Time Travel and the Reality of Spontaneity*

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Abstract

Contrary to the informed consensus, time travel implies spontaneity (as distinct from chance) so that time travel can only be of the second kind.

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No patient who recovers without a physician can logically attribute his recovery to spontaneity. Indeed, under a close examination spontaneity disappears. For everything that occurs will be found to do so through something, and this 'through something' shows that spontaneity is a mere name and has no reality. Medicine, however, because it acts 'through something' and because its results may be forecasted, has reality.

Hippocrates: *The Art**

1 INTRODUCTION

In this paper I consider time travel both as a real physical possibility and as a means of re-examining fundamental assumptions about time. Though stemming from a new mathematical model of the evolutionary equations of physics, the arguments in this paper are robust enough to be stated with the

*cited in S. Sambursky, *Physics of the Stoics*, reprint 1987, Routledge and Keegan Paul, London, p. 51–52.

technicalities only in the background. Such a style of exposition also seems desirable in view of the widespread interest in time-travel.

1.1 Background

Thorne and his consortium have proposed[1] time machines based on 'wormhole' solutions, exploiting the fact that the Hilbert-Einstein equations are silent about the (algebraic) topology of spacetime. While the 'wormhole' solutions involve 'exotic matter'—matter with negative mass and positively amusing properties[2]—Gott[3] has shown that closed timelike curves (CTCs) may also arise with cosmic strings. On the other hand, Hawking[4] has argued that there is excellent empirical evidence for chronology protection since we have not been invaded by hordes of tourists from the future.

1.2 Two kinds of time-travel

For the purposes of this paper it helps to make an informal distinction between two types of time travel: (i) with time-machines and (ii) without machines. An example of the second kind of time travel is transfer of information using a retarded interaction going forward in time and an advanced interaction returning backward in time.[5] Access to advanced interactions[6] is possible under the hypothesis of a microphysical tilt in the arrow of time.[7] Strictly speaking, a 'tilt' does not involve any new hypothesis; the usual hypothesis of 'causality' is rejected, so that the evolution of a many-particle system is governed by a different category of (mixed-type functional differential) equations of motion.

Time travel of the second kind contemplates only transfer of information without involving physical transport of the traveler's body. Nevertheless, some (diminished) kind of intervention in the past is possible, in principle, because information may be transferred from present to past using advanced interactions, though the bandwidth is a very small fraction of the bandwidth for information transfer to the future using retarded interactions.

1.3 Aim

The aim of this paper is to stand on its head the standard conclusion derived from the paradoxes of time travel,[8] especially for the case of time travel without machines.

2 THE PARADOXES OF TIME TRAVEL

2.1 The grandfather paradox

The grandfather paradox is well-known:[9] Tim travels into the past to kill his grandfather when Grandfather was yet a boy; but that would mean that Tim could not have been born and so could not have killed Grandfather. The generally accepted conclusion is as follows. Since Tim did not kill Grandfather in the 'original' 1921, consistency demands that neither does he kill Grandfather in the 'new' 1921. The time traveler must be prepared for unexpected constraints; Tim must fail in the enterprise of killing Grandfather for some commonplace reason. Perhaps some noise distracts him at the last moment, perhaps he misses despite much target practice, perhaps even Tim killed Grandfather only to discover his true antecedents! As summarised by Woodward,[10] 'Time travel makes "free will" irrelevant'.

2.2 Mundane time

One could elaborate the paradox as follows. Mundane time has a structure[11] which is past linear and future branching (Fig. 1). If one bends it around in a circle and joins future to past then either future branching or past linearity must fail, so that one obtains the supercyclic time of Fig. 2.

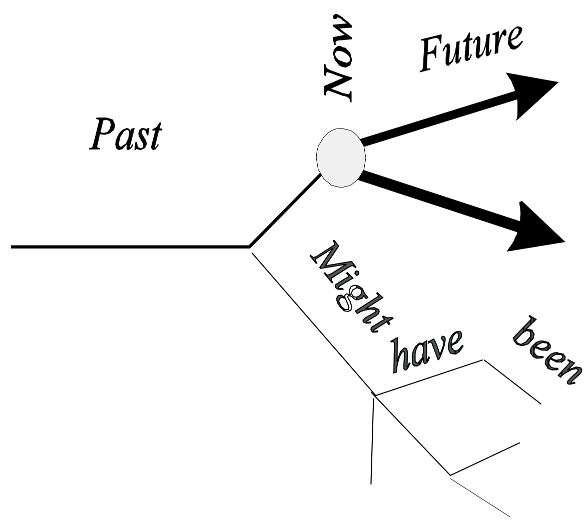

Figure 1: **Mundane time**: In everyday life, one philosophizes about the past but agonizes about the future on the belief that one's actions partly decide the future, but leave the past unaffected.

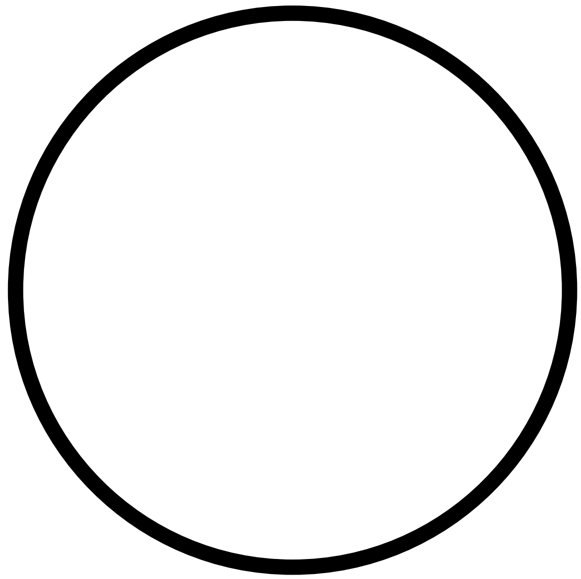

Figure 2: **Supercyclic time**: All instants of time are arranged in a closed cycle, so that any instant 'precedes' any other. Such a situation cannot be readily described with the binary earlier-later relation implicit in the tense-structure of Indo-European languages.

2.3 Popper's record postulate

Why not give up past linearity? This could be problematic, since the significance of experimental records would then diminish, for an experimental record could not, then, claim to represent *the* past. Popper[12] proposed a record postulate, the 'principle of the unbroken connection of world lines' which he formulated in operational terms as follows.

> Any 'observer' (local material system) can begin, at any instant, a record (causal trace); make successive entries into that record; and arrange for the preservation of the record for any desired

> finite period of time. (By 'can' the following is meant; the theoretical possibility of any world-line, to be considered as consistent with the laws of nature, must not entail the impossibility of the operations described in the above principle.)

World lines closed in time now lead to a contradiction, since, for consistency, the closed world line 'must be infinitely and absolutely repetitive,' and hence 'would entail periodic destruction of every single record,' since otherwise the record 'would not be fully repetitive but would constantly enrich itself upon every renewal of the closed journey.'

2.4 The chronology condition

Appeal to the future branching alone is also adequate. 'The same result may be obtained, even less *ad hoc*,' continues Popper, 'by adopting a "principle of indeterminism"; this too would automatically exclude all cosmological solutions permitting closed world-lines.' Hawking and Ellis[13] similarly argue that future branching cannot be lightly rejected, since 'all of our philosophy of science is based on the assumption that one is free to perform any experiment.' Hence, they are 'much more ready to believe' their *chronology condition*, viz. that there are no CTCs. (Hawking's latest position, marks a retreat from postulate to conjecture, and adds the bit about making the universe safe for historians.)

2.5 The paradoxes re-examined

In brief, the informed consensus favours the standard conclusion[14] that time travel is antithetical to spontaneity or 'free will'. I will argue that the exact opposite is true.

Let us re-examine the grandfather paradox, for two of its key features seem to have gone unnoticed. We need to shift our attention from the death of Grandfather to the birth of Tim, that is to the first appearance of Tim in this world. Let us suppose that Tim's 'birth' (i.e. his chronologically earliest appearance in the world) was earlier than his biological birth from his mother's womb. Let us further suppose that the event of Tim's 'birth' did not go unobserved. Say, Tim's house had earlier been occupied by an eccentric scientist, who had called another half-a-dozen scientists for tea. Tim, being a tyro at time travel, appeared bang in the midst of this tea party. The scientists, true to their profession, merely observed and theorised: they did not hop around or interfere in what they took to be a demonstration to challenge their theoretical capabilities, specially arranged by their eccentric host (who had disappeared into the kitchen). Naturally, they were all blase enough to regard it as a magic trick in bad taste. (Tim materialised with one foot on a saucer, and spilled tea on a guest.)

But we know better. We know that, however hard they might have tried, the scientists could not have found an explanation for the fact which was presented to them on a platter—no causal explanation that is. We know that Tim's appearance at the eccentric tea party really had nothing to do with anything prior to the tea party; it was causally inexplicable, spontaneous, so to say. The event of Tim's birth could be explained only with reference to the future.

2.6 Popper's pond

In the non-mechanical mode of time-travel, involving advanced interactions, Tim's 'birth' corresponds exactly to Popper's pond paradox. If a stone is dropped into a pond, ripples usually spread outwards (corresponding to a retarded wave). In the advanced case, the ripples converge spontaneously and throw the stone out of the pond. This sort of thing, though possible according to physical theory, is not usually observed unless one has filmed the sequence and plays the film backwards. But, says Popper,[15] 'no physicist would mistake the end of the film for its beginning; for the creation of a contracting circular wave followed by a zone of undisturbed water would be (causally considered) miraculous.' Popper's own argument involved coherence: for constructive interference of primary wavelets, to produce a converging ripple, by Huyghens' principle, one would need coherence, and this would be practically impossible to arrange without 'organization from the centre'.

One can strengthen the first part of Popper's argument, by giving more general and stronger arguments which show the theoretical impossibility of explaining anticipatory phenomena from the past. A causal explanation of anticipatory phenomena is mathematically impossible for exactly the same reason that a teleological explanation of purely history dependent phenomena is mathematically impossible. Purely anticipatory phenomena may be explained only by reference to the future, just as history dependent phenomena may be explained only by reference to the past, for the reasons sketched in Figs 3, 4, 5, reproduced from Ref. 7, where they are explained in more detail. (A quick exposition is also provided in the appendix to this note.)

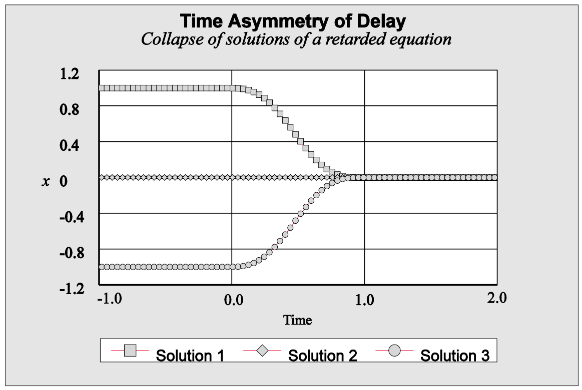

Figure 3: Three solutions of a retarded equation. The different past histories prescribed over $[-1, 0]$ all result in the same future for $t \geq 1$. Retrodiction is hence impossible from future data prescribed over $t \geq 1$. Teleological explanations are impossible, with history-dependent evolution.

The pond paradox is now seen to arise from Popper's metaphysical stipulation that all phenomena must admit a causal explanation, so that phenomena not admitting a causal explanation cannot possibly occur. This position is reminiscent of the Stoics who derived *heimarmene* (fate) from *eiro* (string beads), so that the evolution of the world was analogous to moving beads on a necklace; the slightest spontaneous swerve of the atoms (Epicurean *clinamen*) would break the string: 'the cosmos would break up and be shattered…if some uncaused movement were to be introduced into it.[16] Perhaps it is necessary to restate that a metaphysical stipulation ('everything must have an antecedent cause'), as used e.g. by Hippocrates, may not be used to decide

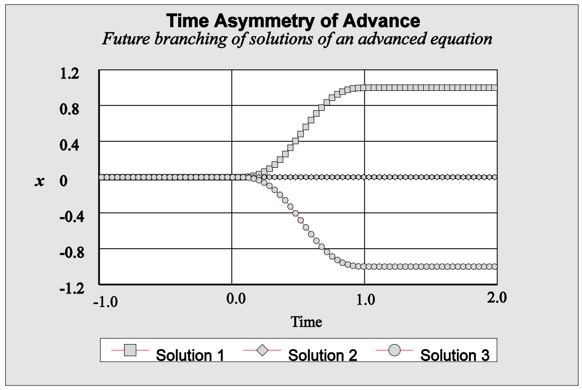

Figure 4: Three solutions of an advanced equation: different futures over [1,2] correspond to the same past for $t \leq 0$. With anticipation past fails to decide the future, for one past may correspond to many futures, in this time reverse of Fig. 3. Hence, causal explanations are impossible with anticipatory evolution.

admissible phenomena. The existence or non-existence of the spontaneous can only be decided by observation.

2.7 The empirical evidence

The absence of hordes of tourists from the future is, therefore, no evidence against time travel of the second kind. It would be enough if we occasionally observe some spontaneous events.

2.8 The mechanization of spontaneity

A key feature of spontaneity in the above sense is that *spontaneity cannot be mechanized*, i.e., though time travel may be possible, time *machines* are not: only time travel of the second kind is possible. Popper's conclusions

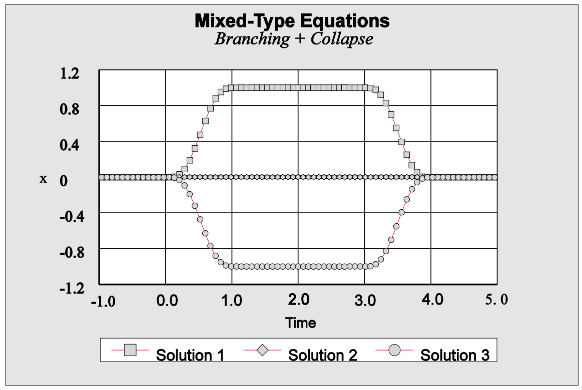

Figure 5: With a realistic mixture of history-dependence and a small amount of anticipation, the past still fails to decide the future. With this model, all phenomena do not admit causal explanations, so that spontaneity really is possible. The existence of a small tilt is exactly the condition for time-travel of the second kind.

from his pond paradox only need to be toned down: while the existence of a causal explanation cannot very well be a precondition for the occurrence of a phenomenon, without a causal explanation one cannot systematically *control* the phenomenon, or *arrange* for it to occur, or *mechanically* reproduce it. The Wellsian time-machine incorporates in its construction the intuitive idea of 'control' from the future. In the physics literature, the same idea was articulated in the context of the tachyonic anti-telephone: if Shakespeare used a tachyonic anti-telephone to dictate *Hamlet* to Bacon then, Benford et al.[17] argued, while Bacon would have chronological priority, Shakespeare remained the author of *Hamlet*—since Shakespeare was the one who had 'control'. But, in a situation where interactions may propagate from future to past, it is not clear that control from the future is any more possible than control from the past, and Fig. 5 sketches a counter-example: in some situations prescribing both past and future data may still be inadequate to determine a unique present. Similarly, the classical argument[18] to exorcise Maxwell's demon excludes only the mechanical form of the demon, which could lead to a controllable, hence possibly unboundedly large, decrease of entropy.

2.9 Spontaneity and chance

The relevance of Maxwell's demon is the following. Spontaneity, in the sense proposed above, differs from the notion of 'chance' in the sense of probabilistic ('stochastic') evolution, such as that of a stochastic process, where the probabilities of future states can be computed once the past states are known. (The meaning we have assigned to 'chance' is related to contemporary customary usage amongst physicists: for the last hundred years or so, it has been argued that probabilistic evolution accounts for entropy increase within classical reversible dynamics.) Mathematically, the difference is that evolution involving such 'chance' may be modeled by stochastic differential equations (Fig. 6), categorically distinct from the mixed-type functional differential equations which model evolution involving a tilt in the arrow of time. In physical terms, a key difference is that chance corresponds to 'mixing' while spontaneity, in the above sense, corresponds to 'sorting'.

That is, if the implicit assumption of some kind of 'mixing' or quasi-ergodicity is acceptable as a characteristic feature of 'chance' in the sense of probabilistic evolution, one might say that 'chance' results in an increase of entropy, whereas spontaneity in the above sense (e.g. a converging ripple)

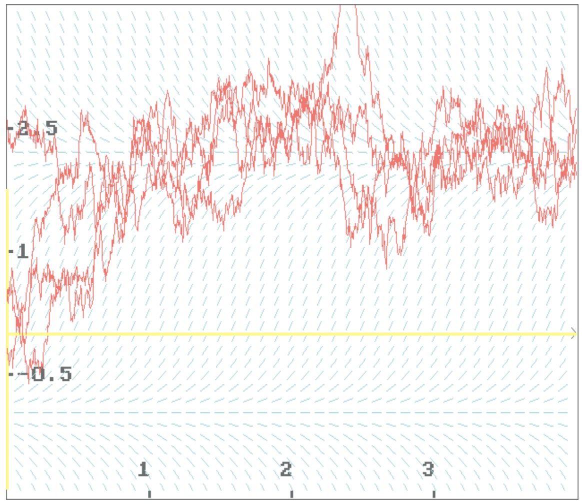

Figure 6: The figure shows some Brownian sample paths. The sample paths mix: the trajectories tend to 'forget' their past and asymptotically become statistically independent of it, unlike the background vector field (dashed lines) which corresponds to solutions of the unperturbed ordinary differential equation. 'Mixing' due to chance is believed to produce entropy rather than order (= negentropy).

clearly corresponds to a reduction in entropy, or to the creation of order. So, time travel of the second kind actually corresponds to spontaneous order creation.

Though general relativistic statistical mechanics (and the statistical basis of the stress-energy continuum) is problematic, the apparently necessary association of exotic matter (hence negative energies) with wormhole spacetimes suggests, in the standard MTW[19] approach to relativistic thermodynamics, that, one should expect a similar association of entropy reduction with time travel in the case of wormholes.

It is natural to conjecture that any macrophysical manifestation of spontaneity would involve, in an essential way, the one thing that has remarkably resisted mechanization: life. Specifically, I expect that a systematic microphysical tilt in the arrow of time would show up in the structure and dynamics of biological macromolecules. At present, solutions of the many-particle equations of motion with a microphysical tilt in the arrow of time are still being simulated, and compared with solutions of a stochastically perturbed form of the classical equations, and only preliminary results are available,[20, 21] so that it would not be in order to make a more definite statement. However, some general arguments connecting spontaneity in the above sense with 'human freedom' in the mundane sense of Fig. 1 may be found in Ref. 7.

At the microphysical level, spontaneity as a necessary correlate of non-locality is especially interesting in the context (Ref. 7) of the structured-time interpretation of quantum mechanics.

3 CONCLUSIONS

Time travel conflicts not with choice but with 'causality': if two-way interaction with the future is permitted, one can no longer hang on to 'causality' in the sense of demanding explanations exclusively from the past. Interactions propagating from future to present imply the occurrence of events that are causally inexplicable. Under the circumstances of time travel, one must allow for the reality of such spontaneous events, which differ from 'chance' events in creating order instead of destroying it. The mechanization of spontaneity, however, is impossible, so that time travel can only be of the second kind.

Appendix

A causal explanation of anticipatory phenomena is mathematically impossible: following the referee's suggestion to keep the paper self-contained, we reproduce here from Ref. 7, some mathematical details of the argument.

First, let us see why *a teleological explanation of history-dependent phenomena is mathematically impossible.* Fig. 3 shows three solutions of the retarded functional differential equation (FDE)

$$x'(t) = b(t)x(t-1) \tag{1}$$

where b is a continuous function which vanishes outside [0, 1], and satisfies

$$\int b(t)\ dt = -1 \tag{2}$$

For example,

$$b(t) = \begin{cases} 0 & : \quad t \le 0 \\ -1 + \cos 2\pi t & : \quad 0 \le t \le 1, \\ 0 & : \quad t \ge 1 \end{cases} \tag{3}$$

For $t \le 0$, the FDE (1) reduces to the ordinary differential equation (ODE) $x'(t) = 0$, so that, for $t \le 0$, $x(t) = k$ for some constant k ($= x(0)$).

Now, for $t \in [0,\ 1]$,

$$x(t) \quad = \quad x(0) + \int_0^t x'(s)ds$$

$$= \quad x(0) + \int_0^t b(s)x(s-1)ds$$

$$= \quad x(0) + x(0)\int_0^t b(s)ds \qquad (4)$$

since $x(s-1) \equiv k = x(0)$ for $s \in [0,1]$. Hence, using (2), $x(1) = 0$, no matter what k was. However, since $b(t) = 0$ for $t \geq 1$, the FDE (1) again reduces to the ODE $x'(t) = 0$, for $t \geq 1$, so that $x(1) = 0$ implies $x(t) = 0$ for all $t \geq 1$. Hence, the past of a system governed by (1) cannot be retrodicted from a knowledge of the entire future; for if the future data (i.e., values of the function for *all* future times $t \geq 1$) are prescribed using a function ϕ that is different from 0 on $[1, \ \infty]$, then (1) admits no backward solutions for $t \leq 1$. If, on the other hand, $\phi \equiv 0$ on $[1, \ \infty]$, then there are an infinity of distinct backward solutions. In either case, knowledge of the entire future furnishes no information about the past.

The actual solutions shown in the graph were obtained numerically, using the RETARD package of Hairer et al.[22]

In the advanced case, as suggested by Fig. 4, the argument is the time-symmetric counterpart of the above argument. In this case, the equation solved was the analogous advanced FDE

$$x'(t) = b(t)x(t+1) \qquad (5)$$

where the function b has the same properties as before, except that

$$\int b(t)dt = 1 \qquad (6)$$

For example,

$$b(t) = \begin{cases} 0 & : \quad t \leq 0 \\ 1 - \cos 2\pi t & : \quad 0 \leq t \leq 1, \\ 0 & : \quad t \geq 1 \end{cases} \tag{7}$$

The reasoning proceeds in an entirely analogous manner. For $t \geq 1$, the FDE (5) reduces to the ODE $x'(t) = 0$, so that, for $t \geq 1$, $x(t) = k$ for some constant k $(= x(1))$.

Now, for $t \in [0,\ 1]$,

$$\begin{aligned} x(t) &= x(1) - \int_0^t x'(s)ds \\ &= x(1) - \int_0^t b(s)x(s+1)ds \\ &= x(1) - x(1)\int_0^t b(s)ds \end{aligned} \tag{8}$$

since $x(s+1) \equiv k = x(1)$ for $s \in [0, 1]$. Hence, using (6), $x(0) = 0$, no matter what k was. However, since $b(t) = 0$ for $t \leq 0$, the FDE (5) again reduces to the ODE $x'(t) = 0$, for $t \leq 0$, so that $x(0) = 0$ implies $x(t) = 0$ for all $t \leq 1$. Hence, the future of a system governed by (5) cannot be predicted from a knowledge of the entire past; for if the past data (i.e., values of the function for *all* past times $t \leq 0$) are prescribed using a function ϕ that is different from 0 on $[-\infty,\ 0]$, then (5) admits no forward solutions. If, on the other hand, $\phi \equiv 0$ on $[-\infty, 0]$, then there are an infinity of distinct forward solutions. In either case, precise knowledge of the entire past furnishes no information about the future. The actual numerical solutions shown were obtained by a time-symmetric modification of the RETARD package.

Fig. 5 shows solutions of the mixed-type equation

$$x'(t) = a(t)x(t-1) + b(t)x(t+1) \tag{9}$$

where b has the same properties as in (6), and the continuous function a now has support on the interval $[2,\ 3]$, and satisfies

$$\int_2^3 a(t)dt = -1 \tag{10}$$

The solutions may be obtained by combining the reasoning used in the preceding two cases.

Physically, retarded FDE arise as the equations of motion of charged particles, using the Heaviside-Lorentz force law, and assuming fully retarded Lienard-Wiechert potentials.[7, 20, 21] Mixed-type equations arise as the equations of motion of charged particles in the case where most electromagnetic radiation is retarded, but some of it may be advanced, i.e., we use a convex combination of retarded and advanced Lienard-Wiechert potentials. This possibility has often been excluded on metaphysical grounds, without studying the immediate empirical consequence (of spontaneity), here and now, of this assumption.

Finally, in the case of Fig. 6 the equation solved was a stochastic differential equation of the type

$$dX_t = a(t,\ X_t)dt + b(t,\ X_t)dw(t) \tag{11}$$

where $w(t)$ is the standard Brownian motion (Wiener process). The background vector field relates to the deterministic part of this equation, obtained using only the drift function $a(t,\ X_t)$ and setting the dispersion function $b(t,\ X_t)$ to zero. The sample paths shown in the figure were obtained using this author's package STOCHODE for the solution of stochastic differential equations (SDE's) driven by Brownian or Lévy motion.

Given the vast difference between the mathematical theory underlying SDE's ('chance') and that underlying mixed-type FDE's ('spontaneity') it is surprising why it should be hard to discriminate between the physical consequences of the two. In the case of SDE's ('chance') the future is *epistemically* uncertain since (a) the past is uncertain, and (b) the relation of past to future is probabilistic rather than deterministic. In the case of mixed-type FDE's ('spontaneity'), the future is *ontically* uncertain, regardless of knowledge of the past, because past does not entirely determine the future

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3. J. R. Gott III, *Phys. Rev. Lett.* **66** (1991) 1126–29.

4. S. W. Hawking, *Phys. Rev. D* **46** (1992) 603–11.

5. Instantaneous transfer of information at superluminal speeds naturally does not contradict the theory of relativity, which only requires that the speed of light should be constant in any local Lorentz frame. The usual

assertion of a conflict between the two implicitly assumes 'causality' which is not quite applicable under the circumstances.

6. Contrary to the claims made in the current literature, the Wheeler-Feynman absorber theory may not be used for this purpose since it is internally inconsistent, while the Hoyle-Narlikar absorber theory may not be used since it is externally inconsistent. See, C. K. Raju, *J. Phys. A: Math. Gen.* **13** (1980) 3303–17.

7. C. K. Raju, *Time: Towards a Consistent Theory*, Kluwer Academic, Dordrecht, 1994. Since retarded interactions dominate, one may still distinguish between forward and backward directions of time.

8. For this purpose, various subtle distinctions such as the distinction between Wellsian and Gödelian forms of time travel do not seem critical. See, John Earman, "Recent work on time travel." In: *Time's Arrows Today*, Steven F. Savitt (ed), Cambridge Univ. Press, 1995.

9. The presentation is adapted from D. Lewis, in: *The Philosophy of Time*, R. L. Poidevin and M. Macbeath (eds), Oxford University Press, Oxford, 1993, pp. 134–46.

10. J. F. Woodward, *Found. Phys. Lett.* **8** (1995) 1–39, p. 2.

11. In the sense of temporal logic, see, e.g. N. Rescher and A. Urquhart, *Temporal Logic*, Springer, Wien, 1973; W. H. Newton Smith, *The Structure of Time*, Routledge and Kegan Paul, London, 1980.

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13. S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime*, Cambridge University Press, Cambridge, 1973, p. 183.

14. A number of differing 'non-standard' views have, however, been documented by P. J. Nahin, *Time Machines: Time Travel in Physics, Metaphysics, and Science Fiction*, American Institute of Physics, New York, 1993.

15. See K. R. Popper cited in Note 12 above. The original discussion was in K. R. Popper, *Nature* **177** (1956) 538; **178** (1956) 382; **179** (1957) 297; **181** (1958) 402.

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