

I would like to advocate to investigate carefully

(i) *what we mean by realism, and whether we should expurgate objectionable ideas taken over from realism as understood in classical physics.*

(ii) *what we mean by quantum mechanics from a contemporary point of view, and whether philosophically important features in our understanding of quantum physics have changed in the last sixty years.*

The Cartesian split, the death of atomism and the limitations of contemporary science

Classical physics and a large part of contemporary science rest on Descartes' idea that nature is intrinsically divided into two parts: mind (*res cogitans*) and matter (*res extensa*). In addition, it is a tacit assumption of all engineering and experimental sciences that nature can be *manipulated* and that the initial conditions required by experiments can be created by interventions using means *external* to the object under investigation. That is, *we take it for granted that the experimenter has a certain freedom of action which is not accounted for by first principles of physics.* Man's free will implies the ability to carry out actions, it constitutes his essence as an actor. Without this freedom of choice, experiments would be impossible. *The framework of experimental science requires this freedom of action as a constitutive though tacit presupposition.* Traditionally, free will is understood as something belonging to the spiritual world, therefore contemporary science cannot dispense lightly with Cartesian dualism.

Many scientists and philosophers praise quantum mechanics as the *fun-*

dament of modern physics, molecular chemistry and molecular biology, but rarely it is stressed that quantum mechanics also put an *end to atomism.* The historical idea that the material world is already structured by some kind of interacting 'elementary systems' is in sharp contradiction to the structure suggested by quantum mechanics. According to quantum mechanics, the material world is a whole, *a whole which is not made out of parts.* If one agrees that quantum mechanics is a serious theory of matter, then one cannot adopt the classical picture of physical reality with its traditional metaphysical presuppositions. In particular, the nonseparability and nonlocality of the material world and its holistic features are not compatible with the ontology usually adopted in classical physics.

The experimentally well-confirmed holistic character of the material world casts severe doubts upon the consistency of the Cartesian separation of the *material* reality from the *spiritual* one—this idea may well be radically in error. Nevertheless, *present-day experimental science* still requires an *epistemological* dualism of subject vs. object. It is true that quantum theory has clearly put in evidence the limitations and the narrowness of today's scientific conception of reality, but *the often heard statement that quantum mechanics has already given up Cartesian dualism is unfounded.* In every experimental investigation of a quantum system, the measuring apparatus is described positively in terms of classical or engineering physics. *In quantum physics man's consciousness does not enter the physical discourse in any other way than in classical physics.* In the words of Wolfgang Pauli: "Die alte Frage, ob unter Umständen der psychische Zustand des Beobachters den äusseren materiellen Naturverlauf beeinflussen kann, findet in der heutigen Physik keinen Platz" [1]. In fact, contemporary quantum mechanics–as it is used by all experimentalists–is still in a kind of "peaceful coexistence" with Cartesian dualism. That does not mean that the Cartesian separation is not misconceived and that we should not try to create a non-Cartesian science. However, today's physics is ill-disposed and technically incapable to start such a project. At present, it would be science fiction to link quantum events to conscious events, or trying to incorporate a representation of conscious processes into physical representations of brain processes. Since there is no sound theory which includes consciousness in the realm of physics, I prefer to acknowledge that there is a gap in the reasonings of present-day science. In this sense, all physical theories at our disposal are *essentially incomplete theories:* they are incapable to deal with the *complementarity* of matter and spirit.

Contemporary quantum mechanics requires an engineering approach with a division into a part "which sees" and a part "which is seen". According to the formalism of quantum mechanics, this cut is *context-dependent* and not identical with the Cartesian cut. The Cartesian separation would require an *intrinsic* separation of the whole reality into *res extensa* and *res cogitans*, while engineering quantum mechanics requires a contextual subject–object tensor-product decomposition of the whole reality such that there are no Einstein–Podolsky–Rosen-correlations between the *observed object* and the *observing tools*. This requirement is a precondition of experimental science. In the formalism of algebraic quantum mechanics, it implies that the observing tools have a representation as *classical* quantum systems[1]. In all engineering applications of quantum mechanics, the conscious human observer is a *part* of the "observing tools" so that the experimenter can be regarded as a "*detached observer*" in the sense of Bohr [2]. Inasmuch as the Cartesian cut is put within the classical domain, a direct conflict between quantum theory and the Cartesian ontology is avoided. This is in accordance with the modern experimental techniques where the observing and recording devices are often completely automated to the extent that the role of the human observer is reduced to simple acts of cognition of the numeric displays of classical measuring instruments. Hence the free will and the awareness of the observing scientist play exactly the same role they have in classical physics and engineering science. Also, in the cosmological or biological evolution there are objective happenings, encodings and registrations which are independent of the existence of beings having a consciousness. For these reasons, we conclude that in general the irreversible transmutation from possibilities to facts cannot depend on anthropogenic preparation and registration procedures, or on the consciousness of a human observer.

Realism

The historical Copenhagen view does not present quantum mechanics as a universal theory, it presupposes observational tools, but does not describe them quantum-mechanically. According to the Copenhagen view, quantum mechanics gives just the rules to calculate the probability of quantum events, but does not describe the events themselves. This attitude was reasonable in the pioneer years of quantum mechanics since at that time the mathematical tools for describing open systems and their interactions with the environment were not available. In order to analyze modern experiments of molecular science and the phenomena of molecular biology from a quantum-

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[1] A quantum system is said to be classical if its algebra of observables is commutative. Note that every classical quantum system depends on Planck's constant. The existence of contextual classical quantum systems has not to be postulated, but is a consequence of a proper mathematical codification of quantum mechanics.

mechanical point of view, *the Copenhagen view is not sufficient.* In molecular and mesoscopic science we need a theory which is *universally valid in the whole molecular domain*, including systems of mesoscopic and macroscopic dimension, having both quantum and classical properties, and which can describe *individual dynamical processes* in an objective way.

Since the atomistic view of classical physics is very different from the holistic view of quantum mechanics, it is plain that the traditional notion of reality used in classical physics and the notion of reality required in quantum theory clash. But these notions only clash because philosophers were not careful enough in their attempts to give an explication what we could mean by realism. A number of views of traditional realist philosophy is incompatible with the results of modern science.

Many formulations of what realism asserts are so vague that it is difficult to evaluate their claims in the domain of science. Often such formulations are unnecessarily coupled with unfounded assumptions about the structure of the material world. For example, it has been said that in a realistic interpretation the theoretical terms genuinely refer (maybe in some approximate way) to objects existing in the world. Such a characterization is inadmissible since it makes a specific assumption about the *physical* structure of the world, namely that the world consists or is built out of well-defined and independently existing objects. From the viewpoint of modern quantum theory, any *a priori* identification of "material objects" (presumably tacitly supposed to be well-localized in physical space) with "material reality" is unacceptable, since–whatever the precise meaning of "material objects" may be–we have to expect that such systems are entangled by Einstein–Podolsky–Rosen-correlations, so that they have no individuality. Quantum mechanics does not describe 'things as they really are' since, according to this theory, there are no things in an absolute sense. Even macroscopic objects are correlated by Einstein–Podolsky–Rosen–correlations. A description corresponding to our inborn pattern-recognition mechanism and common-sense conceptions is possible only if such Einstein–Podolsky–Rosen–correlations are declared as irrelevant. Such a demand is not unreasonable because *without abstractions there is no science.* Every scientific description depends on the decision which effects we consider to be relevant and which effects we decide to ignore. Nevertheless, quantum mechanics allows a *contextual* realistic interpretation, provided we do not claim that matter is *made out* of elementary particles (like electrons). We have to use the more judicious formulation that the material reality can be described–under appropriate circumstances–in terms of elementary systems. Yet it is an *objective property of the material reality* that it can manifest itself under pertinent experimental conditions in a way that is best *described* in terms of elementary systems.

In scientific theories, the *problem of realism* is the question of the ontological status of the material reality while it is not observed. Since the existence of an external reality is not provable with the means available to science, we have to consider *realism as a purely metaphysical regulative principle*, free from any experimentally testable physical content, and without presupposing a particular compartmentalization of the material world. Furthermore, the investigation of the role of potential and actualized properties of physical objects is the business of physics, not of philosophy. In classical physics we are allowed to posit that all potential properties are always actualized but a priori there are no reasons to assume that such a convention is always logically possible.

The *scientific problem* is *not* to prove the existence of an independent reality, but to show that an appropriate regulative principle concerning a reality existing independently of human experience is *useful and compatible with the formalism of a fundamental scientific theory like quantum mechanics, together with all experimental results.* Moreover, the concept of realism should not be combined with structures taken over from classical physics or with specific physical ideas like atomism, localizability, separability, or determinism. I will adopt the following characterization of realism:

(i) *There exists a material world which is independent of our awareness of it.*

(ii) *Our knowledge of the material reality depends also on occurrences external to our consciousness.*

(iii) *Physical theories refer to some intrinsic aspects of the material reality.*

Note that in this characterization, realism does neither assert nor deny the existence of any kind of objects. Furthermore, it is not denied that *some* features of the observable aspects of the material reality may be due to our mental organization. In fact, we have to expect that common-sense descriptions of the outer world always depend on the psychic properties of the observer.

In the framework of theories which include the engineering domain, it is most reasonable to add the following regulative principle:

> *If a universally valid physical theory is restricted to the domain of engineering science, then the adopted realistic interpretation should cum grano salis give the every-day realism of the engineering world.*

Quantum mechanics of mesoscopic, macroscopic and open systems

Practical quantum mechanics–as used by the working scientists–is *not* based on a rigorously specified axiomatization but on some not too well defined 'first principles' and a bunch of working rules. The historical Hilbert-space formalism–as introduced by von Neumann [3] in his book of 1932–is limited to locally compact phase spaces. That is, this theory is restricted to strictly closed systems, and does not, for example, allow a mathematically proper description of the interaction of a charged particle with its electromagnetic field (which is a system having infinitely many degrees of freedom). As a consequence, the axiomatic Hilbert-space formalism does not include genuinely irreversible processes or the possibility of symmetry breakings. An important instance of the breakdown of a fundamental physical symmetry is the emergence of classical observables, that is, observables which commute with all observables and behave like observables in classical mechanics[2]. Von Neumann's Hilbert-space codification is based on the Stone–von Neumann uniqueness theorem for the representations of the canonical commutation relations. It is a simple corollary of this theorem that for finitely many degrees of freedom there exist no spontaneous symmetry breakings and no classical observables. No philosophical conclusions can be drawn from the fact that the traditional Hilbert-space codification cannot explain these features. The resolution is almost trivial: *The uniqueness theorem by Stone and von Neumann says that symmetry breakings and classical observables are impossible in this unnecessarily restricted codification.* That is, von Neumann's Hilbert-space formalism is not an adequate codification of quantum mechanics considered as a universally valid theory. Its straightforward generalization–the Fock-space quantum field theory–is theoretically inconsistent. Clearly, one should not try to conceive a realistic interpretation of quantum mechanics on the basis of a codification which is unable to explain mesoscopic and macroscopic physics. Fortunately, *there is no reason to identify quantum mechanics with the historical Hilbert-space or Fock-space codifications.*

If we consider quantum mechanics as *universally valid* in the atomic, molecular, mesoscopic and engineering domain, then we have to require that a proper mathematical codification of this theory must be capable to describe all phenomena of molecular and engineering science. Already rather small molecules can have classical properties, so that a classical behavior is *not* a characteristic property of large systems. The existence of molecular superselection rules and of molecular classical observables is an

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[2] Note that the classical aspects of quantum systems have nothing to do with the limiting behavior when Planck's constant $\hbar$ can be regarded as "small". Classical quantum systems depend in an essential way on Planck's constant but nevertheless obey the laws of classical mechanics.

empirically well-known fact in chemistry and molecular biology. The chirality of some molecules, the knot type of circular DNA-molecules, and the temperature of chemical substances are three rather different examples of molecular classical observables. Such empirical facts can be described in an ad hoc phenomenological manner, but it is not so easy to explain these phenomena from the first principles of quantum mechanics. A universally valid theory of matter has not only to describe but also to *explain* why the chirality of biomolecules (like the L-amino acids, the D-sugars, lipids, or steroids) is a *classical* observable. The reality of this breakdown of the superposition principle of traditional quantum mechanics on the molecular level is dramatically demonstrated by the terrible Contergan tragedy which caused many severe birth defects. Contergan was the trade name of the drug thalidomide (3–phtalimido–2,6–dioxopiperidin, $C_{13}H_{10}N_2O_4$) which exists in two enantiomeric forms. The left-handed stereoisomer of thalidomide is a powerful and maybe safe tranquilizer, but the right-handed isomer is a teratogenic agent, causing disastrous physiological deformities in the developing embryo and foetus [4].

In the *engineering domain*, quantum theory must in principle be able to provide a description of measuring instruments and of our general experimental laboratory equipment. Therefore, a full-grown codification of quantum mechanics must include the successful engineering theories like classical point mechanics, chaotic nonlinear dynamical systems, continuum mechanics, hydrodynamics, classical stochastic processes, thermostatics including phase transitions, Maxwell's electrodynamics, Newton's gravitation. In the *mesoscopic domain* manifestations of both quantal and classical properties at one and the same object are nothing out of the ordinary, but they cannot be understood by some "correspondence rules"; their description requires a full-blooded theory which includes both traditional quantum mechanics *and* classical mechanics as special cases. For example, DNA-molecules–the material carrier of genetic information– possess important properties which definitely require a quantum-mechanical description, e.g. its photochemical reactivity. On the other hand, every DNA-molecule has a tertiary structure which is manifestly classical, and biologically important for the mechanism of genetic recombination. Moreover, circular DNA-molecules may be knotted, and there are enzymes which can change their knot-type. The knot-type of a DNA-molecule is an example for a classical property which cannot be explained by any variant of a "correspondence principle". Molecular biology is a rich source for such mixed quantal-classical systems. *Enzymes* act as molecular measuring devices and require a classical behavior for their function. The *immune system* is a molecular quantum system with an only classically describable memory, warranting the individual molecular identity. In order to understand such systems, one needs a theory of matter which can describe both the quantal and the classical properties of single individual objects. Since the cross-over from quantum to classical behavior is not given by Bohr's correspondence principle, one of the most important theoretical problems in molecular quantum mechanics is the correct analysis of the interaction of an individual, small, non-isolated quantum object with its environment and with classical degrees of freedom.

In every scientific investigation we divide the universe into an *object system* and its *environment*-which is all the rest. The environment acts as *background* which is indefensibly neglected in historical quantum mechanics. The idea of a physical object without an environment is an outrageous and incongruous abstraction. Eddington, in his posthumous book *Fundamental Theory*, called attention to the inevitability of considering the background: "The environment must never be left out of consideration. It would be idle to develop formulae for the behaviour of an atom in conditions which imply that the rest of matter of the universe has been annihilated. In relativity theory we do not recognise the concept of an atom as a thing complete in itself. We can no more contemplate an atom without a physical universe to put it in than we can contemplate a mountain without a planet to stand on" [5,p.13]. Therefore, the abstract structure of a tough-minded theory must be rich and complex enough to describe the essential features of the environment of an object under study.

A complete, mathematically rigorous and empirically correct theory of *open quantum systems* and of *mesoscopic* and *macroscopic quantum systems* is still a great desideratum, but it seems that most mathematical tools are available in terms of *algebraic quantum mechanics.* Algebraic quantum mechanics is *not a new*, but just a physically and mathematically correct formulation of quantum theory; it is nothing else but a proper codification of the basic principles of quantum mechanics. No ad hoc modifications, no hidden variables, and no quantization procedures are necessary. Algebraic quantum mechanics encompasses all kinds of physical systems, e.g. finite systems (with a locally compact phase space) and infinite systems (whose phase space is not locally compact). There is a dramatic difference between the behavior of finite and infinite systems. According to the uniqueness theorem by Stone and von Neumann, finite systems have a unique Hilbert-space representation while infinite systems have infinitely many *physically inequivalent* W^*-representations which account for the stupendous complexity of observable phenomena in nature.

In the framework of algebraic quantum mechanics, it can be proven that, in general, open quantum systems undergo symmetry breakings and possess *classical observables.* Contextual classical observables are *emergent* in the sense that they are generated by the algebra of intrinsic observables together with a new contextual topology, but they are not functions of the intrinsic observables [6–10]. A typical example for an emergent classical observable is the *temperature* of systems in thermal equilibrium. It turns out that the contextual classical part of a dynamical quantum system is always a *stochastic* dynamical system, it depends in an essential way on Planck's constant but nevertheless obeys the laws of classical mechanics. In addition, the emergence of classical observables does *not* depend on the macroscopic character of the system under investigation, already rather small molecules can have classical properties.

Endophysical and exophysical descriptions

Certainly, present-day quantum mechanics is not the ultimate theory of matter. But even if we had a truly universal ultimate theory it would not give us all the information we need to describe an observed phenomenon. That is, the statement "universally valid" cannot be literally correct since a language which encompasses everything would have to be semantically closed, and hence engender antinomies. The impossibility of a complete description is not a flaw of the theory but a logical necessity. Every theory which attempts to describe its own means of verification is necessarily self-referential. In order to avoid paradoxes of self-reference, we need an at least two-leveled theory where the second level represents the metatheory which must be formulated in another language, a so-called metalanguage. This metalanguage has to be essentially richer than the language of the basic physical theory. If the two languages would be identical (or translatable into each other) we would have a semantically closed language with self-referential sentences [11, 12]. So we have to split the world into two parts, the observed part and the observing part. Our description depends on this cut but *this cut cannot be derived from any kind of an ultimate theory.* Hence the language of a hypothetically posited universal theory can at most describe a *part* of the full reality, perhaps even only a tiny area. Traditionally, the physical sciences exclude the subject of cognizance from their enquiry. No known physical theory deals with the reality of man in his freedom.

In the following I adopt the working hypothesis that quantum mechanics in its algebraic codification is *universally valid* in the atomic, molecular, mesoscopic and non-cosmological macroscopic domain. Our confidence in the trustworthiness of quantum mechanics as a fundamental physical theory is in an essential way based on its confirmation by laboratory experiments. That is, both the validity of engineering physics and the feasibility of experimenters having free will is *presupposed*, and not derived from the first principles of quantum theory. I do not assume that consciousness or free will can be reduced to physical properties of the organism such as brain states. All ideas of choice and purpose must be included in the relevant *regulative principles* which are not derivable from physical first principles. Clearly, such regulative principles play a central role in our picture of the world. These postulates lead to the necessity to distinguish between endophysics and exophysics. This helpful distinction has been made by Otto Rössler [13] and David Finkelstein [14, 15]. Probably misusing their ideas, I adopt nevertheless their way of speaking:

> *A strictly closed physical system without any concept of an observer is called an endosystem.*
>
> *If the endoworld is divided into an observing and an observed part, we speak of an exophysical description.*
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> *The world of the observers with their communication tools is called an exosystem.*

Note that endophysics is different from exophysics. *All fundamental universally valid first principles we know refer to strictly closed systems, hence belong to endophysics.* They are supposed to be universally valid, but they are not operational. Strictly speaking, there is nothing outside an endosystem. The endophysical description is a view without perspective, it is God's panorama, a *"view from nowhere"*.

Already the *formalism* of quantum mechanics predicts that quantum systems like electrons, atoms or molecules are always *entangled* with the rest of the world, so they cannot be possible candidates for individual entities which "really exist". Provided we accept quantum theory as a holistic theory, *a consistent variant of scientific realism cannot postulate an independent existence of building blocks like strings, quarks, electrons, atoms or molecules. We construct* building blocks to describe matter from a particular point of view, but the world is *not made out of* some building blocks. This insight is not in contradiction with the view that quantum mechanics is a story about what there really is. Objectivity does not reside in transcendental entities like molecules, atoms, electrons or quarks, these are just *manifestations* of the material reality. On a fundamental level, we have to emphasize different aspects like symmetries.

First principles are not natural laws but fundamental ideas. To a certain extent it is a matter of taste what we consider as first principles and what as pragmatic working rules. As far as possible and appropriate, first principles should be context-independent. For that reason, first principles are always extravagantly remote from our every-day experience. All popular first principles refer to situations with high intrinsic symmetry. Experience tells us that symmetry is an effective criterion for selecting first principles so we adopt the view that *maximal symmetry* is a typical characteristric of an *endophysical first principle.* Such fundamental symmetries are, as a rule, not manifest in the everyday domain. So it is necessary to break these symmetries, as clearly recognized by Pierre Curie [16]: "C'est la dissymétrie qui crée le phénomène". That is, genuine endophysical symmetries are directly inaccessible by experience, they can empirically be found only by exophysical symmetry breakings. On that account we consider all laws or rules showing *broken symmetries* to be contextual and belonging to a particular *exophysical description.* For example, for endophysics we posit a bidirectional deterministic time evolution distinguished by a time-inversion symmetry, while the *arrow of time* of most exophysical descriptions manifests a broken time-inversion symmetry.

Quantum *endo*physics cannot predict what happens in a physical experiment, since in an endoworld there is not yet any concept of observing tools or observers. It is a strictly deterministic theory, set up to describe the reality existing independently of human observations. Note that the fact that quantum endophysics is deterministic does not imply that it is *determinable* by an internal or an external observer. The *endophysical description refers to an immanent ontology,* it pictures an *independent reality* in a non-operational way. Every operational description of the world requires the transition from the *endophysical* to an *exophysical* description by introducing a cut between the observed and the observing part. The *exophysical description* refers to the *empirical reality* in the sense of d'Espagnat [17, 18]. Yet, *the endophysical first principles are not sufficient for a characterization of exosystems* since every exophysical description depends not only on first endophysical principles but also on the choice of the cut. This fact does not imply that we cannot go from endophysics to an exophysical description, but that for such an enterprise we need additional *regulative principles.* Every exophysical description is therefore *contextual* and at most *weakly objective* (in the sense of an intersubjective agreement of observers choosing the same cut).

The inverse problem is building up a picture of the world independent of the perceiving subject from experimental data, or in our terminology, a logically consistent reconstruction of conjectured endophysics from the operationally accessible exophysical descriptions. The theoretical construction of an endophysically immanent ontology can be considered as a *realization problem.* That is, we are asking for an ontically interpreted theoretical structure which, together with appropriate regulative principles, allows us to *derive* all legitimate exophysical description of all aspects of the material reality encompassed by the basic theory. This realization problem is, in the main, a *consistency problem.* If it has a solution, it has many solutions. We can reduce this nonuniqueness by some *minimality requirements* (Ockham's razor) and by adopting an ontology whose restriction to the engineering domain gives the realism almost universally adopted in classical physics. Therefore, endophysics never can be a literally true story of what the world is like. An endophysical conception of reality must be compatible but cannot be derived from empirical data. In the words of Albert Einstein: " 'Being' is always something which is mentally constructed by us, that is, something which we freely posit (in the logical sense). The justification of such constructs does not lie in their derivation from what is given by the senses. Such a type of derivation (in the sense of logical deducibility) is nowhere to be had, not even in the domain of pre-scientific thinking. The justification of the constructs which represent 'reality' for us, lies alone in their quality of making intelligible what is sensorily given ..." [19, p. 669].

On interpretations

An interpretation always refers to a logically consistent and empirically well-confirmed theoretical formalism. That is, we assume that we have a mathematically rigorous codification of a physical theory (the 'formalism'), a minimal interpretation of the theory which allows an operationalization and an empirical verification of the theoretical predictions. We adopt the following definition:

> *An interpretation of a physical theory is characterized by a set of normative regulative principles which can neither be deduced nor be refuted on the basis of the mathematical codification and the minimal interpretation.*

Since theories are not determined by their empirical consequences, we have some freedom for choosing an interpretation. First of all, we distinguish between epistemic and ontic interpretations. *Epistemic interpretations* refer to our knowledge of the properties or modes of reactions of systems "as we perceive them", while *ontic interpretations* refer to the properties of the "object in itself", regardless of whether we know them or not, and independently of any perturbations by observing acts. An ontic interpretation of quantum mechanics makes assertions about values *possessed* by observables. A realistic world view demands an individual ontic interpretation of *quantum endophysics*, it is intrinsically objective but not operational. The operationalistic view requires an exophysical epistemic interpretation, and usually works with a statistical description. By a proper choice of the regulative principles, one can get a contextually objective and operational *exophysical* description of quantum reality.

To be sure, an ontic interpretation of quantum mechanics does refer only to a fictitious *theoretically immanent reality*, and not to the *ultimate reality.* But under the *working hypothesis*–which nobody really believes–*that quantum mechanics is a universally valid theory*, an ontic interpretation allows us a consistent way of speaking *as if* we would refer to reality.

Individual and statistical descriptions of quantum systems

Both individual and statistical descriptions of material reality are possible, but the appropriate mathematical formulations are fundamentally different. Moreover, a coherent statistical interpretation requires an individual interpretation as a backing. In classical theories this requirement is automatically fulfilled since the convex set of all statistical states is a simplex so that a *unique* decomposition of every mixed state into pure states is warranted. In quantum theories, a mixed state has many feasible realizations in terms of pure states so that it is not at all clear what the *conceptual* meaning of a statistical state is. On the other hand, a complete individual interpretation is always in terms of *ontic* states, mathematically described by pure states. The solution of the equation of motion for this pure state requires a knowledge of the initial conditions of all degrees of freedom of the whole environment. From an experimental point of view, this information is never available so that we are forced to introduce an *epistemic* state by some kind of optimal estimate of the initial conditions of the environment. This procedure leads to a well-defined mixture in terms of ontic states, hence to a conceptually well-defined statistical state. These statistical states are epistemic states, *they refer to our knowledge of the ontic state.*

The usual mathematical formalism of quantum mechanics refers to a *statistical* description, and one would be ill-advised to use this mathematical formalism also for the individual description. *The mathematical formalism required for an individual description is different from the formalism required for a statistical description.* In classical point mechanics, the usual individual description is given in terms of a symplectic phase space Ω, where the individual state of the system at time t is given by a point ω_t of Ω. According to Gelfand's representation [20, p.16], there is a one-to-one correspondence to the algebraic description in terms of the C^*-algebra $\mathcal{C}^\infty(\Omega)$ of continuous functions on Ω which vanish at infinity. In this algebraic description the individual states are given by the extremal elements of the dual of $\mathcal{C}^\infty(\Omega)$. The statistical description of the same mechanical system can be formulated in terms of probability densities, that is of positive and normalized elements of the Banach space $L^1(\Omega)$. The dual of this Banach space is the W^*-algebra $L^\infty(\Omega)$ of bounded Borel-measurable functions on Ω, and is called the algebra of bounded observables. Just as in classical point mechanics, the individual description of an arbitrary quantum system can be given in terms of an appropriate separable *C^*-algebra* $\mathcal{A}$, where the individual states are represented by the extremal elements of the dual $\mathcal{A}^*$ of $\mathcal{A}$. The statistical description of a quantum system has to be given in terms of an appropriate *W^*-algebra* $\mathcal{M}$ with a separable predual $\mathcal{M}_*$. In quantum mechanics the algebras $\mathcal{A}$ and $\mathcal{M}$ are in general noncommutative. In the special case of commutative algebras we speak of classical quantum systems, and we can represent these algebras as in historical classical mechanics by $\mathcal{A} = \mathcal{C}^\infty(\Omega)$ and $\mathcal{M} = L^\infty(\Omega)$, where $\mathcal{M}_* = L^1(\Omega)$.

Ontic interpretation of endo–quantum mechanics

While quantum phenomena require a radical revision of our ideas about physical reality, they do not prevent us from accepting a reasonable realistic individual interpretation. For this we do not require any kind of hidden variables, faster-than light influences, or an exotic continuously splitting many-worlds description. Quantum mechanics does not force us to give up realism, but it forces us to distinguish carefully between *potential* and *actualized* properties. It is a misconception (though one surprisingly widespread among philosophers and scientists) that physical quantities have to be truth-definite. A popular working rule of pragmatic quantum mechanics says that "an observable has *no value* before a measurement"[3]. This is in contrast to the usual metaphysical commitment of classical mechanics that every observable *has* a value at all times. This commitment cannot be transferred to quantum mechanics since there is a theorem saying that for a full set[4] of states of a C^*-algebra $\mathcal{A}$, a hypothetical attribution of definite truth values to *all* elements of $\mathcal{A}$ requires that $\mathcal{A}$ is commutative[5]. However, instead of a positivistic renouncement we can adopt the intrinsic, internally consistent

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[3] Of course, a positivist would not say so much. For example, Reichenbach adopts the following definition: "In a physical state not preceded by a measurement of an entity u, any statement about a value of the entity u is meaningless" [21].

[4] A set $\mathcal{S}$ of states on a C^*-algebra $\mathcal{A}$ is said to be *full* if an element A of $\mathcal{A}$ satisfies $A \geq 0$ if and only if $\rho(A) \geq 0$ for all $\rho \in \mathcal{S}$.

[5] The relevant basic theorem is due to Misra [22]: A C^*-algebra $\mathcal{A}$ (different from the complex numbers) admits a dispersion-free state if and only if it has a nontrivial norm-closed two-sided ideal $\mathcal{I}$ such that the quotient algebra $\mathcal{A}/\mathcal{I}$ is commutative. This theorem implies that in traditional quantum mechanics there are no states which are dispersion-free for *all* observables.

ontic interpretation that at every instant there is a maximal set of truth-definite observables. A truth-definite observable possesses a value–whether we know this value or not, is at this stage of the theoretical discussion entirely irrelevant. This point of view corresponds exactly to the usual interpretation of classical point mechanics, where the ontological question of 'having a value' is clearly separated from the entirely different question how to get empirically some information about this value.

The natural referent for quantum endophysics is a *single system.* A *statistical* interpretation of quantum mechanics presupposes the existence of an *external* measuring system with a *classical irreversible dissipative* behavior, so that it is a topic of quantum *exo*physics. Therefore, a statistical interpretation of quantum endophysics makes no sense, but a non-operational and *intrinsically nonprobabilistic individual ontic interpretation* is possible in a logically consistent way. Algebraic quantum mechanics allows to give a precise definition of an ontic interpretation which is free of inner contradictions. In algebraic quantum mechanics, quantum endophysics is characterized by a C^*-algebra $\mathcal{A}$ of intrinsic observables. The referent of an endophysical ontic interpretation of quantum mechanics is the whole universe of discourse. The *intrinsic potential properties* describe independently of any observation what is physically real, they are represented by the selfadjoint elements of the C^*-algebra $\mathcal{A}$ of intrinsic observables. The *intrinsic ontic state* of an object at time t is characterized by the set of all intrinsic potential properties which are actualized at the instant t. That is, *the intrinsic potential properties characterize the object, while the actualized intrinsic properties characterize the ontic state of the object.* An ontic state can be represented by a positive linear functional and is characterized by the fact that there are no other linear functionals with the same collection of actualized observables. It can be proved that there is a one-to-one correspondence between the ontic states of an object and the *extremal*, normalized positive linear functionals on $\mathcal{A}$ (the so-called 'pure states').

> *Mathematical supplement*
> A selfadjoint operator $A \in \mathcal{A}$ is said to be *dispersion-free* with respect to a state $\rho \in \mathcal{A}^*$ if $\rho(A^2) = \rho(A)^2$. In this case, the observable A is said to *possess* the value $\rho(A)$ with respect to a state ρ. The set of all observables on which a state $\rho \in \mathcal{A}^*$ is dispersion-free, is called the *definite set* $\mathcal{D}_\rho$ of ρ [23],
>
> $$\mathcal{D}_\rho := \{A \in \mathcal{A} \mid A = A^*,\ \rho(A^2) = \rho(A)^2\}.$$
>
> The complex span $\mathcal{A}_\rho$ of the definite set $\mathcal{D}_\rho$
>
> $$\mathcal{A}_\rho := \{A + iB \mid A, B \in \mathcal{D}_\rho\}$$

> is a C^*-algebra with the property [24]
>
> $$\mathcal{A}_\rho := \{A \in \mathcal{A} \mid \rho(AB) = \rho(BA) = \rho(A)\rho(B) \text{ for all } B \in \mathcal{A}\}.$$
>
> We require that $\mathcal{A}_\rho$ is a maximal set of observables which at some instant t possess values, that is we require that the definite set $\mathcal{D}_\rho$ is maximal in the sense that
>
> $$\mathcal{D}_\rho \subseteq \mathcal{D}_\varphi \text{ for some state } \varphi \in \mathcal{A}^* \text{ implies } \rho = \varphi.$$
>
> If $\mathcal{A}$ is a C^*-algebra with identity and with no one-dimensional representation, then a state ρ on $\mathcal{A}$ is pure if and only if its definite set $\mathcal{D}_\rho$ is maximal [25].
>
> The ontic interpretation of a dynamical C^*-system presupposes that at every instant $t \in \mathbb{R}$ there is a maximal definite set $\mathcal{D}_t$ of observables. The corresponding complex span $\mathcal{A}_t \subseteq \mathcal{A}$ defines a unique C^*-homomorphism $\rho_t : \mathcal{A}_t \longrightarrow \mathbb{C}$ which we interpret as a valuation map for the observables that are actualized at the instant t. Any observable $A \in \mathcal{A}_t$ possesses at time t the dispersion-free value $\rho_t(A)$. The functional ρ_t has a unique state extension to an extremal, normalized positive linear functional on the C^*-algebra $\mathcal{A}$ [24]. This uniquely given pure state is called the *ontic state* of the C^*-system at the instant t.

That is, ontic states are represented by (and identified with) pure states. It follows that an intrinsic potential property represented by an observable $A \in \mathcal{A}$ is actualized at time t if and only if $\rho_t(A^2) = \{\rho_t(A)\}^2$ where the extremal normalized positive linear functional $\rho_t \in \mathcal{A}^*$ represents the *ontic state* at time t. This delineation fixes the *ontology* of quantum endophysics. Our reference to an *independent reality* makes only sense as a *theoretical construct*. The *intrinsic ontic interpretation* is a *strongly objective* theory in the sense of d'Espagnat [18] since in the first place it makes no reference to observers or probabilities. It may describe reality in itself *but not the phenomena we observe.* The restriction of this ontic interpretation of algebraic quantum mechanics to the classical part of the system[6] corresponds to the generally adopted realistic individual interpretation of the traditional classical physical theories. Hence the adopted immanent ontology is not radically different from the ontology traditionally accepted for classical physical theories.

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[6] The classical part of a C^*-system with the C^*-algebra $\mathcal{A}$ is given by the center $\mathcal{Z}(\mathcal{A})$ of $\mathcal{A}$. The C^*-system with the commutative C^*-algebra $\mathcal{Z}(\mathcal{A})$ is a classical quantum system.

Epistemic interpretation of exo–quantum mechanics

A theory which describes observable phenomena cannot keep the human means of data processing out of consideration, but these means are not described by the C^*-algebra of intrinsic observables. The observables which describe the outcomes of measurements are context-dependent, they are represented by *positive operator-valued measures* of the W^*-algebra $\mathcal{M}$ of *contextual observables.* This algebra is not intrinsically given but can be *constructed* from the context-independent C^*-algebra $\mathcal{A}$ by a faithful Hilbert-space representation $\pi(\mathcal{A}) \subseteq \mathcal{B}(\mathcal{H})$ of $\mathcal{A}$ by specifying a new contextual topology by selecting a *folium of contextually preferrend intrinsic states.* The weak closure of the C^*-algebra $\pi(\mathcal{A})$ acting on the Hilbert space $\mathcal{H}$ is W^*-isomorphic to the W^*-algebra $\mathcal{M}$ of contextual observables. In this contextual description, the statistical states are represented by the *normal* positive linear functionals on the W^*-algebra $\mathcal{M}$.

The W^*-algebraic formalism describes the *empirical reality,* it is context-dependent hence only *weakly objective,* in the sense that for a given context there is intersubjective agreement[7]. While the nonoperational individual and ontic interpretation is *fully deterministic* and intrinsically richer than an exophysical statistical description, any of the possible operational exophysical statistical descriptions is necessarily contextual but without exceptions *irreducibly probabilistic.* The primary probabilities of quantum mechanics [26] manifest themselves only in the interaction with *external classical systems.*

Our ability to describe the world cannot go farther than our ability to isolate objects. A realistic operational description of quantum systems is possible if and only if there are no Einstein–Podolsky–Rosen–correlations between the object system and the observing system. Only if we can abstract deliberately from these factually existing Einstein–Podolsky–Rosen–correlations, we can investigate the material world by compartmentalization. A realistic description of an individual quantum system is possible if and only if there are no Einstein–Podolsky–Rosen–correlations between the object system and its environment. Therefore I adopt the following definition of an object [27–32]:

> *An object is defined to be an open quantum system, interacting but not Einstein–Podolsky–Rosen–correlated with the environment.*

It follows that objects are exactly those quantum systems for which at every

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[7] The same is true for the quantum-logics approach. The corresponding orthomodular lattice is given by the projection lattice of the contextual W^*-algebra. A representation-independent description (corresponding to the C^*-algebra of intrinsic observables) does not exist in quantum logics.

instant a maximal description in terms of pure states is possible. *An object is something having individuality and potential properties*, so that we can interpret a pure quantum state of an object as an individual state. Here the notion of an 'individual state' refers to a mode of being, describing exophysical characteristics existing independently of any observation, while the notion of a 'pure state' refers to a merely mathematical concept, meaning an extremal positive linear functional on the algebra of observables. Note that the exophysical individual state depends on the breaking of the holistic symmetry of the world by division and abstraction. Over and above, every exophysical description requires a *tensor-product decomposition* but such a decompositition is not God-given. The usual Hamiltonian tensor-product structure refers to *bare* particles and to *bare* fields whereas the object–environment tensor-product structure refers to contextual *dressed* entities. A contextual quantum object appears as an object *not in spite*, but *because* it interacts with its environment. In particular, classical properties are the result of the interaction of an object with its environment. *Without an appropriate background the concept of a quantum object makes no sense.*

It would be unreasonable to expect that the dynamics of an exosystem is governed by a Hamiltonian or a bidirectionally deterministic time evolution. This dynamics cannot be postulated but has to be derived from the intrinsic endophysical time evolution. In an exophysical description, it is in principle possible to eliminate the environmental variables and to write down the dynamics of an individual object in terms of the object observables alone. In general, this reduced dynamics is given by a *stochastic* and *state-dependent* equation of motion. Both the stochastic behavior and the state-dependence have not to be put in by hand, but they can be *derived* from the fundamental *linear* endophysical dynamics. The chaotic behavior arises from the initial values of the unobserved degrees of freedom of the environment, resulting in a stochastic classical force acting on the object. If the spectral distribution of the autocorrelation of this force is absolutely continuous, then the environment forgets the initial conditions completely so that the stochastic force is usually *completely nondeterministic.* The state-dependence is due to feedback effects from the polarization of the environment by the quantum object. If the dynamics of this individual quantum object can be represented in terms of the irreducible Hilbert-space formalism, then the dynamics of the ontic state can be representend by a trajectory $\Psi \mapsto \Psi_t$ of the state vector whose time evolution is given by a *nonlinear stochastic integro-differential equation* for the state vector Ψ_t. In particularly simple models, one gets a nonlinear stochastic Schrödinger equation in the sense of Itô.

All objects we discuss in empirical science are *contextual objects*, their existence depends both on the environment, and on the abstractions we are forced to make in every scientific discussion. It is a theorem of algebraic quantum mechanics that an object exists only if its environment is classical[8]. The meaning of the notion 'classical' depends, however, on our abstractions and is therefore context-dependent. That is, in a quantum world there are no intrinsic context-independent objects besides the whole universe of discourse. Contextual objects are abstraction-dependent, but they are not free inventions. They represent *patterns of reality,* yet they are *not* building stones of reality. Elementary or composed "particles" like electrons, atoms or molecules are not primary but rather secondary and derived. Electrons, atoms or molecules do not simply *exist,* they appear only under special conditions–they are *contextual* systems.

In order to go from the universally valid endophysical description to a contextual exophysical description, one has to introduce in addition *regulative principles like* the Baconian rejection of the existence of final processes, our presupposed freedom to create initial conditions, or the feasibility of "detached observers". The chosen observational tools determine a certain context which in algebraic quantum mechanics is characterized by a *new topology in the space of the intrinsic states*[9]. An exophysical description of contextual objects cannot give us complete knowledge of the endophysical independent reality. Contextual objects depend on the contextually selected topology but are independent of a human consciousness, *they are real relative to the chosen context.* An exophysical description is neither absolutely true nor absolutely false, but we may say that it is correct *relative to the chosen way of describing reality.* Yet exophysical descriptions are not unique, they depend on the neglect of some really existing Einstein–Podolsky–Rosen–correlations. Therefore there are always different exophysical descriptions which according to purely endophysical criteria are logically equivalent. No single exophysical description reveals the whole independent reality with its non-Boolean event structure but projects *some aspects* of this reality onto a *Boolean context.* The material reality has many complementary Boolean descriptions, each being valid from its own perspective. There is only one reality, yet there are many legitimate viewpoints, hence many equally legitimate but complementary descriptions of nature.

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[8] *Theorem: Let $\mathcal{A}$ and $\mathcal{B}$ be two C^*-algebras and $\mathcal{C} = \mathcal{A} \otimes \mathcal{B}$ their minimal tensor product. Every pure state γ on $\mathcal{C}$ is of the form $\gamma = \alpha \otimes \beta$ for some pure states α of $\mathcal{A}$ and β of $\mathcal{B}$ if and only if either $\mathcal{A}$ or $\mathcal{B}$ is commutative* ([20], theorem 4.14). This theorem implies that a nonclassical open C^*-system is an object if and only if its environment is classical. Clearly, every classical C^*-system is an object.

[9] This new topology is different from the intrinsic C^*-topology and can also be characterized by a *folium* of preferred states which in turn characterize the *normal* states of the W^*-closure of the associated Hilbert-space representation.

Conclusions

Except from the fact that present-day physics has nothing to say about the relation between matter and spirit and is not in the position to avoid the Cartesian split, one of the most important open problems of nonrelativistic quantum theory is the proper description of individual open quantum objects in interaction with their environment. This is mainly a problem of *mathematical physics*, not of philosophy. If we are able relinquishing untenable presuppositions and if we accept the holistic structure of the material reality, the philosophical problems associated with quantum mechanics are not radically different from those of science in general. It is not realism that is refuted by quantum mechanics, but atomism and the idea of the existence of context-independent objects. The context-dependence of every description of reality is inevitable, even in classical physics; it is enforced by Tarski's theorem which implies the necessity of an *exophysical metalanguage.* Due to entanglement effects, individual quantum objects are always abstraction-dependent entities. Contextual objects represent *patterns of reality,* yet they are *not* building stones of an independent reality. According to quantum theory, *a consistent variant of scientific realism cannot postulate an independent existence of building blocks like quarks, electrons, atoms or molecules.* The non-Boolean event structure of quantum reality forces us to give up the classical idea that all potential properties of a quantum object can be actualized at the same instant. The nonseparability and nonlocality of the material world are not compatible with the ontology adopted in classical physics. Due to its holistic nature, quantum reality is more elusive and leads to an amazing variety of complementary descriptions.

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