The Argument From Small Improvement is a Red Herring

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ABSTRACT: The much-discussed 'Argument from Small Improvement' has been advanced both as a reason to reject (tripartite) Completeness, one of the standard axioms of decision theory, and to accept the possibility of rationally incomparable choices. But this form of argument cannot be an effective basis for either of these conclusions, unless one already has some prior, independent reason to prefer Transitivity to Completeness as a constraint on rational preferences (or rational values). In particular, I show how a *reverse* argument from small improvement can be constructed, starting from the assumption of tripartite Completeness, to the conclusion that Transitivity fails. I conclude that this form of argument as it has been standardly presented in the literature is a kind of 'red herring'. We can only make progress here by evaluating the reasons *independent* of either such argument to prefer one or other of Transitivity or Completeness.

'**red herring**', *noun*
1: a herring cured by salting and slow smoking to a dark brown color
2: something that distracts attention from the real issue
(Merriam-Webster Dictionary)

1. Introduction

In the theory of rational choice, the 'argument from small improvement'[1] is a much-discussed argument whose immediate conclusion is to *deny* the thesis of *tripartite completeness,* or just 'Completeness' for short. This is the rational requirement that for any two alternative options one must be strictly preferred to the other (or put in terms of value relations: one is 'better than' the other) or else the subject is indifferent between the two (they are both 'equal in value').

- **(TRIPARTITE) COMPLETENESS**: $(x \succ y) \lor (y \succ x) \lor (x \sim y)$

Let's call the denial of (tripartite) Completeness, 'Incompleteness' – viz., the claim that it is possible for a rational agent to find that none of these three relations hold between some pair of options.

[1] The argument is first presented in de Sousa 1974. It receives perhaps its most extensive defence and elaboration in the work of Ruth Chang – see, e.g. Chang 1997, 2002, 2009, 2017. See also Carlson 2010.

One reason that the argument from small improvement has been so frequently discussed is that its conclusion has then often been used in support of the further claim that it is possible for some pairs of options to be rationally *incomparable*[2] or *incommensurable*[3]. If such incommensurability really could be established on this basis it would be a hugely significant result for our understanding both of rationality and of value. However, other explanations/interpretations are possible. For example, you might think that incompleteness is rather due to *indeterminacy* or imprecision or vagueness[4]. Or you might think, as Ruth Chang has influentially argued[5], that tripartite incompleteness is due to the existence of a 4th kind of relation of comparability – being 'on a par'[6].

The argument from small improvement relies (as we will see in sections 2 and 3, below) on the assumption of 'P.I. transitivity' – that is, Preferred-Indifferent transitivity[7].

- **P.I. TRANSITIVITY**: $[(x \succ y) \ \& \ (y \sim z)] \rightarrow (x \succ z)$

P.I. transitivity is entailed by the transitivity of weak preference – or just 'Transitivity' for short – which, like Completeness, is a standard axiom in decision theory:

- **TRANSITIVITY (OF WEAK PREFERENCE)**: $[(x \succcurlyeq y) \ \& \ (y \succcurlyeq z)] \rightarrow (x \succcurlyeq z)$

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2 See de Sousa 1974, Raz 1986, Sinnott-Armstrong 1988. For a useful recent discussion of incomparability and small improvements see Yan 2022.

3 Though the terms 'incomparable' and 'incommensuarable' have sometimes been used interchangeably, one fairly standard way of distinguishing between them is to use 'incomparable' to apply to particular, concrete options, and to use 'incommensurable' to describe different kinds or dimensions of value. See Hsieh (2016), Steele & Stefánsson (2016), Chang (1997).

4 For discussion of this indeterminacy option, see Broome 1997, Espinoza 2008.

5 See Chang 1997, 2002 – see also, Rabinowicz 2008.

6 In a similar vein, Parfit (1984) introduces the notion of 'roughly comparable', whilst Griffin (1986) speaks of 'rough equality'. Espinoza (2008) suggests that these relations are both equivalent to Chang's 'parity'.

7 One could, by simply permuting the labels of the 3 options, formulate the standard argument from small improvement based instead on I.P. transitivity: $[(x \sim y) \ \& \ (y \succ z)] \rightarrow (x \succ z)$. This would be a mere notational variation. (I.P. transitivity is also, of course, entailed by the transitivity of weak preferences.)

Given that the literature on rational choice is generally concerned with Transitivity (of weak preference) rather than with P.I. transitivity in particular, in what follows we will often have occasion to discuss Transitivity (see especially section 3 below), though I will specify P.I. transitivity where necessary. At the risk of stating the obvious, any denial of P.I transitivity must also deny Transitivity (of weak preference).

The argument from small improvement has received much discussion in the literature[8]. It has been described by Chang (1997, 2002) as 'the most powerful' and 'the strongest existing argument for incomparability' and by Espinoza (2008) as 'the strongest argument' in favour of incompleteness (or 'anti-trichotomy'). The argument is presented in Martin Peterson's textbook on Decision Theory (2009) as 'an influential argument against the completeness axiom', whilst Julian Reiss in his introduction to Philosophy of Economics (2013) writes that 'absence of preference should not be interpreted as indifference as the so-called 'argument from small improvement' shows' (p.40). But the argument's reliance on the assumption of Transitivity has been given very little explicit discussion or justification by the argument's proponents – which has tended to create a misleading impression. Whilst it is standardly presented as providing a relatively simple yet serious objection to Completeness, by relying on Transitivity as its starting point the standard argument from small improvement is in fact, I will suggest, somewhat of a red-herring. For, as we will see (in section 4, below), one can easily reverse the direction of argument, starting by assuming Completeness and then concluding that Transitivity fails. So the standard argument cannot be a rationally persuasive basis for rejecting Completeness (much less for accepting rational incomparability) unless one is *already convinced on independent grounds* that Transitivity should be privileged over Completeness as one's starting point. The logical point – that the argument can be reversed – is really pretty elementary, though it has almost never been mentioned in the literature. But the dialectical moral to be drawn from this logical point – that the standard argument from small improvement cannot be a persuasive argument against Completeness unless one has already evaluated a raft of complex and controversial independent arguments for and against both Completeness and Transitivity – is, I suggest,

[8] As well as the works by de Sousa, Chang, Raz and Sinnott-Armstrong, cited in footnotes 1 and 2, above, versions of the argument also appear in influential works by Parfit (1984) and Griffin (1986).

important and is not something that has yet been adequately recognized in the extant presentations of the argument (see section 5, below).

2. Hard Cases and The Standard Argument from Small Improvement

The argument from small improvement starts from the alleged existence of what are now often called 'hard cases' of rational choice. For example, Ruth Chang gives the following scenario:

> "Suppose you are faced with a choice between a particular career as a corporate lawyer and one as a philosopher. Fill out the details of each career so that it is as plausible as possible that neither career is better than the other with respect to goodness as a career. Now if we make the legal career slightly better than it was before with respect to goodness as a career-we might, for example, slightly improve it with respect to goodness as a career by slightly improving it in one relevant aspect like goodness of salary-does it necessarily follow that the improved legal career-the original legal career plus $1,000, for example-is better than the philosophical one with respect to goodness as a career? It seems not." (Chang, 2002, 668)

If the rational permissibility of the preferences in such a hard case is granted then we have a situation involving 3 possible options: a, b and an ever-so-slightly improved version of option a, a^+. The argument is then standardly formalized as follows[9]:

<u>The Standard Argument from Small Improvement</u>

(1) $\neg(a \succ b) \;\&\; \neg(b \succ a)$

(2) $a^+ \succ a$

(3) $\neg(a^+ \succ b)$

(4) $(a^+ \succ a \;\&\; a \sim b) \rightarrow a^+ \succ b$ [by P.I. TRANSITIVITY]

(5) $\neg(a^+ \succ a \;\&\; a \sim b)$ [from 3, 4]

(6) $\neg(a \sim b)$ [from 2, 5]

[9] See e.g. Espinoza 2008, Gustafsson & Espinoza 2010, Peterson 2009.

(7) $\neg(a \succ b)$ & $\neg(b \succ a)$ & $\neg(a \sim b)$ [from 1,6] i.e. INCOMPLETENESS

The first 3 premises of this argument are assumptions that are meant to be accepted on the basis of the existence of a plausible 'hard choice' case. In terms of Chang's 'hard case' scenario, above, option **a** would be the standard legal career, **a+** would be the legal career with a slightly improved salary and **b** would be the philosophical career. If we accept that a rational agent can indeed have pairwise preferences that conform to the first 3 premises and P.I. transitivity is assumed, the conclusion that Completeness fails follows swiftly.

I have here presented the argument in terms of a subject's *preferences*[10]. This is how the argument was originally presented in de Sousa (1974)[11] and how it has often been formulated in the philosophical literature on rational choice and decision theory[12]. However, as the quotation above from Chang[13] illustrates, the argument can also be formulated in terms of the value-relations: better-than, worse-than, equally-as-good-as[14]. In what follows, I will continue to discuss the argument primarily in terms of preferences. But it is also worth clarifying that even if we focus primarily on preferences, there are various different possible interpretations of what a preference is. McGraw, Warren & Boven (2011) identify 30 different 'Definitions and

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[10] Thus I have used the normal notation in rational choice theory: '$\succ$', '$\succeq$', '$\sim$' rather than the notation generally used to denote value relations: '$>$', '$\geq$', '$=$'.

[11] "The obvious thing for me to do now is to get her to the point of clear preference. That should be easy: everyone prefers $1,500 to $1,000, and since she is indifferent between virtue and $1,000, she must prefer $1,500 to virtue by exactly the same margin as she prefers $1,500 to $1,000: or so the axioms of preference dictate. Yet she does not… Her preferences are represented by the following, which obviously violates the transitivity of indifference: $1,000 = V = $1,500, but $1,500 > $1,000. I would prefer to say that the alternatives considered are incomparable." (de Sousa, 1974, 545)

[12] See e.g. Peterson (2006, 2009), Gustaffson & Espinoza (2010), Reiss (2013). As Gustaffson & Espinoza point out, the idea of making a small improvement to one of two options between which a subject is apparently indifferent occurs first in Savage's (1954) seminal work on decision theory.

[13] Ruth Chang herself is perfectly explicit that she takes the argument to also apply to preferences as studied in decision theory: "Perhaps most striking, the possibility of parity shows the basic assumption of standard decision and rational choice theory to be mistaken: preferring X to Y, preferring Y to X, and being indifferent between them do not span the conceptual space of choice attitudes one can have toward alternatives. Put another way, the "partial orderings" sometimes favored by such theorists will underdescribe the range of choice attitudes a rational agent can have toward alternatives." (Chang, 2002, 666)

[14] Gustaffson & Espinoza (2010) comment: "There are both preferential and axiological versions of the argument. The received view, however, seems to be that rationally required preferences and value judgements are closely related, and according to the popular fitting attitudes and buck-passing account of good, the one can even be analysed in terms of the other." (Gustaffson & Espinoza, 2010, 754)

Operationalizations of Preference Construction'. Most importantly, the notion of 'preference' is sometimes simply equated with a subject's actual choice behavior – this is how the term is most commonly used in economics. But sometimes 'preference' is understood as a *hypothetical* choice. And sometimes 'preference' is taken to refer to some kind of mental state (e.g. belief, desire, expectation), which can cause and explain a subject's choice behavior. So once more it should be borne in mind that the argument from small improvement might become more or less persuasive depending on what specific notion of preference one is using. However, as we shall see, these different possible interpretations of the argument from small improvement will not affect the basic point, which concerns the logical structure of the argument no matter how we interpret the specific relation holding between the pairs of options.

3. Clarifying the role of transitivity in the argument

In what follows, I will focus on the assumption of Transitivity and on what would be required to justify taking it as one's starting point rather than starting by assuming Completeness. The formalization given above explicitly included P.I. transitivity as a premise. However, there are alternative formalizations of the argument in Messerli & Reuter 2016, 2017 and in Chang 2017 which do not *explicitly* trade on P.I. Transitivity as a premise. So it is worth briefly clarifying that these alternative formulations do still *implicitly* rely on it. Chang (2017) notes that P.I. transitivity follows from what she calls the principle of 'Substitutability of Equality' (SOE):

- **SOE:** '…if two items are equally good with respect to V, one can always be substituted for the other in comparisons with respect to V.' (Chang, 2017)

So for example: in a situation where and $x \succ y$ and $y \sim z$, according to SOE, since y and z are considered to be equally good, we can substitute z for y in the former expression to get $x \succ z$. P.I. transitivity – $[(x \succ y) \& (y \sim z)] \rightarrow (x \succ z)$ – is thus entailed by SOE. Chang then formulates the argument from small improvement as follows[15]:

[15] Chang actually gives this argument slightly less formally, in numbered, natural language premises, but I trust that this is a faithful formalization – see Chang, 2017, 4.

(1) $\neg(a \succ b) \ \& \ \neg(b \succ a)$

(2) $a^{+} \succ a$

(3) $\neg(a^{+} \succ b)$

(4) $(a \sim b)$	[assumption for *reductio*]
(5) $a^{+} \succ b$	[from 2, 4, by SOE]
(6) $(a^{+} \succ b) \ \& \ \neg(a^{+} \succ b)$	[from 3, 5]
(7) $\neg(a \sim b)$	[from 4, 6, by *reductio*]
(8) $\neg(a \succ b) \ \& \ \neg(b \succ a) \ \& \ \neg(a \sim b)$	[from 1,7] i.e. INCOMPLETENESS

Of course, even though P.I. transitivity is not here *explicitly* formulated as a premise, given that it is entailed by SOE, which is functioning as a crucial assumption/rule of inference, the argument is still reliant on the truth of P.I. transitivity.

In a fascinating empirical study of how real human subjects react when presented with hard case scenarios, which suggests that a significant majority of people would in fact violate transitivity, Messerli & Reuter (2016) remark:

"Gustafsson and Espinoza (2010)… claim that transitivity is one of the core premises of the small-improvement argument. However, this claim can be challenged, since there are alternative formalizations which do not involve transitivity. The following formalization, for instance, does not involve transitivity:

(1) $\neg(x^{+} \succ y)$

(2) $x^{+} \succ x$

(3) $x = y$	assumption for *reductio*
(4) $\neg(x^{+} \succ x)$	1, 3, x=y
(5) $\neg(x^{+} \succ x) \ \& \ (x^{+} \succ x)$	2, 4, & introduction
(6) $\neg(x = y)$	3, 5, by *reductio*

Assuming first that x and y are equally good, we then deduced a contradiction—substituting x and y in premise (1)." (Messerli & Reuter, 2016, 2245)

And in Messerli & Reuter 2017, they add that the ‘the rules of first-order logic suffice to show’ that the conclusion follows here[16]. But the symbol ‘=’ in premise 3 of Messerli & Reuter’s formalization is *not* denoting the (first-order logical notion of the) *identity* of x and y but rather the *indifference of the subject* between choosing x or y in a pairwise comparison. (Or if we are thinking in terms of value-relations, it denotes the equally-as-good-as relation.) It is thus a very substantial, *non-logical* assumption that x and y can be substituted for each other in different pairwise comparisons, such as the comparison with x^+ in premise 1. The transition from the initial three premises to the fourth is thus implicitly relying on precisely the principle that Chang states explicitly, SOE – a principle which, as we have already seen, *entails P.I. transitivity*. It is thus misleading of Messerli & Reuter to suggest that their formalization ‘does not involve transitivity’; it is still committed to P.I. transitivity, even if this principle does not feature explicitly as a premise.

4. Reversing the Argument

Let’s now turn to the core issue with using the argument from small improvement to conclude that Completeness fails (whether or not one takes that failure to show that there can be genuine rational incomparability). This turns on what is really an elementary logical point, though it is one that seems almost never to be made when ‘hard cases’ and the argument from small improvement are discussed[17]. Whilst Transitivity (and hence P.I. transitivity in particular) is no doubt intuitively plausible and perhaps formally attractive as a rational constraint on our preferences, the same could also be said of Completeness. After all, both Transitivity and Completeness are axioms of standard rational choice theory, and so both theses are presumably meant to be intuitively plausible as rational requirements. Now, the simple logical point that needs to be made explicit here is that a friend of Completeness can make a *parallel argument* from small improvement *against* P.I. transitivity, by assuming tripartite completeness:

[16] See footnote 6 of Messerli & Reuter, 2017, 3.

[17] One exception I have found is Hare (2010) who very briefly mentions in a footnote that: ‘One way to accommodate the negative intransitivity of preferences is to drop the transitivity axiom, another is to drop the completeness axiom.’ (Hare, 2010, p239 fn 2).

The *Reverse* Argument from Small Improvement

(1) $\neg(a \succ b)$ & $\neg(b \succ a)$

(2) $a^{+} \succ a$

(3) $\neg(a^{+} \succ b)$

(4) $(a \succ b) \vee (b \succ a) \vee (a \sim b)$ [by COMPLETENESS]

(5) $a \sim b$ [from 1, 4]

(6) $(a^{+} \succ a)$ & $(a \sim b)$ & $\neg(a^{+} \succ b)$ [from 2, 3, 5]

(7) $\neg[(a^{+} \succ a \ \& \ a \sim b) \rightarrow a^{+} \succ b]$ [from 6] i.e. ¬P.I. TRANSITIVITY

The first 3 premises here are exactly the same as before; once more they are supposed to be accepted on the basis of the description of a plausible 'hard choice' case. But this time we assume Completeness and it then follows just as swiftly that P.I. transitivity fails. Effectively then we are faced with the following inconsistent set[18]:

$\{\neg(a \succ b), \neg(b \succ a), (a^{+} \succ a), \neg(a^{+} \succ b)$, P.I. Transitivity, Completeness$\}$

An orthodox decision theorist, who is committed to both Transitivity and Completeness, will simply want to reject the idea that a *fully rational* subject can have these sorts of 'hard choice' preferences in the first place. For example, Donald Regan (1989) argues that a fully rational subject would never be justified in judging that: a is neither better nor worse than b, a+ is better than a but a+ is not better than b. A rational subject should instead judge only that she is *uncertain* as to which relations obtain between these options[19]. But assuming that we hold onto the four claims about the rational agent's pairwise comparisons, there is then a choice whether to give up P.I. Transitivity (and hence Transitivity) or give up Completeness. The existence of 'hard cases' does not, in itself, point towards either Transitivity or Completeness. If we hold onto Completeness and give up Transitivity then we must endorse $a \sim b$. If we hold onto Transitivity and give up Completeness then we must say that $\neg(a \sim b)$. But

[18] Gustafsson & Espinoza (2010) note that the existence of a rational subject with hard-case preferences together with PI-transitivity and Completeness as rational requirements form an inconsistent triad, though they do not discuss the possibility of arguing from Completeness to the denial of PI-transitivity.

[19] See also Gowans (1994) for a similar line of thought.

so then the real action is not with either such argument from small improvement, but with whatever *prior* considerations can be advanced for preferring one or other rational constraint on preferences – Transitivity vs. Completeness. The moral that I suggest we draw from the fact that the argument can be reversed is that the standard way that the argument from small improvement has been presented, as a relatively simple, self-standing argument against Completeness, is misleading. In itself, the standard argument does not give us any reason to favor Transitivity over Completeness; it simply assumes that if we find ourselves with the preferences that occur in a 'hard choice' scenario we will endorse Transitivity and so give up Completeness. But in the reverse argument one assumes that Completeness is endorsed and so Transitivity must be rejected. So neither direction of argument *in itself* can be a rationally persuasive basis for favoring either Completeness or Transitivity. Of course that is not to say that these arguments are therefore totally without philosophical interest. The standard argument from small improvement does serve to usefully illustrate the incompatibility of Transitivity and Completeness given that one has 'hard choice' preferences. But this incompatibility could equally well be illustrated using the reverse argument from small improvement.

5. Transitivity vs. Completeness

At this point one might be tempted to defend the significance of the standard argument from small improvement along the following lines:

> Sure, we can accept the basic logical point – one person's modus ponens is another's modus tollens – and in the abstract one could give up either principle. But there are excellent reasons to endorse Transitivity! Transitivity is clearly correct! And so holding onto Completeness at the expense of Transitivity would be totally implausible! That is why the standard formulation of the argument – moving from P.I. transitivity to the denial of Completeness – is important and interesting, whereas the reverse argument – from Completeness to the denial of P.I. transitivity – is not.

In this section I will explain why such a defense of the significance of the standard argument from small improvement would misrepresent the dialectical situation. It is *not just obvious and uncontroversial* that Transitivity should be privileged over Completeness. There are, as I will very briefly sketch below, many complex considerations for and against both Transitivity and Completeness and no clear theoretical consensus on which are most plausible or important. Now, to be clear, this is *not* to rule out the possibility of weighing all these different complex considerations and coming to a reasonable evaluation that favours Transitivity over Completeness. The point is rather that the standard argument from small improvement leaves all this hard philosophical work still to be done. That is why I have characterized it as a 'red-herring'. The argument is standardly presented as a simple but serious argument against Completeness. But it cannot be a persuasive basis for giving up Completeness unless one has already performed a complex evaluation of a host of other, independent arguments for and against Completeness and Transitivity.

The standard line of thought advanced in favor of Transitivity is the 'money-pump argument' (or family of arguments)[20]. However, it is at least controversial whether such money pump arguments are persuasive. Proponents of non-transitivity have contended in response that so long as a rational subject can *foresee* the danger of becoming a money pump they can rationally refuse to trade one option for another despite preferring one to the other. This is the line taken by Nozick 1993, Anand 1993, 2009 and recently defended in detail by Ahmed 2017, who concludes that this approach can be used to render a subject with cyclic preferences immune to every ingenious variety of money pump argument that currently exists – though see Gustafsson & Rabinowicz (2020) for an even more recent attempt to construct a money pump argument that evades the foresight response.

[20] See Davidson et al (1955), though the idea goes back to Ramsey (1928). It is worth noting that money pump arguments have also been advanced *in favour* of Completeness as a rational constraint on preferences – see e.g. Chang (1997) Broome (1999), Peterson (2007). Gustafsson (2010) points out that these arguments would at best only show that an agent with incomplete preferences would then be rationally *permitted* to behave so as to act as a money-pump when offered various bets, they would not show that the agent with incomplete preferences was rationally *obliged* to act as a money-pump. Whereas, money pump arguments in favor of Transitivity do purport to show that an agent whose preferences flout transitivity would then be rationally *compelled* to behave as a money-pump. In this respect they would seem to offer stronger support for Transitivity than for Completeness. However money-pump arguments for Transitivity *assume Completeness*, so they do not seem to provide grounds for *preferring* Transitivity *over* Completeness if we have to choose between them. Thanks to an anonymous referee for this journal for helpful discussion.

If we are thinking in terms of value relations, I suspect that some theorists might feel that it is hard to even make sense of a non-transitive set such as: ($a^+ \succ a$) & ($a \sim b$) & ($a^+ \sim b$). After all, any relation must be transitive in order for it to provide even a partial ordering of the elements in a set[21]. And one might think that it is just essential or internal to the very notion of *comparative value* that this relation is at least a partial order. However, one should bear in mind that advocates of non-transitive value relations may want to understand value as 'essentially comparative' rather than something intrinsic to the items in question – see Temkin (2012). If the elements in a set do not have unique values assigned *simpliciter*, but only relative to a specific comparison with some specific alternative(s), then it becomes easier to understand how value relations could be non-transitive and could fail to allow for even a partial ordering.

There are also a host of arguments against Transitivity in the literature. Some of these propose counter-examples to Transitivity based on sorites-style series of pairwise comparisons – see e.g. Armstrong 1939, 1948, Luce 1956, Quinn 1990, Rachels 1998, Temkin 2001, 2012. Others construct cases where different properties of the options become more important in different pairwise comparisons – see e.g. Hughes 1980, Schumm 1987, Anand 1993. Of course, there have also been a host of replies seeking to defend Transitivity against these arguments. But in light of these many alleged counter-examples, it would be implausible to take it as just obvious or non-contentious that Transitivity enjoys a settled, unquestionably secure status within the theory of rational choice.

Likewise, various independent reasons both for and against Completeness have also been advanced. For example: Mandler (2001, 2005) and Peterson (2009) both note that denying Completeness would conflict with the orthodox theory of revealed preference in economics, though Levi (1986) argued that this theory's assumption of Completeness is question-begging. Dorr, Nebel & Zuehl (forthcoming), argue on linguistic grounds that preferences are necessarily complete, whereas Broome (2004) argued that the linguistic evidence supports Transitivity but not Completeness. This is

[21] A relation that is reflexive and transitive is a quasi-order (or pre-order). If in addition a relation is also anti-symmetric then it is a partial order. And finally if a relation also satisfies the requirement of Completeness (or 'totality') then it is a total order (or linear order).

certainly not the place to try to adjudicate between these various competing considerations for and against Transitivity and Completeness! The point of this very brief survey is merely that it is at least *not just obvious or uncontroversial* that Transitivity should enjoy a more privileged, unquestionable status as a constraint on rational choice (or rational value judgement) compared with Completeness. And so the standard presentations of the argument from small improvement, which omit to mention that its starting assumption (Transitivity) is, at least prima facie, no more secure than its intended target (Completeness) and that one could just as easily make the reverse argument, create a misleading impression of the argument's importance and persuasive potential.

We should also keep in mind here, as mentioned in section 4, that another possible response to the joint inconsistency of Transitivity, Completeness and the Hard Choice Preferences would be to reject the idea that a fully rational agent could really have those preferences in the first place. So as well as endorsing one or other of the two opposite directions of small improvement argument there is also the third option of endorsing neither. This makes the evaluation that one has to perform before being in a position to endorse one of the arguments rather than the other even more complex, since as well as comparing the respective prior plausibility of Completeness and Transitivity one also has to compare the plausibility that a fully rational agent can have the preferences: $\neg(a>b)$, $\neg(b>a)$, $(a^+>a)$, $\neg(a^+>b)$. And whilst the idea that such preferences could be totally rational no doubt has *some* intuitive appeal, giving up on this intuition is hardly unthinkable[22] and has to be weighed against the theoretical appeal of being able to hold onto both Transitivity and Completeness and so endorse standard formulations of decision theory and rational choice theory. It is worth mentioning here that various non-standard theories of rational choice have been proposed that dispense with one or other of these standard axioms. Just to cite two examples: the 'regret theory' of Loomes & Sugden (1982) offers a non-standard theory of rational choice that allows for intransitive preferences. Whilst van Hees, Jitendranath & Luttens (2021) provide a version of revealed preference theory that dispenses with the assumption of Completeness[23].

[22] Again, see Regan (1989) and Gowans (1994) for explicit arguments against the intuition that these preferences can be fully rational and in defense of tripartite completeness.
[23] I am very grateful to an anonymous referee for this journal for providing this latter reference.

Now, to repeat: I am *not* suggesting that it would be impossible for a philosopher to take all these different considerations into account and reasonably draw the conclusion that Transitivity is to be privileged over Completeness – and also that the 'hard choice preferences' can indeed be fully rational – and so then come to endorse the standard argument from small improvement. For example, perhaps one is impressed by the most recent iteration of the money pump argument in favor of transitivity due to Gustaffson & Rabinowicz (2020). But one also has reasonable replies and objections to the various independent arguments and counter-examples against transitivity and the various independent arguments in favor of completeness. And perhaps one can also make a cogent case for why we should endorse the intuitions that hard-choice preferences can be fully rational despite any costs we might thereby incur by abandoning orthodox decision theory for some non-standard alternative. My point is that all the real philosophical hard work and argumentation concerning Completeness and Transitivity would have to take place *before* we are in a position to endorse the standard argument rather than the reverse argument. In itself the standard argument only illustrates the joint inconsistency of Transitivity, Completeness and the four 'hard choice' preferences, something that could equally well be illustrated with the reverse argument. My hope for this short paper is that it will help to correct the misleading impression that the argument from small improvement provides a relatively simple, free-standing objection against Completeness – when in fact it leaves all the serious philosophical work still to be done. For what it's worth, my own weak intuitions are that Transitivity probably *is* more central to our pre-theoretic notion of choosing rationally than Completeness[24] and so holding on to Transitivity may indeed be preferable to Completeness *if we have to reject one of the two*. But an argument from small improvement, in either direction, cannot be doing any real work in establishing whether such intuitions are correct[25].

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[24] Robert Aumann once commented:
'…of all the axioms of the utility theory, the completeness axiom is perhaps the most questionable. Like others, it is inaccurate as a description of real life; but unlike them we find it hard to accept even from the normative viewpoint. (Aumann, 1962, 446)

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