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Research Article

Robust Stability of Nonlinear Diffusion Fuzzy Neural Networks with Parameter Uncertainties and Time Delays

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In this paper, a class of nonlinear p-Laplace diffusion BAM Cohen-Grossberg neural networks (BAM CGNNs) with time delays is investigated. In the case of $p > 1$ with $p \neq 2$, the authors construct novel Lyapunov functional to overcome the mathematical difficulties of nonlinear p-Laplace diffusion time-delay model with parameter uncertainties, deriving the LMI-based robust stability criterion applicable to computer MATLAB LMI toolbox and deleting the boundedness of the amplification functions. And in the case of $p = 2$, LMI-based sufficient conditions are also inferred for robust input-to-state stability of reaction-diffusion Markovian jumping BAM CGNNs with the event-triggered control, which is different from those of many previous related literature. In particular, the role of diffusion can be reflected in newly acquired criteria. Finally, numerical examples verify the effectiveness of the proposed methods.

1. Introduction

In recent decades, reaction-diffusion neural networks have been the subject of research due to the fact that electrons have diffusion behaviors in an inhomogeneous magnetic field, and the role of diffusion items have always been investigated and discussed in many existing results ([1–4]). Since the conduction velocity of electrons and components is limited, the phenomenon of time delays inevitably appears in various practical projects. Thereby, time-delay reaction-diffusion systems are relatively common objects of study. For example, in [5], the following time-delay reaction-diffusion Cohen-Grossberg neural networks (CGNNs) with impulse was studied (see [7, (7)]),

$$\begin{aligned}\frac{\partial u}{\partial t} &= \nabla \cdot (\mathscr{R} \circ \nabla u) - A(u)\{B(u) \\ &\quad - [Cf(u) + Dg(u(t-\tau(t),x)) + J], \quad t \geq 0, t \neq t_k, \\ u(t_k^+, x) &= Mu(t_k^-, x) + NH(u(t_k^- - \tau(t), x)), \quad k = 1, 2, \dots,\end{aligned} \tag{1}$$

where $\mathscr{R} \circ \nabla u$ is Hadamard product of matrix $\mathscr{R}$ and vector gradient ∇u (see [6] for details).

In [7], the stability of the following BAM Cohen-Grossberg neural networks (BAM CGNNs) with distributed delays was discussed.

$$\begin{aligned}\frac{dx}{dt} &= -A(x(t))\left[B(x(t)) - \left(Cf(y(t-\tau(t))) \right.\right. \\ &\quad \left.\left. + M\int_{-\infty}^{t} K(t-s)f(x(s))ds\right)\right] \\ &\quad + \sigma(t, x(t), y(t-\tau(t)))dw(t), \\ \frac{dy}{dt} &= -\tilde{A}(y(t))\left[\tilde{B}(y(t)) - \left(\tilde{C}g(x(t-\tilde{\tau}(t))) \right.\right. \\ &\quad \left.\left. + \tilde{M}\int_{-\infty}^{t} \tilde{K}(t-s)g(x(s))ds\right)\right] \\ &\quad + \tilde{\sigma}(t, x(t-\tilde{\tau}(t)), y(t))dw(t).\end{aligned} \tag{2}$$

The Cohen-Grossberg-type BAM neural network model was initially proposed by Cohen and Grossberg [8] in 1983.

The model not only generalizes the single-layer autoassociative Hebbian correlator to a two-layer pattern matched heteroassociative circuit but also possesses Cohen-Grossberg dynamics, and it has promising application potentials for tasks of classification, parallel computation, associative memory, and nonlinear optimization problems. Since then, a lot of research has been done on BAM CGNNs models ([7, 9–11]). Besides, owing to biological engineering backgrounds and population dynamics, economics, physical engineering, and other reasons, the stability of nonlinear diffusion systems have received widespread attention [11–17]. For example, in [11], the author studied the following nonlinear diffusion fuzzy system, involved to time-delay BAM Cohen-Grossberg neural networks.

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \sum_{j=1}^{m}\frac{\partial}{\partial x_j}\left(D_{ij}(t,x,u)\left|\nabla u_i(t,x)\right|^{p-2}\frac{\partial u_i}{\partial x_j}\right) \\
&\quad - a_i(u_i(t,x)) \\
&\quad \cdot \left[b_i(u_i(t,x)) - \sum_{j=1}^{n}\tilde{m}_{ij}f_j\left(v_j(t,x)\right)\right. \\
&\quad - \bigwedge_{j=1}^{n}\widehat{m}_{ij}f_j\left(v_j\left(t-\tau_j(t),x\right)\right) \\
&\quad \left. - \bigvee_{j=1}^{n}\check{m}_{ij}f_j\left(v_j\left(t-\tau_j(t),x\right)\right)\right], \quad t\ge 0, x\in\Omega, \\
\frac{\partial u_j(t,x)}{\partial t} &= \sum_{i=1}^{m}\frac{\partial}{\partial x_i}\left(D_{ji}(t,x,u)\left|\nabla u_j(t,x)\right|^{p-2}\frac{\partial u_j}{\partial x_i}\right) \\
&\quad - c_j\left(v_j(t,x)\right) \\
&\quad \cdot \left[d_j\left(v_j(t,x)\right) - \sum_{i=1}^{n}\tilde{n}_{ji}g_i(u_i(t,x))\right. \\
&\quad - \bigwedge_{i=1}^{n}\widehat{n}_{ji}g_i(u_i(t-\rho_i(t),x)) \\
&\quad \left. - \bigvee_{i=1}^{n}\check{n}_{ji}g_i(u_i(t-\rho_i(t),x))\right], \quad t\ge 0, x\in\Omega, \\
&u(\theta,x) = \phi(\theta,x), \\
&v(\theta,x) = \upsilon(\theta,x), \quad (\theta,x)\in(-\infty,0]\times\Omega, \\
&u(t,x) = 0\in R^n, \\
&(t,x)\in R\times\partial\Omega. \qquad v(t,x) = 0\in R^n,
\end{aligned} \tag{3}$$

Under the complex conditions

$$\begin{aligned}
\lambda_1\underline{p}D + \underline{p}\lambda_{\min}(\underline{A}\mathbb{B}) &> \frac{(p-1)\bar{p}}{p}n(|\tilde{m}|+|\widehat{m}|+|\check{m}|)\lambda_{\max}\bar{A}\lambda_{\max}F \\
&\quad + \frac{\bar{p}}{p}n\left(|\tilde{n}| + \frac{|\widehat{n}|+|\check{n}|}{1-\tau}\right)|\lambda_{\max}\bar{C}\lambda_{\max}G, \\
\lambda_1\underline{p}\mathfrak{D} + \underline{p}\lambda_{\min}(\underline{C}\mathbb{D}) &> \frac{(p-1)\bar{p}}{p}n(|\tilde{n}|+|\widehat{n}|+|\check{n}|)\lambda_{\max}\bar{C}\lambda_{\max}G \\
&\quad + \frac{\bar{p}}{p}n\left(|\tilde{m}| + \frac{|\widehat{m}|+|\check{m}|}{1-\tau}\right)\lambda_{\max}\bar{A}\lambda_{\max}F,
\end{aligned} \tag{4}$$

and other conditions, a stability result ([11, Theorem 3.2]) was given, where

$$\begin{aligned}
D &= \min_{jk}\left(\inf_{t,x,u}D_{jk}(t,x,u)\right), \\
\mathfrak{D} &= \min_{ji}\left(\inf_{t,x,v}\mathfrak{D}_{ji}(t,x,v)\right).
\end{aligned} \tag{5}$$

In recent years, some methods and ideas of related literature ([5–45]) inspire our current work. In this paper, we shall discuss the robust stability of nonlinear p-Laplacian diffusion Takagi-Sugeno (T-S) fuzzy system with discrete delays and distributed delays. Actually, T-S fuzzy models provide a successful method to describe certain complex nonlinear system using some local linear subsystems ([31, 32, 46]). Besides, there exist parameter errors unavoidable in factual systems due to aging of electronic components, external disturbance, and parameter perturbations. Therefore, the robustness of the system stability should be investigated, too. Our main objectives are as follows:

(1) Changing (4) into linear matrix inequalities (LMIs) applicable to computer MATLAB LMI toolbox, which can be adapted to large-scale calculation in practical engineering.

(2) Ensure that the nonlinear diffusion term plays a role in the LMI-based stability criterion while in some existing results ([6, Theorem 3.1], [18, Theorem 3.1], [19, Theorem 3.1],), the role of the nonlinear diffusion term was neglected in their LMI-based criteria.

(3) Deleting the boundedness of amplification function $a_i(\cdot)$ in some existing results (see, e.g., [7, 9, 19, 21]).

For these purposes, we need to achieve the following works:

(i) Improve [11, Lemma 3.1] and make it adopted to LMI-based criterion, in which the nonlinear diffusion can play roles.

(ii) Construct a novel Lyapunov functional and design comprehensive applications of variational method, Young inequality, and LMI technique so that LMI-based criterion can be derived for the nonlinear diffusion fuzzy system with parameter uncertainties, discrete delays, and distributed delays.

(iii) Relax the restrictions of amplification function $a_i(\cdot)$ so that the boundedness of $a_i(\cdot)$ is not necessary.

At the same time, employing LMI technique guarantees structuring LMI-based criterion.

(iv) Explore the input-to-state stability of reaction-diffusion Markovian jumping BAM CGNNs with time delays and the event-triggered control

For convenience's sake, we still need to introduce some standard notations:

(i) $A = (a_{ij})_{n\times n} \geq 0(\leq 0)$: a nonnegative (nonpositive) matrix, that is, $a_{ij} \geq 0(\leq 0)$ for all $i, j = 1, 2, \dots, n$.

(ii) $A \geq B(\leq B)$: represents the matrix $C = (A - B)$ satisfying $C \geq 0$ (≤ 4 0).

(iii) $A = (a_{ij})_{n\times n} > 0(<0)$: a positive (negative) definite matrix.

(iv) $A = (a_{ij})_{n\times n} \geq 0(\leq 0)$: a nonnegative (nonpositive) definite matrix.

(v) $A_1 \geq A_2(A_1 \leq A_2)$: this means $A_1 - A_2$ is a nonnegative (nonpositive) definite matrix.

(vi) $A_1 > A_2(A_1 < A_2)$: this means $A_1 - A_2$ is a positive (negative) definite matrix.

(vii) $\lambda_{\max}(\Phi)$ and $\lambda_{\max}(\Phi), \lambda_{\min}(\Phi)$denote the largest and smallest eigenvalue of matrix Φ, respectively.

(viii) Denote $|C| = (|c_{ij}|)_{n\times n}$ for any matrix $C = (c_{ij})_{n\times n}$;

(ix) $|u(t,x)| = (|u_1(t,x)|, |u_2(t,x)|, \dots, |u_n(t,x)|)^T$ for any vector $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))^T$.

(x) $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))^T \geq 0(\leq 0)$ implies $u_i(t,x) \geq 0(\leq 0)$ for all $i = 1, 2, \dots, n$.

(xi) $u(t,x) \geq v(t,x)(\leq v(t,x))$ implies $u_i(t,x) \geq v_i(t,x)$ $(\leq v_i(t,x))$ for all $i = 1, 2, \dots, n$, where $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))^T$ and $v(t,x) = (v_1(t,x), v_2(t,x), \dots, v_n(t,x))^T$

(xii) I: identity matrix with compatible dimension.

(xiii) The Sobolev space $W_{1,p}(\Omega) = \{u \in L^p : \mathcal{D}u \in L^p\}$ (see [28] for details).

(xiv) Denote by λ_1 the lowest positive eigenvalue of the boundary value problem (see [28] for details)

$$\begin{aligned} -\Delta_p \varsigma(t,x) &= \lambda\varsigma(t,x), \quad x \in \Omega, \\ \varsigma(t,x) &= 0, \quad x \in \partial\Omega. \end{aligned} \tag{6}$$

2. Preliminaries

Consider the following Takagi-Sugeno fuzzy p-Laplace partial differential equations with distributed delay.

Fuzzy rule j:

If $\omega_1(t)$ is μ_{j1} and $\cdots\omega_{s*}(t)$ is μ_{js*} then

$$\begin{aligned} \frac{\partial u}{\partial t} &= \nabla\cdot\left(D(t,x,u)\circ\nabla_p u\right) - A(u(t,x)) \\ &\quad \cdot\Bigg[B(u(t,x)) - \Bigg(\left(C_j = \Delta C_j(t)\right)f(v(t-\tau(t),x)) \\ &\quad + \left(M_j + \Delta M_j(t)\right)\int_{t-\rho(t)}^{t} f(v(s,x))ds\Bigg)\Bigg], \\ \frac{\partial v}{\partial t} &= \nabla\cdot\left(\mathfrak{D}(t,x,v)\circ\nabla_p v\right) - \tilde{A}(v(t,x)) \\ &\quad \cdot\Bigg[\tilde{B}(v(t,x)) - \Bigg(\left(\tilde{C}_j + \Delta\tilde{C}_j(t)\right)g(u(t-\tilde{\tau}(t),x)) \\ &\quad + \left(\tilde{M}_j + \Delta\tilde{M}_j(t)\right)\int_{t-\tilde{\rho}(t)}^{t} g(u(s,x))ds\Bigg)\Bigg], \\ u(\theta,x) &= \phi(\theta,x), \\ v(\theta,x) &= \upsilon(\theta,x), \quad (\theta,x) \in [-\tau_*, 0]\times\Omega, \\ u(t,x) &= 0 \in R^n, \\ v(t,x) &= 0 \in R^n, \quad (t,x) \in R\times\partial\Omega, \end{aligned} \tag{7}$$

where $\omega_k(t)(k = 1, 2, \dots, s_*)$ is the premise variable and $\mu_{jk}(j = 1, 2, \dots, r; k = 1, 2, \dots, s_*)$ is the fuzzy set that is characterized by membership function. r is the number of the if-then rules, and s is the number of the premise variables. $D(t,x,u) = \text{diag}\,(D_1(t,x,u), D_2(t,x,u), \dots, D_n(t,x,u))$ and $\mathfrak{D}(t,x,v) = \text{diag}\,(\mathfrak{D}_1(t,x,v), \mathfrak{D}_2(t,x,v), \dots, \mathfrak{D}_n(t,x,v))$ are diffusion coefficients matrices. $D\circ\nabla pu$ is Hadamard product of matrix D and ∇pu (see e.g., [13] for details) and so is $\mathfrak{D}(t,x,v)\circ\nabla pv$. Let $p > 1$ be a given scalar, and $\Omega \subset R^n$ be a bounded domain with a smooth boundary $\partial\Omega$ of class $\mathscr{C}^2$ by Ω. Denote $u(t,x) = (u_1(t,x), u_2(t,x), \dots, u_n(t,x))^T \in R^n$ and $v(t,x) = (v_1(t,x), v_2(t,x), \dots, v_n(t,x))^T \in R^n$. For any given $i \in \mathcal{N} \triangleq \{1, 2, \dots, n\}$, $u_i(t,x)$is the state variable of the ith neuron at time t in space variable x and so is $v_i(t,x)$. $f(v(t-\tau(t),x)) = (f_1(v_1(t-\tau_i(t),x)), \dots, f_i(v_i(t-\tau_i(t),x)), \dots, f_n(v_n(t-\tau_i(t),x)))^T$, and $g(u(t-\tilde{\tau}(t),x)) = (g_1(u_1(t-\tilde{\tau}_1(t),x)), \dots, g_i(u_i(t-\tilde{\tau}_i(t),x)), \dots, g_n(u_n(t-\tilde{\tau}_n(t),x)))^T$, in which $f_i(v_i(t-\tau_i(t),x))$ is the neuron activation function of the ith unit of time $t-\tau_i(t)$ in space variable x and so is $g_i(u_i(t-\tilde{\tau}_i(t),x))$. Both $\tau_i(t)$ and $\tilde{\tau}_i(t)$ are discrete time delays with $0 \leq \tau_i(t) \leq \tau$ and $0 \leq \tilde{\tau}_i(t) \leq \tilde{\tau}, \forall i \in N$. And distributed delays $\rho(t)$ and $\tilde{\rho}(t)$with $0 \leq \rho(t) \leq \rho$ and $0 \leq \tilde{\rho}(t) \leq \tilde{\rho}$. In addition, the positive scalar $\tau_* = \max\{\tau, \tilde{\tau}, \rho, \tilde{\rho}\}$. Here, $\tau, \tilde{\tau}$, and τ_* all may be $+\infty$. Besides, there is a positive scalar $l_0 < 1$ such that $\tau_i'(t) \leq l_0$ and $\tilde{\tau}_i'(t) \leq l_0$ for all $i \in N$. $A(u(t,x)) = \text{diag}\,(a_1(u_1(t,x)), \dots, a_i(u_i(t,x)), \dots, a_n(u_n(t,x)))$, and $\tilde{A}(v(t,x)) = \text{diag}\,(\tilde{a}_1(v_1(t,x)), \dots, \tilde{a}_i(v_i(t,x)), \dots, \tilde{a}_n(v_n(t,x)))$, in which $a_i(u_i(t,x))$ represents an amplification function and so does $\tilde{a}_i(v_i(t,x))$. $B(u(t,x)) = (b_1(u_1(t,x)), \dots, b_i(u_i(t,x)), \dots, b_n(u_n(t,x)))^T$, and $\tilde{B}(u(t,x)) = (\tilde{b}_1(v_1(t,x)), \dots, \tilde{b}_i(v_i(t,x)), \dots, \tilde{b}_n(v_n(t,x)))^T$, in which both $b_i(u_i(t,x))$ and $\tilde{b}_i(v_i(t,x))$ are appropriately behavior functions. C_j, $\tilde{C}_j$, M_j and $\tilde{M}_j$ are connection weight strength coefficient matrices, and $\Delta C_j(t)$, $\Delta\tilde{C}_j(t)$, $\Delta M_j(t)$ and $\Delta\tilde{M}_j(t)$ are real-valued matrix functions which represent time-varying parameter uncertainties.

By means of a standard fuzzy inference method, (7) can be inferred as follows,

$$\begin{aligned}
\frac{\partial u}{\partial t} &= \nabla\cdot\left(D(t,x,u)\circ\nabla_p u\right) - A(u(t,x)) \\
&\quad\cdot\left[B(u(t,x)) - \sum_{j=1}^{r_0} h_j(w(t)) \right.\\
&\quad\cdot\left(\left(C_j+\Delta C_j(t)\right)f(v(t-\tau(t),x))\right.\\
&\quad\left.\left.+\left(M_j+\Delta M_j(t)\right)\int_{t-\rho(t)}^{t} f(v(s,x))ds\right)\right],\\
\frac{\partial u}{\partial t} &= \nabla\cdot\left(\mathfrak{D}(t,x,v)\circ\nabla_p v\right) - \tilde{A}(v(t,x)) \\
&\quad\cdot\left[\tilde{B}(v(t,x)) - \sum_{j=1}^{r_0} h_j(w(t)) \right.\\
&\quad\cdot\left(\left(\tilde{C}_j+\Delta\tilde{C}_j(t)\right)g(u(t-\tilde{\tau}(t),x))\right.\\
&\quad\left.\left.+\left(\tilde{M}_j+\Delta\tilde{M}_j(t)\right)\int_{t-\tilde{\rho}(t)}^{t} g(u(s,x))ds\right)\right],\\
u(\theta,x) &= \phi(\theta,x),\\
v(\theta,x) &= \upsilon(\theta,x),\\
&(\theta,x)\in[-\tau_*,0]\times\Omega,\\
v(t,x) &= 0\in R^n,\\
v(t,x) &= 0\in R^n,\\
&(t,x)\in R\times\partial\Omega,
\end{aligned}\tag{8}$$

where $w(t)=[w_1(t),w_2(t),\dots,w_{s_*}(t)]^T$, $h_j(w(t)) = ((w_j(w(t))/\sum_{k=1}^{r} w_k(w(t))))$, and $w_j(w(t))$: $R^{s_*}\to[0,1]$ $(j=1,2,\dots,r_0)$ is the membership function of the system with respect to the fuzzy rule j. h_j can be regarded as the normalized weight of each if-then rule, satisfying $h_j(\omega(t))\ge 0$ and $\sum_{j=1}^{r_0} h_j(\omega(t)) = 1$.

Particularly in the case of $p=2$, the system (8) is the so-called reaction-diffusion impulsive Markovian jumping BAM Cohen-Grossberg neural networks (BAM CGNNs). Inspired by some methods and conclusions of some related literature ([47–51]), we shall discuss the input-to-state stability reaction-diffusion BAM CGNNs with the event-triggered control in Section 4, for seldom existing literature involved to such complex model with feedback control.

Lemma 2.1. $a^{q-1}b \le ((q-1)/q)a^q + (b^q/q)$, $\forall a,b\in(0,+\infty)$, *and* $q>1$.

Note that Lemma 2.1 is the particular case of the famous Young inequality.

Lemma 2.2 *(Schur complement [52]) Given matrices* Q(t), S(t), *and* R(t) *with appropriate dimensions, where* $\mathcal{Q}(t)=\mathcal{Q}(t)^T$ *and* $\mathcal{R}(t)=\mathcal{R}(t)^T$, *then*

$$\begin{pmatrix} \mathcal{Q}(t) & \mathcal{S}(t)\\ \mathcal{S}^T(t) & \mathcal{R}(t)\end{pmatrix} > 0, \tag{9}$$

if and only if

$$\mathcal{R}(t)>0,\quad \mathcal{Q}(t)-\mathcal{S}(t)\mathcal{R}^{-1}(t)\mathcal{S}^T(t)>0, \tag{10}$$

or

$$\mathcal{Q}(t)>0,\quad \mathcal{R}(t)-\mathcal{S}^T(t)\mathcal{Q}^{-1}(t)\mathcal{S}^T(t)>0, \tag{11}$$

where $\mathcal{Q}(t)$, $\mathcal{S}(t)$, *and* $\mathcal{R}(t)$ *are dependent on t.*

3. Robust Stability on Nonlinear *p*-Laplacian Diffusion System in the Case of $p\neq 2$

Throughout this paper, we assume that $D(t,x,u) = \text{diag}\,(D_1, \dots, D_j \dots D_n)$ and $\mathfrak{D}(t,x,v) = (\mathfrak{D}_1,\dots,\mathfrak{D}_j,\dots,\mathfrak{D}_n)$, where we denote $D_j = D_j(t,x,u)$ and $\mathfrak{D}_j=\mathfrak{D}_j(t,x,v)$ for short. In addition, we always denote $u(t,x)=(u_1(t,x),u_2(t,x),\dots,u_n(t,x))^T$ and $v(t,x)=(v_1(t,x),v_2(t,x),\dots,v_n(t,x))^T$. Denote $u(t,x)$ by u and $u_i(t,x)$ by u_i and so do v and v_j.

Lemma 3.1. *Let* $p>1$ *be a positive real number, and* $Q=\text{diag}\,(q_1,q_2,\dots,q_n)$ *a positive definite matrix. Let u and v be a solution of (8). Then we have*

$$\begin{aligned}
&\int_\Omega \sum_{j=1}^{n} q_j u_i \sum_{k=1}^{n}\frac{\partial}{\partial x_k}\left(D_j|\nabla u_j|^{p-2}\frac{\partial u_j}{\partial x_k}\right)dx\\
&\quad\le \int_\Omega\left(|u_1|^{p/2},|u_2|^{p/2},\dots,|u_n|^{p/2}\right)(-\lambda_1 QD)\begin{pmatrix}|u_1|^{p/2}\\|u_2|^{p/2}\\ \vdots\\ |u_n|^{p/2}\end{pmatrix}dx,\\
&\int_\Omega \sum_{j=1}^{n} q_j v_i \sum_{i=1}^{n}\frac{\partial}{\partial x_i}\left(\mathfrak{D}_j|\nabla v_j|^{p-2}\frac{\partial v_j}{\partial x_i}\right)dx\\
&\quad\le \int_\Omega\left(|v_1|^{p/2},|v_2|^{p/2},\dots,|v_n|^{p/2}\right)(-\lambda_1 Q\mathfrak{D})\begin{pmatrix}|v_1|^{p/2}\\|v_2|^{p/2}\\ \vdots\\ |v_n|^{p/2}\end{pmatrix}dx,
\end{aligned}\tag{12}$$

Proof. Since u is a solution of (8), it follows by Gauss formula and the Dirichlet zero boundary condition that

$$\begin{aligned}
&\int_\Omega \sum_{j=1}^n q_i u_j \sum_{k=1}^n \frac{\partial}{\partial x_k}\left(D_j|\nabla u_j|^{p/2}\frac{\partial u_j}{\partial x_k}\right)dx \\
&\quad = -\sum_{k=1}^n\sum_{j=1}^n \int_\Omega q_j D_j |\nabla u_j|^{p-2}\left(\frac{\partial u_j}{\partial x_k}\right)^2 dx \\
&\quad \le \int_\Omega \left(|u_1|^{p/2}, |u_2|^{p/2}, \dots, |u_n|^{p/2}\right)(-\lambda_1 QD)\begin{pmatrix}|u_1|^{p/2}\\ |u_2|^{p/2}\\ \vdots \\ |u_n|^{p/2}\end{pmatrix} dx.
\end{aligned} \tag{13}$$

Another inequality can be similarly proved. And so the proof is completed.

Remark 1. Lemma 3.1 improves [11, Lemma 3.1] and [18, Lemma 2.3] for the first time, which makes a contribution to the final LMI criterion.

Remark 2. In the case of $\Omega = (0, a) \subset R^1$ or $W_0^{1,p}(0, a)$, the first eigenvalue

$$\lambda_1 = \left(\frac{2}{a}\int_0^{(p-1)^{1/p}} \frac{dt}{(1 - tP/p - 1)^{1/p}}\right)^p \tag{14}$$

(see, e.g., [28]).

Remark 3. If $\Omega = \{(x_1, x_2)^T : 0 < x_1 < a, 0 < x_2 < b\} \subset R^2$ and $p = 2$, the first eigenvalue $\lambda_1 = (\pi/a)^2 + (\pi/b)^2$ (see, e.g., [26]).

In this section, we suppose

(H1) There exist positive definite matrices $\underline{A} = \text{diag}\,(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n)$, $\bar{A} = \text{diag}\,(\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n)$, $\underline{\tilde{A}} = \text{diag}\,(\underline{\tilde{a}}_1, \underline{\tilde{a}}_2, \dots, \underline{\tilde{a}}_n)$, and $\bar{\tilde{A}} = \text{diag}\,(\bar{\tilde{a}}_1, \bar{\tilde{a}}_2, \dots, \bar{\tilde{a}}_n)$ such that

$$\begin{aligned}
0 < \underline{a}_i &\le \frac{a_i(s)}{s^{p-2}} \le \bar{a}_i, 0 < \underline{\tilde{a}}_i \\
&\le \frac{\tilde{a}_i(s)}{s^{p-2}} \le \bar{\tilde{a}}_i, 0 \ne s \in R, \quad i = 1, 2, \dots, n;
\end{aligned} \tag{15}$$

where $A(u) = \text{diag}\,(a_1(u_1), a_2(u_2), \dots, a_n(u_n))$, and $\tilde{A}(u) = \text{diag}\,(\tilde{a}_1(u_1), \tilde{a}_2(u_2), \dots, \tilde{a}_n(u_n))$.

(H2) There exists positive definite matrices $\mathbb{B} = \text{diag}\,(b_1, b_2, \dots, b_n)$ and $\tilde{\mathbb{B}} = \text{diag}\,(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_n)$ such that $b_i(0) = 0 = \tilde{b}_i(0)$ and $((b_i(s))/s) \ge b_i$, $(\tilde{b}_i/s) \ge \tilde{b}_i$, $0 \ne s \in R$, $i = 1, 2, \dots, n$, where $B(u) = (b_1(u_1), b_2(u_2), \dots, b_n(u_n))^T$ and $\tilde{B}(u) = (\tilde{b}_1(u_1), \tilde{b}_2(u_2), \dots, \tilde{b}_n(u_n))^T$.

(H3) There are positive definite matrices $F = \text{diag}\,(F_1, F_2, \dots, F_n)$ and $G = \text{diag}\,(G_1, G_2, \dots, G_n)$ such that $|f_i(s)| \le F_i|s|$, $|g_i(s)| \le G_i|s|$, $\forall s \in R$, $i = 1, 2, \dots, n$, where $f(v) = (f_1(v_1), \dots, f_n(v_n))^T$ and $g(v) = (g_1(v_1), \dots, g_n(v_n))^T$.

Remark 4. The condition (H1) implies that the boundedness of amplification functions a_i and $\tilde{a}_i$ are unnecessary in the case of $p > 1$ with $p \ne 2$, for we may take $a_i(s) = \underline{a}_i s^{p-2}$, which is actually unbounded for $s \in (-\infty, +\infty)$. Below, we denote for convenience

$$\begin{aligned}
C_j(t) &= C_j + \Delta C_j(t) \\
\tilde{C}_j(t) &= \tilde{C}_j + \Delta C_j(t), \\
M_j(t) &= M_j + \Delta M_j(t) \\
\tilde{M}_j(t) &= \tilde{M}_j + \Delta M_j(t),
\end{aligned} \tag{16}$$

where $C_j(t) = (c_{ijk}(t))_{n\times n}$, $\tilde{C}_j(t) = (\tilde{c}_{ijk}(t))_{n\times n}$, $M_j(t) = (m_{ijk}(t))_{n\times n}$, and $\tilde{M}_j(t) = (\tilde{m}_{ijk}(t))_{n\times n}$ are diagonal matrices.

Theorem 3.2. *Suppose that the conditions (H1)–(H3) hold and $p \triangleq (p_1/p_2) > 1$ with p_1 being an even number and p_2 being an odd number. Besides, there are four nonnegative matrices C_*, $\tilde{C}_*$, M_*, and $\tilde{M}_*$ such that*

$$\begin{aligned}
-C_* &\le \Delta C_j(t) \le C_* \\
-\tilde{C}_* &\le \Delta\tilde{C}_j(t) \le \tilde{C}_*, \\
-M_* &\le \Delta M_j(t) \le M_*, \\
-\tilde{M}_* &\le \Delta\tilde{M}_j(t) \le \tilde{M}_*.
\end{aligned} \tag{17}$$

Assume, in addition,

$$\mathfrak{K} \triangleq \begin{pmatrix} \mathbb{B} & -\sum_{j=1}^{r_0}\left[\left(|C_j| + C_*\right)G + \rho\left(|M_j| + M_*\right)G\right] \\ -\sum_{j=1}^{r_0}\left[\left(|\tilde{C}_j| + \tilde{C}_*\right)G + \tilde{\rho}\left(|\tilde{M}_j| + \tilde{M}_*\right)G\right] & \tilde{\mathbb{B}} \end{pmatrix}^{-1} \ge 0, \tag{18}$$

and there is a positive definite matrix $Q = \operatorname{diag}(q_1, q_2, \dots, q_n)$ *such that*

$$\begin{aligned}\lambda_1 QD + Q\underline{A}\mathbb{B} - \sum_{j=1}^{r_0} &\cdot \left[\frac{p-1}{p} Q\bar{A}\left(\left|C_j\right| + C_*\right)F \right.\\ &+ \rho\frac{p-1}{p} Q\bar{A}\left(\left|M_j\right| + M_*\right)F \\ &+ \frac{1}{p(1-l_0)} Q\overline{\tilde{A}}\left(\left|\tilde{C}_j\right| + \tilde{C}_*\right)G \\ &\left.+ \frac{\tilde{\rho}}{p} Q\overline{\tilde{A}}\left(\left|\tilde{M}_j\right| + \tilde{M}_*\right)G\right] > 0,\end{aligned} \tag{19}$$

and

$$\begin{aligned}\lambda_1 Q\mathfrak{D} + Q\underline{\tilde{A}}\tilde{\mathbb{B}} - \sum_{j=1}^{r_0} &\cdot \left[\frac{p-1}{p} Q\overline{\tilde{A}}\left(\left|\tilde{C}_j\right| + \tilde{C}_*\right)G \right.\\ &+ \tilde{\rho}\frac{p-1}{p} Q\overline{\tilde{A}}\left(\left|\tilde{M}_j\right| + \tilde{M}_*\right)G \\ &+ \frac{1}{p(1-l_0)} Q\bar{A}\left(\left|C_j\right| + C_*\right)F \\ &\left.+ \frac{\rho}{p} Q\bar{A}\left(\left|M_j\right| + M_*\right)F\right] > 0,\end{aligned} \tag{20}$$

then there exists the globally asymptotically robust stable unique equilibrium point for (8).

Remark 5. Condition (4) does not complete the matrix form. However, (19)–(20) are complete linear matrices inequalities, which have even gotten better at dealing with the calculation of the large operations involved in the practical engineering by way of computer MATLAB programming.

Proof. There are three steps to the proof.

Step 1. We claim that the null solution is the unique equilibrium point for (8).

In fact, we know from (H2)–(H3) that $b_i(0) = \tilde{b}_i(0) = f_i(0) = \tilde{f}_i(0) = g_i(0) = \tilde{g}_i(0) = 0$, and hence $u = 0$ and $v = 0$ are the equilibrium solution of (8).

Moreover, we prove that the equilibrium point is unique. Indeed, it follows from (H1) that $a_i(s) > 0$. Let (21) be an equilibrium point for (8)

$$\begin{pmatrix} u \\ v \end{pmatrix}, \tag{21}$$

then we get

$$\begin{aligned}\begin{pmatrix} 0 \\ 0 \end{pmatrix} &= \begin{pmatrix} B(u) - \sum_{j=1}^{r_0} h_j(w(t))\left(C_j(t)f(v) + M_j(t)\int_{t-\rho(t)}^{t} f(v)ds\right) \\ \tilde{B}(v) - \sum_{j=1}^{r_0} h_j(w(t))\left(\tilde{C}_j(t)g(u) + \tilde{M}_j(t)\int_{t-\tilde{\rho}(t)}^{t} g(u)ds\right) \end{pmatrix} \\ &= \begin{pmatrix} B(u) - \sum_{j=1}^{r_0} h_j(w(t))\left(C_j(t) + M_j(t)\rho(t)\right)f(v) \\ \tilde{B}(v) - \sum_{j=1}^{r_0} h_j(w(t))\left(\tilde{C}_j(t) + \tilde{M}_j(t)\tilde{\rho}(t)\right)g(u) \end{pmatrix}.\end{aligned} \tag{22}$$

If (23) is another equilibrium point of (8)

$$\begin{pmatrix} \breve{u} \\ \breve{v} \end{pmatrix}, \tag{23}$$

we can actually deduce from (22) that

$$\begin{aligned}&\left|B(u) - B\left(\breve{u}\right)\right| \\ &\le \sum_{j=1}^{r_0}\left(\left|C_j(t)\right|\left|f(v) - f\left(\breve{v}\right)\right| + \left|M_j(t)\right|\int_{t-\rho}^{t}\left|f(v) - f\left(\breve{v}\right)\right|ds\right) \\ &\le \sum_{j=1}^{r_0}\left(\left|C_j(t)\right|\left|f(v) - f\left(\breve{v}\right)\right| + \left|M_j(t)\right|\int_{t-\rho}^{t}\left|f(v) - f\left(\breve{v}\right)\right|ds\right),\end{aligned} \tag{24}$$

and then

$$\begin{aligned}\mathbb{B}\left|u - \breve{u}\right| &\le \sum_{j=1}^{r_0}\left(\left|C_j(t)\right|F\left|v - \breve{v}\right| + \left|M_j(t)\right|\int_{t-\rho}^{t} F\left|v - \breve{v}\right|ds\right) \\ &= \sum_{j=1}^{r_0}\left(\left|C_j(t)\right|F + \rho\left|M_j(t)\right|F\right)\left|v - \breve{v}\right| \\ &= \sum_{j=1}^{r_0}\left(\left(\left|C_j(t)\right| + C_*\right)F + \rho\left(\left|M_j\right| + M_*\right)F\right)\left|v - \breve{v}\right|.\end{aligned} \tag{25}$$

Similarly,

$$\begin{aligned}\tilde{\mathbb{B}}\left|v - \breve{v}\right| &\le \sum_{j=1}^{r_0}\left(\left|\tilde{C}_j(t)\right|G + \tilde{\rho}\left|M_j(t)\right|G\right)\left|u - \tilde{u}\right| \\ &= \sum_{j=1}^{r_0}\left[\left(\left|\tilde{C}_j\right| + \tilde{C}_*\right)G + \rho\left(\left|\tilde{M}_j\right| + \tilde{M}_*\right)G\right]\left|v - \breve{v}\right|.\end{aligned} \tag{26}$$

Combining (25) and (26) implies

$$\begin{pmatrix} \mathbb{B} & -\sum_{j=1}^{r_0}\left[\left(|C_j|+C_*\right)F+\rho\left(|M_j|+M_*\right)F\right] \\ -\sum_{j=1}^{r_0}\left[\left(|\tilde{C}_j|+\tilde{C}_*\right)G+\tilde{\rho}\left(|\tilde{M}_j|+\tilde{M}_*\right)G\right] & \tilde{\mathbb{B}} \end{pmatrix}\begin{pmatrix} |u-\breve{u}| \\ |v-\breve{v}| \end{pmatrix} \leq 0 \in R^{2n}, \quad (27)$$

and (18) yields

$$\begin{pmatrix} |u-\breve{u}| \\ |v-\breve{v}| \end{pmatrix} = 0 \in R^{2n}, \quad (28)$$

or

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \breve{u} \\ \breve{v} \end{pmatrix}. \quad (29)$$

Thereby, the null solution is the unique equilibrium point for (8).

Remark 6. In ordinary differential systems, the uniqueness of the equilibrium solution can be determined by the existence of the equilibrium solution and its global asymptotic stability. However, (8) is a partial differential system, including two different variables: t and x. Since the existence of the equilibrium solution and its global asymptotic stability only determines the equilibrium solution which is unique about variable t, but it may be not unique on variable x. Hence, it is necessary to verify the uniqueness of the equilibrium solution.

Step 2. To derive LMI-based criterion in which the nonlinear diffusion terms can play roles, we need to construct new Lyapunov-Krasovskii functionals as follows:

$$\begin{aligned} V(t) &= V_1(t)+V_2(t)+V_3(t)+V_4(t) \\ &\quad + V_5(t)+V_6(t), \end{aligned} \quad (30)$$

where

$$\begin{aligned} v_1(t) &= \int_\Omega u^T(t,x)Qu(t,x) \\ dx &= \sum_{i=1}^{n}\int_\Omega q_i u_i^2 dx, \\ v_2(t) &= \int_\Omega v^T(t,x)Qv(t,x) \\ dx &= \sum_{j=1}^{n}\int_\Omega q_i v_j^2 dx, \end{aligned} \quad (31)$$

$$\begin{aligned} v_3(t) &= \frac{2}{1-l_0}\sum_{i=1}^{n}\sum_{j=1}^{r_0}\sum_{k=1}^{n}\bar{a}_i q_i\left(|c_{ijk}|+c_{*ijk}\right)F_k\int_\Omega\int_{t-\tau_k(t)}^{t} \\ &\quad \cdot \frac{|v_k(s,x)|^p}{p}dsdx, \end{aligned}$$

$$\begin{aligned} v_4(t) &= \frac{2}{1-l_0}\sum_{i=1}^{n}\sum_{j=1}^{r_0}\sum_{k=1}^{n}\bar{a}_i q_i\left(|\tilde{c}_{ijk}|+\tilde{c}_{*ijk}\right)G_k\int_\Omega\int_{t-\tilde{\tau}_k(t)}^{t} \\ &\quad \cdot \frac{|u_k(s,x)|^p}{p}dsdx, \end{aligned} \quad (32)$$

$$\begin{aligned} v_5(t) &= \frac{2}{p}\sum_{j=1}^{r_0}\int_\Omega\int_{-\rho}^{0}d\sigma\int_{t+\sigma}^{t}\left(|v_1(s,x)|^{p/2},\dots,|v_n(s,x)|^{p/2}\right) \\ &\quad \cdot \left(Q\bar{A}\left(|M_j|+M_*\right)F\right)\begin{pmatrix} |v_1(s,x)|^{p/2} \\ |v_2(s,x)|^{p/2} \\ \vdots \\ |v_n(s,x)|^{p/2} \end{pmatrix}dsdx, \end{aligned} \quad (33)$$

$$\begin{aligned} v_6(t) &= \frac{2}{p}\sum_{j=1}^{r_0}\int_\Omega\int_{-\tilde{\rho}}^{0}d\sigma\int_{t+\sigma}^{t}\left(|u_1(s,x)|^{p/2},\dots,|u_n(s,x)|^{p/2}\right) \\ &\quad \cdot \left(Q\bar{\bar{A}}\left(|\tilde{M}_j|+\tilde{M}_*\right)G\right)\begin{pmatrix} |u_1(s,x)|^{p/2} \\ |u_2(s,x)|^{p/2} \\ \vdots \\ |u_n(s,x)|^{p/2} \end{pmatrix}dsdx, \end{aligned} \quad (34)$$

Remark 7. The uncertainty of parameters brings a difficulty to design the Lyapunov functions. If imitating the previous Lyapunov functions in existing literature, for example, let

$$v_3(t) = \frac{2}{1-l_0}\sum_{i=1}^{n}\sum_{j=1}^{r_0}\sum_{k=1}^{n}\bar{a}_i q_i\left|c_{ijk}(t)\right|F_k\int_\Omega\int_{t-\tau_k(t)}^{t}\frac{|v_k(s,x)|^p}{p}dsdx, \quad (35)$$

one can find it impossible that the sufficient conditions of stability criterion can be derived. In addition, Lyapunov functions (33) and (34) help us to derive the complete linear matrix inequality condition for the stability criterion of nonlinear diffusion system (8).

Step 3. We claim that the null solution is globally asymptotically robust stable.

Evaluating the time derivation of V1(t) along the trajectory of the (8), we can derive from Lemma 3.1

$$v_1'(t) \le \int_\Omega 2\left[-\lambda_1 U^T(t,x)QDU(t,x) - u^T QA(u)B(u)\right]dx + 2\int_\Omega \sum_{j=1}^{r_0} \left|h_j(w(t))\right|\left|u\right|^T QA(u)\left|C_j(t)\right| \cdot \left|f(v(t-\tau(t),x))\right|dx + 2\int_\Omega \sum_{j=1}^{r_0}\left|h_j(w(t))\right|\left|u\right|^T QA(u) \cdot \left|M_j(t)\right|\int_{t-\rho(t)}^t \left|f(v(s,x))\right|dsdx, \tag{36}$$

where we simply denote

$$U(t,x) = \begin{pmatrix} |u_1(t,x)|^{P/2} \\ |u_2(t,x)|^{P/2} \\ \vdots \\ |u_n(t,x)|^{P/2} \end{pmatrix}, \quad v(t,x) = \begin{pmatrix} |v_1(t,x)|^{p/2} \\ |v_2(t,x)|^{p/2} \\ \vdots \\ |v_n(t,x)|^{p/2} \end{pmatrix}, \tag{37}$$

and

$$U(t-\tau(t),x) = \begin{pmatrix} |u_1(t-\tau_1(t),x)|^{P/2} \\ |u_2(t-\tau_2(t),x)|^{P/2} \\ \vdots \\ |u_n(t-\tau_n(t),x)|^{P/2} \end{pmatrix}, \quad V(t-\tilde{\tau}(t),x) = \begin{pmatrix} |v_1(t-\tilde{\tau}_1(t),x)|^{P/2} \\ |v_2(t-\tilde{\tau}_2(t),x)|^{P/2} \\ \vdots \\ |v_n(t-\tilde{\tau}_n(t),x)|^{P/2} \end{pmatrix}. \tag{38}$$

It follows by (H1), (H2), and the conditions on the parameter p that

$$sa_i(s)b_i(s) \ge a_i b_i s^p, \forall s \in R. \tag{39}$$

So combining (H1), (H2), and (39) results in

$$2\int_\Omega u^T QA(u)B(u)dx = 2\int_\Omega \sum_{i=1}^n u_i q_i a_i(u_i)b_i(u_i)dx \ge 2\int_\Omega \sum_{i=1}^n q_i \underline{a}_i b_i u_i^p dx = \int_\Omega U^T(t,x)2Q\underline{A}\mathbb{B}U(t,x)dx, \tag{40}$$

From Lemma 2.1, (H1), and (H3), we get

$$\begin{aligned}
&2\int_\Omega \sum_{j=1}^{r_0} h_j(w(t))u^T QA(u)C_j(t)f(v(t-\tau(t),x))dx \\
&\le 2\sum_{j=1}^{r_0}\int_\Omega |u|^T Q\left|A(u)\right|C_j(t)\left|f(v(t-\tau(t),x))\right|dx \\
&\le 2\sum_{i=1}^n\sum_{j=1}^{r_0}\sum_{k=1}^n \bar{a}_i q_i \left|c_{ijk}(t)\right|F_k\int_\Omega \cdot \left[\frac{p-1}{p}|u_i|^p + \frac{|v_k(t-\tau_k(t),x)|^p}{p}\right]dx \\
&= \sum_{j=1}^{r_0}\int_\Omega U^T(t,x)\left(2\frac{p-1}{p}Q\bar{A}\left|C_j(t)\right|F\right)U(t,x) + \sum_{j=1}^{r_0}\int_\Omega V^T(t-\tau(t),x)\left(\frac{2}{p}Q\bar{A}\left|C_j(t)\right|F\right)V \\
&\cdot (t-\tau(t),x)dx \le \sum_{j=1}^{r_0}\int_\Omega U^T(t,x) \cdot \left(2\frac{p-1}{p}Q\bar{A}\left|C_j(t)\right|F\right)U(t,x) + \sum_{j=1}^{r_0}\int_\Omega V^T(t-\tau(t),x) \\
&\cdot \left(\frac{2}{p}Q\bar{A}\left(\left|C_j\right| + C_*\right)F\right)V(t-\tau(t),x)dx,
\end{aligned} \tag{41}$$

where $C_j(t) = (c_{ijk}(t))_{n\times n}$.

Besides, we can conclude from (H2), (H3), and Lemma 2.1 that

$$\begin{aligned}
&2\int_\Omega \sum_{j=1}^{r_0} h_j(w(t))u^T QA(u)M_j(t)\int_{t-\rho(t)}^t f(v(s,x))dsdx \\
&\le 2\sum_{j=1}^{r_0}\int_\Omega\int_{t-\rho(t)}^t |u|^T Q\left|A(u)\right|M_j(t)\left|f(v(s,x))\right|dsdx \\
&\le 2\sum_{i=1}^n\sum_{j=1}^{r_0}\sum_{k=1}^n \bar{a}_i q_i\left|m_{ijk}(t)\right|F_k\int_\Omega\int_{t-\rho}^t |u_i|^{p-1}|v_k(s,x)|dsdx \\
&\le \sum_{j=1}^{r_0}\int_\Omega U^T(t,x)\left(2\rho\frac{p-1}{p}Q\bar{A}\left|M_j(t)\right|F\right)U(t,x)dsdx \\
&+ \sum_{j=1}^{r_0}\int_\Omega\int_{t-\rho}^t V^T(s,x)\left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right| + M_*\right)F\right)V \cdot (s,x)dsdx,
\end{aligned} \tag{42}$$

where $M_j(t) = (m_{ijk}(t))_{n\times n}$.

So we have

$$v_1'(t) \le \int_\Omega U^T(t,x) \cdot \left[-2\lambda_1 PD - 2Q\underline{A}\mathbb{B} + \sum_{j=1}^{r_0}\left(2\frac{p-1}{p}Q\bar{A}\left|C_j(t)\right|F + 2\rho\frac{p-1}{p}Q\bar{A}\left|M_j(t)\right|F\right)\right]U(t,x) + \sum_{j=1}^{r_0}\int_\Omega V^T(t-\tau(t),x)\left(\frac{2}{p}Q\bar{A}\left(\left|C_j\right|+C_*\right)F\right)V \cdot (t-\tau(t),x)dx + \sum_{j=1}^{r_0}\int_\Omega\int_{t-\rho}^{t} V^T(s,x) \cdot \left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right)V(s,x)dsdx. \quad (43)$$

Besides, we get by (32)

$$\begin{aligned} v_3'(t) \le{}& \frac{2}{1-l_0}\sum_{i=1}^{n}\sum_{j=1}^{r_0}\sum_{k=1}^{n}\bar{a}_i q_i\left(\left|c_{ijk}\right|+c_{*ijk}\right)F_k\int_\Omega \cdot \frac{\left|v_k(t,x)\right|^p}{p}dx - 2\sum_{i=1}^{n}\sum_{j=1}^{r_0}\sum_{k=1}^{n}\bar{a}_i q_i \cdot \left(\left|c_{ijk}\right|+c_{*ijk}\right)F_k\int_\Omega\frac{\left|v_k(t-\tau_k(t),x)\right|^p}{p}dx \\ ={}& \sum_{j=1}^{r_0}\int_\Omega V^T(t,x)\left(\frac{2}{p(1-l_0)}Q\bar{A}\left(\left|C_j\right|+C_*\right)F\right)V(t,x)dx - \sum_{j=1}^{r_0}\int_\Omega V^T(t-\tau(t),x)\left(\frac{2}{p}Q\bar{A}\left(\left|C_j\right|+C_*\right)F\right) \cdot V(t-\tau(t),x)dx, \end{aligned} \quad (44)$$

One can deduce from (33) that

$$\begin{aligned} v_5'(t) ={}& \sum_{j=1}^{r_0}\int_\Omega\int_{-\rho}^{0}V^T(t,x)\left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right)V(t,x)dx - \sum_{j=1}^{r_0}\int_\Omega\int_{-\rho}^{0}V^T(t+s,x)\left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right) \cdot V(t+s,x)dsdx \\ ={}& \sum_{j=1}^{r_0}\int_\Omega V^T(t,x)\left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right)V(s,x)dx - \sum_{j=1}^{r_0}\int_\Omega\int_{t-\rho}^{t}V^T(s,x)\left(\frac{2}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right)V \cdot (s,x)dsdx. \end{aligned} \quad (45)$$

Hence,

$$\begin{aligned} &v_1'(t)+v_3'(t)+v_5'(t) \\ &\le 2\int_\Omega U^T(t,x) \cdot \left[-\lambda_1 QD - Q\underline{A}\mathbb{B} + \sum_{j=1}^{r_0}\left(\frac{p-1}{p}Q\bar{A}\left|C_j(t)\right|F + \rho\frac{p-1}{p}Q\bar{A}\left|M_j(t)\right|F\right)\right]U(t,x) + 2\int_\Omega V^T(t,x)\sum_{j=1}^{r_0} \cdot \left(\frac{1}{p(1-l_0)}Q\bar{A}\left(\left|C_j\right|+C_*\right)F + \frac{\rho}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right) \cdot V(t,x)dx. \end{aligned} \quad (46)$$

Similarly, we can deduce from $\mathscr{V}(2)$, $\mathscr{V}(4)$, and $\mathscr{V}(6)$ that

$$\begin{aligned} &\mathscr{V}_2'(t)+\mathscr{V}_4'(t)+\mathscr{V}_6'(t) \\ &\le 2\int_\Omega V^T(t,x) \cdot \left[-\lambda_1 Q\mathfrak{D} - Q\underline{\tilde{A}}\tilde{\mathbb{B}} + \sum_{j=1}^{r_0}\left(\frac{p-1}{p}Q\bar{\tilde{A}}\left|\tilde{C}_j(t)\right|G + \tilde{\rho}\frac{p-1}{p}Q\bar{\tilde{A}}\left|\tilde{M}_j(t)\right|G\right)\right]V(t,x) + 2\int_\Omega U^T(t,x)\sum_{j=1}^{r_0} \cdot \left(\frac{1}{p(1-l_0)}Q\bar{\tilde{A}}\left(\left|\tilde{C}_j\right|+\tilde{C}_*\right)G + \frac{\tilde{\rho}}{p}Q\bar{\tilde{A}}\left(\left|\tilde{M}_j\right|+\tilde{M}_*\right)G\right) \cdot U(t,x)dx. \end{aligned} \quad (47)$$

Therefore, (17), (19), and (20) yield

$$\begin{aligned} \mathscr{V}'(t) \le{}& 2\int_\Omega U^T\left[-\lambda_1 QD - Q\underline{A}\mathbb{B} + \sum_{j=1}^{r_0}\left(\frac{p-1}{p}Q\bar{A}\left|C_j(t)\right|F + \rho\frac{p-1}{p}Q\bar{A}\left|M_j(t)\right|F + \frac{1}{p(1-l_0)}Q\bar{\tilde{A}}\left(\left|\tilde{C}_j\right|+\tilde{C}_*\right)G + \frac{\tilde{\rho}}{p}Q\bar{\tilde{A}}\left(\left|\tilde{M}_j\right|+\tilde{M}_*\right)G\right)\right]U \\ &+ 2\int_\Omega V^T\left[-\lambda_1 Q\mathfrak{D} - Q\underline{\tilde{A}}\tilde{\mathbb{B}} + \sum_{j=1}^{r_0}\left(\frac{p-1}{p}Q\bar{\tilde{A}}\left|\tilde{C}_j(t)\right|G + \tilde{\rho}\frac{p-1}{p}Q\bar{\tilde{A}}\left|\tilde{M}_j(t)\right|G + \frac{1}{p(1-l_0)}Q\bar{A}\left(\left|C_j\right|+\tilde{C}_*\right)F + \frac{\rho}{p}Q\bar{A}\left(\left|M_j\right|+M_*\right)F\right)\right]V \le 0. \end{aligned} \quad (48)$$

It follows by the standard Lyapunov functional theory that the null solution of (8) is globally asymptotically robust stable. And the proof is completed.

Remark 8. There have been some other approaches removing boundedness of amplification functions. For example, in [53], an appropriate Lyapunov-Krasovskii functional is set up to derive the LMI-based μ-stability for discrete time-delay system. This is really a good result. However, in this paper, our system model (8) is the continuous system different from the discrete system ([54, (1)]). Of course, the main results of this paper are inspired by some methods and ideas of these documents.

Remark 9. The Lyapunov functionals (33) and (34) are similar to the quadric form different from those of [11, 13, 14, 20]. Actually, the quadric form and matrix form help us to derive the LMI-based criterion.

Remark 10. The boundedness of amplification functions a_i and $\tilde{a}_j$ may be unbounded while amplification functions are always proposed to be bounded in many existing results (see, e.g., [7, 9, 18–21]).

If the diffusion phenomena are ignored, (8) degenerates into the following BAM CGNNs with discrete and distributed time-varying delays:

$$\begin{aligned} \frac{dx}{dt} &= -A(x(t))\Bigg[B(x(t)) - \sum_{j=1}^{r_0} h_i(w(t)) \\ &\quad \cdot \Bigg(\left(C_j + \Delta C_j(t)\right) f(y(t-\tau(t))) \\ &\quad + \left(M_j + \Delta M_j(t)\right)\int_{t-\rho(t)}^{t} f(y(s))ds\Bigg)\Bigg], \\ \frac{dy}{dt} &= -\tilde{A}(y(t))\Bigg[\tilde{B}(y(t)) - \sum_{j=1}^{r_0} h_i(w(t)) \\ &\quad \cdot \Bigg(\left(\tilde{C}_j + \Delta\tilde{C}_j(t)\right) g(x(t-\tilde{\tau}(t))) \\ &\quad + \left(\tilde{M}_j + \Delta\tilde{M}_j(t)\right)\int_{t-\tilde{\rho}(t)}^{t} g(x(s))ds\Bigg)\Bigg], \\ x(s) &= \phi(s), \\ y(s) &= \upsilon(s), \\ &\quad s \in [-\tau_*, 0]. \end{aligned} \tag{49}$$

Since in ordinary differential systems, the uniqueness of the equilibrium solution can be determined by the existence of the equilibrium solution and its global asymptotic stability, and the diffusion items disappear, we can directly deduce the following corollary from Theorem 3.2:

Table 1: Comparisons of amplification function $a_j(u_j(t,x))$ in related results.

Theorems and examples	Boundedness conditions of a_j
Theorem 3.2, (Corollary 3.3), Example 5.1	No requirements
[9, Theorem 2.1, Theorem 3.1, and Example 1]	$0 < \underline{a} < a_j(r) < \bar{a}, \quad \forall r \in R$
[2, Theorem 1 and Example 1]	$0 < \underline{a}_j \le a_j(r) \le \bar{a}_j, \quad \forall r \in R$
[3, Theorem 3.1 and Example 4.1]	$0 < \underline{a}_j \le a_j(r) < \infty \text{with } a_j'(r) r \ge 0, \quad \forall r \in R$
[3, Theorem 3.1 and Example 4.1]	$0 < \underline{a}_j \le a_j(r), \quad \forall r \in R$
[5, Theorem 4 and Example 1]	$0 < \underline{a}_j \le a_j(r) \le \bar{a}_j, \quad \forall r \in R$
[6, Theorem 3.1 and Example 4.1]	$0 < \underline{a}_j \le a_j(r) \le \bar{a}_j, \quad \forall r \in R$

Corollary 3.3. *Suppose that the conditions (H1)–(H3) hold. Besides, there are four nonnegative matrices C_*, $\tilde{C}_*$, M_*, and $\tilde{M}_*$ such that*

$$\begin{aligned} -C_* &\le \Delta C_j(t) \le C_*, \\ -\tilde{C}_* &\le \Delta\tilde{C}_j(t) \le \tilde{C}_*, \\ -M_* &\le \Delta M_j(t) \le M_*, \\ -\tilde{M}_* &\le \Delta\tilde{M}_j(t) \le \tilde{M}_*, \end{aligned} \tag{50}$$

and there is a positive definite matrix $Q = \mathrm{diag}\,(q_1, q_2, \dots, q_n)$ *such that*

$$\begin{aligned} &Q\underline{A}\mathbb{B} - \sum_{j=1}^{r_0}\Bigg[\frac{p-1}{p}Q\bar{A}C_jF + \rho\frac{p-1}{p}Q\bar{A}M_jF \\ &\quad + \frac{1}{p(1-l_0)}Q\bar{\tilde{A}}\tilde{C}_jG + \frac{\bar{\tilde{\rho}}}{p}Q\bar{\tilde{A}}\tilde{M}_jG\Bigg] > 0, \end{aligned} \tag{51}$$

and

$$\begin{aligned} &Q\underline{\tilde{A}}\tilde{\mathbb{B}} - \sum_{j=1}^{r_0}\Bigg[\frac{p-1}{p}Q\bar{\tilde{A}}\tilde{C}_jG + \tilde{\rho}\frac{p-1}{p}Q\bar{\tilde{A}}\tilde{M}_jG \\ &\quad + \frac{1}{p(1-l_0)}Q\bar{A}C_jF + \frac{\rho}{p}Q\tilde{A}M_jF\Bigg] > 0, \end{aligned} \tag{52}$$

then there exists the unique globally asymptotically robust stable equilibrium point for (49).

Remark 11. For the BAM CGNNs (53), Corollary 3.3 deletes the boundedness of amplification functions a_i and $\tilde{a}_j$, improving related results (see, e.g., [7, 9, 18, 19, 21]). This is also shown below (Table 1).

4. Input-to-State Stability of Markovian Jumping Reaction-Diffusion BAM CGNNs with Event-Triggered Control in the Case of $p = 2$

In this section, we consider the following Markovian jumping reaction-diffusion BAM CGNNs with event-triggered control under Dirichlet zero-boundary value.

$$\begin{aligned}\frac{\partial u}{\partial t} &= \nabla\cdot(D(t,x,u)\circ\nabla u) - A(u(t,x)) \\ &\quad\cdot\Bigg[B(u(t,x)) - \zeta(\tilde{v}(t,x)) \\ &\quad - \sum_{j=1}^{r_0} h_i(w(t))\big((C_{rj}+\Delta C_{rj}(t))f(v(t-\tau(t),x)) \\ &\quad + M_{rj\varphi}(\widehat{v}(t,x))\big)\Bigg], \\ \frac{\partial u}{\partial t} &= \nabla\cdot(\mathfrak{D}(t,x,v)\circ\nabla v) - \tilde{A}(v(t,x)) \\ &\quad\cdot\Bigg[\tilde{B}(v(t,x)) - \zeta(\tilde{u}(t,x)) \\ &\quad - \sum_{j=1}^{r_0} h_i(w(t))\Big(\big(\tilde{C}_{rj}+\Delta\tilde{C}_{rj}(t)\big)g(u(t-\tilde{\tau}(t),x)) \\ &\quad + \tilde{M}_{rj}\varphi(\widehat{u}(t,x))\Big)\Bigg],\end{aligned} \tag{53}$$

for all $t > 0$, the initial value is $u(\theta,x) = \phi(\theta,x)$, $v(\theta,x) = \psi(\theta,x)$, and $\forall(\theta,x)\in[-\tau_*,0]\times\Omega$, where $\tilde{u},\tilde{v}$ represent feedback, and $\tilde{u},\tilde{v}$ represent the unknown exogenous disturbance of the neuron. For any $k = 0,1,2,\ldots$, the time t_k is the triggering time or update time. Between the triggering times t_k and t_{k+1}, the feedback control is designed as

$$\begin{aligned}\widehat{u}(t,x) &= \xi u(t_k,x), \\ \widehat{v}(t,x) &= \eta v(t_k,x), \\ &\quad t\in[t_k,t_{k+1}),\end{aligned} \tag{54}$$

where $t_0 = 0$, $\widehat{u}(t,x) = (\widehat{u}_1(t,x),\ldots,\widehat{u}_n(t,x))^T$, $\widehat{v}(t,x) = (\widehat{v}_1(t,x),\ldots,\widehat{v}_n(t,x))^T$, $\xi = \operatorname{diag}(\xi_1,\ldots,\xi_n)$, and $\eta = \operatorname{diag}(\eta_1,\ldots,\eta_n)$. Here ξ_i and η_i are constants for all i.

Let $(\breve{\Omega},\Upsilon,\mathbb{P})$ be the given probability space where $\breve{\Omega}$ is the sample space, Υ is σ, the algebra of subset of the sample space, and $\mathbb{P}$ is the probability measure defined on Υ. Let $S = \{1,2,\ldots,n_0\}$ and the random form process $\{r(t)\colon [0,+\infty)\to S\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories with generator $\Pi = (\gamma_{ij})_{n_0\times n_0}$ and transition probability from mode i at time t to mode j at time $t+\Delta t$, i, and $j\in S$.

$$\mathbb{P}(r(t+\delta) = j \mid r(t) = i) = \begin{cases}\gamma_{ij}\delta + o(\delta), & j\neq i, \\ 1+\gamma_{ij}\delta + o(\delta), & j\neq i,\end{cases} \tag{55}$$

where $\gamma_{ij}\geq 0$ is transition probability rate from i to $j(j\neq i)$ and $\gamma_{ii} = -\sum_{j=1,j\neq i}^{n_0}\gamma_{ij}$, $\delta > 0$ and $\lim_{\delta\to 0} o(\delta)/\delta = 0$.

Let $e(t,x) = (e_1(t,x),\ldots,e_n(t,x))^T$ and $\tilde{e}(t,x) = (\tilde{e}_1(t,x),\ldots,\tilde{e}_n(t,x))^T$ be the error signal defined by

$$\begin{aligned}e(t,x) &= u(t_k,x) - u(t,x), \\ \tilde{e}(t,x) &= v(t_k,x) - v(t,x), \\ &\quad t\in[t_k,t_k,+1),\end{aligned} \tag{56}$$

then we actually get

$$\begin{aligned}\widehat{u}(t,x) &= \xi[u(t,x)+e(t,x)], \\ \widehat{v}(t,x) &= \eta[v(t,x)+\tilde{e}(t,x)], \\ &\quad t\geq 0.\end{aligned} \tag{57}$$

Define the event-triggering mechanism by

$$\begin{aligned}t_{k+1} = \inf\Big\{t > t_k : {}&\theta\big(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\big) + \varepsilon\big(\|u\|_{L^2}^2 + \|v\|_{L^2}^2\big) \\ &- \big(\|u(t-\tilde{\tau}(t))\|_{L^2}^2 + \|v(t-\tau(t))\|_{L^2}^2\big) \\ &- W\big(\|e\|_{L^2}^2 + \|\tilde{e}\|_{L^2}^2\big) < 0\Big\},\end{aligned} \tag{58}$$

where $\theta > 0$, $\varepsilon\in(0,1)$, and $W > 0$.

Remark 12. Such that t_{k+1} is always defined well on many occasions. For example, let initial value $u(\theta,x) = \phi(\theta,x)\equiv 0$, $v(\theta,x) = \psi(\theta,x)\equiv 0$, and $\forall(\theta,x)\in[-\tau_*,0]\times\Omega$, then we must get $t_1 > 0$.

In this section, we assume that the conditions (H1)–(H3) hold still in the case of $p = 2$.

Besides, suppose that

(H4) $\zeta(\tilde{v}) = (\zeta_1(\tilde{v}_1),\zeta_2(\tilde{v}_2),\ldots,\zeta_n(\tilde{v}_n))^T$ with a positive definite matrix $L = \operatorname{diag}(l_1,l_2,\ldots,l_n)$ such that

$$|\zeta_i(s) - \zeta_i(t)| \leq l_i|s-t|, \quad \forall s,t\in R^n,\ i = 1,2,\ldots,n. \tag{59}$$

(H5) $\varphi(\widehat{v}) = (\varphi_1(\widehat{v}_1),\varphi_2(\widehat{v}_2),\ldots,\varphi_n(\widehat{v}_n))^T$ with a positive definite matrix $\widehat{L} = \operatorname{diag}(\widehat{l}_1,\widehat{l}_2,\ldots,\widehat{l}_n)$ such that

$$|\varphi_i(s) - \varphi_i(t)| \leq \widehat{l}_i|s-t|, \quad \forall s,t\in R^n,\ i = 1,2,\ldots,n. \tag{60}$$

For any mode $r\in S$,

$$\begin{aligned}C_{rj}(t) &= C_{rj} + \Delta C_{rj}(t), \\ \tilde{C}_{rj}(t) &= \tilde{C}_{rj} + \Delta C_{rj}(t),\end{aligned} \tag{61}$$

which do not have to be diagonal matrices or other special matrices.

In addition, we assume that

$$f(0) = g(0) = 0 = \zeta(0) = \varphi(0), \tag{62}$$

which can guarantee that $u = 0$, and $v = 0$ is a trivial solution of (53).

Besides, there are nonnegative matrices C_{r*} and $\tilde{C}_{r*}$ such that

$$\begin{aligned} -C_{r*} &\le \Delta C_{rj}(t) \le C_{r*}, \\ -\tilde{C}_{r*} &\le \Delta \tilde{C}_{rj}(t) \le \tilde{C}_{r*}. \end{aligned} \tag{63}$$

Before giving the man result of this section, we need the following lemma:

Lemma 4.1 *([54]) Let $x \in R^n$, $y \in R^n$, and $\varepsilon > 0$. Then we have*

$$x^T y + y^T x \le \varepsilon x^T x + \varepsilon^{-1} y^T y \tag{64}$$

Definition 4.2. System (53) is called robust stochastic input-to-state in mean square stable for all admissible uncertainties satisfying (63), if for $t > 0$, there exist function $\beta \in \mathscr{KL}$ and $\gamma \in \mathscr{K}$ such that

$$\begin{aligned} \mathbb{E}\left(\|u\|_{L^2}^2 + \|v\|_{L^2}^2\right) \le \beta\big(t, \mathbb{E}\left(\|u(0,x)\|_{L^2}^2 + \|v(0,x)\|_{L^2}^2\right)\big) \\ + \gamma\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right), \end{aligned} \tag{65}$$

where $\mathscr{K} = \{\gamma(\cdot)|\gamma : R_+ \to R_+$ is continuous strictly increasing with $\gamma(0) = 0\}$ with $R_+ = [0, \infty)$, $\mathscr{KL} = \{\beta(\cdot,\cdot)|\beta(t,\cdot) \in \mathscr{K}$ for each fixed $t, \beta(t,y)$ is decreasing for fixed y and $\lim_{t\to\infty} \beta(t,y) = 0\}$.

Theorem 4.3. *Assume the conditions (H1)–(H5) hold. Suppose that there is a sequence of positive definition matrices $P_r (r \in S)$ and positive scalars α_1, α_2, α_3, and α_4 such that the following LMI conditions hold for any mode $r \in S$:*

$$\begin{pmatrix} \mathcal{Q}r & P_r\bar{A}(|C_{r1}| + C_{r*})F & \cdots & P_r\bar{A}(|C_{rr0}| + C_{r*})F & \frac{1}{\alpha_2}P_r\bar{A}|M_{r1}| & \cdots & \frac{1}{\alpha_2}P_r\bar{A}|M_{rr0}| \\ * & -I & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & 0 & 0 & \cdots & 0 \\ * & * & \cdots & -I & 0 & \cdots & 0 \\ * & * & \cdots & * & -I & \cdots & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & \cdots & -I \end{pmatrix} > 0,$$

$$\begin{pmatrix} \tilde{\mathcal{Q}}r & P_r\bar{\bar{A}}\left(|\tilde{C}_{r1}| + \tilde{C}_{r*}\right)G & \cdots & P_r\bar{\bar{A}}\left(|\tilde{C}_{rr0}| + \tilde{C}_{r*}\right)G & \frac{1}{\alpha_2}P_r\bar{\bar{A}}|M_{r1}| & \cdots & \frac{1}{\alpha_2}P_r\bar{\bar{A}}|M_{rr0}| \\ * & -I & \cdots & 0 & 0 & \cdots & 0 \\ * & * & \ddots & 0 & 0 & \cdots & 0 \\ * & * & \cdots & -I & 0 & \cdots & 0 \\ * & * & \cdots & * & -I & \cdots & 0 \\ * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & \cdots & -I \end{pmatrix} > 0, \tag{66}$$

where I represents the identity matrix with suitable dimension under different cases for convenience.

$$\begin{aligned} \mathcal{Q}_r &= 2\lambda_1 P_r D + 2P_r\underline{A}\mathbb{B} - \sum_{j=1}^{n_0}\left|\gamma_{rj}\right|P_j - \frac{1}{\alpha_1}I - 2r_0\alpha_4\xi^2\hat{L}^2 - \varepsilon r_0 I, \\ \tilde{\mathcal{Q}}_r &= 2\lambda_1 P_r\mathfrak{D} + 2P_r\underline{\tilde{A}}\tilde{\mathbb{B}} - \sum_{j=1}^{n_0}\left|\gamma_{rj}\right|P_j - \frac{1}{\alpha_3}I - 2r_0\alpha_2\eta^2\hat{L}^2 - \varepsilon r_0 I. \end{aligned} \tag{67}$$

If, in addition,

$$\max\left\{\lambda_{\max}\left(2r_0\alpha_2\eta^2\hat{L}^2\right), \lambda_{\max}\left(2r_0\alpha_4\xi^2\hat{L}^2\right)\right\} \le r_0 W, \tag{68}$$

then (53) is a robust stochastic input-to-state stable in mean square.

Proof. Construct the Lyapunov-Krasovskii functionals as follows:

$$\mathbb{V}(t,r)=\mathbb{V}_1(t,r)+\mathbb{V}_2(t,r),\quad r\in S,$$
$$\mathbb{V}_1(t,r)=\int_\Omega u^T(t,x)P_r u(t,x)dx, \tag{69}$$
$$\mathbb{V}_2(t,r)=\int_\Omega v^T(t,x)P_r v(t,x)dx,$$

where each $P_r(r\in S)$ is positive definition diagonal matrix.- Due to

$$\widehat{v}(t,x)=\eta[v(t,x)+\tilde{e}(t,x)], \tag{70}$$

we get

$$|\widehat{v}(t,x)|^T|\widehat{v}(t,x)|\le 2\left[|v|^T\eta^2|v|+|\tilde{e}(t,x)|^T\eta^2|\tilde{e}(t,x)|\right], \tag{71}$$

and

$$\begin{aligned}
&2\int_\Omega |u|^T P_r A(u)\sum_{j=1}^{r_0}|M_{rj}||\varphi(\widehat{v}(t,x))|dx\\
&\le 2\sum_{j=1}^{r_0}\int_\Omega |u|^T P_r\bar{A}|M_{rj}|\widehat{L}|\widehat{v}(t,x))|dx\\
&\le \sum_{j=1}^{r_0}\int_\Omega\left[\frac{1}{\alpha_2}|u|^T P_r\bar{A}|M_{rj}||M_{rj}|^T\bar{A}P_r|u|\right.\\
&\qquad\left.+\alpha_2|\widehat{v}(t,x))|^T\widehat{L}^2|\widehat{v}(t,x))|\right]dx\\
&\le \sum_{j=1}^{r_0}\int_\Omega\left[\frac{1}{\alpha_2}|u|^T P_r\bar{A}|M_{rj}||M_{rj}|^T\bar{A}P_r|u|+2\alpha_2|v|^T\eta^2\widehat{L}^2|v|\right.\\
&\qquad\left.+2\alpha_2|\tilde{e}(t,x)|^T\eta^2\widehat{L}^2|\tilde{e}(t,x)|\right]dx,
\end{aligned} \tag{72}$$

where $u=u(t,x)$ and $v=v(t,x)$.

Similarly, we get

$$\begin{aligned}
&2\int_\Omega |u|^T P_r L|\tilde{v}(t,x)|dx\\
&\le\int_\Omega\left(\frac{1}{\alpha_1}|u|^T|u|+\alpha_1|\tilde{v}(t,x)|^T LP_rP_rL|\tilde{v}(t,x)|\right)dx.
\end{aligned} \tag{73}$$

Next,

$$\begin{aligned}
&2\sum_{j=1}^{r_0}\int_\Omega |u|^T P_r A(u)|C_{rj}(t)||f(v(t-\tau(t),x))|dx\\
&\le 2\sum_{j=1}^{r_0}\int_\Omega |u|^T P_r\bar{A}|C_{rj}(t)|F|v(t-\tau(t),x)|dx\\
&\le\sum_{j=1}^{r_0}\int_\Omega |u|^T P_r\bar{A}|C_{rj}(t)|FF|C_{rj}(t)|^T\bar{A}P_r|u|dx\\
&\quad+r_0\int_\Omega |v(t-\tau(t),x)|^T|v(t-\tau(t),x)|dx.
\end{aligned} \tag{74}$$

Let L be the weak infinitesimal operator, then we get

$$\begin{aligned}
\mathscr{L}\mathbb{V}_1(t,x)\le&\int_\Omega |u(t,x)|^T\\
&\cdot\left[-2\lambda_1P_rD-2P_r\underline{A}\mathbb{B}+\sum_{j=1}^{n_0}|\gamma_{rj}|P_j+\frac{1}{\alpha_2}I\right.\\
&+\sum_{j=1}^{r_0}P_r\bar{A}|C_{rj}(t)|FF|C_{rj}(t)|^T\bar{A}P_r\\
&\left.+\sum_{j=1}^{r_0}\frac{1}{\alpha_2}P_r\bar{A}|M_{rj}||M_{rj}|^T\bar{A}P_r\right]|u(t,x)|dx\\
&+\int_\Omega|\tilde{v}(t,x)|^T(\alpha_1LP_rP_rL)|\tilde{v}(t,x)|dx\\
&+r_0\int_\Omega|v(t-\tau,x)|^T|v(t-\tau(t),x)|dx\\
&+\int_\Omega|v(t,x)|^T\left(2r_0\alpha_2\eta^2\widehat{L}^2\right)|v(t,x)|\\
&+\int_\Omega|\tilde{e}(t,x)|^T\left(2r_0\alpha_2\eta^2\widehat{L}^2\right)|\tilde{e}(t,x)|dx
\end{aligned} \tag{75}$$

Similarly,

$$\begin{aligned}
\mathscr{L}\mathbb{V}_2(t,x)\le&\int_\Omega |u(t,x)|^T\\
&\cdot\left[-2\lambda_1P_r\mathfrak{D}-2P_r\underline{\tilde{A}}\tilde{\mathbb{B}}+\sum_{j=1}^{n_0}|\gamma_{rj}|P_j+\frac{1}{\alpha_3}I\right.\\
&+\sum_{j=1}^{r_0}P_r\bar{\bar{A}}|\tilde{C}_{rj}(t)|GG|\tilde{C}_{rj}(t)|^T\bar{\bar{A}}P_r\\
&\left.+\sum_{j=1}^{r_0}\frac{1}{\alpha_4}P_r\bar{\bar{A}}|\tilde{M}_{rj}||\tilde{M}_{rj}|^T\bar{\bar{A}}P_r\right]|v(t,x)|dx\\
&+\int_\Omega|\tilde{u}(t,x)|^T(\alpha_3LP_rP_rL)|\tilde{u}(t,x)|dx\\
&+r_0\int_\Omega|u(t-\tilde{\tau}(t),x)|^T|u(t-\tilde{\tau}(t),x)|dx\\
&+\int_\Omega|u(t,x)|^T\left(2r_0\alpha_4\xi^2\widehat{L}^2\right)|u(t,x)|\\
&+\int_\Omega|e(t,x)|^T\left(2r_0\alpha_4\xi^2\widehat{L}^2\right)|e(t,x)|dx.
\end{aligned} \tag{76}$$

Hence, we get

$$\begin{aligned}
\mathscr{L}\mathbb{V}(t,r)\le&\int_\Omega |u(t,x)|^T\\
&\cdot\left[-2\lambda_1P_rD-2P_r\underline{A}\mathbb{B}+\sum_{j=1}^{r_0}|\gamma_{rj}|P_j\right.\\
&+\frac{1}{\alpha_1}I+\sum_{j=1}^{r_0}P_r\bar{A}|C_{rj}(t)|FF|C_{rj}(t)|^T\bar{A}P_r\\
&\left.+\sum_{j=1}^{r_0}\frac{1}{\alpha_2}P_r\bar{A}|M_{rj}||M_{rj}|^T\bar{A}P_r+2r_0\alpha_4\xi^2\widehat{L}^2\right]
\end{aligned}$$

$$\cdot |u(t,x)|dx + \int_\Omega |v(t,x)|^T$$
$$\cdot \left[-2\lambda_1 P_r \mathfrak{D} - 2P_r \underline{\tilde{A}} \tilde{\mathbb{B}} + \sum_{j=1}^{n_0} \left|\gamma_{rj}\right| P_j + \frac{1}{\alpha_3} I \right.$$
$$+ \sum_{j=1}^{r_0} P_r \overline{\tilde{A}} \left|\tilde{C}_{rj}(t)\right| GG \left|\tilde{C}_{rj}(t)\right|^T \overline{\tilde{A}} P_r$$
$$\left. + \sum_{j=1}^{r_0} \frac{1}{\alpha_4} P_r \overline{\tilde{A}} \left|\tilde{M}_{rj}\right| \left|\tilde{M}_{rj}\right|^T \overline{\tilde{A}} P_r + 2r_0 \alpha_2 \eta^2 \hat{L}^2 \right]$$
$$\cdot |v(t,x)|dx + \int_\Omega |\tilde{u}(t,x)|^T (\alpha_3 L P_r P_r L) |\tilde{u}(t,x)|dx$$
$$+ \int_\Omega |\tilde{v}(t,x)|^T (\alpha_1 L P_r P_r L) |\tilde{v}(t,x)|dx$$
$$+ r_0 \int_\Omega |u(t-\tilde{\tau}(t),x)|^T |u(t-\tilde{\tau}(t),x)|dx$$
$$+ r_0 \int_\Omega |v(t-\tau(t),x)|^T |v(t-\tau(t),x)|dx$$
$$+ \int_\Omega |e(t,x)|^T \left(2r_0 \alpha_4 \xi^2 \hat{L}^2\right) |e(t,x)|dx$$
$$+ \int_\Omega |\tilde{e}(t,x)|^T \left(2r_0 \alpha_2 \eta^2 \hat{L}^2\right) |\tilde{e}(t,x)|dx. \tag{77}$$

That means

$$\begin{aligned} \mathscr{L}\mathbb{V}(t,x) \le & -\Theta_r \left(\|u\|_{L^2}^3 + \|v\|_{L^2}^3\right) \\ & + r_0 \left(\|u(t-\tilde{\tau}(t))\|_{L^2}^2 + \|v(t-\tau(t))\|_{L^2}^2\right) \\ & + \Psi\left(\|e\|_{L^2}^3 + \|\tilde{e}\|_{L^2}^3\right) + \Phi_r \left(\|\tilde{u}\|_{L^2}^3 + \|\tilde{v}\|_{L^2}^3\right), \end{aligned} \tag{78}$$

where

$$\begin{aligned} \Theta_r = \min & \left\{ \lambda_{\min} \left[\mathcal{Q}_r - \sum_{j=1}^{r_0} P_r \bar{A} \left(\left|C_{rj}\right| + C_{r*}\right) FF \left(\left|C_{rj}\right| + C_{r*}\right)^T \bar{A} P_r \right. \right. \\ & \left. - \sum_{j=1}^{r_0} \frac{1}{\alpha_2} P_r \bar{A} \left|M_{rj}\right| \left|M_{rj}\right|^T \bar{A} P_r \right], \lambda_{\min} \\ & \cdot \left[\tilde{\mathcal{Q}}_r - \sum_{j=1}^{r_0} P_r \overline{\tilde{A}} \left(\left|\tilde{C}_{rj}\right| + \tilde{C}_{r*}\right) GG \left(\left|\tilde{C}_{rj}\right| + \tilde{C}_{r*}\right)^T \overline{\tilde{A}} P_r \right. \\ & \left.\left. - \sum_{j=1}^{r_0} \frac{1}{\alpha_4} P_r \overline{\tilde{A}} \left|\tilde{M}_{rj}\right| \left|\tilde{M}_{rj}\right|^T \overline{\tilde{A}} P_r \right] \right\}, \Psi = \max \\ & \cdot \left\{ \lambda_{\max}\left(2r_0 \alpha_2 \eta^2 \hat{L}^2\right), \lambda_{\max}\left(2r_0 \alpha_4 \xi^2 \hat{L}^2\right) \right\} \le r_0 W, \end{aligned} \tag{79}$$

and

$$\Phi r = \max \left\{ \lambda_{\max}(\alpha_3 L P_r P_r L), \lambda_{\max}(\alpha_1 L P_r P_r L) \right\}. \tag{80}$$

In addition, for any $t \in [t_k, t_{k+1})$, the definition of t_{k+1} derives

$$\begin{aligned} & \theta\left(\|\tilde{u}\|_{L_2}^2 + \|\tilde{v}\|_{L_2}^2\right) + \varepsilon\left(\|u\|_{L_2}^2 + \|v\|_{L_2}^2\right) \\ & - \left(\|u(t-\tilde{\tau}(t))\|_{L^2}^2 + \|u(t-\tau(t))\|_{L^2}^2\right) \\ & - W\left(\|e\|_{L^2}^2 + \|\tilde{e}\|_{L^2}^2\right) \ge 0. \end{aligned} \tag{81}$$

So we get

$$\begin{aligned} \mathscr{L}\mathbb{V}(t,r) \le & -(\Theta_r - \varepsilon r_0)\left(\|u\|_{L^2}^2 + \|u\|_{L^2}^2\right) \\ & + (\Phi_r + \theta r_0)\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right), \end{aligned} \tag{82}$$

or

$$\mathscr{L}\mathbb{V}(t,r) \le -\beta_1\left(\|u\|_{L^2}^2 + \|v\|_{L^2}^2\right) + \beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right), \tag{83}$$

where (66) and Schur complement lemma yield that $\Theta_r - \varepsilon r_0 > 0$ and hence $\beta_1 > 0$ with $\beta_1 = \min_{r\in S}(\Theta_r - \varepsilon r_0)$ and $\beta_2 = \max_{r\in S}(\Phi_r + \theta r_0) > 0$.

Furthermore, Dynkin's formula yields

$$\begin{aligned} \frac{d}{dt}\mathbb{E}\mathbb{V}(t,r) = & \mathbb{E}\mathfrak{L}\mathbb{V}(t,r) \\ \le & -\beta_1\left(\|u\|_{L^2}^2 + \|v\|_{L^2}^2\right) + \beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right) \\ \le & -\frac{\beta_1}{\lambda_{\max} P_r} EV(t,r) + \beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right) \\ \le & -\beta_3 EV(t,r) + \beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right), \end{aligned} \tag{84}$$

where

$$\beta_3 = \min_{r\in S} \frac{\beta 1}{\lambda_{\max} P_r} > 0. \tag{85}$$

Applications of the Comparison principle to (84) reaches

$$\begin{aligned} \mathbb{E}\mathbb{V}(t,r) \le & \mathbb{E}\mathbb{V}(0,r)e^{-\beta_3 t} - \frac{\beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right)}{\beta_3}\left(e^{-\beta_3 t} - 1\right) \\ \le & \mathbb{E}\mathbb{V}(0,r)e^{-\beta_3 t} + \frac{\beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right)}{\beta_3} \\ \le & (\lambda_{\max} P_r)\left(\|u(0,x)\|_{L^2}^2 + \|v(0,x)\|_{L^2}^2\right)e^{-\beta_3 t} \\ & + \frac{\beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right)}{\beta_3}, \end{aligned} \tag{86}$$

which derives

$$\begin{aligned} \left(\|u\|_{L^2}^2 + \|v\|_{L^2}^2\right) \le & \frac{\lambda_{\max} P_r}{\lambda_{\max} P_r}\left(\|u(0,x)\|_{L^2}^2 + \|v(0,x)\|_{L^2}^2\right)e^{-\beta_3 t} \\ & + \frac{\beta_2\left(\|\tilde{u}\|_{L^2}^2 + \|\tilde{v}\|_{L^2}^2\right)}{\beta_3 \lambda_{\min} P_r}, \end{aligned} \tag{87}$$

which together with Definition 4.2 implies that (53) is robust stochastic input-to-state stable in mean square.

Remark 13. Theorem 4.3 provides a new stability criterion which is different from the existing criteria of [55–59]. In addition, to the best of our knowledge, it is the first time to investigate input-to-state stability of reaction-diffusion time-delay system with event-triggered control. Especially, the diffusion items play roles in the criterion.

5. Numerical Examples

Example 5.1. Consider (8) with the following parameters: $p = 8/3$, $u = (u_1(t,x), u_2(t,x))^T$, $v = (v_1(t,x), v_2(t,x))^T \in R^2$, $x \in \Omega = (0, \pi)$, and then the first eigenvalue

$$\lambda_1 = \left(\frac{2}{\pi} \int_0^{((8/3)-1)^{(1/(8/3))}} \frac{dt}{(1 - t^{8/3}/(8/3))^{(1/(8/3))}} \right)^{8/3} = 9558 \tag{88}$$

(see Remark 2).

Let $a_1(u_1) = 0.1\sqrt[3]{u_1^2}(1 + \cos^2(u_1 + 1))$, $a_2(u_2) = 0.2\sqrt[3]{u_2^2}$, $\tilde{a}_1(v_1) = 0.2\sqrt[3]{u_1^2}$, $\tilde{a}_2(v_2) = 0.1\sqrt[3]{u_2^2}(1 + \sin^4(v_2 - 10))$, $b_1(u_1) = 2_{u_1}(1 + \sin^2 u_1)$, $b_2(u_2) = 2.5_{u_2}$, $d_1(u_1) = 2_{v_1}(1 + \cos^2 u_1)$, $d_2(u_2) = 2.7_{u_2}$, $f_1(v_1) = 0.16 v_1 \sin v_1$, $f_2(v_2) = 0.166 v_2$, $g_1(u_1) = 0.15 u_1 \sin u_1$, $g_2(u_2) = 0.17 u_2$, $\tau_1(t) = \tau_2(t) = (t/2) = \tilde{\tau}_1(t) = \tilde{\tau}_2(t)$, $\rho_1(t) = \rho_2(t) = \tilde{\rho}_1(t) = 5 = \tilde{\rho}_2(t)$, and $l_0 = 0.5$, $\rho = \tilde{\rho} = 5$,

$$\begin{aligned}
\underline{A} &= \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, \\
\bar{A} &= \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix} = \overline{\tilde{A}}, \\
\underline{\tilde{A}} &= \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix}, \\
\mathbb{B} &= \begin{pmatrix} 2 & 0 \\ 0 & 2.5 \end{pmatrix}, \\
\tilde{\mathbb{B}} &= \begin{pmatrix} 2 & 0 \\ 0 & 2.7 \end{pmatrix}, \\
D(t,x,u) &= \begin{pmatrix} 0.003 & 0 \\ 0 & 0.006 \end{pmatrix}, \\
\mathfrak{D}(t,x,u) &= \begin{pmatrix} 0.0033 & 0 \\ 0 & 0.0057 \end{pmatrix}, \\
F &= \begin{pmatrix} 0.16 & 0 \\ 0 & 0.166 \end{pmatrix}, \\
G &= \begin{pmatrix} 0.15 & 0 \\ 0 & 0.17 \end{pmatrix}.
\end{aligned} \tag{89}$$

Let $r_0 = 2$, and

$$\begin{aligned}
C_1 &= \begin{pmatrix} -0.053 & 0.0011 \\ 0.0018 & 0.085 \end{pmatrix}, \\
C_2 &= \begin{pmatrix} 0.086 & 0.0009 \\ -0.0011 & 0.0085 \end{pmatrix}, \\
\tilde{C}_1 &= \begin{pmatrix} 0.036 & 0.001 \\ -0.0011 & 0.085 \end{pmatrix}, \\
\tilde{C}_2 &= \begin{pmatrix} 0.035 & 0.0011 \\ -0.0009 & 0.088 \end{pmatrix}, \\
M_1 &= \begin{pmatrix} -0.023 & 0.0013 \\ 0.0008 & 0.072 \end{pmatrix}, \\
M_2 &= \begin{pmatrix} 0.076 & 0.0003 \\ -0.0002 & 0.0072 \end{pmatrix}, \\
\tilde{M}_1 &= \begin{pmatrix} 0.036 & 0.0003 \\ -0.0009 & 0.036 \end{pmatrix}, \\
\tilde{M}_2 &= \begin{pmatrix} 0.032 & 0.0003 \\ -0.0002 & 0.077 \end{pmatrix}, \\
C_* &= \begin{pmatrix} 0.033 & 0.0011 \\ -0.0018 & 0.0063 \end{pmatrix}, \\
\tilde{C}_* &= \begin{pmatrix} 0.0063 & 0.0013 \\ 0.0012 & 0.00036 \end{pmatrix}, \\
M_* &= \begin{pmatrix} 0.0033 & 0.0061 \\ -0.0019 & 0.0013 \end{pmatrix}, \\
\tilde{M}_* &= \begin{pmatrix} 0.0033 & 0.0011 \\ 0.0039 & 0.0066 \end{pmatrix}.
\end{aligned} \tag{90}$$

So we can use MATLAB software to compute (18), obtaining

$$\begin{aligned}
\mathfrak{K} &= \begin{pmatrix} 2.0000 & 0 & -0.1078 & -0.0693 \\ 0 & 2.5000 & -0.0644 & -0.0813 \\ -0.0685 & -0.1081 & 2.0000 & 0 \\ -0.0734 & -0.1251 & 0 & 2.7000 \end{pmatrix}^{-1} \\
&= \begin{pmatrix} 0.5014 & 0.0018 & 0.0271 & 0.0129 \\ 0.0009 & 2.4012 & 0.0130 & 0.0121 \\ 0.0172 & 0.0217 & 0.5016 & 0.0011 \\ 0.0137 & 0.0186 & 0.0013 & 0.3713 \end{pmatrix} \geq 0.
\end{aligned} \tag{91}$$

Moreover, utilizing MATLAB LMI toolbox to solve LMIs (19)–(20) reaches the feasibility data as follows:

Table 2: Comparisons of our results with other results related to reaction-diffusion.

Related results	Value of p	Diffusion plays a role	Applicable to MATLAB LMI toolbox
Our Theorem 3.2	$p > 1$	Yes	Completely applicable
[19, Theorem 3.1]	$p > 2$	No	Not
[8, Theorem 3.1]	$p = 2$		Yes
[11, Theorem 3.2]	$p > 1$	No	Not
[16, Theorem 1–3]	$p = 2$	Yes	Not
[18, Theorem 3.1]	$p > 1$	No	Yes

$$Q = \begin{pmatrix} 46.6134 & 0 \\ 0 & 46.9921 \end{pmatrix}. \quad (92)$$

Now, one can conclude from Theorem 3.2 that there exists the globally asymptotically robust stable unique equilibrium point for (8).

Remark 14. From Table 1, we know, our new results (Theorem 3.2 and Corollary 3.3) is novel because the boundedness of amplification functions becomes unnecessary.

Remark 15. From Table 2, we know, our Theorem 3.2 is novel, different from those of existing results.

Example 5.2. Consider (63) with the following parameters: $\Omega = (0,10) \times (0,10)$ and $u = (u_1, u_2)^T$, $v = (v_1, v_2)^T \in R^2$, and $x = (x_1, x_2) \in \Omega \subset R^2$. And so $\lambda_1 = 0.02\pi^2 = 0.1974$ (see Remark 3).

Let $a_1(u_1) = 0.1\sqrt[3]{u_1^2}(1+\cos^2(u_1+1))$, $a_2(u_2) = 0.2\sqrt[3]{u_2^2}$, $\tilde{a}_1(v_2) = 0.2\sqrt[3]{v_1^2}$, $\tilde{a}_2(v_2) = 0.1\sqrt[3]{v_2^2}(1+\sin^4(v_2-10))$, $b_1(u_1) = 2u_1(1+\sin^2 u_1)$, $b_2(u_2) = 2.5u_2$, $d_1(v_1) = 2v_1(1+\cos^2 v_1)$, $d_2(v_2) = 2.7v_2$, $f_1(v_1) = 0.16 v_1 \sin v_1$, $f_2(v_2) = 0.166 v_2$, $g_1(u_1) = 0.15 u_1 \sin u_1$, $g_2(u_2) = 0.17 u_2$, $\tau_1(t) = \tau_2(t) = (t/2) = \tilde{\tau}_1(t) = \tilde{\tau}_2(t)$,

$$\underline{A} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix},$$

$$\bar{A} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix} = \bar{\tilde{A}},$$

$$\underline{\tilde{A}} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.1 \end{pmatrix}$$

$$\mathbb{B} = \begin{pmatrix} 2 & 0 \\ 0 & 2.5 \end{pmatrix},$$

$$\tilde{\mathbb{B}} = \begin{pmatrix} 2 & 0 \\ 0 & 2.7 \end{pmatrix},$$

$$D(t,x,u) = \begin{pmatrix} 0.003 & 0 \\ 0 & 0.006 \end{pmatrix},$$

$$\mathfrak{D}(t,x,u) = \begin{pmatrix} 0.0033 & 0 \\ 0 & 0.0057 \end{pmatrix},$$

$$F = \begin{pmatrix} 0.16 & 0 \\ 0 & 0.166 \end{pmatrix},$$

$$G = \begin{pmatrix} 0.15 & 0 \\ 0 & 0.17 \end{pmatrix}. \quad (93)$$

Let $r_0 = 2 = n_0$, and

$$C_{11} = \begin{pmatrix} -0.053 & 0.0011 \\ 0.0018 & 0.085 \end{pmatrix},$$

$$C_{12} = \begin{pmatrix} 0.086 & 0.0009 \\ -0.0011 & 0.0058 \end{pmatrix},$$

$$\tilde{C}_{11} = \begin{pmatrix} 0.036 & 0.001 \\ -0.0011 & 0.016 \end{pmatrix},$$

$$\tilde{C}_{12} = \begin{pmatrix} 0.035 & 0.0011 \\ -0.0009 & 0.088 \end{pmatrix},$$

$$M_{11} = \begin{pmatrix} -0.023 & 0.0013 \\ 0.0008 & 0.072 \end{pmatrix},$$

$$M_{12} = \begin{pmatrix} 0.076 & 0.0003 \\ -0.0002 & 0.0027 \end{pmatrix},$$

$$\tilde{M}_{11} = \begin{pmatrix} 0.036 & 0.0003 \\ -0.0009 & 0.036 \end{pmatrix},$$

$$\tilde{M}_{12} = \begin{pmatrix} 0.032 & 0.0003 \\ -0.0002 & 0.077 \end{pmatrix},$$

$$C_{1*} = \begin{pmatrix} 0.033 & 0.0011 \\ -0.0018 & 0.0063 \end{pmatrix},$$

$$\tilde{C}_{1*} = \begin{pmatrix} 0.0063 & 0.0013 \\ 0.0012 & 0.00036 \end{pmatrix},$$

$$\tilde{C}_{2*} = \begin{pmatrix} 0.0058 & 0.0013 \\ 0.0012 & 0.00053 \end{pmatrix},$$

$$C_{21} = \begin{pmatrix} -0.049 & 0.0014 \\ 0.0018 & 0.085 \end{pmatrix},$$

$$C_{22} = \begin{pmatrix} 0.089 & 0.0009 \\ -0.0018 & 0.0058 \end{pmatrix},$$

$$\tilde{C}_{21} = \begin{pmatrix} 0.054 & 0.001 \\ -0.0011 & 0.016 \end{pmatrix},$$

$$\tilde{C}_{22} = \begin{pmatrix} 0.043 & 0.0011 \\ -0.0009 & 0.088 \end{pmatrix},$$

$$M_{21} = \begin{pmatrix} -0.032 & 0.0013 \\ 0.0008 & 0.072 \end{pmatrix},$$

$$M_{22} = \begin{pmatrix} 0.083 & 0.0003 \\ -0.0002 & 0.0027 \end{pmatrix},$$

$$\tilde{M}_{21} = \begin{pmatrix} 0.043 & 0.0003 \\ -0.0009 & 0.036 \end{pmatrix},$$

$$\tilde{M}_{22} = \begin{pmatrix} 0.039 & 0.0003 \\ -0.0002 & 0.077 \end{pmatrix},$$

$$C_{2*} = \begin{pmatrix} 0.0043 & 0.0011 \\ -0.0018 & 0.0063 \end{pmatrix},$$

$$\xi = \begin{pmatrix} 0.0010 & 0 \\ 0 & 0.0015 \end{pmatrix},$$

$$\eta = \begin{pmatrix} 0.0021 & 0 \\ 0 & 0.0033 \end{pmatrix},$$

$$\begin{aligned} \zeta(\tilde{v}) &= (\zeta_1(\tilde{v}_1), \zeta_2(\tilde{v}_2))^T \\ &= \left(0.01 \sin \tilde{v}_1, 0.02 \sin^2 \tilde{v}_2\right) \varphi(\hat{u}) \\ &= (\varphi_1(\hat{u}_1), \varphi_2(\hat{u}_2))^T \\ &= \left(0.01 \sin^3 \hat{u}_1, 0.02\hat{u}_2\right). \end{aligned} \tag{94}$$

Let $\theta = 0.01$, $\varepsilon = 0.001$, and $W = 0.5$ and then we can compute and verify that (68) is satisfied. Now using computer MATLAB LMI-toolbox to solve LMI (66) gives the feasibility data as follows:

$$\begin{aligned} P_1 &= \begin{pmatrix} 7.7856 & 0 \\ 0 & 6.899 \end{pmatrix}, \\ P_2 &= \begin{pmatrix} 6.6189 & 0 \\ 0 & 6.9973 \end{pmatrix}, \\ \alpha_1 &= 1.5346, \\ \alpha_2 &= 1.5986, \\ \alpha_3 &= 1.1323, \\ \alpha_4 &= 0.9869. \end{aligned} \tag{95}$$

According to Theorem 4.3, (53) is robust stochastic input-to-state stable in mean square.

Remark 16. This paper is inspired by the methods and conclusions of the previous literature [55–59]. But the sufficient conditions of Theorem 4.3 is easier to be verified than those of existing results.

6. Conclusions

In this paper, we mainly provided two novel conclusions for p-Laplace diffusion BAM CGNNs. In the case of $p > 1$ with $p \neq 2$, the authors construct novel Lyapunov functional to overcome the mathematical difficulties of nonlinear p-Laplace diffusion time-delay model with parameter uncertainties, deriving the LMI-based robust stability criterion applicable to computer MATLAB LMI toolbox, deleting the boundedness of the amplification functions. On the other hand, when $p = 2$, LMI-based sufficient conditions are also inferred for robust input-to-state stability of reaction-diffusion Markovian jumping BAM CGNNs with the event-triggered control, which is different from those of many previous related literature. As far as we are concerned, seldom literature involved a reaction-diffusion stochastic system with time delays and the event-triggered control. It is the first time to explore the method for the stability analysis of this system. Finally, numerical examples illustrate the effectiveness and feasibility via computer MATLAB LMI toolbox.

Conflicts of Interest

The authors declare that they have no competing interests.

Authors' Contributions

All authors typed, read, and approved the final manuscript.

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