The Unnameable

Limits of Language in Early Analytic Philosophy

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A thesis submitted for the degree of Doctor of Philosophy

2015

Contents

Acknowledgements

My foremost thanks go to my primary supervisor, Adrian Moore. For four years, my thesis and I have benefited enormously from his encouragement, sympathetic conversation, depth of insight, and astonishing breadth of learning; I am extremely grateful to him. My second thanks go to my secondary supervisor, Timothy Williamson, whose thoughtful, immensely astute and always constructive supervision has taught me a huge amount and greatly improved my work. It's been a privilege to be a student of Adrian and Tim's.

I've had the good fortune to discuss the ideas and issues of this thesis with many friends, teachers, fellow graduate students, and faculty members, in Oxford and elsewhere. These include Luke Brunning, Bill Child, Neil Dewar, Paul Egré, Robert Ellis, Viryanaya Ellis, Rachel Fraser, Peter Fritz, Peter Hacker, Louise Hanson, Edward Harcourt (who kindly acted as examiner for my transfer of status), Natalia Waights Hickman, Daniel Isaacson, Cameron Domenico Kirk-Giannini, Ieuan Lloyd, Ofra Magidor, Stephen Mulhall (who generously acted as examiner for both my transfer and confirmation of status), Beau Mount, Yuuki Ohta, James Openshaw, Alexander Roberts, James Studd, Trevor Teitel, Andy Yu, and Andy West.

My special thanks go to Robert Trueman, whose work has provided much stimulus for this thesis. I've gained a great deal from corresponding with Robert on the issues surrounding this thesis and from his extremely insightful comments on several drafts.

I'm grateful to those who have attended talks I have given at Leeds, Salzburg, and the Welsh Philosophical Society, as well as audiences in Oxford at the Ockham Society, the DPhil Seminar,

a CRNAP workshop at Queen's College, and my lectures on Frege during Hilary Term 2015.

I'm indebted most of all to my parents, who have been there for me in so many ways. Finally, I thank Lisa Clausen for sharing the experience of doing a DPhil with me, and for enriching it so much with her love and support.

Abstract

This thesis concerns the view, shared by Frege, Russell and the early Wittgenstein, that there are entities that cannot be named. Chapter 1 clarifies the particular form this commitment takes in the work of these three authors. The chapter also details a distinctive cluster of philosophical difficulties attending the view certain entities are unnameable, and explores the relation between unnameability and inexpressibility. The remaining chapters are devoted to investigating what grounds there are for countenancing the unnameable. The particular focus throughout is Frege's thesis that *concepts* cannot be named. Chapters 2 and 3 are devoted to giving a detailed hearing to two arguments for Frege's thesis distinguishable in the *locus classicus*, 'On Concept and Object'. The first argument concerns the relationship between co-reference and intersubstitutability; the second concerns the unity of thought. It is contended that these arguments fail to substantiate Frege's thesis. Chapters 4 and 5 examine two further arguments for Frege's rejection of singular reference to concepts. The first is based upon the alleged impossibility of expressing identities between objects and concepts; the second draws on upon considerations pertaining to diagonalization and Russell's paradox. It is contended that each of these arguments can be resisted in defence of singular reference to concepts.

Word count: 74,996

To my parents

He granted for the sake of argument that some unnatural monster had really existed, but reminded me that even the most morbid perversion of Nature need not be *unnamable…*

H. P. Lovecraft, 'The Unnamable'

Chapter 1

Limits of Naming and Saying in Early Analytic Philosophy

The fact would seem to be, if in my situation one can speak of facts, not only that I shall have to speak of things of which I cannot speak, but also, which is even more interesting, but also that I, which is if possible even more interesting, that I shall have to, I forget, no matter. And at the same time I am obliged to speak. I shall never be silent. Never.

Samuel Beckett, *The Unnamable*

It is a remarkable fact about the early history of the analytic tradition that its three most important protagonists all held, at least during significant intervals of their respective careers, that there are entities that cannot be named. This shared commitment on the part of Frege, Russell and the early Wittgenstein is the topic of this thesis. My principal concern in the present chapter will be to clarify the particular form this commitment takes in the work of these three authors. I will also be concerned to detail a distinctive cluster of philosophical difficulties attending the view that certain entities that lie beyond the limits of naming. A central theme in this connection will be the relation between unnameability and inexpressibility—between the limits of naming and the limits of saying.

The unnameable is liable to be considered—though perhaps not the *ultimate abomination*, as

in Lovecraft's tale—pretty unsavoury.[1] For there is an eminently natural assumption to make about what it is possible to name: *anything*. One might, as James Shaw [Shaw, 2013, p 64] does, make the parallel point concerning the question of what it is possible to *express* in language. But in this case the answer 'anything' stands immediately in need of qualification; for there plainly are things which, though describable, are categorially unfit to be linguistically *expressed*—bricks, boats and bridges, for example. Concerning what can be named, however, no qualification of this kind seems necessary: it really is natural to assume that anything whatsoever can be named.[2] The modality expressed by 'can' here is crucial, of course: there are senses of that term—strong varieties of possibility—on which we should be hesitant to claim that everything can be named. It seems improbable, for example, that for anything at all, it is *humanly possible* to name that thing. Some things are presumably just too remote, small, undifferentiated and/ or causally insulated from us for our linguistic practices and determinations to establish them as the referent of some name.[3] Providing it is the weakest variety of possibility that is at stake, however, the assumption is compelling: nothing, it seems, is such that it is *absolutely impossible* that it be named. Nonetheless, in the work of Frege, Russell and the early Wittgenstein alike, one finds claims inconsistent with this compelling assumption.

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[1]The definite generic, 'the unnameable', which features as the grammatical subject of this sentence, is free of the odd property possessed by the same noun phrase as it occurs in Lovecraft's story, as an appellation for an amorphous monstrosity: namely, the property of being either empty or a misnomer. Incidentally, unlike Lovecraft and Beckett, I favour spelling 'unnameable' with two e's.

[2]Cf. [Anderson, 1980, p. 224]: 'And is it not obvious that every entity whatever *is* nameable?'

[3]Looming in the background here, on the one hand, is the disquieting contention that matters are no different close to home: perhaps our practices are impotent even to establish medium-sized terrestrial objects as things to which certain names determinately refer. On the other hand, matching this extreme diffidence about our power to name, it is easy to feel great confidence; for one might think that we can name anything whatsoever by simply ordaining, in Lagadonian fashion, that everything be a name of itself (see [Lewis, 1986, pp. 145-46]). Concerning the extreme diffidence, I intend to (almost entirely) bracket considerations to do with the indeterminacy of reference for the purposes of this study. Concerning the extreme confidence, I simply note that the unnameables of Frege, Russell and Wittgenstein will be (held to be) no exception to their own unnameability: they can no more be named by themselves than they can be named by anything else.

1.1 Frege

1.1.1 The *Bedeutungen* of Incomplete Expressions

Unnameability is perhaps most familiarly a feature of Frege's philosophical semantics. Frege maintained that the *Bedeutungen*—or as I shall say, the *referents*[4]—of linguistic expressions divide exhaustively and exclusively into *objects* and *functions*.

> *Objects* are opposed to functions. Accordingly, I count as *objects* everything that is not a function[.] [Frege, 1997b, p. 213]

The categories of *object* and *function* are sharply disjoint: no function is an object. The division between these two kinds of entity, Frege claims, is 'of the highest importance' [Frege, 1997h, p. 192] and is 'not made arbitrarily, but is founded deep in the nature of things' [Frege, 1997e, p. 148]:[5] 'functions are fundamentally different from objects' [Frege, 1997e, p. 146]. A special case of this distinction, which will be a central concern in subsequent chapters, is that between object and *concept*, concepts being one species of function—namely, those whose value is always a truth-value [Frege, 1997e, p. 139]. In particular, then, Frege holds that 'the properties of being a concept and of being an object are mutually exclusive' [Frege, 1997h, p. 182]: no concept is an object.

The significance, for present concerns, of Frege's thesis that functions in general and concepts in particular are not objects is that, on his conception of objecthood, it is a sufficient (and a necessary) condition for something's being an object that it be capable of being the referent of a proper name (*Eigenname*). That functions are not objects therefore entails, for Frege, that functions are *in*capable of being the referent of a proper name: functions are *unnameable*. Frege explicitly draws this consequence in the special case of concepts:

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[4]The question of how to translate the Fregean term of art, 'Bedeutung', and its cognates, 'bedeuten', 'bedeutungsvoll', 'bedeutungslos', etc., has been has been a locus of considerable controversy (for discussion see Michael Beaney's introduction in [Frege, 1997d, pp. 36-46]). The noun has been rendered variously as 'referent', 'reference', 'meaning', 'denotation', 'indication', 'semantic value', etc. I shan't enter into this controversy here, only acknowledge it and give notice of my own general policy: the noun will be rendered as 'referent' or else retained in the German (particularly in quotations); the verb will be translated as 'refer' or 'designate'; the adjectives will generally be retained in the German.

[5]Frege is, in this latter passage, actually speaking directly of the distinction between first- and second-level functions; but preceding remarks—e.g. [Frege, 1997e, p. 146]—leave absolutely no doubt that Frege ascribes the same fundamentality to the distinction between objects and functions generally.

> [T]he phrase 'is a concept' requires a proper name as grammatical subject; and so, strictly speaking, it requires something contradictory, since *no proper name can designate a concept* ... [Frege, 1979d, p. 178, my emphasis]

(We will return in §1.1.2 to Frege's point about the contradictory requirements of 'is a concept'.) Proper names, in Frege's terminology, include not only expressions that are customarily designated as such—e.g. 'Boston', 'Gottlob Frege', etc.—but also definite descriptions, for example, and complex terms for the values of arithmetical functions (e.g. '73+16'). Indeed, the class of Fregean proper names includes any singular term[6]—any expression that can feature as the grammatical subject of a singular proposition [Frege, 1997c, p. 281]. '[A]n object', Frege explains, 'is something that. . . can be the *Bedeutung* of a subject' [Frege, 1997h, p. 187].[7] His denying that functions are objects amounts, therefore, to a repudiation of the possibility of singular reference—reference with a singular term—to functions.

Where an object is, for Frege, anything that can be the referent a proper name, a function is anything that can be the referent of an *incomplete* expression. Incomplete expressions are so called because they feature at least one empty place; they result from the omission of one or more expressions from a proper name or sentence.[8] In the simplest case, on which we concentrate, such an expression is the result of omitting one or more *proper names* from a proper name or sentence [Frege, 1997b, p. 218-219]. For instance, omission of the proper name 'Sweden' from 'the capital of Sweden', a complex proper name for Stockholm, yields an incomplete expression, 'the capital of ζ', designating a function the value of which for any country as argument is that country's capital.[9] Zeta, it is crucial to note, here functions solely to indicate the empty place—the *argument place*—left by the omission: it is not itself part of the expression for the function [Frege, 1997b, p. 212]. From a sentence or complex proper name featuring multiple constituent

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[6]Frege uses 'proper name' (*Eigenname*) interchangeably with 'singular term' (*Einzelname*): see [Frege, 1997a, p. 172].

[7]This conception of objecthood is also to be found in [Frege, 1953, p. 77], though formulated in earlier nomenclature.

[8]In fact, there is a redundancy in this description which will emerge momentarily.

[9]Actually, for Frege, if this expression designates at all, it designates that function which maps any object whatsoever to the capital of that object; for Frege requires that functions be defined over the entire domain of objects. He stresses, however, that providing some determination is made, it is a matter of comparative indifference what, for example, the capital of Proxima Centauri is settled to be [Frege, 1997e, p. 148]. See also [Frege, 1997a, p. 178] where it is indicated that *bedeutungslos* incomplete expressions are precisely those which do not have a determinate value for every object.

proper names, we may omit multiple such proper names, obtaining thereby expressions with multiple argument places, designating polyadic functions. As a limiting case we may also omit a proper name from itself, leaving a bare argument place, 'ζ'—an expression designating the identity function. *Predicates* are those incomplete expressions that result from the omission of one or more constituent proper names from a *sentence* in particular. They designate *concepts (Begriffe)*: functions whose domain is the set of all objects, and whose value for each object as argument is one of (classical) the truth-values—either 'the truth-value of what is true' [Frege, 1997e, p. 137], which Frege dubs *the True*, or the truth-value of what is false, which he dubs *the False*.[10] For example, omitting '2' from '2 is a prime number' yields 'ζ is a prime number', which designates that concept whose value for any prime number as argument is the True, and whose value for all other objects is the False. Consonant with the Fregean conceptions of objecthood and functionhood, a concept is understood to be anything that can feature as the referent of a predicate [Frege, 1997h, p. 187].[11, 12]

Frege calls incomplete expressions *names* of functions—*function names (Funktionsname)* for short [Frege, 1997b, p. 218-19]. This jars somewhat with our formulation of Frege's view as the thesis that functions are unnameable. However, 'unnameable' is here intended as a cognate of Frege's 'proper name', and function names, it is essential to recognise, are emphatically not proper names of functions. Proper names altogether lack the empty places possessed by function names. Frege is explicit that it is only with an expression possessed of such empty places that reference to a function is possible:

> [T]he expression for a function must always show one or more places that are intended to be filled up with the sign of the argument. [Frege, 1997e, p. 134]

The *Bedeutungen* of incomplete expressions can hence be *Bedeutungen only* of incomplete expressions. Objects, by contrast, are precisely those things to which reference can be made with,

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[10] Frege actually reserves 'concept' for *monadic* functions whose codomain is the set of truth-values, calling dyadic functions with the same codomain 'relations'. For simplicity, I shall ignore this terminological distinction and call any function of any number of (object) arguments with this codomain a concept.

[11] Again, see [Frege, 1953, p. 77] for an earlier formulation of this conception of concepthood.

[12] We have not here broached the higher echelons of Fregean type theory, which are populated by functions whose arguments include functions of lower levels and whose values are objects. Higher-level functions needn't concern us just yet; the issues can be introduced on the lowest floors of the hierarchy.

and only with, a complete expression—an expression devoid of empty places.

> An object is anything that is not a function, so that an expression for it does not
> contain any empty places. [Frege, 1997e, p. 140]

Since anything referred to with an expression lacking empty places is thus deemed an object, and since Frege 'call[s] anything a proper name if it is a sign for an object' [Frege, 1997h, p. 185], the Fregean category of proper names is yet more expansive than hitherto indicated: it includes any *bedeutungsvoll* expression (any expression possessed of a *Bedeutung*) containing no empty place. Frege regards sentences as *bedeutungsvoll*:[13] the *Bedeutung* of a sentence is taken to be its truth-value. Moreover, a sentence contains no empty place. Thus, truth-values are counted objects, and sentences, remarkably, are classified as proper names [Frege, 1997i, p. 158], [Frege, 1979c, p. 195].

1.1.2   The Concept *Horse* Problems: Self-Stultification, Paradox and Inexpressibility

Frege's view that the *Bedeutungen* of incomplete expressions are not objects engenders a cluster of perturbing problems usually gathered under the rubric of 'the concept *horse* paradox'.[14] To illustrate, let's focus, without loss of generality, on Frege's thesis that concepts are not objects. Consider, for a given concept, the relevant instance of the generalization that concepts are not objects. For the concept to which 'ξ is a horse' refers, the relevant instance is presumably formulable as follows:

(i) The concept to which 'ξ is a horse' refers is not an object.

Or more concisely:

(ii) The concept *horse* is not an object.

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[13]Strictly, Frege holds that some sentences—like some non-sentential proper names and some incomplete expressions—suffer reference failure: they lack *Bedeutung*. But we can set this consideration to one side for the moment; we return to it below on page 59.

[14]See [Proops, 2013] for an excellent discussion of the distinguishable difficulties. The concept *horse* paradox has enjoyed a swell of interest in recent years. Notable contributions include [Hale, 2010], [Hale and Wright, 2012], [Hale, 2013], [Hale and Linnebo, TS], [Jolley, 2007] [MacBride, 2011], [Noonan, 2006], [Proops, 2013], [Rayo, TS], [Textor, 2010] and [Trueman, 2015].

But as Frege intends 'object', (ii) is equivalent to,

  (iii) The concept *horse* cannot be referred to with a proper name.

But (iii) appears glaringly self-stultifying: it seems itself conspicuously to accomplish just that which it asserts to be impossible, and thus to bear witness to its own falsity. For 'the concept *horse*' is a proper name; hence, whatever it designates *can* be referred to with a proper name.[15] By a natural disquotational principle governing reference, 'the concept *horse*' designates the concept *horse*. Thus, *pace* (iii), the concept *horse* can be referred to with a proper name. Therefore, by Fregean lights, and *pace* (ii), the concept *horse* is an object (and by parity of reasoning, *pace* (i), so is the concept to which 'ξ is a horse' refers). However, granting Frege's view that the properties of being an object and of being a concept are mutually exclusive, it follows that the concept *horse* cannot then be a concept:

  (iv) The concept *horse* is not a concept.

But (iv) is apt to be regarded as paradoxical. It seems that no sentence of the form ⟨the ϕ α is a ϕ⟩—e.g. 'the city Berlin is not a city', 'the volcano Vesuvius is a volcano' [Frege, 1997h, p. 185]—ought to be counted false.

We took (i)-(iii) for instances of Frege's general thesis that no concept is an object; but in that case, since they are false, so must the general thesis be. Indeed, further reflection seems to confirm as much. 'ξ is a concept' requires a proper name to fill its argument place. It is extremely natural to suppose that where a predicate truly applies to something, there could, in principle, be a true sentence consisting of an expression referring to that thing in the argument place of that predicate. But in that case, anything to which 'ξ is a concept' does truly apply is something to which a proper name can, in principle, refer—i.e. an object. But 'ξ is a concept' truly applies to something just in case it is a concept. So anything that is a concept *is* an object. Granted that there *are* concepts, as Frege maintains, it follows that it is *false* that no concept is an object.

The problem here is in essence the one acknowledged by Frege in the quotation above on page 11. As Frege there puts it, 'ξ is a concept', strictly speaking, requires something contradic-

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[15] '[T]he singular definite article', Frege tells us, 'always indicates an object' [Frege, 1997h, p. 184].

tory: on the one hand it requires, in his judgement, that anything to which it truly applies *not* be an object, since being a concept excludes being an object; on the other, it requires that anything to which it applies *be* an object, since this is a condition of its being used to *say* of that thing that it is a concept. The same problem arises with 'ξ is not an object'.

It appears, then, that the attempt to give expression to Frege's general thesis, or to its instances, produces sentences—e.g. 'concepts are not objects', 'the concept *horse* is not an object'—which emerge as false by Frege's own lights; for the crucial predicates to which we appeal in attempting to articulate that thesis ('ξ is a concept', 'ξ is not an object') are applicable only to objects. Worse, in having to acknowledge those sentences as false, Frege seems forced therewith to acknowledge as true certain sentences, like (iv), that seem downright paradoxical. If Frege is not simply to concede the falsity of his view, it seems he must hold that the sentences with which we attempt to express it in fact expressively misfire: they do not, strictly, succeed in articulating his intended claims. And indeed Frege does hold this:

> By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that I was relying on a reader who would be ready to meet me halfway—who does not begrudge a pinch of salt. [Frege, 1997h, p. 192]

Elsewhere he confesses that there is 'a great obstacle in the way of expressing ourselves correctly and making ourselves understood', for 'language, with an almost irresistible force, compels me to use an inappropriate expression which obscures—I might almost say falsifies—the thought' [Frege, 1997a, p. 174].

But this confession raises the question of how—indeed, whether—Frege's thesis *can* correctly be expressed. What is it possible to say that does not miss, or falsify, Frege's thoughts? How, without simply remaining silent on the matter, *could* Frege go about resisting that *almost* irresistible compulsion to produce sentences that must subsequently be repudiated as expressively off-target and false? Or must Frege concede that his thesis on the (non-)objecthood of concepts is simply ineffable?

It is not only that thesis itself that appears inexpressible by Frege's own lights. The expressive difficulties engendered by Frege's position are far-reaching. It becomes unclear how large

swathes of semantics—notably, the semantics of predicates—might be so much as expressed. Suppose, for example, we attempt to frame a sentence assigning to a predicate its *Bedeutung*. The strong temptation is to produce the likes of

(v) 'ζ is a horse' refers to the concept *horse*.

But again, by virtue of being a singular term, the expression following 'refers to' designates, if anything, an object; it fails, therefore, to pick out the non-object concept to which, according to Frege, the predicate refers. So (v) fails to specify the *Bedeutung* of that predicate; it miscarries as an attempt to express the semantic thesis we intend. It seems, moreover, that any such attempt must founder; for in order to say that the predicate has the *Bedeutung* it does, we should apparently have to pick out both the predicate and its *Bedeutung* by the use of singular terms. This, at any rate, is a condition of our using a construction of the form 'a refers to b' (or a similar construction employing a kindred transitive semantic verb like 'designates', 'stands for', 'denotes' etc.); for this dyadic predicate requires singular terms in each of its argument places. But then, by what other means might we go about specifying a predicate's *Bedeutung*?

It is true that 'ξ refers to ζ' can grammatically combine with expressions that are not singular terms—for example, the quantifier phrase 'every predicate' and the predicative expression 'a concept':[16]

(vi) Every predicate refers to a concept.

But a general sentence like (vi), featuring 'ξ refers to ζ', surely admits of paraphrase such that the argument places of that dyadic predicate are visibly occupied by (variable or constant) singular terms.[17] (vi) would be naturally formulated in logician's English (though we might equally exploit the singular pronouns of standard English) as follows: for all x, if x is a predicate, then for some y, y is a concept and x refers to y. But if the referents of predicates cannot be referents

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[16] A predicative expression (the terminology is Dummett's [Dummett, 1981a, p. 214ff]) is, roughly, what remains of a full predicate when it is stripped of its argument place, and the copulative form of 'to be' is dropped or the predicate's main verb is converted to the relevant participle form: so from 'ξ is a concept' we obtain the predicative expression 'a concept'; from 'ξ sings' we obtain 'singing' (see also [Wright, 1998, p. 79-80])

[17] Cf. [Frege, 1979d, p. 178] where Frege, having claimed that 'ξ is a concept' requires a proper name as grammatical subject, rebuts the objection that such a predicate can also combine with a quantifier phrase.

of proper names, then they cannot count among the values of variable expressions whose substitution instances are proper names. Naming and first-order quantification are constitutively related such that the unnameable—that which is not an object—cannot lie within the range of a variable bound by a first-order quantifier. In particular, then, the referents of predicates cannot count among the values of the foregoing term variable, y. But in that case, although (vi) is precisely the kind of thing one might say in the course of attempting to articulate a general semantic thesis characteristic of Frege's philosophy of language,[18] it too emerges as false by Frege's lights, since it implies, *contra* Frege, that every predicate refers to some *object*.

The alleged non-objecthood of the referents of incomplete expressions therefore gives rise to serious obstacles to expressing both particular and general Fregean theses about the semantics of predicates.

Finally, Frege's view also presents expressive obstacles to reporting the very expressive obstacles presented by his view. Consider what I said above about (v). I claimed that 'the concept *horse*', qua singular term, fails to pick out the non-object concept to which, according to Frege, 'ξ is a horse' refers. But, of course, 'the non-object concept to which, according to Frege, "ξ is a horse" refers' is itself just another singular term. Whatever it refers to *can* be picked out by singular term, and no reason has been given for thinking that 'the concept *horse*' does not succeed in doing so. Frege's pinch of salt is even required when attempting to convey his need of a pinch of salt.

1.1.3 Incomplete Expressions and Their Senses

When we ascribe to Frege the view that the *Bedeutungen* of incomplete expressions cannot be named, we must inevitably be prepared subsequently to qualify that ascription in light of Frege's plea for a pinch of salt. Nevertheless, there is clear sense in which that view is incontestably present in Frege's work: there simply are recurrent remarks in Frege's writings clearly to the effect that *Bedeutungen* of incomplete expressions are not objects, the kind of things that can be

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[18]Subject to the qualification mentioned in footnote 13 above.

named.[19],[20] What *can* be contested, on the other hand, is Frege's position on the nameability of the *senses* (*Sinne*) of incomplete expressions, and of those expressions themselves. Some commentators have held that Frege conceives of those senses or those expressions, or both, as kinds of function, and therefore as incapable of being designated with a proper name. The functions with which some commentators[21] take Frege to identify incomplete expressions are functions mapping proper names to proper names.[22] For instance, the predicate featuring in the sentence 'Shergar is a horse' is to be identified with that linguistic function whose value for any name, A, as argument is the sentence $\ulcorner A$ is a horse$\urcorner$. The functions with which some commentators[23] take Frege to identify the *senses* of incomplete expressions are functions mapping senses of proper names to senses of proper names.[24] For instance, the sense of the predicate featuring in 'Shergar is a horse' is to be identified with that function whose value for the sense of any name, A, is the sense of—the *thought (Gedanke)* expressed by—the sentence $\ulcorner A$ is a horse$\urcorner$.

Before considering the tenability these interpretations, note what is, and what is not, at stake in this dispute. What is not at stake is whether Frege *recognises* the functions with which these commentators wish to identify incomplete expressions and their senses: he does. Neither is Frege's position on the nameability of these functions at issue: for Frege, *no* function can be named. What is at stake is what position(s) these functions occupy in the tripartite structure of language, sense, and *Bedeutung*, and in consequence how pervasive unnameability is within Frege's philosophy of language. Frege's recognition of the functions in question *is* recognition of their appearance at the level of *Bedeutung*; for, as Dummett puts it, 'All things are *Bedeutungen*'

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[19]In addition to those remarks already cited, consider: 'Value-ranges of functions are objects, whereas functions themselves are not ... Extensions of concepts likewise are objects, though concepts themselves are not' [Frege, 1997e, p. 140-41]

[20]Cf. Adrian Moore's [Moore, 2013, p. 239-40] discussion of the sense in which the ascription of a certain view to the Tractarian Wittgenstein can receive decisive textual justification, even if 'in his most authentic mode' Wittgenstein would claim that the sentences purporting to express that view are nonsensical.

[21]These include Geach ([Geach, 1976b, pp. 59-61], [Geach, 1961, p. 144], [Geach, 1976a, p. 440]), Hugly [Hugly, 1973, pp. 236-242], Rumfitt [Rumfitt, 1994, p. 601ff], and Sullivan [Sullivan, 1992]

[22]As we ignored higher-level functions above (see footnote 12), similarly, for simplicity, we here ignore higher-level function names. In fact, according to the commentators under discussion, higher-level function names *are* higher level functions for Frege; they are functions taking lower-level incomplete expressions (themselves functions) as arguments and returning proper names as values. The functional treatment of higher-level incomplete expressions will need to be subtler than in the first-level case, but Sullivan [Sullivan, 1992] shows how it can be done.

[23]These include Baker and Hacker [Baker and Hacker, 1984, p. 324-6], Geach ([Geach, 1976a], [Geach, 1961], Jackson [Jackson, 1963], and Parsons [Parsons, 1981].

[24]Again, we set aside the senses of higher-level incomplete expressions.

[Dummett, 1995, p. 8] for Frege—everything is designated by some possible expression or other. If functions appear moreover at the levels of sense and of language, additional obstacles confront the expression of truths about language. If the predicate of 'Shergar is a horse' is a linguistic function, then (v) is in fact *doubly* referentially unsuccessful: the quotation preceding 'refers to' fails to designate the predicate whose *Bedeutung* we sought to specify; for it is a proper name and therefore designates, if anything, an object and not a function. The unnameability of predicates would appear to prohibit not only the assignment of a particular *Bedeutung* to a predicate, but also the declaration of that predicate's *having* a *Bedeutung*, of its being *bedeutungsvoll*. That predicates are referential expressions, however, is a vital—and enduringly controversial[25]—tenet of Fregean semantics.[26] The instances of this tenet seem, however, to be rendered ineffable; for it is unclear how one might say of a predicate that it is *bedeutungsvoll* without designating it with a singular term. Similar considerations hold for the senses of incomplete expressions, functionally conceived.

Imputing to Frege the functional conception of the senses of incomplete expressions might be thought untenable in light of the following assertion in Frege's 'On Sense and *Bedeutung*':

> In order to speak of the sense of an expression 'A' one may simply use the the phrase 'the sense of the expression "A" '. [Frege, 1997i, p. 154]

If one assumes, as Dejnožka [Dejnožka, 2007, p. 83] does, that this principle applies to *all* expressions—that any expression can be substituted for 'A'—then, by Frege's thesis that the singular definite article always indicates an object (footnote 15 above), it follows that *all* senses are objects, and thus that all senses are nameable. I think it is clear, however, that Dejnožka's assumption is mistaken—that, specifically, the principle does not apply to incomplete expressions. We saw above that, for Frege, incomplete expressions essentially feature argument places; but Frege additionally claims that even schematic letters used to indicate functions indefinitely must be accompanied by at least one empty place [Frege, 1997a, p. 176]. Thus, if 'A' were to count incomplete expressions among its substitution instances it would have to feature an empty place,

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[25]See [MacBride, 2006] for discussion.
[26]See, for example, Frege [Frege, 1979c, p. 193], for Frege's case for the referentiality of predicates. Note that he describes the contrary possibility as 'inconceivable'.

which it does not. There is every reason to suppose that Frege would insist on the parallel constraint for schematic letters counting proper names among their instances: that such letters are unaccompanied by empty places. Thus, for Frege, a schematic letter counting *all* expressions as instances—which is what 'A' is intended to be if Dejnožka is to be believed—is an impossibility, and any such letter purporting to do so is an abomination. Furthermore, in 'On Sense and *Bedeutung*' Frege, by his own admission, 'distinguished between sense and *Bedeutung* in the first instance only for the case of proper names (or if one prefers singular terms)' [Frege, 1997a, p. 172]. (In fact, it was Frege's only having drawn the distinction for proper names in that piece that permitted controversy in the early scholarship concerning whether Frege even intended to make the sense/ *Bedeutung* distinction for incomplete expressions—a dispute conclusively settled by the subsequent publication of Frege's *Nachlaß* and correspondence). Frege's above principle about speaking of senses was intended to apply, therefore, only to proper names, and is consistent with the functional conception of the senses of incomplete expressions.

A further challenge to that conception is that Frege repeatedly claims that the senses of the constituents of a sentence are constituents of the sense of the sentence—that the former enter into the *composition* of the latter:

> If a name is part of the name of a truth-value, then the sense of the former is part of the thought expressed by the latter. [Frege, 1997b, p. 222]

The sense of 'Barack Obama', for instance, is a part of the thought expressed by 'Barack Obama is mortal'—as is the sense of the predicate of that sentence. In this respect, the level of sense differs from that of *Bedeutung*. For the *Bedeutungen* of the constituents of a sentence are not parts of the Bedeutung of the sentence—its truth-value. Barack Obama himself, for example, is not part of the True, though his name is part of the above name of the True.[27] More generally, it is not the case that the *Bedeutung* of the constituent of a name is always part of the *Bedeutung* of that name. To use a counterexample of Frege's, it is not the case that Sweden is part of Stockholm, though the former is the *Bedeutung* of a part a name of which the latter is the *Bedeutung*: 'The capital of

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[27]This point will prove particularly significant in our discussion of Frege's most explicit argument for the unnameability of concepts, in §3.1.1.

Sweden'.[28] (This is not to say, of course, that the contrary—as opposed to the contradictory—is the case: it is *sometimes* the case that the *Bedeutungen* of the part is part of the *Bedeutung* of the whole. This is always the case, for example, when the name-part in question is the improper part.) But Frege, it has been alleged, cannot coherently sustain both a conception of the thought as composed of the senses of names and predicates, and a conception of the thought as the value of the sense-function of a predicate for the senses of names as arguments; for together these conceptions require that the sense of a predicate be a function whose values are sums of which both it and the relevant argument(s) are parts. Sullivan puts the point thus:

> Where our intuitions invite us to think in terms of parts combined into a whole they thereby resist the functional model, since it would in general be absurd to suppose that the value of a function for some argument is a whole in which argument and function are combined. [Sullivan, 1992, pp. 91-92]

It is not evident to me, however, why it must be incoherent to sustain these two conceptions at once. I suspect Sullivan's remark benefits unduly from the scope ambiguity of 'in general'. If what is meant is that it would be absurd to suppose *of functions in general* that their value for some argument is a whole in which argument and function are combined, then this is quite true; but whether *all* functions are parts of their values is not what is at issue. If, on the other hand, what is meant is that the supposition that there are functions the values of which are wholes in which argument and function are combined *is in general absurd*, then it remains unclear to me in what that absurdity consists. Certainly, urging that functions are not *typically* parts of their values will not settle the matter; nor, I think, will saying that 'it would seem to be quite a mystery' [Klement, 2002, p. 68] how the sense-function of a predicate could both map the sense of a name to a thought and enter into the composition of that thought. But even if there is a theoretical tension here, it strikes me that, charity notwithstanding, this is a tension we will have to impute to Frege's position.[29] For his explicit declarations of the exhaustiveness of the object/ function division ([Frege, 1997b, p. 213], [Frege, 1997e, p. 140]) make clear that, for Frege, if the senses of incomplete expressions are not functions, then they are objects. The supposition

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[28]The example is given in Frege [Frege, 1997g, pp. 364-65]. The passage actually represents a recantation of earlier remarks [Frege, 1997i, p. 159] endorsing the part-whole conception at the level of Bedeutung.

[29]Cf. [Baker and Hacker, 1984, pp. 322-40].

that such senses are objects, however, conflicts with what Frege says about the natures of such senses and of objects, respectively. In addition to being called 'incomplete', expressions designating functions are described as *in need of supplementation* (*ergänzungsbedürftig*) and *unsaturated* (*ungesättigt*). Frege also applies these descriptions both to the senses and the *Bedeutungen* of those expressions. They are all, in some sense, unsaturated. There thus exists, for Frege, a parallelism between these three: the incomplete expression both expresses and designates items that are also incomplete.[30] It is the unsaturatedness of the senses of incomplete expressions that accounts for the *unity* of the thoughts of which they are constituents: it is responsible for the integration into a single thought of senses that would otherwise 'hold aloof from one another' as a mere congeries:

> [N]ot all the parts of a thought can be complete; at least one must be unsaturated or predicative; otherwise they would not hold together. [Frege, 1997d, p. 193]

Proper names, their senses and their *Bedeutungen* belong to a complementary parallelism of *saturated* entities:

> The names of objects, *proper names*, thus carry no argument places with them, they are saturated, like the objects themselves. [Frege, 1997b, p. 213][31]

The problem for the classification of the senses of incomplete expressions as objects is that, for Frege, they contrast with objects in respect of saturatedness. The problem isn't immediately fatal because although functional expressions, their senses and their *Bedeutungen* are all in *some* sense unsaturated, it is not clear that they are all unsaturated in the very same sense. Moreover, there remains the possibility that the unsaturatedness of items at one level is derivative upon that of items at another. Dummett, the pre-eminent advocate of the objectual conception of the senses of incomplete expressions[32], appeals to that possibility in response to the present problem. He argues that these senses are

> incomplete only in that it would be necessary, in order to grasp the sense of an incomplete expression, to understand it [the incomplete expression] as an expres-

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[30]Textor [Textor, 2010] dubs this the *mirroring principle.*

[31]Also [Frege, 1997e, p. 134].

[32]See Dummett [Dummett, 1981a, pp. 291-94], [Dummett, 1981b, pp. 249-54]; although see his [Dummett, 2007, p. 122] for a recantation of his earlier position, indeed a *volte face* to the opposite position that *no* senses are objects.

> sion containing argument places which, when these were filled by singular terms,
> yielded a singular term. [Dummett, 1981a, p. 291]

That is, the unsaturatedness of such a sense 'consist[s] merely in its being the sort of sense appropriate to an incomplete expression' [ibid]. Dummett's account conflicts, however, with Frege's insistence that it is the unsaturatedness of the *senses* of incomplete expressions that has primacy:

> It is really in the realm of sense that unsaturatedness is found, and it is transferred
> from there to the symbol. [Frege, 1984a, p.393]

It is in fact the unsaturatedness of incomplete expressions that is derivative upon the unsaturatedness of their senses, rather than conversely, as Dummett maintains. It would, therefore, be baffling if Frege had conceived of these senses, whose unsaturatedness is considered basic, as objects; since, for Frege, objects are essentially saturated, complete. It strikes me therefore, that not only the *Bedeutungen* but also the senses of incomplete expressions are conceived as functions, and that the latter, like the former, are thus unnameable.

That the unsaturatedness of incomplete expressions is derivative upon that of their senses, is to be explained, I think, by reversing Dummett's account: their unsaturatedness consists merely in their being the sort of expression appropriate to expressing a (non-derivatively) unsaturated sense. This derivative unsaturatedness may well be entirely compatible with the saturatedness of objects; and this opens the possibility of classifying them as objects and not functions. There is, moreover, strong pressure to think that Frege did so.

Firstly, Frege defines a function name as the result of removing a proper name from a proper name [Frege, 1997b, p. 218-19]. But removing a proper name from a proper name does not yield a *function* but a string of words or signs with an empty space hitherto occupied by the removed name. It is as the latter that Frege consistently characterises incomplete expressions.[33] I am thoroughly in agreement with Oliver [Oliver, 2010, pp.128-31] when he claims that there is nothing Frege says that recommends anything other than a literal reading of these characterisations. Secondly, and most importantly, Frege just plainly indicates, in a letter to Russell, that

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[33]See, for example, [Frege, 1997a, pp. 173-74], [Frege, 1997d, p. 134].

he takes incomplete expressions to be possible referents of proper names. Specifically, they are designated by the proper names that result from simply enclosing them in quotation marks:

> While '( ) • 3 + 5' is a function name, ' "( ) • 3 + 5" ' is a proper name, and its referent is the function name '( ) • 3 + 5'. [Frege, 1980, p. 136].

Since incomplete expressions can be designated by proper names, they must, unlike their senses and referents, be objects. Thus, although we apparently cannot say of the referent of an incomplete expression that, for example, it is a function, we can say of the expression itself that it is a function name:

> Instead of using the imprecise expression 'ξ is a function', we can say: ' "( ) • 3 + 5" ' is a function name. [Frege, 1980, p. 136]

In summary, then, while Frege considered incomplete expressions themselves to be capable of being named, he held that their *content*—bifurcated in his mature philosophy into a sense and a *Bedeutung*—is unnameable.

1.2 Russell

1.2.1 The Logical Atomist Russell: Qualities, Relations and Facts

During his logical atomist period, Russell shared with Frege a recognition of that which cannot be named. Unnameability forms a recurrent theme both in his 1918 lecture series, 'The Philosophy of Logical Atomism' [Russell, 2010b] and in his later essay of 1924, 'Logical Atomism' [Russell, 2010a]. Russell concludes the penultimate lecture of that series, for example, lamenting '[t]he trouble that. . . arises from our inveterate habit of trying to name what cannot be named' [Russell, 2010b, p. 108].

Russell's commitment to the unnameable during this period takes a form in one respect very closely parallel to Frege's. Russell holds that *qualities*, which are characterized as the kind of entities designated by predicates [Russell, 2010b, p. 34], and *relations*, their polyadic counterparts, cannot be named. A relation, for example, cannot feature as the subject (in the non-linguistic sense) of a sentence [Russell, 2010b, p. 35]—i.e. as that which the grammatical subject of a sentence designates. Russell stresses [Russell, 2010a, p. 143-44] that sentences in which qualities

(*attributes* as he calls them in 1924) or relations *seem* to be the named subject are either nonsense or else capable of being brought into a form in which the attribute or relation does not appear as subject. He insists that only *particulars*, with which qualities and relations contrast, can feature as the named subject of a sentence: 'Strictly speaking, only particulars can be named' [Russell, 2010b, p. 108]. Russell's view that the entities semantically correlated with predicates and relational expressions cannot be named is strikingly similar to Frege's position, even if Russell's conception of those entities differs in other respects from Frege's.

Another important unnameability thesis in Russell's logical atomist writings, however, finds no counterpart in Frege's work. Russell maintains that *facts*, the kind of things that make sentences true or false, are unnameable:

> You must not run away with the idea that you can name facts...; you cannot. You cannot name them at all. You cannot properly name a fact. The only thing you can do is to assert it, or deny it...You can never put the sort of thing that makes a proposition to be true or false in the position of a logical subject. [Russell, 2010b, pp. 13-14] [34]

Although this claim finds no parallel in Frege's writings, it is echoed in Wittgenstein's early philosophy, as I shall illustrate in §1.3.1. Actually, it would be more appropriate to describe Russell's claim as an echo of Wittgenstein's, since Russell's lectures are, as the author indicates in the preface [Russell, 2010b, p. 1], principally an exposition of ideas he had learned from Wittgenstein. There remains the question of whether Russell's lectures are a faithful exposition of Wittgenstein's views, not least because at the time of their delivery Russell had had no contact with Wittgenstein since 1914. But I shall argue below that, at least on the issue of the unnameability of qualities, relations and facts, Russell's exposition is faithful.

There is, however, a further Fregean echo in Russell's discussion of unnameability: namely, a confession, unmistakeably reminiscent of Frege's on page 15 above, of the serious expressive obstacles confronting his view, and a plea for indulgence on the part of his readership:

> The topic is one with which language, by its very nature, is peculiarly unfitted to deal. I must beg the reader, therefore, to be indulgent if what I say is not exactly

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[34]This thesis is reiterated in [Russell, 2010b, pp. 110-11] and [Russell, 2010a, p. 141].

what I mean, and to try to see what I mean in spite of unavoidable linguistic obstacles to clear expression. [Russell, 2010a, p. 142]

1.2.2 *The Principles of Mathematics*: **Propositional Functions and Concepts**

There are strong grounds for thinking that commitment to unnameables is also present in writings of Russell's that pre-date his logical atomist period—notably in his 1903 *The Principles of Mathematics* [Russell, 2009]. It seems quite clear, in particular, that Russell maintains that *propositional functions*—which he describes as 'nearly the same thing[s]' [Russell, 2009, §481] as Fregean concepts—cannot not be the named logical subject of a sentence. He argues [Russell, 2009, §85] that if a propositional function *were* an object, it could be its own argument:[35] there could be a sentence in which that propositional function is asserted of itself. But with this possibility, he argues, there arises a contradiction that we would now describe as a version of Russell's paradox. (This kind of motivation for denying objecthood will be the central focus of chapter 5.)

There is no doubt that Russell is acutely aware during this period that any view to the effect that such-and-such cannot be designated by the subject of a sentence confronts grave problems as to how that view might coherently be stated. In contemporaneous correspondence with Frege, Russell writes:

> If there can be something which is not an object, then this fact cannot be stated without contradiction; for in the statement, the something in question becomes an object. [Frege, 1980, p. 134]

Indeed, in the *The Principles of Mathematics* [Russell, 2009, §85], Russell admits, concerning his position that propositional functions are not objects, that he is 'highly doubtful whether such a view does not lead to a contradiction', but claims that 'it appears to be forced upon us' [ibid.].

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[35]Russell in fact uses 'distinguishable entity' rather than 'object', but we retain the Fregean expression. Cf. the following passage from the first volume of *Principia Mathematica*:

> A function, in fact, is not a definite object, which could be or not be a man; it is a mere ambiguity awaiting determination, and in order that it may occur significantly it must receive the necessary determination which it obviously does not receive if it is merely substituted for something determinate in a proposition. [Russell and Whitehead, 1963, p. 48]

Richard Gaskin [Gaskin, 2008, pp. 148-153], has argued that Russell's commitment to the un-nameable during this period goes beyond his denying that propositional functions are objects. Gaskin maintains that there is an unacknowledged such commitment, concealed beneath Russell's nomenclature. Russell, like Frege, holds that the referents of linguistic expressions belong to one of two categories: *things* are those entities referred to by proper names, while *concepts* are those entities referred to by all other expressions—for instance, by adjectives and verbs. Concepts give *propositions* their unity—where propositions are conceived, unlike Frege's *Gedanken*, as situated at the level of reference. Concepts are, in Fregean terms, *incomplete* or *unsaturated* propositional constituents, for Russell. *Contra* Frege, Russell claims [Russell, 2009, §52] that concepts can indeed be named—can indeed be subjects of propositions. The concept to which the verb 'denotes' refers, for example, is named by the abstract substantive 'denotation'. *Naming* concepts, however, appears to strip them of their unifying capacity within the proposition. This shows forth in the fact that analysing a declarative sentence, by indicating each of the constituents of the proposition it expresses, yields merely a list of disparate and unintegrated items. Russell concludes is that the concept *as designated by a verb* and 'embody[ing] the unity of the proposition' [Russell, 2009, §54] is distinguishable from the concept *as designated by a name*, and thus that concepts have a 'twofold nature' [ibid.]. So whilst Russell's terminology allows him to *say* that concepts are nameable, the divergence from Frege, Gaskin [Gaskin, 2008, p. 151ff] argues, is slight.[36] The concept *qua unsaturated propositional unifier* cannot be named. Frege claims that 'it is a mere illusion to suppose that a concept can be made an object without altering it' [Frege, 1953, p. X]; in short, on Gaskin's reading, Russell departs from Frege in being prepared to call the item post-alteration a concept; the unaltered concept continues necessarily to elude naming. I think Gaskin is likely correct in his account of Russell's position on concepts; though I'm afraid a full discussion of the exegetical issues must be left for another occasion.

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[36]See also [Ishiguro, 1981, p. 51] on the similarity between Frege's and Russell's positions.

1.3 Early Wittgenstein

1.3.1 Properties, Relations and Facts

I claimed in §1.2.1 that Russell's logical atomism is faithful to Wittgenstein's early philosophy in respect of the unnameability of qualities (or as I shall now call them *properties*), relations and facts. I defend that claim in the present section. First, however, some general remarks on naming and objecthood in Wittgenstein's early thought.

Wittgenstein's fundamental commitment on naming in *Tractatus Logico-Philosophicus* is as follows:

> A name means [*bedeutet*] an object. The object is its meaning [*Bedeutung*]...(3.203)[37]

On the face of it, Wittgenstein has, in this connection, simply taken over Fregean doctrine:

> The *Bedeutung* of a proper name is the object it designates or names. [Frege, 1997a,
>
> p. 173]

As will emerge below, however, Wittgenstein's conception of names and objects diverges dramatically, in several respects, from Frege's. 3.203 is clearly a universal generalization: it asserts that if *anything* is meant by a name that thing is an object. Moreover, it is equally clear that it is a generalization Wittgenstein intends to apply all *possible* names: just as Wittgenstein is centrally concerned in the *Tractatus* to give the essential nature of the proposition, and therewith the essence of all description (5.471, 5.4711), he is here concerned to characterise the essential nature of naming. 3.203 expresses the thesis that only objects *can* be the meanings of names. Everything nameable is, for Wittgenstein, an object. This accords with Frege's deeming it sufficient for something's being an object that it be capable of being the referent of a proper name.

Does Wittgenstein also hold that every object is nameable? Does he coincide with Frege in also counting it a necessary condition of something's being an object that it be capable of being named? I am essentially in agreement with Hintikka and Hintikka [Hintikka and Hintikka, 1986, p. 77] when they claim that, for Wittgenstein, all (Tractarian) objects are nameable. But,

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[37]References to the *Tractatus* are made using the decimal number of the relevant passage. I draw upon both the Ogden [Wittgenstein, 1981] and the Pears & McGuinness [Wittgenstein, 1974] translations.

to my mind, the ascription of that view merits two caveats. In fact, the first caveat is equally applicable to Frege. For Wittgenstein, objects cannot, as it were, *just* be named. One can only name an object in the course of asserting a proposition featuring a name for that object; for 'only in the nexus of a proposition does a name have meaning' (3.3).[38] No object is nameable in isolation from the sentential context. The second caveat does not apply to Frege; for it is required partly in consequence of a departure from Frege on Wittgenstein's part. It is this: Wittgenstein holds that there is at least one kind of circumstance in which even certain objects are, in some strict sense, unnameable:

> Either a thing has properties which no other has, and then one can distinguish it straight away from the others by a description and refer to it; or, on the other hand, there are several things which have the totality of their properties in common, and then it is quite impossible to point to any one of them.
>
> For if a thing is not distinguished by anything, I cannot distinguish it—for otherwise it would be distinguished. (2.02331)

The case at issue here, in which multiple objects have precisely the same properties, is one Frege (like Russell) deems impossible. For he endorses, and indeed adopts as a definition of identity, the Leibnizian principle that a is numerically identical to b iff a and b have precisely the same properties, the right-to-left direction of which precludes the existence of distinct, indiscernible objects [Frege, 1997a, p. 175], [Frege, 1953, §65]. Wittgenstein repudiates this account of identity (5.5302), insisting that it is possible that multiple objects share all their properties.[39] In such a case, he thinks, none of the indiscernible objects can be named, for there is nothing in virtue of which any possible name might refer to one among them and not another; if there were, then that would distinguish the referent from the others, contrary to the supposition that they are indiscernible. Granting the possibility of distinct indiscernibles, Wittgenstein is, in my view, quite right. The unnameability in question is strict in the following sense: holding fixed the indiscernibility of objects a and b, it is absolutely impossible to name either a or b.[40] But note that since Tractarian objects possess both essential ('internal', 'formal') and inessential ('external',

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[38] Frege advances this thesis, known as the context principle, in his *Grundlagen* [Frege, 1953, pp. X, 71].

[39] That Wittgenstein deems this possible is witnessed by his granting *sense* to the proposition that a and b have all their properties in common.

[40] Cf. Timothy Williamson's notion of an *elusive object* [Williamson, 2007b, p. 16-17].

'material') properties (e.g. 2.01231, 2.0233), a and b must share properties of the latter kind, which each can lack or possess independently of the of the other's lacking or possessing them. The indiscernibility of a and b must, therefore, be a mere contingency. Ceasing to hold fixed the contingent indiscernibility of a and b, then, it is not absolutely impossible to name either a or b. It strikes me that there is a clear sense, then, in which all objects are nameable for Wittgenstein: none are essentially resistant to being named.

In fact, an upshot of my argument below will be that a third caveat is required in connection with the claim that, for Wittgenstein, all objects are nameable: there is, as I shall explain, an important *narrower* sense of nameability, on which certain Tractarian objects are unnameable.

In light of 3.203, one might suppose that the question whether the Tractarian Wittgenstein is committed to unnameable properties and relations is to be settled by establishing whether the objects of the *Tractatus* are uniformly particulars. Some commentators have indeed held that Tractarian objects are particulars one and all.[41] However, whilst adopting this reading certainly involves denying that the *Tractatus* allows for the possibility of names for properties and relations, it does not involve imputing to the text a commitment to unnameable properties and relations: for a proponent of that reading will deny that the *Tractatus* recognises properties and relations, and attribute to the text a nominalistic ontology in which, as Anscombe puts it, 'there is nothing but [particular] objects in configuration' [Anscombe, 2001, p. 99]. Universals are not, she claims, for Wittgenstein, 'a kind of thing that is to be found in the world' [ibid.].

So Russell's exposition of Wittgenstein's views is not to be vindicated on the Anscombian, nominalistic reading. I think that reading is mistaken, however: for Wittgenstein, objects *do* include properties and relations.[42] Wittgenstein plainly says as much in the wartime notebooks he kept during the period in which the *Tractatus* was being composed:

Relations and properties, etc. are *objects* too. [Wittgenstein, 1979, p. 61]

He also explicitly said as much retrospectively, with regard to relations, when explaining the

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[41]This view is held by Anscombe [Anscombe, 2001, Ch.7], Copi [Copi, 1966], Sellars [Sellars, 1962] and Carruthers [Carruthers, 1989, Ch.11].

[42]Commentators who share this view include Stenius [Stenius, 1960, pp. 61-69], Allaire [Allaire, 1966], Hintikka and Hintikka [Hintikka and Hintikka, 1986, pp. 30-39], and McGuinness [McGuinness, 2002].

opening passages of the *Tractatus* in lectures recorded by Desmond Lee:

> 'Objects' also include relations; a proposition is not two things connected by a relation. 'Thing' and 'relation' are on the same level. The objects hang together as it were in a chain. [Wittgenstein, 1980, p. 120]

If properties and relations do count among objects, then in the Tractarian sense of 'name', properties and relations are nameable. However, as I say, I think there is an important narrower notion of nameability, which is closer to the notion operative in Frege's and Russell's work, on which properties and relations cannot, by the lights of the *Tractatus* be named. This is best brought out, I think, by consideration of a key passage in Wittgenstein's exposition of his theory of the proposition:[43]

> Instead of, 'The complex sign "aRb" says that a stands to b in the relation R', we ought to put, '*That* "a" stands to "b" in a certain relation says *that aRb*'. (3.1432)

That which asserts a fact, we are being told, is itself a fact. It is the obtaining of a state of affairs among signs—one name's being related to another—that symbolises the obtaining of a state of affairs among the objects for which those names stand; and it is the fact that those names are so related that is the propositional sign, not the complex consisting of "a" concatenated with "R" concatenated with "b" (3.14). But if this is so, what significance are we to attribute to 'R'? Is 'R' the name of a relation? If it were, we would be left wondering about its absence from the symbolising linguistic fact; we would expect that fact to be not that "a" *stands in a certain relation to* "b", as Wittgenstein has it, but rather *that* "a", "R" *and* "b" *stand in a certain (triadic) relation.* Given that "R" is not, in Wittgenstein's estimation, a constituent of the symbolising linguistic fact, are we to understand him as deeming its presence otiose? I think not. The most natural role to attribute to "R" in light of 3.1432 is that of *establishing* that relation-among-names the obtaining of which symbolises the relevant fact. "R" is not a name, nor indeed a non-nominal symbol; rather, it is the obtaining of the relation between "a" and "b" *by virtue of their appearing to the left and to the right, respectively, of the inscription* "R" that symbolises the

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[43]I confine my attention in what follows to relations, but my argument carries over, mutatis mutandis, to properties.

obtaining of the coordinate relation between a and b.[44] This is made clear in the predecessors of 3.1432 in the 1913 'Notes on Logic' and the 1914 'Notes Dictated to G. E. Moore in Norway', where Wittgenstein explicitly remarks on the status of "R":

> In "aRb", "R" looks like a substantive, but it is not one. What symbolises in "aRb" is that R occurs between a and b. [Wittgenstein, 1979, p. 98]

> [I]n "aRb", "R" is *not* a symbol, but *that* "R" is between one name and another symbolises ...The true analysis is: "R" is no proper name, and, that "R" stands between "a" and "b" expresses a *relation*' [Wittgenstein, 1979, pp. 109-10].

Having settled the status of "R", there remains a question about the status of the relation in which two names stand by virtue of flanking "R". *That* this relation holds between names symbolises *that* a corresponding relation holds between their meanings; but does the relation *itself* symbolise that corresponding relation? If, as Wittgenstein says, relations are objects too, I submit that it must. Indeed, as the Hintikkas [Hintikka and Hintikka, 1986, pp. 37-39] rightly stress, that relation between names is to be regarded as the *name* of that corresponding relation. For that corresponding relation is then one of the objects entering into the constitution of the situation represented by the proposition; and it is a cardinal tenet of Wittgenstein's picture theory of the proposition that to each object that is a constituent of the represented situation there corresponds, in the proposition, a name that is that object's representative (esp. 4.04, 3.21-3.22, 2.13-2.131): the name of that relation can only be the relation obtaining between "a" and "b". However, once it is granted that, for Wittgenstein, there are propositions in which a relation is named by a relation among names, I struggle to see how one might resist the conclusion that, according to the *Tractatus*, relations are exclusively nameable by relations. That the names of relations are themselves always relations is required, it strikes me, by the picture-theoretic insistence upon commonality of form between a proposition and the situation it represents (4.12, 2.18, 2.2); for this demands that the form of an object, its combinatorial capacity to occur as a constituent of situations (2.0141), be precisely paralleled by the combinatorial capacity of its name to occur in propositions representing situations.

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[44]This view is defended by Sellars [Sellars, 1962] and in lucid detail by Long [Long, 1969]. Long calls "R" the *index* of a relation.

That relations are exclusively nameable by relations, however, means that relations cannot be named by *particular* names—names that are not themselves relations or properties. It is in this narrower, but significant sense that they are unnameable by early Wittgensteinian lights. (We will return to the significance of this narrower species of unnameability in §1.3.2.) We saw in §1.1.3 that according to some commentators, functions are, for Frege, exclusively designatable by linguistic functions. The ascription of that view to Frege was, I believe, shown untenable; but Wittgenstein *does* hold a view of this kind, on which relations are designatable only by members of their own ontological category. This difference notwithstanding, there are important parallels between Wittgenstein and Frege here. As Geach puts it, Wittgenstein's view that relations can only be named by relations means that he "would no more tolerate a relation's being presented by a free-standing namelike expression like "the relation *on top of*" than Frege would have' [Geach, 1976b, p. 65].

The question whether *facts* are unnameable according to the author of the *Tractatus* can, I think, more straightforwardly be answered in the affirmative. That facts are unnameable is explicitly stated more than once in the 1913 'Notes on Logic' [Wittgenstein, 1979, p. 96, 107]:

Facts cannot be *named*. [Wittgenstein, 1979, p. 107, original emphasis]

In the *Tractatus*, the only explicit unnameability thesis we receive is that *situations* (*Sachlagen*) cannot be named (3.144). The word 'situation' belongs to a set of contentious Tractarian terms of art including 'state of affairs' ('*Sachverhalt*') and 'fact' ('*Tatsache*'). There is disagreement both about how to translate the relevant German expressions (I give the Pears and McGuiness translations here) and, relatedly, how to conceive of the items satisfying them. What is clear, I think, is that these terms are bound-up sufficiently intimately (Black, for instance, argues that 'Sachlage' and 'Tatsache' should sometimes be understood as approximate *synonyms* [Black, 1964, p. 45]) that for Wittgenstein to hold that items satisfying one can be named, while those satisfying another cannot, would be deeply odd. I see no evidence that Wittgenstein changed his mind about the nameability of facts between the *Notes on Logic* and the *Tractatus*. In truth, the matter is independently settled by 3.203. Situations, states of affairs and facts are all unnameable according to the Tractatus, for it is abundantly clear from numerous passages that none of these

items are Tractarian objects. Objects, for instance, are simple (2.02); situations, facts and states of affairs however, must be complex. The latter two, for example, are directly described as *structured* (2.034). Situations are pictured by propositions (4.031) and can only be so pictured by a proposition if the latter is articulated (4.032) and precisely shares its complexity with the situation (4.04).

1.3.2 Unnameability, Ineffability and the *Tractatus*

We saw, in our discussion of the concept *horse* problems in §1.1.2, that Frege's unnameability commitments appear to have the consequence that certain (putative) truths—notably, certain insights concerning language itself—are rendered inexpressible. This apparent connection between unnameability and ineffability—between the limits of naming and the limits of saying—takes on particular significance in Wittgenstein's case. For, as P. M. S. Hacker puts it, '[t]hat there are things that cannot be put into words, but which *make themselves manifest* (*Tractatus* 6.522) is a leitmotif running through the whole of the *Tractatus*' [Hacker, 2000, p. 353]. Many of the ineffability theses to be found in the *Tractatus*, moreover, concern language itself, its logic and semantics (see, for example, the first few categories of Tractarian ineffabilia listed in [Hacker, 2000, pp. 353-4]). The question arises, therefore, whether the ineffability of that which, according to the *Tractatus*, cannot be said is perhaps to be explained in terms of the unnameability of properties, relations and facts.

Consider the case of relations first.[45] There is a *prima facie* case to be made that the unnameability of relations by particular names prevents the expression of claims about relations. I can assert the obtaining of a relation between particular objects *a* and *b*, by establishing a relation between "*a*" and "*b*". I might do so by writing their names in some direct configuration, or by placing each in spatial relation to a further inscription like "*R*". But if relations are exclusively nameable by relations, I cannot assert, for example, the obtaining of a higher-order relation between relations—say one relation's *naming* another—by establishing a relation between their names in either of these two ways. Their names, as relations themselves, cannot be *written down*

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[45] Again, I think the following considerations carry over, mutatis mutandis, to properties.

in order that they might be directly configured or placed in relation to a further inscription. Nor indeed can I use the oral counterparts of these methods, which would involve (say, temporally) configuring utterances of names and perhaps additional sounds; for I cannot *speak* the name of a relation (for I cannot *speak* a relation!). Relations, then, are neither inscribably nor utterably nameable; and this appears to foreclose the possibility of *saying* of a given relation that it is named by a certain linguistic relation, for doing so would seem to require naming the relations in question with particular names and thereby naming them with non-relations.

The key question, concerning this *prima facie* argument from the unnameability of relations to the ineffability of certain claims, is whether, for Wittgenstein, articulating the claims in question *does* necessitate, as appearances would suggest, naming the relations in question with non-relations. There is, in fact, good reason to think that there is no such necessity. On the present account, one relation can name another; and since naming is itself a relation, and one which *we* (somehow) establish between the symbol and the symbolised, we can clearly establish higher-order relations between relations. Moreover, despite not being able to inscribe or utter the names of relations, we can nonetheless establish a relation between those names in script or speech. For example, the linguistic relation in which "a" and "b" stand, by virtue of flanking "R" is *itself* brought into relation with the linguistic relation in which "a" and "b" stand by virtue of flanking "S", when I, say, establish the former linguistic relation between "a" and "b" on a page and establish the latter linguistic relation between "a" and "b" immediately there below. Indeed, these linguistic relations are hereby multiply related. The former is then co-instantiated by "a" and "b" with the latter; the former is instantiated immediately above an instantiation of the latter, and so on. What is to prevent our settling by convention that one of *these* relations is the name of a higher-order relation (e.g. the relation of *naming* obtaining between a relation and its name), so that the fact whose existence I bring about by the above method—the obtaining of higher-order relation between one relation-name and another—amounts to *the proposition that* the relation named by the former stands in a higher-order relation to the relation named by the latter? If there is nothing to prevent our making higher-order relations names of higher-order relations in this way, then while noteworthy unnameability of relations can be found in the *Tractatus*, it does not suffice for ineffability. The relations in question need not be named in the

way they cannot be named in order to express facts about them. The ineffability of Tractarian unsayables is therefore not explicable, I suggest, by reference to the unnameability of relations.

The unnameability of facts by the lights of *Tractatus* shows particular promise for casting light on the ineffability claims found in that text. That facts are unnameable entails that propositions are unnameable, for according to the *Tractatus* the propositional *sign* is a fact (3.14), and a proposition is just a propositional sign in its projective relation to reality (3.12). (This, in turn, entails the unnameability of thoughts, since a thought, for Wittgenstein, is a proposition with a sense (4).) Those ineffability claims in the *Tractatus* which concern language itself—its semantics and logic etc.—almost all admit of at least *prima facie* justification by reference to the unnameability of facts. For almost all concern the proposition; and it is natural to suppose that in order to say anything about a proposition—that it concerns a particular object (4.1211), that it is a tautology (6.127), that it has a certain modal status (5.525), etc.—it would have to feature as the named subject of another proposition, which is, in Wittgenstein's estimation, impossible. Indeed, in some such cases, expressing that which is deemed ineffable would apparently involve multiple feats of impossible naming: in order, for instance, to state that a certain situation is represented by a certain proposition, one should have to—impossibly—name both.

This account of the grounds for Wittgenstein's logico-semantic ineffability claims forges a close connection between those claims and what Wittgenstein deemed to be his fundamental insight in the *Tractatus*. His *Grundgedanke*, he tells us, is 'that the "logical constants" are not representatives' (4.0312). This thesis is a repudiation of the Fregean conception of the logical connectives as predicates of propositions, signifying functions from propositions (or n-tuples thereof) to truth-values. A key argument for the thesis in the *Notes on Logic* is as follows: 'Logical indefinables [logical constants] cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations'. Why does a proposition's having sense mean that we cannot predicate things of it? Because it is a condition of a proposition's having sense that it is a fact (3.142) and facts cannot be named. Predicating something of a proposition, however, would apparently require naming it.

The account also furnishes an explanation for the inexpressibility of the logical form shared by a proposition and the reality it depicts; for, again, a proposition could not be named in order

that form be attributed to it by means of a proposition. The ineffability of logical form has been explained by some, however, in terms of the unnameability of logical form *itself*, rather than its bearer. Thus Candlish and Damnjanovic suppose for *reductio* that there could be a proposition featuring the name of a logical form, and reason as follows:

> [A]ccording to the picture theory, for such a proposition to be true, there would need to be a fact which contains some thing, the logical form, as a constituent. Yet Wittgenstein's position relies on forms not being possible constiuents of facts: they are instead the mode of combination of the objects which are the constituents of facts. Therefore, we cannot name logical forms. [Candlish and Damnjanovic, 2012, p. 93] [46]

The authors are right to attribute to Wittgenstein the view that logical forms of facts are unnameable. He says as much in the 'Notes on Logic': 'There is no thing which is the form of a proposition, and no name which is the name of a form' [Wittgenstein, 1979, p. 105]. The *account* of Tractarian logical form embedded in their argument, however, is mistaken, and once we're clear on the nature of logical form, the account becomes unsatisfactory. The logical form of a fact (proposition) is definitely *not* the mode of combination of its constituent objects (names).[47] This is, rather, the *structure* of the fact (proposition), as we are told in 2.03-2.032.[48] The *form* of a fact is defined immediately thereafter as the *possibility* of this structure (2.033). This definition in fact presents the third notion of form we've received at this early stage in the text. The second was that of the form of the *world*, where form of this kind is identified with that which the world shares with every imaginable world—namely, the totality of objects (2.022, 2.023). The first was that of the form of an *object* (2.0141), where form of this kind is identified with the possibility of its (the object's) occurring in states of affairs.

The first notion, I submit, is fundamental. A fact's having the form it does, and indeed the world's having the form it does, consist precisely in their constituent objects' having the forms they do. Anscombe is quite right to urge that '[w]e must remember that the original seat of form

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[46]The same explanation is offered by Priest [Priest, 2002, p. 188-89].

[47]The same misconception appears in Priest's account, where logical form is characterised as 'the way that objects are put together' [Priest, 2002, p. 189].

[48]Actually, in these passages the modes of combination of objects are said to constitute structures of *states of affairs*; but two remarks later (2.034) Wittgenstein confirms that the structure of a fact just consists in the structure of the corresponding state(s) of affairs. It is safe, I think, to assume that the same holds of form.

is the objects themselves' [Anscombe, 2001, p. 110]. The form of an object comprises its capacities for combining with other objects in the constitution of states of affairs. The combinatorial capacities of objects taken collectively thus settle which maximal sums of such combinations—that is, which *worlds*—are possible: they determine logical space (2.014). What the actual world has in common with every imaginable world is its being situated in this space—its being one such maximal sum. But the world's being so situated—its having the form it does—is just a matter of objects having the combinatorial capacities, the forms, they do. The same is true of facts. The form of a fact cannot be the mode of combination of its constituents, for it is a condition of the possibility of a fact's (e.g. a proposition's) depicting the (portion of) reality it does that it share form with that reality (2.18, 4.12). But if form were the mode in which constituents are combined, then a proposition could only depict reality if it combined its constituents as the objects for which they stand are in reality combined. This, however, would make false representation—i.e. incorrect depiction—impossible; for a fact could not depict the objects for which its constituents stand as being combined in a manner in which they are not combined. Commonality of form between depicting fact and reality requires not that the modes in which they configure their constituents be the same but rather that their respective constituents *could* be configured in the same way (2.151). A fact's (proposition's) having the form it does, therefore, is just a matter of its constituent objects' (names') having the combinatorial capacities, the forms, they do.

But this latter consideration renders Candlish and Damnjanovic's account unsatisfactory; for it is now simply unclear that in order to express that a fact (proposition) has the form it does it would be necessary to *name* that form. It now seems, on the contrary, that it would suffice to name each of its constituents and say of each of them that they have the combinatorial capacities they do. However, this also appears to tell equally against the foregoing explanation of the ineffability of form by reference to the unnameability of facts. For it now appears that in order to state a fact's logical form one need only be able to name the constituents of that fact, not the fact itself, and to say something of each constituent.

The broader concern of which this particular worry is an instance is that, contrary to initial appearances, we do not need to name facts in order to say things about them. This is,

after all, precisely the Tractarian position on *at least* the vast majority of things we ordinarily take ourselves to name. Carfax Tower and Tom Tower, for example, are, uncontroversially, not Tractarian objects—both are composite—and neither, therefore, can be named according to the *Tractatus*. But that the former is to the north of the latter is not thus rendered ineffable; for neither tower *need* be named in order that that fact be expressed. It suffices, according to the *Tractatus*, to frame a certain (perhaps infinitely) complex truth-function of elementary propositions each of which asserts the obtaining of a single state of affairs, and none of which feature a name for either tower, only names for the simple constituents of states of affairs. The putative names 'Carfax Tower' and 'Tom Tower' in fact disappear upon analysis.

But why not accord the same status to expressions ostensibly naming facts—expressions like 'the fact that aRb' naturally employed in attempt to articulate claims about facts? Indeed Wittgenstein does just that in the 'Notes Dictated to Moore' [Wittgenstein, 1979, p. 112]. Moreover, in the *Tractatus* itself (5.541-2) we receive a proposal—albeit programmatic in the extreme—as to how the analysis of propositions having facts (specifically propositions) at least superficially as their subject matter will proceed. The propositions in question are those ascribing propositional attitudes to subjects, such as 'A thinks (believes, knows) that p'. Such propositions, we are told, are all of the form ' "p" says that p'. According to Wittgenstein, someone's thinking that p is a matter of their harbouring a mental picture—itself a fact whose elements are psychical constituents related to the world in the same manner as lexical names ((3), [Wittgenstein, 1979, p. 131])—with the representational content that p. To say that A thinks that p, Wittgenstein is claiming then, is to say of some such mental picture of A's that it has that content—that it says that p. This is to assert a correlation between facts (not between a subject and a fact as appearances would suggest). But crucially, Wittgenstein tells us that that correlation really amounts to a correlation between their respective *constituents*. If so, the facts themselves need not be named in order to express that correlation, only their constituents. Accordingly, while ' "p" says that p' appears to feature as its subject a name for a proposition, that expression would disappear upon analysis.

The inference from the unnameability of facts to the ineffability of claims concerning them is therefore thoroughly insecure given Tractarian commitments. The unnameability of ordinary

composite objects is, by Wittgenstein's lights, no impediment to the expression of that which
we express when we employ unanalysed propositions in which composite objects are ostensibly
named; and it is unclear why facts should differ in this regard. Moreover, Wittgenstein outlines a
programme of analysis for a whole swathe of claims concerning facts—namely, propositional at-
titude ascriptions—according to which those claims are expressible without recourse to naming
facts. Promising though the suggestion initially seems, I suggest, then, that the ineffability theses
found in the *Tractatus* are not to be explained by reference to the commitments to unnameability
also present in that text.

1.4 Looking Ahead

It is an important question whether Frege too is in a position to resist the *prima facie* argument,
presented in §1.1.2, to the effect that his unnameability commitments render various semantic
insights ineffable. Can Frege, for example, deny, on principled grounds, the appearance that in
to order specify the *Bedeutung* of a predicate, one would have to, *per impossibile*, designate that
Bedeutung with a singular term?

Though these questions unquestionably deserve careful investigation, I do not intend to
pursue them here. Rather, I propose to examine, in the chapters that follow, what grounds
there are for accepting, with Frege, Russell and Wittgenstein, that there is that which cannot be
named. My particular focus throughout will be Frege's thesis that *concepts* cannot be named. My
central concern will be to examine in detail the grounds on which Frege advanced this thesis,
and to investigate whether it can be effectively substantiated.

Chapter 2

Substitution and Co-reference

If we are concerned to establish Frege's own grounds for accepting the thesis that no singular term can designate a concept, we do well to turn to the *locus classicus* of that thesis—his 1892 essay 'On Concept and Object'. In this chapter and the next we will be centrally concerned with examining arguments drawn from this essay.

2.1   Concept and Object

It was Frege's view that the categories of *concept* and *object* are disjoint—that no concept is an object. In his 'On Concept and Object' Frege replies to his critic Benno Kerry, who had contested this view. According to Frege's diagnosis [Frege, 1997h, p. 182, 185], Kerry's opposition to the view results from his neglecting to take the word 'concept' in Frege's intended sense. Frege accordingly undertakes to clarify the senses in which he intends 'concept' and 'object' and to demonstrate that, thereupon, putative examples of concepts that are also objects—such as those adduced by Kerry—are not genuine such examples.

This, at any rate, would be a fairly standard, albeit adumbrative description of Frege's position and purposes in that seminal paper. It is noteworthy, however, that Frege's explanation of the intended senses of the expressions in question *stipulatively* precludes any such counterexample: it is true simply *in virtue of* the senses he confers upon these expressions that no concept is an object:

> [T]aking 'subject' and 'predicate' in the linguistic sense: a concept is the *Bedeutung*
> of a predicate; an object is something that can never be the whole *Bedeutung* of a
> predicate, but can be the *Bedeutung* of a subject' [Frege, 1997h, pp. 186-187].[1]

The stipulative exclusivity of concepthood and objecthood, on Frege's 1892 use of the expressions, is also apparent in his treatment of Kerry's purported counterexamples. Concerning Kerry's contention that the concept 'horse' is an object, Frege replies:

> Quite so; the three words 'the concept "horse"' do designate an object, but on that
> very account they do not designate a concept, as I am using the word [Frege, 1997h,
> p. 184].

This is odd, because Frege's explanation is advertised [Frege, 1997h, p. 186] as a mere terminological revision of the explanation given in *The Foundations of Arithmetic* of 1884, updated to accord with Frege's distinguishing *thought* and *truth-value* in what he had formerly designated 'judgeable content'; the *Foundations* explanation, however, does *not* build exclusivity into the senses of 'concept' and 'object' (here 'subject' and 'predicate' are obviously not being used in the linguistic sense):

> A concept is for me a possible predicate of a singular judgeable content, an object a
> possible subject of such a content. [Frege, 1953, p. 77e] [2]

Since for all this says something can be *both* a predicate of a singular judgeable content and a subject of such a content, this is quite consistent with the thesis that something is both a concept and an object. So however Kerry might have misunderstood Frege's use of 'concept', his misunderstanding could not have been a matter of his failing to appreciate that on Frege's use

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[1]Frege's thesis that no concept is an object is, of course, a corollary of his more general thesis that no *function* is an object. His earlier explanation, in 'Function and Concept', of the intended senses of 'function' and 'object' yet more plainly renders it true in virtue of those senses that no function is an object:

> [T]he question arises what it is that we are here calling an object. I regard regular definition as impossible...It is only possible to indicate what is meant. Here I can only say briefly: an object is anything that is not a function ... [Frege, 1997e, p. 140]

In 'On Concept and Object' Frege likewise cautions that his 'explanation' (*Erklärung*) [Frege, 1997h, p. 182, p. 186] is not advanced as a 'proper definition' [p. 182]; he denies that the terms in question admit of such. It wouldn't be appropriate, therefore, to say that Frege's remarks render it true *by definition* that no concept is an object. Nevertheless, it seems clear that if Frege's explanation succeeds at all in indicating what is meant by the expressions, it settles that, in virtue of what those expressions mean, nothing satisfies both of them.

[2]I've modified Austin's translation as in [Frege, 1997d, p. 113]

of the expressions, if something is object, it is *on that very account* not a concept; for that is not true of Frege's use of the expressions as explained in the work Kerry was criticising. In any case, though Frege's purported target in 1892 is Kerry's 'mak[ing] out that the distinction between concept and object is not absolute' [Frege, 1997h, p. 182], and though he rounds off his rebuttal with 'Thus Kerry does not succeed in filling the gap between concept and object' [Frege, 1997h, p. 187], there is in fact no substantive issue that can be formulated in Frege's 1892 nomenclature as the issue whether there is absolute distinction, or unfilled gap, between concept and object. In the 1892 nomenclature, the issue of substance is shifted elsewhere; it becomes instead whether, as Frege maintains, everything that can be the *Bedeutung*—or as I shall say, the *referent*—of a singular term is an object. Similarly, the challenge posed by the concept *horse* and its kin becomes not that it seems that these entities should be both concepts and objects, but that it seems that these entities should be concepts, and thus (trivially) not objects, and thus non-objects that are nevertheless referents of singular terms.

We can circumvent these terminological subtleties, of course, by framing the issue without use of 'concept' and 'object' as follows: Are any of the things to which predicates can refer also things to which singular terms can refer? Frege's answer is negative: no possible referent of a singular term is a possible referent of a predicate—or, the categories of *possible referent of a singular term* and *possible referent of a predicate* are disjoint. However, rather than relinquishing 'object' and 'concept', I suggest we simply omit that feature of Frege's explanations which builds exclusivity into the senses of these terms. The following explanations result: an object is a possible referent of a singular term, a concept a possible referent of a predicate. These explanations, it seems to me, *can* lay claim to being simply updated renditions of the *Foundations* explanations of the meanings of these expressions. Moreover, it lets us honour the standard description of Frege's position and purposes in 'On Concept and Object' with which we began. With 'concept' and 'object' thus understood, we do formulate a substantive philosophical thesis in saying that no concept is an object; and we can credibly describe Frege as concerned to *defend* the thesis— rather than merely stipulate—that no concept is an object. This, in any case, will be the use to which we shall here put 'concept' and 'object'.

How, then, *does* Frege defend the thesis that no concept is an object? His response to chal-

lenges to that thesis looks, for much of 'On Concept and Object', like mere bullet-biting. He simply accepts, for example, the counter-intuitive consequence that 'expressions like 'the concept F' designate not concepts but objects' [Frege, 1997h, p. 187], from which follows the paradoxical-sounding claim that the concept *horse* is not a concept. Furthermore, he seems simply to accept the expressive impediments engendered by the thesis that concepts are unnameable [Frege, 1997h, p 192] (page 15 above).

There are, I submit, just two positive arguments for Frege's thesis to be discerned in 'On Concept and Object'. The passage most explicitly advertised as such is the article's penultimate paragraph, in which, having acknowledged the referential and expressive difficulties to which Frege's thesis gives rise, the author appears to argue that the recognition of 'such an unmanageable thing' [Frege, 1997h, p. 192] as an unnameable concept is unavoidable if we are satisfactorily to account for the unity of thought. The next chapter will be devoted to an examination of this argument. The other argument, on which we focus in the present chapter, is found in the following brief passage:

> Thus the words 'the concept *square root of 4*' have an essentially different behaviour, as regards possible substitutions, from the words 'square root of 4' …; i.e. the *Bedeutungen* [referents[3]] of the two phrases are essentially different.
>
> What has been shown here in one example holds good generally … [Frege, 1997h, p. 189]

The thrust of the argument is clear: for any singular term, t, and any predicate, p, t and p will exhibit different behaviour as regards possible substitutions; therefore, t and p do not co-refer.[4] That no singular term co-refers with a predicate—a claim I shall in this chapter refer to simply as *Frege's thesis*—is, given the above terminological considerations, equivalent, for Frege, to the claim that no concept is an object. Fleshing out this argument involves indicating exactly what kind of difference of behaviour as regards possible substitutions is being presented

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[3] Quoted translations will sometimes be modified by rendering 'Bedeutung' as 'referent'.

[4] Frege often informally speaks as though concepts were referents of predicative expressions, like 'a square root of 4', or even merely common-nominal phrases, like 'square root of 4', rather than referents of fully-fledged predicates, like 'ξ is a square root of 4'. The passage being discussed is an example of this habit. Frege's official position, however, is that it is the predicate entire—including the copula or finite verb form—that refers to a concept. This is entailed, for example, by his definitional remarks in the *Grundgesetze* [Frege, 1997d, pp. 218-19]. I take it, therefore, that it is, strictly, 'the concept *square root of 4*' and 'ξ is a square root of 4' whose referents Frege is claiming must differ.

as inconsistent with co-reference. A very natural proposal is that Frege is here invoking an intersubstitutability principle that he asserts in the contemporaneous 'On Sense and Reference':

> [T]he truth-value of a sentence remains unchanged when an expression in it is replaced by another with the same referent. [Frege, 1997i, p.159]

Alternatively put:

Salva Veritate Co-referential expressions are intersubstitutable *salva veritate* in all contexts.

Frege thinks that if this principle did not hold, his identification of the referent of a sentence with its truth-value would be untenable:

> If our supposition that the referent of a sentence is its truth-value is correct, the latter must remain unchanged when a part of the sentence is replaced by an expression with the same referent. And this is in fact the case. [Frege, 1997i, p.158]

It is no mystery why he thinks this: it is a cardinal doctrine of Frege's semantics that, quite generally, the referent of a complex expression is determined by the referents of its constituents. This is one of two compositionality theses Frege advances—the other being that the *sense* of a complex expression is determined by the senses of its constituents.

Predicates and singular terms, however, fail to satisfy the condition of intersubstitutability *salva vertitate*: replacing a predicate in a sentence with a singular term *does* precipitate a change in the truth-value of the sentence. This is true even of predicate-term pairs that appear very good candidates for co-reference: for example, pairs such that the term is a nominalization of the predicate ('ξ is a horse', 'horsehood') and pairs such that the term is an expression of the form $\ulcorner$ the concept F $\urcorner$, where $\ulcorner$ F $\urcorner$ is the result of omitting the copula and indefinite article from the predicate ('ξ is a horse', 'the concept *horse*') or is the participle of the predicate's main verb ('ξ runs', 'the concept *running*'). The change brought about by the substitution of a term for a predicate is not a switch but a *loss* of truth-value:

(1) Shergar is a horse.

(2) * Shergar the concept *horse.*

Can Salva Veritate, then, be satisfactorily adduced in justification of Frege's thesis? The difficulty with doing so is that it is a matter of considerable controversy whether Salva Veritate is even true. There *appear* to be counterexamples. Familiarly, contexts featuring intentional verbs (e.g. 'knows', 'believes', 'thinks', 'says') seem to fail to sustain the intersubstitution *salva veritate* of co-referring expressions. (Substitute 'Charles Dodgson' for 'Lewis Carroll' in 'John knows that Lewis Carroll composed "Jabberwocky"' and you may well obtain a false sentence from a true one, or vice versa.) There are, of course, strategies for reconciling Salva Veritate with intensional contexts. Frege's own response to putative counterexamples of this kind, as is well known, was to deny that customarily co-referring terms share their referents in such contexts. When occurring within the scope of an intentional verb, he claimed, an expression does not possess its ordinary referent, but rather designates the item that is customarily its sense. Since ordinarily co-referring expressions may differ in sense, they may in consequence have different referents when in the scope of an intentional verb. According to Frege, then, failures of intersubstitutability in such contexts are, initial appearances notwithstanding, the result of failures of co-reference. One might alternatively seek to reconcile Salva Veritate with intensional contexts by denying the appearance Frege accepts: that intersubstitution of customarily co-referring terms in such contexts fails to preserve truth-value.

The controversy attending intensional contexts, in which Salva Veritate embroils us, is, however, largely extraneous to the issue of whether Frege's thesis is defensible. The disturbance to truth-value brought about by the substitution in a sentence of a singular term for a predicate is not explicable by reference to intensionality. The crucial consideration concerning such substitutions seems to be that they yield something that is not even grammatically well-formed. Salva Veritate precludes co-reference between a singular term and a predicate because, very plausibly, the preservation of grammatical well-formedness is a condition of the preservation of truth-value. It is natural, therefore, to isolate the preservation of grammatical well-formedness as a condition on co-reference.

2.2   Intersubstitutability *Salva Congruitate*

The result of doing so is the substance of Crispin Wright's [Wright, 1998] *Reference Principle*:

(RP) Co-referential expressions are intersubstitutable *salva congruitate* (preserving grammaticality) in all contexts.

Granting that preservation of grammaticality is a condition of preservation of truth-value, (RP) is a corollary of Frege's Salva Veritate. It was Frege's commitment to (RP), Wright thinks [Wright, 1998, p. 73], that drove him to insist that no singular term co-refers with a predicate; for singular terms and predicates are not pairwise intersubstitutable *salva congruitate* in all contexts. [5]

Can (RP) be appealed to as justification for Frege's thesis? My concern in this section will be to show that it cannot. Straightforwardly read, (RP) is false and therefore justifies nothing. David Dolby [Dolby, 2009] has mounted a defence of (RP), and of its adduction in support of Frege's thesis, by appeal to an alternative reading of the principle. I shall argue that Dolby's defence is not undermined by an objection raised by Robert Trueman [Trueman, 2012], but fails nevertheless; for the reading of (RP) upon which it depends both is inadmissible and renders (RP) impotent to justify Frege's thesis.

To see that (RP), straightforwardly read, is false, note that there are, as Alex Oliver [Oliver, 2005] demonstrates, numerous cases in which grammaticality is lost by substituting an expression for another with which it uncontroversially co-refers. Examples are to be found, for instance, where the substitutional context features a pre-modifying adjective: substitution of 'the Big Apple' for the co-referring 'New York' in 'Bustling New York is a centre of finance' results in the ungrammatical 'Bustling the Big Apple is a centre of finance'. Two further kinds of example—on which we will focus in what follows—are to be found where co-referring expressions differ in grammatical case or in gender. Such examples are particularly prevalent in more inflected languages like German. An example involving a difference of grammatical case:

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[5]In Wright's view, (RP), though implicated in Frege's lapse into the concept *horse* paradox, itself points the way out. Very briefly: Wright argues 'purely by appeal to [(RP)]' [Wright, 1998, p. 84], that predicates do not refer to concepts (rather, he thinks, they ascribe them) and thus that (RP) 'puts no barrier before the idea that *singular terms* can after all *refer* to concepts' [Wright, 1998, p. 85, emphasis original]. However, both Wright and Wright's Frege accept Frege's thesis on the basis of (RP).

(3) John fällt in den Rhein.[6]

(4) * John fällt in der Rhein.

'den Rhein' in (3) is appropriately inflected for the accusative case; 'der Rhein' is not, and (4) is consequently ungrammatical. But the two terms obviously co-refer. An example involving a difference of gender:

(5) Die Ostsee ist bekannt für ihre Schiffswracks.[7]

(6) * Das Baltische Meer ist bekannt für ihre Schiffswracks.

In (5), the possessive adjective 'ihre' is in agreement with the feminine subject; in the ungrammatical (6), 'ihre' is at odds with the neuter subject. The subjects of the two sentences co-refer, however; they are both names of the Baltic Sea. *Pace* (RP), then, not all co-referential expressions are intersubstitutable *salva congruitate* in all contexts. That singular terms and predicates are not thus intersubstitutable does not indicate, therefore, that they never co-refer.

2.2.1 Dolby's Defence of the Reference Principle

David Dolby [Dolby, 2009] has sought to defend (RP) by appeal to an alternative reading of the principle, invoking a richer notion of substitution.[8] The counterexamples to (RP) above were set up on the assumption that what it is to substitute one expression, β, for another, α, in a sentence, ϕ, is nothing more than to omit α from ϕ and insert β in its place: substitution is a matter of simple cut-and-paste. Dolby's contention is that there is legitimate and standard sense of 'substitution' not so austere, and that when, in (RP), the cognate 'intersubstitutable' is intended in the corresponding sense, cases like the above are in fact consistent with (RP). For in *this* sense the examples are not cases of an expression being substituted for a co-referential expression.

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[6]John falls into the Rhine.

[7]The Baltic Sea is known for its shipwrecks.

[8]Dolby thinks that some of the counterexamples raised in Oliver's paper can be dealt with by restricting (RP) to referential positions [Dolby, 2009, pp. 287-9]. The counterexamples we shall focus on, however, are of the kind Dolby thinks are 'the most challenging' [Dolby, 2009, p. 289] and necessitate appeal to an alternative notion of substitution.

'We arrive at such a notion [of substitution]' Dolby says, 'if we consider the way in which competent speakers substitute expressions according to grammatical rules' [Dolby, 2009, p. 290]. When charged with substituting an expression, Dolby suggests, competent speakers will make alterations requisite for grammaticality as part of the substitution. This sense of 'substitution', he claims, is also 'the sense in which students in a language class might be asked to to substitute various phrases into a sentence. Such students would be expected to make all the changes necessary for [grammatical] agreement as part of the substitution' [Dolby, 2009, p. 290-1]. *What it is* to substitute, in this sense, β for α in ϕ, may *involve* making alterations to ϕ other than the omission of α and insertion of β in its place. It may, indeed, involve making changes to β. When it does involve such additional alterations, they are precisely those that are required to ensure grammaticality. Substituting 'Das Baltische Meer' for 'Die Ostsee', in this sense, would involve changing the anaphoric possessive adjective to accord with the gender of the substituend, and would yield not (6), but

(7) Das Baltische Meer ist bekannt für seine Schiffswracks.

Substituting 'der Rhein' for 'den Rhein' in (3) would involve changing the definite article in the former to 'den'. Doing so just yields the expression that is to be replaced, so the substitution results in (3) once more. On this richer notion of substitution, then, we've yet to be given a case in which the substitution of co-referential terms fails to preserve grammaticality.

Small wonder, one might think, given that the variety of substitution being invoked involves making those alterations required to retain grammaticality. If 'intersubstitutable' is correspondingly intended in (RP), one might think, the principle is vacuous; for substitution of this kind will, trivially, never yield an ungrammatical result. Thus, failure of intersubstitutability *salva congruitate* can no longer serve as a test of difference of reference; for intersubstitutability *salva congruitate* will never fail.

It is indeed so that substitution of this kind will never produce an ungrammatical result. Nevertheless, Dolby claims [Dolby, 2009, p. 294-5], it not true that intersubstitutability *salva congruitate* will never fail. There will be failures; however, they will not be cases in which two expressions *can* be substituted for one another, but sometimes (or always) only with the loss of

grammatical well-formedness. Rather, they will be cases in which two expressions just cannot, in the present sense, be substituted for one another. This is the case when there do not exist grammatical rules for the intersubstitution of two expressions. Dolby gives as an example the pair: 'quickly', 'the number seven' (ibid). Since intersubstitutability—and *a fortiori*, intersubstitutability *salva congruitate*—requires that there be such rules, (RP) entails that there are such rules for any pair of co-referential expressions. (RP) therefore does furnish a test of difference of reference, and indeed does, Dolby thinks [Dolby, 2009, p. 295], justify Frege's thesis that no singular term co-refers with a predicate: 'there are no rules', he claims, 'for the substitution of 'is a horse' for 'the concept horse'' [Dolby, 2009, p. 295].

Crucially, Dolby claims that the grammatical rules for substitution 'are also the rules for the formation of generalizations from particular statements and for the specification of these generalizations' [Dolby, 2009, p. 290]. They are the rules, he thinks, for quantifying into the position of the expression to be replaced in the sentence and then instantiating the result for the expression to be substituted in. On this conception, the substitution of 'Das Baltische Meer' for 'Die Ostsee' in (5) proceeds according to the grammatical rules for (existentially) generalizing from (3), to obtain,

(8) Etwas ist bekannt für seine Schiffswracks.[9]

and then specifying that generalization for 'Das Baltische Meer', giving (7). Generally, substitution of β for α in ϕ is taken to consist in generalizing ϕ with respect α and specifying the resultant generalisation for β.

2.2.2 Trueman's Circularity Objection

Robert Trueman [Trueman, 2012] has objected that this last aspect of Dolby's account of substitution—the identification of the rules for substitution with those for generalization and specification—renders Dolby's account viciously circular. The worry is that Dolby's account presupposes an understanding of the notions of generalization and specification; but these notions must themselves be explained partly in terms of the very notion of substitution an account of which Dolby

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[9]Something is known for its shipwrecks.

is seeking to give. For example, existential generalization with respect to a particular term involves substituting a quantifier phrase for that term. Specifying the the resultant generalisation with a particular term involves substituting that term for the quantifier phrase. But Dolby must deny that the variety of substitution involved in generalization and specification is the simple, cut-and-paste variety; for if it were, existential generalization with respect to 'Die Ostsee' in (5) would yield

(9) * Etwas ist bekannt für ihre Schiffswracks.

in which we have simply omitted 'Die Ostsee' and inserted the quantifier phrase 'Etwas' in its place. Specification of (9) with 'Das Baltische Meer' would merely involve pasting the latter in place of 'Etwas', giving the ungrammatical (6). We would then be back with a counterexample to (RP). If, as Dolby requires, substitution followed by specification is to take us from (5) to (7), and not to (6), the substitution involved in generalization and specification must be the richer variety of substitution an account of which Dolby is trying to give. Thus in giving an account of substitution Dolby 'has appealed to generalization and specification, which are themselves to be accounted for in terms of substitution' [Trueman, 2012, p. 3]: the account, Trueman concludes, is viciously circular.

Though Trueman has located a circularity in Dolby's account, I don't think he has done anything to establish that the circularity is vicious. Notwithstanding the challenge of giving a theory of 'how exactly quantification works in natural languages' [Trueman, 2012, p. 2], we do have a good grip on generalization and specification in our home language. If this means that we have an implicit grasp of the notion of substitution to which Dolby is appealing in his defence of (RP), so much the better for Dolby. Indeed, that we already have a pre-theoretic grip on that notion of substitution seems to be precisely what Dolby is claiming: he claims that the notion is 'a standard sense of ['substitution']' [Dolby, 2009, p. 290] and characterises it, in the first instance, by reference to 'the way in which competent speakers substitute expressions according to grammatical rules' [Dolby, 2009, p. 290]. Trueman objects that '[e]ven if...Dolby's account is materially adequate, it could never be used to introduce substitution in English to someone in the first place' [Trueman, 2012, p. 3]. But I fail to see why we ought to impose this extraordinarily

demanding constraint on Dolby's account. I take it that the person to whom Trueman imagines substitution being introduced 'in the first place' is someone lacking the conceptual apparatus of generalization and specification; it's tempting to echo the sentiment David Lewis expressed in a different connection: 'since such unfortunates are rare, even among philosophers, we needn't worry if their condition is incurable' [Lewis, 1986, p. 193].

2.2.3 Against Dolby's Defence of the Reference Principle

The fact that generalization and specification involve the richer variety of substitution to which Dolby wishes to appeal *does*, I think, present a serious problem for Dolby's defence of (RP); but it is not a problem of vicious circularity as Trueman alleged. Rather, the problem is this. If substitution of β for α in ϕ consists in existentially generalizing ϕ with respect to α and specifying the resultant generalisation for β, then it is impossible to substitute expressions we use to *form* generalisations—namely, quantifier phrases. It is impossible, for example, to substitute the quantifier 'something' for 'Germany' in 'Germany is a nation'; for one cannot use the quantifier to *specify* a generalisation that results from generalizing that sentence with respect to 'Germany'. But generalisation, as Trueman notes, itself requires just such substitution of quantifier phrases; and according to Dolby's account, substitution of any expression in turn involves generalization. The upshot is that Dolby's account makes substitution of any expression impossible. Similarly, it is impossible to substitute an expression *for* a quantifier phrase, for one cannot generalize a sentence with respect to a quantifier phrase. Since specification involves just such substitution and, for Dolby, substitution of any expression in turn involves specification, we again have the impossibility of substituting any expression. On Dolby's account, then, expressions are *never* intersubstitutable, *salva congruitate* or otherwise—not even with themselves! Clearly, though, there is no admissible reading of 'intersubstitutable' in (RP), on which expressions are never intersubstitutable. I conclude that Dolby's reading of (RP) is inadmissible. It also renders (RP) false, and therefore impotent to justify Frege's thesis; for (RP) makes intersubstitutability a condition of co-reference, and therefore (since on Dolby's reading, no expressions are intersubstitutable) has the obviously false consequence that expressions never co-refer.

Perhaps Dolby could respond by denying that the variety of substitution involved in gener-

alisation and specification is the same variety of which he is trying to give an account. I cannot see how this response might be motivated or developed, but suppose it can be. There remain strong grounds for rejecting Dolby's account. Dolby entertains the objection that the procedure of generalisation and specification 'is too complicated to be called 'substitution'', but responds that 'it is ... a standard sense of the word' [Dolby, 2009, p. 290]. Actually, the problem isn't that the procedure Dolby calls 'substitution' is too complicated to merit the name, but that it is far too restricted in application. It is not only quantifier phrases that, in Dolby's sense of 'substitution', one can neither substitute for an expression nor substitute an expression for; the same, obviously, is true of any expression occurring in a sentence in a position into which one cannot quantify. One cannot, for example, substitute a possessive determiner, like 'our' or 'the people's', for the indefinite article in 'Germany is a nation'; for one cannot quantify into the position of the indefinite article to produce a generalisation one might specify with a possessive determiner. The reader can readily enumerate examples of types and tokens of expressions excluded from substitution for this reason. No reading of 'substitution' that renders substitution of these expressions impossible should be deemed standard. Indeed, no such reading should be deemed even admissible. Again I conclude that Dolby's reading of (RP) is inadmissible.

Two options remain for Dolby, I think. Each involves jettisoning the claim that there is a sense of 'substitution' on which substitution consists in generalization followed by specification. The first is to fall back on the more impressionistic characterization of substitution as proceeding according to grammatical rules and involving alterations required for grammaticality. The reading of (RP) that results from this characterisation of substitution may be admissible, but I submit that it renders (RP) impotent to justify Frege's thesis. For someone who maintains that some singular terms do co-refer with predicates can coherently accept (RP) by claiming that there *do* exist grammatical rules for the intersubstitution of singular terms and predicates. Towards maintaining that 'ξ is a horse' and 'the concept horse' co-refer, I might claim that the result of substituting the latter for the former in (1) is not (2) but

(10) Shergar falls under the concept *horse.*

In (10), an expression ('falls under') occurs that was a part neither of the substituend nor the

sentential context into which it was substituted. (Accordingly, call this approach to substituting

a singular term for a predicate the *auxiliary material strategy*.) But the introduction of auxiliary

material does not alone disqualify (10) from being the result of substituting the singular term for

the predicate; for the substitution of 'Das Baltische Meer' for 'Die Ostsee' in (5) also involved

the introduction of auxiliary material—in that case, 'seine'. Alternatively, I might claim that (1)

itself is the result of substituting 'the concept horse' for 'ξ is a horse'. The idea would be that this

is one of those cases in which the substitution involves alteration of the substituend, where the

alteration involved is just that which, as it were, turns the substituend into the very expression

for which it is to be substituted. Like the substitution of 'der Rhein' for 'den Rhein' in (3), the

present case (the thought goes) is what we may call a *null substitution*: a substitution that yields

the very sentence into which it is made.[10] In the former case, the alteration required was that of

inflecting the substituend for the accusative case; in the present case, the alteration might readily

be called 'de-nominalisation': it is that alteration that turns 'the concept horse', for example, into

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[10]We might alternatively call this a *fixed point substitution*, by analogy with a fixed point of a function. The analogy raises the question whether, given a substituend, β, and expression for which it is to be substituted, α, Dolby substitution of β for α is a function (from sentences to sentences). If so, the sentences for which this substitution is a null substitution will be fixed points of this one-place function. More generally, we might ask whether Dolby substitution is a three-place function taking as arguments a sentence, ϕ, into which the substitution is to be made, a substituend, β, and an expression, α, for which the substitution is to be made. Such a function might be symbolised, $\phi\,[\beta/\,\alpha]$. The answer to this second question is negative, since there will often be more than one way of Dolby-substituting β for α in ϕ, by virtue of there being more than one way of securing grammaticality. Consider, for example, substituting 'the Titanic' for 'the Buddhas of Bamiyan' in the sentence 'The Buddhas of Bamiyan amazed all who saw them'. The substitution would demand replacing the plural anaphoric pronoun 'them', but as replacements both 'her' and 'it' would suffice to secure grammaticality. The answer to the first question depends, consequently, on the particular α and β being held fixed. For some α, β, Dolby-substitution of β for α will be a function. For example, assuming that alterations are only to be made as part of the substitution where this is necessary to secure grammaticality, and that substitution is only made into grammatical sentences, Dolby-substitution of β for α will be a function (though not a very interesting one) if, for example, α and β are one and the same proper name.

It is worth noting in this connection that even the austere, cut-and-paste variety of substitution, to which Dolby-substitution is intended to be an alternative, is not obviously a function. The reason is that in some sentences into which a substitution is to be made there will be multiple, overlapping occurrences of the expression for which the substitution is to be made. Consider cut-and-paste substitution of 'colleague' for 'friend of a friend' in the sentence 'A friend of a friend of a friend is coming to stay'. The sentences 'A colleague of a friend is coming to stay' and 'A friend of a colleague is coming to stay' ought equally, it seems, to be counted results of this substitution. Perhaps another result is the ungrammatical 'A colleague colleague is coming to stay', in which each of the overlapping occurrences of 'friend of a friend' have been replaced by an occurrence of 'colleague'. Yet another kind of result would be available if the substituend was itself susceptible of overlapping occurrences: substituting 'neighbour of a neighbour' for 'friend of a friend' may give 'A neighbour of a neighbour of a neighbour is coming to stay', in which the two overlapping occurrences of the substituend have been substituted for the two overlapping occurrences of the expression to replaced. The functionality of cut-and-paste substitution could be obtained by fairly simple stipulation concerning such cases: we might decree that substitution is always to be made only for the left-most occurrence wherever there is overlap. (I'm grateful to Beau Mount for bringing cases of this kind to my attention, and for discussion.) By contrast, it seems unlikely that simple stipulation will secure functionality for Dolby substitution.

'ξ is a horse'. (Call this approach to substituting a singular term for a predicate, then, the *null substitution strategy*.) Whether I propose that the substitution of a singular term for a predicate is an auxiliary material substitution or a null substitution, a corresponding proposal will be available to me with regard to substitution in the other direction—of a predicate for a singular term. Each strategy allows me coherently to accept (RP), on the reading we are now supposing Dolby to advance, whilst maintaining that some singular terms co-refer with predicates.

The second option would be for Dolby to abandon the project of defending (RP) and instead claim only that the principle he had (erroneously) advanced as a reading of (RP) is true and justifies Frege's thesis. That principle can be formulated thus:

(DP) If expressions α and β co-refer, then for any sentence, ϕ, in which α occurs, ϕ can be existentially generalized with respect to α and the resultant generalization can be specified with β to produce a grammatical result.

However, it strikes me, again, that someone who denies Frege's thesis can coherently accept (DP), and therefore that (DP) is impotent justify Frege's thesis. (DP) implies that if 'the concept horse' and 'ξ is a horse' co-refer, then one can (a) generalize with respect to 'ξ is a horse' in (1) and (b) specify the resultant generalization with 'the concept horse'. Now, it is a matter of controversy whether English (indeed, natural language generally) permits second-order generalization of the kind required to discharge (a). But there are those of us who think it does.[11] We hold that some English generalizations featuring adjectival, common-nominal and/ or adverbial quantifiers deserve to be counted second-order generalizations. The transition from 'Bucephalus was a warhorse and Shergar was not a warhorse' to 'Bucephalus was something Shergar was not' is an example of such generalization. Accordingly, we may submit that the second-order

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[11]See, e.g. Dummett [Dummett, 1981a, pp. 214-22] ('the use of higher-level quantification is extremely common in natural language' 218)), Strawson [Strawson, 1961, p. 404ff] and more recently Rayo and Yablo [Rayo and Yablo, 2001]. Surprisingly, Dolby makes no mention of the issue of whether natural language permits second-order quantification; nevertheless, I believe he commits himself to accepting that it does. Dolby defends Wright's solution to the concept *horse* paradox, mentioned in footnote 5, which involves claiming that predicates do not *refer to* but rather *ascribe* concepts [Dolby, 2009, p. 295]. He [ibid.] proposes defending Wright's [Wright, 1998, p. 87] *Ascription Principle* (co-ascriptive expressions are intersubstitutable *salva congruitate* in all contexts) against counterexamples raised by Oliver [Oliver, 2005, p. 184] in a way that parallels his defence of (RP). That must mean understanding the Ascription Principle as equivalent to what results from replacing 'co-refer' in (DP) with 'co-ascribe': but if second-order quantification is unavailable, the Ascription Principle, thus construed, would imply that natural language predicates don't even co-ascribe with *themselves*: that would be a disastrous result for the Wrightian view Dobly wishes to defend.

existential generalization of (1) is

(11) Shergar is something.

(Or: There is something Shergar is.)[12]

It might be thought that the rub lies in now discharging (b). However, I think the proponent of co-reference between singular terms and predicates can again appeal to versions of the auxiliary material and null substitution strategies canvassed above. She can maintain that the result of specifying (11) with 'the concept horse' is (10). If so, the specification involves the introduction of an expression additional to the expression for which the specification is being made—namely, 'falls under'. But, again, this cannot disqualify (10) from being the result of this specification; for specification in natural language frequently does involve the introduction of such auxiliary material: e.g. specifying (8) with 'Die Ostsee' to yield (5) involves the introduction of the anaphoric pronoun 'ihre'. Alternatively, she can maintain that the result of specifying (11) with 'the concept horse' is (1). If so, the specification involves the alteration—namely, the de-nominalisation—of the expression with which the generalization is being specified. But, again, this cannot disqualify (1) from being the result of this specification; for specification in natural language frequently does involve the alteration of the expression with which we are specifying: e.g. specifying the generalization 'John fällt in etwas' with 'der Rhein' would involve altering the latter to 'den Rhein', so that it is appropriately inflected for the accusative case.

Of course, these strategies for accepting both (DP) and the negation of Frege's thesis cannot be considered uncontroversial. But in the absence of an argument against them, (DP) is impotent to justify Frege's thesis. Since Dolby's reading of (RP) makes (RP) equivalent to (DP), his reading is not only inadmissible (as I hope to have shown above) but also renders (RP) impotent to justify Frege's thesis.

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[12]Alternatively, appealing to adverbial quantifiers and pro-forms in the way Rayo and Yablo (2001) recommend, we might proffer the more prolix and semi-technical 'Somehow an object is-or-isn't determined such that Shergar is so determined'.

2.2.4 Weakening the Reference Principle

I close my discussion of intersubstitutability *salva congruitate* by briefly considering a relative of (RP) that might be considered as an alternative ground for Frege's thesis. ('Substitution' is to be understood in its cut-and-paste sense for the remainder of the essay; accordingly for its cognates.) It results from weakening the condition 'in all contexts' in (RP), and might be dubbed the *Weak Reference Principle*:

(WRP) Co-referential expressions are intersubstitutable *salva congruitate* in some contexts.

However, it can quickly be been seen that (WRP) will not justify Frege's thesis; for there *are* contexts in which singular terms and predicates are intersubstitutable *salva congruitate*. Contexts featuring quotation are obvious examples:

(12) John uttered "is a horse".

(13) John uttered "the concept horse".

But these are not the only such contexts:

(14) What John likes about the monastery is the reverent atmosphere and the humble are peaceful.

(15) What John likes about the monastery is the reverent atmosphere and the humble peacefulness.

(14) is an odd-sounding but perfectly grammatical conjunction, whose latter conjunct is to be read in the spirit of Matthew 5:5—those who are humble are peaceful.

2.3 Intersubstitutability *Salva Significatione*

(RP), recall, was supposed to be a corollary of Salva Veritate. If that supposition is correct, the counterexamples to the (RP) are also counterexamples to Salva Veritate. The former was thought to be a consequence of the latter because intersubstitutability *salva veritate* was thought to *entail* intersubstitutability *salva congruitate*. This was based on the thought that in order for a string to be truth-valued at all, it must be grammatically well-formed; and I hazard that this thought

was based, in turn, upon the assumptions that in order for a string to be truth-valued it must be meaningful (i.e. possessed of sense), and that in order for a string to be meaningful, it must be grammatically well-formed.

If one wishes to defend Frege's Salva Veritate, the best course is to resist this line of thought and to deny that Salva Veritate entails (RP). The best way to do that, I submit, is to deny the last assumption, that meaningfulness requires grammaticality. (We shall adopt the other assumption, that a string must be meaningful in order to be truth-valued.) One should insist that ungrammatical strings can be meaningful and indeed truth-valued. One should claim, further, that an ungrammatical string that results from substituting an expression for a co-referring expression in a sentence is meaningful, truth-valued and materially equivalent to the sentence into which the substitution was made. To do so is to deny that counterexamples to (RP) are counterexamples to Salva Veritate. To do so is also, however, to concede that the loss of grammaticality brought about by substituting a singular term for a predicate fails to show that no singular term co-refers with a predicate. If one wished not only to defend Salva Veritate, but to adduce it in support of Frege's thesis, one would need to make the case, afresh, that substitution of a singular term for a predicate fails to preserve truth-value. The natural way to make that case, given our assumption that truth-valuedness requires meaningfulness, is to argue that such substitution precipitates a loss of meaningfulness: the result of substituting a singular term for a predicate in a sentence is *nonsense*. To do so is, in effect, to present Frege's thesis as a consequence of the following substitution principle.

Salva Significatione Co-referential expressions are intersubstitutable in all contexts *salva sig-*

nificatione.[13],[14]

Salva Significatione may appear, on first inspection, to be a straightforward corollary of Salva Veritate, given the auxiliary assumption that truth-valued-ness requires meaningfulness. In fact, however, it is not. For Salva Veritate and the auxiliary assumption could both be true, and Salva Significatione be false. This is the case in the following circumstance: All true sentences and all false sentences are meaningful (the auxiliary assumption is true); substitution of an expression for a co-referring expression in a true or a false sentence only ever results in a sentence with the same truth-value as the sentence into which the substitution was made (Salva Veritate is true); some sentences are truth-value-less;[15] among the truth-value-less sentences, some are meaning-less and some meaningful; substitution of co-referring expressions within sentences in the truth-value-less category sometimes takes one from a meaningful but truth-value-less sentence to a meaningless and truth-value-less sentence (Salva Significatione is false). Figuratively: We suppose that the space of sentences is tripartitely partitioned into truth-value regions, populated by the true sentences, the false sentences, and the truth-value-less sentences, respectively.[16] Salva

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[13]Some remarks on terminology: Firstly, the expression '*salva significatione*' appears to be variously used in the literature to mean 'preserving meaningfulness' (e.g. [Szabo, 2013, §1.6.1]) and 'preserving meaning' (e.g. [Kripke, 1979, p. 240]. Here I am, of course, using the expression in the former sense. Secondly, I shall be using 'meaning' in this section as equivalent to 'sense' (and the cognates of former as equivalent to the corresponding cognates of the latter) as the latter is used in exposition of Frege's semantic doctrines—i.e. as a translation of Frege's 'Sinn'. The chief motivation for doing so is that it permits us to bring our terminology into line with that of literature germane to present concerns. The policy has the subsidiary benefit of granting us an agreeably unremarkable adjective meaning 'possessed of sense'—viz. 'meaningful'—together with its nominalisation—viz. 'meaningfulness'—for which we will have much use. (It has been suggested to me that English suffers a lexical lacuna in lacking an adjectival cognate of 'sense'. This is not entirely true: 'senseful', though uncommon, is recognised by the Oxford English Dictionary.) The policy has a not insignificant disadvantage, however: 'meaning' is one of the main expressions used to translate Frege's 'Bedeutung'—indeed, a particularly important one, for it was the translation agreed upon by a committee of eminent Frege scholars for the various translations of Frege's works published by Blackwell (see [Frege, 1997d, p. 36]) (On the other hand my use of 'meaning' accords with Russell's discussion of Frege's views in 'On Denoting'.) I trust that these clarificatory remarks, together with the fact that I shall avoid rendering 'Bedeutung' in Frege's mature work as 'meaning', will suffice to forestall any potential confusion.

[14]Like the Reference Principle, Salva Sigification is free of the controversial consequences concerning intensional contexts carried by Salva Veritate: the disturbance to truth-value brought about by the intersubstitution in intensional contexts of expressions that uncontroversially (at least customarily) co-refer is not due to a loss of meaningfulness.

[15]It is clear that Frege accepts that that there are sentences that lack truth-value. He must, since, firstly, he accepts, for better or for worse, that there are subsentential expressions that have sense but no referent (this is, admittedly, not completely uncontroversial among commentators; but see [Frege, 1997i, p. 153] for recognition of *bedeutungslos* proper names, and [Frege, 1997a, p. 178] for recognition of *bedeutungslos* concept words.); secondly, he holds—indeed, is committed to holding by his thesis of the compositionality of *Bedeutung*—that 'reference-failure is upwardly contagious', as Hale puts it [Hale, 2010, p. 418].

[16]I want to stress that the posited truth-value-less sentences are not to be conceived as possessing a third, non-classical truth-value ('neuter', say), which Frege explicitly denounces [Frege, 1997d, p. 157-8]. Rather, as the descrip-

Veritate implies that if one jumps around in the space of sentences using a particular jumping technique, *j*, (viz., swapping co-referential expressions), one will never jump from the true region to any other truth-value region or from the false region to any other truth-value region.[17] The auxiliary assumption implies that if one jumps around (using any jumping technique) in the union of the true and false regions, one will never jump from a meaningful sentence to a meaningless one. But neither of these claims imply, either individually or in conjunction, that if one jumps around using the jumping technique *j* within the truth-value-less region, one will never jump from a meaningful sentence to a meaningless sentence; and this latter is a condition of the truth of Salva Significatione.

Though Salva Significatione is not a corollary of Frege's Salva Veritate, it is credible to assume, as Max Black does [Black, 1954, p. 235], that Frege would accept it. Can Salva Significatione, then, be satisfactorily adduced in justification of Frege's thesis?

There do appear to be grounds for supposing that, though each may precipitate a loss of grammaticality, intersubstitution of uncontroversially co-referring expressions preserves meaningfulness, while substitution of singular terms for predicates does not. The ungrammatical strings obtainable by swapping uncontroversially co-referring expressions (strings like 'Bustling the Big Apple is a centre of finance', or (4) and (6) above) appear to be mere *grammatical solecisms*, *mild ungrammaticalities*, or what Wilfrid Hodges dubs mere *perturbations* of perfectly grammatical sentences [Hodges, 2001, p. 7]. They are the kind of ungrammatical strings that might be produced by a language user who has yet to achieve fluency; with them, he shall in all likelihood make himself understood: a fluent audience will have little difficulty in divining his expressive intentions and identifying, by way of correction, a grammatical sentence that fulfills them. The results of substituting singular terms for predicates in a sentence, on the other hand, seem to be of a different, and greatly *worse*, order of linguistic aberration. They are strings

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tion indicates, they are to be conceived as simply *lacking* a truth-value. My saying that they collectively populate a 'truth-value region' should not be understood as suggesting otherwise. If they did possess a third truth-value, our supposition that some among them are meaningless would be inconsistent with the auxiliary assumption.

[17]Salva Veritate certainly implies this, but it is a question of interest whether (continuing to speak figuratively), for *any* truth-value region, the principle should be understood, further, to exclude the possibility that substitution of co-referring expressions takes one from that truth-value region to another—i.e. whether it implies that substitution of an expression for a co-referring expression in a *truth-value-less* sentence never results in a *truth-valued* sentence. I'm confident Frege would accept this claim in any case.

whose constituent expressions are not even *broadly* of the right syntactic types to constitute a sentence—strings Ofra Magidor calls *grammatical type confusions* (GTCs) [Magidor, 2009b].[18] These strings differ not only in the severity of their grammatical infelicity, but in respect of their intelligibility to a fluent audience. If I address the string 'Germany nationhood' (obtained from 'Germany is a nation' by replacing the predicate with its nominalisation) to a fluent audience, there will simply be no meaning of which my audience will reliably judge that *that* is what I am getting at.

The fact that, for a given, say, English, mild ungrammaticality, θ, auditors fluent in English will reliably, and without special preparation or co-ordination, converge upon some one content as what was meant by a tokening of θ indicates, one might claim, that the semantics of English determines some content as the meaning of θ. One might correspondingly propose that the fact that GTCs are, by contrast, consistently found unintelligible by fluent language users indicates that they are meaningless. The former claim is on somewhat firmer ground than the latter, I suspect; for it is not hard to find examples of perfectly meaningful English sentences that fluent speakers consistently find unintelligible. Familiar examples are sentences featuring multiple centre-embedding, such as (16), which is both grammatical and meaningful:

(16) The postman the cat the dog the horse kicked bit licked jumped.

The fact that such sentences are found unintelligible is plainly a consequence, however, of their

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[18]Magidor declines to offer a precise definition of a GTC, instead taking as paradigm cases ungrammatical strings such that 'the source of their ungrammaticality has something to do with using expressions of the wrong syntactic type' [Magidor, 2009b, p. 2]. She leaves unsettled, in particular, the question whether mild ungrammaticalities like the above count as GTCs. I shall assume that they do not, and that GTCs always, not just paradigmatically, owe their ungrammaticality to their constituents' being of the wrong broad syntactic types to constitute a sentence. The reason for describing the ungrammaticality of GTCs as owed to their constituents' being of the wrong *broad* syntactic types, rather than simply the wrong syntactic types, is this: There is a common account of the syntactic types (or grammatical categories) on which two expressions are of the same syntactic type just in case they are intersubstitutable in all contexts *salva congruitate*. Jan Westerhoff calls this 'the standard account' [Westerhoff, 2005, pp. 42-43]. In light of what emerged in our discussions of substitutability *salva congruitate*, this account has the consequence that, for example, 'New York' and 'The Big Apple' (or 'Holland' and the 'The Netherlands', in an established colloquial use of the former) belong to different syntactic types, and that those mild ungrammaticalities resulting from their intersubstitution owe their ungrammaticality to a mismatch of grammatical type. In this case, saying merely that GTCs owe their grammaticality to their constituents' being of the wrong syntactic types will fail to capture their distinctive grammatical infelicity. Now, that 'New York' and 'The Big Apple' are assigned to different syntactic types just shows, one might conclude, that the account is wrong (that appears to be Westerhoff's verdict). Alternatively, one might think it shows that the account distinguishes but one of several good notions of syntactic type. In any case, *some* sense must be recognised in which these two proper names are of the same syntactic type, and it is just such a sense that the modifier 'broad' is intended to capture.

high degree (and distinctive kind) of syntactic complexity; and no such explanation is available
in the case of GTCs like 'Germany nationhood' which are syntactically quite simple.[19] So we
have, I think, at least a *prima facie* case that Salva Significatione is inconsistent with the sup-
position that some singular terms co-refer with predicates, though not falsified by the cases of
substitution that falsify (RP).[20]

2.3.1 The Semantic Status of Grammatical Type Confusions

The view that GTCs, like those that result from switching predicates for singular terms, are
meaningless, has recently been subjected to a surprisingly formidable challenge in work by Ofra
Magidor [Magidor, 2009b]. Magidor canvasses a range of putative reasons for accepting this
view, which she calls the 'last dogma of type confusions', and concludes that none is compelling.
For our purposes, the most important part of her argument is to the effect that several dominant,
general approaches to semantics, including the functional framework of which Frege's semantics
is a variety, fail to substantiate 'the last dogma'. Each, Magidor claims, will render at least some
GTCs meaningful: for some GTC, there will be some content, such that the tenets of the semantic
approach make it natural and principled to assign that content to the GTC as its meaning. Let
us concentrate on the Fregean functional framework, on which singular terms refer to objects,
first level *n*-adic predicates to functions from *n*-tuples of objects to truth-values (i.e. concepts),
second-level monadic predicates to functions from functions from *n*-tuples of objects to truth-
values to truth-values, and so on. She notes that the *meaning* of a subject-predicate sentence, like

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[19]I suppose one might here balk at the assignment of a degree of syntactic complexity to a string that is radically
syntactically ill-formed. I grant that it is a substantive question whether a syntactic *structure* can be ascribed to
such strings. But two points on this: Firstly, even if one denied that GTCs like 'Germany nationhood' have any
syntactic structure, and on this basis judged talk of their syntactic complexity incoherent, one should still accept the
present point (though on different grounds): that their unintelligibility is not explicable by reference to their syntactic
complexity. Secondly, it is not clear that denying that such GTCs have syntactic structure really gives one reason to
altogether repudiate talk of their syntactic complexity. Certainly, one could deny that e.g. 'Germany nationhood' has
determinate syntactic structure, and still coherently accept that, in some sense, it is relatively syntactically simple:
for one might think that there is a range of candidate syntactic structures, such that it is determinate that it exhibits
one or other of them, but it is indeterminate which, and each such structure is relatively simple (see [Magidor,
2009b, § IV] for discussion). Moreover, one could accept that there is a simple sense in which 'Germany nationhood'
is syntactically quite simple: it only contains two expressions. Again, I think even this kind of syntactic simplicity
sustains the point I wish to make in the main text.

[20]Salva Significatione is briefly mentioned by Oliver [Oliver, 2005, p. 182] as a 'close cousin' of (RP), but is dismissed
on the grounds that it also falls foul of many of the counterexamples to (RP). However, it is not clear that Salva
Significatione does fall foul of those examples.

'John runs', on this approach, is not, and cannot be, simply the value of the function to which the predicate 'runs' refers for the object to which 'John' refers as argument, lest all materially equivalent subject-predicate sentences have the same meaning. Rather, she claims, the meaning is 'some sort of intentional specification of this entity' [Magidor, 2009b, p. 10]. She explains:

> Suppose that the semantic value of 'John' is John, and of 'runs' is a function f from individuals to truth-values ... one might argue that 'John runs' means (roughly) that the result of applying the function f to John is the value 'true'. (ibid.)[21]

On this account of the meaning of sentences, Magidor argues, there are principled meaning assignments to be made to GTCs. 'Let[ing] f be the semantic value of 'runs' and g be the semantic value of 'eats'' [Magidor, 2009b, p. 11], the GTC 'Runs eats', she claims, will be assigned as its meaning the thought that the result of applying the function g to the function f is the True.[22] She concedes that since 'the result of applying the function g to the function f' is presumably an empty definite description (the function f is not even in the domain of the function g) this thought, and in consequence the GTC to which it is assigned as meaning, might be truth-value-less; nevertheless, she claims, the assignment endows the GTC with meaning.

Though she does not explicitly address this case in connection with Fregean semantics, Magidor presumably also holds that GTCs consisting solely of singular terms can naturally be assigned meanings. 'John Jack' will presumably be assigned the thought that the result of (functionally) applying Jack to John is the True. Again, she will no doubt concede that the subject of the preceding that-clause is empty, since there is no such result (Jack is not even a function), but claim that this makes 'John Jack' (at worst) meaningful but truth-value-less.

It may be thought at this juncture that Frege's response to these proposals of Magidor's can and will be blunt: there *are no such thoughts* as the thoughts Magidor proposes should be acknowledged as the meanings of those GTCs on a Fregean functional semantics. Evidence that

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[21] Henceforth I shall sometimes follow Magidor in using 'semantic value' instead of 'referent'.

[22] I've deviated from Magidor's exposition here, for what she actually says is that 'the GTC ['Runs eats'] can receive as its meaning some intentional specification such as 'The result of applying the function g to the function f is the value *true*" [Magidor, 2009b, p. 11]. Strictly, what she has proposed here is that the meaning assigned to 'Runs eats' is a certain *sentence*. That would be altogether implausible and entirely out of keeping with Fregean semantics. She must intend that the meaning assigned to 'Runs eats' be *that of* the sentence in question. To refer to that meaning I have, in Fregean spirit, used the result of prefixing 'the thought that' to that sentence (unquoted).

Frege would respond thus is to be found in '[Comments on Sinn and Bedeutung]', in which Frege claims that 'concepts cannot stand in the same relations as objects. It would not be false but impossible to think of them as doing so' [Frege, 1997d, p. 175]. It follows from this remark that it is impossible that the referent of 'runs' be thought to be functionally mapped to the True by the referent of 'eats'; for this would entail thinking that a concept (the referent of 'runs') stands in a relation to the referent of 'eats' in which (certain) objects stand—that of *being mapped to the True by*. Similarly, it follows that it is impossible that the referent of 'Jack' be thought to functionally map the referent of 'John' to the True; for this would entail thinking that an object (the referent of 'Jack') stands in a relation to the referent of 'John' in which concepts stand—that of *mapping to the True*. In each case, the impossibility cannot, in Frege's estimation, be owed to the contingent cognitive shortcomings of thinkers like ourselves: it must be owed, rather, to there being, in each case, *no such thought*.

However, there are several reasons to be concerned about giving Magidor's proposals such short shrift. The first is quite general: claims to the effect that some particular thing is unthinkable, or that some particular thought does not exist, are attended by a glaring problem of self-stultification. If the claimant and his or her audience are even to understand such a claim, they must be in a position to apprehend just *what* it is that is being claimed unthinkable, and such apprehension threatens to demand the very thinkability of the subject of the claim; in which case, the claim cannot be at once intelligible and true. The problem is that identified in the preface of the *Tractatus*: 'in order to set a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought)' [Wittgenstein, 1981, p. 27]. Frege could seek to address this concern by adopting Wittgenstein's response to it: commuting unthinkability claims to nonsensicality claims. Frege might say, not that it is unthinkable that the result of applying the function g to the function f is the True, but rather that 'the result of applying the function g to the function f is the True' is devoid of sense. (Whether the Wittgensteinian response is satisfactory is a complex issue. At first blush, there is reason to doubt that it is; for it seems to etiolate the claim in question beyond all recognition: what was initially presented as a deep thesis about essential limits of thought, becomes a meagre claim about the (presumably contingent) matter of whether some particular

string of expressions possesses sense.)

A second concern is this. The interest of Magidor's proposal in the present dialectic is that it is readily extendable to the GTCs of particular concern: those that result from replacing a predicate in a sentence with a purportedly co-referring singular term. 'Germany the concept *nation*', for example, will be assigned as meaning the thought that the result of (functionally) applying the concept *nation* to Germany is the True. Now, the opponent of Frege's thesis—one who holds that some singular terms co-refer with predicates—may well hold, for example, that 'the concept *nation*' genuinely co-refers with 'ξ is a nation'; and, by Frege's lights, the function to which 'ξ is a nation' refers *can* be thought to map Germany to the True, *can* be thought to stand in that relation to Germany. Suppose, then, that Frege's opponent adopts Magidor's proposal concerning the GTCs of particular concern. Frege is in no position to protest that there are no such thoughts as the putative thoughts his opponent wishes to assign as meanings to those GTCs on the grounds that those thoughts would be to the effect that some entity stands in a relation in which it is unthinkable that that entity stands: for, by his opponent's lights, those thoughts are to the effect that some entity stands in a relation such that, by Frege's lights, it *is* thinkable that that entity stands in that relation.

A related, third concern is this. The blunt response to Magidor's proposal seems simply to beg the question against Frege's opponent. For one will only hold that it is unthinkable that a concept stands in the same relation as an object if one holds that no concept is an object; and this is simply equivalent to the claim that no singular term co-refers with a predicate, the very claim from which Frege's opponent dissents. If Frege adopts the blunt response, then, he gets no purchase on his opponent's position.

If Magidor is right, Frege's *own* functional semantics provides grounds for classifying as meaningful those GTCs that result from the pairwise intersubstitution of singular terms and predicates. Frege's opponent can thus appeal to those grounds in order coherently to insist that predicates *are* intersubstitutable *salva significatione* with the relevant singular terms. In that case, Frege seems to be in no position effectively to substantiate Frege's thesis by appeal to Salva

Significatione.[23]

It appears then, that if Salva Significatione is to be adduced as grounds for Frege's rejection of co-reference between singular terms and predicates, the case needs to made that Magidor is *not* right. There are several objections that might, to this end, be raised against Magidor's view on Frege's behalf. The first, indeed, is considered on the last page of Magidor's essay [Magidor, 2009b, p. 25], and is as follows. Magidor's proposed assignment of meanings to the GTCs in question involves, as it were, semantically treating one of the constituent expressions of a GTC as though it belonged to the broad syntactic category of expression the absence of which from that GTC is sufficient to make that GTC ungrammatical. For example, in 'John Jack', 'Jack' is semantically treated like a predicate, in that (continuing to confine our attention to Fregean functional semantics, as construed by Magidor) *its* semantic value is what, according to the thought assigned as meaning to this GTC, yields the True when applied to the semantic value of 'John', rather than the other way round. Similarly, in 'Runs eats', 'Runs' is semantically treated like a singular term, in that *its* semantic value is what, according to the thought assigned as meaning to that GTC, is mapped to the True by the semantic value of 'runs', and not the other way round. The objection is that since in each case we just have two expressions of the same syntactic category, it is arbitrary to semantically treat one, and not the other, as belonging to the missing syntactic category. One could equally well treat 'John' as though it were a predicate in 'John Jack'; likewise, 'eats' might just as well have been semantically treated as though it were a singular term in 'Runs eats'. Were we to do so, we would assign different thoughts as meanings to these GTCs. So Magidor's assignment of meanings, the objection goes, is arbitrary and therefore illegitimate.

On reflection, however, this objection emerges as unconvincing. To echo Magidor's response, it is, firstly, not clear that the admissibility of multiple alternative meaning assignments

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[23]It is worth noting that if Frege's opponent does adopt the suggested approach of appealing to Magidor's case for the meaningfulness of GTCs, she secures her against the expressive difficulties that afflict Frege's semantic theory, from two different directions: Firstly, and most obviously, she avoids them by permitting herself singular reference to concepts (and functions more generally). Secondly, she permits herself the possibility of articulating semantic theses shared with Frege without the use of singular terms to refer to concepts; instead, she can use certain meaningful GTCs. For example, she can specify the referent of 'ζ is a horse' either by using "'ζ is a horse' refers to the concept *horse*', or by using the GTC "'ζ is a horse' refers to is a horse'.

makes each such assignment arbitrary. Some may prove more natural than others, and indeed one may be distinguished as most natural. Among various considerations that might bear upon the naturalness of alternative assignments, the fact, for example, that it is considerably more common for predicates to occur to the right, rather than to the left of a singular term in an atomic sentence of English might make it more natural semantically to treat the right hand expression of an English GTC as though it were a predicate, rather than the left hand expression. Secondly, even if no one admissible assignment is distinguished as most natural, the GTC in question need not be judged meaningless; we ought, on the contrary, to conclude that the GTC suffers a *surfeit* of meaning, rather than a total lack of it—that the GTC is structurally ambiguous, not meaningless.

A second objection, not canvassed in Magidor's paper, similarly concerns the fact that, for a GTC consisting solely of two singular terms or of two predicates, Magidor's proposed meaning assignment involves semantically treating a constituent expression, *e*, of that GTC as though it were an expression of a different syntactic type, *t*. This fact shows, one might object, that Magidor's assignment represents an attempt underhandedly to introduce an expression, e', of type *t*, though homonymic with *e*, and to construe the GTC in question as containing not *e* but e', in just that position that it had been supposed to contain *e*. Magidor's assignment to 'John Jack', for example, represents an attempt to construe 'John Jack' as containing not the proper name 'Jack', but a coined predicate 'Jack', semantically related thereto in the following way: for any object, the predicate 'Jack' is true of that object just in case the result of functionally applying Jack (the man) to that object is the True. To the extent that the meaning assignment is successful, then, that to which meaning is assigned is not a GTC after all, but a grammatical, albeit neologistic, subject-predicate sentence. Magidor's treatment of 'Runs eats' correspondingly represents an attempt to construe that string as containing not the predicate 'Runs', but a coined singular term 'Runs', that names the semantic value of its predicate homonym. Interpreted in the way Magidor proposes, then, 'Runs eats' is not a GTC after all. In neither case, therefore, does Magidor give reason to think that *GTCs* are meaningful.

The heart of the objection is the thought that the difference, in respect of semantic contribution, between, for example, 'Jack', as it occurs in 'Jack runs', interpreted by Fregean semantics,

and 'Jack' as it occurs in 'John Jack', interpreted in the way Magidor proposes, is such as to warrant the conclusion that these two occurrences are in fact occurrences of distinct expressions. It strikes me, however, that Magidor is at liberty to respond to this objection as follows: "There is indeed a sense in which 'Jack' as it occurs in 'Jack runs' differs in respect of semantic contribution from 'Jack' as it occurs in 'John Jack'. That difference can indeed be brought out by saying that in the former case, the referent of 'Jack' is what, according to the thought assigned as meaning to the string in question, is mapped to the True (by a certain function), whereas in the latter case, the referent of 'Jack' is what, according to the thought assigned as meaning to the string in question, does the mapping to the True (given a certain argument). But there is also a sense in which 'Jack' as it occurs in 'Jack loves Jill', differs in respect of semantic contribution from 'Jack' as it occurs in 'Jill loves Jack'. That difference can be brought out by saying that in the former case, the referent of 'Jack' is what, according to the thought assigned as meaning to the string in question, bears a certain relation to a certain thing, whereas in the latter case, the referent of 'Jack' is that to which, according to the thought assigned as meaning to the string in question, a certain relation is borne by a certain thing. This latter difference raises no doubt that we are dealing here with two occurrences of a single expression; and the reason is that this latter difference is explained by the semantically significant difference in the two occurrences' respective positions in the respective modes of composition of the two sentences. However, I (Magidor) maintain that the former difference is of the same kind as the latter: the difference in semantic contribution between 'Jack' in 'Jack runs' and 'Jack' and in 'John Jack' is likewise owed to a semantically significant difference in these occurrences' respective positions in the respective modes of composition of the two strings. This difference should likewise cast no doubt that we have here two occurrences of one expression—in this case, the proper name 'Jack'."

Simply put, this response amounts to explaining the difference of semantic contribution between the two occurrences of 'Jack' by reference to the fact that, in the one case 'Jack' occurs as the left hand expression in a string of two expressions, while in the other 'Jack' occurs as the right hand expression in a string of two expressions. This requires claiming that the mode of composition both strings in question exhibit—namely, the concatenation of two expressions, each drawn from the union of the categories of singular term and predicate, one to the left and

one to the right—has the significance that the right hand expression is applied to the left hand expression, in the sense in which 'runs' is applied to 'Jack' in the sentence 'Jack runs'. It is not clearly mistaken to impute this kind of semantic significance to concatenation. We have, then, yet to find an effective challenge that Frege can pose against Magidor's argument.

A third, and more promising objection is as follows. Magidor's idea, one might surmise, is this: Given a subject-predicate sentence $\ulcorner \alpha\psi \urcorner$, Fregean functional semantics will assign to it as meaning the thought that the result of applying the referent ψ to the referent of α is the True. ('α' and 'ψ' are metalinguistic variables whose values are, respectively, singular terms and predicates; corners are being used as a device of quasi-quotation [Quine, 1951, p. 33-37].) But this recipe for assigning meanings is naturally generalized to any string $\ulcorner \Sigma\Pi \urcorner$, where '$\Sigma$' and '$\Pi$' are metalinguistic variables ranging over *both* singular terms and predicates: $\ulcorner \Sigma\Pi \urcorner$ will be assigned the thought that the result of applying the referent of Π to the referent of Σ is the True. However, this idea—so the objection proceeds—issues from a fateful misrepresentation of Fregean semantics: it is *not* the case that a subject-predicate sentence $\ulcorner \alpha\psi \urcorner$ is assigned as meaning the thought that the result of applying the referent of ψ to the referent of α is the True. The sentence 'John runs', for example, is not held by Frege to mean that the result of applying the referent of 'runs' to the referent of 'John' is the True; for the sentences 'John runs' and 'the result of applying the referent of 'runs' to the referent of 'John' is the True' will, for Frege, differ in sense. It is, to be sure, a delicate exegetical question what exactly Frege's criteria are for sameness of sense. As Beaney [Beaney, 1996, p. 225-34] explains, Frege seldom sets forth clear criteria for one sentence's possessing the same sense as another; and in those passages in which he does advance such criteria, the criteria advanced are in fact (externally) inconsistent. In a letter to Husserl of 1906, he claims that two sentences, A and B, possess the same sense just in case it is possible, with the aid only of purely logical laws, to prove that A and B are materially equivalent. In 'A Brief Survey Of My Logical Doctrines', on the other hand, he claims that A and B possess the same sense just in case anyone who recognises A as true must straight away recognise B as true, and vice versa (if A and B satisfy this condition they are said to be *equipollent*). (The passages in question are quoted in [Beaney, 1996, p.228-9]) These criteria will

clearly yield different results.[24] But according to neither criterion will 'John runs' and 'the result of applying the referent of 'runs' to the referent of 'John' is the True' express the same sense. A proof of their material equivalence could not be prosecuted solely with the aid of purely logical laws. Similarly, one could recognise the former to be true without straight away recognising that the latter to be true. This might be so if, for example, I simply lacked the notion of *referent*, but might be so even in the circumstance that I understand both sentences: I could recognise that the former is true, and yet both understand and dissent from the latter, say because I just disagree with Frege that predicates refer to functions whose values are truth-values. A semantics on which a subject-predicate sentence $\ulcorner \alpha\psi \urcorner$ is assigned as meaning the thought that the result of applying the referent ψ to the referent of α is the True may indeed be very naturally generalized such that GTCs are assigned meanings; but Fregean semantics is not such a semantics, and so no grounds have been given for thinking that the Fregean semanticist should recognise GTCs as meaningful.

Now, as it stands, this objection is itself open to a charge of misrepresentation. For Magidor in fact does not quite claim that according to Frege the meaning of e.g. 'John runs' is the thought that the result of applying the referent of 'runs' to the referent of 'John' is the True. Rather, what she does (in the quotation above from [Magidor, 2009b, p. 10]) is to first introduce a term 'f', stipulated to refer to the referent of 'runs',[25] and then to specify the thought ascribed by Fregean semantics to 'John runs' as the thought that the result of applying f to John is the True. One might be tempted to think this an inconsequential circuity—tempted to assume that 'f' is, by stipulation, just an abbreviation of 'the referent of 'runs'' and therefore that the argument in the previous paragraph counts without further ado against identifying the thought expressed by 'John runs' with the thought that the result of applying f to John is the True. But this would

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[24]As Beaney illustrates, Frege's vacillation on this front was very likely a response to a very real tension in Frege's thought. His wish to secure the sameness of sense of the two sides of the abstraction principles at the heart of his logicism—most notably Axiom V—seems to pull in the direction of the provable-material-equivalence criterion; while his concern to account for the informativeness of identity statements and to allow, in the presence of his logicism, that arithmetical truths may differ in sense, seems to pull in the direction of the equipollence criterion. It is to the latter, as Beaney notes, that Frege seems for the most part to be drawn.

[25]Of course, the possibility of introducing a singular term to refer to the referent of a predicate is precisely what Frege denies. But what is at issue is whether someone who *opposes* Frege on just this point can, by appeal to Magidor's argument, nevertheless coherently accept Salva Significatione.

be overhasty. For although '*g*' has been introduced by means of the definite description 'the referent of 'runs' ', it does not follow that it has been stipulated to be a mere abbreviation of or otherwise equivalent to, that description. That a name may be introduced using a description without being determined to be equivalent to it was stressed by Geach [Geach, 1968, p. 122-124]. If '*g*' does not express the same sense as 'the referent of 'runs' ', then the difference between identifying the thought assigned to 'John runs', on the one hand, with the thought that the result of applying the referent of 'runs' to the referent of 'John' is the True, and on the other, with the thought that the result of applying *f* to John is the True, is not so clearly inconsequential after all. For 'the result of applying the referent of 'runs' to the referent of 'John' is the True' is clearly semantically ascended, whereas we have seen no reason to think 'the result of applying *f* to John is the True' is likewise. Semantic ascent does not preserve sense; perhaps, then, the argument in the previous paragraph only works by misrepresenting Magidor as specifying the thought assigned to e.g. 'John runs' with a semantically ascended clause.

It can quickly be seen that this is not so. It is equally plain that 'John runs' is not held by Frege to express the thought that the result of applying *f* to John is the True; for it is equally plain that the sentences 'John runs' and 'the result of applying *f* to John is the True' will, for Frege, differ in sense. A proof of their material equivalence could not be prosecuted solely with the aid of purely logical laws; and one could recognise the former to be true without straight away recognising the latter to be true, even if one understood the latter (suppose I believe that neither *f* nor the True exists!). At this stage the question seems pressing which thought Frege *does* take to be the meaning of 'John runs'; but the answer is immediate: *the thought that John runs!* The only sure-fire procedure for monolingually specifying the thought expressed by a sentence is to use the sentence itself [26]—to assert the relevant instance of the disquotational schema,

$$\ulcorner \phi \urcorner \text{ expresses the thought that } \phi.$$

However, this procedure is certainly not naturally generalized to GTCs. Towards convincing me that 'John Jack' expresses a thought, it would be hopeless for you to specify *which* thought it

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[26]Cf. Wittgenstein in *Culture and Value*: 'The limit of language is shown by its being impossible to describe the fact which corresponds to a sentence, without simply repeating the sentence' [Wittgenstein, 1998, p. 13].

expresses by uttering,

> 'John Jack' expresses the thought that John Jack.

I would only accept that you had hereby succeeded in specifying a thought that you claim to be expressed by 'John Jack' if I already accepted that 'John Jack' expresses a thought.

Once due care is taken, then, in specifying *which* thought is recognised by Fregean semantics as the meaning of a sentence, we find that no reason has been given to think that Fregean semantics renders GTCs meaningful. Frege *is*, therefore in a position to respond to Magidor in defence of the claim that the substitution of a singular term for a predicate precipitates a loss of meaningfulness. Thus, as matters stand, Frege *does* seem to be in a position to argue, by appeal to Salva Significatione, that no singular term co-refers with a predicate.

2.3.2   Meaningful Grammatical Type Confusions in Fregean Semantics?

There is, however, a twist in the tale, to which we turn in the present section. It is this: there is a strong argument to the effect that Salva Significatione commits Frege himself to the meaningfulness (indeed, the truth) of a formidable array of GTCs. In the following paragraph I speak in the voice of a proponent of this argument.

Frege holds that sentences refer to their truth-values; he calls the truth-values to which true and false sentences respectively refer 'the True' and 'the False' ([Frege, 1997i, p. 157-8], [Frege, 1997e, p. 137], [Frege, 1964, p. 35]). On Frege's view, then, '6 is a perfect number' and '6 is the sum of its proper divisors' each refer to the True. But if '6 is a perfect number', '6 is the sum of its proper divisors' and 'the True' all have the same referent, Salva Significatione implies that, for example, 'If the True, then the True' is meaningful; for the latter is obtainable by substitution of co-referring expressions from 'If 6 is a perfect number, then 6 is the sum of its proper divisors'. But 'If the True, then the True' is a GTC! [27] [28] The singular term 'the True' is

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[27] Max Black [Black, 1954, p. 235-34] makes the point that Frege is committed to meaningfulness of strings of this kind. In Black's estimation, the point yields 'a sufficient refutation of Frege's view that sentences are designations of truth-values' since, he thinks, such strings are nonsense. Whether the point does yield a refutation of Frege's view is an issue on which the discussion below will bear.

[28] This also means, of course, as Crispin Wright and Bob Hale [Hale and Wright, 2012, p. 86] note, that the supposition that 'the True' co-refers with true sentences runs afoul of the Reference Principle; but we are presently supposing Frege not to be committed to the Reference Principle.

not even broadly of the right syntactic type to occupy the antecedent and consequent positions of the English indicative conditional construction.[29] Salva Significatione similarly implies that '6 is a perfect number is not a perfect number' and '6 is a perfect number is numerically identical to 6 is the sum of its even divisors' are meaningful, though they too are GTCs; for they are respectively obtainable by the substitution of co-referring expressions from 'The True is not a perfect number' and 'The True is numerically identical to the True'. Salva Significatione commits Frege, therefore, to recognizing at least some GTCs as meaningful—namely, those resulting from the intersubstitution of sentences with the singular terms 'the True' and 'the False'. These are, moreover, not the only kinds of GTCs to the meaningfulness of which Frege is committed by Salva Significatione. There is nothing to prevent one from introducing a singular-term-forming functor,[30] 'the parity of ξ', syntactically of a piece with 'the cube root of ξ', with the following stipulation: the parity of x is the True if 2 is a factor of x; otherwise, the parity of x is the False. Thus introduced, 'the parity of ξ' refers to a function whose value-range (*Wertverlauf*) is precisely that of the function to which the predicate '2 is a factor of ξ' refers. By Fregean lights, therefore, the former co-refers with the latter. But coupled with Salva Significatione, this co-reference claim entails that 'There is something such that the parity of it' is meaningful; for it is obtainable by substitution of co-referring expressions from 'There is something such that 2 is a factor of it'. But 'There is something such that the parity of it' is a GTC; it is syntactically of a piece with 'There is something such that the cube root of it'. Similarly, there is nothing to prevent one from introducing a *dyadic* term-forming functor that, by design, co-refers with a

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[29]There are cases that, on initial inspection, may seem to indicate, on the contrary, that the English indicative conditional does admit singular terms into antecedent and/or consequent positions. Imagine that you hear Helen speaking on the phone to her PhD supervisor. The conversation concerns when she will send her supervisor a long-due chapter. 'If not Thursday, then Friday', you hear Helen say. Helen's utterance sounds perfectly acceptable; but therein the singular term 'Friday' occurs alone in the consequent position of the conditional—or so it seems. It is clear, though, that this is a case of ellipsis. Helen's utterance is a substantially elided version of the sentence 'If I do not send the chapter on Thursday, then I will send it on Friday'. I am inclined to say that Helen is to be considered as having in fact tokened that sentence, but in an abbreviated fashion that leaves certain constituents of the sentence harmlessly unpronounced (Cf. [Dummett, 1981a, p. 298]), such that in the expression tokened, 'Friday' does not occur alone in the consequent position of the conditional construction. (Strictly, I am inclined to say that, though Helen has tokened a sentence, it may be indeterminate just *which* sentence she has tokened among a range of similar candidates; in no member of that range, however, does 'Friday' occur unaccompanied in consequent position.) This is obviously not the only position one might take on ellipsis of this kind; it lies beyond the scope of this essay to treat of the phenomenon in any detail. I very much doubt, however, that Frege could, by appeal to this phenomenon, glean any succour in connection with the argument being presented in the main text.

[30]Henceforth I shall shorten 'singular-term-forming functor' to 'term-forming functor'.

relational predicate. 'the parity of the sum of ξ and ζ', syntactically of a piece with 'the square of the sum of ξ and ζ', may be introduced with the following stipulation: the parity of the sum of x and y is the True if 2 is a factor of the sum of x and y; otherwise, the parity of the sum of x and y is the False. Thus introduced, this dyadic term-forming functor co-refers with the relational predicate '2 is a factor of the sum of ξ and ζ'. Frege must, therefore, recognise the GTC 'There is one thing and another, such that the parity of the sum of the one and the other' as meaningful, for it is obtainable by substitution of co-referring expressions from 'There is one thing and another, such that 2 is a factor of the sum of the one and the other'. Likewise, there is nothing to prevent one from introducing a *second-level* term-forming functor that, by design, co-refers with a given second-level predicate (e.g. 'Something ϕ'). The substitution of that second-level term-forming functor for that second-level predicate in a third-level predication (e.g. a second-order generalisation[31]) will result in a GTC to whose meaningfulness Frege is committed. Quite generally, we can repeat this procedure along the dimensions of adicy and level as far as English predicates and term-forming functors go in tandem together: If, for all $k \leq l$, there are in English k-level predicates and k-level term-forming functors, and if, for all $m \leq n$ there are in English m-adic predicates and m-adic term-forming functors, then Frege is compelled to recognize at least $l \times n$ different kinds of meaningful English GTCs that do not result from the intersubstitution of sentences with singular terms like 'the True' and 'the False'. Furthermore, Frege is committed not only to the meaningfulness but also to the *truth* of some GTCs. This is the case for each of the examples presented in this paragraph; each results from the substitution of co-referring expressions in a true sentence, so must, by Salva Veritate, likewise be true. Frege must, for example, accept that 'There is something such that the parity of it' is true. In this important respect, Frege's commitment to the good standing of the aforementioned GTCs outstrips Magidor's defence of the good standing of GTCs; for Magidor only defends the thesis that GTCs are meaningful.

It strikes me that there are three ways one might respond to this argument on Frege's behalf. Before we discuss these responses in detail, however, an initial misgiving about the argument

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[31]If such there is—see footnote 11 above and the main text on which it comments. On the status of second-order generalisations as third-level predications see [Dummett, 1981a, p. 49].

needs to be addressed. The misgiving can perhaps be voiced as follows: "All the strings cited above are indeed strings to whose meaningfulness Frege is committed. But—at least by Fregean lights—the argument fails to establish that Frege is committed to the meangingfulness of some GTCs, because Frege will not accept that the strings in question *are* GTCs: indeed, in his estimation, they will be grammatically quite well-formed. Frege thinks that sentences *are proper names*; so by his lights, we really find in 'If the True, then the True' just what we find in 'If 6 is a perfect number, then 6 is the sum of its proper divisors'—viz. the conditional connective 'If __ then . . .' with each of its argument places occupied by a proper name. Surely then, in Frege's view, if the latter is grammatically well-formed, so is the former. Similarly, Frege thinks that predicates *are functors*; so by his lights, we really find in 'There is something such that the parity of it' just what we find in 'There is something such that 2 is a factor of it'—viz. the second-level predicate 'There is something such that __ it' with its argument place occupied by a first-level functor. Again, in Frege's view, the former is grammatically well-formed if the latter is."

I submit that this misgiving does not impede the above argument. Frege's (intimately related) theses that sentences are proper names and that predicates are functors are *semantic* theses, not grammatical claims. Roughly, they are respectively to the effect that the theory of reference is to treat of sentences uniformly with (complex) singular terms like 'the capital of Germany' and that it is to treat of predicates uniformly with term-forming functors like 'the capital of ξ': sentences refer to objects just as singular terms do;[32] predicates refer to functions just as term-forming functors do. The theses are emphatically *not* respectively to the effect that sentences are uniform with singular terms, and predicates uniform with term-forming functors, in respect of their *syntactic* profiles. They better not be to that effect, moreover; for those grammatical claims are immediately visible as beyond the pale: sentences manifestly do not share their syntactic profiles with singular terms, nor do predicates share theirs with term-forming functors. An analogy: Suppose someone advanced a claim in chess strategy according to which a uniform strategic treatment is to be given of the deployment of both bishops and rooks. We might have

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[32]The 'just as' here of course raises the important question whether the reference relation that obtains betwixt a sentence and its truth-value is the very same reference relation, or only an analogue thereof, that obtains between a singular term and its referent. The 'just as' is in fact Frege's, however [Frege, 1997e, p. 137]—'*ebenso*' in the original.

a discussion about this heterodox strategic view, even if we were swiftly and conclusively to find it ill-conceived. But if someone were to submit that bishops and rooks actually have the same movement profiles—that the same moves are permissible for the two kinds of pieces—the manifest falsity of his claim would likely lead us to suppose that he was actually making an assertion about a different game played with chess pieces.[33] Frege's semantic assimilation of sentences to proper names and of predicates to functors is comparable to the strategic claim; the view on the imputation of which to Frege the above misgiving is based is comparable to the claim about movement profiles.[34] The point is perhaps obscured by the fact that in Frege's own symbolism in the *Grundgesetze*, sentences and predicates (or *Grundgesetze* translations of sentences and predicates) clearly are syntactically undistinguished from proper names and term-forming functors, respectively. It is indeed true that in that symbolism both the conditional [Frege, 1964, §12] and the identity sign [Frege, 1964, §7] (see also [Frege, 1997e, p. 137]) admit both sentences, e.g. '$3 > 2$', and singular terms, e.g. '2', into each of their argument places (both sentences and singular terms are substitution instances of 'Γ' and 'Δ', [ibid.]). Likewise, the quantifiers admit both predicates, e.g. '$\xi^2 = 1$', and term-forming functors, e.g. '$(2 + 3 \cdot \xi^2) \cdot \xi$' into their argument places [Frege, 1964, p.](both predicates and term-forming functors are substitution instances of 'Φ' [ibid.]). So the *Grundgesetze* counterparts of the English GTCs indicated in the above argument are themselves grammatical. That, however, does not mean that those English strings are themselves grammatical. They are not.

As I say, I see three possible responses on Frege's behalf to the above argument. The first would be to accept that Frege must recognise the aforementioned GTCs as meaningful, but to argue that this position is unproblematic. This response is taken by Dummett, at least with

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[33]There are those who would deny that our speaker's remark even enjoys the rank of falsity—who would count the remark, to use Wolfgang Pauli's famous phrase, *not even wrong* (*nicht einmal falsch*)!

[34]One point at which the analogy breaks down, I concede, is that informed disagreement *is* possible concerning the grammaticality of certain, say, English strings, whereas informed disagreement concerning the legality of moves in chess is not.∗ Intuitions about grammaticality, though evidentially highly significant, are not sacrosanct, and, in any case, may yield no firm judgement about the grammatical status of some string. Theoretical considerations—of explanatory strength, unity and simplicity, etc.—may warrant transcension, or indeed transgression, of intuitive judgements of grammaticality. Informed disagreement about these theoretical considerations and their import is possible; consequently, so is disagreement about the grammaticality of certain strings. The point in the main text is not disturbed, however. It is altogether incredible to claim that sentences and singular terms have the same syntactic profiles.

∗There are, I confess, some extremely rare exceptions to this claim about chess legality: see [Krabbé, 1985, pp. 8-9].

respect to one of the kinds of GTCs indicated above, in his 1959 paper, 'Truth':

> Nor need we waste time on the objection raised by Max Black that on Frege's theory certain sentences become meaningful which we should not normally regard as such, e.g., 'If oysters are inedible, then the False'. If sentences stand for truth-values, but there are also expressions standing for truth-values which are not sentences, then the objection to allowing expressions of the latter kind to stand wherever sentences can stand and vice versa is grammatical, not logical. We often use the word 'thing' to provide a noun where grammar demands one and we have only an adjective, e.g., in 'That was a disgraceful thing to do'; and we could introduce a verb, say 'trues', to fulfil the purely grammatical function of converting a noun standing for a truth-value into a sentence standing for the same truth-value. [Dummett, 1978, p. 1-2]

We could profitably spend a lot of ink discussing Dummett's claims that allowing singular terms like 'the True' to appear in sentence position is only grammatically, not logically, objectionable, and that a verb may be introduced that fulfils a purely grammatical role of converting such terms into sentences. However, here I want only to highlight that this response leaves Frege in a very poor position effectively to argue by appeal to Salva Significatione for his thesis that no singular term co-refers with a predicate. For if Frege is indeed committed to the meaningfulness of the aforementioned GTCs, he is, I suggest, in basically the same dialectical position as the opponent of his we encountered in §2.3.1—she who maintains, by appeal to Magidor's proposal, that the GTCs resulting from the pairwise intersubstitution of singular terms and predicates are meaningful, and who (thereby coherently) accepts both Salva Significatione and the negation of Frege's thesis. Frege, no less than his opponent, must count as meaningful certain strings that look for all the world like nonsense. He faces, moreover, the same predicament in defending that view in light of the challenge, concerning any such putatively meaningful GTC, "Very well—*what* does it mean?" Recourse to the appropriate instance of the disquotational schema discussed at the end of §2.3.1 will obviously not satisfy the challenger. If Frege bites the bullet and accepts some GTCs as meaningful, he cannot make any simple appeal to the common sense tribunal of meaningfulness in an attack on his opponent's view. Against his opponent's recognition of the likes of 'Sings runs' and 'Germany the concept *nation*' as meaningful, Frege cannot credibly say simply 'Look, these strings are plainly nonsense'; for that complaint is equally cogent against the likes of '6 is a perfect number is not a perfect number'. Frege must seek, then, to distinguish his commitments to the meaningfulness of apparent nonsense from those of his opponent. But,

to return to a theme encountered on p. 65, here is the rub: the natural thing for Frege to say
to this end seems simply to beg the question against his opponent. The natural thing for Frege
to say is that his opponent's GTCs violate logical type distinctions, whereas his own do not:
'Sings runs' is meaningless, for example, because the referent of 'sings' belongs to the very
same logical type—*first-level monadic function*—as the referent of 'runs', whereas 'runs' can only
be significantly applied to something of immediately lower type—i.e. an object; on the other
hand, '6 is a perfect number is not a perfect number' is logico-typically entirely in order; for
the referent of '6 is a perfect number' is of immediately lower type than the referent of 'is not
a perfect number'. But Frege's opponent holds that the referent of 'sings' *is* an object; and,
thus, she will either maintain that 'Sings runs' does satisfy the constraint that 'runs' is only
significantly applicable to something of immediately lower type; or, if 'logical type' is so used
that it is analytically true that nothing belongs to more than one logical type, she will deny the
constraint on meaningfulness.

The second response leaves Frege in a better position to appeal to Salva Significatione in de-
fence of Frege's thesis. Unlike the first, it involves resisting the conclusion of the above argument
on Frege's behalf. It begins with the observation that each sub-argument above, to the effect that
some particular class of GTCs must be counted meaningful by Frege, rests, more or less directly
(as will emerge momentarily), upon the assumption that Frege subscribes to the following: there
are singular terms that refer to the referents of sentences. The grounds for accepting that as-
sumption are just that in the course of expounding his semantic doctrines Frege uses certain
singular terms—most notably, 'the True' and 'the False'—that purport to, and look obviously
intended to, refer to the referents of sentences. However, it is also true that in the course of ex-
pounding his semantic doctrines Frege uses singular terms that purport to, and look obviously
intended to, refer to the referents of *predicates* (or functional expressions more generally); yet
Frege does not subscribe to the view that there are singular terms that refer to the referents of
predicates; it is his subscription to the negation of that view that is the focus of our discussion.
A peculiar feature of the present dialectical context is that the author's using singular terms that
purport to, and look obviously intended to, refer to the referents of expressions of a certain type
is not a conclusive reason to think that the author accepts that it is *possible* to use singular terms

to refer to the referents of expressions of that type. If pressed—the response continues—Frege will in fact accord 'the True' and 'the False' the same status as the singular terms with which, in the course of his exposition, he purports to pick out concepts and other functions: by the lights of his own semantic doctrines, these expressions referentially misfire—they designate something they were not intended to, or nothing at all—but are thrust upon him, 'by a kind of necessity of language' [Frege, 1997d, p. 192] in endeavouring to communicate those very doctrines. If that is so, then the sub-arguments concluding that, by Salva Significatione, Frege must recognise the likes of 'If the True, then the True' and '6 is a perfect number is not a perfect number' as meaningful are blocked; for Frege will deny, for example, that '6 is a perfect number' and 'the True' have the same referent. What is more, the sub-arguments concluding that Frege must similarly count as meaningful GTCs resulting from the substitution of term-forming functors for predicates (of various adicies and levels) are likewise blocked; for those arguments also rest, albeit less directly, upon the assumption that Frege recognises the possibility of singular reference to the referents of sentences. Those arguments appeal to the possibility of introducing term-forming functors that, by design, co-refer with predicates. But, firstly, the successful introduction of these term-forming functors relies upon 'the True' and 'the False' (or alternative such singular terms) co-referring with true and false sentences, respectively; observe, for example, the use of the 'the True' and 'the False' in the introductory stipulation for 'the parity of ξ' above. Moreover, secondly, if the introduction of these term-forming functors *were* successful, then the results of filling their argument places with singular terms would be singular terms co-referring with sentences; 'the parity of 8', for example, would co-refer with '8 is even'. So if Frege really holds that no singular term co-refers with a sentence, just as he holds that no singular term co-refers with a predicate, he will deny that term-forming functors co-referential with predicates have been, or could be, successfully introduced. The upshot is that Frege can simply accept the appearances concerning the GTCs he allegedly had to count meaningful: they are nonsense, as they seem to be. Unburdened by commitments of his own to meaningful GTCs, Frege is in a much more respectable position to censure such commitments on the part of his opponent.

If this second response is right, the extent of the concept *horse* paradox is greater than is usually appreciated: the truth-values too, for Frege, necessarily elude the reach of singular

reference. In consequence, the expressive difficulties attending the specification of the referents of particular predicates and attending generalisation about predicate reference are replicated in connection with the specification of the referents of particular sentences and in connection with generalisation about sentence reference. '6 is a perfect number' is held to refer to a truth-value; but concerning the question *which* truth-value it refers to, no strictly satisfactory answer is forthcoming: answers like, 'Its own', 'The truth-value of what is true' [Frege, 1997e, p. 137], or 'the circumstance that it is true' [Frege, 1997i, p. 157] all go referentially astray, just like 'the True'; that much is guaranteed by their being singular terms. But the good standing of the general claim that '6 is a perfect number' refers to a truth-value surely hangs on the possibility, in principle, of our saying, for some particular truth-value, that it refers to that truth-value. Moreover, the apparently paradoxical consequence that the concept *horse* is not a concept finds its counterpart in the apparently paradoxical consequence that the truth-value *true* is not a truth-value.[35,36]

The case for ascribing to Frege the view that no singular term co-refers with a sentence is, in my judgement, very poor. Though Frege uses singular terms that purport to, and look obviously intended to, refer to concepts, Frege explicitly and repeatedly cautions the reader that things are not as they seem. He plainly confesses that such singular terms do not in fact refer to concepts [Frege, 1997h, p. 184ff.], that they 'fail of their intended target' [Frege, 1997g, p. 365], that in using such expressions he 'mentions an object, when what [he] intend[s] is a concept' [Frege, 1997h, p. 184ff.]. He insists that each such singular term is 'an inappropriate expression which obscures—I might almost say falsifies—the thought' [Frege, 1997a, p. 174], an expression that 'belies the predicative nature of a concept' [ibid. p. 177] and whose inappropriateness must always be born in mind [ibid. p. 174]. Most importantly, he explicitly and repeatedly

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[35]The impossibility of singular reference to the referents of sentences furthermore overdetermines the referential failure of many singular terms purporting to refer to concepts: namely, those that depend upon singular reference to the referents of sentences. For example, the expression 'the function that maps any perfect number to the True and anything that is not a perfect number to the False' not only fails to refer to what it purports to refer to by virtue of being a singular term; but it also suffers such referential failure because certain of its constituents suffer such referential failure—viz. 'the True' and 'the False'.

[36]It is also notable that imputing to Frege the view that the truth-values cannot be named by non-sentential names forges a connection between Frege's views on the limits of singular reference and those of Russell and Wittgenstein; for Russell and Wittgenstein held, at least during their respective logical atomist phases, that the entities with which sentences are semantically correlated—namely, states of affairs or situations—cannot be named.

claims, with full generality, that no possible singular term refers to a concept (e.g. [Frege, 1997h, passim], [Frege, 1979d, p. 178], [Frege, 1997a, passim]). Matters are altogether different, however, with regard to Frege's use of singular terms that purport to, and seem obviously intended to, refer to truth-values. To the best of my knowledge, the reader is never cautioned that *these* terms are not what they appear to be, and Frege never advances the general claim that no singular term refers to a truth-value. He had, moreover, ample opportunity to do both. His use of 'the True' and 'the False', for example, sometimes occurs in immediate proximity to discussions of the referential failure of singular terms purporting to designate concepts (e.g. [Frege, 1997g, p. 365], [Frege, 1997a, p. 174]). It strains credulity to suppose that Frege neglected ever to record that he held that singular terms like 'the True' fail of their intended target, when he took such care to make plain that he thought singular terms like 'the concept *horse*' fail of theirs.

In fact, it is not merely that there is a dearth of evidence that Frege thought the truth-values unnameable. There is strong evidence to the contrary. Frege is quite clear that the truth-values are, in his view, *objects* [Frege, 1997i, p. 158] [Frege, 1964, p. 35-36]. But something is an object if, and only if, it can be the referent of a singular term. Perhaps a proponent of the second response might balk at this last point.—"Sure, in the present discussion *we* are so using 'object' that something is an object just in case it can be the referent of a singular term. But for Frege, an object is anything that can be the referent of a *proper name*, where he counts as proper names both singular terms and sentences. So, for Frege, truth-values may count as objects even if they cannot be referents of singular terms, because they can be referents of sentences."—This is not suasive, however. Consider Frege's explanation, in the *locus classicus* of the object/ concept distinction, of the sense in which he intends 'object': 'taking 'subject' and 'predicate' in the linguistic sense: . . . an object is something that can never be the whole *Bedeutung* of a predicate, but can be the *Bedeutung* of a subject' [Frege, 1997d, p. 186-7]. Now, I think it is simply incredible to suppose that Frege so intends 'subject' here that *sentences* count as subjects. That would be a recklessly non-standard use of the expression, if unaccompanied by any acknowledgement of deviation from ordinary use. But Frege's sensitivity to the way in which his readership is likely to read 'subject' is very clear: immediately after giving the above explanation, he addresses the fact that his reader may well count the likes of 'All mammals', 'Any man' and 'No true Scotsman'

as subjects, and explains why they should not be so counted. So Frege's explanation in fact leaves no room for objects that can only be referents of sentences. If something is an object, then it can be the referent of a subject; and it is clear that on Frege's use of 'subject', something can be the referent of subject if, and only if, it can be the referent of a singular term.[37]

The situation is much the same with regard to ascribing to Frege the view that no term-forming functor co-refers with a predicate. There is, I believe, both a dearth of textual evidence for that ascription, and evidence to the contrary. The second response, we ought to be clear, requires that ascription, though; because although our introduction, above, of term-forming functors designed to co-refer with predicates would arguably be blocked if Frege held that no singular term refers to a truth-value, if there are *any* term-forming functors that co-refer with predicates, Frege will be committed to the meaningfulness of GTCs.

One might, of course, jettison the exegetical claims of the second response in favour of corresponding revisionary recommendations for Frege: Frege ought to accord 'the True', 'the False' and their kin the same status as the singular terms he uses that purport to refer to concepts; quite generally, he ought to deny the possibility of singular reference to truth-values as he denies the possibility of singular reference to concepts; likewise, he ought to deny that term-forming functors that purport to co-refer with predicates do so co-refer, and to deny, quite generally, that concepts are possible referents of term-forming functors. Interestingly, this position can also be found in Dummett's work. Dummett seems to have undergone a change of mind between 'Truth' of 1959 and the first edition of *Frege: Philosophy of Language* of 1973. Where, in the former work, he insisted that we need not 'waste time' worrying about Frege's commitment to the meaningfulness of GTCs resulting from the intersubstitution of singular terms and sentences, in the latter work he calls that commitment a 'ludicrous deviation', a 'gratuitous blunder' and laments: 'It is tragic that a thinker who achieved the first really penetrating analysis of the structure of our language should have found himself driven into such absurdities' (this appears unrevised in the second edition: [Dummett, 1981a, p. 184]). He claims that 'it would have been in

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[37]It is not unreasonable to think, moreover, that if truth-values were to count as objects by virtue of being possible referents of sentence, but were not possible referents of singular terms, there would indeed be a sense in which the designation of the truth-values as objects would be a 'mere play on words' [Frege, 1997d, p. 158]

line with everything that Frege had said to date if he had held that sentences were of a different logical type from names, and that therefore truth-values were no more objects than concepts are' [Dummett, 1981a, p. 184].

The changes to Frege's semantic theory counselled by the revisionary version of the second response make for a significantly altered ontology. The ontology is, firstly, altered in respect of its typical or categorial structure. It is tempting to describe the change in this regard by saying that there are double the types: where once there were only objects and functions, now there are objects, truth-values, non-concept functions and concepts. But, of course, only one of the former pair of totalities (viz. objects), and only two of the latter four totalities (viz. objects and truth-values), form a type. In fact, the ontology does not gain *two* additional types, but denumerably infinitely many; for among the concepts are the nth-level m-adic concepts, for every finite n, m, which are now taken collectively to form a type, where previously they were counted only some among the nth-level m-adic functions, which were previously taken collectively to form a type. From this it can be seen that the number of types does not double, but remains the same: there remain exactly denumerably infinitely many types in the ontology. Nevertheless, the addition of as many new types as there were old types is significant.

It would be easy to suppose that the new types are, as it were, populated by old elements—by functions that already belonged to the ontology, but were previously counted as belonging to types formed by super-totalities of the totalities now taken to form types. On this picture, the change wrought upon the ontology is merely one of hiving off old elements into finer categories. In fact, this is not so. The ontology is radically changed in respect of its elements. In fact, none of the functions that belong to the old ontology belong to the new. For ease of exposition, allow me to focus just on monadic functions and let us grant for now that truth-values are not objects. Note first that there are no functions in the new ontology to whose domain belong both objects and truth-values. For Frege, types just are domains of functions; so no function can count things of different types in its domain. But that means that none of the first-level functions of the old ontology belong to the new ontology; for *all* of those functions count both truth-values and objects in their domains, and difference of domain membership between functions

f and f' suffices for the distinctness of f and f'.[38] But if none of the old first-level functions are elements of the new ontology, none of the old *second*-level functions are elements of new ontology; because the members of the domain of the old second-level functions are precisely the old first-level functions. More generally, and for parallel reasons, if none of the old n^{th}-level functions are elements of the new ontology, none of the old $n+1^{th}$-level functions are elements of new ontology. (This reasoning generalizes to functions of adicy greater than 1.) So none of the functions belonging to the old ontology, belong to the new. [39]

Just as the new ontology differs from the old in containing no functions to whose domain belong both objects and truth-values, so the new differs from the old in containing no functions to whose *range* belong both objects and truth-values. Speaking in a way that is, from the perspective of the revised Fregean theory, doubly pinch-of-salt-requiring, there were in the old ontology functions such as the following: the function whose value for a given argument is Venus, if the argument has a non-zero mass less than 80kg, the True if the argument has mass greater than 80kg, and the False otherwise.[40] There is reason for thinking that the elimination of such functions from Frege's ontology is a desirable result; for their presence, given other Fregean assumptions, yields, as Peter Sullivan [Sullivan, 1994] shows, a paradoxical consequence. The stipulation

$g(x)$=Venus, if the argument has a non-zero mass less than 80kg;
$g(x)$=the True if the argument has mass greater than 80kg;
$g(x)$=the False otherwise.

apparently suffices to secure a referent, and thus a sense, for 'g(Barack Obama)'. What is more,

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[38]This remark is one among several in the present paragraphs that need to be taken *cum grano salis*. For Frege, functions cannot be thought of as standing in the relations of identity or distinctness [Frege, 1997a, p. 175]. Functions can be thought of as standing in higher-level analogues of the identity and distinctness relations; but where functions f and f' have different domains, and thus are of different types, there will be no such higher-level analogues in which they can be thought to stand. This raises deep and important issues; for now, I must set these to one side and ask for the reader's patience.

[39]It might be thought that although none of, e.g., the old first-level functions belong to the new ontology, each old function at least has a 'counterpart pair' in the new ontology. Where f and f' are first-level functions of the new ontology, and f^{Old} a first-level function of the old ontology, let $< f, f' >$, be a *counterpart pair* of f^{Old} if, and only if, the domain of f is the type *object*, the domain of f' is the type *truth-value*, and $f^{Old}(x) = f(x)$ if x is an object while $f^{Old}(x) = f'(x)$ if x is a truth-value. (More salt is required here, of course. The variable x could not count both objects and truth-values among its values if objects and truth-values belong to different types.) But in fact, there are, as we shall see momentarily, first-level functions in the old ontology that lack a counterpart pair in the new.

[40]This is an example of a first-level function of the old ontology that has no counterpart pair in the new.

my acquaintance with the above stipulation apparently allows me to grasp that sense. How-
ever, not knowing whether Barack Obama has mass greater than 80kg, I do not know to what
'g(Barack Obama)' refers. Indeed, I do not even know whether 'g(Barack Obama)' refers to a
truth-value. But, for Frege, an expression's referring to a truth-value is a sufficient condition for
that expression's having a thought as its sense:

> The names, whether simple or themselves composite, of which the name of a truth-
> value consists, contribute to the expression of the thought, and this contribution of
> the indiviual [component] is its sense. If a name is part of the name of a truth-value,
> then the sense of the former name is part of the thought expressed by the latter
> name. [Frege, 1964, p. 90]
>
> Every such name of a truth-value *expresses* a sense, a *thought*. [Frege, 1964, p. 89]

An expression's referring to something that is not a truth-value is, moreover, a sufficient condi-
tion of that expression's not having a thought as its sense. So I do not know whether 'g(Barack
Obama)' has a thought as its sense. Yet I grasp that sense. So I can grasp a sense without
knowing whether that sense is a thought—the kind of thing that I can suppose, believe, suspend
judgement upon, and so forth—and indeed not even be in a position to remedy my ignorance
without undertaking an *a posteriori* inquiry (in this case, into Obama's mass). This is absurd.
However, if there simply are no functions like $g(x)$, whose ranges contain both objects and
truth-values, then this paradox does not arise.

The third possible response to the above argument concerning Frege's commitment to the
meaningfulness of GTCs can perhaps best be introduced by way of an alternative response to the
paradox highlighted by Sullivan. It is natural to respond that the real source of the paradoxical
consequence is not Frege's recognition of functions like $g(x)$, although refusing to recognise
such functions does indeed obviate the paradox; rather, the source is his counting it a sufficient
condition of an expression's having a thought as its sense that it refer to a truth-value.[41] It is

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[41] According to Sullivan, the source of the paradoxical consequence is that according to Frege 'whether a sense
is a thought is a matter, not of its internal form or nature, but of its particular semantics. It is a matter of what
object in particular it determines as its referent, and it cannot be a requirement of understanding that I must know
that' [Sullivan, 1994, p. 479]. This is clearly extremely close to the diagnosis just tabled in the main text; but there
is a subtlety concerning Sullivan's imputation to Frege: What is required for a sense to *determine* an object as its
referent? Does it require that the object in question *be* the referent of that sense? We do find in Frege's work—or,
rather, translations thereof—the verb 'refer' being used for the relation that obtains between a sense and the referent

here, one naturally surmises, that Frege's theory goes awry. (Indeed, when accompanied by the view that truth-values are objects, there is an even shorter route from this position to implausible consequences than is taken in the above paradox: the position has the consequence that 'the True', for example, must express a thought. But *which* thought does it express? No satisfactory answer to this question seems forthcoming.) Frege ought to make room—the response to the paradox continues—for expressions that designate a truth-value, but do not express a thought. Just as he denies that it is a necessary condition of an expression's having a thought as its sense that it refer to a truth-value (see footnote 41), he ought also to deny that it is a sufficient condition. Doing so would leave him in a position to deny that, given the above stipulation for $g(\xi)$, 'g(Barack Obama)' expresses a thought if Obama's mass exceeds 80kg—a premise required for the above paradox. (It similarly permits him simply to deny that 'the True' and its kin express thoughts.)

This sketch, on Frege's behalf, of a revisionary response to Sullivan's paradox leads us to a third response to the above argument concerning Frege's commitment to the meaningfulness of GTCs. If Frege sheds the view that an expression's referring to a truth-value is a sufficient condition of its expressing a thought, then—since an expression's referring to anything at all is, for Frege, a sufficient condition of its having a sense—the semantics that results is one on which co-referential expressions can have very different *kinds* of sense. Two expressions—e.g. '6 is a perfect number' and 'the True'—can refer to the same thing, though the sense of the one is a thought and the sense of the other is not a thought. But this means—I claim—that the resulting

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of an expression that expresses that sense: 'The constituents of the thought do refer to the object and concept, but in a special way' [Frege, 1997g, p. 363]. (The verb translated as 'refer' here is not 'bedeuten' but 'hinweisen'.) But on the reading of 'determine' most readily to hand, it too picks out that very same relation. On that reading, the object determined by a sense must (trivially) be the referent of that sense. But then, the claim that a sense's being a thought 'is a matter of what object in particular it determines as its referent' is not imputable to Frege; for Frege recognizes thoughts that are neither true nor false, and thus refer neither to the True nor to the False, and thus determine neither the True, nor the False (nor anything else) as their referents. Perhaps there is a sense of 'determine' on which it can be true that a *bedeutungslos* expression can nevertheless determine an object as its referent; this would require 'determine' to be an intensional transitive verb like 'seek'. Perhaps the sense of 'the largest prime' determines as its referent the largest prime, though there is none, just as perhaps John seeks the fountain of youth, though there is none. In that case, the fact that some thoughts are *bedeutungslos* wouldn't be inconsistent with Sullivan's imputation. But even on such a reading of 'determine', Sullivan's reading seems wrong: a sense's being a thought cannot be a matter of which object *in particular* it determines as its referent; because, presumably, a thought may determine no *particular* object as its referent, though must determine that its referent is nothing that is not a truth-value. In light of these subtleties, the Fregean commitment from which the paradox springs is better and more cautiously identified as in the main text, or equivalently, as follows: if an expression refers to a truth-value, then its sense is a thought.

semantics must violate Salva Significatione; for the difference in the kind of sense possessed by certain co-referential expressions will be borne out in failures of intersubstitutability *salva significatione*. Expressions that express a thought are such that they can occur meaningfully in isolation; expressions that do not are not. So '6 is a perfect number' is itself a context in which the substitution for it of 'the True' fails to preserve meaningfulness. Thus, on the revised semantics, the argument for the meaningfulness of GTCs resulting from the intersubstitution of sentences with 'the True', 'the False', etc., is blocked: that two expressions refer to the same truth-value does not imply that they are intersubstitutable *salva significatione*. Moreover, once it has been accepted that expressions referring to the same truth-value may possess different kinds of sense, such that there are contexts in which they fail to be intersubstitutable *salva significatione*, it becomes natural similarly to allow that expressions co-referring elsewhere in the ontology may possess different kinds of sense. It becomes natural to accept that predicates and term-forming functors, for example, may refer to the same function but feature different kinds of modes of presentation of that function as their sense, such that there are contexts in which they too fail to be intersubstitutable *salva significatione*. The contexts discussed above, in which the intersubstitution of predicates and term-forming functors produced GTCs, are excellent candidates; so the argument for the meaningfulness of those GTCs appears similarly blocked.

The third response, then, relieves Frege of the obligation to recognise apparent nonsense as sense. What it clearly sacrifices, however, is any hope of his justifying the thesis that singular terms and predicates never co-refer by appeal to their failure to be intersubstitutable *salva significatione*. Since, on the revised semantics, the failure of singular terms and sentences, and of predicates and term-forming functors, to be respectively intersubstitutable does not imply that they never respectively co-refer, it is entirely unclear why the failure of singular terms and predicates to be thus intersubstitutable should count as reason to think they never co-refer. Frege is left without a complaint against one who claims that singular terms and predicates may refer to one and the same object-concept, but feature different kinds of modes of presentation of that object-concept as sense.

Of these three responses, the first would very likely be Frege's own. He would presumably insist, as Dummett of 1959 claims he is entitled to, that the GTCs which are meaningful on his

semantics are defective only in a superficial, purely grammatical way. However, as I argued, this response really leaves Frege in no dialectical position to argue for his rejection of co-reference between terms and predicates by appeal to Salva Significatione; since his opponent is equally entitled to claim that GTCs resulting from term-predicate intersubstitution are defective only in a superficial, purely grammatical way. The first response does nothing, moreover, to avert Sullivan's paradox. The latter two, revisionary responses have the significant merit of forestalling that paradox. But only the second leaves Frege in any position to argue from Salva Significatione that terms and predicates never co-refer; for the third response involves accepting failures of intersubstitutability *salva significatione* between co-referential expressions. However, it strikes me that, by Fregean lights, the second response compares unfavourably to the third. The second response involves, we saw, subjecting Frege's ontology to a major overhaul. Indeed it involves repudiating some Frege's most distinctive ontologico-semantic doctrines: that the truth-values are objects; that sentences, in referring to objects, are semantically of a piece with singular terms; that concepts are functions; that, in referring to functions, predicates are semantically of a piece with term-forming functors. And the response significantly exacerbates Frege's concept *horse* problems. The third response, however, preserves the foregoing doctrines, avoids worsening Frege's equine troubles, and is, in one important respect, deeply Fregean in spirit: for it is another of Frege's most distinctive doctrines in the philosophy of language that two expressions can designate the same referent under very different modes of presentation. In essence, the third response just amounts to allowing that this difference in mode of presentation, e.g. between a sentence and a singular term designating a truth-value, may be a difference in *kind*, such that co-referring expressions need not be everywhere meaningfully intersubstitutable.

Frege's unrevised position does not sustain the argument for Frege's thesis by appeal to Salva Significatione, because of its own commitment to meaningful GTCs; but neither does the most favourable way of revising Frege's position in order to abolish that commitment, since the revision itself involves countenancing violations of Salva Significatione.

2.4   Conclusion

An effective, fleshed-out argument for Frege's thesis has not emerged from his remarks concerning the 'different behaviour, as regards possible substitutions,' of singular terms and predicates. If Frege's remarks are taken to concern intersubstitutability *salva congruitate*—if they are understood as an invocation of the Reference Principle—then the argument falls foul of an array of counterexamples in which co-referential expressions fail to be grammatically intersubstitutable. Or at least, this is the case if the notion of substitution at issue is a simple and standard one. A suggested richer notion of substitution may disarm the counterexamples, but, at best, renders it unclear that terms and predicates *fail* to be intersubstitutable *salva congruitate*. Frege's remarks might instead be understood as concerning intersubstitutability *salva significatione*, the key contention being that intersubstitution of terms and predicates results in nonsensical grammatical type confusions (GTCs). In that case, however, Frege confronts the objection that, by the lights of his own semantics, those allegedly nonsensical GTCs are in fact quite meaningful. That objection can be countered, I sought to show; but less tractable difficulties result from the fact that other GTCs, which equally seem nonsensical, *are* recognized as meaningful on Frege's semantics. This leaves Frege in no position to rule out co-reference between terms and predicates on the grounds that their intersubstitution results in intuitively nonsensical GTCs. There is independent pressure to revise Frege's philosophy of language in a manner that eliminates his commitment to meaningful GTCs; but the most favourable such revision involves accepting that co-referential expressions need not be intersubstitutable *salva significatione*: intersubstitutability of this kind then cannot be used as a test of co-reference with which to substantiate Frege's thesis.

Chapter 3

Unity, Unsaturatedness and Objecthood

3.1   The Argument from Unity

It is Frege's remarks in the closing paragraphs of 'On Concept and Object' which are most clearly advanced as an argument for the thesis that concepts are not objects. That argument is the topic of this chapter. The argument immediately follows Frege's acknowledgement (quoted on page 15 above) of the serious expressive impediments engendered by his insistence upon the unnameability of concepts.

> Somebody may think that this [the aforementioned expressive difficulty] is an artificially created difficulty; that there is no need at all to take account of such an unmanageable thing as what I call a concept; that one might, like Kerry, regard an object's falling under a concept as a relation, in which the same thing could occur now as object, now as concept. The words 'object' and 'concept' would then serve only to indicate the different positions in the relation. This may be done; but anybody who thinks the difficulty is avoided this way is very much mistaken; it is only shifted. For not all the parts of a thought can be complete [*abgeschlossen*]; at least one must be unsaturated [*ungesättigt*] or predicative [*prädikativ*]; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link [*Bindemittel*]. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'falls under', which need to be completed in two ways—by a subject and an accusative; and only because their sense is thus unsaturated are they capable of serving as a link [*Bindeglied*]. Only

when they have been supplemented in this twofold respect do we get a complete sense, a thought. I say that such words or phrases stand for a relation. We now get the same difficulty for the relation that we were trying to avoid for the concept. For the words 'the relation of an object to the concept it falls under' designate not a relation but an object; and the three proper names 'the number 2', 'the concept *prime number*', 'the relation of an object to a concept it falls under', hold aloof from one another just as much as the first two do by themselves; however we put them together, we get no sentence. It is thus easy for us to see that the difficulty arising from the unsaturatedness of one part of the thought can indeed be shifted, but not avoided. 'Complete' and 'unsaturated' are of course only figures of speech; but all that I wish or am able to do here is to give hints. [Frege, 1997h, p 192-93]

Recognising unnameable concepts embroils one, Frege acknowledges, in serious expressive difficulties. It is natural to seek, therefore, to shake off such difficulties by maintaining that concepts can be named and can thus be first, as well as second relata of the *falls under* relation. Doing so, Frege is claiming, will permit one only to shift but not eliminate the difficulty engendered by the unnameability of concepts. That is, though it may indeed allow one to shift the limits of singular reference, it will fail to divest one of a commitment to entities transcending them. The tacit moral appears to be that the shift, since profitless, should not be made in the first place; rather, we should bite the bullet, and embrace the conclusion that singular reference to concepts is impossible.

The consideration motivating Frege's claim that unnameability can only be shifted, not banished, is that 'not all the parts of a thought can be complete; at least one must be 'unsaturated', or predicative; otherwise they would not hold together'. The phenomena to which Frege appears to be appealing here are each often designated 'the unity of the proposition': a sentence is no mere *list* or *congeries* of names, and the thought expressed by a sentence is no mere list or congeries of the senses of names. Both sentence and thought possess a *unity* lacked, respectively, by a mere congeries of names and a mere congeries of senses of names. (The problem of accounting for that unity, and of explaining in what the difference consists between the complex entities instantiating it and the mere pluralities that do not, is the problem of the unity of the proposition.) For example, the pair of names 'the number 2' and 'the concept *prime number*', hold aloof from one another in failing to form a sentence; and the pair of senses they express correspondingly hold aloof from one another in failing to constitute a thought. Taking account

of these phenomena, Frege thinks, requires accepting that for any thought, at least one of its parts is 'unsaturated' or 'predicative'—elsewhere Frege uses 'incomplete' ('*unvollständig*') and 'in need of supplementation' ('*ergänzungsbedürftig*')—and accordingly, that for any sentence, at least one of its parts expresses an unsaturated sense. Moreover, though it is not made entirely explicit in the quoted passage, there can be little doubt that Frege will make the parallel claim with respect to sentences themselves: taking account of the unity the sentence likewise requires recognizing at least one of its constituents as unsaturated.

3.1.1   Unities at the Level of Reference

For at least two reasons, it is deeply puzzling that Frege should have thought that these phenomena give us reason to think that concepts are unnameable. Firstly, adverting to these phenomena seems simply to change the subject. Concepts are *never* constituents of thoughts and obviously never constituents of sentences. Why then, even if we are compelled to recognise unsaturated constituents of the unities in question—sentences and thoughts—should this reveal anything about the nature of *concepts*? In particular, why should it reveal that *they* must be unsaturated? Secondly, even supposing that considerations of this kind force us to recognise concepts as unsaturated, it is unclear why this should have any bearing upon the possibility of making singular reference to them. In what does the unsaturatedness of concepts consist, such that their being unsaturated is incompatible with their being objects?

A natural attempt to respond to the first worry is as follows. Just as there are unities at the level of language, the constitution of which by subsentential expressions can only be accounted for by recognizing at least one constituent expression as unsaturated, and just as there are unities at the level of sense, the constitution of which by the senses of subsentential expressions can only be accounted for by recognizing at least one constituent sense as unsaturated, *so*—Frege means to point out—there are unities at the level of reference, the constitution of which by the referents of subsentential expressions can only be accounted for by recognizing at least one constituent referent as unsaturated. Concepts are just these unsaturated referents; objects, their saturated counterparts.

If the consideration that there are such unities at the level of reference were really the op-

erative part of Frege's argument, it would certainly be odd that he neglects explicitly to advert to such unities in the quoted passage, but rather treats only of unities at the level of language and sense. However, this response has the merit of according with certain of Frege's remarks elsewhere. The following is particularly strong evidence that Frege would respond in the way just adumbrated:

> [T]he unsaturatedness of the concept brings it about that the object, in effecting the saturation, engages immediately with the concept, without need of any special cement. Object and concept are fundamentally made for one another, and in subsumption we have their fundamental union. [Frege, 1979d, p. 178]

Dummett expounds the point thus:

> A concept and an object, or a relation and two objects, need no glue to fit them together: they fit together naturally, in a way we can think of as analogous to that in which a predicate and a proper name, or a relational expression and two proper names, fit together to form a sentence. [Dummett, 1981a, p. 174-175]

Without need of any additional mediating element to act as adhesive, concept and object fit together into a fundamental union. The concept's being saturated by the object is precisely this unmediated coming together; and this is possible only because the concept admits of saturation—that is, because the concept is unsaturated. The problem that this conception of the relation between concept and object is clearly intended to obviate is familiar as 'Bradley's regress'[1]. Suppose we think of an object and a concept under which it falls as united by a third element: the relation of subsumption. We now confront the question of how the subsumption relation unites object and concept; and if we thought that an auxiliary relation was required to unite

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[1]Consider the sentences immediately preceding the quotation just given from [Frege, 1979d, p. 178]: 'In the sentence, 'Two is prime' we find a relation designated: that of subsumption. We may also say the object falls under the concept *prime*, but if we do so, we must not forget the imprecision of the linguistic expression we have just mentioned. This also creates the impression that the relation of subsumption is a third element that occurs as something additional to the object and the concept. This is not the case...'. (I have modified the translation given in [Frege, 1979d, p. 178]. Long and White render the sentence 'Diese erweckt auch den Anschein, als ob die Beziehung der Subsumption ein Drittes wäre, was zu dem Gegenstand und dem Begriffe hinzukomme.' [Frege, 1969, p.193] as 'This also creates the impression that the relation of subsumption is a third element supervenient upon on the object and the concept.' There is indeed a sense of 'to supervene' on which it is an adequate translation of 'hinzukommen', and when the adjectival cognate 'supervenient' has the cognate sense, Long and White's construction is, I suppose, acceptable too. But 'supervene' and 'supervenient' are seldom used with these senses. Their chief use is as philosophical terms of art; and on the sense they possess in the philosophical vernacular, they may not be used to translate the former sentence. It strikes me, therefore, as best to avoid them.)

concept and object, we ought, for parallel reasons, to think that a fourth relation unites object and concept, on the one hand, and subsumption, on the other. These, plainly, are the initial stages of an infinite regress, and one apt to appear vicious. At any stage, n, we can ask, of the uniting relation posited at stage $n - 1$, what unites it with the entities recognised at stage $n - 2$; and a further uniting relation seems requisite at stage n, if one was requisite at stage $n - 1$. Resisting the regress seems to require, at some stage, insisting that the combination of the entities in question is immediate. The only stage at which it seems plausible to halt proceedings in this way is the very first: concept and object engage immediately with one another, without need of an additional mediating relation as cement. Locating the immediate combination at some later stage, $m > 1$, seems arbitrary; indeed, one trades on this fact in setting up the regress: why is an additional uniting relation required at $m - 1$, but not at m? Although, in the paragraphs from 'On Concept and Object' above, the concern is with the holding together of parts of *thoughts*, something closely resembling the dialectic we have just rehearsed does indeed seem to be present: Frege seems to making the parallel point that, if a sense is genuinely to unite itself and one or more other senses, it must link immediately with those constituents, lest a further sense be required to unite that original link with those other senses. This fact, together with the cited remarks of Frege's elsewhere, constitute a good case that Frege would respond in the manner adumbrated above.

On reflection, however, it emerges that the response is deeply problematic. A (subject-predicate) sentence, *s*, is composed of those expressions the respective saturatedness and unsaturatedness of which is required for the unity of *s*. A thought expressed by such a sentence, *t*, is composed of those senses the respective saturatedness and unsaturatedness of which is required for the unity of *t*. But there is nothing composed of objects and concepts whose unity requires their respective saturatedness and unsaturatedness. Proper names and predicates compose sentences; their senses compose senses of sentences; but their *referents*—objects and concepts—do not compose the referents of sentences. The referents of sentences are truth-values, and though a concept is a function the *value* of which for a given object as argument is a truth-value, the concept and the object are not parts of that truth-value. The value of the concept *mortal*, for myself as argument, is the True; but myself and the concept *mortal* are not parts of the True.

Moreover, even if it were the case, on the contrary, that

(*) The value, $c(o)$, of any concept, c, for any object, o, as argument has c and o as parts.

c and o would still not *compose* $c(o)$. Some things, the *Xs*, compose something, *y*, just in case each of the *Xs* is a part of *y* and every part of *y* overlaps (i.e. shares at least one part with) at least one of the *Xs*. However, c and o would not be such that, for any part of $c(o)$, that part overlaps with c or o. For example, though myself and the concept *mortal* would be parts of the True, we would not compose the True; for Betelgeuse and the concept *star* would also be parts of the True, and neither overlaps with either myself or the concept *mortal*. Indeed, the True would have as parts not only myself and the concept *mortal*, but absolutely every object, since every object is such that, for some concept, the value of that concept for that object as argument is the True. Since every object would be part of the True, the False would be part of the True; and since, by parity of reasoning, every object would also be part of the False, the True would be part of the False. But the True and the False must be distinct. So we have distinct objects that are parts of one another. This is in violation of the core principle that parthood is a partial order—in particular the view that parthood is anti-symmetric. (For discussion of deviation from the orthodoxy, its motivations and consequences, see [Cotnoir and Bacon, 2012]). In fact, it is inevitable that, given (*), the True would contain *absolutely everything* as a part: (*) is surely only true if the parallel principle holds for second-level concepts—i.e. second-level functions whose range is {the True, the False}:

(**) The value, $c_2(f_1)$, of any second-level concept, c_2, for any first-level function, f_1, as argument has c_2 and f_1 as parts.

What grounds could one have for accepting (*) but not (**)? But equally, what grounds could one have for holding both (*) and (**) but denying the analogue for third-level concepts? Indeed, it seems altogether arbitrary to affirm only a finite set of the principles captured in the following schema

($*^\infty$) The value, $c_n(a_{n-1})$, of any n^{th}-level concept, c_n, for any argument, a_{n-1}, has c_n and a_{n-1} as parts.

I conclude that one must affirm all instances of ($*^\infty$) or none. Since we are supposing ($*$), we must accordingly suppose all instances. But now, for anything whatsoever—any object or function— some n^{th}-level concept is such that its value is the True for that thing as argument, and so, by the appropriate instance of ($*^\infty$), it would be part of the True. However, by parity of reasoning, absolutely everything would be part of the False. The True and the False, therefore would have precisely the same parts. But the True and the False must be distinct. So we have distinct objects with precisely the same parts. This, at the very least, makes it profoundly puzzling how the True and the False manage to be distinct objects.[2,3]

These considerations show, furthermore, that even supposing ($*$), saturation of a concept by an object could not account for the unity of the True. My saturating the concept *mortal*, for example, could not account for the unity of the True. The concept *mortal* and I would be but one humble couple among the parts of the True, of which absolutely everything is one, and of which most are distinct from, and non-overlapping with, the two of us. My saturating the concept *mortal* could only account for the unity of the True if it could account for the holding together of absolutely everything in the True; but for that it could not account: at best, it could account for *our* holding together. Generally, saturation of concept by object, could at best likewise account for the holding together of certain parts of the True—namely, those pairs $\{c_n, a_{n-1}\}$, such that c_n

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[2]This is, moreover, in violation of a further principle of classical mereology—extensionality—according to which distinctness entails a difference of parts. I don't wish to place much weight on this (more controversial) principle concerning parthood. For one thing, where terms and predicates are conceived as parts of sentences, and their senses parts of thoughts, there is reason to think that sentences and thoughts violate extensionality anyway. I think it suffices to stress the mystery of how the True and the False manage to be distinct on the present suppositions.

[3]The foregoing reasoning is very closely related to a *reductio* given in [Sullivan, 1992, p.107]. We can yet more closely follow Sullivan's reasoning to the same unhappy destination as follows. To hold ($*$) is to hold that composition of the referents of parts of sentences at least partly parallels the composition of the senses of parts of sentences. It seems ad hoc to stop short of asserting a full parallel between composition of the former kind and composition of the latter kind. The following is, according to Frege, true of composition of the latter kind: the sense of a part of a sentence is a part of the sense of the whole of the sentence. The parallel principle for composition of the latter kind is as follows: the referent of a part of a sentence is part of the referent of the whole of the sentence. This principle entails that the True and the False have precisely the same parts; for an expression with a sense occurs as part of a true sentence just in case it occurs as part of a false one.

is an nth-level concept, a_{n-1} an argument thereof, such that $c_n(a_{n-1})$= the True. But that leaves a vast amount of holding together to account for! What accounts for the holding together of all the objects, for instance, or the first-level concepts and the eighteenth-level concepts?

These reflections bring out the strange and extravagant consequences of supposing that concepts and objects are parts of truth values. They show, moreover, that not even embracing those consequences would permit one claim that, for Frege, the referents of sentences are to objects and concepts as the senses of sentences are to senses of proper names and predicates, and as the sentences themselves are to proper names and predicates. In particular, the saturation of concept by object could not account for the unity of the referents of sentences.

3.1.1.1 States of Affairs

However, for two reasons, these reflections do not, I think, altogether put paid to the argument of the penultimate paragraph of [Frege, 1997h]. The first reason is that there is a natural revision of Fregean doctrine that, plausibly, has the consequence that there *are* unities at the level of reference comparable to those at the levels of language and sense. The revision is to this effect: the referent of a sentence is not the truth-value of that sentence; it is rather a state of affairs that obtains just in case that sentence is true and of which (in the case of an atomic subject-predicate sentence) the referents of subject and predicate are parts. (This revision of Frege's mature view is arguably also a reprise of his earlier doctrine that sentences stand for judgeable contents. As Mark Textor puts the point, 'Jugeable contents seem to be nothing other than states of affairs' [Textor, 2012, §5.2] [4]. One might, in fact, regard the view that sentences stand for states of affairs as a constant in Frege's philosophy of language, but conceive of the mature Frege as individuating states of affairs in a spectacularly coarse-grained fashion, such that any states of affairs, S and S', are identical just in case the proposition that S obtains is materially equivalent to the proposition that S' obtains (cf. [Williamson, 2003b, p. 700]).) The referent of 'Mike is mortal', for example, will be the state of affairs of Mike's being mortal and will be understood to count Mike and the concept of mortality as parts. On this revised view there credibly *is* a

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[4] See also [Beaney, 1996, p.155]: '...'state of affairs' would probably be the best term to use in characterizing what Frege meant [in the *Begriffsschrift* and the *Grundlagen*] by the 'content' of a sentence.'

problem of unity concerning the referent of a sentence parallel to those concerning the sentence and its sense: just as a sentence is no mere congeries of subsentential expressions, and a thought no mere congeries of senses thereof, so a state of affairs is no mere congeries of referents thereof; it is an integrated whole, a unity. This revision eliminates several of the difficulties that arise on Frege's mature view if one supposes that the referent of a sentence is composed of the referents of its subject and predicate. For any (atomic) sentence, S, there is no longer the worry that the referent of S will have parts that overlap neither the referent of the subject of S nor the referent of the predicate of S. There is no longer the worry that the referent of 'Mike is mortal' must also contain Betelgeuse and the concept *star* as parts; for we've seen no reason to count the latter two as parts of the state of affairs of Mike's being mortal. Similarly, there is no longer the worry that if the referent of S is composed of the referents of S's constituent expressions, then the referent of S must be a bizarre entity that counts absolutely everything as a part, while managing to be distinct from another entity counting absolutely everything as a part, and that (consequently) violates the antisymmetry of parthood. The revision thus avoids the worry that saturation of an unsaturated concept by a saturated object could not explain the unity of the referent of a sentence because of the abundance of parts of the referent of the sentence that overlap neither concept nor object.

This is not to say that it is unproblematic to maintain that a sentence refers to a state of affairs in which object and concept feature as parts.[5] Indeed, we can here bring to bear misgivings

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[5]Dummett remarks,

> It is generally agreed that, if Frege had to ascribe reference to sentences at all, then truth-values were by far the best thing he could have selected as their referents: at least, he did not go down the dreary path which leads to presenting facts, propositions, states of affairs or similar entities as the referents of sentences. [Dummett, 1981a, p. 182]

The comment is puzzling, however: it is neither clear what underpins Dummett's confidence that general consensus markedly favours truth-values as referents of sentences over states of affairs, nor clear what Dummett has in mind with his 'dreary path' aspersion.

One philosophical current brought to mind by the remark is that of the *slingshot argument*: a remarkably frugal piece of reasoning purporting to establish that sentences with the same truth-value co-refer, lending clear support, therefore, to the identification of the referent of a sentence with its truth-value. Versions of the argument have variously been developed by Church [Church, 1956, p. 25], Gödel [Gödel, 1944], Quine [Quine, 1976, p 163-64] and Davidson [Davidson, 1967], but their seed is in Frege's less-than-compelling defence, in [Frege, 1997i, p.157ff], of the thesis that sentences refer to their truth-values. (Frege asks, 'What feature except the truth-value can be found that belongs to sentences, if the referent of their component parts is at all relevant, and remains unchanged by substitutions [of co-referential expressions for its component parts]?' [Frege, 1997i, p.158-59, translation altered]. Precisely the answer 'The corresponding state of affairs' seems forthcoming (cf. [Barwise and Perry, 1981, p. 395])).

Frege himself expressed, in correspondence with Wittgenstein, concerning the ontology of the *Tractatus.* Frege highlights a problem that issues from another fundamental principle concerning the parthood relation: parthood is transitive. This has the consequence that where an object is a part of a state of affairs, the parts of that object are also parts of that state of affairs. This gives results that are apt to seem wrong. Frege:

> I would like to have an example which illustrates that Vesuvius is a part of a state
> of affairs. Then it appears that the parts of Vesuvius must also be a part of this fact;
> the fact will therefore also consist of hardened lava. That does not seem right to
> me. [Frege et al., 1989, p. 20](Mark Textor's translation [Textor, 2012, §4.1])

This difficulty does not obviously afflict the views that proper names and predicates compose sentences and that the sense of names and predicates compose thoughts. This, perhaps, places some strain on the supposed parallelism between unities at the level of reference and those at the level of sense and language even on the present revision to Fregean doctrine.

3.1.2   Unsaturatedness and Objecthood

The second reason I think the argument of the penultimate paragraph of [Frege, 1997h] remains of interest is this. Even if the passage were to give us no grounds for thinking that *concepts* are not objects, it may yet give us grounds for thinking that there are parts of *thoughts* or *sentences* that are not objects. It may yet be that accounting for unity at the level of sense or language demands recognition of unsaturated thought-parts or unsaturated sentences-parts and, therewith, recognition of parts that are not objects. That, I take it, would be a conclusion as significant as the thesis that concepts are not objects.

This brings us to the second puzzling feature of Frege's argument, mentioned on page 92 above. Even if considerations of unity force us to recognise unsaturated entities, why should this entail recognising entities that are not objects? *In what does unsaturatedness consist, such that unsaturatedness is incompatible with objecthood?* The question is a vital one for the present

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I regret that a discussion of the slingshot argument lies beyond the scope of this essay. As developed by Church et al., the argument is more compelling than Frege's aforementioned defence; but if one is looking for the general agreement of which Dummett speaks, the slingshot doesn't fit the bill: it's a controversial argument and there's a substantial literature on resisting it ([Barwise and Perry, 1981] is a good place to start).

inquiry, since it is natural, if asked what grounds Frege's rejection of singular reference to concepts, to simply advert to the unsaturatedness of concepts and to contrast it with the saturatedness of objects: what motivates the Fregean claim 'that there can be no such thing as a proper name of a concept', as David Bell puts it, 'is that objects are complete in themselves, saturated or self-subsistent, while concepts are, by contrast, incomplete, unsaturated, and unable to stand by themselves' [Bell, 1979, p. 29]. This proposal has compelling textual support: Frege explicitly claims that a concept cannot play the part of the referent of a grammatical subject 'because of its predicative nature [*wegen seiner prädikativen Natur*]' [Frege, 1967, p. 171], and explains that '[w]hat [he] here call[s] the predicative nature of concepts is just a special case of the need of supplementation, the unsaturatedness, that [he] gave as the essential feature of a function' [Frege, 1997h, p. 186]. But the proposal nevertheless immediately invites versions of our question: *Why cannot that which is unsaturated be named? Why must objects invariably be saturated?*

Perhaps the reader is already inclined to protest on Frege's behalf. After all, Frege repeatedly cautions that 'unsaturated' and its antonym are only figures of speech, that definition is impossible in this territory, and that all he is able to do in this connection is to give hints. He confessedly 'count[s] on the co-operative understanding of the reader' [Frege, 1984b, p. 281], upon her 'agreeing to meet [him] half-way' [Frege, 1984c, p. 292] and not begrudging a pinch of salt [Frege, 1997h, p. 192]. Indeed, he claims that the saturated/ unsaturated distinction 'must simply be accepted' as a 'logically primitive phenomenon' [Frege, 1984b, p. 281]. Are we not, then, already asking too much of Frege?

No. We need not object to granting, for the sake of discussion, Frege's contention that the notion of unsaturatedness is too fundamental to admit of definition. Hints will suffice. But the hints must extend to furnishing us with at least a reasonably firm grip on the attribute towards which Frege is gesturing, on its incompatibility with objecthood, and on why accounting for unity demands recognising entities that possess it. To the extent that expecting hints of that calibre is incompatible with Frege's insistence that the saturated/ unsaturated distinction must simply be accepted, we should reject that insistence. We need not begrudge a pinch of salt; but we are entitled to expect that our co-operative understanding be reciprocated by a co-operative

explanation on the part of the author. If *no* light can be cast on the incompatibility of unsaturatedness with objecthood, and upon the need to countenance the unsaturated in order to account for unity, then I suggest that we are being asked, unreasonably, to meet Frege *much* further than half-way. Let us, then, examine whether that light can be cast.

3.1.2.1 The Unsaturatedness of Linguistic Expressions

It can seem that the notion of unsaturatedness is meant, in the first instance, to be explained by reference to the unsaturatedness of linguistic expressions. It can seem, furthermore, that the unsaturatedness of expressions is simply a matter of their containing gaps or empty places. Let's take these two points in reverse order.

The second point is readily illustrated with two passages from 'Function and Concept':

> We can split ['$\acute{\epsilon}(\epsilon^2 - 4\epsilon) = \acute{\alpha}(\alpha(\alpha - 4)$'] into '$\acute{\epsilon}(\epsilon^2 - 4\epsilon)$' and '$(\ ) = \acute{\alpha}(\alpha(\alpha - 4)$'.
> This latter part is in need of supplementation, since on the left of the 'equals' sign it contains an empty place. [Frege, 1997e, p. 140, translation altered]

> Thus, e.g., we split up the sentence 'Caesar conquered Gaul' into 'Caesar' and 'conquered Gaul'. The second part is unsaturated—it contains an empty place; [Frege, 1997e, p. 139]

The clear impression given is that the unsaturatedness (or need of supplementation) of the predicates in question is just a matter of their containing empty places into which singular terms can be inserted. This hardly seems like a mysterious attribute.

The first point is strongly suggested by the following passage of the posthumously published paper dubbed 'Comments on *Sinn* and *Bedeutung*':

> [O]ne can always speak of the name of a function as having empty places, since what fills them does not strictly belong to them. *Accordingly*, I call the function itself unsaturated, or in need of supplementation, *because* its name has first to be completed with the sign of an argument if we are to obtain a *Bedeutung* that is complete in itself. [Frege, 1997a, p. 174, my emphasis] [6]

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[6]Rather jarringly, Frege calls incomplete expressions names of functions—function names (*Funktionsname*) for short. It is essential to recognise, however, that function names are not proper names of functions.

Quoting this same passage with the same added emphases, Heck and May conclude: 'So, in the end, it is the unsaturatedness of the *expression* that is basic' [Heck and May, 2013, p. 844]. Frege's account here of the unsaturatedness of functions does indeed seem to be based on their being referents of expressions that are themselves unsaturated or incomplete. Now, what is disappointing about the passage is that the explanation *invokes* the notion of completeness at the level of reference with the condition 'if we are to obtain a *Bedeutung* that is complete in itself', because if the notion of completeness at the level of reference were grasped securely enough for it to be invoked in the explanans of a profitable explanation, we would not need an explanation of *in*completeness at the level of reference, which is precisely what we seek; for a referent could be understood to be incomplete just in case it were not complete. Let us ignore, then, this invocation of the complete/ incomplete distinction at the level of reference, and instead put our second point above to use. Here is the resultant proposal. The unsaturatedness of concepts (we focus on that case), and of senses that are modes of presentation thereof, is to be explained in terms of the possession, by certain expressions, of empty places.

Flesh can be put on the bones of this rather programmatic proposal in two ways, I think. The first involves taking 'accordingly' and 'because' in the above quotation as seriously as possible: At the level of reference, for something to be unsaturated is for it to be such that any expression of which it is the referent feature at least one empty place. This account has the considerable merit of immediately providing an excellent explanation of the incompatibility of referential unsaturatedness with objecthood. To be an object is to be capable of being the referent of a proper name. Proper names lack empty places. To be unsaturated is to be capable of being the referent only of expressions featuring empty places ("gappy expressions"): this immediately follows from the present account of unsaturatedness if the quantifier phrase 'any expression' therein is read—as I suggest it must be in this context—as ranging over all possible expressions. On the present account, the incompatibility of unsaturatedness and objecthood is explicable thus: unsaturatedness *consists*, inter alia, in not being an object.

This merit notwithstanding, the account has several serious shortcomings. Firstly, it does little, as it stands, to cast light on the connection between unsaturatedness and unity: why does accounting for the unity of states of affairs require recognising constituents of states of affairs

which can only be referents of gappy expressions? Secondly, the account scuppers the proposal, mentioned on page 100 above, that what grounds or motivates Frege's thesis that concepts cannot be named is that they are unsaturated: if the unsaturatedness of concepts partly *consists* in their being unnameable, it would be question-begging to adduce their unsaturatedness as grounds for deeming them unnameable. Thirdly, the account is not satisfactorily extensible to the level of sense. Extending the account to unsaturatedness at the level of sense ("sential unsaturatedness")[7] must yield the following: at the level of sense, for something to be unsaturated is for it to be such that any expression of which it is the sense features at least one empty place. This account of sential unsaturatedness accords very closely with the view, for which Dummett argues, that the unsaturatedness of a sense 'consist[s] merely in its being the sort of sense appropriate to an incomplete expression' [Dummett, 1981a, p. 291]. The account is unsatisfactory, however. It provides no explanation, as it stands, of the incompatibility of sential unsaturatedness with objecthood, because, for all that has been said so far, it may be that that which can only feature as the *sense* of a gappy expression can nevertheless feature as the *referent* of a gapless expression; in particular, it may be that it can feature as the referent of a proper name.[8] (It might be suggested that Frege could invoke his theory of indirect reference to answer this challenge. According to that theory, in *oratio obliqua*, an expression refers not to that to which it customarily refers, but rather to that which it customarily expresses—what is ordinarily its sense. Thus, an unsaturated sense must also be something that can feature as the *referent* of a gappy expression, since it is the referent of any gappy expression of which it is customarily the sense when that expression appears in indirect speech. These considerations do not suffice to answer the challenge, however. They only establish that, granting Frege's theory of indirect reference, the sentially unsaturated *can* feature as the referent of an gappy expression, not that it *can only* feature as the referent of an gappy expression—and, in particular, not that it is incapable of featuring as the referent of a proper name. That gappy and gapless expressions never co-refer cannot permissibly be *assumed* in the present context.) The account conflicts, furthermore, with

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[7]I owe the neologism 'sential', meaning 'of or relating to sense', to Michael Bench-Capon.

[8]Though I say 'in particular', this last clause is, for Frege, no more particular than the one that precedes it; for the gapless expressions are, by his lights, just the proper names. We will not follow Frege in this respect, however, but will rather count sentences as gapless expressions that are not proper names.

an important thesis of Frege's concerning unsaturatedness:

> It is really in the realm of sense that unsaturatedness is found, and it is transferred from there to the symbol. [Frege, 1984a, p.393]

According to Frege, the unsaturatedness of expressions ("linguistic unsaturatedness") is in fact derivative upon sential unsaturatedness. However, the present account of sential unsaturatedness, and the Dummettian view with which it closely accords, precisely invert that order of priority. (The point equally confutes Heck and May's conclusion mentioned on page 102 above.) It seems clear, moreover, that in Frege's view, the applicability of the saturated/ unsaturated distinction at the level of sense is prior, not only to its applicability at the level of language, but also to its applicability at the level of reference:

> The words 'unsaturated' and 'predicative' seem more suited to the sense than the *Bedeutung*; still there must be something on the part of the *Bedeutung* which corresponds to this, and I know of no better words. [Frege, 1997a, p. 174]

This primacy of sential unsaturatedness on Frege's conception makes all the more dissatisfying the account's failure to explain the incompatibility of sential unsaturatedness with objecthood. When fleshed out in this first way, then, the proposal that unsaturatedness is to be explained in terms of the gappiness of certain expressions leaves much to be desired.

The second way of fleshing out that proposal is to this effect: unsaturatedness at the levels of sense and reference is to be understood *by analogy with* or *on the model of* gappiness at the level of linguistic expressions. Much that Frege says recommends this account. For example:

> I also call the first constituent ['Two'] saturated; the second ['is a prime number'], unsaturated. To this difference in the signs there of course *corresponds an analogous one* in the realm of *Bedeutungen.* [Frege, 1984b, p. 281, my emphasis][9]

The suggestion is that we are to think of the unsaturated at the levels of sense and reference as possessed of empty places, in some sense analogous to the sense in which unsaturated expressions possess empty places. A concept, on this picture, contains an empty place which can be occupied by an object to constitute a state of affairs, and the sense of a (monadic) predicate

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[9] Similar remarks are to be found in [Frege, 1979d, p. 177], [Frege, 1980, p. 142], and [Frege, 1984c, p. 292].

contains an empty place which can be occupied by the sense of a name to constitute a thought, much as a predicate contains an empty place which can be occupied by a name to constitute a sentence.

One immediate concern is whether this picture can be anything more than 'hopelessly metaphorical', as Magidor describes it [Magidor, 2009b, p. 5]. She continues: 'No semanticist seriously thinks that the semantic values of predicates are literally entities which contain gaps into which we try to fit other entities' [ibid.]. I am not quite so sure. There are entities taken seriously by semanticists and other theorists of intentionality that seem to be literally describable as containing gaps at least to approximately the extent that we speak the literal truth in describing predicates and other incomplete expressions as containing gaps. Perhaps we fall well short of speaking the literal truth in describing predicates as containing gaps; but then, our comfort with describing them thus, and our confidence that we know what we mean in doing so, ought to extend to describing predicate-senses and concepts similarly—making the foregoing picture metaphorical, but certainly not hopelessly so. The entities I have in mind are variously known as *gappy propositions*, *gappy contents* and *unfilled propositions*. Propositions—-the things expressed by declarative sentences—are commonly conceived as complex entities that possess a structure comprising a number of positions or places in relation to one another. These positions are taken to be occupiable by propositional constituents—be they, as Russell maintained, the very objects, properties and relations the proposition is about, or, as a Fregean would maintain, only modes of presentation thereof.[10] *Gappy propositions* differ from propositions thus conceived just in having at least one position that is unoccupied or unfilled. They may be posited for various theoretical purposes. The gappy cousins of Russellian structured propositions, for example, are accepted by some direct reference theorists as the contents expressed by sentences featuring empty names (see e.g. [Braun, 1993], [Braun, 2005]). While the proposition expressed by 'Obama is mortal' is understood, on this view, to be a structured entity whose two positions are respectively occupied by Obama and mortality, the gappy proposition expressed by 'Vulcan is a planet' is an entity with the same structure whose two positions are respectively unoccupied—since 'Vulcan'

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[10]I will assume that Russellian propositions are the same things as states of affairs.

is empty—and occupied by planethood. Gappy propositions have also been proposed as the contents of certain perceptual experiences [Siegel, 2010, §5.1]. If gappy propositions are accepted, it is natural similarly to accept gappy sentences: We conceive of sentences as complex entities that possess a syntactic structure comprising a number of positions or places in relation to one another. These positions are taken to be occupiable by subsentential expressions. Gappy sentences differ from sentences just in having at least one position in this syntactic structure that is unoccupied. Here, then, is a proposal: predicates are to be identified with gappy sentences; their referents, concepts, are to be identified with gappy Russellian propositions; and their senses are to be identified with gappy Fregean propositions.

That predicates are gappy sentences is an independently attractive interpretation of talk of predicates as possessing empty places; and on the present proposal, the referents and senses of predicates possess empty places in a sense highly akin to that in which predicates possess empty places. It strikes me, therefore, that on this proposal, we speak the literal truth in describing concepts and predicate senses as possessing empty places to at least approximately the degree that we speak the literal truth in describing predicates likewise.

The foregoing proposal represents a reputable semantic picture which sustains Frege's claim that the linguistic distinction between the saturated and the unsaturated is paralleled by a corresponding and analogous distinction in the realm of *Bedeutungen*. (Again, this is not to say that picture is unproblematic. One of several questions hanging over it is this. If we are identifying predicates with gappy sentences, how are we to conceive of the subsentential expression actually *occupying* what we would intuitively characterize as predicate position? The syntactic structure of 'Obama is mortal', for example, presumably contains two positions, the first of which is occupied by the name 'Obama'. What is it that occupies the second? It cannot be the predicate of 'Obama is mortal', for whatever occupies the second position of the sentence also occupies that same position in the predicate itself (and I take it that we would not want to say that one of a predicate's multiple syntactic positions is occupied by itself). Towards answering this question, some inspiration can be perhaps be found in a distinction Frege draws in discussing the unsaturatedness of the sentential connective 'and'. This, he says, 'is doubly unsaturated: to saturate it we require both a sentence preceding and another following' [Frege, 1984a, p. 393], but qualifies

this remark by saying, 'As a mere thing, of course, the group of letters 'and' is no more unsaturated than any other thing'. One might, in this spirit, contrast the bare orthographic item 'is mortal' with the partially vacant sentence structure that is the predicate, and propose that the former occupies the filled position of the latter (though to draw the contrast, as Frege does, in terms of *thinghood* is tendentious, as will emerge below, since thinghood is simply objecthood). Similar questions, however, arise at the levels of sense and reference. For example, if we identify concepts with gappy states of affairs, how are we to conceive of the entities occupying what we would intuitively characterize as property position?)

What the proposal fails to do, I submit, is to cast *any* light on the incompatibility between unsaturatedness and objecthood. Why should a structured entity's possessing an empty place render it unnameable? The gappiness under discussion invites comparison with the possession of vacua of a different kind by certain material objects. We feel no qualms about the possibility of singularly referring to incomplete jigsaw puzzles, pieces of Emmental or molars in need of a filling. How does the gapped nature of concepts differ from the holed nature of these material objects, such the latter enjoy objecthood, but the former do not? Any air of silliness attending this question attaches to the thought that possession of holes could be an obstacle to being named.[11]

The mysteriousness of the incompatibility of unsaturatedness, on the present account, with objecthood is compounded by the fact that, for Frege, linguistic unsaturatedness is *not* incompatible with with objecthood. It is clear that Frege deems unsaturated expressions objects. He plainly indicates, for example, in a letter to Russell, that he takes incomplete expressions to be possible referents of proper names. Specifically, they are referents of the proper names that result from simply enclosing them in quotation marks (a fact he exploits in the passages from 'Function and Concept' on page 101):

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[11]While holed material entities have an excellent claim to objecthood, I note that at least one metaphysician—namely, C.B. Martin—has defended the thesis that the holes in them are not objects. The parallel with Frege's view about concepts is striking in that Martin's denial of the objecthood of holes (he defends the same position with regard to *absences* or *lacks*) is combined with a robust commitment to their reality. While decrying the 'deontologizing' [Martin, 1996, p. 58] of holes or voids, he says nonetheless 'An absence or lack of something or a hole are not things' [Martin, 1996, p. 58] and 'A void is not a *thing* . . .' [Martin, 1996, p. 62, orignal emphasis].

> While '( ) • 3 + 5' is a function name, "'( ) • 3 + 5'" is a proper name, and its referent
> is the function name '( ) • 3 + 5'. [Frege, 1980, p. 136].[12]

It might be thought that our conception of referential and sential unsaturatedness ought to be guided by a looser analogy with the gappiness of certain linguistic expressions. Perhaps the consideration to focus upon is that a monadic predicate, for example, must be supplied with a name in order that a sentence be produced. Martha I. Gibson pursues the analogy as follows.

> The function itself is incomplete in that it requires supplementation by a certain number of arguments in order to produce a value. … The incompleteness of the concept is just its need to be supplemented with objects in order to produce the value true or false. [Gibson, 2004, p. 4-5]

My concern is that this conception of unsaturatedness does not make for a contrast with the nature of objects. It would, after all, be very puzzling to extend Gibson's remarks by saying 'On the other hand, by contrast, objects do *not* require supplementation by a certain number of arguments in order to produce a value'. The best that could be made of that remark, I think, would be a statement of the consideration that if an object is given then a value of *some* function is thereby given; for the object is the value of some function for some argument. But it is not clear that functions lack this property: Frege himself does not recognise functions whose values are functions, but it is not clear that we should follow him in this respect. As for the consideration that a concept, in particular, needs to be supplied with an object in order to produce a truth-value, it is likewise not clear that objects lack that property. If the concept *horse* alone is given, no truth-value has yet been determined until an argument is also given; but similarly, if the object Earth is given, no truth-value has yet been determined until a concept is also given. I struggle to see why we are not equally entitled, therefore, to say that an object must be "supplemented" with a function in order to produce a truth-value, and hence to characterise objects as unsaturated or incomplete in this sense.

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[12]Some commentators, including Geach ([Geach, 1976b, pp. 59-61], [Geach, 1961, p. 144], [Geach, 1976a, p. 440]), and Hugly [Hugly, 1973, pp. 236-242], have maintained that, for Frege, incomplete expressions are themselves functions mapping singular terms or sentences to singular terms or sentences. This is clear evidence to the contrary.

3.1.2.2   Unsaturatedness as Existential Dependence

Another important conception of Fregean unsaturatedness finds textual support in the very earliest explicit appearance in Frege's corpus of the thesis that concepts are unsaturated. This is in Frege's letter to Anton Marty of 29/8/1882:

> A concept is unsaturated [*ungesättigt*] in that it requires something to fall under it; hence it cannot exist on its own. [Frege, 1997f, p.81]

The succeeding remarks further confirm that Frege wishes, at this stage, to deny the 'independent existence of concepts' [ibid.].

Now, his assertion that a concept requires something to fall under it, and that it is in its requiring this that its unsaturatedness consists, will strike anyone acquainted with Frege's logicism as very odd. That a concept need *not* have anything falling under it—that there *are* concepts under which absolutely nothing falls—is exploited by Frege himself in his definition of the natural numbers. In the *Foundations of Arithmetic*, appearing two years after the letter to Marty, zero is defined as the number which belongs to the concept *non-self-identical*, under which precisely nothing falls. Nevertheless, Frege's conception of concepts, at this stage, as lacking independent existence and as existentially dependent, in particular, upon the objects that fall under them, connects intimately with an important tradition in the metaphysics of properties. It is a conception apt to be echoed, for example, by those metaphysicians who conceive of properties as *ways things are.* (See e.g. [Armstrong, 1997b, §3.6] and [Lowe, 2006, p. 90ff]. Armstrong explicitly draws the connection between conceiving of properties as ways things are and considering them unsaturated (he is thinking of properties as universals): 'the conception of universals as ways . . . quite naturally goes along with, the important Fregean idea . . . that universals are 'unsaturated' entities' [ibid.]). Ways things are, the thought goes, depend for their existence upon things that are those ways. They cannot exist in isolation, as free-floating denizens of the world, any more than a feline grin could exist without a cat (though a cat could exist without a feline grin). They lack—to use an expression of Frege's from the *Grundlagen*—*self-subsistence* (*Selbständigkeit*)[13]. A way something is is always a way *something* is.

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[13]Most literally: the quality of being self-standing.

Frege's apparent alignment with this tradition of considering properties to be non-self-subsistent can in fact be reconciled, I suggest, with his subsequent commitment to concepts whose extensions are empty. This can be accomplished by first extending the conception of properties as ways things are to the case of relations: we understand a binary relation, for example, to be a way two things are, one with respect to the other [Lowe, 2006, p. 91]; and we consider that relation to depend for its existence upon two things being that way, one with respect to the other. Secondly, we conceive of functions as relations of a particular species, in the customary way—namely, as relations, R, such that, for all x, y, z, if Rxy and Rxz, then $y = z$. Combining these two conceptions with Frege's conception of concepts as functions from objects to truth-values we secure a position on which concepts, though existentially dependent upon objects, may nevertheless have an empty extension: they are existentially dependent upon objects since they are ways object–truth-value pairs are, one with respect to the other; but they need not have a non-empty extension, since they need not be a way some object and the True are, one with respect to the other.

Frege can, then, in this regard consistently maintain that concepts are existentially dependent entities and that it is in this dependence that their unsaturatedness consists.

This account makes for a noteworthy equation of unsaturatedness with *insubstantiality*—the quality of not being a *substance*—at least on one important conception of the latter quality. To be a substance, on said conception, is precisely to enjoy existential *in*dependence. This is the Cartesian conception, for example: 'by substance we can understand nothing other than a thing which exists in such a way as to depend on no other thing for its existence' ([Descartes, 1984, Vol. I, p. 210], quoted in [Lowe, 2010, §1])

To underscore an emerging pattern, however, the account of unsaturatedness as existential dependence shows little promise of furnishing a satisfactory explanation of the incompatibility of unsaturatedness with objecthood; for existential dependence appears rife among objects: sets depend for their existence upon their members; trees upon carbon atoms; water waves upon molecules of H_2O; assassinations upon those assassinated; and, as we have already seen, particular grins, feline or otherwise, upon those who wear them. None of these dependents seem incapable of being the referents of singular terms. Indeed, later in his career, Frege himself, as

I. Angelelli notes [Angelelli, 1967], explicitly ascribes *unselbständigkeit* [Frege, 1997j, p. 339] to ideas and sensations, though it seems quite clear that Frege would classify the latter as species of object:

> It seems absurd to us that a pain, a mood, a wish, should go around the world without an owner, independently [*selbständig*]. …ideas need an owner. Things of the outer world are on the contrary independent [*selbständig*] [Frege, 1997j, p 334].

The conception of unsaturatedness as existential dependence, coupled with the thesis that objecthood is incompatible with unsaturatedness, seems to yield a wildly un-Fregean classification of entities as objects and non-objects.[14] Further reflection suggests that the alleged existential dependence of properties and relations upon objects would be altogether reciprocated by the objects themselves. Objects must instantiate some properties; they cannot exist alone, thoroughly unpropertied. They must stand in some relations too; they cannot exist alone, thoroughly unrelated. Or at least, these things are to be granted by anyone who countenances properties and relations in the plenitude required by Fregean doctrine.[15]

The kind of dependence we have so far been considering ascribing to properties and relations would be, as it were, a *horizontal* dependence relation among the constituents of unities at the level of reference. One kind of constituent of a state of affairs has been supposed dependent upon another kind of constituent of a state of affairs. The generalized supposition would

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[14]With unsaturatedness conceived as existential dependence, there is a certain historical incongruity associated with the Fregean view that unsaturatedness is, as it were, inversely correlated with objecthood: The God of classical theism becomes, on this conception, the paragon of saturatedness: that God is absolutely saturated becomes one way of stating the doctrine of *divine aseity*, which is explained as follows by Jeffrey E. Brower:

> …traditional theists also habitually think of [God] as an absolutely independent being—that is, as a being who is *first* or *primary* in the sense that he does not depend on anything distinct from himself. Such a being, it is often said, exists entirely from himself (*a se*). Hence, …his *aseity*. [Brower, 2009, §1.1]

And yet there is a long and robust tradition in theism of *denying* that God is an object. In fact, Paul Tillich specifically maintains [Tillich, 1978, p. 236] that God can only be possessed of aseity if he is *not* an object—not *a* being, but being itself.

[15]There may still be scope for maintaining that *concepts*—when conceived as ways object–truth-value pairs are, one with respect to the other, as suggested on page 110—are asymmetrically dependent for their existence upon objects. This could be maintained if it were held that the truth-values are contingent existents. In that case, there could not be concepts without objects, but there could be objects without concepts. It's hard to see what significance this asymmetric dependence of concepts on objects could have, though, given that concepts are (presently being conceived as) just a special kind of relation. Just as there are special kinds of relation that depend asymmetrically upon objects, so there are special kinds of object that depend asymmetrically upon relations: tennis balls cannot exist without relations, though relations can exist without tennis balls.

be that the unsaturated constituents of a unity of a certain kind generically depend upon saturated constituents of unities of that kind. A rather different supposition that also deserves our attention, however, is that there obtains a *vertical* existential dependence on the part of the unsaturated constituents of unities upon the unities themselves. As an interpretation of Frege's doctrine of the unsaturated, this supposition is developed in lucid detail by Peter Simons in his 'Unsaturatedness' [Simons, 1981]. Concerning unities at the level of sense, Simons reads Frege thus:

> Thoughts are, for Frege, self-sufficient entities but any thought which contains parts, in particular other self-sufficient parts such as other thoughts, or the senses of proper names, must contain at least one part which is unsaturated or supplement-demanding: in our terms, a dependent part. [Simons, 1981, p. 80]

Simons defines *dependent part* [Simons, 1981, pp. 78-80] by way of a series of definitions inspired by those of Husserl, whom Simons considers to have advanced an account of unity similar to that which he distinguishes in Frege. When Simons' definitions are traced back, it emerges that to say that an unsaturated constituent (say, a monadic predicate-sense), u, of a thought, t, is a dependent part of t is to say this:

1. u is a part of t.

2. For some kinds α, β:

    (a) u is of kind α;

    (b) t is of kind β;

    (c) Nothing of kind α can exist unless something of kind β exists;

    (d) t meets u's need for something of kind β. [16, 17]

Presumably, the particular kinds, α, β, that are supposed to witness (2) are precisely the kinds *monadic predicate-sense* and *thought*, the claim being that no monadic predicate-sense could

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[16] In general, a's meeting b's need for a β is meant to be consistent with other things' also meeting b's need for a β.

[17] I have ignored one complication in the definitions which, roughly, allows for something to be a dependent part of a whole by depending on a part of that whole rather than the whole itself—i.e. I have ignored the disjunct 'or by some part of b' in definition D3. This will not matter for our concerns.

exist without some thought existing and that t meets u's need for a thought. But now it surely becomes clear that we have similar and equally compelling grounds for likewise classifying e.g. name-senses—which Frege considers to be saturated objects—as dependent parts of thoughts: for no name-sense can exist unless some thought exists. Similarly, at the level of reference, while there is a good case for classifying properties and relations as dependent parts of states of affairs, there is an equally good case for classifying objects likewise. Again, objects cannot exist altogether unpropertied, so cannot exist without being part of—and, therefore, without there also existing—some state of affairs. So, although Simons notes early in his exposition that the unsaturated is meant to be distinguished 'utterly from all objects' [Simons, 1981, p. 74] by its unsaturatedness, the account he develops of unsaturatedness in terms of dependent parthood fails to preserve this feature in any recognizably Fregean fashion.

For what it is worth, I also doubt that positing horizontal or vertical existential dependence within a complex whole can account for the unity of that whole. Consider horizontal dependence first. We are told that entities of one kind, k, cannot exist without entities of kind k', where wholes of a certain sort feature constituents from both k and k'. However, this alone does not preclude entities of kinds k and k' merely, as it were, *disparately co-existing*; so it does not alone explain their being united into integrated wholes.

Vertical dependence seems to obviate this problem of disparate co-existence, since it is the integrated wholes themselves whose existence is required by the relevant kind of constituent. But the genericity of the vertical dependence relation in question seems to me to prevent it from accounting for unity. Simons writes,

> It is characteristic of a dependent part that it cannot be detached or isolated from
> the whole of which it is part, ... [Simons, 1981]

However, straightforwardly read, this is wrong. Where w is a whole of which a is a dependent part, a may well be detachable or isolable from w: it may be possible for a to exist and yet not be a part of w (though perhaps the only circumstances in which this is so are circumstances in which w does not exist). Qua dependent part, a cannot exist unless wholes (at least one) of w's *kind* exist; but since a's need for something of w's kind may have been met, or may in fact be met, by something of that kind *other* than w, it need not be the case that a cannot exist except as

a part of w. In this sense, a dependent part is only generically dependent upon the kind of whole of which it is in fact a part. Thus, Simons is closer to the mark when he summarizes (what he takes to be) Frege and Husserl's shared response to the problem of unity:

> [W]hat holds the parts of a complex whole together? Their common answer is: such a whole contains at least one part which cannot exist outside that *sort* of whole. [Simons, 1981, p. 80, my emphasis]

(In fact, strictly, the definitions do not even entail that a dependent part of a whole cannot exist outside that *sort* of whole. A dependent part requires the *existence* of wholes of that sort, but the definitions do not imply that it cannot exist without being *part* of one of them. But I will assume that this implication is secured by further stipulation.) However, supposing that a whole has a dependent part does not seem to account for its unity; for even that dependent part may be capable of existing detached from *that very whole*; and thus the question remains: by virtue of what do it and its fellow parts in fact exist attached?

Some, I suspect, will view the considerations of this subsection simply as indications of the folly of conceiving of ontological dependence according to what Kit Fine calls the *modal/existential account* [Fine, 1995]. On this account, one (kind of) thing depends on another (kind of) thing just in case the former (kind of) thing cannot exist unless the latter (kind of) thing exists—i.e. iff, necessarily, if the former exists, the latter exists. Inspired by remarks of Frege's, the idea of this subsection has been to identify unsaturatedness with ontological dependence as conceived by the modal/existential account. The idea is problematic, it has emerged, since, given that identification, many entities which are meant to be saturated turn out to be unsaturated. This might be seen as symptomatic of the implausible weakness of ontological dependence on the modal/existential account: the account makes dependence appear where it intuitively ought not to [Fine, 1995, p. 270-72]. Perhaps, then, the problem lies with the modal/existential account. Where that account characterises dependence in terms of a necessary connection between the existence of the dependent and the dependee, perhaps, as Fine argues, we ought rather to characterise dependence in terms of an *essential* connection between the *essence* of the dependent and the dependee, and refrain from construing essence in modal terms. I don't know whether this is the right course. I am confident, though, that neither the Finean essentialist account, nor

any other alternative account of which I am aware, will forfend the difficulty confronting the identification of unsaturatedness with dependence—namely: there are objects exhibiting such dependence, so unsaturatedness emerges as compatible with objecthood. In this sense, the problem does not merely lie with the modal/existential account.

3.1.2.3   The Primacy of the Whole

The claim that some constituents of unities are vertically dependent upon the unities of which they are constituents is closely related to an important current in Frege's philosophy of language. In some significant sense, Frege gives primacy to the whole thought, to the sense of a whole sentence. In his view, it is the thought entire that is primary, basic, fundamental. Its constituents, the senses of subsentential expressions, are secondary, derivative: they are, as it were, abstractions from the complete thought—the products of subjecting the thought to analysis or decomposition. The primacy of the thought goes hand in hand with the semantic primacy of the vehicle of its expression, captured in Frege's *context principle*:

> [W]e ought always to keep before our eyes a complete sentence [*Satz*]. Only in a complete sentence have the words really a meaning. [Frege, 1953, p. 71, translation altered]

Now, passages in which Frege clearly affirms this whole-first order of priority can give the impression that it is characteristic of the *unsaturated* to be the derivative product of decomposition of the whole:

> . . . I start out from judgements and their contents, and not from concepts. . . . And so instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement. [Frege, 1979b, pp.16-17]

> I do not believe that concept formation can precede judgement, because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content. [Frege, 1997f, p. 81]

> What is distinctive about my conception of logic is that I begin by giving pride of place to the content of the word 'true', and then immediately go on to introduce a thought as that to which the the question 'Is it true?' is in principle applicable. So I

> do not begin with concepts and put them together to form a thought or judgement;
> I come by the parts of a thought by analysing the thought. [Frege, 1997g, p. 362]

In these passages, which span almost forty years of Frege's career (the first written in 1880 or 1881, the third in 1919), the recurrent focus is on the status of unsaturated *concepts* as posterior to, and arrived at only by analysis of, the contents of whole sentences.[18,19] In remarks almost immediately following the first quotation, Frege seems to stress, in particular, the semantic posteriority of expressions designating concepts, as reflected in a perspicuous symbolism:

> Hence in the concept-script [concepts'] designations never occur on their own, but always in combinations which express contents of possible judgement. ... A sign for a property never appears without a thing to which it might belong being at least indicated ... [Frege, 1979b, pp.16-17]

The idea can suggest itself, therefore, that the unsaturatedness of concepts *consists* in their being derivative abstractions from the contents of whole sentences; or—equivalently, one might propose—that their unsaturatedness consists in their designations' only being capable of occurring meaningfully in the nexus of an entire sentence.[20] The idea certainly animates the image of unsaturatedness as a kind of lack of self-sufficiency.

The idea rapidly founders, though. The context principle applies quite generally to expressions, not merely to those whose *Bedeutungen* are classified as unsaturated. By its reckoning, proper names, as much as predicates, are capable only of occurring meaningfully in the context of a complete sentence. Frege makes quite clear in the *Grundlagen* that the species of self-subsistence he has in mind, and with which we are supposing unsaturatedness to contrast, is *not* a matter of being designatable independently of the sentential context:[21]

> The self-subsistence which I am claiming for number is not to be taken to mean that

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[18]The notion of unsaturatedness has not quite appeared in earnest in Frege's work at the time of the first quotation; but, as Heck and May point out [Heck and May, 2013, p 840], the germ of that notion is surely present in [Frege, 1979b].

[19]Note the connection made, in the second quotation, with existential dependence.

[20]The idea appears in Gaskin's exposition of Fregean unsaturatedness: '[T]here is no way of referring to the unsaturated item introduced by a concept-expression other than by use of that very expression (or a synonym) in a sentence; concepts cannot merely be *mentioned*; reference cannot be achieved in a context which falls short of the full act of assertion. [Gaskin, 1995, p. 165].

[21]James Conant [Conant, 2002, p. 433] also stresses this point.

> a number word signifies something when removed from the context of a sentence … [Frege, 1953, p. 72e, translation altered]

For Frege, it is, accordingly, *all* parts of a thought falling short of *being* a thought—not just those classified as incomplete—which are posterior to the thought in toto (as the final clause in the quotation from [Frege, 1997g, p. 362] on page 116 above confirms). It would, therefore, be natural for Frege to consider states of affairs—now thought of as the referents of sentences—as prior to all of their constituents that fall short of being a state of affairs: objects, as much as concepts, would naturally be considered abstractions from the whole. Something very close to this view of states of affairs and their constituents seems to be defended by Armstrong. '[S]tates of affairs come first' [Armstrong, 1997b, p. 118], he claims; their constituents, particulars and properties, are 'vicious abstractions (in the non-Quinean sense of 'abstraction', of course!)' [Armstrong, 1997a, pp. 109-10] therefrom.[22]

The prospects look poor, then, for explicating unsaturatedness by reference to the primacy of the whole. Conceiving of the unsaturatedness of a constituent as a matter of its being posterior to the whole to which it belongs does not seem to vindicate Frege's sharp segregation of the unsaturated from (entities he wishes to classify as) objects.

An alternative view, which I cannot see how to preclude, would be that the relevant unities are prior only to some of their constituents. Perhaps it would be viable to hold, for instance, that states of affairs and the particulars occurring in them are, so to speak, ontological contemporaries, whereas properties and relations are secondary—as Armstrong puts it, just 'everything that is left in the state of affairs after the particular particulars involved in the state of affairs have been abstracted away in thought' [Armstrong, 1997b, p. 29]. Combining this view with the conception of unsaturatedness as posteriority to the whole fares better in classifying entities as (un)saturated in a way acceptable to Frege. Even when combined with this view, however, this conception of unsaturatedness appears to offer no explanation of its incompatibility with objecthood. For one thing, the notions in which this conception trades are, as they stand, rather murky: one thing's being prior or posterior to another; its being more or less basic or funda-

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[22] Armstrong in fact explicitly proposes that, in accordance with this view, particulars, as well as properties, should be thought of as unsaturated [Armstrong, 1997a, pp. 110].

mental than another; its being a mere abstraction from another; its being only arrived at by the analysis or decomposition or splitting up of another. It is far from being immediately clear how to understand these (perhaps equivalent) notions. I don't wish to deny that they admit of elucidation. (It may be that they are to be clarified in terms of some notion of dependence, returning us to the discussion of §3.1.2.2.) What I cannot see, though, is how these notions might be suitably connected with nameability in order to provide the explanation we seek. Armstrong's mention above of 'the non-Quinean sense of 'abstraction'' is apposite: if that which is an abstraction (posterior, non-basic, arrived at only by analysis, etc.) is nevertheless *real* (which it is in Armstrong's non-Quinean sense of 'abstraction'), why should its being an abstraction constitute an essential obstacle to its being referred to with a singular term?

It might be suggested that unsaturatedness ought not to be identified simply with being *a* result of splitting up a previously given whole, but rather with being a particular *kind* of result of such decomposition. Certainly, there are remarks of Frege's with which this suggestion closely accords—e.g. in 'Function and Concept', [Frege, 1997e, p 134]. The challenge, however, when faced with the inevitable follow-up, "*Which* kind of result?", is not to resort merely to the reply "The unsaturated kind". One might try to appeal here to an illustrative geometric comparison Frege offers in the aforementioned passage of 'Function and Concept'. The comparison is with the clean division of a line by a point. The dividing point must be counted as belonging to one of the resulting line segments or the other, but not both, lest the dividing point be counted twice over. The segment to which that point is taken to belong becomes, Frege says, "fully complete in itself (*völlig in sich abgeschlossen*)" [Frege, 1997e, p 134]; by contrast, the other segment "is lacking in something (*etwas fehlt*)" [ibid.]—namely, the dividing point—and only becomes something complete in itself if supplemented with an endpoint or a line with two endpoints. It is hard to see how the comparison might help, however. After all, an open ray, no less than a closed ray, is perfectly available to singular reference: its openness is perfectly compatible with its objecthood.[23]

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[23]See also [Gaskin, 2008, p. 148].

A suggestion related to the one introduced in the previous paragraph is that the unsaturatedness of an entity—its referential unsaturatedness, though the suggestion could be duly extended to cover sential and linguistic unsaturatedness—is a matter, not merely of its designation's being semantically posterior to the whole sentence, but of the particular *kind* of semantic posteriority to the sentence its designation exemplifies. As Dummett explains, the semantic primacy Frege attributes to the sentence does not require that 'every explanation of a word must make an explicit allusion to its occurrence in sentences' [Dummett, 1981a, p. 6]. Frege would not deny, Dummett continues, that the referent of a *proper name* can be specified (and therewith, its sense shown) without such allusion. One can, for example, proffer the likes of,

(i) "London" refers to London.

One might claim that predicates differ from proper names in just this regard. They are semantically posterior to sentences not only in that they lack meaning outside of the context of a sentence—an attribute they share with proper names—but in that their semantics is to be given in a manner that explicitly alludes to their occurrence in sentences. One might, furthermore, identify the unsaturatedness or incompleteness of a concept with its designation's exhibiting precisely this kind of posteriority to the sentence. A version of this account of unsaturatedness is, in fact, proposed by Heck and May:

> [T]he claim that concepts are 'incomplete' is far more adequately expressed by the semantic thesis that the meaning of a predicate should be given by stating the meaning of an arbitrary atomic sentence in which it occurs … [Heck and May, 2013, p. 848]

The kind of semantic clause for predicates they intend is as follows:

(ii) $\ulcorner \Delta$ swims$\urcorner$ denotes the True iff for some x, Δ denotes x and x swims.

For our concerns, the crucial question concerning Heck and May's account is this: what is the import of 'should' in the claim 'the meaning of a predicate should be given by stating the meaning of an arbitrary atomic sentence in which it occurs'? On its most natural reading, the claim that the meaning of a predicate *should* be given thus is consistent with its being *possible*, in a relevant alethic sense, that the semantics of a predicate be given otherwise. In particular, it

is consistent with the possibility of specifying the referent of a predicate in the manner of (i), as follows:

(iii) 'ξ swims' refers to the concept *swimming*.

Something may significantly speak against treating predicate semantics in this way. Perhaps it fosters a false impression of the logical structure of atomic sentences—an impression that the atomic sentence asserts the obtaining of a *relation* between the referent of its subject and the referent of its predicate [Heck and May, 2013, p. 847]. This is consistent, nevertheless, with its being possible to specify the referents of predicates in this way; and the claim that it *is* possible, I submit, is roughly equivalent to the claim that concepts are objects. Thus, on the most natural reading of Heck and May's interpretation of the Fregean claim that concepts are unsaturated, that claim need not clash with the thesis that concepts are objects. Again, the alleged incompatibility of unsaturatedness with objecthood remains unexplained.

On the other hand, if the import of 'should' is really that of an alethic 'must', such that the claim that predicate semantics should be given in the manner of (ii) amounts to a denial of the *possibility* of treating predicate semantics in the manner of (iii), then we *are* granted an explanation of the incompatibility of unsaturatedness with objecthood; since, to reiterate, the denial of that possibility is roughly equivalent to the denial of the objecthood of concepts. However, on this reading, Heck and May's account confronts some of the same difficulties discussed on page 102 in connection with an earlier account. As yet, it leaves unclear why we must accept that concepts are unsaturated if we are adequately to account for the unity of the proposition (although we will return to this issue in the following subsection). Moreover, because of the aforementioned rough equivalence, it renders question-begging any adduction of the unsaturatedness of concepts as *grounds* for denying that concepts are objects. Someone who maintains that concepts are objects will, qua proponent of that view, hold that the meaning of a predicate *can* be given in the manner of (iii), and so will, qua proponent of that view, reject the claim that concepts are unsaturated, on its present interpretation.

One final point deserves to be made before we leave the topic of the primacy of the whole. Frege's view that the thought in its entirety is primary or basic does not seem to do his argu-

ment from the unity of the thought any favours; since, as several authors have argued,[24] that view seems to *obviate* the problem of the unity of the proposition, whereas Frege's argument from unity seems to be to the effect that the problem requires solution by the invocation of unsaturated entities. If the parts of a thought only arrive on the scene, so to speak, as a result of the decomposition of the complete and antecedently given thought, then there is, plausibly, no problem about how those parts hold together in the whole.[25]

3.1.2.4 The Unsaturated as *Bindemittel*

Notwithstanding this apparent tension between the whole-first current in Frege's thought, on the one hand, and his argument from unity on the other, it is natural to propose a conception of unsaturatedness drawn straightforwardly from the role assigned to the unsaturated in that argument. The unsaturated is allegedly required to discharge the role of *Bindemittel* or *Bindeglied*. In Geach's translation of the argument from unity quoted on page 90 above, both German words are rendered as 'link'. However, similar uses of 'Bindemittel' elsewhere in Frege's writings have alternatively been translated as 'means of connection' and 'cement' (see below), and the expression might equally be rendered as 'bonding agent' or 'binding material'. The unsaturated sense of the relational predicate 'falls under', for example, serves, Frege tells us, to bind together the saturated senses of the singular terms 'the number 2' and 'the concept *prime number*' into a unitary thought, where these senses would otherwise hold aloof from one another. This picture of the unification of senses is reflected, Frege elsewhere seems to indicate, at the level of reference:

> An object, e.g. the number 2, cannot logically adhere to another object, e.g. Julius Caesar, without some means of connection [*Bindemittel*]. This, in turn, cannot be an object but rather must be unsaturated. [Frege, 1984b, p. 281]

Where the *monadic* unsaturated is concerned—e.g. the sense or referent of a monadic predicate— Frege stresses the lack of necessity for any additional bonding agent. To return to a passage we've already met:

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[24] See [Textor, 2009, p 63], [Hale and Wright, 2012, p. 112] and [Travis, TS, §1.1].
[25] Cf. Textor [Textor, 2009, p 63]: '[T]he problem of the unity of the proposition simply does not arise for [Frege].'

> [T]he unsaturatedness of the concept brings it about that the object, in effecting
> the saturation, engages immediately with the concept, without need of any spe-
> cial cement [*eines besonderen Bindemittels*]. Object and concept are fundamentally
> made for one another, and in subsumption we have their fundamental union. [Frege,
> 1979d, p. 178]

The concept is such as to allow unmediated, limpet-like union with the object: it is itself a limiting case of a *Bindemittel*, binding not a plurality of other entities, but binding itself to one other or—equivalently—one other to itself. (Frege's claim that it is in *subsumption* that one finds the fundamental union of concept and object is surely amiss. If we acknowledge compound unities—viz. states of affairs—as the referents of sentences, such that the union of the parts of these unities needs to be accounted for, then we must not identify subsumption as the locus of that union; for we must account for the unity of the referents of *false* sentences as well as true sentences, and where false sentences are concerned, subsumption will not obtain: for example, in the referent of 'Obama is immortal'—viz. the state of affairs of Obama's being immortal—the union of Obama with the concept of immortality cannot be found in the subsumption of the former under the latter, since the former is not subsumed under the latter. We are, instead, led to suppose that an object is united with—saturates—a concept regardless of whether that object is subsumed under that concept.[26])

Frege claims in the argument from unity that it is 'only because' of a sense's unsaturatedness that it is capable of serving as a bonding agent uniting other senses into a complete thought [Frege, 1997h, p 193]. On the conception of unsaturatedness I now want to discuss, this consideration has a simple explanation: unsaturatedness just *is* that very capacity to serve as bonding agent; it is precisely the kind of adhesive or copulative potency that permits an entity to consolidate elements—of language, thought, or extra-representational reality—that would, left to their own devices, remain aloof.

Now, this conception immediately invites the complaint that Frege has really failed to give any substantial account of the unity of the proposition. Jeffrey C. King objects, for example, that Frege's position 'seems to essentially amount to saying that the parts hold together because

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[26]This view is ascribed to Frege by Gibson [Gibson, 2004, p 11-12]

some of them are "sticky" ' [King, 2007, p. 18], without the provision of a "substantial theory of *stickiness*" [ibid.] to give the account genuine content. Similarly, Donald Davidson charges that Frege's classification of certain meanings as unsaturated, towards preventing the disintegration of the meaning of a sentence into a mere congeries of disparate elements, 'seems to label a difficulty rather than solve it' [Davidson, 1967, p 304]. There is no doubt a good measure of justice in this kind of objection, as I suspect Frege would be prepared to concede. But one respect in which the charge does not stick—no pun intended—is this: Frege insists that that which serves as bonding agent *cannot be an object* [Frege, 1984b, p. 281]. This *is* a substantial thesis and, in at least this respect, his account of unity does not merely label the phenomenon of which explanation was sought. And it is precisely this respect which here concerns us.

Why, then, cannot that which serves as *Bindemittel* be an object? I turn now to two contemporary authors who might be thought to have answered this question: Graham Priest and Richard Gaskin.

According to Priest, Frege was quite right to insist that 'if something is to perform the role of explaining how it is that a unity of objects is achieved, it cannot just be another object' [Priest, 2014, p. 9]. Priest offers two, at least presentationally distinct arguments for Frege's view. Concerning the posited bonding agent holding together parts in a unity, Priest argues that

> ... it cannot be an object. If it were, the collection of parts *plus* the [bonding agent] constitute a plurality, just as much as the original. So the problem of binding would not be solved. [Priest, 2014, p. 9]

In somewhat more detail, Priest reasons as follows:

> Take any thing, object, entity, with parts, $p_1, \ldots p_n$. ... A thing is not merely a plurality of parts: it is a unity. There must, therefore, be something which constitutes them as a single thing, a unity. Let us call it, neutrally (and with a nod in the direction of particle physics), the *gluon* of the object, g. Now what of this gluon? Ask whether it itself is a thing, object, entity? ... [I]t is not, since, if it is, $p_1, \ldots p_n, g$, would appear to form a congeries, a plurality, just as much as the original one. If its behaviour is to provide an explanation of unity, it cannot simply be an object. [Priest, 2014, p. 9]

This is, in my judgement, a non sequitur. To be sure, if a complex unity's *Bindemittel*—its *gluon*, in Priestian terminology—is an object, then it and the (other) parts of the unity form a plurality

(or congeries). Of course they do; for Priest so intends 'plurality' that to say of some things that they form a plurality is just to say that they are *some things* (and does not imply that they form some *one* thing) [Priest, 2014, p. 9]. But that they form a mere plurality does not entail that they *merely* form a plurality. They may *also* form a unity, and may do so, for all that Priest has said, by virtue of the unifying action of the gluon. That is, the fact that $p_1, \ldots p_n, g$ form a plurality is perfectly compatible with the circumstance that $p_1, \ldots p_n, g$ also constitute a unity and are bound together in that unity by g. Equally, it is consistent with the circumstance that $p_1, \ldots p_n$ constitute a unity and are bound together in that unity by g. Indeed, for Frege, these circumstances will be one and the same, for the plurality $p_1, \ldots p_n, g$ will be none other than the plurality $p_1, \ldots p_n$: the gluon of a unity is, on Frege's account, itself a part of the unity. (For example: 'This unsaturatedness *of one of the components* is necessary, since otherwise the parts do not hold together' [Frege, 1979d, p. 177, my emphasis]. Recall also: '[N]ot all the parts of a thought can be complete; *at least one* must be 'unsaturated' ' [Frege, 1997h, p 193, my emphasis].)

Priest's second argument proceeds as follows:

> It will pay to become clearer about why a gluon cannot be an object. A vicious regress stands behind this. ... Suppose that we have a unity comprising the parts, a, b, c, d, for example. There must be something which, metaphysically speaking, binds them together. This is the object's gluon, g. But then there must be something which binds g and a, b, c, d together, a hyper-gluon, g'. There must, then, be something which binds g', g and a, b, c, d together, a hyper-hyper-gluon, g''. Obviously we are off on an infinite regress. Moreover, it is a vicious one. ... [Priest, 2014, p. 9-11]

Is it Priest's assumption here that Frege was mistaken in supposing that the gluon of a unity is itself a part of that unity? This is the strong implicature of, e.g., his use of the (otherwise redundant) plural term 'g and a, b, c, d'. At any rate, let us grant this assumption for the moment: g is not one of a, b, c, d. But now, what justifies Priest's contention that 'then there must be something which binds g and a, b, c, d together'? Suppose, by way of illustration, we accept an account of unity according to which what binds a, b, c, d together as a unity is the fact of their being related in a certain manner—the fact that $Rabcd$, where '$R\alpha\beta\gamma\delta$' is some quaternary predicate. Moreover, this fact is, *pace* the Wittgenstein of the *Tractatus*, itself considered an object. If Priest now attempts to ignite a vicious regress by insisting that we must recognize some

additional binding agent unifying a, b, c, d and the fact that $Rabcd$, the proper response on our part would be to return a puzzled stare and remind Priest that we would only need to recognize such an additional binding agent if we thought that a, b, c, d and the fact that $Rabcd$ *were* bound together into a unity: but we need not think that. We have only agreed to recognizing a, b, c, d— a distinct plurality on the present supposition—as constituting a unity. Indeed, if a, b, c, d are not themselves facts, common sense favours refusing to recognise any one thing of which a, b, c, d and the fact that $Rabcd$ are parts. On the present account of unity, therefore, the regress seems to be forestalled at this early stage. I do not claim that such an account is ultimately tenable. I only wish to illustrate that on the un-Fregean supposition that the gluon of a unity is not a part of that unity, it is far from obvious that one must recognize a second gluon binding the first together with the parts of the original unity. The ignition of the regress depends on that claim, however.

On the other hand, let us now instead grant Frege's view that the gluon of a unity is itself a part of that unity: g is one of a, b, c, d. If we are asked what binds a, b, c, d together, we reply that g does. If we are then asked what binds g and a, b, c, d together, we should simply reply that our questioner has repeated himself, for the plurality of g and a, b, c, d is none other than the plurality of a, b, c, d. We simply refer him back to our first answer. If we really wish, we can nominally proceed with the regress, answering this second question by saying that some gluon g' effects the binding, answering the follow-up by saying that some gluon g'' effects the binding, and so on. But as soon as it is alleged that the regress on which we have embarked is vicious, we must reply that there is no question of its being anything other than benign, since $g \overbrace{'\ldots'}^{n} = g \overbrace{'\ldots'}^{n+1}$, for all n. Equally, therefore, Priest does not seem to have given reason to think that a vicious regress ensues when the Fregean view is granted.

We might alternatively suppose that a, b, c, d are only the *non-gluon* parts of the unity in question. By Fregean lights, it is appropriate to describe the gluon of that unity, g—also a part of that unity, we now suppose—as binding together a, b, c, d. (Recall, for example, Frege's claim that the unsaturated sense of 'falls under' binds together the saturated senses of 'the number 2' and 'the concept *prime number*' into a thought: the latter two senses are the non-gluon parts, so

to speak, of the thought in question.) But perhaps Priest might now attempt to ignite the regress as follows. "If some additional part g must be acknowledged to account for the holding together of the parts a, b, c, d, some yet additional part g' must, by the same token, be acknowledged to bind the parts a, b, c, d and g together." Of course, it is exactly this conditional (or rather, the instances of this conditional schema) that Frege denies: he maintains that the gluon of a unity copulates immediately—precisely without the help of additional glue—with the non-gluon parts of that unity. The immediacy of this copulation is reflected in its being appropriate not only to describe that gluon as binding together the non-gluon parts of a unity but also to describe that gluon as binding itself together with those parts. Crucially for our purposes, it remains unclear that only by classifying the gluon of a unity as a non-object is one able to affirm its immediate copulation with the remaining parts. The conditional in quotes above seems to amount not to an assertion of the specific incompatibility of objecthood with the capacity for immediate copulation, but rather to a general rejection of that capacity.

It strikes me that the foregoing arguments of Priest's do not, therefore, satisfactorily answer the question of why that which serves as *Bindemittel* or gluon cannot be an object. Now, what I have so far suppressed in my quotations from [Priest, 2014] on the issue of the (non-)objecthood of gluons is Priest's equal and opposite conviction that a gluon *must* be an object. "[W]e can refer to it, quantify over it, talk about *it*", he stresses. "If this does not make something an object, I am at a loss to know what could" [Priest, 2014, p. 15]. (This leaves no doubt that if an entity lacks objecthood by Priest's lights, then it lacks objecthood by Frege's. This suffices for present purposes; I shan't pursue the question whether the converse holds, for our principal concern here is with the putative *non*-objecthood of gluons.) Remarkably—though not altogether surprisingly, for if anyone is known for his willingness to contain multitudes it is Priest—he proposes simply to bite this bullet and accept that gluons both are and are not objects. Gluons have contradictory properties. Thus, the truth about gluons witnesses dialetheism: the thesis that some contradictions are true.[27]

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[27]That gluons both are and are not things to which singular reference can be made is really just one instance of Priest's broader view that the limits of language are inconsistent. He also holds, for example, that the limits of *expression* are loci of true contradictions: there are things which cannot be, and yet *are*, expressed [Priest, 2002].

Priest goes on [Priest, 2014, p. 16-37] to develop a detailed account of *how* it is that gluons succeed in binding together the parts of a complex unity. (Frege's own failure to provide such an account met with criticism above.) This account, it emerges, corroborates both Priest's judgement that gluons are possessed of contradictory properties and (in consequence, as I shall shortly explain) his conviction that gluons are not objects. Indeed, in a sense, this account of how gluons glue constitutes the most important argument in Priest's book for the non-objecthood of gluons. The essence of the account can be introduced by again pressing the question whether the gluon of a unity is itself a part of that unity. Priest's answer is that not only is the gluon one of the parts, it is *each* of the parts. The gluon binds together the parts into a unity by being numerically *identical* to each of them. It is this, Priest thinks, that prevents the precipitation of the regress described above. No further gluon, g', is required to unite g with a, b, c, d; for g is identical to each of a, b, c, d. There is no 'metaphysical space' [Priest, 2014, p. 17] between, for example, a and g, across which a further join must be made; likewise for b, c and d.

The immediate objection to this account is this: The phenomenon of which we sought an explanation is the holding together of *distinct* parts as a unity—the case in which, e.g., $a \neq b \neq c \neq d$. Thus, the phenomenon cannot be explained by the supposition that some one part is identical to each of the parts, since if that were so, the parts would not be distinct: e.g., if $g = a$ and $g = b$, then also, by the symmetry and transitivity of identity, $a = b$, contrary to the hypothesis. Priest's bold response to this objection is to deny the transitivity of identity. So in the present case, he maintains that while $g = a$ (hence $a = g$) and $g = b$, it is not the case that $a = b$. Though the gluon is identical to each of the parts, the parts may nevertheless be distinct.

This is, of course, a radical departure from the orthodoxy concerning the identity relation. The definition of identity Priest adopts, though, is a standard Leibnizian one, also adopted by Frege:

(L) $\alpha = \beta \equiv_{df} \forall X(X\alpha \equiv X\beta)$

However, as a dialetheist, Priest is working with a paraconsistent logic—a logic that does not validate the inference from a contradiction to an arbitrary proposition—in which the relation of material equivalence (sharing the same truth-value) expressed by '$\equiv$' is not transitive. (Let P be

just true, let Q be a *dialetheia* (both true and false), and let R be just false; then, $P \equiv Q$ and $Q \equiv R$ but $\neg(P \equiv R)$.) Consequently, the relation asserted between α and β by '$\forall X(X\alpha \equiv X\beta)$' is also not transitive, and neither is that asserted between α and β by the definitionally equivalent '$\alpha = \beta$'. It can quickly be seen that, granted (L), Priest's account of how gluons glue entails that gluons do indeed possess contradictory properties.[28] Let a and b be distinct parts of some unity, u, of which g is the gluon. So $\neg a = b$ and thus, by (L) and the substitution of definitional equivalents, $\neg\forall X(Xa \equiv Xb)$. Therefore, $\exists X \neg(Xa \equiv Xb)$. Hence, either $\exists X(\neg Xa \wedge Xb)$ or $\exists X(Xa \wedge \neg Xb)$. Assume the latter (the former case is alike). Let 'P' denote an arbitrary witness of this latter second-order existential generalisation, so that $Pa \wedge \neg Pb$. Now, according to Priest's account, $g = a$ and $g = b$. Hence, $\forall X(Xg \equiv Xa)$ and $\forall X(Xg \equiv Xb)$, and in particular, $Pg \equiv Pa$ and $Pg \equiv Pb$. Since $Pa \wedge \neg Pb$, it follows that $Pg \wedge \neg Pg$.[29] Thus, g has contradictory properties.

From here, there is a very strong case that g is not an object. Since $Pg \wedge \neg Pg$, we have that $\neg(Pg \equiv Pg)$ and thus that $\exists X \neg(Xg \equiv Xg)$. Hence, by (L), $\neg g = g$. But if g is not identical to itself, it is not identical to anything! That is, $\neg\exists x(g = x)$. To see this, consider an arbitrary thing, e. Granted the Law of Excluded Middle (which Priest accepts), $Pe \vee \neg Pe$. If Pe, then, since $\neg Pg$, $\neg(Pe \equiv Pg)$. Hence $\exists X \neg(Xe \equiv Xg)$ and therefore $\neg e = g$. On the other hand, if $\neg Pe$, then since Pg, $\neg(Pe \equiv Pg)$. Hence $\exists X \neg(Xe \equiv Xg)$ and therefore $\neg e = g$. So in either case $\neg e = g$. But e was arbitrary. Thus, quite generally, $\neg\exists x(g = x)$. But this latter proposition very plausibly amounts to a denial of the objecthood of g. Singular reference and first-order quantification are constitutively connected, such that to be a possible referent of a singular term is to be ranged over by—to be a value of a variable bound by—the unrestricted first-order quantifiers. Since in the present context '$\exists$' [30] is to be understood as absolutely unrestricted, '$\neg\exists x(g = x)$' implies that g is not ranged over by the unrestricted first-order quantifiers. Hence, it is not a possible referent of a singular term, which is to say, not an object.

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[28]The following draws particularly on [Priest, 2014, §2.8].

[29]Strictly: this follows granted the assumption that a and b are not themselves contradictory with respect to P.

[30]Priest in fact uses a different symbol for the particular quantifier, for reasons connected with his acceptance of *noneism*: the thesis that some objects do not exist. [Priest, 2014, p. xxii]

Thus, Priest can, in summary, be understood to offer the following answer to our question of why that which serves as *Bindemittel* must not be an object: something can only bind together distinct parts into a unity—can only copulate immediately with them in the manner Frege envisaged, such as to forestall a vicious infinite regress of auxiliary *Bindemittel*—if it is numerically *identical* to each of those distinct parts; that requires that it have contradictory properties; and that, in turn, requires that it not be an object.

We will find this answer satisfying only if we are prepared to accept Priest's account of the unification effected by the *Bindemittel*; and there is simply no getting away from the fact that this entails acquiescing in the dual heresies of dialetheism and the denial of the transitivity of identity. (The shock of the latter is perhaps mitigated in some measure by the former, since, as we have seen, the failure of the transitivity of material equivalence in paraconsistent logic, coupled with the definition of identity (L), does indeed have the consequence that identity is not transitive.) There are those for whom the essential role of dialetheism in Priest's answer is alone sufficient reason to reject it. "[D]ialetheism", Williamson avers, "is a fate worse than death" [Williamson, 2007a, p. 387]. For my part, I counsel a weaker form of logical conservatism. In my judgement, it would be a grave decision indeed to relinquish consistency. Though we should not rule out doing so, it should take a formidable case indeed to induce us to do so—the kind of case which cannot, I submit, be made (entirely) in one small neighbourhood of metaphysics, such as the problem of the unity of the proposition, where consistent theories appear still to be in the running. The extremely high price (in my estimation) of tolerating true contradictions should buy us more philosophical amenity than an explanation of unity. Of course, Priest's wider body of work makes precisely the case that dialetheism buys us much more—*inter alia*: a unified response to the semantical, set-theoretical and soritical antinomies; relatedly, an account of the limits of cognition, conception, expression and iteration; an explanation of the metaphysics of change; an account of various puzzling legal and moral situations.[31] It evidently lies well beyond the scope of this essay to evaluate this wider case for dialetheism. Thus, I cannot accept Priest's answer to our question concerning the objecthood of *Bindemittel*, though nor can I reject it: I

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[31]See particularly [Priest, 2006] and [Priest, 2002]; also, e.g. [Priest, 2010].

must place it in abeyance pending the outcome of debates elsewhere.

A rather different, but similarly innovative treatment of the unity of the proposition is developed by Gaskin [Gaskin, 1995].[32] Gaskin takes an approach reminiscent of that which Frege says 'may be done' in the passage on page 90 above. Taking his cue from [Wiggins, 1984], he essentially proposes to identify the *copula* of a sentence—paradigmatically (though not invariably), the finite form of 'to be' in its predicative sense, or the finite ending of the relevant verb—rather than the whole predicate, as that which, by virtue of its unsaturatedness, accounts for propositional unity. Unsaturatedness is shifted to the copula away from the concept expression, which is now understood to be simply a species of proper name—its referent, thus, simply a species of object. This shift is not profitless, Gaskin claims *contra* Frege, since it permits us to talk about concepts—to singularly refer to and quantify over them. *Pace* [Wiggins, 1984], Gaskin maintains that the copula is itself referential. He proposes to capture the referentiality of the copula with the following semantic clause:[33]

(C) If $R^n(x_1 \ldots x_n)$ then $x_1 \ldots x_n$ instantiate^{n+1} R^n.

Thus, for example, if Shergar is a horse, then Shergar instantiates^2 horsehood. (One immediately worries that (C) isn't grammatically well-formed, since 'R^n' features in incompatible syntactic positions therein: predicate position in the antecedent and term position in consequent. In the example just given, I have nominalized the concept expression to resolve this tension (thus, "horsehood"), and though the issue goes unremarked in Gaskin's paper, I shall assume that in the consequent of (C) 'R^n' is simply elliptical for some nominalization of the predicate in the antecedent.) Accordingly, a first pass at specifying the referent of the copula would be to say that the copula refers to the (dyadic) relation of instantiation. But the consequent of our Shergar

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[32]I shall focus on the account developed in [Gaskin, 1995], rather than Gaskin's later treatise, [Gaskin, 2008]. The account developed in the paper is more germane to our investigation of Frege's views, in that Gaskin there seeks to explain unity by proposing, with Frege, a distinction between saturated and unsaturated features of the sentence, and by ascribing a special kind of unifying referent to the unsaturated feature of a sentence. By contrast, in [Gaskin, 2008], Gaskin departs from Frege in maintaining that the significant components of a sentence are *all* unsaturated if any are [Gaskin, 2008, p. Ch. 3], and in stopping short of ascribing a *referent* to that aspect of a sentence which supplies its unity [Gaskin, 2008, p. Ch. 6]. The treatise is, moreover, a work of astonishing breadth, depth and systematicity; to echo [Vallicella, 2010, p. 265], these virtues have the consequence that criticism of it is liable to either be lengthy (which space constraints here prohibit) or fail to do the work justice.

[33]I take it that the copula is here represented by sheer concatenation of the concept expression 'R^n' with its argument expressions 'x_1' ... 'x_n'. I indicate adicy with a numerical superscript, whereas Gaskin uses a subscript.

example can equally be plugged into (C) as the antecedent, yielding as consequent that Shergar and horsehood instantiate^3 instantiation^2. Putting these two instances of (C) together, we have that if Shergar is horse, then Shergar and horsehood instantiate^3 instantiation^2. Thus, by the same token, a more considered specification of the referent of the copula would be that it refers to the (triadic) relation of the instantiation of (dyadic) instantiation. By similar reasoning, a third and in Gaskin's estimation even better effort to specify the copula's referent would be that it refers to the (quatenary) relation of the instantiation of the (triadic) instantiation of (dyadic) instantiation. We can continue this process *ad infinitum*, obtaining ever more complex referent-specifications for the copula. The parallel with Bradley's regress is unmistakeable: indeed the possibility of repeating this process, Gaskin says, 'is just the possibility of generating Bradley's regress' [Gaskin, 1995, p. 174]. Repeating the process yields referent-specifications of ever greater fidelity to the real situation; but each such specification, Gaskin claims, is 'inevitably inchoate' [Gaskin, 1995, p. 176], a yet more complete specification always lurking over the horizon. The final statement of what the copula refers to irremediably 'keeps eluding our grasp' [ibid.]. And it is precisely in the inevitable inchoateness of any specification of its referent that the unsaturatedness of the copula consists. Since this inchoateness is a matter of its being possible to generate Bradley's regress, the unsaturatedness of the copula is in fact underwritten by that regress. Moreover, since it is, in Gaskin's view, by virtue of the unsaturatedness of the copula that the proposition has unity, the regress similarly underwrites that unity: so far from being vicious, 'Bradley's regress is [actually] the metaphysical ground of the unity of the proposition' [ibid.]. It is the infinitism introduced into the proposition by the unsaturated copula, in the form of the regress, that constitutes the proposition as a unity. In a slogan (with a confessedly paradoxical air): 'what stops a proposition from being a 'mere list' is that it is an *infinite* list' [ibid.].

Though Gaskin does not explicitly say as much, it is very natural to surmise that his account of propositional unity is, like Frege's, one on which the unsaturated component of a sentence, though it refers, does not refer to an object. For I suggest that if the copula did refer to something to which, possibly, some singular term, t, also refers, then we could not take fully seriously Gaskin's claim that a final specification of the referent of the copula always eludes our grasp—

that specification of its referent is 'inevitably inchoate': for the claim that the copula refers to t would then, on the contrary, seem finally and choately [34] to specify the copula's referent.[35] Thus Gaskin's account of propositional unity does appear to furnish an answer to our question of why that which serves as *Bindemittel* is to be deemed a non-object—at least in the case of the referent of the unsaturated component of the sentence.

I suggest that answer shouldn't satisfy us, however; for Gaskin overestimates the inevitability of inchoateness in specifying the copula's referent. The n-adic instantiation relations in terms of which Gaskin proposes to explain the semantics of the copula can be well-ordered by making dyadic instantiation first, triadic instantiation second and, generally, n+1-adic instantiation n^{th}. Let 'I_i' name the ith such relation. (Gaskin accepts that each such relation is a 'perfectly good object' [Gaskin, 1995, p. 173].) Now define an n+1-adic relational expression 'copulates' as follows:

(C*) $x_1 \ldots x_n$ copulate^{n+1} R^n iff$_{df}$

1. $x_1 \ldots x_n$ instantiate^{n+1} R^n, and

2. if $x_1 \ldots x_n, R^n, I_1 \ldots I_m$ instantiate^{n+1+m} I_{m+1},

   then $x_1 \ldots x_n, R^n, I_1 \ldots I_m, I_{m+1}$ instantiate$^{n+1+m+1}$ I_{m+2}.

Roughly, $x_1 \ldots x_n$ copulate^{n+1} R^n iff $x_1 \ldots x_n$ instantiate R^n, and $x_1 \ldots x_n$ and R^n instantiate instantiation, and $x_1 \ldots x_n, R^n$ and instantiation instantiate instantiation, ... *and so on.* But the ellipsis—the inchoateness—of this rough explanation is eliminated in (C*) by the second, inductive clause. We can now nominalize 'copulate', using whatever device of nominalization we accept—as, say, 'copulation'. As far as I can see, the singular term 'copulation' then designates that to which, on Gaskin's account, the copula refers. The referent of the copula is, therefore,

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[34] Apologies to the etymology of 'inchoate'.

[35] That Gaskin would accept that the copula does not refer to an object is also strongly suggested by his remarks on the concept *horse* paradox: "What Frege's paradox shows is that one cannot both talk about concepts... and continue to insist on their essentially unsaturated... nature." [Gaskin, 1995, p. 166]. On the other hand, Gaskin's exposition does feature *unapologetic* use of singular terms purporting to co-refer with the copula: e.g. 'what the copula refers to' [p. 175], 'the reference of the copula' [p. 176] (he also uses 'referent'). Indeed, he even adduces, as a reason for deeming the copula referential, that 'we want, as philosophers, to talk about predicative being' [p. 177]. I fear this just represents a tension in Gaskin's paper.

finally specifiable: it refers to copulation. And copulation, too, is a perfectly good object.

On the conception we've been examining, unsaturatedness is the capacity to serve as a bonding agent securing the unity of a sentence, thought or state of affairs. That conception presents a now familiar problem for Frege's argument from unity: we have found no acceptable answer to the question of why unsaturatedness, so conceived, must be incompatible with objecthood.

3.2 Taking Stock: Can Anything Be Made of the Argument from Unity?

Frege's argument from the unity of thought for the non-objecthood of concepts is, I hope to have shown, deeply problematic. In essence, it proves exceedingly difficult simultaneously to substantiate, on the one hand, the claim that unsaturated entities must be recognized in order to account for unity and, on the other, the claim that unsaturatedness is incompatible with objecthood.

Is it the case, then, that nothing can be made of Frege's argument? Two considerations might somewhat temper one's confidence in an affirmative answer to this question.

The first is simply that we have proceeded by trying to find a satisfactory reconstruction of the argument and failing. Obviously, we have no full assurance that some persuasive reconstruction has not been neglected: our investigation of Fregean incompleteness may be incomplete.

The second is that there is, I suggest, *a* sense in which Frege's non-reism[36] might after all glean some support by reference to the problem of the unity of the proposition. As Gaskin puts it, by 'the unity of the proposition' is traditionally meant 'the ability of a proposition (or, as we would more naturally say now, a sentence) to say something ... rather than merely list its referents' [Gaskin, 1995, p.162]. The problem of the unity of the proposition is then to explain a sentence's differing from such a list in being capable of saying something. But there is a sense in which, granted Frege's non-reism, this problem simply does not arise; because there *can be no such list* as the kind of list a proposition's difference from which allegedly needs to be explained.

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[36]Non-reism is the thesis that there are non-objects.

Some of a sentence's referents are, Frege holds, not objects. They are unnameable. On an utterly standard conception of *list*, the unnameable is *unlistable*. Thus, the particular instances of the problem cannot strictly even be posed. If we ask Frege how it is that 'Shergar is a horse' manages to say something, while 'Shergar, the concept *horse*' does not, we have in fact, by Frege's lights, not inquired about the list we'd intended: 'Shergar, the concept *horse*' is, strictly, *not* a list of the referents of 'Shergar is a horse'. The problem of the unity of the proposition is one of accounting for the difference between two kinds of linguistic production. The Fregean non-reist is relieved of this problem, since s/he simply does not recognize one of these kinds of linguistic production. Thus, an argument for Fregean non-reism: to accept the view is to be unburdened by this vexing philosophical problem.

I suspect that this, if anything, is what can be made of the argument from unity. It is not much. Allow me to conclude the discussion by offering three reasons for considering this last take on the argument from unity to be unpersuasive.

Firstly, the argument ought to pull us towards Fregean non-reism only to the extent that we are pessimistic about the prospects for a satisfactory solution to the problem of the unity of the proposition. For, I propose, if such a solution is to be had, it wins out over the non-reist *dis*solution of the problem, simply because it grants an intuitive datum that the non-reist response denies—viz. that it is possible to list the referents of a sentence's constituent expressions. However, it is not at all clear that such pessimism is warranted. Certainly, a number of impressive attempts at a solution have made.[37]

Secondly, evading the problem of propositional unity by appeal to the unlistability of (the totality of) the referents of a sentence's sub-expressions is a strategy equally available one who accepts that those referents are all objects. A reist can, to this end, appeal to Frege's own context principle. As fomulated in the *Grundgesetze*: 'We can inquire about reference only if the signs are constituent parts of sentences expressing thoughts'.[38] The kind of list that is supposed to contrast with a proposition would appear to involve just what the context principle, straight-forwardly understood, rules out: reference in isolation from the complete sentential context. A

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[37] See, for example, the works I have cited by Gaskin, Griffin and King.
[38] Quoted in [Heck and May, 2013, p.849].

stringent insistence upon the context principle forestalls the problem of unity in essentially the same way as Fregean non-reism, but leaves open the possibility of singular reference (within a complete sentence) to concepts. Thus, the argument lends no clear advantage to Frege's non-reism.

Thirdly, the argument would, it seems, involve Frege in a kind of dialectical foul play. Recall Frege's plea for a pinch of salt over his own use of singular terms purporting to co-refer with predicates and other incomplete expressions: taken literally, he acknowledged, his expressions go referentially astray. Well, if we attempt to pose an instance of the problem of propositional unity, we are, by Frege's lights, merely in the very same bind; and if we did not begrudge the said pinch of salt, and consented to meet Frege halfway, we can surely ask for the same in return. That same pinch will allow us to pose the unity problem precisely in so far as it allows Frege to communicate his own philosophical semantics. The only option for Frege would be, when confronted with an attempt to pose the problem of unity, to begrudge the very pinch of salt for which he had himself pleaded. That would surely be foul play.

Chapter 4

Naming the Concept *Horse*

> . . . Coriolanus
> He would not answer to: forbad all names;
> He was a kind of nothing, titleless,
> Till he had forged himself a name o' the fire
> Of burning Rome.

> *Coriolanus*, Act V, Scene I

4.1   Frege Against Singular Reference to Concepts

Frege rejects singular reference to concepts. So far in this essay, we have—on firm textual grounds, I believe—understood that rejection to consist in the claim that reference to a concept with a singular term is *impossible*. In the present chapter I shall dub this claim the *impossibility thesis*. To echo remarks made at the start of §1.1.3, the impossibility thesis is, in a clear sense, incontestably present in Frege's work. However, it strikes me that there are, in fact, three further claims discernible in Frege's writings in each of which his rejection of singular reference to concepts might alternatively be taken to consist. Consider again, for example, the passage quoted on page 11, in which Frege clearly affirms the impossibility thesis:

> . . . the phrase 'is a concept' requires a proper name as grammatical subject; and so, strictly speaking, it requires something contradictory, since *no proper name can designate a concept* . . . [Frege, 1979d, p. 178, my emphasis].

What immediately follows that passage in fact appears to amount to a cautious recantation of the impossibility thesis just affirmed:

> ...or perhaps better still, something nonsensical. [Frege, 1979d, p. 178]

Better than saying that singular reference to concepts is impossible, Frege ventures, is to say that there is *no sense to be made of* reference to a concept with a singular term. This latter is the first alternative claim I have in mind: it is *nonsense* to say that a concept might be the referent of a singular term, nonsense to say that a concept is an object ('the nonsensicality thesis'). The second alternative claim is that it is impossible to *think* of a concept as being the referent of a singular term—impossible to *suppose* that a concept is an object ('the unthinkability thesis'). This follows from Frege's general pronouncement that

> ...concepts cannot stand in the same relations as objects. It would not be false, but impossible to think of them as doing so. [Frege, 1997a, p.175],

since to think of a concept as being the referent of a singular term *would* be to think of a concept as standing in a relation in which objects stand. The third alternative claim is that it is impossible to *state* that a concept is an object ('the inexpressibility thesis'). In 'On Concept and Object', Frege insists that

> ...what is suitably stated of the concept does not suit the object ...I do not want to say it is false to state concerning an object what is stated here concerning a concept; I want to say it is impossible, senseless, to do so. [Frege, 1997h, p. 188-9]

Since, in particular, it is possible to state concerning an object that it *is* an object, it seems clear that Frege deemed it impossible to state concerning a *concept* that it is an object. The import of 'senseless' in this passage is made more explicit in an earlier draft of the paper:

> ...a sentence which tried to express such a thing [i.e. which tried to do the impossible and state concerning a concept what can be stated concerning an object] would be absolutely devoid of sense; ... [Frege, 1979a, p. 109]

Since 'A concept is an object' is presumably just such a sentence, we have here a further affirmation of the nonsensicality thesis.

Of these four theses (the impossibility thesis and the three aforementioned alternatives), each of which constitutes a position on which singular reference to concepts is rejected, three face a pressing threat of self-stultification. We have, in §1.1.2, already encountered this threat in the case of the impossibility thesis. However, it is also conspicuous in the case of the inexpressibility thesis. Saying 'It is impossible to state that a concept is an object' is liable to be compared to saying out loud (and with correct pronunciation) 'I cannot pronounce the word "banana" '.[1] Merely voicing the thesis appears to expose it as false. A similar problem attends the unthinkability thesis: if we are even to understand the thesis, we must be in a position to apprehend just what it is that is being claimed unthinkable, and such apprehension threatens to demand the very thinkability of the subject of the claim. The problem is that identified in the preface of the Tractatus: 'in order to set a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought)' [Wittgenstein, 1981, p. 27].

The nonsensicality thesis stands out on this count, I submit: it is innocent of self-stultification. For it must be understood as a claim *about a certain linguistic expression*, 'a concept is an object', and as being to the effect that that expression lacks sense—i.e. expresses no thought (this is the relevant notion of nonsensicality in the present context). (Better: it must be understood as claim about a cluster of linguistic expressions—'a concept is an object', 'a concept can be the referent of a singular term', 'a singular term can co-refer with a predicate', etc.—that are, by virtue of our semantic stipulations, possessed of the same sense if possessed of sense at all.) As Wittgenstein stressed, '[w]hen a sentence is called senseless, it is not, as it were, its sense that is [being claimed] senseless' [Wittgenstein, 2010, p. 147e]; it is the sentence itself. Accordingly, when we formulate the thesis as 'It is nonsense to say that a concept is an object', the sentence 'a concept is an object' therein must be understood as mentioned and not used. Mentioned nonsense, as opposed to used nonsense, is quarantined: it does not infect its surroundings with nonsensicality. So the thesis can simply be true, much as it can simply be true to say 'Finally Carroll said "And the mome raths outgrabe" '.[2] Whereas the inexpressibility of a thought threatens to

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[1]The example is Peter Sullivan's [Sullivan, 2002, p. 76], though he deploys it to illustrate a different point.
[2]See also [Moore, 1997, p. 157].

render it inexpressible *that* that thought is inexpressible, so that one's success in stating that that thought is inexpressible would confute that very statement, and whereas the unthinkability of a thought threatens to render it unthinkable *that* that thought is unthinkable, so that one's success in supposing that that thought is unthinkable would confute that very supposition, the nonsensicality of a sentence, by contrast, does not risk rendering it nonsensical to say *that* that sentence is nonsensical; so one's success in sensefully saying so does not confute one's saying.[3]

4.2   Trueman Against Singular Reference to Concepts

If one is going to argue for a Fregean rejection of singular reference to concepts, there are, then, *prima facie* grounds for arguing for it in the form of the nonsensicality thesis. In this chapter, I propose to examine an impressive recent defence of Frege's rejection of singular reference to concepts which does indeed take the form of an argument for the nonsensicality thesis.

The defence in question is Robert Trueman's in 'The concept *horse* with no name' [Trueman, 2015]. The conclusion of Trueman's paper is 'that it is nonsense to say that a property is an object' [Trueman, 2015, §8]. (Trueman favours 'property' over 'concept'; we'll switch to the former in what follows to accord with his formulations. Frege himself often does use 'property' ('*Eigenschaft*') as equivalent to 'concept'. A particularly clear example in 'On Concept and Object': "I call the concepts under which an object falls its properties..." [Frege, 1997h, p. 189].) Interestingly, though, he sets about establishing that conclusion by arguing, in the first instance, for the inexpressibility thesis: we are prevented, in principle, he argues, 'from so much as expressing the thought that a property is an object' [Trueman, 2015, §3]. If, for the moment, we prescind from the self-stultification problem afflicting it, the inexpressibility thesis does look fit to serve as intermediate conclusion en route to the nonsensicality thesis. For suppose that, contrary to

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[3]The two uses of 'threatens' in this sentence may look like undue caution. One reason I think caution is due, however, is that there are ways of referring to and apprehending thoughts that do not require the expressibility or thinkability of those thoughts. Suppose, for example, that the truth fathomed by Siddhārtha Gautama on the night of his enlightenment is inexpressible and unthinkable (at least to the benighted likes of us). *That* that truth is inexpressible and unthinkable, though, need be neither inexpressible nor unthinkable (even to us), providing that truth can be referred to, and apprehended under, the definite description in the preceding sentence. It is quite unclear, however, how this expedient for making inexpressibility claims and unthinkability claims respectively expressible and thinkable (even if true) could be used to surmount the charge of self-stultification facing Frege's inexpressibility and unthinkability theses above.

the nonsensicality thesis, it *does* make sense to say that a property is an object—i.e. that one does succeed in expressing a thought in saying so. Whatever thought is thereby expressed is, trivially, not inexpressible; but, 'It is impossible to express that a property is an object' says of that thought that it *is* inexpressible, and is thus false. If 'It is impossible to express that a property is an object' is false, then it is not impossible to express that a property is an object. So if it makes sense to say that a property is an object, it is possible to express that a property is an object. By contraposition, if it is not possible to express that a property is an object, then it is nonsense to say that a property is an object: the inexpressibility thesis implies the nonsensicality thesis.[4,5]

However, even ignoring the *self*-stultification of the inexpressibility thesis, we are now confronted with the fresh oddity that the inexpressibility thesis is stultified by the conclusion for which it serves as lemma—the nonsensicality thesis: after all, 'a property is an object' is *used*, not mentioned, in 'it is impossible to express that a property is an object'; so, if the former is nonsense, the latter is too. But if the latter is *nonsense*, how can it serve as a lemma en route to the nonsensicality thesis? Trueman acknowledges that he is in 'a precarious dialectical position' [Trueman, 2015, §8]. He denies that his position need be 'ultimately unstable', however. The argument is to be understood, according to Trueman, as an attempt to show his opponents— those who think it does make sense to say that a property is an object—*by their own lights* that it does not make sense. It need not, to that end, succeed in showing this by the lights of those sharing his view, who indeed need not (and as we've seen, cannot) even accept the meaningfulness of the argument. Trueman accordingly prefaces the argument with a warning: that he will show his opponents, by their own lights, that it does not make sense, 'in the only way we can: by jumping into [their] way of speaking with both feet' [Trueman, 2015, §4].

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[4]Cf. [Moore, 1997, p. 198].

[5]The unthinkability thesis also features in Trueman's paper. Like the nonsensicality thesis, it is affirmed as a consequence of the inexpressibility thesis: "unless we can find a way of expressing Benno's thought [the thought that a certain concept is an object], we cannot suppose that it might actually be true" [Trueman, 2015, p. S4]. The inference of the unthinkability thesis from the inexpressibility thesis is mediated by a 'ground rule' Trueman lays down [Trueman, 2015, §4] of not entertaining the suggestion that the thinkable outstrips the expressible. Though my main focus in this paper will be with Trueman's argument for the inexpressibility thesis as intermediate conclusion en route to the nonsensicality thesis, what I say will equally bear on whether Trueman has established the unthinkability thesis via the same route. Incidentally, Trueman ascribes the unthinkability thesis to Frege, claiming that 'he was clear' that this thesis was 'the proper formulation of his position' [Trueman, 2015, p. S4]. I suggest that the brief survey of relevant passages in §4.1 does not in fact disclose a single, official formulation of Frege's position.

For what it is worth, I'm sympathetic to Trueman's response. We *should*, I believe, recognize the legitimacy of arguments that are only intelligible by the lights of those on whose position they are an attack. Such arguments are, I think, just extreme examples of good ad hominem arguments: arguments whose cogency depends upon assumptions accepted by opponents of, but not proponents of, the view they are intended to support. In any case, it is not with the ultimate stability of Trueman's dialectical position that I shall take issue here. Rather, my concern is to argue that Trueman does *not* show, even by his opponent's lights, that it is impossible to express that a property is an object. Let us turn, then, to the details of the argument that purports to show that.

4.2.1   The Basic Argument

Suppose that it is possible, in principle, to express the general thought that a property is an object—or equivalently, that a property is identical to some object. (Throughout, 'a property is an object' is to be understood as an existential generalisation—i.e. as 'some property is an object'—rather than a universal generalisation or a generic; similarly for its equivalents.) In that case, it must also be possible, in principle, to express the thought (call it *Benno's thought*, in honour of Frege's critic) that a particular property—say, the one 'ξ is a horse' refers to—is identical to a particular object—say, the one 'the property *horse*' refers to. Benno's thought is an identity, so the natural way to attempt to express it is by use of the identity predicate, '$\xi = \zeta$'. But while 'the property *horse*' can be introduced into an argument place of the identity predicate, 'ξ is a horse', whose referent we wish to identify with the property *horse*, cannot be introduced into the remaining argument place. The point is not, Trueman is keen to stress, merely that the parochial grammatical strictures of English forbid substituting 'ξ is a horse' for 'ξ' in 'ξ = the property *horse*'. Rather, the kind of *sense* possessed by 'ξ is a horse', qua predicate, is such that, unlike a singular term, it cannot combine solely with 'ξ = the property *horse*', given the kind of sense *it* has, to yield the expression of a thought. This, Trueman thinks, is just a consequence of the general consideration that the contrasting kinds of sense respectively possessed by terms and predicates prevent them from being substituted for one another. (It is worth pausing here to connect this aspect of Trueman's argument with our discussion in chapter 2. It *is* a premise of

Trueman's argument that terms and predicates are not intersubstitutable *salva significatione*—indeed that they are *nowhere* thus intersubstitutable. Trueman puts this by saying that 'it is impossible to substitute terms and predicates *at the level of sense*' [Trueman, 2015, §3, original emphasis]. But Trueman is crucially not *assuming* that this failure of intersubstitutability implies that terms and predicates never co-refer. Neither the principle we called Salva Significatione in chapter 2, nor the Reference Principle, is a premise of his argument.)

If we are to express Benno's thought, then, we must complete 'ξ = the property *horse*' with a singular term. Clearly, we must select one that co-refers with 'ξ is a horse'. Let 'a' be whichever such term we select. The problem, however, Trueman claims, is that even if 'a' indeed co-refers with 'ξ is a horse' (and to assume that it could not, or that is nonsense to say that it does, would be to beg the question in the present context), 'a = the property horse' will nevertheless fail to express Benno's thought. The reason is that 'a', qua singular term, will not *present its referent* in the right way—namely, *as a property*, something to which predicate reference can be made. Trueman illustrates this point by way of comparison with the following case. Suppose that there is a philosopher, the Misguided Metaphysician, who maintains that the number 3 is identical to Julius Caesar. The Misguided Metaphysician attempts to express that view not with the sentence 'Julius Caesar = 3' but rather by saying '2+1=3'. The natural verdict is that the Misguided Metaphysician's attempt to express her claim in this way is unsuccessful. Moreover, her attempt would be unsuccessful even if she were right, and '2+1' did indeed refer to Julius Caesar. If she were right, her utterance would say *de re* of Julius Caesar that he is identical to the number 3; but her utterance would nevertheless fail to express the thought that Julius Caesar is identical to the number 3. This is witnessed by our being able to accept what the Misguided Metaphysician says without agreeing with her that Julius Caesar and the number 3 are identical. Trueman's diagnosis of the failure is that '2+1', even if it refers to Julius Caesar, does not *present its referent* as Julius Caesar. The attempt to use the sentence 'a = the property *horse*' to express Benno's thought is comparable, Trueman thinks, to the Misguided Metaphysician's attempt to express her thought using '2+1=3'. Benno's thought is that a certain property—something that can feature as the referent of a predicate—is identical to a certain object. But the singular term 'a', even if it succeeds in picking out the intended property, does not present its referent as

something to which predicate reference can be made. Thus, while 'a = the property *horse*'
may succeed in saying *de re* of the referent of 'ξ is a horse' that it is identical to a certain
object, it nevertheless fails to express Benno's thought. In order, Trueman thinks, to present
the property we wished to identify with an object *as* a property, we would have to refer to it
with a predicate. But we cannot both do this and employ the predicate 'ξ = the property horse'
to express Benno's thought. Quite generally, we cannot do this and employ the identity sign
'$\xi = \zeta$', or any expression with the same sense, to express Benno's thought: '$\xi = \zeta$' requires—
once again, as a matter of sense—that each of its argument places be supplied with a singular
term. But then how *can* Benno's thought be expressed? Conclusion: it cannot. And if it cannot,
nor can the general thought that a property is an object.

4.2.2   Can Terms Not Present Their Referents as Properties?

Trueman considers two broad kinds of objection to this argument. The first is to the effect that
a singular term *can* present its referent as a property, so that Benno's thought can after all be
expressed by use of the identity predicate. The second is to the effect that Benno's thought
admits of expression without use of the identity predicate, by some means that allows for the
use of 'ξ is a horse' itself to pick out the property to be identified with an object, rather than a
surrogate singular term. I shall, in fact, ultimately push versions of both these objections against
Trueman's argument, though my version of the second objection will be quite different from the
version Trueman treats and will, in a sense, be parasitic upon my defence of the first objection.[6]
Let us then, give substance to the first objection and hear Trueman's response to it.

Two terms that do appear to present their referent as a property, and are excellent candidates
for co-referring with 'ξ is a horse', are 'the referent of "ξ is a horse"' and 'the property *horse*'.
It appears that the latter quite explicitly presents its referent as a property, while the former
presents its referent as a property—something to which a predicate *can* refer—by presenting it
as something to which a particular predicate *does* refer. If we substitute either for 'a' in 'a = the

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[6]The version of the second objection Trueman discusses consists in the proposal that Benno's thought can be
stated by means of a *lopsided* analogue of the identity predicate, '$\Phi \doteq \xi$', admitting a predicate into one argument
place and a term into the other. I shan't pursue this proposal here.

property *horse*', won't the result suffice to express Benno's thought? For closely related reasons
Trueman thinks neither expression is fit to task.

I shall focus on 'the referent of "ξ is a horse"'. This looks like a singular term, one natu-
rally analysed as the result of using a definite description operator to bind a term variable in the
argument place of the predicate '"ξ is a horse" refers to ζ', where this predicate is in turn the
result of inserting the predicate name '"ξ is a horse"' into the first argument place of 'ξ refers
to ζ'.[7] The problem with this analysis, Trueman thinks, is that 'refers', used when speaking of
predicates referring, should not be understood as the very same first-level dyadic predicate, 'ξ
refers to ζ', used when speaking of *terms* referring. The reason has to do with the possibility of
specifying, by disquotation, what an expression refers to. Disquotation sentences like

(i)   The referent of 'Julius Caesar' = Julius Caesar.

capture, Trueman maintains, something essential about reference—roughly, that when we pass
from using a referring expression to talking explicitly about what that expression refers to, we
continue to talk about the same thing. So if we are to count predicates as referring expressions,[8]
it must correspondingly be true that when we pass from referring to something using a predi-
cate to explicitly speaking of what that predicate refers to, we continue to talk about the same
thing; and it must be possible to capture this for any particular predicate, as (i) captures this
for the term 'Julius Caesar'. It must be possible, that is, to furnish a disquotation sentence that
is to 'ξ is a horse', for example, as (i) is to 'Julius Caesar'. However, if 'refers to' in discourse
about predicate reference is to be understood as 'ξ refers to ζ', Trueman thinks, it would not
be possible to furnish any such sentence. For in that case, 'the referent of "ξ is a horse"' would
indeed be the singular term it appears to be, but then in order to state an analogue of (i) for 'ξ is a
horse' we would have to express the thought that something referred to with that singular term

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[7]I shall assume, in Fregean spirit, that if 'the referent of "ξ is a horse"' is the definite description it appears to be,
it is a referring expression and is not to be analysed in Russellian fashion.

[8]It is, of course, not uncontroversial that we *are* to count predicates as referring expressions. However, again in
Fregean spirit, we will follow Trueman [Trueman, 2015, §2] in assuming that, in some significant sense, at least some
predicates do refer. We will shortly see that Trueman himself presents an argument to the effect that, in another
significant sense, predicates do *not* refer.

is identical to something referred to with a predicate, 'ξ is a horse'. That is, we would have to express an object-property identity. So we are confronted once more with the above argument that this cannot be done. We cannot use the predicate 'ξ is a horse' to pick out the property whose identity with a certain object we wish to express: the sense of the identity predicate prohibits that. So we would have to resort to the use of a co-referring singular term in its stead; but no singular term we choose will be fit for the expression of our target thought: even if the term we select refers to the property, it will not present its referent as a property, as would be required to express the thought.

The possibility of giving an analogue of (i) for a predicate can only be secured, Trueman thinks, if the relational expression for predicate reference permits us to specify what a predicate refers to using a predicate—that is, if 'refers to' for predicates is to be understood as 'ξ refers to Φ' (where 'Φ' marks an argument place open to predicates). There is no uncontroversial rendering of 'ξ refers to Φ', Trueman acknowledges;[9] but unless talk of predicates referring can be interpreted in this way, he thinks, an essential feature of reference, captured in the case of terms by the likes of (i), cannot even be said to obtain in the case of predicates. However, we cannot understand 'the referent of "ξ is a horse"' as the result of using a definite description operator to bind a term variable in the argument place of '"ξ is a horse" refers to Φ', because only a *predicate* variable could be bound in that argument place. Trueman notes that we may wish to countenance

> . . . a higher-level definite description operator which binds (first-level) predicate variables rather than term variables; such an operator would take a second-level predicate and return a first-level predicate standing for the unique property which satisfies that second-level predicate, if there is such a unique property. We could then use this operator to form a higher-level definite description by binding the variable in '"ξ is a horse' refers to Φ', which we could write as 'the F such that "ξ is a horse" refers to F'. But crucially, 'the F such that "ξ is a horse" refers to F' would itself be a predicate, and so we could no more plug it into the gap in 'ξ = the property horse', which is a gap for singular terms, than we could plug 'ξ is a horse' into that gap. [Trueman, 2015, §5]

Thus, Trueman thinks, with 'refers to' for predicates understood as it must be, the appearance

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[9]One candidate, Trueman notes, is Dummett's '$\forall x(x$ is what ξ refers to $\leftrightarrow \Phi x)$', where the 'is' therein is meant to be the 'is' of predication rather than identity [Dummett, 1981a, p. 217].

that 'the referent of "ξ is a horse"' is a singular term presenting its referent as a property dissolves.

4.3   Initial Reflections

Oddly, Trueman does not actually deliver the predicate analogue of (i) that is allegedly only available if the relational expression for predicate reference takes a predicate in accusative position. This omission can be remedied, however, using resources he accepts. We've just seen Trueman countenance a second-level analogue of the definite description operator; later in the paper he follows Frege in countenancing a second-level analogue of the identity predicate, which takes a predicate in each of its argument places and which I shall notate '$=_2$'. (As with 'ξ refers to Φ', there is no uncontroversial rendering of '$=_2$'. Frege favoured '$\forall x(\Phi x \leftrightarrow \Psi x)$'; Trueman suggests '$\Box \forall x(\Phi x \leftrightarrow \Psi x)$'.) Using the second-level analogues of the identity predicate and the definite description operator, an analogue of (i) for "ξ is a horse" may be given as follows:

(ii)   The F such that "ξ is a horse" refers to $F =_2$ is a horse.

The admission of a second-level analogue of the identity predicate opens up a possibility for pursuing the second of the two kinds of objection mentioned at the start of §4.2.2. Specifically, it presents the option, not discussed in Trueman's paper, of stating Benno's thought using '$=_2$' in conjunction with two predicates. Using '$=_2$', we could employ 'ξ is a horse' itself to pick out the property to be identified with an object, rather than a surrogate singular term. Of course, this strategy of stating Benno's thought as a higher-level identity confronts a mirror image of the problem set out in §4.2.1: a singular term cannot fill an argument place of '$=_2$', only a predicate; so in this case it is requisite to use a surrogate *predicate* to pick out the *object* we wish to identify with a property. But whichever predicate we select, Trueman will presumably claim, it will not present its referent as an object; so the resulting second-level identity sentence will fail to express Benno's thought. The challenge, if we are to maintain that Benno's thought can be expressed with a second-level identity sentence, is to find a predicate that presents its

referent as an object, just as the challenge, if we are to maintain that Benno's thought can be expressed with a first-level identity sentence, is to find a term that presents its referent as a property. I hope to show in §4.4 that both of these challenges can be met, that Benno's thought is accordingly susceptible of expression either as a first- or as a second-level identity claim, and hence that Trueman's case against singular reference to properties fails.

First, though, I want to raise a lesser objection to Trueman's argument: the argument does not, I submit, establish that it is nonsense to say that a property is an object; at most it establishes the disjunctive thesis that it is either nonsense or *false* to say that a property is an object. This disjunction of the nonsensicality and impossibility theses would still be a remarkable result. If Trueman were to have established the disjunction, he would have succeeded in ruling out singular reference to concepts (as either impossible or unintelligible). Nevertheless, it is worth making clear that, for all Trueman has shown, saying that a property is an object may at least escape the ignominious fate of being not even wrong. This emerges from an observation that will also provide inspiration in §4.4: even granting Trueman's claim that " 'refers' for predicates must be read as 'ξ refers to Φ' " [Trueman, 2015, §5], we can still insert the name of a predicate into the first argument place of 'ξ refers to ζ'; and when the second argument place is also filled with a singular term, the result will, at least, *make sense.* The sense of 'ξ refers to ζ' is only such as to require that the expressions introduced into its argument places have the characteristic kind of sense possessed by singular terms;[10] the name of a predicate has that kind of sense.[11]

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[10]Might it be objected that 'ξ refers to ζ' semantically requires more of the expressions admitted into its argument places than that they have the sense of singular terms? Perhaps: the introduction of certain singular terms into the first argument place of 'ξ refers to ζ' will produce sentences that might seem to be category mistakes—e.g. 'The Baltic Sea refers to Socrates'—and there are those who have wished to classify category mistakes as nonsense. However, firstly, it is not clear that the likes of 'The Baltic Sea refers to Socrates' really *are* category mistakes, at least if a category mistake represents an attempt to ascribe a property to an object that is of the wrong the *kind* to exemplify that property. After all, can't any object, in principle, serve as a name? (See e.g. [Lewis, 1986, p. 145-6].) If so, the Baltic Sea is *not* of the wrong kind to instantiate the property of referring to Socrates. Secondly, there is a strong case to be made against the view that category mistakes are nonsense (see [Magidor, 2009a], [Magidor, 2013]), and the popularity of the view has certainly waned. Thirdly, even if there were categorial restrictions on the kind of thing a name of which can meaningfully appear in the first argument place of 'ξ refers to ζ'—restrictions that exclude objects like the Baltic Sea—those restrictions would have to be *very* stringent to exclude predicates: a predicate is a *linguistic expression*; is a linguistic expression not the kind of thing that can refer to Socrates, for example? Though I cannot address these issues in detail, I think this suffices to indicate the difficulties confronting the view that names of predicates cannot be meaningfully substituted for 'ξ' in 'ξ refers to ζ'.

[11]I am, of course, here assuming that predicates can be named—i.e. that predicates are objects. Trueman also makes this assumption, though he evinces a suspicion that it is false [Trueman, 2015, fn. 11]. I shall only say that this is not a suspicion I share. As we saw in chapter 1, some commentators, of whom Geach is a notable example ([Geach,

The sentence ' "ξ is a horse" refers to Socrates', for example, is perfectly meaningful. In that case, so is the generalization '$\exists x(x$ is a predicate $\wedge$ x refers to Socrates)', as is its modalization '$\Diamond \exists x(x$ is a predicate $\wedge$ x refers to Socrates)'. The conjunction

(iii) $\Diamond \exists x(x$ is a predicate $\wedge$ x refers to Socrates) $\wedge$

        $\Diamond \exists y(y$ is a singular term $\wedge$ y refers to Socrates)

must also make sense, since there is no doubt about the meaningfulness of the latter conjunct. Moreover, the result of existentially generalizing this conjunction in the position of 'Socrates' also makes sense:

(iv) $\exists z$ $(\Diamond \exists x(x$ is a predicate $\wedge$ x refers to $z)) \wedge$

        $\Diamond \exists y(y$ is a singular term $\wedge$ y refers to $z))$

However, given the way in which 'object' and 'property' were introduced, (iv) is a very natural reading of 'A property is an object'. So there is a very natural way of taking 'A property is an object' on which it is perfectly meaningful.[12] What, then, is the import of Trueman's claim that 'refers' for predicates must be read as 'ξ refers to Φ'? This: when we speak of predicates referring, it is a condition of the *truth* of what we say (*not* the meaningfulness of what we say) that 'refers' mean 'ξ refers to Φ'. Thus, the complaint against (iv) to which Trueman is entitled is not that it is nonsense, but that it is *false*—false because nothing satisfies the predicate '$\Diamond \exists x(x$ is a predicate $\wedge$ x refers to $\xi)$'.

These considerations also bear upon the interpretation of 'the referent of "ξ is a horse" ' and the possibility of employing this expression to articulate Benno's thought. This expression *can* be interpreted as the result of first inserting the predicate name ' "ξ is a horse" ' into the

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1976b, pp. 59-61], [Geach, 1961, p. 144], [Geach, 1976a, p. 440]), have maintained that Frege himself deemed this assumption false and conceived of incomplete expressions (of which predicates are a species) as linguistic functions mapping singular terms or sentences to singular terms or sentences. To repeat, I think these commentators are wrong. Several things Frege says indicate that he conceived of incomplete expressions as objects—e.g. [Frege, 1980, p. 136].

    [12]'A property is identical to some object' would be more naturally read as:

    $\exists z \exists v$ $(\Diamond \exists x(x$ is a predicate $\wedge$ x refers to $z)$ $\wedge \Diamond \exists y(y$ is a singular term $\wedge$ y refers to $v) \wedge z = v)$.

first argument place of 'ξ refers to ζ', and then using a definite description operator to bind a term variable in the argument place of the resulting predicate, ' "ξ is a horse" refers to ζ'. Moreover, if 'the referent of "ξ is a horse" ', thus interpreted, were to refer, it would, in one sense, present its referent as something to which a predicate can refer. Letting 'refers$_1$' mean 'ξ refers to ζ' and 'refers$_2$' mean 'ξ refers to Φ', 'the referent of "ξ is a horse" ', thus construed, would present its referent as something to which a predicate can refer$_1$. Moreover, if 'referred' in the stipulation, 'a *property* is anything that can be referred to with a predicate' [Trueman, 2015, p. §2], is understood to be 'referred$_1$', the resulting sense of 'property' is one on which 'the referent of "ξ is a horse" ', as presently construed, would present its referent as a property. Let 'property$_1$' mean 'property' in this sense. If Benno's thought concerns the property$_1$ referred to by 'ξ is a horse', there is, then, no problem in presenting it as such with a singular term, and therewith identifying it with an object, as follows: the referent of 'ξ is a horse' = the property *horse*. But the import of Trueman's claim that 'refers' for predicates must be read as 'ξ refers to Φ' is that (a) there *is* no property$_1$ referred to by 'ξ is a horse', since (b) there are no properties$_1$ at all, because predicates do not refer$_1$, and hence (c) 'the referent of "ξ is a horse" ', on its present reading, lacks a referent, as 'ξ is a horse', in particular, does not refer$_1$. It does not follow that 'the referent of "ξ is a horse" = the property *horse*' is nonsense, but the emptiness of the singular term left-flanking the identity sign renders the sentence at least untrue, a fate shared by Benno's thought if this sentence indeed expresses it.

4.4   In Defence of Singular Reference to Concepts

4.4.1   How To State Object-Property Identities I: Second-Level Operators

If Benno's thought is to be neither inexpressible nor untrue, what appears to be requisite is a singular term that presents its referent as something to which a predicate can refer$_2$. I propose that there is a way of construing 'the referent of "ξ is a horse" ', neglected in Trueman's paper, on which this expression is just such a singular term. This expression cannot, we have seen, be understood as the result of applying the standard term-variable-binding definite description operator to the predicate ' "ξ is a horse" refers to Φ': the argument place of this predicate cannot

accommodate a term variable. But nor can it be understood, if it is to be singular term, as the result of applying to this predicate the second-level definite description operator Trueman countenances in the quotation at the end of §4.2.2: though this operator *can* bind a predicate variable in the argument place of ' "ξ is a horse" refers to Φ', it produces a predicate, not a singular term. However, these two operators do not, I submit, exhaust our options. In addition to a term-variable-binding, term-forming operator and a predicate-variable-binding, predicate-forming operator, we can recognize a predicate-variable-binding, *term*-forming operator—one that (roughly) yields a complex name of the unique value of the predicate variable it binds (if there is one) that satisfies the relevant second-level predicate. This operator would, firstly, be fit to bind a variable in the argument place of the second-level predicate ' "ξ is a horse" refers to Φ'; secondly, produce, when applied to that second-level predicate, a singular term that co-refers with "ξ is a horse"; and thirdly, produce, crucially, a singular term that *presents* its referent as a property in the required sense—that is, as something to which a predicate might refer$_2$. If there is such an operator, 'the referent of "ξ is a horse" ' may be understood as the result of applying *this* operator to the second-level predicate ' "ξ is a horse" refers to Φ'.

If Trueman's argument is to go through, therefore, he requires that *there can be no such operator*. But what reason have we to preclude such a term-forming operator? Needless to say, variable-binding operators need not yield expressions of the same logical type as the variables they bind.[13] *Prima facie*, the term-forming operator seems just as intelligible as the predicate-forming operator Trueman recognises. Neither operator is readily rendered in English, though I note that 'the F such that "ξ is a horse" refers to F' is much more straightforwardly heard as a singular term than a predicate.

Are there any grounds for denouncing the operator I propose? There is at least one reason

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[13]More than one predicate-variable-binding, term-forming operator appears in Frege's own work. Firstly, since he thinks that sentences are names, the second-order quantifiers, on Frege's analysis, will surely count as such operators. Secondly, Frege will presumably want to recognise a second-level counterpart to his first-level notation for the value-range of a first-level function. The first-level notation features an operator (a Greek letter with a smooth breathing) that binds a first-order variable (the same letter without the smooth breathing) in the argument place of a function expression, 'f'. Applied to 'f' the operator yields a name of the value-range of f: e.g. $\acute{\varepsilon} f(\varepsilon)$. (See [Frege, 1997e, p. 136-7]). The second-level counterpart of this notation will feature an operator that binds a function variable (e.g. a predicate variable) in the argument place of a second-level function expression 'Q', and yields a name of the value-range of Q. This operator will thus be both predicate-variable-binding and term-forming.

one might be concerned about it. Let's introduce some notation for the two predicate-variable-binding operators now on the table. Let I_p be the higher-level definite description operator Trueman recognizes (the subscript indicating that it is predicate-forming) and let I_t be the higher-level operator I propose that we recognize (the subscript indicating that it is term-forming). Call an I_p-*predicate* any predicate that results from binding a predicate variable in the argument place of a second-level predicate with I_p. Similarly, call an I_t-*term* any term that results from binding a predicate variable in the argument place of a second-level predicate with I_t. The proposal is to state the desired object-property identity, Benno's thought, using an I_t-term as follows:

(v) $I_t F(\text{`}\xi \text{ is a horse' refers to } F) =$ the property *horse.*

Here is the concern.[14] If we are to introduce I_t then we will have to state its semantics. That means giving a principle that settles, given an arbitrary I_t-term, to what it refers. In the case of the standard definite description operator—which we now symbolize as ι—we give some such principle as the following:

(vi)   For all x ($\text{`}\iota x(Fx)\text{'}$ refers to x iff for all y (Fy iff $x = y$)).

This tells us that the result of applying ι to a first-level predicate Fx refers to the object that uniquely satisfies Fx. Likewise, in the case of I_p we can give the following principle:

(vii)   For all F ($\text{`}I_p F(QF)\text{'}$ refers to F iff for all G (QG iff $G =_2 F$)).

This tells us, intuitively, that the result of applying I_p to a second-level predicate, Q, refers to the property that uniquely satisfies Q. However, we run into difficulty when we attempt to state a corresponding principle for I_t. The temptation is to offer something like the following:

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[14]I'm grateful to Robert Trueman for impressing the following kind of worry upon me in correspondence.

(viii)   For all x ('$I_t F(QF)$' refers to x iff for all G (QG iff $G = x$)).

But this is nonsense: no sense has been given to '$=$' that would permit it to receive a predicate (variable or constant) into one argument place and a singular term (variable or constant) into the other. So then how *are* we to give the intended semantics of I_t?

A more promising attempt to state a corresponding principle for I_t is as follows:

(ix)   For all F ('$I_t F(QF)$' refers to F iff for all G (QG iff $G =_2 F$).

In the left-hand clause of the bi-conditional, 'refers to' is 'ξ refers to Φ'. This might seem aberrant, since '$I_t F(QF)$' is intended to be a singular term, not a predicate. However, there is, of course, no *syntactic* obstacle to our inserting the name of a singular term into the first argument place of 'ξ refers to Φ', and the results of doing so are perfectly meaningful. Note that the strategy of inserting the name of a singular term into the first argument place of 'ξ refers to Φ' also affords a way of pursuing the neglected approach mentioned in §4.3: expressing an object-property identity using two predicates in conjunction with the second-level analogue of the identity predicate. In particular, it seems to enable us to produce a predicate that presents its referent as an object. We may insert, for example, 'the property *horse*'—the name of an object with which we wish to identify a property—into the first argument place of 'ξ refers to Φ'. To the predicate that results—' "the property *horse*" refers to Φ'—we may then apply the predicate-forming operator I_p to yield: '$I_p F$("the property *horse*" refers to F)'. This predicate appears to present its referent as something to which singular reference can be made, and to allow us to state the desired object-property identity as follows:

(x) $I_p F$('the property *horse*' refers to F) $=_2$ is a horse.

The concern, though, about inserting the name of a singular term into the first argument place of 'ξ refers to Φ', and thus about the proposed strategies for stating the I_t operator's semantics and articulating Benno's thought with the higher-level identity sentence, (x), will be

this: Trueman's argument that 'refers' in talk of predicates referring must be read as 'ξ refers to Φ' will be paralleled by an argument that 'refers' in talk of singular terms referring must be read as 'ξ refers to ζ'. In brief: it is only possible to state a disquotation sentence, like (i), for a singular term using 'ξ refers to ζ', not 'ξ refers to Φ'.

However, I believe we now have the resources to reply to this argument and to vindicate the use of 'ξ refers to Φ' in speaking of singular terms referring. The challenge is to formulate a disquotational identity, analogous to (i), for a singular term, using 'ξ refers to Φ'. Letting 'a' be a singular term, I propose that the following meets this challenge:

(xi) $\quad I_t F(\text{'}a\text{' refers to } F) = a.$

Roughly, we here employ our term-forming operator I_t to form a term that picks out the unique property to which 'a' refers—if there is such a property—and say that that property is identical to a. The reference predicate we use is 'ξ refers to Φ'.

The reader will have noticed that there is a circularity here: we have sought to vindicate the use of 'ξ refers to Φ' to specify the referent of a singular term, by giving a disquotational identity that makes use of the I_t operator; but we gave the semantics of I_t using a principle—(ix)—that itself uses 'ξ refers to Φ' to specify the referent of a singular term. However, I simply see no reason to think that this circularity is vicious. It is certainly not generally to be expected that the semantics for an expression may be given in a manner completely devoid of circularity. Absent a reason for thinking the circularity in this particular instance a cause for concern, I conclude that the use of 'ξ refers to Φ' in speaking of singular terms referring satisfies Trueman's requirement concerning the possibility of stating disquotation sentences.

I have now adduced two methods of expressing Benno's thought, each exploiting a second-level variable-binding operator: firstly, with the first-level identity sentence, (v), using I_t; and secondly, with the second-level identity sentence (x), using I_p. So far, no reason has been given to doubt that these methods succeed.

4.4.2   How To State Object-Property Identities II: First-Level Operators

Having recognized a predicate-variable-binding, term-forming operator in addition to a term-variable-binding, term-forming operator and a predicate-variable-binding, predicate-forming operator, it is natural to complete the square and recognize a term-variable-binding, *predicate-*forming operator. This first-level operator, which we symbolise ι_p, binds a term variable in the argument place of a first-level predicate and, informally, yields a predicate referring to the unique value of that term variable (if there is one) that satisfies that first-level predicate. The semantics for ι_p can more formally be captured by the following principle:

(xii)   For all x ('$\iota_p x(Fx)$' refers to x iff for all y (Fy iff $x = y$)).

'$\iota_p x(Fx)$' is a *predicate*, and the principle tells us, intuitively, that it is a predicate whose referent is the unique object that satisfies the predicate 'Fx'. But since it is a predicate, (xii) involves the appearance of a predicate in the first argument place of 'ξ refers to ζ'. Surely this falls foul of Trueman's argument that 'refers' in talk of predicates referring must be read as 'ξ refers to Φ'.

However, I believe we now have the resources to reply to this argument and to vindicate the use of 'ξ refers to ζ' in speaking of predicates referring. The challenge is to formulate a disquotational identity, analogous to (i), for a predicate, using 'ξ refers to ζ'. Letting '$F\xi$' be a predicate, I propose that the following meets this challenge:

(xiii) $\iota_p x(\text{'}F\xi\text{' refers to } x) =_2 F$.

Roughly, we here employ our predicate-forming operator ι_p to form a predicate that picks out the unique object to which '$F\xi$' refers—if there is such an object—and flank the second-level analogue of the identity sign with this predicate and the disquoted predicate '$F\xi$'. The reference predicate we use is 'ξ refers to ζ'.

We find a circularity here, parallel to the circularity involved in our vindication of the use of 'ξ refers to Φ' in speaking of singular terms referring: we seek to vindicate the use of 'ξ refers to

ζ' in speaking of predicates referring, by giving a disquotational identity that makes use of the ι_p operator; but we gave the semantics of ι_p using a principle—(xii)—that itself uses 'ξ refers to ζ' to specify the referent of a predicate. But again, I just see no reason to think that this circularity is vicious. I conclude that the use of 'ξ refers to ζ' in speaking of predicates referring satisfies Trueman's requirement concerning the possibility of stating disquotation sentences.

Having recognized the ι_p operator and vindicated the use of 'ξ refers to ζ' for predicates, we are, I submit, afforded two further methods for stating Benno's thought. Firstly, we may use the ι_p operator to construct a predicate that presents its referent as an object: '$\iota_p x$('the property *horse*' refers to x)'. Therewith, we may formulate Benno's thought as the following second-level identity:

(xiv) $\iota_p x$('the property *horse*' refers to x) $=_2$ is a horse.

Secondly, we may use the ordinary definite description operator to construct a singular term that presents its referent as a property: 'ιx('ξ is a horse' refers to x)'. Therewith, Benno's thought can be naturally stated as the following first-level identity:

(xv) ιx('ξ is a horse' refers to x) $=$ the property *horse*.

Exploiting the first-level operators ι and ι_p, we can, I conclude, again either express Benno's thought as a first-level identity claim or as a second-level identity claim.

4.5   Conclusion

I hope to have established that, *pace* Trueman's argument, Benno's thought does admit of expression. It can be expressed, I claim, as a first-level identity, using (v) or (xv), or as a second-level identity, using (x) or (xiv). Benno's thought was just an arbitrary object-property identity, so I conclude that it is possible to express object-property identities. If I am right, Trueman has failed to establish that it is nonsense to say, generally, that a property is identical to some object. But

nor has he has established that it is *false* to say so: for no reason has been given for thinking that (v), (xv), (x) and (xiv), each of which can be understood as instances of this existential generalization, are false. I hold, then, that singular reference to properties/ concepts survives Trueman's assault. If the Fregean position is to be upheld, justification must be sought elsewhere.

4.6 Coda: Identity and the Proposed Operators

I would like, in closing this chapter, to address a possible objection to my defence of singular reference to concepts in the face of Trueman's argument.[15] In §4.6.1 I imagine a critic speaking in defence of Trueman's argument; in §4.6.2 I address that critic.

4.6.1 An Objection

The additional operators you have proposed to introduce—'I_t' and 'ι_p'—are essentially meant each to signify a certain function: 'I_t' is supposed to signify a function mapping second-level properties to first-level properties; 'ι_p' is supposed to signify a function mapping first-level properties to first-level properties. Now Trueman and Frege are of course free to acknowledge functions of each of these kinds. However, if your operators are to serve the expressive purposes to which you wish to put them in answering Trueman's challenge, then they must signify the *right* such functions. Taking a pinch of salt, you have to maintain that the functions they signify are such that, in particular: $I_t G(G =_2 F)$ is always the same entity as F, and $\iota_p y(y =_1 x)$ is always the same entity as x.[16] Unless this is the case, 'I_t' and 'ι_p' are not the operators they are supposed to be—i.e. the respectively term- and predicate-forming counterparts of 'I_p' and 'ι'. In other words, you must hold $I_t G(G =_2 \Phi)$ and $\iota_p y(y =_1 \alpha)$ are identity functions. But how are you to even go about expressing that this is the case? It is tempting to attempt to express this as follows:

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[15]The objection was separately brought to my attention by Robert Trueman and James Studd in their comments on drafts of this chapter, for which I thank them. I am heavily indebted, in my exposition of the objection, to correspondence with Trueman, though I stress that it may or may not reflect his own view on the matter.

[16]For the sake of clarity in what follows, I shall notate the first-level identity predicate as '$=_1$'.

(xvi) $\forall F(I_t G(G =_2 F) = F)$

(xvii) $\forall x(\iota_p y(y =_1 x) = x)$

However, these are both meaningless, since in each case, the rightmost identity sign is flanked by a predicate variable on one side and a term variable on the other; whether that sign is taken to be the be the first- or the second-level identity sign, nonsense results. But then how *are* you to express your claim that $I_t G(G =_2 \Phi)$ and $\iota_p y(y =_1 \alpha)$ are identity functions?

It's likely that you consider yourself to be in a familiar predicament in this regard. As in the case of Benno's thought, you find yourself tasked with expressing an identity between something referred to with a singular term and something referred to with a predicate. Presumably, therefore, you will attempt to simply deploy your new operators once more. A natural way of doing this would be to offer, not (xvi) and (xvii), but

(xviii) $\forall F \forall x(\iota_p y(y =_1 I_t G(G =_2 F)) =_2 F)$

(xix) $\forall F \forall x(I_t G(G =_2 \iota_p y(y =_1 x)) =_1 x)$

The rationale behind (xviii) would be that because $\iota_p y(y =_1 x)$ is always the same thing as x, in particular $\iota_p y(y =_1 I_t G(G =_2 F))$ is the same thing as $I_t G(G =_2 F)$, so that to claim, as (xviii) does, that, in general, $\iota_p y(y =_1 I_t G(G =_2 F))$ is identical to F is to claim that, in general, $I_t G(G =_2 F)$ is identical to F, as required. The rationale behind (xix) is similar: because $I_t G(G =_2 F)$ is always the same thing as F, in particular $I_t G(G =_2 \iota_p y(y =_1 x))$ is the same thing as $\iota_p y(y =_1 x)$, so that to claim, as (xix) does, that, in general, $I_t G(G =_2 \iota_p y(y =_1 x))$ is identical to x is to claim that, in general, $\iota_p y(y =_1 x)$ is identical to x, as required. In a sense, this response to the difficulty is circular, since you are assuming just those claims whose very expressibility is in question. But, no doubt, you will claim that this circularity, like that noted on pages 153 and 154 above, is benign.

The problem, however, is this. Frege and Trueman are perfectly at liberty to accept that there

is a function I_t from second-level properties to first-level properties which makes (xviii) true; and they are perfectly at liberty to accept that there is a function ι_p from first-level properties to first-level properties which makes (xix) true. All they require is that these functions be such that $I_t G(G =_2 F)$ and $\iota_p y(y =_1 x)$ are one another's inverse. Thus, although you require it to be the case that $I_t G(G =_2 F)$ and $\iota_p y(y =_1 x)$ are identity functions, you cannot actually even *express* that this is the case; for your best attempt to do so amounts to saying things that even Frege and Trueman are at liberty to accept—namely, (xviii) and (xix). Perhaps you'll protest that Frege and Trueman cannot accept (xviii) and (xix) as *you* mean them—that they cannot accept them when 'I_t' and ι_p mean what *you* say they are to mean. But the whole point is that you do not seem to be in a position *to* say what they mean.

4.6.2   Reply

The problem you pose is how to even express that $I_t G(G =_2 F)$ and $\iota_p y(y =_1 x)$ are identity functions. I confess I cannot see how to directly assuage your worries about the possibility of doing so. But I hope to get myself off the hook by asking you this: What has become of the method of Trueman's argument at this stage in the dialectic? I thought that the method of Trueman's argument, directed at opponents like myself, was to jump into our way of talking *with both feet*, as he put it (see page 140 above), and to show us *by our own lights* that we cannot even articulate certain putative thoughts—initially, the putative general thought that a property is an object, and now the putative thoughts that $I_t G(G =_2 F)$ is the same thing as F and $\iota_p y(y =_1 x)$ is the same thing as x. However, at this stage in the discussion, it seems that you, the proponent of Trueman's argument, really only have one foot, as it were, in your opponents' idiom. Really, your concern about deploying (xviii) and (xix) to express the required claims (and you represent me quite rightly in anticipating that I will wish deploy the proposed operators once more, reapplying the general strategy I used to state Benno's thought) is that there are ways of interpreting the functors '$I_t G(G =_2 F)$' and '$\iota_p y(y =_1 x)$' such that they do not signify identity functions and yet (xviii) and (xix) come out true. Agreed; but as you rightly anticipate, I rejoin that '$I_t G(G =_2 F)$' and '$\iota_p y(y =_1 x)$' are *not* to be so interpreted! I assert (xviii) and (xix) *on their intended interpretation*, and by *my* lights they succeed in stating

my distinctive claims. Your worry about this rejoinder seems to be that I cannot say *what* that intended interpretation is in a manner you can accept. But it seems to me that this makes for a *very* different dialectical situation. The challenge you're now posing to your opponents, it seems to me, is to show you by *your* Fregean lights that we can express various putative thoughts; and showing that we can do so by *your* lights means showing that we can do so without granting ourselves the intended interpretations of certain expressive resources that we accept. As I say, I cannot see how to directly meet that challenge; but I contend that I am under no obligation to meet that challenge. I am inclined to concede that the thoughts in question cannot, by your lights, be expressed, but from this I conclude: so much the worse for your lights!

We are basically returning here to the circularity involved in my response to Trueman's argument, which I claim is entirely benign. In essence: I am at ease with the necessity of using my proposed operators—using them with their intended interpretations, mind—in stating the semantics of those operators. Or, to put the point slightly differently: I am comfortable with my operators being expressively indispensable. Moreover, I can appeal to precedent here. We are comfortable with the necessity of using the truth-functional connectives in stating the semantics of the truth-functional connectives. We are usually comfortable with necessity of using the quantifiers in stating the semantics of the quantifiers. Indeed, the case of the quantifiers allows me to illustrate that Trueman himself confronts an analogue of the problem which you have posed against my response. After all, just as I must claim that $I_t G(G =_2 F)$ is the same thing as F, Trueman must claim that (note the different subscript) $I_p G(G =_2 F)$ is the same thing as F, since he countenances the operator I_p as a second-level counterpart of the standard definite description operator. I think that in order to express this claim, he would offer the following:

(xx) $\quad \forall F(I_p G(G =_2 F) =_2 F)$

However, all that is required for (xx) to come out as true is that the $I_p G(G =_2 \Phi)$ function map any property as argument to a property that bears the $\Phi =_2 \Psi$ relation to that argument. Thus, a sceptic concerning the (of course, not uncontroversial) higher-order expressive resources 'I_p' and '$=_2$' could equally well claim to be at liberty to accept (xx) without accepting

that $I_pG(G =_2 \Phi)$ is an identity function. Would Trueman concede, on these grounds, that (xx) does not succeed in articulating his claim that $I_pG(G =_2 F)$ is always the same thing as F? I don't think he ought to.

My suspicion is that your objection takes us well into the fresh territory of arguments concerning the (in)determinacy of meaning. There are always alternative interpretations of what we say—are there not?—that nevertheless make what we say true. These arguments present deeply troubling problems, but—surely—problems for everyone, not just those of use who consider singular reference to concepts intelligible.

Chapter 5

Diagonalization and Russell's Paradox

5.1 Are Concepts Objects? A Diagonal Argument

Suppose, for *reductio*, that (first-level) concepts are (one and all) objects. In that case, there is an injective function (henceforth, *injection*) from all concepts to objects: a function, f, such that for every concept, c, f maps c to some object, $f(c)$, and for any concept c', if $c \neq c'$ then $f(c) \neq f(c')$. For, if concepts are objects, the identity function mapping each concept to itself is such an injection. However, there is no such injection. Let g be any injection from concepts to objects. Let c^d be the concept *object to which g maps a concept under which it (the object) does not fall.* That is, for any object, o, o falls under c^d just in case, for some concept, c, $o = g(c)$ and o does not fall under c. Now c^d is a concept that is not mapped to any object by g; since if, on the contrary, there is some object, o, such that $o = g(c^d)$, then either o falls under c^d or o does not fall under c^d. Consider each case in turn. Suppose that o falls under c^d; then for some concept, c, $o = g(c)$ and o does not fall under c. But since g is an injection and $o = g(c^d)$, for any concept c, if $o = g(c)$, then $c = c^d$. Thus, o does not fall under c^d. On the other hand, suppose that o does not fall under c^d; then for all concepts, c, if $o = g(c)$, then o falls under c. Since $o = g(c^d)$, o falls under c^d. Therefore, o falls under c^d if, and only if, it does not fall under c^d. Contradiction.

Therefore, c^d is a concept not mapped to any object by g. Thus, g is not an injection from *all* concepts to objects. But g was arbitrary. So there is no injection from all concepts to objects. Therefore, it is not the case that concepts are objects.

The foregoing argument is intimately related to both Cantor's Theorem and Russell's paradox, and these latter to one another. The argument deploys the technique of *diagonalization* to define, given an arbitrary injection from concepts to objects, a ('diagonal') concept demonstrably not mapped to any object by that injection.[1] The same technique was exploited by Georg Cantor [Cantor, 1892] to establish the theorem bearing his name: that every set is of strictly lesser cardinality than its powerset. Indeed, it is natural to attempt to apply this result itself towards establishing the Fregean conclusion drawn in the foregoing argument. We might reason as follows. The number of sets of objects (the cardinality of the powerset of the set of objects) must, by Cantor's theorem, strictly exceed the number of objects (the cardinality of the set of objects). But the number of concepts must be at least as great as the number of sets of objects, since for each such set, s, there is the concept of belonging to s (the concept *element of s*). Thus, the number of concepts is strictly greater than the number of objects. (Specifically, where κ is the number of objects, the number of concepts is at least 2^κ, which is greater than κ.) Therefore, it is not the case that concepts are objects.

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[1] The reason the technique is dubbed 'diagonalization', and both the defined concept and the argument itself called 'diagonal', can be seen if we imagine the arguments of our given injection listed vertically on the left of a tabular array, one argument per row, and its values correspondingly listed horizontally along the top of the array, one value per column, such that the value above the nth column is the value of the given injection for the argument appearing to the left of the nth row. For any cell, C_{mn}, of the array intersecting the mth row and the nth column, C_{mn} contains a 1 if the value above the nth column falls under the argument to the left of the mth row; otherwise, it contains a 0. The 'diagonal' concept, c^d, can then be defined with reference to the entries on the diagonal in this array, in bold below:

	$g(c_1)$	$g(c_2)$	$g(c_3)$	$\ldots$
c_1	**0**	1	0	$\ldots$
c_2	0	**1**	1	$\ldots$
c_3	1	1	**0**	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

The concept c^d is, in effect, defined as follows: an object falls under c^d just in case it (the object) appears in the horizontal list above the table and the number on the diagonal directly below it is 0. It then follows that c^d cannot appear to the left of any row in the vertical list of arguments, since this would place an impossible demand on the cell at the intersection of c^d's row with the diagonal—viz. that the number that cell contains be 0 iff it is 1.

The connection with Russell's paradox emerges when we consider the case in which the function g is the mentioned injection whose existence immediately follows from the supposition that concepts are objects—to wit, the identity function mapping each concept to itself. In that case, c^d is the *Russell property*—the concept *concept that does not fall under itself.* This concept engenders the property version of Russell's paradox: for if it falls under itself, then it does not fall under itself, and if it does not fall under itself, then it falls under itself. Thus, the diagonal argument above against the thesis that concepts are objects might be only slightly reframed in summary as follows: it cannot be the case that concepts are objects, for if they were, Russell's property paradox would ensue.[2]

5.2   The Diagonal Argument and Frege's Views on Objecthood

To my knowledge, Frege never advanced such an argument to substantiate his doctrine that it is not the case that concepts are objects. It is overwhelmingly likely, at any rate, that no such argument occurred to Frege prior to 16th June 1902, the date of Russell's [Frege, 1980, p. 130-31] historic missive apprising Frege of (what would come to be called) Russell's paradox in its property- and class-theoretic versions, the latter version representing a catastrophic contradiction in the foundations of Frege's logicist edifice. For, given the conceptual proximity of the diagonal argument to Russell's paradox, if Frege had contemplated such an argument for the non-objecthood of concepts, it would be unaccountable that he should have been, by his own admission, 'surprised beyond words' and even 'thunderstruck' [Frege, 1980, p. 132] by Russell's discovery of the paradox. The closest Frege comes, as far as I'm aware, to appealing to the threat of Russell's property paradox towards establishing the non-objecthood of concepts is in his appealing to the non-objecthood of concepts to stave off the threat of that version of Russell's paradox. Russell presents that version by letting 'w be the predicate of being a predicate which cannot be predicated of itself' and asking 'Can w be predicated of itself?' [Frege, 1980, p. 130]. (Russell is using 'predicate' in the non-linguistic sense here, as equivalent to 'property' or 'con-

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[2]Several portions of Russell's *The Principles of Mathematics* are extremely pertinent here: [Russell, 2009, §§85, 499, 500].

cept'; 'can be predicated of' expresses the relation of subsumption—i.e. the converse of the relation expressed by 'falls under'.) Frege replies that '[a] predicate is as a rule a first-level function which requires an object as argument and which cannot therefore have itself as argument (subject)' [Frege, 1980, p. 132]: By Frege's lights, sense can only be made of a (first-level) property's being instantiated by, or failing to be instantiated by, an object. Since the property is itself not an object, no sense can be made of a property's instantiating or failing to instantiate itself. Thus, no sense can be made of the proposed conditions for the instantiation of the putative property w. Russell's proffered definition simply fails to define any property/ concept.

By contrast, no such objection is to be made against talk of the *extension* of a concept falling or not falling under that concept, since, in Frege's judgement, 'extensions of concepts...are objects, although concepts themselves are not' [Frege, 1997e, p. 141].[3] On the contrary, given Frege's Principle of Completeness—that for any concept Φ, '[a]ny object Δ that you choose to take either falls under the concept Φ or does not fall under it; *tertium non datur*' [Frege, 1997c, p. 259]—the extension of a concept must indeed either fall under that concept or not fall under it. We can thus consider the concept under which fall all and only extensions that do *not* fall under the concept of which they are the extension—i.e. the concept *extension of a concept under which it does not fall*. When we ask whether the extension of *that* concept falls under that concept, the contradiction that ensues is one against which the non-objecthood of concepts provides no bulwark: the extension of that concept falls under that concept if, and only if, it doesn't. This, of course, is just the class version of Russell's paradox formulated in Fregean vernacular.

We turn now in earnest to the topic of this section: namely, the question of how congenial the diagonal argument is to Frege's views on objecthood—how favourable that reasoning is to Frege's judgements about what is and what is not an object. Though he did not advance such an argument, might he profitably have done so? The considerations of the previous paragraph already bring us to one important respect in which that reasoning is profoundly uncongenial to Frege's views. It was Frege's belief, at least prior to Russell's disclosure of The Contradiction,

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[3]Generally: 'Value-ranges of functions are objects, whereas functions themselves are not' [Frege, 1997e, p. 140].

that for each concept there exists a unique extension. This is the import (in the special case of concepts) of Frege's infamous Basic Law (Vb): the distinctness of the concept Φ from the concept Ψ (I shall say more about distinctness of concepts momentarily) implies the distinctness of the extension of the concept Φ from the extension of the concept Ψ. Coupled with his position that extensions are objects, this implies precisely what the diagonal argument's main lemma rules out: the existence of an injection from all concepts to objects. For the function mapping each concept to its extension would be just such an injection. The precipitation of Russell's class (extension) paradox is thus made inevitable. So, holding fixed Frege's belief concerning the abundance of extensions (one for each concept), the reasoning of the diagonal argument can equally well be used to establish, *pace* Frege, that it is not the case that extensions are (one and all) objects.

Whether Frege's aforementioned belief *is* to be held fixed, though, is an important question. Frege's own response to the paradox, presented in a hurriedly prepared appendix to the second volume of the *Grundgesetze* [Frege, 1997c, p. 279-89], was indeed to relinquish that belief rather than sacrifice the claim that 'extensions of concepts, or classes, [are] objects in the full and proper sense of the word' [Frege, 1997c, p. 282]. Essentially, Frege proposed to modify the criterion of identity for extensions (more generally, value-ranges) embodied in Basic Law (V), such as to allow that distinct concepts may nevertheless have the same extension, providing their distinctness is solely a matter of their shared extension *itself* falling under the one but not the other. Frege's response proved vain, since the modified criterion, it subsequently emerged, still engenders a contradiction, granted the assumption that there is more than one object (!).[4] Though we shan't dwell further on the objecthood of extensions and the inconsistency of Basic Law (V), the idea that the diagonal argument might be applicable, *mutatis mutandis*, to other elements of Frege's ontology is one to which, for good and ill, we will shortly return.

Now, though, I want to raise two initial doubts concerning the congeniality of the diagonal argument to Frege's views on objecthood. The first is simply whether the diagonal argument is sufficiently strong for Frege's purposes. The conclusion of the argument is that it is not the case

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[4]See [Dummett, 1981a, pp. xli, 656], [Quine, 1955], [Geach, 1956].

that concepts are objects—i.e. the particular negative thesis that some concept is not an object. Frege's position is the logically stronger claim that concepts are not objects—i.e. the universal negative thesis that no concept is an object. So, in principle, even if one were wholly swayed by the diagonal argument, one might nevertheless hold, contra Frege, that *some* concepts are objects. Indeed, it might be held that almost all of those concepts to which we make reference with natural language predicates are objects. (Almost but not quite all, presumably, since the predicate 'ξ is a concept that does not fall under itself' refers—does it not?—to the Russell property, and if the latter were an object, paradox would ensue.)

The suggestion that some, but not all, concepts are objects is, I regret, not one I am able to consider in detail. Nevertheless, the suggestion is apt to seem unattractive. It is very natural to suppose that concepts are broadly alike in kind. It would seem mysterious if some among them were of the right kind to be the referents of singular terms, whilst others among them were not. The suggestion ought, moreover, to be backed up by some principled specification of *which* among the concepts lack objecthood. This is not easy to come by. Simply denying objecthood to those concepts (like the Russell property) whose objecthood would engender paradox would seem *ad hoc*; for, on that criterion, very closely kindred concepts would differ profoundly in respect of their status as objects—e.g. the Russell property's non-paradoxical cousin, the concept *concept that falls under itself*, would enjoy object status. I, for one, am prepared to grant that if Frege can, by appeal to the diagonal argument, establish that some concepts are not objects, then he can, by the same appeal, provide a strong case that no concept is an object.[5]

The second doubt is whether the diagonal argument is even available to Frege. The concern here has to do with talk, which I rather casually used in the preceding paragraphs, of concepts being *distinct*. Such talk featured centrally in the diagonal argument, in the explanation of

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[5]By a similar token, I am prepared to grant that the diagonal argument, if cogent, also provides a strong case for Frege's more general thesis that no first-level function is an object, since concepts are merely some among said functions and it is natural to suppose that if any such function is a potential referent of a singular term, each such function is. Frege's yet more general thesis that no function whatsoever is an object is another matter: even setting aside the dubiousness of its purportedly level-transcendent generality, the transition to this universal negative from the corresponding particular negative is problematic, since we do not seem entitled to suppose that functions are broadly alike in kind. Nevertheless, for each functional level, a diagonal argument will be forthcoming to the effect that not all functions inhabiting that level are objects, and it will be natural to conclude therefrom that no function of that level is an object. We will continue to be primarily concerned with the lowest levels of the Fregean hierarchy in what follows.

the injectivity of a function. A key claim in the argument was that if concepts were objects, there would be a function from all concepts to objects which always mapped *distinct concepts* to distinct objects. But recall that for Frege 'identity, [and thus distinctness,] can only be thought of as holding for objects, not concepts' [Frege, 1997a, p. 175]. One might surmise that Frege is obliged, therefore, to consider the diagonal argument improper, since that key claim and the whole notion of an injective function from all concepts to objects are unintelligible. Frege does, in fact, express something very much like this view when in correspondence Russell remarks,

> It is easy to prove (as concerns the contradiction) that there is no one-one relation between all objects and all functions. [Frege, 1980, p. 139]

Frege replies,

> [T]he proof that there is no one-one relation between all objects and all functions strikes me as dubious. I believe that even the idea of objects standing in a one-one relation to functions is not quite clear. For standing in a one-one relation presupposes identity, and the relation of identity is a first-level relation which can hold only between objects and not between functions. [Frege, 1980, p. 143]

(The issue is the same, though it is a bijection from all objects to all functions that is under discussion, rather than an injection from all concepts to objects.)

However, we should not surmise that Frege must consider the diagonal argument improper. Firstly, even if in light of these considerations about the typical inapplicability of identity to concepts the argument is, for Frege, not just 'dubious' but strictly nonsensical, it may nevertheless serve as an effective *ad hominem*. After all, Frege's opponent, someone who thinks concepts are objects, will, qua proponent of that view, deny that identity is typically inapplicable to concepts; so the argument is intelligible by *their* lights, even if not by Frege's, and may therefore suffice to convince them that their own view disintegrates into incoherence. Secondly, the talk of distinctness of concepts in the diagonal argument can be replaced, or reinterpreted, with talk of concepts standing in Frege's proffered second-level analogue of the distinctness relation: namely, the relation holding between concepts F and G just in case some object falls under F but not G, or vice versa. Consider a function from concepts to objects, designated by the second-level functor '$M(\Phi)$', which admits a predicate into its argument place and forms a singular term. The claim that that function is injective could be expressed as follows:

$$\forall X, \forall Y (\neg \forall x (Xx \leftrightarrow Yx) \rightarrow M(X) \neq M(Y))$$

We speak here only of distinctness between objects. It strikes me, therefore, that the talk of distinctness of concepts in the diagonal argument is no serious obstacle to Frege's appealing to it.

There is further good news for Frege. Frege denies the objecthood not only of concepts but also of senses that are modes of presentation of concepts—senses that can feature as the sense of a *bedeutungsvoll* predicate. The diagonal argument can readily be modified to support this position too. The main lemma will be that there is no injection from all senses presenting concepts ("concept-senses") to objects. The argument that there is no such injection, which there would be if concept-senses were themselves objects, is as follows. Let f be any injection from concept-senses to objects. Let c^{d*} be the concept *object to which f maps a concept-sense presenting a concept under which it (that object) does not fall.* That is, for any object, o, o falls under c^{d*} just in case, for some concept-sense, s, and some concept, c, $o = f(s)$, s presents c, and o does not fall under c. Finally, let s^d be any concept-sense presenting c^{d*}—for example, the sense of the predicate 'ξ is an object to which f maps a concept-sense presenting a concept under which ξ does not fall'. Now s^d is a concept-sense that is not mapped to any object by f; for suppose, on the contrary, that there is some object, o, such that $o = f(s^d)$. Then either o falls under the concept, c^{d*}, which s presents, or o does not fall under c^{d*}. Consider each case in turn. Suppose that o falls under c^{d*}; then for some concept-sense, s, and some concept, c, $o = f(s)$, s presents c, and o does not fall under c. But since f is an injection and $o = f(s^d)$, for any concept-sense s, if $o = f(s)$, then $s = s^d$. Moreover, a concept-sense presents at most one concept,[6] and s^d presents c^{d*}; so for any concept, c, if s^d presents c, then $c = c^{d*}$. Thus, o does not fall under c^{d*}. On the other hand, suppose that o does not fall under c^{d*}; then for all concept-senses, s, and all concepts, c, if $o = f(s)$, and s presents c, then o falls under c. Since $o = f(s^d)$, and s^d presents c^{d*}, o falls under c^{d*}. Therefore, o falls under c^{d*} if, and only if, it does not fall under c^{d*}. Contradiction. Therefore, s^d is a concept-sense not mapped to any

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[6]This is an instance of a principle concerning Fregean senses which Kevin C. Klement dubs the *Principle of Determinacy:* no sense presents (i.e. is a mode of presentation of) more than one entity [Klement, 2003, p. 305].

object by f. Thus, f is not an injection from *all* concept-senses to objects. Since f was arbitrary, there is no such injection.

Again, the argument that it is not the case that concept-senses are objects can be slightly recast in summary as follows: if concepts-senses were objects, a version of Russell's paradox would arise. Consider the concept *concept-sense which presents a concept under which it does not fall*. Now consider any concept-sense that presents that concept—for example, the sense of the predicate 'ξ is a concept-sense which presents a concept under which ξ does not fall'. Call that sense the *Russell concept-sense*. If the Russell concept-sense is an object, either it itself falls under the concept it presents, or it does not. However, either supposition implies the other. Contradiction.

There is also bad news for Frege, however.[7] The diagonal argument is *also* readily modified to produce arguments against Frege's own classification of certain other elements of his ontology as objects. Specifically, not only does diagonal reasoning appear to refute the anti-Fregean claim that senses fit to be expressed by predicates are objects, it also appears to refute the *Fregean* claim that senses fit to be expressed by *proper names* are objects.

At least some senses fit to be expressed by proper names (in Frege's broad sense) are what we may call *definitely descriptive* senses: senses fit to be expressed by definite descriptions. These senses present, for some concept, c, whichever object, if any, uniquely falls under c. The definitely descriptive sense of 'the even prime' presents, if anything the unique even prime, while that of 'the horse' presents, if anything, the unique horse. Let the *associated concept* of a given definitely descriptive sense, d, be that concept, c, such that d presents, if anything, the unique object falling under c. Now if definitely descriptive senses were objects, there would be an injection from all of them to objects (again consider the identity function on such senses). But diagonal reasoning once more seems to show that there can be no such injection. Let h be any injection from definitely descriptive senses to objects. Let c^{d**} be the concept *object to which h maps a definitely descriptive sense under whose associated concept it (that object) does not fall*. That is, for any object, o, o falls under c^{d**} just in case, for some definitely descriptive sense,

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d, and some concept, c, $o = h(d)$, c is the associated concept of d, and o does not fall under c.

Finally let d^d be any definitely descriptive sense whose associated concept is c^{d**}—for example,

the sense of the definite description 'the object to which h maps a descriptive sense under whose

associated concept it (that object) does not fall'. Now d^d is a definitely descriptive sense that is

not mapped to any object by h; for suppose, on the contrary, that there is some object, o, such

that $o = h(d^d)$. Then either o falls under d^d's associated concept, c^{d**}, or o does not fall under

c^{d**}: I leave it to the reader to derive each supposition from the other, yielding a contradiction.

And once more, the argument can alternatively be cast as an appeal to a version of Russell's

paradox. Consider the concept *definitely descriptive sense which does not fall under its associated*

concept. Now consider any definitely descriptive sense whose associated concept is this latter

concept—for example, the sense of the definite description 'the definitely descriptive sense which

does not fall under its associated concept'. Call that sense the *Russell definitely descriptive sense.*

If the Russell definitely descriptive sense is an object, it either falls under its associated concept

or it does not. But if it does, it doesn't, and if it doesn't, it does.[8]

Further bad news: similar diagonal reasoning appears to refute Frege's view that *thoughts*—

senses fit to be expressed by *sentences*—are objects. For brevity's sake, we proceed straight to

the attendant version of Russell's paradox. Some thoughts are to the effect that everything falls

under some particular concept[9]. The thought that everything is a horse is one such: it is to the

effect that everything falls under the concept *horse.* We can say that that thought *universalizes*

the concept *horse.* Generally, we say that a thought universalizes a concept, c, exactly if it is

to the effect that everything falls under c. Consider the concept *thought which universalizes*

a concept under which it (the thought) does not fall. Call this concept U. (The thought that

everything is a horse falls under U, for example, since it universalizes the concept *horse*, but is

not a horse.) But now consider any thought which universalizes U—for example, the sense of

the sentence 'Everything is a thought which universalizes a concept under which it does not

fall'. Call this the *Russell thought.* If the Russell thought is an object, either it falls under U or it

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[8]See [Klement, 2009].

[9]…where *everything* ranges as widely as possible, over all (possible) arguments of the concept in question, and
not just some contextually relevant subcollection thereof.

doesn't. But once more, either supposition implies the other: contradiction.[10, 11]

The foregoing is really one form of the *propositional* version of Russell's paradox—now commonly known as the Russell-Myhill antinomy—which Russell in fact communicated to Frege later in 1902 [Frege, 1980, p. 147ff].[12] In the correspondence that followed, Frege apparently holds that the propositional paradox is forestalled by considerations having to do with the distinction between sense and reference. Though we cannot here pursue the details of that exchange, I think it is clear that the sense-reference distinction does not suffice to stave off the paradox of the previous paragraph, and thus that Frege failed to appreciate its import.[13]

The diagonal argument is, I conclude, both available to Frege and, if cogent, strong enough to furnish significant support for his classification of concepts and concept-senses as non-objects. With minor modification, however, the argument spills over into other regions of Frege's ontology, where its conclusion is quite unwelcome. The upshot, I suggest, is that if Frege were to advance the diagonal argument, he could only do so at the cost either of incoherence, or of hypocrisy, or of substantial revision to his ontology and philosophy of language: incoherence if the diagonal reasoning were deemed cogent, but the claim that (e.g.) thoughts are objects were retained; hypocrisy if the diagonal argument were deemed unconvincing but impressed nevertheless upon Frege's opponent who regards concepts as objects; revision if the reasoning were deemed cogent and its wider consequences duly heeded. Surely revision would be the only tenable option.[14]

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[10]See [Klement, 2003, p. 307]. A similar argument is presented in [Rieger, 2002], but seems to suffer a critical flaw, as highlighted by [Denyer, 2003]. The argument in the main text seems to suffer from no such defect, however, as [Klement, 2005, p. 47] points out.

[11]A further Fregean view on objecthood that is apparently confounded by a version of Russell's paradox is the view, whose ascription to Frege I argued for in chapter 1, that *predicates* (in the linguistic sense) are objects. The version in question is what is usually known as Grelling's antinomy or the paradox of heterologicality, but can, as Sullivan [Sullivan, 2003, p. 34] indicates, justly be called the predicate version of Russell's paradox. Consider the predicate 'ξ is not true of itself'. Call that the *Russell predicate*. If predicates were objects, names for them could be introduced into the argument place of the Russell predicate. Introducing a name of the predicate 'ξ is a horse' would yield a true sentence, since that predicate is not itself a horse. But introducing a name of the Russell predicate itself into its own argument place would yield a sentence that is true exactly if it is false. I suspect that these considerations are less unfavourable to Frege than those raised in the main text, since although the view that predicates aren't objects is, as I have argued, inconsistent with the letter of Frege's writings, I admit that the view has some consonance with their spirit. See [Sullivan, 2003].

[12]The propositional paradox is also presented in the *The Principles of Mathematics* [Russell, 2009, §500].

[13]See [Klement, 2001] for detailed discussion and a defence of this judgement.

[14]I've perhaps not seemed much of an enemy of philosophical hypocrisy, having more than once been open to the propriety of arguments which cannot even be recognised as intelligible by those who would advance them. But there

But if so, the question immediately arises as to how well Frege's philosophy would tolerate the requisite revision. This question must be left for another occasion; but my suspicion is that the strain caused by revising the view that thoughts are objects, for example, would be considerable. Moreover, there is an important sense in which the revision cannot *merely* involve relinquishing the objecthood of thoughts:[15] it would not suffice for Frege just to assign thoughts to some logical type other than that of *object*. For, in fact, if there is any *one* logical type to which all thoughts belong, even if not the type *object*, the relevant version of Russell's paradox will still arise. There is some one logical type to which all thoughts belong just in case there is some (not necessarily first-level) function, f, such that, for every thought, t, f takes t as argument. If there is such a function, Frege must surely acknowledge, then there are functions whose arguments include all thoughts and whose value for any argument is always a truth-value. Generalizing 'concept' from its application to the first level of the type-theoretic hierarchy, such functions are naturally described as *concepts applicable to thoughts* (I will hyphenate this phrase for convenience). We may say that a thought *universalizes* a concept-applicable-to-thoughts, c^t, exactly if it is to the effect that everything belonging to the same logical type as thoughts falls under (is mapped to the True by) c^t. But now, in particular, there is the concept-applicable-to-thoughts, U', under which an argument falls just in case that argument is a thought which universalizes a concept-applicable-to-thoughts under which it does not fall. Finally, there is the thought that everything (belonging to the same logical type as thoughts) falls under U'. This thought, which we call the *Russell thought'*, universalizes U'. Since U' is a concept-applicable-to-thoughts, it takes the Russell thought' itself as argument. Does the Russell thought' fall under U'? It does if, and only if, it doesn't.

Regarding thoughts as objects is, then, a sufficient but not a necessary condition for engendering the propositional version of Russell's paradox. The question of what further measures are best taken in order to avert the paradox is both open and extremely difficult. Let me very

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is a vital difference between (i) pressing an argument one considers unconvincing (even unintelligible) against an opponent whose own position gives them reason to consider the argument convincing, and (ii) pressing an argument one considers unconvincing against an opponent whose position gives them no such reason. It is the legitimacy of (i) that I am open to, but (ii) that I have in mind when I speak here of hypocrisy and which I consider an untenable option.

[15]We focus here on the case of thoughts, but similar considerations pertain to definitely descriptive senses.

briefly adumbrate two possible responses deriving, respectively, from Russell and Wittgenstein.

The first, Russell-inspired response would be to *ramify* the type-theoretic hierarchy.[16] In particular, where thoughts were previously conceived as belonging to some one type, and the concepts applicable to them likewise, in the ramified hierarchy thoughts would belong to various types, as would the concepts applicable to them. Thoughts would belong to types according to their *order*: thoughts which do not feature quantification over other thoughts are *first-order* thoughts; those which feature quantification over thoughts of the first order, but of no higher order, are *second-order* thoughts; those which quantify over thoughts of the second order, but of no higher order, are *third-order* thoughts; and so on. Thoughts of distinct orders would belong to distinct types and concepts applicable to thoughts would only ever take thoughts of some one type as arguments. How would this avert the paradox? A universalizing thought quantifies over all and only arguments of the concept it universalizes:[17] the thought that everything Φs quantifies over all and only arguments of the concept Φ. A universalizing thought either quantifies over thoughts or it doesn't. If it doesn't, then the concept it universalizes does not take thoughts as arguments, and thus the universalizing thought itself is not an argument of that concept. If it does quantify over thoughts, then, according to ramified type theory, it must belong to a higher order than—and thus a distinct type from—the thoughts over which it quantifies: since concepts never take thoughts of distinct types as arguments, the universalizing thought again cannot be among the arguments of that concept. Thus, in either case, there is simply no question as to whether a universalizing thought falls under the concept it universalizes. Recall that it was precisely that question concerning the Russell thought$'$ that engendered the contradiction. Indeed, ramified type theory blocks the paradox not only by precluding that question; it precludes the Russell thought$'$ itself, since it recognizes no such concept as the concept the Russell thought is supposed to universalize: the concept U' is defined in terms of a thought's falling or failing to fall under a concept it universalizes—terms now deemed destitute of sense.[18]

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[16]See especially [Russell, 1908]. The essential idea is tabled, with misgivings, in the penultimate paragraph of the *Principles* [Russell, 2009, p. 540].

[17]We equivocate somewhat in speaking of a *thought*, rather than a sentence, as quantifying over things; I don't think this is much cause for concern, however. The equivocation parallels that involved in speaking of senses, rather than expressions, as *referring* to things, which was discussed above on page 85, footnote 41.

[18]See also [Klement, 2003, p. 317ff.], [Klement, 2010, p. 32].

The second, radical, Wittgenstein-inspired response would be to deny that thoughts belong to *any* logical type. They are, as it were, off the the type-theoretic map: not only is there no *first-level* function of which thoughts are arguments, there is no function *whatsoever* of which thoughts are arguments. Thoughts (indeed, propositions in general, of which thoughts are some, for Wittgenstein[19]) only ever occur as *bases of operations*, never arguments of functions, functions and operations being fundamentally different.[20] There are thus no concepts applicable to thoughts and can be no sentence in which an expression designating a thought features in the argument place of a predicate (of any level): '[P]ropositions , owing to sense, cannot have predicates or relations' [Wittgenstein, 1979, p. 99]. This Wittgensteinian position undermines the propositional paradox at a yet more basic level than the Russellian one. If a thought is not an argument of *any* concept, then it is, a fortiori, not an argument of any concept it may universalize: thenceforth, the paradox is blocked as before.[21]

I cannot here offer an evaluation of these two possible responses. But note that each appears to impose striking expressive limitations. In Russell's case, limitations are imposed on the potential breadth of our generalizations about thoughts or propositions: no sentence could express a thought generalizing over *absolutely all* thoughts, since any such thought would have to be among the thoughts generalized over, and would thus have to be of a higher order than itself. (We return to the limits of generalization below.) In Wittgenstein's case, the possibility of talking about thoughts at all seems altogether lost: talking *about* thoughts would require that thoughts be the *Bedeutung* of (as opposed to expressed by) some kind of expression; but denying that thoughts belong to any logical type is precisely inconsistent with that.

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[19]'A thought is a senseful proposition.' (TLP 4)

[20]'Operations and functions must not be confused with one another' (TLP 5.25). See particularly the 5.2s of the *Tractatus* for the distinction. That propositions cannot be arguments of functions according to Wittgenstein is perhaps somewhat obscured by the author's retention of the expression 'truth-function'. In fact, what we would call truth-functions (negation, disjunction etc.) are, for Wittgenstein, the paradigm examples of operations [White, 2006, p. 87].

[21]Sullivan's paper [Sullivan, 2000] has convinced me that it is Wittgenstein's denying that there are functions taking propositions as arguments that lies behind his purporting (in TLP 3.333) to resolve Russell's paradox with full generality. One point on which I depart from Sullivan, however, is this. Sullivan argues that 'Wittgenstein's account of the logical constants provides for propositions to form a single logical type' [Sullivan, 2000, p. 190]. It strikes me that the key point here is that Wittgenstein is *not* to be understood as implicitly stratifying propositions in the manner of ramified type theory. But I propose that rather than understanding Wittgenstein as holding that propositions (and so thoughts) belong to one type, we should understand him as denying that propositions belong to *any* logical type, since I consider it preferable to conceive of types as totalities of arguments of functions.

5.3   The Limits of Naming and the Limits of Generality

Setting aside the question of how congenial the diagonal argument is to Frege's views on objecthood more generally, does the argument at least establish that Frege was right to deny that concepts are objects? I believe that it does not establish that—at least, not as Frege intended that denial, as I shall explain below. I want to suggest that a different lesson could be taken from the considerations at the heart of the diagonal argument. I cannot hope to offer a full response to the argument here: the issues it raises are massive and vexed, intertwining as they do with the logico-mathematical and semantic paradoxes. But by adumbrating an alternative moral which can be drawn from diagonalization, and by suggesting certain respects in which drawing that moral may be preferable to drawing the Fregean conclusion that concepts are not objects, I hope at least to indicate that the argument falls short of deciding the matter in favour of the Fregean view.

The lesson of the diagonalization involved in the argument might not be that concepts are not objects, but that *absolutely general* quantification over objects is impossible.

To illustrate, we return to the way in which the diagonal argument may be cast in terms of the threat of Russell's property paradox. We define the Russell property (R for short) as follows:

(R) For any object, o, o falls under R just in case o is a concept which does not fall under o.

If concepts are objects, then R is an object, since it is a concept; in which case, the argument goes, we can instantiate (R) by letting o be R itself, yielding

(R^R) R falls under R just in case R is a concept which does not fall under R.

Since R is a concept, we have

($\bot$) R falls under R just in case R does not fall under R.

The proponent of the diagonal argument takes this for a *reductio* of the supposition that concepts are objects. However, we might instead regard this reasoning as demonstrating that the initial quantifier in (R), 'For any object, o', cannot have ranged *completely unrestrictedly* over objects. R cannot, on pain of contradiction, be among the values of that quantifier's variable, 'o'; the

step from (R) to (R^R) must be invalid. On this count we would agree with the proponent of the diagonal argument. But where s/he infers that R is not an object, we would infer that the quantifier in (R) turns out to have been a *restricted* quantifier over objects: expectations to the contrary notwithstanding, at least one object—namely R itself—cannot have lain within its purview.

It might be objected that it is incoherent to claim of R that it *is* an object and yet that it cannot be used validly to instantiate a universal generalisation beginning 'For any object . . . '. But it needn't be. "Every student passed the logic exam", reports a lecturer. It is perfectly coherent to deny that the lecturer thereby commits herself to the falsehood that Bill, the logic-averse undergraduate on a different continent, has passed the logic exam, though he too is a student. When a restricted universal quantifier is at issue, something's satisfying that quantifier's constituent noun phrase ('student', 'object') is no guarantee that a generalization featuring that quantifier may validly be instantiated with that thing.

There is nothing special about the quantifier used in (R): any first-order universal quantifier, q, might replace it to define a concept which, by the same token, cannot belong to q's domain: viz., the concept of being a concept belonging to q's domain which does not fall under itself. Again, the alternative moral that could be drawn is that q is thereby revealed to be a restricted quantifier over objects: the diagonal concept defined with respect to q is an object, but one over which q itself cannot have ranged. Thus, according to the suggested alternative to the Fregean view, diagonalization betrays the general impossibility of absolutely unrestricted quantification over objects. In denying the possibility of such quantification, the alternative response is a species of the position known as generality-relativism [Williamson, 2003a, p. 416], restrictivism [Button, 2010], or limitavism [Fine, 2006, p. 21]. It is certainly not a novel thought that the strongest grounds for generality-relativism derive from reflection upon Russell's paradox and related antinomies.[22]

The relativist response involves affirming the objecthood of a diagonal concept—a Russell

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[22]Arguments based on Russell's paradox against absolutely unrestricted quantification have recently been advanced, e.g. in [Glanzberg, 2004] and [Fine, 2006]. A particularly general and forceful such argument, developed in terms of interpretations of expressions in a language, is due to Timothy Williamson [Williamson, 2003a, §IV], though Williamson himself is not persuaded of the relativist conclusion.

property—defined in terms of a given quantifier q. The response therefore involves asserting, *contra* Frege, that *some* concept is an object. But is the relativist in a position to go further in their disagreement with Frege and affirm the contrary—in the sense of the traditional square of opposition—of the Fregean doctrine that no concept is an object? That is, are they in a position to claim that *all* concepts are objects? There are some delicate issues here. Firstly, the relativist with aspirations to dissent from Frege's view in this further way confronts a problem. She seems unable, by her own lights, to give full and final expression to the contrary of Frege's view. Suppose the relativist announces 'All concepts are objects'. The quantifier in that universal generalization (suppressed in 'Concepts are objects') is, the relativist will hold, a quantifier over objects: after all, precisely what she aspires to claim is that concepts are some among the objects. However, her own commitments compel her to concede that that quantifier does not range over absolutely all concepts: for the familiar diagonal recipe will locate a concept which, on pain of contradiction, cannot belong to that quantifier's domain. There is some concept— absolutely some, so to speak—over which it does not range. Concerning the (non-)objecthood of any such concept, the relativist's universal generalization must, therefore, remain silent. Thus, in asserting that generalization, what she says is in fact compatible with the *non*-objecthood of the diagonal concept defined with respect to its quantifier. She has therefore, by her own lights, not managed to affirm the genuine contrary of Frege's view; for Frege's view is that absolutely no concept is an object.

There need be nothing stopping the relativist from going on to assert that the abovementioned diagonal concept, omitted from her generalization, is an object too. Indeed, there need be nothing preventing her from consolidating this assertion with her prior generalization in a new universal generalization—say, 'All^{+} concepts are objects'[23]—whose quantifier ranges over everything previously ranged over by that of the old generalization and, additionally, the above-mentioned diagonal concept. But now the diagonal recipe will locate a *further* concept which, on pain of contradiction, must be absent from the domain of the quantifier of this new generalization. We can repeat the process, invoking a yet more expansive universal quantifier; but

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[23]Of course, the new generalization could in fact be homographic with the old, differing only in interpretation.

diagonalization can once more be brought to bear on this quantifier to show its restrictedness too. These considerations display what Dummett calls the *indefinitely extensibility* of the concepts of *object* and *concept* on the relativist view:

> An indefinitely extensible concept is one such that, if we can form a definite conception of a totality all of whose members fall under that concept, we can, by reference to that totality, characterize a larger totality all of whose members fall under it. [Dummett, 1993, p. 441][24]

It appears that, by the lights of the relativist response, the best one could accomplish towards committing oneself to indiscriminately recognising concepts as objects would be to strap oneself in for the long haul and *keep* subsuming the concepts located by diagonalization under one's ever more expansive generalizations. But the haul isn't just long; it's utterly without end. The relativist could really only hope to simulate taking the contrary position to Frege by winning a kind of dialogical war of attrition against any diagonalizing interlocutor: to assert that all concepts are objects and to successively expand that generalization until one's interlocutor grows tired of diagonalizing.

A further issue concerning the possibility of the relativist's affirming the contrary of Frege's view is that there may well be a sense in which even the relativist must concede that not all concepts are objects. Consider the question whether it is the case, according to the relativist, that a quantifier, q, over objects, always comes accompanied by an associated quantifier, Q, which ranges over all concepts applicable to objects in q's domain. A second-order quantifier with the standard semantics based on q's domain would fit Q's description. Suppose that the relativist accepts that this is the case. Q would then range, *inter alia* over the Russell property defined in terms of q, which cannot itself belong to q's domain. In this case, it seems that the relativist must concede, roughly, that there is, in the sense of Q, a concept which is, in the sense of q, no object. She can proceed to introduce a more expansive objectual quantifier q^+, which can coherently be taken to range over everything belonging to q's domain and, additionally, the

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[24]The connection between deeming *object* and *concept* indefinitely extensible and denying the possibility of absolutely general quantification over objects and concepts is undoubtedly intimate, but not entirely straightforward (see [Rayo and Uzquiano, 2006, p. 6]. Dummett himself in fact claims that '[t]here can be no objection to quantifying over all objects falling under some indefinitely extensible concept' [ibid.], but holds that any statement doing so will fail to satisfy the laws of classical logic. See [Shapiro and Wright, 2006, p. 286ff.] for discussion.

Russell property defined in terms of q: q's Russell property then is, in the sense of q^+, an object. But on the present supposition, given q^+, we are *therewith* given a quantifier, Q^+, ranging over all concepts on the domain of q^+, including the Russell property defined in terms of q^+. Thus, the relativist now seems obliged to concede, roughly, that there is, in the sense of Q^+, a concept which is, in the sense of q^+, no object. To appeal again to the temporal metaphor of strapping oneself in for the long haul and making ever more expansive generalizations over objects, it appears that, on the present supposition, at any stage in the haul, the resources of generalization given at that stage are such that the concepts over which one can quantify outstrip those one can recognize as objects.

One option for the relativist would be to reject the above supposition that whenever a quantifier, q, over objects is given, an associated quantifier, Q, is given therewith which ranges over all concepts on q's domain. The relativist might reject the standard semantics for second-order quantifiers, and insist upon a non-standard or "general" semantics, on which the domain of concepts over which those quantifiers range must be determined separately from the first-order domain [Boolos et al., 2007, p. 280-81], [Enderton, 2008, §§2,3]. That separately determined domain might be taken not to include the Russell property defined relative to the first-order domain. The relativist might, in this way, hope to be in a position, at any stage, to recognize as an object any concept over which they then have the resources to quantify. The significant ramifications of pursuing this option are unfortunately not something we can here explore. However, even if this option is not pursued, and the relativist must concede a sense in which not all concepts are objects, it strikes me that she does nonetheless register an anti-Fregeanism that goes beyond claiming that some concept is an object. She does so, I suggest, by persistently taking the following course: when confronted with a concept over which her prior objectual quantifications cannot have ranged, she proceeds, by appeal to a more expansive objectual quantifier, to classify that concept too as an object; at no point, as it were, does the relativist rest content with being unable to recognize a given concept as one among the objects.

This expressive problem confronting an attempt, on the part of the relativist, to fully give voice to the contrary of Frege's view on the objecthood of concepts is related to another, arguably more disquieting one: by the lights of relativism, it appears that relativism itself cannot be fully

articulated. We've formulated (the present species of) relativism as the thesis that absolutely general quantification over objects is impossible. Absolutely general quantification over objects is, if anything, quantification over *absolutely all* objects. So our formulation is equivalent to

(ℜ) It is impossible to quantify over absolutely all objects.

The problem, of course, is that (ℜ) itself features a quantifier, 'absolutely all objects', purporting to accomplish precisely the kind of quantification the possibility of which relativism is meant to deny—namely, quantification over absolutely all objects! If it *does* accomplish that kind of quantification, then (ℜ) itself bears witness to relativism's falsity. So the relativist must deny that it accomplishes that, and consider even 'absolutely all objects' a restricted quantifier over objects. (The presence of the adverb 'absolutely' is alone no guarantee that the quantifier is not restricted [Williamson, 2003b, p. 416].) But in that case (ℜ) doesn't express the relativist's intended claim; for (ℜ) is then to the effect that a certain variety of *restricted* quantification over objects is impossible. Such quantification is not supposed to be ruled out by the relativist's thesis.[25] Absolutely general quantification seems needed in order to state its impossibility, such that relativism is either inexpressible or false.[26]

Concerning this expressive difficulty, the relativist can, after a fashion, claim some partnership in guilt with his opponent (the generality-absolutist [Williamson, 2003a], generalist [Button, 2010], or universalist [Fine, 2006]). After all, absolutely general quantification seems equally needed in order to state its *possibility*; and since the relativist maintains that such quantification cannot be had, he will regard absolutism as equally inexpressible. However, as Williamson argues [Williamson, 2003b, p. 433], the respective circumstances of the relativist and the absolutist are crucially different in this regard. Relativism appears inexpressible *by its own lights*. By the lights of the absolutist, however, absolutism is perfectly expressible; for the kind of quantification required to articulate absolutism is just what the absolutist considers possible. Whereas relativism seems either inexpressible or false, absolutism is apparently either inexpressible or *true*—no dilemma at all for the absolutist.

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[25]The relativist may *also* consider certain species of restricted quantification over objects to be impossible; but the claim at issue is not to that effect.

[26]See [Williamson, 2003b, p. §V] and [Button, 2010, p. §II] for vivid presentations of this foregoing difficulty.

There is no question that this is a disconcerting challenge to relativism. Several authors have argued that the challenge is in fact surmountable and have proposed means of expressing relativism consistently with relativist scruples.[27] We unfortunately cannot review these proposals here. We must take it that at least a *prima facie* case has been made that relativism is inexpressible by its own lights. However, in the present context, this cannot be regarded as decisive against the relativist response to the diagonal argument. For, let us not forget, we equally find in the concept *horse* problems a *prima facie* case that disconcerting expressive limitations are engendered by the Fregean doctrine that concepts are not objects. Indeed we find, in particular, grounds to doubt that that doctrine itself, or its instances, can be expressed in a manner consistent with its truth. And in Frege's case too, recall, the attempt to articulate certain of his semantic and metaphysical views tends to issue in pronouncements that, by Fregean lights, are actually untrue. Both relativism, a thesis about the limits of the potential generality of discourse, and Fregean non-reism, a thesis about the limits of naming, appear inexpressible without transgressing the limits they affirm.

Something of a stand-off results, I think, between the Fregean proponent of the diagonal argument and the relativist who resists it. The situation is well captured by Michael Glanzberg's characterisation of an intimately related dialectical situation:

> Both sides in the standoff have some embarrassments. I have to insist that apparently unrestricted quantification is not really so. My opponent, on the other hand, has to insist that apparently successful nominalization is not so. [Glanzberg, 2004, p. 556]

Frege's view and the relativist view each have unpalatable consequences for certain of our linguistic ambitions. The relativist view implies that the apparently reasonable ambition to articulate a completely all-encompassing generalization about objects will, of necessity, always be frustrated. The Fregean view implies that the apparently reasonable ambition to refer to a

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[27]Fine [Fine, 2006] argues, for example, that the view may given a *modal* formulation, by appeal to a distinctive *postulational* modality. Shaughan Lavine [Lavine, 2006] and others have argued that the view may be given *schematic* expression. A more radical response to the expressibility challenge is defended by Tim Button [Button, 2010], who proposes that restrictivism (relativism) ought not to be conceived as positive *doctrine* at all, but rather as a species of *quietism*: so conceived, the restrictivist refuses to draw a positive conclusion about absolute generality, instead simply issuing a combative challenge to those with ambitions to quantify over "absolutely everything".

concept with a singular term—for example, by nominalization of a predicate—will, of necessity, always be frustrated.

There is reason to think, though, that the relativist enjoys some dialectical advantage in this stand-off. The reason is this: the Fregean view *also* has unpalatable consequences for the potential breadth of our generalizations. It implies that the apparently reasonable ambition to quantify, at once, over objects and concepts will, of necessity, always be frustrated. And yet it is exactly that ambition that one has when one attempts to express the Fregean view itself, as follows: everything is such that if it is a concept, then it is not an object. That 'everything' simply cannot, by Fregean lights, range with the generality necessary for the intended claim to be expressed. The ambition is recurrently manifest in Frege's own exposition. It is particularly conspicuous, for example, in his claim that 'an object is anything that is not a function' [Frege, 1997e, p. 140], in which 'anything' strains—forlornly, he must admit—to encompass objects, concepts, and functions more generally.[28] While Frege may be able to countenance quantification over absolutely all *objects*, he cannot countenance even restricted simultaneous quantification over objects *and* concepts. *A fortiori*, he cannot countenance type-transcendent quantification over *absolutely everything* in his ontology—as we might put it, quantification over all *entities* whatsoever. This gives rise to an expressive problem very similar to that confronting the relativist response: even to express the thesis that it is impossible to quantify unrestrictedly over all entities appears to require the use of just such quantification.

The relativist response, on the other hand, *can* allow for the possibility that what appears to be simultaneous quantification over concepts and objects—for example, in the course of articulating general principles in semantics—really is such; for the proposal is to recognize concepts as some among the objects. Such quantification will never range absolutely unrestrictedly over concepts and non-concept-objects, but it can embrace both, and its generality can be extended indefinitely.

The relativist view, then, involves denying that we can achieve a kind of generality in what

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[28]A less conspicuous example among many: concerning the sentence 'Jupiter is larger than Mars' Frege generalizes both over the objects introduced by the proper names 'Jupiter' and 'Mars' and over the relation introduced by the predicate 'is larger than', to claim that all belong to the 'realm of meanings [*Bedeutungen*]' [Frege, 1979c, p. 193].

we say which we *seem* able to achieve and which appears necessary, inter alia, to give expression to that view. However, the Fregean view *also* involves denying that we can achieve a kind of generality in what we say which we *seem* able to achieve and which appears necessary, inter alia, to give expression to that view. Moreover, the Fregean view involves, *in addition*, denying that we can achieve a certain kind of *reference* which we seem able to achieve: reference to a concept with a singular term. This denial generates disconcerting expressive difficulties in its own right.

In my judgement, this comparison gives provisional grounds for favouring the relativist position over the Fregean view; for, on the face of it, relativism does less violence to the expressive and referential powers of language.

I stress that these are only provisional grounds. Further investigation might reveal that the expressive impediments facing the Fregean view are more tractable, or less disquieting, than those facing relativism. We cannot undertake that investigation here. I do hope to have indicated, however, that an alternative view to Frege's may be proffered as the lesson of the considerations central to the diagonal argument; that, at first blush, that view compares favourably with Frege's; and thus that the diagonal argument ought not to be regarded as deciding matters to Frege's advantage.

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