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How does a tautology say nothing?¹

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Introduction

In Wittgenstein's wartime *Notebooks* logic is still very much 'in the melting pot'.² Nonetheless, one lump of nascent doctrine does seem to have appeared in the melt. This is the idea that tautologies 'say nothing' (NB 8, October 3, 1914).³ Having settled upon this idea, Wittgenstein turns to considering *how* tautologies might accomplish their muteness. On the 5th of December 1914, he asks: "How does " $p \vee \sim p$ " say nothing?" (NB 35).⁴ Since here ' $p \vee \sim p$ ' is an arbitrarily chosen expression for the unique tautological 'truth condition' the question he means to be asking is general: "How does *any* expression of the tautological truth condition manage to say

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² Letter to Russell, 22nd June 1912 (NB 120).

³ The idea is a settled position by October 1914. It is stated in slightly different terms at the opening of the *Moore Notes* of April 1914 (NB 108) and repeated in the *Notebooks* entry for October 6, 1915 (NB 58). It survives into the *Tractatus* at 5.142 and 5.513.

⁴ In my translations of remarks from the *Notebooks* I will mainly, but not always follow, Anscombe.

nothing?”.⁵ Exactly six months later, still worrying away at the problem, he poses the sister question: “Why does tautology say nothing?” (NB 55). And by ‘tautology’ in this context he plausibly means any instance of a valid schema of first-order logic without identity.

As we will eventually see, each of these questions seeks an answer that is metaphorically modelled on a certain kind of causal explanation. Wittgenstein, the philosopher-engineer, wants to know how a tautology *works*. Comparison with the causal case suggests that little of importance is likely to turn on the difference between the ‘how-’ and ‘why-’ formulations. After all, if we confine ourselves to efficient causation, it is a matter of indifference whether one asks *how* the sun warms the atmosphere or *why* the atmosphere is warmed by the sun. Wittgenstein’s question, however, is more akin to a request for an explanation of a mechanism than to a request for an account of an event. He wants to know what *kind* of expression a tautology is such that it should say nothing. (Compare: ‘What kind of mechanism is an Edwardian watch such that it should keep time?’).

Wittgenstein’s initial answer in the *Notebooks*, recorded in an entry that predates by a month his first recorded posing of the question, is that a tautology says nothing because it is a part of the language that brings about a cancellation of truth conditions or ‘presenting relations’ (NB 24, November 2nd, 1914).⁶ This answer is, I take it, the deepest answer that Wittgenstein can think of at this stage; for a later entry makes clear that he envisages a chain of answers, apparently of increasing depth, of which the present answer is not the most superficial member: “Why does

⁵ My talk of the ‘tautological truth condition’, while in agreement with 4.46, is also in *prima facie* in tension with 4.461. I address this apparent tension in section 5.

⁶ Wittgenstein was in the habit of re-posing questions to which he’d already offered tentative answers. His question, though unstated there, seems to have been already contemplated in the *Moore Notes* where he had offered a tentative – and rather unclear – answer, namely that the simple parts of a tautology “paralyse” or “destroy” one another (NB 118).

tautology say nothing? Because every possibility is admitted in it in advance; because ...” (NB 55, entry for May 6th, 1915).

I will argue that Wittgenstein’s ‘cancellation of truth-conditions’ answer is subsequently refined to the claim that a tautology says nothing (or equivalently is ‘without sense’ [*sinnlos*]) because it expresses an equilibrium of opposed senses. This claim is not made explicitly but emerges from the metaphors to which Wittgenstein resorts in trying to convey his conceptions—or so, at least, I will argue.

At any rate, I will argue that Wittgenstein’s view in the period from the *Prototractatus* to the *Tractatus* is that a tautology is without sense not because it contains a word that lacks meaning but rather because it combines opposed senses that cancel out. It thus resembles less a broken electrical circuit than an intact circuit in which the opposed potentials, being in equilibrium, cancel out. As we will see, Wittgenstein explains the idea of a cancellation of senses both by means of a certain electrical metaphor and through a certain analogy borrowed from applied mathematics: he implicitly compares what a tautology expresses to a *null* value of a vector quantity. The cancellation involved in a tautology is thus conceived of on the model of a cancellation of equal and opposite vectors or -- staying with the causal-explanatory metaphor -- physical vector-quantities. In virtue of effecting a cancellation of senses, a tautology, on Wittgenstein’s conception, winds up possessing a zero quantity of ‘what is said’—in other words, a zero quantity of sense.

The idea that a tautology possesses a zero quantity of sense is not spelled out in the *Tractatus*, but Wittgenstein does later imply that he had once been guided by this conception. “When I called tautologies ‘senseless’”, he says in lectures delivered at Cambridge in the 1930s,

“I meant to stress a connection with a quantity of sense, namely 0.”⁷ In other words, Wittgenstein chose to call tautologies ‘senseless’ [*sinnlos*] precisely because he wanted to convey – presumably through the privative suffix ‘-less’ (-los) -- the idea of possessing a zero quantity of sense (compare the way that ‘motionless’ expresses the possession of a zero quantity of motion).

I will argue that Wittgenstein’s report in the 1930s of his earlier view is a correct characterization of his view in the *Tractatus*. According to the Tractarian Wittgenstein, the way in which a tautology says nothing is indeed by possessing a zero quantity of sense. This idea, I shall claim, is in fact central to the *Tractatus*’s conception of logic, and the term ‘*sinnlos*’ is an apt way of expressing it. I will further argue that although the vector (or ‘arrow’) analogy, which is first suggested in the *Notes on Logic*, is unquestionably present in the *Tractatus*, it jostles there with another guiding metaphor, namely: that of a degenerate conic section. Both pictures shape Wittgenstein’s understanding of tautology but without ever, I think, being fully articulated or completely reconciled.

1 The *Notebooks* on the “amount-of-what-is-said”

Wittgenstein sometimes puts the idea that a tautology possesses a zero quantity of sense by saying or implying that it has a zero quantity of “what-is-said”. For example, in the *Notebooks* entry for the 2nd of June 1915, he says:

[In order to bring out that the proposition *must* have two poles I would have to] find an expression in the language [of my theory] for HOW MUCH *a proposition says*. And this would have to yield the result that tautologies say NOTHING [*NICHTS*].

But how can we find this measure of amount-of-what-is said [*Maß Vielsagendheit*]? (NB 53–54)

⁷ *Wittgenstein’s Lectures Cambridge, 1930–35* (from the notes of Alice Ambrose and Margaret MacDonald), Alice Ambrose (ed.), Midway reprint, Chicago: University of Chicago Press, 1989, 137.

Eight days later, ‘NOTHING [*NICHTS*]’ is replaced in the German by ‘zero’:

Here it is clear that $p \vee \sim p$ plays the role of a true proposition, which however says nought [*zero*].
So we have again arrived at the quantity of what is said [*Quantität des Sagens*]. (10th, June, 1915, NB 58, emphasis in the original)

Tautologies, then, cohabit with senseful propositions on a scale of how much is said. They lie at the silent extreme of this scale because they express a zero quantity of what-is-said. However, in contrast to nonsense, they are at least *in the running* for the possession of a non-zero quantity of what-is-said. They are at least on the scale.

Wittgenstein’s talk of having arrived “again” at the quantity of what is said is a nod to the entry for the 3rd of June, where he had already been considering how to specify a rule for ordering propositions according to how much they say. Reversing the expected order of ‘p’ and ‘q’, he says: “But doesn’t it work like this: If p follows from q but not q from p, then q says more than p?”⁸ In other words, what counts in favour of one proposition’s saying more than another is unrequited logical entailment. On this picture, since contradictions entail every proposition while tautologies entail only tautologies, the former must say more than the latter. Indeed, on this conception “contradiction would have to say more than all other propositions” (*ibid.*). Such, at least, is Wittgenstein’s thinking at this stage; and, as he recognizes, it provides a reason to doubt that tautology and contradiction could both be zero-points on the scale of propositions:

⁸ NB 54. This rule survives into the *Tractatus* as 5.14: “If one proposition follows from another, then the latter says more than the former, the former less than the later.”. However, the tension between this claim and his claim that tautology *and contradiction* each say nothing (4.461) goes unacknowledged.

We cannot say that both tautology and contradiction say *nothing* in the sense that they are both, say, zero points [*Nullpunkte*] on the scale of propositions. For at least they are *opposite* poles. (May 3rd, 1915, NB 45)

If a tautology says nothing, then a contradiction would surely have to say a great deal, indeed, it would have to say more than any other proposition. But, despite registering this (entirely correct) point in the *Notebooks*, the Wittgenstein of the *Tractatus* does treat contradiction as on a par with tautology in also saying nothing (4.461), and this certainly looks like treating them both as *Nullpunkte* on the scale. Why does the *Tractatus* set aside the *Notebooks*'s well-considered reservations? The answer, it seems, is that Wittgenstein had persuaded himself of the “two *Nullpunkte*” view by means of the following bad argument:

If a proposition saying a great deal is false, it ought to be interesting that it is false. It is disconcerting [*befremdend*] that the negative [*das Negativ*] of a proposition that says a great deal should say absolutely nothing. (June 11th, 1915, NB 58–9)

Since he holds firmly that a tautology says nothing, this reasoning persuades Wittgenstein that a contradiction cannot, after all, say a great deal.

The problem with the argument is that its crucial premise—namely, the assumption that the negation of a proposition that says a great deal must itself say a great deal—is simply false. If I were to say that it is not raining and not hailing and not cloudy and not snowing and not sleeting, then I would have said a good deal about the weather. But if I were to assert the negation of this claim, I would have said very little. What Wittgenstein finds disconcerting, therefore, is simply not disconcerting.

What might have prompted such an apparently crude mistake? One likely possibility, I think, is that Wittgenstein was seduced by a certain picture of the proposition—one already in

place in the 1913 *Notes on Logic*. There he had remarked: “Names are points, propositions arrows—they have *sense*” (NB 101). This remark—as every scholar of the *Tractatus* knows—involves a pun on ‘sense’. This term has the twin connotations of ‘meaning’ and ‘direction’. The pun is stronger in German than in English, but it exists in both languages and in English it is especially pronounced in mathematical contexts, where ‘sense’ is used both for direction and—more technically—for a component or determinant of direction. The latter, less familiar, usage is described in the following entry from the *Oxford English Dictionary* (2nd edition):

Sense, *sb.* 29b. Chiefly *Math.* Either of the two possible directions with respect to which certain quantities or objects can be defined; *esp.* either of the two directions along an orientated line a vector can take, as determined by which of the vector’s end points is its head. *The direction of a vector is determined by both its sense and orientation.* (emphasis added).

The entry goes on to illustrate this idea with an example from one Lieutenant-Colonel H. W. L. Hime, a mathematician and artillery officer (the author of *Gunpowder and Ammunition: Their Origin and Progress*). In his less explosive work, *The Outlines of Quaternions* (1894), the good Colonel writes:

No two vectors are equal unless they have, first equal lengths, and secondly, similar directions—the phrase ‘similar directions’ meaning ‘parallel directions with the same sense’.⁹

Hime’s remark contains a three-fold distinction between: (a) a vector’s *length*; (b) the *orientation* of the line forming its ‘tail’; and (c) its *sense* (its ‘direction’ along the orientated line—from tail to

⁹ Hime H.W.L. (1894), *The Outlines of Quaternions*, London: Longmans, 2.

head). To grasp the distinction between (b) and (c) it suffices to note that parallel vectors which point in opposite directions have the same orientation but opposite senses.¹⁰

The metaphorical comparison between a sense and a direction made in the *Notes on Logic* is made again in the *Tractatus*, where, replacing a metaphor with a simile, Wittgenstein repeats the claim from the *Notes on Logic* almost word for word. “Names”, he says, “resemble points; propositions resemble arrows [*Pfeilen*], they have sense” (3.144). It is possible that the ‘arrows’ mentioned here are, more specifically, *vectors*; for in Wittgenstein’s idiolect one meaning of ‘*Pfeil*’ is ‘vector’.¹¹

Wittgenstein’s attachment to this comparison suggests a possible explanation of why he should have mistakenly assumed that the negation of a proposition that says a great deal must itself say a great deal. The idea would be that in the summer of 1915 he had been, first, conceiving of propositions on the model of vectors and, second, thinking of negation as analogous to the operation of reversing a vector’s direction while leaving its magnitude (the length of the arrow) unchanged.¹² If he had further conceived of the magnitude of a vector as analogous to the amount that is said by a proposition, it would have been natural for him to think of the negation operation as leaving the amount said by a proposition unchanged. This idea might have led Wittgenstein to the mistaken idea that tautology and contradiction say the same amount. And since he was already wedded to the idea that tautology says nothing it would have then led to the compounding mistake of supposing that a contradiction says nothing. This aspect of the vector or ‘arrow’ analogy is

¹⁰ The alert reader will have noticed that the notion of orientation identified in (b) is the same as the notion of direction contextually defined by Frege in section 65 of *The Foundations of Arithmetic*.

¹¹ See, for example, *Remarks on the Foundations of Mathematics*, 337.

¹² It is unclear whether the vector’s magnitude is better thought of as analogous to *how much* is said by the proposition or to *what* is said by the *unnegated* proposition (as is perhaps suggested by 4.0621). But this detail is immaterial to the present point because how much is said supervenes on what is said.

therefore deeply unfortunate, but, as we will see, the analogy partially redeems itself through its helpfulness elsewhere.

2 The ‘cancellation of truth conditions’ as a summing to zero of senses

By the November of 1914, Wittgenstein had arrived at the view that a tautology says nothing because it expresses the result of a cancellation of truth conditions or ‘presenting relations’. At this stage, however, he had not yet explicitly combined this idea with the vector analogy. “In the tautology”, he says,

the conditions of agreement with the world (the truth conditions)—the presenting relations [*die darstellenden Beziehungen*]*—cancel one another out [heben auf], so that it does not stand in any presenting [darstellenden] relation to reality (says nothing).*¹³

Once the idea that tautologies express a cancellation of truth conditions had got a foothold, a natural next step—at least for a thinker (a) open to the vector analogy (and its attendant pun) and (b) disinclined to distinguish between senses and truth conditions—would have been to suppose that the cancellation in question might be thought of on the model of a balancing of oppositely directed senses. And the next step after that—a small one for a thinker already persuaded, as Wittgenstein was, that a tautology “says *zero*”—would have been to think of these balanced, vector-like senses as *summing to zero*. Although in the *Notebooks* everything was in place for Wittgenstein to have taken these steps, he seems not to have taken them—or at least not to have registered them in a surviving document—until the *Prototractatus*, where, as we will see in more

¹³ November 2nd, 1914, NB 24; compare *Tractatus* 4.462.

detail shortly, the ideas in question feature in his discussion of a ‘zero method’ (*PT* 6.1211 and *PT* 6.1212). The same ideas are, I think, also obliquely present in the *Tractatus*’s famous comparison:

Tautology and contradiction are without sense [*sinnlos*].
(Like the point from which two arrows go out in opposite directions.) (4.461)

If we suppose that Wittgenstein had been thinking of these ‘arrows’ as linear vectors, then the geometric ‘point’ from which the arrows ‘go out’—a point that is directionless, hence within the analogy without ‘without sense’—would be the *null* vector.¹⁴

The analogy between the sense of a proposition and a vector—or, more precisely, between the former and the *direction* of a vector—is also present, albeit less conspicuously, at *Tractatus* 6.121, where Wittgenstein develops the *Prototractatus*’s idea that the method of logic is a zero-method. “The propositions of logic”, he says,

demonstrate the logical properties of propositions, by combining them into propositions which say nothing. This method could be called a zero-method [*Null-methode*]. In a logical proposition propositions are brought into equilibrium with one another [*ins Gleichgewicht gebracht*], and the state of equilibrium then shows how these propositions must be logically constructed. (6.121)

Wittgenstein’s explanation of this remark to Ogden makes clear that the example tacitly involves an appeal to vectors:

“Null-methode” in German is an expression used in physics; when—for instance—you measure an electric resistance by regulating another resistance until the galvanometer points to 0 again we call this a “Nullmethode”. Now there is sure to be an English word for it; but is it “null-method”? [Perhaps it] should rather be “zero-method” but I don’t know.¹⁵

¹⁴ The morpheme ‘*sinn*’ is often used for rotational direction. ‘*Urzeigersinn*’, for example, refers to the rotational direction of clock hands. But, as Wittgenstein was aware, ‘*sinn*’ is also used for linear direction (see, for example, the *Notebooks* entry for October 12, 1916 (NB, 84)).

¹⁵ *Letters to C. K. Ogden*, 34. Wittgenstein is right that in English the method of taking null measurements is called the ‘null-method’. It is mildly regrettable, therefore, that in the end Ogden opted for ‘zero-method’.

When taken together, the remark and its explanation suggest that Wittgenstein is comparing propositional sense specifically to a vector-*quantity*, namely, electric field.

The vector analogy is also arguably discernible in the *Tractatus*'s claim that negation [*Verneinung*] (which Ogden renders 'denial') "reverses the sense of a proposition" (5.2341).¹⁶ This remark poses a *prima facie* difficulty for Wittgenstein's conception of tautology and contradiction as analogues of the null vector. For he conceives of the null vector as analogous to a geometrical point, hence as something apparently directionless. But since he also conceives of tautology and contradiction as capable of being negated the question arises: how can something supposedly directionless be *reversed*?

One way out of this difficulty would be to conceive of the null vector not as directionless but rather as possessing *every* direction.¹⁷ Although this is not Wittgenstein's declared position, it does seem to be Tractarian in spirit. For when incorporated into the analogy it yields the satisfactory result that negation operates on a *sinnlos* contradiction to yield a *sinnlos* tautology (and *vice versa*). Or rather, it yields this result so long as we think of reversing an omnidirectional vector as a matter of reversing each of its directions. For then the operation of reversal takes the omnidirectional null vector to itself.

At any rate, the tendency of these reflections is that the vector analogy survives robustly, if not all that overtly, into the *Tractatus*. This point, however, is not uncontroversial. Michael

¹⁶ Compare the related thought at 5.253. Also compare Hime's explanation of the function of the minus sign in his notation: "The minus sign reverses the direction of a vector" (*Outlines*, 3). The context makes clear that the notion of 'direction' involved here is the one corresponding to Hime's talk of 'sense'—our (c). See page 2 of *Outlines*. (To ward off possible misunderstanding, I should emphasize that I am *not* claiming that Wittgenstein was influenced by Hime.)

¹⁷ I am indebted to Josh Dever for this suggestion.

Potter, for example, presents the vector analogy—or at least the ‘arrow-direction’ analogy—as being on the way out in *Tractatus*. He says:

[While in the *Notes on Logic* the analogy that guides Wittgenstein’s account of propositional expression is that of (something like) a compass needle, in the *Tractatus*] that analogy is almost wholly absent. Although propositions still have sense, the punning suggestion that this is to be thought of as like the direction of an arrow has been all but dropped.¹⁸

Potter is certainly right that the specific idea of a compass needle does not figure in the *Tractatus*, but it does not seem correct to say that the punning suggestion that sense is to be thought of as like the direction of an arrow has been ‘all but dropped’. Indeed, that suggestion seems only to grow in prominence in the run-up to the *Tractatus*. For one thing, the ‘zero-method’ passage (TLP 6.121), which suggests the idea of an equilibrium of opposed, hence oppositely directed, senses, is absent from the earliest pre-*Tractatus* manuscripts and appears for the first time in the *Prototractatus* (PT 6.1212). For another, the overtly punning remark: “Denial reverses the sense of a proposition” (TLP 5.2341) occurs in neither the *Notebooks* nor the *Prototractatus*. Though it does pick up on a *thought* first expressed in the *Notes on Logic*, it is a relatively late remark, having been added to the *Bodleianus* manuscript only after the *Prototractatus* had been completed.¹⁹ In truth, then, the ‘arrow-direction’ analogy and its attendant pun on ‘*Sinn*’ are waxing rather than waning in the run-up to the *Tractatus*.

¹⁸ Potter 2009, 226.

¹⁹ The thought in the *Notes on Logic* is that ‘p’ has an opposite sense to ‘not-p’ (*Notes on Logic*, NB 95; cf. TLP 4.0621). The remark “Denial reverses ... etc.” occurs on page 113 of the *Bodleianus* manuscript and so after the phase of that manuscript, subsequently dubbed the ‘*Prototractatus*’, which ends on page 103. For this last point, and for an explanation of Brian McGuinness’s term, ‘*Bodleianus*’, see Michael Potter, “Wittgenstein’s pre-*Tractatus* manuscripts: a new appraisal”, in P. Sullivan and M. Potter, eds., *Wittgenstein’s Tractatus: History and Interpretation*, Oxford University Press, 2013, 13–39.

I have suggested that tautologies, for Wittgenstein, sit on the scale of sense-possessing items and so ‘have sense’, albeit to a zero degree. In the light of Wittgenstein’s apparent endorsement of Ogden’s translation of ‘*sinnlos*’ as ‘without sense’, such an idea might at first seem implausible. However, if we attend to historical precedent—and, in particular, to a certain idea of Kant—this impression can be softened. The idea I have in mind relates to Kant’s distinction between ‘qualitative’ and ‘quantitative’ negation—a distinction that might well have filtered down to Wittgenstein through the tradition, though by which route is unclear.

As Ralf Bader has observed, Kant conceives of a body’s state of *rest* as a particular state of *motion*. It is that state of motion in which the quantity of motion is zero.²⁰ In Kant’s terminology *rest* is the “quantitative negation” of *motion* (Reflection 5816; compare Reflection 5815). A body at rest has a *quantitative* lack. It has motion but has it to a zero degree. The *qualitative* negation of motion, by contrast, is the lack of motion enjoyed by something which by its very nature cannot be in motion. The number two, for example, is unmoving but not at rest; and in claiming that it is unmoving I am qualitatively negating motion in relation to the number two. According to this same conception, when I describe a body that is at rest as ‘motionless’, the privative suffix ‘-less’ expresses quantitative negation. As we have noted, Wittgenstein seems to be implying in his Cambridge lectures that he chose to call tautologies ‘*sinnlos*’ because he envisaged a parallel role for the privative suffix ‘-los’. If that is right, then when he calls a linguistic string ‘*unsinnig*’ in the *Tractatus* he might be thought of as expressing qualitative negation; and when he calls it ‘*sinnlos*’ he might be thought of as expressing quantitative negation—though it would be too strong to suppose that he had Kant’s distinction consciously in mind.²¹

²⁰ Bader, “Kant and the Table of Nothing”, 15.

²¹ Would the author of the *Tractatus* have been familiar with this idea from Kant? It is hard to say. The idea appears in the *Metaphysical Foundations of Natural Science* (4: 486), so it is not hidden

As Bader notes, because Kant conceives of the state of rest as a null state of motion he takes it to be: (a) governed by the laws of motion; (b) mathematically constructible; and (c) combinable with other motions. We may say, similarly, that because Wittgenstein conceives of *Sinnlosigkeit* as a null quantity of sense (a null state of sensefulness) he takes it to be: (a) governed by the laws of logic; (b) capable of being constructed in the *TF*-notation; and (c) combinable with other senses.²² These ideas are, I think, part of what Wittgenstein means when he says that tautology and contradiction belong to the symbolism in a way similar to the way in which ‘0’ belongs to the symbolism of arithmetic (4.4611; compare *Notebooks* entry for June 10th, 1915, NB 58).²³

A clue that the *Tractatus* is treating tautology as possessing of a zero quantity of sense is provided by Wittgenstein’s conventions governing capitalization. When he claims that the nonsense-string ‘Socrates is identical’ says nothing because we’ve given no adjectival meaning to ‘identical’ (5.4733) the word ‘*nichts*’ is not capitalized. When, however, he says that tautology—or a proposition of logic—says nothing (5.142, 5.43, 6.11) the same word is usually capitalized.²⁴ This suggests that for Wittgenstein the presence or absence of capitalization serves to mark – or comes to do so – two different ways in which a claim says nothing, and it arguably suggests that

in some obscure corner of Kant’s corpus; but I know of no direct evidence that the early Wittgenstein read this work.

²² For Wittgenstein, however, there are no *laws* of logic in the usual sense; so claim (a) needs to be reformulated. Suitably refined, it becomes the claim that we can generate substitution instances of logically valid schemata by uniformly substituting tautologies for one or more of the sentence letters occurring in these schemata. Only in this attenuated sense is *Sinnlosigkeit* subject to the laws of logic.

²³ In thus glossing 4.4611 I am guided by the German, which suggests that in the two cases the mode of belonging is *similar* [‘*ähnlich wie*’], rather than by Ogden’s translation, which suggests it is identical.

²⁴ An exception to this rule occurs at 4.461, where the expected capitalization is absent. At 5.513 and 6.121 the capitalization is again absent, but in these cases this is attributable merely to the fact that in these remarks ‘*nicht*’ forms part of the compound verb ‘*nichtssagen*’.

the capitalized ‘*Nichts*’, which would naturally be construed as a term roughly equivalent to the English word ‘naught’, is being treated as the name of a zero quantity of sense.²⁵

3 A Note on the Ramsey-Ogden-Wittgenstein translation

These reflections suggest that it would be more accurate to render ‘*sinnlos*’ as ‘senseless’ than ‘without sense’. After all, the fact that the English adjective ‘senseless’ shares a privative suffix with its German cousin makes the former peculiarly apt for expressing the property of possessing a zero quantity of sense. And Ogden, who was working closely with Wittgenstein on the translation of the *Tractatus*—and more distantly with Ramsey, does render ‘*sinnlos*’ as ‘senseless’ a couple of times (5.132 and 5.1362). Unfortunately, however, he fails to do so uniformly. And, in particular, this is not the translation he offers when translating the thesis, expressed at 4.461, that tautology and contradiction are *sinnlos*. At this crucial juncture he instead offers ‘without sense’.

Since that is so, a question arises for my line of interpretation. If I’m right, why should Ogden have failed to exploit the relevant parallel between the two languages? The answer, I think, is just that a certain feature of English blocks the exploitation. This is the fact that ‘senseless’, while it *can* mean ‘lacking in sense’, can also mean ‘unmeaning’ and even ‘foolish’—and it often means a blend of the two. But ‘unmeaning’, being close to ‘nonsensical’, is precisely *not* what is intended by ‘*sinnlos*’ in the *Tractatus* (compare 4.461). Plausibly, then, it is in order to avoid this unwanted connotation that Ogden opts for ‘without sense’ in preference to ‘senseless’ at 4.461.²⁶

²⁵ Compare: ‘It came to naught’. As we have already seen, in the *Notebooks* entry for the 2nd of June 1915, when wondering how to express the idea that tautology possess a zero amount of what-is-said, Wittgenstein gets the same effect by setting ‘NICHTS’ in capitals (NB 54).

²⁶ Strong evidence that Ogden is aware that ‘senseless’ can mean ‘unmeaning’—hence something close to ‘nonsense’—is provided by the fact that he translates ‘*unsinnig(keit)*’ as ‘senseless(ness)’

A question also arises about Ogden's translation of '*unsinnig*'. He usually favors 'senseless' over 'nonsense' (or 'nonsensical') as a translation of this word. Why so? The answer, I think, is that 'nonsense' is itself awkwardly ambiguous. On the one hand, it *can* mean 'gibberish', 'unmeaning', or 'meaningless'; but, on the other, it can also mean 'absurd' or 'plainly false'. (Witness the mildly pompous phrase: "What utter nonsense!", which is roughly equivalent to the archaic: "Balderdash!". Since these terms and phrases express extreme incredulity they convey perceived or imagined falsehood rather than the recognition of vacuity of sense.) Since this usage of 'nonsense' is not uncommon, Ogden had reason to choose another word to translate '*unsinnig*'. He seems to have plumped for 'senseless' in the hope that the reader would hear its connotation of 'unmeaning'.²⁷ However, once it had been assigned the job of translating '*unsinnig*', the word 'senseless' could hardly *also* be given the job of signalling the quantitative negation of 'having sense'. It is for this reason too, I think, that Ogden's translation leaves an otherwise helpful connection between 'less' and '*los*' on the cutting-room floor.

4 'Tautology' in the *Tractatus*

According to traditional, non-philosophical usage, a 'tautology' is a statement that repeats a statement, word, or phrase. Its extension includes statements that repeat an attribution, sometimes through the piling up of adjectives, and without necessarily employing the same words to express the repetition. One amusing example is the tongue-in-cheek remark of Sir W. B. Brett: "[T]he

no fewer than ten times in the *Tractatus*. See 4.003, 4.124, 4.1272 (four occurrences), 4.12724, 5.473, 6.51, and 6.54.

²⁷ It is notable that on those occasions when Ogden does render '*unsinn (unsinnige)*' as 'nonsense (nonsensical)' (namely, 3.24, 4.4611, 5.5303, 5.5351, 5.5422, 5.5571) the 'falsehood' reading of 'nonsense' is—if we assume charitable interpretation—for one reason or another, unavailable.

legislature intended in this case to be verbose and tautologous, and to say the same thing twice”.²⁸ Obviously, the *Tractatus* employs the term ‘tautology’ with a meaning strikingly different from this traditional one. It explains a tautology as a proposition that is true for all truth possibilities, where a truth possibility is an assignment of truth or falsity to each of the elementary propositions (4.46).

The fact that Wittgenstein bestows a wholly new meaning on an old term has been astutely noted by Burton Dreben and Juliet Floyd, who imply – with considerable plausibility -- that because his innovation was picked up by lexicographers Wittgenstein did nothing less than change the English language.²⁹ An attempt to change, if not the German language, then at least the philosophical lexicon had previously been made by Kant, who in his lectures on logic had tried to narrow the extension of the concept *tautology* by demanding that the repetition in question should involve the use of the very same words rather than mere synonyms.³⁰ But Kant’s proposal failed to catch on, and was, in any case, less radical than Wittgenstein’s. Whereas Kant had tried to shrink the extension of the concept *tautology* to a certain subset of traditional tautologies—namely, those possessing a privileged syntax, Wittgenstein offered a semantic characterization that deemed some traditional tautologies non-tautological (e.g., instances of ‘p and p’, where ‘p’ is contingent), while also counting some traditional non-tautologies tautological (e.g., instances of ‘p or not-p’).

One detail that is seldom noted about Wittgenstein’s use of ‘tautology’ is that in his writings as a whole—though not in the *Tractatus*—he uses the term in more than one way. In the *Notebooks* entry for the 2nd of November, 1914, for example, he says: “a=a is not a tautology in

²⁸ *OED* (2nd edition) entry on ‘tautologous’. The quip stems from 1884.

²⁹ See Dreben and Floyd 1991.

³⁰ For Kant, a tautology is an *explicitly* identical proposition. The standard example is the quantifier-deficient ‘Man is man’, but explicitly quantified claims of the form ‘All *F*s are *F*s’ are also tautologies in Kant’s sense (Kant 1900, 9: 111).

the same sense as $p \supset p$ " (NB 24). The sense in which the sentence 'a=a' is a tautology is, I think, approximately the traditional one: it is a statement involving *some* kind of repetition. We find a more radical stretching of this traditional usage in the *Moore Notes*, where Wittgenstein describes the purported proposition 'M is a thing' as "superfluous (tautologous)". For him this claim is—in aspiration at least—tautologous because it already "shows" what it attempts to say and thus effects—or at least tries to effect—a certain (rather *recherché*) kind of repetition. Of course, 'a=a' might be construed as a tautology in *Kant's* sense too, but I know of no evidence that Wittgenstein was familiar with the Kantian usage.

Wittgenstein is using the term 'tautology' in his own innovative and technical sense when he advances the thesis that tautology says nothing (5.142). *Verbally*, this thesis is not original to him: it already occurs in the first *Critique* (A 597/B 625). But since Kant uses 'tautology' with a different meaning from Wittgenstein, Kant's thesis is no scoop.

5 Tautologies as extreme cases of truth conditions

In the *Tractatus*, Wittgenstein describes tautology as one of two "extreme cases [*extreme Fälle*]" of truth conditions (the other being contradiction) (4.46). He means that tautology is the final member of a *series* of progressively less stringent truth conditions (compare 4.45). Tautology is a 'condition' so lax that it demands of reality nothing at all. Since it makes no demands on reality, a tautology fails to represent the world as being one way in contrast to another. And it is presumably for this reason that Wittgenstein denies that it is a picture of reality (compare 4.462).³¹ But if a tautology doesn't depict reality as thus-and-so, why should we think of it as *true*? This

³¹ The stated rationale for denying that a tautology pictures reality is that in *allowing* every possible state of affairs [*Sachlage*], the tautology *presents* [*darstellen*] none (compare 4.462).

question, already natural, is rendered urgent by a remark in the *Tractatus*: “A proposition can be true or false only in virtue of being a picture of reality” (4.06). This remark has suggested to some commentators that—in spite of Wittgenstein’s saying the opposite at 4.461 and 4.464—a tautology is not after all the kind of symbol that can be true.³² However, when we put it under the microscope, 4.06 is not as clear-cut as it might seem. For Wittgenstein uses ‘proposition’ in narrower and broader senses (I expand on this point below). In the present context he may merely mean that a proposition *in the narrow sense*—what in the *Tractatus* he calls a ‘significant [*sinnvollen*] proposition’ (3.13), and the *Moore Notes* ‘a proposition proper’ (NB 108) —can be true or false only in virtue of being a picture of reality. Such a reading is in fact suggested by the *Notebooks*’s precursor to 4.06:

Only in this way can *the proposition* be true or false: It can agree or disagree with reality only by being *a picture* of a state of affairs [*Sachverhaltes*]. (NB 8, October 2nd, 1914, emphasis in the original)

Here the emphasis on ‘the proposition’ is plausibly a clue that the Tractarian remark it prefigures – viz., 4.06 – is concerned exclusively with propositions proper, as opposed to tautologies, contradictions, or identity claims (compare the entries for November 11th, 1914 (NB 28) and June 2nd, 1915 (NB 53)). And propositions proper would seem to be contingent propositions (using ‘proposition’ now in the broad sense). Both at 4.06 and in the corresponding passage in the *Notebooks* we are told that a proposition proper can be true or false only by being a picture of reality. The question of whether and how propositions in the loose sense can be true is left open. Wittgenstein does think that tautologies are true, so in virtue of what are *they* true? One answer runs as follows. Since tautology belongs to a series of progressively less stringent truth conditions

³² See for example Fogelin 1987, 45.

as its least stringent member it inherits the truth-aptness of the other members of the series. (Contradiction also belongs to this series, but it lies at the opposite extreme.) And since the propositions (proper) in the series are (roughly speaking) true on increasingly many truth possibilities as one moves toward the extreme at which tautology lies, there is reason to think that it too is true.

Tautology is a ‘limiting case’ – or perhaps better ‘boundary case’ [*Grenzfall*] -- of this series (cf. 4.466) in something like the way in which a geometrical point is the limiting case of a series of concentric circles of diminishing radius centred on that point. A point in a three-dimensional space can be thought of as a degenerate ‘circular’ conic section, and a tautology can be thought of as degenerate truth condition or degenerate picture. A geometrical point, conceived of as a degenerate ‘circular’ conic section, is a circle with a radius of zero; and, analogously, a degenerate truth condition (or degenerate picture) is a truth condition (or picture) with a zero quantity of sense. Of course, *strictly speaking*, a geometrical point is not a circle and *strictly speaking* a tautology is not a picture of reality (4.462) – and neither is it strictly speaking the possessor of a truth condition (4.461). However, we can speak more loosely, and when one does so one may characterize a point as a degenerate circle and a tautology as a null or degenerate picture or truth condition. The latter, at any rate, is one conception by which I take Wittgenstein to have been guided in the *Tractatus*.

In the *Tractatus* the concept of degeneracy is not explicitly invoked in connection with the notion of a tautology, but Wittgenstein did eventually come to invoke it. In his lectures at

Cambridge for the year 1934–35, he says: “A tautology is a degenerate case of a proposition”.³³ And later, in notes written between 1937 and 1944 and published as *Remarks on the Foundations of Mathematics*, he describes a tautology as a “degenerate proposition [*degenerierten Satz*], which is on the side of truth”.³⁴ In the *Philosophical Investigations* when he describes the command ‘Slab!’ as a “degenerate proposition” he is clear that the notion of degeneracy to which he is metaphorically appealing there is that of a degenerate conic section—and specifically that of a degenerate hyperbola. In this particular case his grounds for treating the linguistic item in question as degenerate differ in obvious ways from those motivating his description of tautology as a degenerate case of a proposition.³⁵ Nonetheless, these remarks do suggest the possibility that a notion of degeneracy that is apt for conic sections might have informed his thinking about tautology and contradiction.

Obviously, to think of tautology as an extreme case of a series of propositions is not yet to think of it on the model of two oppositely directed vectors of equal magnitude. Nonetheless, Wittgenstein’s two guiding models are to some extent unified insofar as they each take a tautology to have a zero-quantity of sense. According to the linear-vector model, tautology has a zero quantity of sense because it is the result of adding together oppositely directed vector-like quantities of equal magnitude. According to the extreme case/degenerate-conic-section model, on the other hand, tautology has a zero quantity of sense because it lies at one extreme of a series of truth conditions of increasing laxness.

³³ 1979 [AM], *Wittgenstein’s Lectures Cambridge, 1930–35* (from the notes of Alice Ambrose and Margaret MacDonal), Alice Ambrose (ed.), Midway reprint, Chicago: University of Chicago Press, 1989, 137.

³⁴ *Remarks on the Foundations of Mathematics*, 167.

³⁵ Part I, section 19, line 6.

We can, I think, demystify the apparently conflicting things that the *Tractatus* has to say about tautology if—but only if—we suppose that its author is oscillating between these two suggestive mathematical models. The linear-vector model, for example, struggles to make sense of the following remark:

Tautology and contradiction are the limiting cases [*Grenzfälle*] of the combination of symbols, namely their dissolution. (4.466)³⁶

A physical balance that is brought into equilibrium is not, after all, dismantled in the process. If we switch to the extreme case/degenerate-conic-section model, however, this remark begins to make sense. Or rather, it does so if we think of certain symbols as being combined into a significant proposition only in virtue of that proposition's being a picture of reality (in the strict sense of 'picture'). For then we will be able to think of tautology, since it is the limiting case of a series of pictures of diminishing informational richness as the dissolution of this combination (since it fails to be a picture in the strict sense). Such an idea, although playfully paradoxical, is not, I think, unintelligible.³⁷

In 1911 Alfred North Whitehead wrote a short book entitled, *An introduction to mathematics*. He wrote it for the same series as Russell's better-known "shilling shocker", *The Problems of Philosophy*. In it Whitehead makes an illuminating remark about "modern mathematics". Speaking of degenerate conic sections, which he calls 'special cases', he says:

³⁶ Here I correct Ogden's plural 'combinations'.

³⁷ Paradox-skirting playfulness is not unusual in philosophy. *TLP* 4.466 might be regarded as belonging to the same genre as, for example, Kant's self-consciously paradoxical claim that only the permanent is altered while the transitory suffers not alteration but only a change (A 187/B 230–1). That Kant should have indulged in such whimsy shows that one needn't possess an especially keen sense of humor to write in this way. The early Wittgenstein's apparent lack of a funny bone, therefore, need present no obstacle to our interpreting 4.466 as similarly playful.

They are certainly included among the curves represented by the general algebraic form of the second degree. This fact is worth noting; for it is characteristic of modern mathematics to include among general forms all sorts of particular cases which would formerly have received special treatment. This is due to the pursuit of generality.³⁸

Had he written his book a decade later, Whitehead might have made a similar remark—*mutatis mutandis*—about Tractarian tautologies and contradictions. They are special cases that are ‘included among general forms’ when we pursue one particular kind of generality, namely: the general form of the proposition.

Against the present line of interpretation, which develops this idea, it might be objected that Wittgenstein explicitly denies that tautology (and contradiction) are pictures of reality (4.462, compare 2.225). How is that compatible with their being degenerate cases of pictures? This is a fair question. To answer it one would need to concede that there is indeed a perfectly natural sense in which tautologies are not pictures of reality, just as there is a perfectly natural sense in which geometrical points are not circles. Nonetheless, this is compatible with maintaining that there is also a somewhat Pickwickian sense in which a tautology *is* a picture: it is the limit of a series of truth conditions of decreasing stringency, hence a maximally sketchy or washed out picture. In denying that tautology (and contradiction) are pictures, therefore, Wittgenstein is actually telling only half of his own story: the part told when using ‘picture’ in its strict and familiar sense.

In this connection, we should note that insofar as he uses a range of technical terms in both a narrow and a broad sense Wittgenstein is participating in a long tradition of using terms ‘simpliciter’ (narrowly) and ‘secundum quid’ (broadly, or ‘by courtesy’). As we have seen, ‘truth condition’ has a broad sense according to which tautologies possess truth conditions in virtue of

³⁸ *An introduction to mathematics*, 144.

being extreme cases of them (4.46). But the phrase also has a narrow sense according to which tautologies lack truth conditions because they are unconditionally true (4.461). To take another example, in the broad sense of ‘proposition’, tautologies are propositions; indeed, they are *defined* to be propositions of a certain kind (consider the combination of 5 with 5.101). But in the narrow sense they are not propositions (4.464–4.465). In view of these facts, it would not, I think, be unreasonable to suppose that Wittgenstein’s took even the expression ‘has sense’ to possess broad and narrow meanings. In the broad sense of ‘having sense’, tautologies and contradictions have sense (though they have it only to a zero degree), but in the narrow sense, they don’t have sense because they fail to have it to a non-zero degree.

6 Where the vector analogy breaks down

These considerations suggest that in the *Tractatus* the ‘null vector’ model of tautology already developed in the pre-*Tractatus* writings comes to be supplemented (though not supplanted) by the extreme case/degenerate-conic-section model. And it is fortunate that Wittgenstein doesn’t rely exclusively on the vector analogy; for that analogy breaks down at several points, and does so in fairly obvious ways. One such point involves the intended parallelism between tautology and contradiction. As we have seen, Wittgenstein maintains that contradiction parallels tautology in saying nothing. But the way he tries to bring out this taciturnity in connection with tautology does not in fact carry over to contradiction. In the *Notebooks* entry for December 12, 1914, he says:

$p \ \& \ \text{Taut} = p$ (i.e., Taut says nothing)³⁹

³⁹ NB 36. Compare the entry for the 3rd of October, 1914 (NB 8), which suggests that ‘p’ is understood to be replaced by a proposition proper, and 4.465. At this stage Wittgenstein seems to

The idea seems to be that to add what is said by a tautology to what is said by a proposition proper (by conjoining the former with the latter) is to add nothing. Such an idea is suggested by the vector analogy because the addition of the null vector to any non-null vector yields the original vector. To sustain the parallel in the case of contradiction, one would have to say: ' $p \ \& \ \text{CON} = p$ '. But, of course, such a claim is false. Unfortunately for Wittgenstein, contradiction cannot be happily compared to a null vector because it simply does not behave like an additive identity.⁴⁰

By the spring of 1915 Wittgenstein seems to have come to appreciate this problem. For in the *Notebooks* entry for May 25th of that year we find him struggling to establish a certain parallel which—as his formulation seems tacitly to acknowledge—is undermined by this very problem. He says: “[O]ne could append ‘and’ and some tautology to any proposition without altering its sense; and equally *the negation of a contradiction*.” (NB 51, second emphasis added). Since the negation of a contradiction just *is* a tautology, the ‘equally’ only *purports* to extend an observation that holds of tautology to the new case of contradiction. In truth, the claim it introduces merely repeats the first observation in different words. Somewhat ironically, then, the remark is itself a tautology in the *traditional* sense—but the irony is wholly unintended.

This is the first point at which the analogy breaks down and it reflects the implausibility of maintaining that a contradiction says nothing. The second concerns the idea of cancellation. The problem is that, whereas in the case of vector-addition you can get a *partial* cancellation—suppose

be operating with the classification: tautology, proposition, contradiction. Recall the emphasis we noted on ‘proposition’ in the entry for the previous day.

⁴⁰ One of the requirements on a vector space is that it should be a set of vectors, V , such that there exists an element of V that is an additive identity. That is, there should be an element of V , 0 , such that for all v in V , $v + 0 = v$.

the added vectors have opposite directions but different magnitudes—in the case of conjunction nothing analogous holds. The proposition ‘ $(p \ \& \ q) \ \& \ \text{not-}q$ ’, for example, is not, for Wittgenstein, a *significant* claim weaker than ‘ $p \ \& \ q$ ’, but rather a *sinnlos* contradiction. The third point at which the analogy breaks down is that the conjunction of a proposition with itself is, for Wittgenstein, just that very same proposition, and yet the result of adding a non-null vector to itself is a different vector: one that has twice the magnitude of the original vector and the same direction.

Nonetheless, in spite of these disanalogies, there are clear interpretive advantages to taking the vector analogy seriously. I close with a brief review of these advantages.

Conclusion

First, the vector analogy can help to explain how (and why) a tautology says nothing: a tautology does so by combining elementary propositions in such a way that their senses cancel out or sum to zero, with the result that it possesses a zero quantity of ‘what is said’—a zero quantity of sense. The explanation works by assimilating the unfamiliar to the familiar: the muteness of tautology is explained as an instance of, so to speak, *zero upshot*: just as a balance in equilibrium, turned by no net force, will display no motion, so a proposition in equilibrium (a tautology), combining as it does equal and opposite senses, will express no information.

Second, the vector analogy makes possible the provision of a unitary answer to the question: “Upon what exactly does a logical operator operate?” The vector analogy allows one to treat the item that is logically operated upon as being in *every* case a sense. This is possible so long as one also maintains that some of these operands are *null* senses (compare null vectors).

Third, thinking of negation as analogous to the operation of reversing the direction of a vector helps Wittgenstein to cement his idea that we should not think of negation as specifically

the transition from p to not- p , for the opposite transition—from not- p to p —results from this very same operation of reversal. Relatedly, the analogy helps to underscore the point that negation for Wittgenstein is not a matter of building a new proposition out of an old one: the proposition that is negated is not a component of its negation. It follows from this point, taken in conjunction with the previous one about transitions, that of two contradictorily opposed propositions neither one is intrinsically negative. As the *Tractatus* puts it, “That negation occurs in a proposition, is no characteristic of its sense ($\sim\sim p = p$).” (4.0621). The vector analogy helps to bring out this idea because, if we analogize negating a sense to reversing a vector, then the fact that (when considered in abstraction from any particular co-ordinate system) neither one of a pair of oppositely directed vectors of equal magnitude is intrinsically negative, can serve as a model for the parallel point concerning propositions.

Taken together, the first two of these points help to convey a deep sense in which tautology is in Wittgenstein’s phrase “part of the symbolism” (4.4611). Tautology is like the 0 of arithmetic, but, more specifically, it is like the 0 that results from a certain Kantian quantitative negation. It is like the zero- or null-vector; for as the null vector possesses a zero magnitude, tautology possesses a zero quantity of sense.

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