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# ON WITTGENSTEIN'S PHILOSOPHY OF MATHEMATICS

## Hilary Putnam and James Conant

## II—James Conant

Putnam says, in his contribution to this symposium, that his Wittgenstein is neither a philosophical realist, nor a an anti-realist, but only someone who is concerned to defend 'commonsense realism'.[1] What does *that* mean?

The following represents an attempt to get clearer about the approach to reading Wittgenstein that Putnam recommends. I begin by canvassing a fairly standard proposal for what is involved in offering a 'realist' interpretation of Wittgenstein[2] and a proposal for how to oppose such an interpretation; and I try to see why Putnam's Wittgenstein is not the one who figures in either of these proposals. I then try to underscore some of the differences between Putnam's approach to Wittgenstein and that of various other commentators by elaborating some of his suggestions concerning how to read (and how *not* to read) Lectures XXV and XXVI of Wittgenstein's 1939 *Lectures on the Foundations of Mathematics* and, in particular, Wittgenstein's enigmatic discussion in those lectures of G. H. Hardy's claim that mathematical propositions 'correspond to a reality'.

### I

*Alternative ways of reading Wittgenstein.* Simon Blackburn offers a helpful overview of what he takes to be the three different clusters of interpretations of Wittgenstein's later work that dominate 'the

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1. Putnam's contribution is published in *Proceedings of the Aristotelian Society*, Supplementary Volume LXX (July 1996), pp. 243–264. It, in turn, is a shortened version of a longer paper. All subsequent unqualified references to page numbers are to the version in *Proceedings of the Aristotelian Society*.

2. When I say 'a fairly standard proposal for what is involved in offering a "realist" interpretation of Wittgenstein', I mean standard mostly among those who reject such an interpretation and prefer to read Wittgenstein as some sort of anti-realist.

contemporary scene'.[3] The first cluster—as Blackburn summarizes it—'makes Wittgenstein into a... "realist" of a fairly specific kind'—one who affirms that 'we need not blush to talk about truth, facts, knowledge, and certainty' in areas such as ethics any more than in any other area in which 'realistic thought and language is perfectly in place'; the second cluster takes Wittgenstein to be committed to some sort of anti-realist (or quasi-realist) position with respect to 'areas' such as ethics;[4] the third cluster insists that Wittgenstein is rightly seen as neither a realist nor an anti-realist but a quietist who eschews the resolution of philosophical problems.

Blackburn seems to take these three alternatives, as he characterizes them, to exhaust the standing alternatives for interpreting Wittgenstein. Blackburn himself opts without hesitation for the second, on the grounds that the first alternative is exegetically and the third philosophically unsustainable. Of the three, the second is the option which Putnam, in his contribution to this symposium, is most clearly concerned to reject.[5] Where then are we to locate Putnam's Wittgenstein on Blackburn's grid? Well, Putnam starts off by saying that his 'aim in this essay is to show that Wittgenstein's work as a whole defends commonsense realism' (p. 243), and, indeed, this fits a recent trend in Putnam's work of purporting to expound Wittgenstein while, all the while, proudly flying the banner of (something Putnam calls) realism.[6] All this suggests that his interpretation falls squarely into Blackburn's first category. But later on, in his contribution to this symposium, we find a number of remarks which will seem to Blackburn to tend in a contrary direction. The following passage will suffice as a preliminary example: 'The problem with commonsense realism, many will say, is that it isn't a philosophical position at all. Wittgensteinians will agree; Wittgenstein himself tells us that it is not his *intention* to put forward "theses"' (p. 251). Now this would

3. Simon Blackburn, 'Review of Paul Johnston, *Wittgenstein and Moral Philosophy*' (*Ethics*, April 1993); p. 588.

4. Commentators who fall within this cluster are permitted to differ over the nice question whether Wittgenstein himself realized that he was so committed.

5. I will not attempt, in what follows, to rehearse or assess Putnam's reasons for rejecting this option. My business lies elsewhere: in trying to get clearer about the option he doesn't reject.

6. See, for example, chapters 12–15 of Putnam's *Words and Life* (Cambridge, Ma.: Harvard University Press, 1994).

seem to smack more of Blackburn's third category: the quietist interpretation. If one accepts Blackburn's way of specifying the options for reading Wittgenstein, then the simultaneous presence in Putnam's essay of remarks of the former kind (which suggest that he opts for a 'realist' interpretation) and remarks of the latter kind (which suggest that he opts for a 'quietist' interpretation) will seem to provide grounds for concluding that Putnam does not really know who he wants his Wittgenstein to be.

I shall assume otherwise. If Putnam's reading of Wittgenstein can be so easily seen not to fall neatly into any of Blackburn's three standing alternatives for how to read Wittgenstein (and, indeed, if Wittgenstein's own remarks about what he is up to, exhibit a similar irresoluteness) then, before concluding that Putnam (or Wittgenstein himself) does not know what he wants, we should take a closer look at the philosophical assumptions built into Blackburn's way of carving up the logical space of possible ways of reading of Wittgenstein.

## II

*A kind of 'realist' Wittgenstein is **not**.*  Blackburn says that those who belong to the first category of interpreters 'make Wittgenstein into a "realist" of a fairly specific kind'—and he cites John McDowell, Sabina Lovibond and Susan Hurley as exemples of commentators who make Wittgenstein into a 'realist' of this kind. Is Putnam's Wittgenstein a further example of a 'realist' of this fairly specific kind? Judging from Blackburn's preliminary characterization of the kind (as resting on the affirmation that 'we need not blush to talk about truth, facts, knowledge, and certainty' in areas such as ethics and mathematics any more than in any other area), the answer would appear to be affirmative. But, as Blackburn goes on to refine his characterization of the kind, it becomes less and less clear that this is the right answer. The crucial refinement comes when we are further told that proponents of this interpretation hold that for Wittgenstein in ethics and mathematics 'truth, knowledge, and the rest are in place as firmly and *in the same way* as everywhere else' [my emphasis].

Let us begin by considering a Wittgenstein who at least at first can seem to be—and who Blackburn takes to be—a 'realist' of this

fairly specific kind: Sabina Lovibond's Wittgenstein.[7] Lovibond writes:

> What Wittgenstein offers us... is a homogenous or 'seamless' conception of language. It is a conception free of invidious comparisons between different regions of discourse.... On this view, the only legitimate role for the idea of 'reality' is that in which it is coordinated with... the metaphysically neutral idea of 'talking about something'.... It follows that 'reference to an objective reality' cannot intelligibly be set up as a target which some propositions—or rather, some utterances couched in the indicative mood—may hit, while others fall short. If something has the grammatical form of a proposition, then it *is* a proposition: philosophical considerations cannot discredit the way in which we classify linguistic entities for other, non-philosophical, purposes....
>
> The only way, then, in which an indicative statement can fail to describe reality is by *not being true*—i.e. by virtue of reality not being as the statement declares it to be....
>
> Thus Wittgenstein's view of language confirms us—provisionally, at least—in the pre-reflective habit of treating as 'descriptive', or fact-stating, all sentences which qualify by grammatical standards as propositions. Instead of confining the descriptive function to those parts of language that deal with a natural-scientific subject-matter, it allows that function to pervade all regions of discourse irrespective of content....
>
> Wittgenstein's view of language implicitly denies any meta-physical role to the idea of 'reality'; it denies that we can draw any intelligible distinction between those parts of assertoric discourse which do, and those which do not, genuinely *describe* reality.[8]

Lovibond's Wittgenstein refuses to draw any metaphysically invidious distinctions between different regions of assertoric discourse; he takes language to be 'metaphysically homogenous'. He acknowledges that there are various dimensions along which we might distinguish regions of discourse; but he holds that all of

7. I say 'who at least *at first can seem* to be a "realist" of this fairly specific kind' because as one reads further on (than the passages quoted below) in Lovibond's book—into the portions of her book in which she attempts to account for the differences between ethical and other kinds of knowledge, and to do justice to what she takes to be the genuine insights underlying non-cognitivism about ethics (concerning which the non-cognitivist himself provides a metaphysical misconstrual)—it becomes increasing doubtful that her Wittgenstein qualifies as a 'realist' of Blackburn's 'fairly specific kind'. But, since my present business is to defend Putnam's Wittgenstein, I will confine my remarks about 'Lovibond's Wittgenstein' to the passages from her quoted below, pretending (along with Blackburn) that her Wittgenstein does so qualify and thus riding roughshod over many of the details of her elegant and nuanced reading of Wittgenstein.

8. Sabina Lovibond, *Realism and Imagination in Ethics* (Oxford: Blackwell, 1983), pp. 25–27, 36.

these regions stand in the same relation to reality. Each region of discourse describes or represents what is the case. Ethical discourse represents ethical features of reality, and so on. The differences between different regions of discourse lie not in *how* they bear on reality, but only in *what* they bear on. Ethical, mathematical and empirical discourse are each concerned with a different kind of feature or aspect of reality.

Blackburn's response to this would seem to be: '*This* is supposed to be a reading of Wittgenstein?' Here is what he says:

> This interpretation would be nice if there were evidence for it. Unfortunately it flies in the face of innumerable texts, and indeed of the whole spirit of the later Wittgenstein. Far from finding a fundamental identity in our different assertoric activities, Wittgenstein wants to force the difference between different 'language games' right down our throats. He is constantly suggesting that underneath a superficial similarity of linguistic form there is a deep difference of function. There is no area he considered in the later work where this approach is not found.[9]

Blackburn evidently has trouble taking what he calls a 'realist' interpretation seriously as a *reading* of Wittgenstein. What is the source of his incredulity? Here are two things Blackburn says about Wittgenstein: (1) he constantly wants to force the difference between different language games right down our throats, and (2) he is constantly suggesting that underneath a superficial similarity of linguistic form there is a deep difference of function. He notices remarks of Lovibond's which appear to deny (2),[10] and he thus takes her to be concerned to deny (1) as well. With respect to the generic denial of (1), he seems to me to be quite mistaken about Lovibond;[11] but with respect to the denial of (2), Blackburn does have his finger on a substantive difference between himself and Lovibond—a difference that is made all the sharper by the (unnecessarily) extreme terms in which Lovibond expresses her denial of (2).

9. Blackburn, *op. cit.*, p. 589.

10. Remarks such as 'the descriptive function pervades all regions of discourse irrespective of content'.

11. In so far as it is important to Blackburn to claim that a 'realist' reading of Wittgenstein fails to acknowledge that 'Wittgenstein wants to force the difference between different "language games" right down our throats', it is by no means clear that Lovibond's Wittgenstein can furnish an example of what Blackburn wants to call a 'realist'. For her Wittgenstein is as interested as Blackburn's is in the difference between different 'language games', but locates the source of the differences in a different place (namely, in what Lovibond calls 'differences in relationships of intellectual authority').

In her eagerness to deny (2), Lovibond allows herself to wander into formulations of the doctrine she attributes to Wittgenstein which are very difficult to sustain exegetically. The clash between what Wittgenstein himself says and what she says on his behalf is perhaps most evident in her attribution to Wittgenstein of the principle that 'if something has the grammatical form of a proposition, then it *is* a proposition'.[12] Wittgenstein, on Lovibond's reading of him, would seem to be positively hostile to the idea that there could so much as be a significant philosophical task which consists in the investigation of whether, in a given case, an indicative sentence which appears to be a proposition is only misleadingly of that appearance. More to our present purpose, he would seem to be equally hostile to the idea that there could be a significant philosophical task which consists in the investigation of whether, in a given case, a proposition which appears to be of a particular sort— say, for example, a description—is only misleadingly of that appearance.[13] Yet, if any credence is to be lent to Wittgenstein's own descriptions of what he is up to, the prosecution of these twin philosophical tasks would appear to form the central business of the better part of his corpus (both early and late). Hence Blackburn's incredulity. The first of these tasks is summarized in §464 of *Philosophical Investigations*: 'My aim is: to teach you to pass from a piece of disguised nonsense to something that is patent nonsense.' In so far as this is Wittgenstein's professed aim in philosophy, it is arguable that no principle is more basic to his philosophical practice than the negation of the one which Lovibond ascribes to him: not everything which appears to be a proposition is one.[14] Allusions to the second task can be found in Wittgenstein's warnings to his reader against mistaking a certain sort of proposition 'whose form makes it look like an empirical proposition' for an empirical proposition, or against construing the grammar of a certain sort of expression 'on the model of "object and designation"'.[15] Arguably the second most basic principle to Wittgenstein's philosophical practice might be put

12. Lovibond, *op. cit.*, p. 26.

13. This shortcoming of Lovibond's reading of Wittgenstein is discussed by Cora Diamond in her article 'Wittgenstein, Mathematics and Ethics' (in *The Cambridge Companion to Wittgenstein*, edited by Hans Sluga and David Stern, Cambridge University Press: Cambridge, 1997). Throughout the remainder of this essay, I am indebted to this article and to conversations with Cora Diamond on these topics.

14. See, for example, *Philosophical Investigations*, §520.

15. As in §251 and §293 respectively of *Philosophical Investigations*.

somewhat tendentiously as follows: not every proposition which appears to be a description is one.

In so far as Lovibond's formulations tend towards denying that Wittgenstein holds to either of these principles, it is difficult not to sympathize with Blackburn's sense that what we are being offered is no longer a reading of Wittgenstein. For Wittgenstein is constantly concerned to show us how we can be misled by the similarities of grammatical appearance between empirical propositions, on the one hand, and ethical or mathematical or psychological propositions, on the other; and how these superficial similarities hide from us the distinctive kind of relation each of these latter kinds of proposition bears to reality. His point is not that we cannot use the word 'description' to encompass all of these different cases.[16] But that we will mislead ourselves if we suppose that, among all these diverse sorts of proposition, there runs some common thread which is usefully characterized as *the* descriptive function.[17]

One hint as to why Lovibond might feel wedded to the claim that it is Wittgenstein's view that 'the descriptive function pervades all regions of discourse' emerges in her characterization of what she takes the alternative to her own reading of Wittgenstein to be. She does not see how to allow for the passages in Wittgenstein's work which run contrary to her reading without ascribing to Wittgenstein a conception of reality that she justly feels he would wish to set himself against: a conception of reality as a *target* ('a target which some propositions—or rather, some utterances couched in the indicative mood—may hit, while others fall short'). She appears to think that the alternative to (what she calls) 'a homogenous view of language' must be such a conception: a conception which assigns (what she calls) 'a metaphysical role to the idea of "reality"'—so that any reading of Wittgenstein which denies that he holds the principle she ascribes to him (that everything which appears to be a description is one) thereby ascribes to him an intention of seeking to rank sentences into those that are metaphysically first-class (and really do represent things

16. This is what is 'somewhat tendentious' about my formulation of the second most basic principle to Wittgenstein's philosophical practice: it makes it seem as if his concern were to legislate how to use the word 'description'.

17. See, for example, *Philosophical Investigations*, §24 ('think how many different kinds of thing are called "description"'), and §§290–291 ('the word "describe" tricks us here').

as they are) and those that are metaphysically second-class (and purport to—but do not really—represent things as they are).

Locating where Putnam's Wittgenstein stands in regard to the disagreement between Lovibond and Blackburn is a tricky matter. To the extent that Blackburn is merely concerned to affirm that Wittgenstein does indeed hold to his two 'basic principles' (enunciated in the preceding paragraph but two), Putnam will want to side with Blackburn. But there is something in Lovibond's 'realism' which Putnam wants, nevertheless, to try to hang on to— the idea that ethical and mathematical propositions are *bona fide* instances of assertoric discourse: ethical and mathematical thought represent forms of reflection that are as fully governed by norms of truth and validity as any other form of cognitive activity. But he is not friendly to the idea that, in order safeguard the cognitive credentials of ethics or mathematics, one must therefore suppose that ethical or mathematical thought bears on reality *in the same way* as ordinary empirical thought; so that, in order to safeguard talk of the truth of propositions such as 'it is wrong to break a promise' or '2+2=4', one must suppose that, like ordinary empirical propositions, such propositions, in each sort of case, 'describe' their own peculiar sort of state of affairs. There is an assumption at work here that Putnam wants to reject—one which underlies Blackburn's way of distinguishing 'realism' and 'anti-realism'—the assumption that there are just two ways to go: either (i) we accept a general philosophical account of the relation between language and reality according to which all indicative sentences are to be classified equally as 'descriptions of reality'; or (ii) we accept an alternative philosophical account of the relation between language and reality which rests on a metaphysically-grounded distinction between those sentences which do genuinely describe reality (and whose cognitive credentials are therefore to be taken at face value) and those which merely purport to describe reality (and whose claims to truth are therefore to be taken as chimerical).

## III

*A kind of 'anti-realist' Wittgenstein is* **not**. Putnam's interpretation clearly does not qualify as 'realist' in Blackburn's ('fairly specific') sense, in so far as it explicitly repudiates a central tenet of that realism—that language is (as Lovibond puts it) 'metaphysically

homogenous', that in ethics and mathematics (as Blackburn puts it) 'truth, knowledge, and the rest are in place in the same way as everywhere else'. This repudiation can be found in a passage such as the following:

> The problem in all of these cases... is that we wish to impose a *pattern* of what it is to be true, a pattern devised largely from the successes of the physical science, on all of our discourse.... In contrast, the Wittgensteinian strategy, I believe, is to argue that while there is such a thing as correctness in ethics... [and] in mathematics, the way to understand that is not by trying to model it on the ways in which we get things right in physics, but by trying to understand the life we lead with our concepts in each of these distinct areas. The problems in the philosophy of mathematics are not precisely the *same* as the problems in metaethics... because the way the concepts work is not the same in these different areas, but what drives the sense that there is a problem—a problem which calls for either a 'skeptical solution' or an absurd metaphysics— can be the very same preconceptions about what 'genuine' truth, or 'genuine' reference must look like. (pp. 262–4)

Putnam's Wittgenstein denies here precisely what Blackburn's 'realist' is most concerned to affirm: that the kind of relation propositions bear to reality conforms throughout all regions of assertoric discourse to a single pattern. Putnam insists rather that in ethics, in mathematics, and in physics 'the way the concepts work is not the same', and that the avoidance of philosophical confusion concerning the differences between such concepts can be won only at the cost of a painstaking philosophical labour of 'trying to understand the life we lead with our concepts in each of these distinct areas'. Putnam's Wittgenstein thus shares with the Wittgenstein who figures in Blackburn's second cluster of interpretations a commitment to the following two ideas: (1) that superficial similarities of linguistic form can mask profound differences in function; (2) that we must come to *understand* these differences if we are to avoid being misled, in our philosophizing, by them.

   To see where the differences lie between Putnam's and Blackburn's preferred ways of refusing (what Blackburn calls) 'realism', consider how Blackburn characterizes the philosophical morals which he thinks are to be drawn from Wittgenstein's efforts

'to force the difference between different "language-games" right down our throats':

> [Wittgenstein] is constantly suggesting that underneath a superficial similarity of linguistic form there is a deep difference of function. There is no area he considered in the later work where this approach is not found. Philosophical statements are not what they appear; they are rules of grammar. Mathematical statements are often not what they appear; they do not have the use of statements but of rules. Apparent self-descriptions are not what they appear; avowals are forms of self-expression. Statements attributing consciousness to others, or describing acts as voluntary, have the use of expressions of attitude. Ethical and aesthetic and theological assertions are not what they appear; the form of life in which they are found is not that of describing how the world is but of reacting emotionally to it.

> Bully for Wittgenstein, say I. But now remember too that 'philosophy leaves everything as it is.' So can we continue to talk of truth, fact, knowledge, and the rest in these non-descriptive areas without blushing? It seems a good question, and I do not think Wittgenstein ever confronted it squarely. His answer is going to be that we can, but it is not at all plain how he gets to it, for the difference of activity he harps on is introduced precisely by *contrast* with describing and representing the way of the world, and those are the activities that most obviously must conform to norms of truth and fact. Wittgenstein seems to leave unfinished business.... The business would be understanding how ejaculations, expressions of emotion, rules, and the rest can properly don the garb of assertions in the first place.[18]

There are two tendencies at play in how Blackburn characterizes what emerges from Wittgenstein's grammatical investigations which Putnam is bound to regard as unfaithful to Wittgenstein.

The first is a tendency to characterize non-descriptive forms of linguistic activity so that (in refusing to assimilate them to the category of descriptions) one assimilates them instead to some non-discursive category—so that they no longer seem to qualify even as regions of assertoric discourse. Blackburn does not say—as Wittgenstein is forever saying—that it helps to mitigate certain puzzling features of a given language-game if one *compares* (an aspect of) what one is doing in thus using language with certain non-linguistic forms of behaviour. Blackburn suggests rather that what Wittgenstein is concerned to claim is that certain forms of

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18. Blackburn, *op. cit.*, p. 589.

words that appear to be descriptions are (not only not descriptions, but) not even, in any obvious sense, assertions at all—what they *really* are is, *mere* ejaculations, or *mere* expressions of feeling, or *mere* forms of emotional reaction. If one thinks *this* is what Wittgenstein is saying, then one is bound to think that he left unfinished philosophical business (namely, the business of addressing the question: how can one 'continue to talk of truth, fact, knowledge, and the rest' in connection with mere ejaculations or sub-cognitive emotional reactions or other bits of intrinsically non-assertoric behaviour?). Such a characterization of these (non-descriptive) regions of discourse invites us to conceive of them as being, in the first instance, utterly without connection to those regions of our discourse that most obviously do conform to norms of truth and fact, and hence threatens to render them no longer *regions* of discourse at all. Once this threat is realized, any subsequent rescue attempt (which allows one to continue to speak along with the vulgar of truth, fact, knowledge, and the rest in connection with these 'regions') comes too late. It will amount to nothing more than an attempt to show how various forms of non-assertoric linguistic behaviour can (as Blackburn puts it) 'don the garb of assertions', while remaining in substance, underneath the misleading outward garb, a wholly distinct kind of performance—one that falls outside the game of truth and falsity altogether.

The second of the two tendencies (at play in how Blackburn characterizes what emerges from Wittgenstein's grammatical investigations) is a tendency to set-up a single overarching contrast: on the one hand, there is language which 'describes and represents the way of the world', and, on the other hand, there is all this other stuff—and then examples follow, such as *mere* ejaculations, *mere* forms of emotional reaction, etc. A general positive character-ization of 'all this other stuff' is thereby insinuated through the character of the examples. The bulk of this other stuff looks as if it could be summed up in some single heading such as 'all this *merely expressive* stuff'. It then looks as if there is a single outstanding philosophical question: how can talk of truth, knowledge and the rest gain a foothold with respect to this latter sort of (merely expressive) stuff. So we appear to have an obligation to look for *the* story which answers this question (and it can seem as if Wittgenstein has inexcusably evaded this obligation).

The differences Wittgenstein seeks to teach are not (as Blackburn tends to suggest) primarily ones that obtain between one central kind of statement (namely descriptive statements) and the rest of language, but, more often than not, ones that obtain *within* the very broad category of uses of language that are not primarily concerned with representing or describing the world. Where Blackburn seeks to erect a single broad (non-descriptive) category and pose a single question about all of the cases which fall under it ('how do *these* sorts of proposition all manage to don the garb of assertions?'), Wittgenstein is concerned to show us how different the various cases which fall under this category are from each other. If one looks at what Wittgenstein has to say about the examples which Blackburn himself adduces—avowals, mathematical theorems, ethical statements—it is evident that he thinks that each of these regions of discourse has its own distinctive grammar, and that precisely what confuses us is our tendency to run these cases together with each other (as well as with ordinary empirical statements). Each of these non-descriptive regions, according to Wittgenstein, is caught up in the business of speaking truth in a different way. The anti-realist's determination to find *the* story (about how all these regions of our discourse are to be brought within the scope of a single contrast with empirical description) is thus identified by Wittgenstein as being as great a source of mischief as the determination of (what Blackburn calls) the 'realist' to find a single overarching story about how each sort of proposition corresponds to its own region of reality.

## IV

*A preliminary example: avowals.* To further see the difference between Blackburn's and Putnam's Wittgensteins, it will help to explore for a moment a preliminary example—one that Blackburn himself adduces, that of avowals.

Wittgenstein's interest in avowals rests exclusively on an interest in dispelling the philosophical puzzlement occasioned by certain features of avowals. *One* such feature is that they possess first-person authority. If we take this authority to be continuous with the sort of authority that ordinary empirical reports possess—yet somehow extraordinarily less corrigible —we are bound to become puzzled. Wittgenstein suggests that, if we wish to understand the

nature of the authority of avowals, we do well to consider the sort of (internal) relation that obtains between a smile and the joy which the smile expresses, and then to consider the way in which linguistic avowals often play the same sort of expressive role in our lives that such pre-linguistic non-verbal forms of behaviour, such as smiles, play. The point of the comparison is to bring out the expressive dimension of avowals (because it is partly a neglect of that dimension, Wittgenstein thinks, that renders the phenomenon of first-person authority puzzling).

The point of such a comparison however is not thereby to strip avowals of their assertoric dimension. If one sees non-descriptive uses of language as being simply a form of non-assertoric language-use, one will obscure from view how such uses of language interweave with those regions of our discourse that most obviously do conform to norms of truth and fact. When I say 'I am happy', I am expressing my happiness through a linguistic mode of behaviour. But such an utterance cannot simply be equated with a cry of joy. For when I say 'I am happy' there is something which is my telling the truth with respect to this matter. If I say 'I am happy', I issue an inference-license, and you may now say of me 'He is happy'; and what you say of me, assuming I am telling the truth, will be true. The grammatical and logical relations that obtain between my statement and yours cannot obtain between a non-assertoric bit of behaviour and a statement—say, between my smile or my cry of joy and your saying of me 'He is happy'. A smile or a cry of joy is neither true nor false, whereas an avowal is. The philosophical difficulty here, on Wittgenstein's view, lies in obtaining a perspicuous overview of the interplay between the various functions of avowals (among which are its expressive and assertoric functions).

Blackburn is certainly right that Wittgenstein thinks that the linguistic form of avowals masks their expressive function. Blackburn's formula for reading Wittgenstein, however, tends in the direction of hearing him say that philosophical clarity is achieved by simply exchanging one understanding of *the* function of avowals for another. *The* function of a bit of language is not only not simply what it appears to be; according to Blackburn's Wittgenstein, it is in reality not at all what it appears to be. *The* function of a bit of language is rather to be identified with that function which is masked by its linguistic form. Putnam's formula

for reading Wittgenstein rests on trying to bring out how much philosophical trouble we get into when we start talking about '*the* function of a bit of language'. Those language-games which puzzle us most when we are doing philosophy, according to Putnam's Wittgenstein, are precisely those that possess the most complex and multi-layered structure—a structure which enables them to serve various kinds of function at once. The philosophical trick then lies in seeing how, in a given case, within a set of apparently mutually exhaustive accounts of a particular region of discourse (ethics, mathematics, avowals), each alternative account contains, and each occludes, a piece of the whole truth.

Putnam says 'the Wittgensteinian strategy' is to try to show how there can be such a thing as *correctness* in ethics and in mathematics, but not by trying to impose a single *pattern* or *model* on all regions of discourse, but rather by trying to understand the life we lead with our concepts in each of these distinct areas. But an appreciation of what correctness comes to in each of these distinct areas turns on an appreciation of the distinctive way in which each of these regions of our discourse is interwoven with those portions of our discourse where talk of truth, knowledge and the rest seems least problematic. When Wittgenstein, in his *Remarks on the Foundations of Mathematics*, says that a certain species of realism is the hardest thing in philosophy,[19] this does not, according to Putnam, refer to the task of finding a way to shoehorn the diverse regions of our discourse into a single general 'realistic' account of the relation between language and reality (such that each region of language is accorded its own region of reality for it to be 'about'); rather it refers to the task of providing a fully *realistic depiction* of the ways in which the different regions of our discourse are inextricably entangled in one another. Thus when Wittgenstein speaks of realism in philosophy, he means the word not in the philosopher's sense (as an account of some domain of fact that is out there anyway and which provides a target for thought to hit or fall short of), but rather in something more like the novelist's sense (i.e. a faithfulness to the complexity of everyday reality in depicting our lives with one another and in the world).[20]

19. *Remarks on the Foundations of Mathematics* (Oxford: Blackwell, 1967), VI, §23.

20. For an elaboration of this suggestion concerning how to understand what Wittgenstein means when he says he aspires to realism in philosophy, see chapter I of Cora Diamond *The Realistic Spirit* (Cambridge Ma.: MIT Press, 1991).

## V

*Quietism?* Given that Putnam's Wittgenstein falls into neither the first nor the second of the clusters of interpretations of Wittgenstein's later work that Blackburn allows for, the question arises as to how he fares with respect to the third. Here is how Blackburn characterizes the third:

> The third cluster of interpretations wants elements from the second and from the first. It properly remembers Wittgenstein's love of motley. But it leaves no unfinished business—after all, how can Wittgenstein, with his hostility to the idea that philosophy is trying to find explanations of anything, admit to a feature of our language games that needs philosophical explanation? The true Wittgensteinian reaction is just to find *more* motley.... [This] is true to much in Wittgenstein, yet its problem is obvious: it denies Wittgenstein any words to say what he wanted about the differences that the position starts by celebrating.[21]

The third cluster wants to combine an element from the second cluster with an element from the first: from the second it keeps the idea that Wittgenstein wants to show us how very different ethical or mathematical propositions are from empirical descriptions (that he wants to teach us differences), from the first it keeps the idea that he takes us as far as we should want or need to go in philosophy (that he leaves no unfinished business). The problem, according to Blackburn, is that these elements cannot be combined without further ado: a mere exhibition of differences cannot by itself solve the very problems which force themselves upon us as soon as we take these differences to heart (problems such as: how can mere ejaculations don the garb of assertions?). Blackburn congratulates Wittgenstein on having unearthed some interesting and important philosophical problems; but for this very reason he is perplexed by Wittgenstein's apparent refusal to engage in the sort of constructive philosophizing that (Blackburn takes it) these very problems cry out for.

This renders the following exegetical question urgent for Blackburn: if grammatical investigation consists in a mere exhibition of differences, and if Wittgenstein's grammatical investigations unearth puzzling philosophical questions, then why

21. Blackburn, *op. cit.*, pp. 589–90.

does Wittgenstein think that, after having completed his grammatical investigations, he leaves no unfinished business? There is one answer to this question which has of late become something of an interpretative commonplace: Wittgenstein, we are told, espouses *quietism*. This is evidently the answer which Blackburn has in mind here. Elsewhere he offers the following characterizations of quietism:

> [T]he attitude which I christen *quietism* or *dismissive neutralism*... urges that at some particular point the debate is not a real one, and that we are only offered, for instance, metaphors and images from which we can profit as we please.[22]

> [Q]uietism... [is] the doctrine associated with Wittgenstein that there is no standpoint from which to achieve the traditional philosophical goal of a theory about some concept or another (e.g. truth, experience).[23]

Is Putnam's Wittgenstein a quietist? If one rests with the characterizations of quietism provided in these passages, the question is difficult to answer. For there is an unclarity in expressions such as 'urging that the debate [about X] is not a real one' and '[asserting] that there is no standpoint from which to achieve the goal [of providing a theory about X]'—an unclarity which has done much to obstruct the possibility of an encounter with Wittgenstein's philosophy.

One way of understanding these expressions is to take quietism to be (to borrow Blackburn's apt phrase) a form of *dismissive neutralism*. There is a debate we might engage in, but, while remaining neutral towards the parties currently so engaged, we quietists roundly dismiss the entire activity of so engaging oneself. Quietism, thus understood, is a call for renunciation: there is something which would be partaking in the debate in question, but the quietist urges us instead not to partake; there is something which would be attaining to the philosophical 'standpoint' in question, but the quietist urges us instead not to try to attain it. 'Quietism' is an apt label for such a call for renunciation. The original quietists, followers of the Spanish mystic Michaelis de Molinos, were participants in the seventeenth-century Catholic

22. *Spreading the Word* (Oxford: Oxford University Press, 1984), p. 146.
23. Simon Blackburn, *The Oxford Dictionary of Philosophy* (Oxford: Oxford University Press, 1984), p. 315.

Counter Reformation. They sought to withdraw from the world of the senses, to abstain from outward activity, and to absorb themselves in continuous passive devotional contemplation and prayer, hoping by this means gradually to effect an annihilation of the will (and therewith to attain beatitude). For the original quietists, there was, of course, a clear standing alternative to the rigours of their ascetic spiritual discipline: namely, indulging in an ordinary life of activity and absorption in the world of the senses— an alternative which most people embraced, but which they chose to renounce.[24] A quietist reading of Wittgenstein similarly takes there to be a clear standing alternative to the master's ascetic credo: indulging in an activity of constructive philosophizing—an alternative which most philosophers embrace, but which the true Wittgensteinian chooses to renounce. This fits in nicely with the thought that Wittgenstein's teaching means to leave us with no unfinished business. If we are true disciples of Wittgensteinian quietism we will have no stomach for the business that the unconverted feel we leave unaddressed. To say that Wittgenstein espouses quietism, so understood, is to say that there is a form of activity that he thinks we might engage in, but he urges us instead not to engage in it, urging us instead just to *stop*—and to renounce the activity altogether.[25]

Given such an understanding of quietism, is Putnam's Wittgenstein a quietist? No. To see why not, consider an alternative way of resolving the unclarity in the expressions that Blackburn employs in his characterization of quietism (such as 'urging that the debate [about X] is not a real one' and '[asserting] that there is no standpoint from which to achieve the goal [of providing a theory about X]'). On this alternative understanding, the aim is not merely

24. The term 'quietism' first came into professional philosophy, as far as I know, through Schopenhauer who thought the followers of Molinos were on to something, and was thus happy to declare: '[I]f, in the judgement of contemporaries, the paradoxical and unexampled agreement of my philosophy with quietism... appears an obvious stumbling block, yet I, on the other hand, see in this very agreement a proof of its sole accuracy and truth.' (*The World As Will and As Representation*, Vol. II, New York, NY: Dover, 1958; p. 615).

25. Various reasons can be (and have been) given for why Wittgenstein might think we should heed such a call for renunciation (e. g. there are reasons to think we will never be able to succeed in the activity in question, the activity itself is a frivolous one, etc.). It is immaterial to my purposes to enter into this level of detail. As soon as one thinks one owes an answer to the question 'Why does Wittgenstein think we ought to try to leave off philosophizing?', one's reading of Wittgenstein qualifies as quietist in the sense here outlined.

to *urge* that we leave off debating a certain question, but to *show* that there is no question to debate—to show that a certain apparent debate 'is not a real one', that the forms of words we avail ourselves of when we seek to describe the standpoint in question only apparently describe a standpoint. Thus the aim is not simply to withdraw from the debate, but rather to attempt to bring the debate to a satisfactory resolution. Thus Putnam writes:

> Wittgenstein... did *not* wish to sweep any problem under the rug; what he was rather trying to do is see just what picture 'holds us captive'—to find the roots of our conviction that we *have* a genuine problem, and to enable us to see that when we try to state clearly what it is, it turns out to be a nonsense problem. (p. 252)

A very different way of understanding Wittgenstein, from any of the three for which Blackburn allows, is encoded in this passage.

Putnam's Wittgenstein diverges from (what I have been calling) quietism in the following seven respects: (1) his philosophical practice is not exhausted by the activity of merely describing the differences between language-games; partly because (2) genuinely grasping what such differences come to itself requires an extensive positive effort of *understanding* (one of 'trying to understand the life we lead with our concepts in each of these distinct areas'), and because (3) the achievement of such understanding is itself in service of an ulterior elucidatory aim—one of *enabling us* to see something about ourselves:[26] that sometimes we mean nothing when we think we mean something (if 'we try to state clearly what the problem is, it turns out to be a nonsense problem');[27] this activity of elucidation itself presupposes an equally extensive task of *diagnosis* ('trying to see just what picture "holds us captive"— to find the roots of our conviction that we *have* a genuine problem'); thus (5) when a problem is made to disappear it is not because we succeed in averting our gaze from the problem, leaving it unaddressed, but because *the problem itself* is made to disappear; hence (6) we are not called upon to renounce anything (but rather to see that *there is nothing to renounce* where we thought that there

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26. Where it is important that this is something that each person, in each such case, must come to see for him- or herself.

27. None of Blackburn's three clusters allows one to see how Wittgenstein's own descriptions of his aim in philosophy (in passages such as *Philosophical Investigations*, §464) could be anything but misdescriptions.

was something);[28] and, finally, (7) these twin philosophical tasks of understanding and elucidation never come to an end—each of us, necessarily always under the pressure of taking thought, will necessarily always provide the philosopher (in each of us and in each other) with plenty of unfinished business.[29]

## VI

*Wittgenstein on Hardy.* The text on which Putnam leans most heavily in his discussion is Lectures XXV and XXVI of Wittgenstein's 1939 Cambridge *Lectures on the Foundations of Mathematics*. It is clear why the 1939 lectures provide an attractive text for Putnam's purposes. In those lectures, Wittgenstein is concerned both to deny precisely what 'realists' (in Blackburn's sense of the word) affirm and to deny what anti-realists affirm. Or, to put the point positively, in those lectures Wittgenstein is concerned to affirm both (1) that the kind of relation that mathematical propositions bear to reality is of an entirely different sort than that of empirical propositions; and (2) that this does not mean that we should conclude that talk of the truth or falsity of mathematical propositions is a misleading *façon de parler.*

In Lecture XXV, Wittgenstein begins by considering a remark of G. H. Hardy's:

> Consider Professor Hardy's article ('Mathematical Proof') and his remark that 'to mathematical propositions there corresponds—in some sense, however sophisticated —a reality'.[30] (The fact that he

28. The following passages can be read as a disavowal of (what I have been calling) quietism:

[P]hilosophy does not lead me to any renunciation, since I do not abstain from saying something, but rather abandon a certain combination of words as senseless....

If I am correct, then philosophical problems must be completely solvable....

If I say: here we are at the limits of language, then it always seems as if resignation were necessary, whereas on the contrary complete satisfaction comes, since *no* question remains.

The problems are dissolved in the actual sense of the word—like a lump of sugar in water. (*Philosophical Occasions*, Indianapolis, IN: Hackett, 1993, pp. 161, 181, 183).

29. Wittgenstein never comes close to saying 'My aim in philosophy is to leave no unfinished business' or anything of the sort! In my introduction to Putnam's *Realism with a Human Face* (Cambridge, Ma.: Harvard University Press, 1990), I discuss some of the passages in his work which have occasioned such a misreading and why it is essential to (Putnam's understanding of) Wittgenstein's conception of philosophy that it necessarily leave unfinished business.

30. What Hardy actually says is: '[Mathematical theorems] are, in one sense or another, however elusive and sophisticated that sense may be, theorems concerning reality...' (Hardy, 'Mathematical Proof', *Mind* 38 (1929); p. 18). Hardy does not anywhere in this article speak of a '*correspondence* to reality'!

said it does not matter; what is important is that it is a thing which lots of people would like to say.)

Taken literally, this seems to mean nothing at all—*what* reality? I don't know what this means. But it is obvious what Hardy compares mathematical propositions with: namely physics.[31]

Hardy thinks that mathematics is about mathematical features of reality in the same way that physics is about physical features of reality. The view that Wittgenstein here ascribes to Hardy nicely parallels the one that we earlier saw Lovibond ascribe to Wittgenstein: the truth or falsity of a particular sort of statement depends on whether the statement in question accurately describes or represents the relevant sorts of features of reality. Hardy says that the truth (or falsity) of mathematical statements depends on their *correspondence* (or lack of correspondence) to mathematical reality. Hardy's picture is the same as that of Blackburn's 'realist': mathematics and physics have different subject-matters, but in each of these regions of discourse language functions in a parallel way. Its function in each case is to describe the corresponding region of reality. Wittgenstein continues:

Suppose we said first, 'Mathematical propositions can be true or false.' The only clear thing about this would be that we affirm some mathematical propositions and deny others. If we then translate the words 'It is true...' by 'A reality corresponds to...'—then to say a reality corresponds to them would say only that we affirm some mathematical propositions and deny others. We also affirm and deny propositions about physical objects.—But this is plainly not Hardy's point. If this is all that is meant by saying that a reality corresponds to mathematical propositions, it would come to saying nothing at all, a mere truism: if we leave out the question of *how* corresponds, or in what sense it corresponds.[32]

According to Wittgenstein, when Hardy says that a reality corresponds to mathematical propositions, he wants to be saying more than just that we affirm some mathematical propositions and deny others. He wants to be saying how, or in what sense, a reality corresponds; and his answer to that question is: *in the same way* that empirical features of reality correspond to empirical propositions.

31. *Wittgenstein's Lectures on the Foundations of Mathematics: Cambridge, 1939*, ed. Cora Diamond (Chicago: University of Chicago Press, 1976) [henceforth referred to as *LFM*]; p. 239.
32. *LFM*, p. 239.

It is Wittgenstein's criticism of this idea that Putnam is centrally concerned to elucidate in his essay:

> Wittgenstein was scornful of the idea that talk of numbers or sets is analogous to talk of objects.[33] We should not think that set theory has discovered an unimaginably large 'universe' of intangible objects; this is all confusion....
>
> Wittgenstein claimed that it is nonsense to say that following rules involves a special relation to mental (or Platonic) objects. Can't one respond, 'then what does make these sorts of claims true'? Isn't that a *real* question? Are you praising Wittgenstein for ignoring it?
>
> But this supposed 'real question' rests on the following picture: truth is what results when a statement 'is made true by' (or 'corresponds to') *something*. But, while that picture fits *some* statements, for example, the statement that a sofa is blue may 'correspond' to a certain blue sofa on a particular occasion—it doesn't fit other familiar statements without strain. (pp. 247, 253)

Putnam's way of summarizing Wittgenstein's point here is to say that we have a certain picture, derived from the case of empirical description (and, in particular, from physics), of what it is for truth to result when a statement 'is made true by' (or 'corresponds to') *something*; and, although we do (and should feel free to continue to) speak of mathematical statements as being true or false, we go awry when we try to extend this picture of what it is for something 'to correspond to a reality' to mathematics.

A little later on, Putnam writes:

> [W]hat of the question, 'Are mathematical propositions about reality?' I remind you of Wittgenstein's remark that our paradigm of a reality is the thises and thats we can point to.[34] As we get farther and farther away from away from these, our 'hold' on the notion weakens. This is not to say that the only realities there are [are] observables, but it is to say that the less what we are talking about is analogous to the thises and thats we can point to, the less sense it has to ask about whether what we are talking about is or is not a 'reality'. (pp. 261–2)

This passage suggests that insofar as we are talking about 'the thises and thats we can point to' it makes perfect sense to speak of a reality to which our talk corresponds; but, as we move farther and farther away from this particular language game—from this particular

33. Putnam here footnotes Lecture XXVI of *LFM*.
34. Putnam here footnotes the beginning of Lecture XXV, *LFM*, p. 240.

'paradigm of a reality'—it makes less and less sense (even in those regions of our talk where we distinguish between truth and falsity) to speak of a reality to which our talk corresponds.

Though this touches on a central feature of what Wittgenstein is up to in the *Lectures on the Foundations of Mathematics*, it is not quite a faithful paraphrase of what Wittgenstein says in the passage in question. What he says is:

> We have here a thing which constantly happens. The words in our language have all sorts of uses; some very ordinary uses which come into one's mind immediately, and then again they have uses which are more and more remote.... A word has one or more nuclei of uses which come into everyone's mind first....
>
> So if you forget where the expression 'a reality corresponds to' is really at home—
>
> What is 'reality'? We think of 'reality' as something we can *point* to. It is *this*, *that*.
>
> Professor Hardy is comparing mathematical propositions to propositions of physics. This comparison is extremely misleading.[35]

Wittgenstein's point is not that there is a paradigmatic context in which the expression 'a reality corresponds to' is at home and if we deviate too far from that context we necessarily fail to make sense. Rather his point is that we need to distinguish different kinds of things we can (and do) mean by this expression. We can deviate very far from this (paradigmatic) context and still make sense. There are perfectly good things that we can mean by calling on this expression in the context of talking about mathematics. Since we (rightly) feel that there is something to be meant when we call upon this expression in the context of talking about mathematics, we are not going to be (nor should we be) satisfied by someone who comes along and tells us that we fail to make sense when we attempt to use this expression in this context.

Putnam is quite right, nevertheless, that Wittgenstein thinks that there is a danger that, in our philosophizing about mathematics, we will end up employing the expression 'a reality corresponds to' in a way that fails to make sense; and this is indeed what he thinks happens in Hardy's case. But his complaint about Hardy is not that he strayed too far from a certain paradigmatic use of the expression. (In a sense it is the opposite: that Hardy is unable to free himself,

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35. *LFM*, pp. 239–40.

when employing the expression in an utterly different context, from imagining that he is somehow still employing it in a way that is closely akin to its paradigmatic use.) Wittgenstein's point, when cautioning us about the possibility of straying into nonsense when philosophizing, never takes the form of an injunction to the effect that 'you must use this expression in this way and no other or else you will be speaking nonsense.'[36] The source of our failure to mean something, on Wittgenstein's view, never lies with the form of words itself. (Indeed, Wittgenstein is a master at finding a context which reveals that there is something we might mean after all by some unusual—and at first not obviously intelligible—combination of words.) The trouble, according to him, comes when we unwittingly run together distinct sorts of uses to which such a form of words might be put.

Wittgenstein touches on this topic at the beginning of the above passage. ('A word has one or more nuclei of uses which come into everyone's mind first.') Whenever we fail carefully to distinguish between the possible uses to which an expression can be put, the tendency will be to have a certain 'nucleus of use' come into our minds, occluding and distorting our view of the expression's alternative possible uses.[37] What happens when we are thus 'bewitched' by a nuclear use of an expression is, according to Wittgenstein, that we end up trying to be in two language-games at once (and therefore not quite in either): in the (philosophically puzzling) language-game which we are trying to understand and, at the same time, in the (nuclear) one which the expression—viewed in isolation from its use—calls immediately to our mind. We view the former use through the lens of the latter, and become puzzled by the features of the latter sort of case which are missing from the former.

When, in doing philosophy of mathematics, we call upon the expression 'a reality corresponds to', then what we inevitably think of, Wittgenstein says, is the sort of 'reality' that we can *point* to (and about which we can say: 'It is *this, that*.').[38] When we then try to

36. To the extent that he has an injunction to offer at all at this level of generality, it would be better put thus: 'Say whatever you like, as long as you don't confuse yourself!'.

37. This can happen even when the core or nuclear use in question has an utterly unrelated grammar from that of the use of the expression to which we presently wish to attend; and even when the alternative use is of a firmly established and familiar nature.

38. This is the point of the remark ('What is "reality"? We think of "reality" as something we can *point* to. It is *this, that*') which Putnam misunderstands.

go on and understand what it means when one says 'mathematics corresponds to reality', we try to understand what is meant on the model of the sort of relation that 'There is a sofa in my living-room' bears to the state of affairs that it is about. The solution here, according to Wittgenstein, lies not in denying (à la the logical positivists) that there is something to be meant by such a way of talking, but by showing what *can* be meant by it. Wittgenstein devotes the next several lectures of *Lectures on the Foundations of Mathematics* to providing an overview (what he, in *Philosophical Investigations*, later calls a perspicuous representation) of the different things that one might mean by the expression 'statements of such-and-such type *correspond to* (or: are *responsible to*) a reality'. This requires distinguishing, among other things, the various sorts of things this expression can mean when it is applied to mathematical statements[39], and the various sorts of things it can mean when it is applied to other sorts of statements. The use to which Hardy attempts to put the expression is to be seen in the end as a confused attempt to amalgamate several of these available possibilities of use in such a way as to fail in the end to be saying anything at all.

## VII

*The importance of mixed statements.* 'Science', Putnam observes, 'doesn't divide into a part which is empirical and a *different* part which is mathematics. It contains 'mixed statements', statements which are empirical but which speak of... functions and their derivatives as well as of physical entities' (p. 250).

Even if we concede that propositions of pure mathematics are not about reality in the same way that propositions of physics are about reality, what about the case of mixed statements? Are we to conclude that such statements are 'about' physical entities, but are *not* 'about' functions and their derivatives? In Lecture XXVI of the 1939 lectures, to which Putnam refers us in this connection,

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39. There is one thing that such an expression might perfectly well be taken to mean that Wittgenstein mentions in these lectures only in order to put aside—the case of (what he calls) 'mathematical responsibility': 'Given certain principles and laws of deduction, you can say certain things and not others.' He takes this to be a kind of responsibility which mathematical propositions can only bear to one another, rather than to something over and above the mathematical framework itself. Thus he goes on to say: 'But it is a totally different thing to ask: "And now what is *all* this responsible to?"' (*LFM*, p. 240). By the time he writes *Philosophical Investigations*, he has gained a much deeper appreciation of how puzzlement about the sheer possibility of normative constraint as such can help fuel a platonist appeal to a 'super-reality'.

Wittgenstein resists this conclusion: '[M]athematical propositions do not treat of numbers. Whereas a proposition like "There are three windows in this room" *does* treat of the number 3.'[40] Wittgenstein insists, however, that we must be careful here to distinguish between the sense in which mixed statements 'treat of' numbers (or functions and their derivatives) and the sense in which they 'treat of' physical entities. His route to this point is a characteristically roundabout one.

Lecture XXVI begins, rather peculiarly, as follows:

> If one talks about a reality corresponding to mathematical propositions and examines what that might mean, one can distinguish two very different things.
>
> (1) If we talk of an experiential proposition, we might say a reality corresponds to it, if it is true and we can assert it.
>
> (2) We may say that a reality corresponds to a *word*, say the word 'rain'—but then we mean something quite different. This word is used in 'it rains', which may be true or false; and also in 'it doesn't rain'. And in this latter case if we say 'some phenomenon corresponds to it', this is queer. But you might still say something corresponds to it; only then you have to distinguish the sense of 'corresponds'.[41]

What does Wittgenstein mean when he says 'we may say that a reality corresponds to a *word*'? He is here inventing a new way of speaking—one which he hopes will provide a useful point of comparison when considering what it might mean to say of mathematical propositions that they correspond to—or are responsible to—a reality. Wittgenstein continues:

> We *can* explain the *use* of the words 'two', 'three', and so on. But if we were asked to explain what the reality is which corresponds to 'two', we should not know what to say. This? [Wittgenstein raised two fingers and pointed to them.] But isn't it also six, or four?
>
> We have certain words such that if we were asked, 'What is the reality which corresponds?', we should all point to the same thing—for example, 'sofa', 'green', etc. But 'perhaps', 'and', 'or', 'two', 'plus' are quite different.[42]

If you were asked what reality corresponds to words such as 'sofa' and 'green', then you might answer by simply pointing to a sofa

40. *LFM*, p. 250.
41. *LFM*, p. 247.
42. *LFM*, p. 248.

or to something green; but in the case of words such as 'perhaps', 'and', 'plus' or 'two', you would be at a loss as to what to point to. Wittgenstein suggests that we say of such words that the reality which corresponds to them is our having a use for them; and he suggests that this is analogous to something one might mean in talking of the reality which corresponds to a proposition of mathematics:

> So with these words 'and', 'or', etc., we can say that the reality which corresponds to them is that we have a use for them.
>
> What I want to say is this. If one talks of the reality corresponding to the propositions of mathematics or of logic, it is like speaking of a reality corresponding to these *words*—'two' or 'perhaps'—more than it is like talking of a reality corresponding to the *sentence* 'It rains'....
>
> To say 'A reality corresponds to "$2 + 2 = 4$"' is like saying 'A reality corresponds to "two"'. It is like saying a reality corresponds to a rule, which would come to saying: 'It is a useful rule, *most* useful—we couldn't do without it for a thousand reasons, not just *one*.'[43]

Wittgenstein, at the end of Lecture XXV, suggests that we can speak of 'a reality corresponding to a rule'. Yet, he says there of this correspondence that it won't be 'of the kind we first expect', but rather will lie in the rule being of such a sort that it is rendered important and justified by all sorts of facts—facts about the world and about us—so that we shall not want (and perhaps may not even know what it would mean) to do without it. We are thus presented with the following suggestion for what it is for there to be a reality corresponding to a rule: for there to be any number of facts (mostly of a very general—and therefore easily overlooked—nature) about us and about the world which make it very useful to have the rule as part of the overall package of thought and language which provides us with our means of description. Wittgenstein takes this way of thinking about how rules can be responsible to reality to provide a way of understanding what it might mean to say of a mathematical proposition that it corresponds to a reality. Building on this suggestion, in Lecture XXVI, he draws attention to some of the differences between activities in which we first lay the groundwork for our means of description and employments of language which then go on to deploy those means of description.

43. *LFM*, p. 249.

'You might say' that mathematical propositions are in this respect more akin to '*preparations* for a use of language', and can be thought of in this respect as 'part of the *apparatus* of language' rather than 'part of the application of language'.[44] Wittgenstein goes on to say:

> It is the whole system of arithmetic which makes it possible for us to use '900' as we do in ordinary life. It *prepares* '900' for the work it has to do. In this sense, mathematical propositions do not treat of numbers. Whereas a proposition like 'There are three windows in this room' *does* treat of the number 3.[45]

In this sense of 'propositions treat of numbers', coming to understand the sense in which propositions can treat of numbers requires first understanding the role mathematics plays in mixed statements—even such elementary mixed statements as 'There are three windows in this room'.

There are diverse kinds of mixed statements—diverse ways in which mathematical rules and methods are integrated into empirical statements, and diverse ways in which mathematical and non-mathematical language-games interweave with one another— as many as the sorts of application mathematics has in our lives. It is this diversity of function to which Wittgenstein thinks we need to attend, not merely in order to wallow in the motley, but because it is the interplay of these functions which occasions our puzzlement. Despite the tremendous differences at the level of detail, Wittgenstein's treatment of mathematics can in this respect be seen in its general approach to parallel his treatment of avowals (outlined in section IV). In both cases, he seeks to show how non-descriptive and descriptive dimensions of language are intertwined in—and interdependent on—one another. In the case of mathematics, what this means is that he seeks to uncover the diverse kinds of ways in which mathematics becomes embedded in forms of description. Two kinds of example come in for special attention in this regard in *Lectures on the Foundations of Mathematics*: (1) ways in which the formulation of a kind of ideal case enables us to describe various actual cases as departures of one or another sort from the ideal; and (2) ways in which mathematics is integrated into the body of standards for carrying out methods of arriving at descriptive propositions (as, for example, in locating miscounts or mistakes in

44. *LFM*, p. 249.
45. *LFM*, p. 250.

measurement). The point of such examples is to help us 'break free', as Putnam puts it, 'of the picture that if a statement is true there must be a *something* which 'makes' it true' (p. 252)—the picture that there must be a separate region of reality to which each region of assertoric discourse corresponds. Each of these examples are meant to underscore how mathematical propositions can be brought to bear on 'a reality' (not by describing how things are in some extra-empirical region of reality, but rather) by enabling the myriad kinds of application which constitute integral aspects of the framework within which the activity of empirical description takes place.

## VII

*Conclusion.* The point of providing the preceding brief overview of Lectures XXV and XXVI of *Lectures on the Foundations of Mathematics* is to provide textual evidence for the following exegetical claim: in his discussion of the question of whether mathematical propositions 'correspond to' a reality, Wittgenstein aims to conclude neither (1) that they simply do correspond in the same way that any other proposition does, nor (2) that they simply don't, nor (3) that it would be a good idea to abstain from answering the question. Each of Blackburn's three interpretative strategies fails to account for large stretches of the text. Yet each of them can also point to stretches of the text which, taken in isolation from Wittgenstein's larger aims, will appear to support their interpretation. All three strategies share a common assumption: that Wittgenstein takes us to understand 'the question' as posed. (Hence the three interpretative strategies appear to exhaust the logical space of possible responses: the question as posed can be answered in only one of three ways—affirmatively, negatively, or evasively.) Whereas what Wittgenstein does instead is to begin by showing us the diverse questions we might be asking, when calling upon the words which we take to express the question—showing us that on some understandings of what we might be asking the answer is affirmative, on others negative; and that there is no answer to 'the question' as posed, because there is no clear question but only a form of words hovering indeterminately between these diverse possibilities of use.[46]

46. I am indebted to David Finkelstein for conversations on these topics.