Analysis, 66.3, July 2006

The Eleatic Non-Stick Frying Pan

SIMON PROSSER

1. A strange surface

In his book *Infinity: An Essay in Metaphysics* José Benardete described a rather unusual book (1964: 236–37). The book has an infinite number of pages, all made from infinitely divisible matter. The first page is half a unit thick; page 2 is a quarter unit thick; page 3 is one eighth of a unit thick, and so on to infinity. The book therefore has a thickness of one unit and although it has a first page it has no last page. For simplicity I shall discuss a version of Benardete's book that has no cover and the pages are not fastened together; it is nothing more than a pile of pages. Because of the way it is constructed, the back of the book has a *topologically open* surface (henceforth *open surface*, i.e. it has no outermost layer of points). The front of the book has a *topologically closed* surface (henceforth *closed surface*, i.e. it has an outermost later of points) provided the pages themselves have closed surfaces, which I stipulate to be the case. I shall stipulate, further, that each page is perfectly rigid and non-porous and that no page is transparent, no matter how thin it is.[1] Benardete asks the

[1] Benardete also describes other objects with the same properties such as a pile of slabs of stone and a board made of sheets of wood, and some equally (perhaps more) bizarre phenomena with analogous temporal properties (1964: chapter 6).

following question: if the book is placed face-down with the first page at the bottom and one looks at the book from above, what does one see? Since there is no last page, one does not see a page. Benardete concluded that one sees nothing.

I would like to consider a different question. Take a piece of sticky tape with a closed surface, where 'sticky' means roughly that if the sticky surface of the tape made contact with any surface made of the same material as the pages then an extra force of magnitude F (>0) would be required in order to separate the two surfaces.[2] Let F be greater than the downward force of gravity on the whole book (this is inessential; any value of F strictly greater than zero would do). In the actual world stickiness consists in various electromagnetic properties of matter, but it seems prima facie intelligible to consider a property of brute stickiness that does not require the sticky object, or anything it sticks to, to

---

[2] Here is a more precise definition:

$$\Box \forall x \forall t (\mathrm{S}_{\mathrm{M}} xt \leftrightarrow \Diamond \exists y(\mathrm{M}y \,\&\, \mathrm{C}xyt) \,\&\, \forall z((\mathrm{M}z \,\&\, \mathrm{C}xzt) \mathbin{\Box\!\!\rightarrow} \mathrm{F}xzt))$$

x, *y* and *z* range over surfaces of objects, *t* ranges over times, '$\mathrm{S}_{\mathrm{M}}xt$' means that *x* is sticky at time *t* relative to material M (the material to which *x* can stick) and 'C' stands for contact (I say more below about what counts as contact). 'F*xzt*' means that an extra force F (>0) is required to separate *x* from *z* at time *t* ('extra' in the sense that the net force required to separate *x* from *z* is the vector sum of F and whatever other forces hold *x* and *z* together independently of their making contact). The counterfactual should be understood standardly as ignoring distant worlds such as those in which there is a bizarre person who tries to hold surfaces together whenever they make contact, or whenever a force is applied that would otherwise separate them. The first clause after the biconditional is to prevent vacuous stickiness of surfaces that could not make contact with anything.

have an internal structure. Now, place the book face-down as described above, with the open surface facing upwards. Press the tape, sticky side down, on to the top of the book. Lift the tape back up. What on Earth happens?

What happens varies from world to world. For any metaphysically possible sequence of events there is a world in which that sequence occurs as soon as the upward force is applied to lift the tape. The tape could be subject to a sudden downward force preventing it from being lifted; the world could immediately come to an end; the book and the tape could leap up together and dance the tango. None of this teaches us anything very interesting. But suppose we limit our attention to some of the closer worlds in which infinitely divisible matter occurs and the scenario as a whole is possible. Consider, for example, worlds governed by laws similar to those of Newtonian mechanics. Newtonian worlds are probably not the only ones in which the phenomenon to be described occurs; I restrict attention to them just for simplicity. Newtonian mechanics includes Newton's first and second laws, which together entail that an object initially at rest moves when and only when a net force is applied to it. Together with Newton's third law we get the result that if there were an upward force on the tape greater than would account for the upward acceleration of the tape then the upward force must act on some other matter as well. Given conservation laws, and ruling out heavy objects accidentally falling on the tape and such like, the only candidate for the extra matter would be a part of the book.

So in these worlds, what happens when the tape is lifted? We have just seen that if an extra force had to be applied in order to lift the tape then some part of the book would have to lift up with it. But since the book has no last page, the tape cannot lift the last page. Neither is there any reason to think that the tape could lift a multiplicity of pages together; by stipulation, the pages are not stuck together so for all values of *n*, if page number *n* is lifted then page number *n*-1 will remain in place. This also rules out the possibility that the whole book is lifted. I rule out, by stipulation, worlds in which there is a further, non-Newtonian law entailing additional forces not predicted by Newton's laws that would make the whole book lift while the pages stayed together. We can only conclude that *nothing* is lifted; the tape does not stick to anything and no extra force is required to separate it from the top of the book.[3] The book has a surface to which it is logically impossible for anything to stick, even though a sticky thing would stick to any of its individual pages.

Does the tape fail to stick because it does not make contact with anything? The claim that nothing could make contact with the book might seem odd, given that the book would halt the downward motion of an object dropped on it. How could the motion of the object be stopped without making contact with anything? Benardete's answer to this was to posit a 'field of force' that acts without contact (1964: 258). John Hawthorne (2000: 623–26), discussing a similar case, however, denies this. Suppose we drop a ball, whose surface is closed, on to the book so that it lands on the open surface. The book stops the motion of the ball (it may or may not bounce off the book depending on how the further details of the situation, such as the elasticity of the ball, are specified). But does the book stop the motion of the ball by making contact with it? Hawthorne accepts that the ball does not make contact with any of the pages. But he argues, correctly I think, that it does make contact with the book, where the book is conceived of as the fusion of the pages. There are, he suggests, three ways for surfaces to make contact:

---

[3] Benardete (1964: 275) does appear to very briefly hint at a similar point in a slightly different context, but does not develop it.

> A closed surface contacts an open surface insofar as there is no unoccupied space in between the two surfaces. Call this open-closed contact.
>
> An open surface contacts an open surface insofar as there is no more than a line's breadth of unoccupied space between them (the line can then be called the boundary of the two surfaces). Call this open-open contact.
>
> A closed surface contacts a closed surface insofar as the outer skin of each overlaps. Call this closed-closed contact. (Hawthorne 2000: 626)

By thus broadening the notion of contact we can say that the book (i.e. the fusion) stops the ball because it makes open-closed contact with the ball even though the skin of the ball does not overlap with the skin of any of the pages; there is no closed-closed contact.

Another way to look at this is that in moving toward the book the ball reaches a point where it tries to occupy a region of space already occupied by the fusion. There is no more empty space for the ball to occupy once the closed surface of the ball makes open-closed contact with the fusion, so unless the ball can pass right through the book it can move no further. I mention this second way of looking at the matter because it helps avoid any temptation to think of open-closed contact as capable of bringing about all the same kinds of interaction between objects as closed-closed contact. Just because both kinds of contact allow the motion of a ball to be stopped it does not follow that both kinds of contact would allow sticking or other kinds of interaction. The case of the book shows that there are at least some interactions, objects and worlds for which open-closed contact cannot facilitate the same interactions between objects as closed-closed contact. Moreover, in the example under discussion this is entailed by laws of nature that say nothing explicit about types of contact.

Hawthorne thus makes a questionable move when he extends his reasoning about the ball to some of Benardete's (1964: 255–59) temporal cases, such as the infinite series of peals of a gong with no first peal, each peal being so loud that it would instantly deafen the hearer upon being heard (cf. making contact). This case seems paradoxical because of the apparent implication that the 'hearer' is deafened yet never hears a single peal because each peal is preceded by another deafening peal. Hawthorne suggests thinking of the fusion of the peals as analogous to a fusion of walls of sound moving toward the hearer and making open-closed contact with the hearer. Now, Hawthorne is right insofar as he is merely saying that one can give a coherent description of the events by saying that the hearer becomes deaf *upon* making open-closed contact with the fusion of peals (i.e. at that very instant). The apparent paradox is thus dissolved. But, given the reasoning above, there is no reason for saying, as Hawthorne appears to, that the hearer is deafened *by* making open-closed contact with the fusion of peals in a manner analogous to the way in which the motion of the ball is stopped by making open-closed contact with the fusion of the pages. This kind of contact cannot bring about deafness in the same way in which the hearer would have been deafened by making closed-closed contact with, and thus hearing, an individual peal. Many things could have happened *upon* making open-closed contact with the fusion of peals; deafness was just one possibility and was not entailed by the nature of the world as described up to that point. The only way to ensure that deafness would occur would be to stipulate a further law of nature that explicitly determined the outcome in such cases.

Given Hawthorne's distinction between different kinds of contact the definition of stickiness given above must be refined; in the worlds under consideration there is no property of stickiness such that a sticky thing would stick to anything with which it made any kind of contact whatsoever. There can, however, be a property of sticking to anything of a given material with which closed-closed contact is made. In the world in question the sticky tape could only have had this kind of stickiness.

2. *What about suction?*

Assume there is an atmosphere present, and recall that the pages were stipulated to be rigid and non-porous. Take a suction cup and press it down on the open surface of the book. Lift up. What happens?

Given the arguments above it might seem, though somewhat counterintuitive, that the suction cup does not attach to the book – it can be lifted without any force being required to separate it from the book. But in fact this is not correct. The suction cup does indeed become attached to the book by suction, and just as much force is required to remove it as would be required to remove it from any one of the pages. Yet when it is lifted no page is removed from the book, even if the first page is fastened to the ground, and no matter how strong the suction is. How come?

The answer is simple, but to understand it one must understand how suction works. First consider a single page on its own, held above the ground by a suction cup. Inside the suction cup the air pressure is lower than outside. Consequently, on the part of the page covered by the suction cup the upward force from the air below is greater than the downward force from the air inside the cup. Hence there is a net upward force and the page is held against the suction cup. Secondly consider a suction cup pressed against a flat, smooth region of the Earth's surface. Force is required to remove the suction cup. There is no air pressure from underneath, so nothing pushes the Earth's surface up against the suction cup. There is, however, air pressure exerting a downward force on the upper surface of the suction cup, and this exceeds the upward force from the lower pressure air inside the suction cup. Hence there is a net downward force on the suction cup and an upward force greater than this would be required in order to remove it.

There are two cases to consider regarding the book. Firstly, there is air underneath the book; it is standing on something porous, for example. Pressing the suction cup on the top of the book and then lifting it causes there to be lower pressure air above the book than below, so there is a net upward force on the book. But this force is exerted at the base, against the first page of the book, and therefore lifts the whole book provided the pressure below is high enough and the pressure in the suction cup is low enough. Will gravity not cause the lower pages to fall away? Well, if the book were in a vacuum this would be correct (and that is why the arguments above work straightforwardly for stickiness). But when there is an atmosphere, which there must be for suction to work, the situation is more complex. Suppose we press the suction cup to the top of the book, lift the book by hand from underneath and then release it while holding the suction cup in place. The pages will fall away, but not all at once. Page 1, being the thickest and therefore the heaviest, will exert the greatest downward force. This may pull it away from page 2 by a tiny amount, thus allowing air in between pages 1 and 2. At this point page 1 will drop. Other pages will drop as well, but the heaviest ones will drop the quickest because they will open a gap of a given size more quickly and will expand the gap more quickly as air begins to enter.[4] Hence we can

[4] Real air is of course made of molecules of non-zero size and would therefore require a gap of a specific size before any air could enter. But we can imagine instead a kind of perfect fluid not composed of particles, or composed of point particles. The fluid is confined within a finite volume but

imagine the pages falling away in a kind of expanding concertina, with lower pages falling more quickly. At some point the pages would cease from falling away because the lighter ones would not have a sufficient downward force to open a gap from the next page at all, and a top section of the book would remain stuck together by suction. But even if this were not the case, the pages would just continue falling away from the suction cup forever, and they would fall more and more slowly due to their reducing weight. The suction cup would never come free of the book.

The second case to consider is when there is no air beneath the book. Suppose the book were placed on a perfectly flat part of the Earth's surface. For simplicity assume that the Earth's surface is closed at that point. When the suction cup is pressed to the top of the book, force is required to remove it. But this is entirely due to the downward pressure from the atmosphere pressing against the upper surface of the suction cup, which produces a downward force greater than the upward force from the low air pressure inside the suction cup. Although it may seem counterintuitive when straining to pull the suction cup up from the book, the suction cup exerts no upward force on any part of the book whatsoever. If there is any air at all in the suction cup then in fact it exerts a small downward force on the book.

In summary, it is possible to attach something to an open surface by suction because suction does not rely on any particular kind of contact with the surface but rather by the manipulation of the forces of air pressure on the external surfaces of the object.

---

it is its nature to try to expand and it thus exerts pressure when prevented from doing so. While this is not really air it will suffice for our conceptual purposes.

3. Conclusion

By thinking about matter in the classical way that seems natural (albeit probably false in the actual world) we sometimes find that it can exhibit unexpected and almost magical-seeming behaviour. Our surprise at such behaviour probably illustrates a confusion built into our intuitive concepts. The non-stick surfaces that I have described are of interest because of the surprising way in which they are constructed. It is no surprise that logically non-stick surfaces exist in certain far-off worlds; of course they do. But what is more surprising is that some intuitive, classical laws of nature in much closer worlds entail that one can assemble some objects, each of which can be stuck to, and make a fusion to which it is logically impossible for anything to stick.

Philosophical curiosities can be worth collecting because of the unexpected consequences they sometimes have for subsequent debates. But in any case we can draw two rather general conclusions straight away. Firstly, even when the laws of nature say nothing explicit about types of contact it is not always the case that all types of contact facilitate the same kinds of interaction between objects. Secondly, the properties of an object do not always reflect the properties of its parts in quite the way we would intuitively expect. No one thought that objects shared all of the properties of their parts, of course; but we might have supposed that for every object at least one part of its surface must share the properties of at least one of the surfaces of one of its parts. This supposition would be false (for example there could be an object with only non-stick surfaces yet composed entirely from objects with no non-stick surfaces).

Think of the applications of the new Eleatic technology if the actual world turned out to contain infinitely divisible matter after all. By considering other Eleatic possibilities Josh

Parsons (2004) has already shown us how to cure a hangover by completing a supertask. Now, with the new Eleatic non-stick technology frictionless railways and permanently clean clothes would just be the start. And forget Teflon; each Eleatic non-stick frying pan will come with a *logical* guarantee that food will not stick to it.[5, 6]

University of St. Andrews

Fife KY16 9AL, UK

sjp7@st-andrews.ac.uk

References

Benardete, J. A. 1964. *Infinity: An Essay in Metaphysics*. Oxford: Clarendon Press.

Hawthorne, J. 2000. Before-effect and Zeno causality. *Noûs* 34: 622-33.

Parsons, J. 2004. The Eleatic hangover cure. *Analysis* 64: 364-66.

[5] The guarantee is void when cooking calamari, octopus or anything else with suction cups.

[6] Thanks to Patrick Greenough, Josh Parsons and Robert Williams for very helpful comments and discussion on this material and to an audience at St Andrews for helpful feedback.