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Research Article

Stability Analysis of Impulsive Stochastic Reaction-Diffusion Cellular Neural Network with Distributed Delay via Fixed Point Theory

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This paper investigates the stochastically exponential stability of reaction-diffusion impulsive stochastic cellular neural networks (CNN). The reaction-diffusion pulse stochastic system model characterizes the complexity of practical engineering and brings about mathematical difficulties, too. However, the difficulties have been overcome by constructing a new contraction mapping and an appropriate distance on a product space which is guaranteed to be a complete space. This is the first time to employ the fixed point theorem to derive the stability criterion of reaction-diffusion impulsive stochastic CNN with distributed time delays. Finally, an example is provided to illustrate the effectiveness of the proposed methods.

1. Introduction

In 1988, cellular neural networks (CNN) were originally introduced in [1, 2]. Since then, dynamic neural networks have received extensive attention due to their classification, associative memory, and parallel computing tasks and the ability to solve complex optimization problems. It is generally known that almost all neural networks have similar applications ([3–12]), but the key to the success of these applications lies in the stability of the system. In fact, there are a number of literatures involved in the stability analysis of CNN ([5, 7, 12–14]). In practical engineering, time delay and pulse are unavoidable. Since neural networks usually have spatial properties, due to the existence of parallel paths of various axonal sizes and lengths, it is necessary to introduce continuous distributed delays to simulate them over a given time horizon. Besides, many evolutionary processes, especially the biological neural network in biological systems and bursting rhythm models in pathology, frequency-modulated signal processing systems, are characterized by abrupt changes of states at certain time instants. In addition, electrons have diffusion behavior in inhomogeneous media. Noise disturbance is unavoidable in real nervous systems, which is a major source of instability and poor performance in neural networks. A neural network can be stabilized or destabilized by certain stochastic inputs. The synaptic transmission in real neural networks can be viewed as a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes. Hence, the above influent factors should be also taken into consideration in stability analysis of neural networks. So, in this paper, we consider a class of impulsive stochastic reaction-diffusion cellular neural networks with distributed delay. Lyapunov function method is one of the common techniques to solve the stability of neural networks in recent decades. However, every method has its limit. Different methods lead to different criteria for stability criteria which may imply innovations. Fixed point theory and method is one of the alternative methods ([15–22]). Unlike the known literature, we try to employ Banach fixed point theory in this paper to derive the stability of impulsive stochastic reaction-diffusion cellular neural networks with distributed delay. In the next sections, we shall give some model descriptions and preliminaries and employ Banach fixed point theorem, Hölder inequality, Burkholder-Davis-Gundy inequality, and the continuous semigroup of Laplace operators to derive the stochastically exponential stability criterion of the complex system. Of course, to overcome the difficulty of the complex mathematical model, we need to formulate a new contraction mapping on a product space. Moreover, in order to guarantee the completeness of product space, we need to give a reasonable definition of distance. Finally, an example is provided to illustrate the effectiveness of the proposed result.

2. Model Description and Preliminaries

Consider the following reaction-diffusion impulsive stochastic cellular neural networks under Dirichlet boundary value:

$$\begin{aligned} du_i(t,x) &= -q_i \operatorname{div}\nabla u_i(t,x)\,dt - \Bigg[a_i u_i(t,x) \\ &\quad - \sum_{j=1}^{n} b_{ij} f_j\big(u_j(t,x)\big) - \sum_{j=1}^{n} c_{ij} f_j\big(u_j(t-\tau(t),x)\big) \\ &\quad - \sum_{j=1}^{n} h_{ij}\int_{t-\rho(t)}^{t} f_j\big(u_j(s,x)\big)\,ds\Bigg]dt \\ &\quad + \sigma_i\big(u_i(t,x)\big)\,dw_i(t), \\ &\qquad\qquad t\neq t_k,\ x\in\Upsilon,\ i\in\mathcal{N} \\ u(t_k^+,x) &= u(t_k^-,x) + g\big(u(t_k,x)\big), \\ &\qquad\qquad x\in\Upsilon,\ k=1,2,\ldots \\ u_i(t,x) &= \zeta_i(t,x), \quad \forall (s,x)\in[-\tau,0]\times\Upsilon \\ u(t,x) &= 0, \quad \forall (t,x)\in[0,+\infty)\times\partial\Upsilon, \end{aligned} \tag{1}$$

where $\Upsilon \subset R^m$ is a bounded domain with the smooth boundary $\partial\Upsilon$. $u_i(t,x)$ is the state variable of the ith neuron at time t and in space variable x for $i \in \mathcal{N}$ with $\mathcal{N} \triangleq \{1,2,\ldots,n\}$. f_i denotes the active function of neuron. a_i is the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and the external inputs. b_{ij}, c_{ij}, and h_{ij} are elements of feedback template. Let $\{w_i(t),\ t \geqslant 0\}$ be a real-valued Brownian motion defined on the complete probability space $\{\Omega,\mathcal{F},\mathbb{P}\}$ which has natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$. Denote by $\mathcal{L}^2(\Upsilon)$ the space of all real-valued square integrable functions with the inner product $\langle\xi,\eta\rangle = \int_\Upsilon \xi(x)\eta(x)dx$, for $\xi,\eta \in \mathcal{L}^2(\Upsilon)$ which derives the norm $\|\xi\| = (\int_\Upsilon \xi^2(x)dx)^{1/2}$ for $\xi \in \mathcal{L}^2(\Upsilon)$. $\sigma_i(\cdot)$ is a Borel measurable function. Denote by $\Delta = \sum_{j=1}^{m}(\partial^2/\partial x_j^2)$ the Laplace operator, with domain $\mathcal{D}(\Delta) = W_0^{1,2}(\Upsilon) \cap W_0^{2,2}(\Upsilon)$, which generates a strongly continuous semigroup $e^{-q_i t\Delta}$, where $W_0^{1,2}(\Upsilon)$ and $W_0^{2,2}(\Upsilon)$ are the Sobolev spaces with compactly supported sets. $\operatorname{div}\nabla u_i(t,x)$ denotes the divergence of $\nabla u_i(t,x)$ (see, e.g., [25, 26]). q_i is the diffusion coefficient, and time delays $\tau(t),\rho(t) \in [0,\tau]$. Besides, initial value $\zeta_i(t,x)$ is continuous for $(s,x)\in[-\tau,0]\times\Upsilon$. The fixed impulsive moments t_k ($k = 1,2,\ldots$) satisfy $0 < t_1 < t_2 < \cdots$ with $\lim_{k\to+\infty} t_k = +\infty$. $u(t_k^+,x)$ and $u(t_k^-,x)$ stand for the right-hand and left-hand limit of $u(t,x)$ at time t_k, respectively. Further, suppose that $u(t_k^-,x) = \lim_{t\to t_k^-} u(t,x) = u(t_k,x)$, $k = 1,2,\ldots$.

In this paper, we assume that

(H1) $\|e^{-q_i t\Delta}\| \leqslant Me^{-\gamma t}$, where $M > 0$ and $\gamma > 0$ are constants;

(H2) f_i, g_i, and σ_i are Lipschitz continuous with Lipschitz constants $L_i > 0$, G_i, and $T_i > 0$ for $i \in \mathcal{N}$, respectively. In addition, $f_i(0) = g_i(0) = 0 = \sigma_i(0)$, $\forall i \in \mathcal{N}$.

Definition 1. For any $T > 0$ and $x \in \Upsilon$, a stochastic process $u = \{(u_1(t,x),u_2(t,x),\ldots,u_n(t,x))\}_{[0,T]}$ is called a mild solution of impulsive system (1) if, for any $i \in \mathcal{N}$, $u_i(t,x) \in \mathcal{C}([0,T];\mathcal{L}^2(\Upsilon))$ and, for any $t\in[0,T]$, $u_i(t,x)$ is adapted to $\mathcal{F}_t$ with

$$\mathbb{P}\left\{\omega : \int_0^t \int_\Upsilon |u_i(s)|^2\,dx\,ds < \infty\right\} = 1, \tag{2}$$

and the following stochastic integral equations hold for all $i \in \mathcal{N}$, a.s. for any $t\in[0,T]$ and $x\in\Upsilon$,

$$\begin{aligned} u_i(t,x) &= e^{-q_i t\Delta}\zeta(0,x) - \int_0^t e^{-q_i(t-\theta)\Delta}\Bigg[a_i u_i(\theta,x) \\ &\quad - \sum_{j=1}^{n} b_{ij} f_j\big(u_j(\theta,x)\big) - \sum_{j=1}^{n} c_{ij} f_j\big(u_j(\theta-\tau(\theta),x)\big) \\ &\quad - \sum_{j=1}^{n} h_{ij}\int_{\theta-\rho(\theta)}^{\theta} f_j\big(u_j(s,x)\big)\,ds\Bigg]d\theta \\ &\quad + \int_0^t e^{-q_i(t-\theta)\Delta}\sigma_i\big(u_i(\theta,x)\big)\,dw_i(\theta) \\ &\quad + e^{-q_i t\Delta}\sum_{0<t_k<t} e^{q_i t_k\Delta} g_i\big(u_i(t_k,x)\big), \quad t\geqslant 0 \\ u_i(t,x) &= \zeta_i(t,x), \quad \forall (s,x)\in[-\tau,0]\times\Upsilon, \\ u(t,x) &= 0, \quad \forall (t,x)\in[0,+\infty)\times\partial\Upsilon. \end{aligned} \tag{3}$$

Remark 2. In Definition 1, the mild solution of impulsive system (1) is well defined due to [24, Lemma 3.1].

Lemma 3 (Hölder inequality). *Assume that* $1/p + 1/q = 1$ *with* $p > 1$*, and* $\varphi(x) \in \mathcal{L}^p(\Upsilon)$, $\phi \in \mathcal{L}^q(\Upsilon)$*; then,*

$$\begin{aligned} &\int_\Upsilon \varphi(x)\phi(x)\,dx \\ &\quad \leqslant \left(\int_\Upsilon |\varphi(x)|^p\,dx\right)^{1/p}\left(\int_\Upsilon |\phi(x)|^q\,dx\right)^{1/q}. \end{aligned} \tag{4}$$

Lemma 4 (Banach contraction mapping principle). *Let* Θ *be a contraction operator on a complete metric space* Γ*; then there exists a unique point* $u\in\Gamma$ *for which* $\Theta(u) = u$.

3. Main Result: Stochastically Exponential Stability

Theorem 5. *Assume that (H1) and (H2) hold. Then, CNN (1) is stochastically exponentially mean square stable if the following condition holds:*

$$0 < \kappa < 1, \tag{5}$$

where $\mu = \inf_{k=1,2,\ldots}(t_{k+1} - t_k) > 0$ *and*

$$\begin{aligned}\kappa \triangleq 6M^2 \Bigg[& \frac{1}{\gamma^2}\left(\max_{i\in\mathcal{N}} a_i^2\right) \\ & + n\frac{1}{\gamma^2}\max_{i\in\mathcal{N}}\left(\sum_{j=1}^{n}\left(\left|b_{ij}\right|^2 + \left|c_{ij}\right|^2\right)L_j^2\right) + \frac{n\tau^2}{\gamma^2} \\ & + 2M^2\left(1 + \frac{1}{\gamma^2\mu^2}\right)\left(\max_{i\in\mathcal{N}} G_i^2\right) + \frac{2}{\gamma}\left(\max_{i\in\mathcal{N}} T_i^2\right)\Bigg].\end{aligned} \tag{6}$$

Proof. Firstly, we need to formulate a contraction mapping on a product space.

Let Γ_i be the Banach space of all $\mathcal{F}_t$-adapted mean square continuous processes consisting of functions $u_i(t,x)$ at $t \geqslant 0$ with $t \neq t_k$ such that $\mathbb{E}(e^{\alpha t}\|u_i(t,x)\|^2) \to 0$ as $t \to +\infty$, where $\alpha \in (0,\gamma)$ is a positive scalar. Now, we construct an operator $\Theta \triangleq (\Theta_1, \Theta_2, \ldots, \Theta_i, \ldots, \Theta_n)$ with $\Theta_i : \Gamma_i \to \Gamma_i$ as follows:

$$\begin{aligned}\Theta_i(u_i)(t,x) = {} & e^{-q_i t\Delta}\zeta(0,x) - \int_0^t e^{-q_i(t-\theta)\Delta}\Bigg[a_i u_i(\theta,x) \\ & - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) - \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\ & - \sum_{j=1}^{n} h_{ij}\int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right)ds\Bigg]d\theta \\ & + \int_0^t e^{-q_i(t-\theta)\Delta}\sigma_i\left(u_i(\theta,x)\right)dw_i(\theta) \\ & + e^{-q_i t\Delta}\sum_{0<t_k<t} e^{q_i t_k\Delta} g_i\left(u_i(t_k,x)\right), \quad t \geqslant 0,\end{aligned} \tag{7}$$

$$\begin{aligned}&\Theta_i(u_i)(t,x) = \zeta_i(t,x), \quad (s,x) \in [-\tau, 0]\times\Upsilon \\ &\Theta_i(u_i)(t,x) = 0, \quad \forall (t,x) \in [0,+\infty)\times\partial\Upsilon.\end{aligned} \tag{8}$$

Equipped with the following distance:

$$\begin{aligned}\operatorname{dist}(u,v) = \left(\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i(t,x) - v_i(t,x)\right\|^2\right)^{1/2}, \\ \forall u, v \in \Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n,\end{aligned} \tag{9}$$

$\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$ becomes a complete metric space, where $u = u(t,x) = (u_1(t,x), u_2(t,x), \ldots, u_n(t,x))^T$, $v = v(t,x) = (v_1(t,x), v_2(t,x), \ldots, v_n(t,x))^T$.

Next, we are to apply contractive mapping theory to complete the proof via three steps.

Step 1. From (7), for $t \in [0,+\infty)\setminus\{t_k\}_{k=1}^{\infty}$, we claim that $\Theta_i(u_i)(t)$ is mean square continuous. Indeed, let δ be a small enough scalar:

$$\begin{aligned}&\mathbb{E}\left\|\Theta_i(u_i)(t+\delta,x) - \Theta_i(u_i)(t,x)\right\|^2 \\ &\quad\leqslant 4\mathbb{E}\left\|e^{-q_i(t+\delta)\Delta}\zeta(0,x) - e^{-q_i t\Delta}\zeta(0,x)\right\|^2 \\ &\quad + 4\mathbb{E}\Bigg\|\int_0^{t+\delta} e^{-q_i(t+\delta-\theta)\Delta}\Bigg[a_i u_i(\theta,x) \\ &\quad - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) - \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\ &\quad - \sum_{j=1}^{n} h_{ij}\int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right)ds\Bigg]d\theta \\ &\quad - \int_0^t e^{-q_i(t-\theta)\Delta}\Bigg[a_i u_i(\theta,x) - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) \\ &\quad - \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\ &\quad - \sum_{j=1}^{n} h_{ij}\int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right)ds\Bigg]d\theta\Bigg\|^2 \\ &\quad + 4\mathbb{E}\Bigg\|\int_0^{t+\delta} e^{-q_i(t+\delta-\theta)\Delta}\sigma_i\left(u_i(\theta,x)\right)dw_i(\theta) \\ &\quad - \int_0^t e^{-q_i(t-\theta)\Delta}\sigma_i\left(u_i(\theta,x)\right)dw_i(\theta)\Bigg\|^2 \\ &\quad + 4\mathbb{E}\Bigg\|e^{-q_i(t+\delta)\Delta}\sum_{0<t_k<t+\delta} e^{q_i t_k\Delta} g_i\left(u_i(t_k,x)\right) \\ &\quad - e^{-q_i t\Delta}\sum_{0<t_k<t} e^{q_i t_k\Delta} g_i\left(u_i(t_k,x)\right)\Bigg\|^2.\end{aligned} \tag{10}$$

Firstly, we estimate

$$\begin{aligned}&\mathbb{E}\left\|e^{-q_i(t+\delta)\Delta}\zeta(0,x) - e^{-q_i t\Delta}\zeta(0,x)\right\|^2 \\ &\quad\leqslant \mathbb{E}\left\|\left(e^{-q_i\delta\Delta} - 1\right)e^{-q_i t\Delta}\zeta(0,x)\right\|^2 \longrightarrow 0, \\ &\qquad\qquad \text{if } \delta \longrightarrow 0.\end{aligned} \tag{11}$$

Next, we evaluate

$$\begin{aligned}&\mathbb{E}\Bigg\|\int_0^{t+\delta} e^{-q_i(t+\delta-\theta)\Delta}\Bigg[a_i u_i(\theta,x) - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) \\ &\quad - \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right)\end{aligned}$$

$$
\begin{aligned}
&-\sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right) ds \Bigg] d\theta \\
&- \int_0^t e^{-q_i(t-\theta)\Delta} \Bigg[a_i u_i(\theta,x) - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) \\
&- \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\
&- \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right) ds \Bigg] d\theta \Bigg\|^2 \\
&\leqslant 2\mathbb{E} \Bigg\| \int_t^{t+\delta} e^{-q_i(t+\delta-\theta)\Delta} \Bigg[a_i u_i(\theta,x) \\
&- \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) - \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\
&- \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right) ds \Bigg] d\theta \Bigg\|^2 \\
&+ 2\mathbb{E} \Bigg\| \int_0^t e^{-q_i(t-\theta)\Delta} \Bigg[a_i u_i(\theta,x) - \sum_{j=1}^{n} b_{ij} f_j\left(u_j(\theta,x)\right) \\
&- \sum_{j=1}^{n} c_{ij} f_j\left(u_j(\theta-\tau(\theta),x)\right) \\
&- \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right) ds \Bigg] \left(e^{-q_i\delta\Delta} - 1\right) d\theta \Bigg\|^2 \\
&\longrightarrow 0, \quad \text{if } \delta \longrightarrow 0.
\end{aligned} \tag{12}
$$

Via Burkholder-Davis-Gundy inequality, we can conclude that if $\delta \to 0$,

$$
\begin{aligned}
&\mathbb{E} \Bigg\| \int_0^{t+\delta} e^{-q_i(t+\delta-\theta)\Delta} \sigma_i\left(u_i(\theta,x)\right) dw_i(\theta) \\
&- \int_0^t e^{-q_i(t-\theta)\Delta} \sigma_i\left(u_i(\theta,x)\right) dw_i(\theta) \Bigg\|^2 \\
&\leqslant 8\mathbb{E} \int_0^t M^2 e^{-2\gamma(t-\theta)} \left\| \sigma_i\left(u_i(\theta,x)\right) \left(e^{-q_i\delta\Delta} - 1\right) \right\|^2 d\theta \\
&+ 8\mathbb{E} \int_t^{t+\delta} M^2 e^{-2\gamma(t+\delta-\theta)} \left\| \sigma_i\left(u_i(\theta,x)\right) \right\|^2 d\theta \longrightarrow 0.
\end{aligned} \tag{13}
$$

Due to $t \neq t_k$, it is obvious that

$$
\begin{aligned}
&\mathbb{E} \Bigg\| e^{-q_i(t+\delta)\Delta} \sum_{0<t_k<t+\delta} e^{q_i t_k \Delta} g_i\left(u_i(t_k,x)\right) \\
&- e^{-q_i t\Delta} \sum_{0<t_k<t} e^{q_i t_k \Delta} g_i\left(u_i(t_k,x)\right) \Bigg\|^2 \longrightarrow 0, \\
&\qquad \text{if } \delta \longrightarrow 0.
\end{aligned} \tag{14}
$$

So, we have proved from (10)–(14) that $\Theta_i(u_i)(t)$ is mean square continuous at $t \geqslant 0$ with $t \neq t_k$.

Next, we claim that

$$
\begin{aligned}
&\lim_{\delta\to 0^+} \Theta_i(u_i)(t_k+\delta) = \Theta_i(u_i)(t_k) + g\left(u_i(t_k)\right), \\
&\lim_{\delta\to 0^-} \Theta_i(u_i)(t_k+\delta) = \Theta_i(u_i)(t_k).
\end{aligned} \tag{15}
$$

Indeed, obviously, (11)–(13) hold for all $t = t_k$, too. In addition, let $\delta > 0$ be small enough:

$$
\begin{aligned}
&e^{-q_i(t_k+\delta)\Delta} \sum_{0<t_j<t_k+\delta} e^{q_i t_j \Delta} g_i\left(u_i\left(t_j,x\right)\right) \\
&- e^{-q_i t_k\Delta} \sum_{0<t_j<t_k} e^{q_i t_j \Delta} g_i\left(u_i\left(t_j,x\right)\right) \\
&= g_i\left(u\left(t_k,x\right)\right), \quad \delta \longrightarrow 0^+.
\end{aligned} \tag{16}
$$

On the other hand, let $\delta < 0$ be small enough:

$$
\begin{aligned}
&e^{-q_i(t_k+\delta)\Delta} \sum_{0<t_j<t_k+\delta} e^{q_i t_j \Delta} g_i\left(u_i\left(t_j,x\right)\right) \\
&- e^{-q_i t_k\Delta} \sum_{0<t_j<t_k} e^{q_i t_j \Delta} g_i\left(u_i\left(t_j,x\right)\right) = 0, \\
&\qquad \delta \longrightarrow 0^-.
\end{aligned} \tag{17}
$$

This together with (16) implies that (15) holds.

Step 2. We claim that

$$
\mathbb{E}\left(e^{\alpha t} \left\|\Theta_i\left(u_i(t,x)\right)\right\|^2\right) \longrightarrow 0, \quad \text{if } t \longrightarrow +\infty. \tag{18}
$$

Indeed, we have the following inequality similar to (10):

$$
\begin{aligned}
&\mathbb{E}\left(e^{\alpha t} \left\|\Theta_i(u_i)(t,x)\right\|^2\right) \leqslant 4\mathbb{E}\left(e^{\alpha t} \left\|e^{-q_i t\Delta} \zeta(0,x)\right\|^2\right) \\
&+ 4\mathbb{E} \Bigg\{ e^{\alpha t} \Bigg\| \int_0^t e^{-q_i(t-\theta)\Delta} \Bigg[a_i u_i(\theta,x)
\end{aligned}
$$

$$- \sum_{j=1}^{n} b_{ij} f_j \left(u_j (\theta, x)\right) - \sum_{j=1}^{n} c_{ij} f_j \left(u_j (\theta - \tau (\theta), x)\right)$$
$$- \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j \left(u_j (s, x)\right) ds \Bigg] d\theta \Bigg\|^2 \Bigg\}$$
$$+ 4\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sigma_i \left(u_i (\theta, x)\right) dw_i (\theta) \right\|^2\right)$$
$$+ 4\mathbb{E}\left(e^{\alpha t} \left\| e^{-q_i t\Delta} \sum_{0<t_k<t} e^{q_i t_k \Delta} g_i \left(u_i (t_k, x)\right) \right\|^2\right), \quad t \geqslant 0. \tag{19}$$

Condition (H1) yields

$$\mathbb{E}\left(e^{\alpha t} \left\| e^{-q_i t\Delta} \zeta (0, x)\right\|^2\right) \leqslant \mathbb{E}\left(M^2 e^{-(2\gamma-\alpha)t} \left\|\zeta (0, x)\right\|^2\right) \longrightarrow 0, \quad \text{if } t \longrightarrow +\infty. \tag{20}$$

For any given $\varepsilon > 0$, the assumption $\mathbb{E}(e^{\alpha t}\|u_i(t, x)\|^2) \to 0$ tells us that there exists $t_* > 0$ such that

$$\mathbb{E}\left(e^{\alpha t} \left\|u_i (t, x)\right\|^2\right) < \varepsilon, \quad \forall t \geqslant t_*. \tag{21}$$

Moreover, Hölder inequality gives

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} a_i u_i (\theta, x) \, d\theta \right\|^2\right) \leqslant \frac{M^2 a_i^2}{\gamma} \cdot \mathbb{E}\left(e^{\alpha t} \int_0^t e^{-\gamma(t-\theta)} \left\|u_i (\theta, x)\right\|^2 d\theta\right) \leqslant \frac{M^2 a_i^2}{\gamma} \cdot \mathbb{E}\left(e^{-(\gamma-\alpha)t} t^* e^{\gamma t^*} \max_{\theta\in[0,t_*]} \left(\left\|u_i (\theta, x)\right\|^2\right) + \varepsilon \frac{1}{\gamma - \alpha}\right), \tag{22}$$

which together with the arbitrariness of ε derives

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} a_i u_i (\theta, x) \, d\theta \right\|^2\right) \longrightarrow 0, \quad \text{if } t \longrightarrow +\infty. \tag{23}$$

Besides,

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} b_{ij} f_j \left(u_j (\theta, x)\right) d\theta \right\|^2\right) \leqslant \mathbb{E}\left(M \sum_{j=1}^{n} \left|b_{ij}\right| L_j e^{-(\gamma-\alpha)t} \int_0^t e^{\gamma\theta} \left\|u_j (\theta, x)\right\| d\theta\right)^2. \tag{24}$$

Using similar methods of (21) and (22), we can deduce from (24) that

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} b_{ij} f_j \left(u_j (\theta, x)\right) d\theta \right\|^2\right) \longrightarrow 0, \quad \text{if } t \to +\infty. \tag{25}$$

Similar to that of (24) and (22), we can also obtain

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} c_{ij} f_j \left(u_j (\theta - \tau (\theta), x)\right) d\theta \right\|^2\right) \leqslant \mathbb{E}\left[M \sum_{j=1}^{n} \left|c_{ij}\right| L_j \frac{1}{\gamma} \left(e^{-(\gamma-\alpha)t} e^{\gamma\tau} (t_* + \tau) \cdot \max_{s\in[-\tau,t^*+\tau]} \left(e^{\gamma s} \left\|u_j (s, x)\right\|^2\right) + \varepsilon e^{\gamma\tau} \frac{1}{\gamma - \alpha}\right)\right]. \tag{26}$$

Now, similar to that of (22), we know from (26) that

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} c_{ij} f_j \left(u_j (\theta - \tau (\theta), x)\right) d\theta \right\|^2\right) \longrightarrow 0, \quad \text{if } t \longrightarrow +\infty. \tag{27}$$

Similarly, Hölder inequality yields

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j \left(u_j (s, x)\right) ds \, d\theta \right\|^2\right) \leqslant M^2 \mathbb{E}\left[\sum_{j=1}^{n} \left|h_{ij}\right| L_j \frac{\tau}{\gamma} \cdot \left(e^{-(\gamma-\alpha)t} \tau \max_{\theta\in[-\tau,t_*+\tau]} \left\|u_j (s, x)\right\|^2 \frac{1}{\gamma} e^{\gamma(t_*+\tau)} + \varepsilon\tau e^{\alpha\tau} \cdot \frac{1}{\gamma - \alpha}\right)\right]. \tag{28}$$

Similar to (22), we can conclude from (28) that

$$\mathbb{E}\left(e^{\alpha t} \left\| \int_0^t e^{-q_i(t-\theta)\Delta} \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j \left(u_j (s, x)\right) ds \, d\theta \right\|^2\right) \longrightarrow 0, \quad \text{if } t \longrightarrow +\infty. \tag{29}$$

Hence,

$$\mathbb{E}\Bigg\{e^{\alpha t} \Bigg\| \int_0^t e^{-q_i(t-\theta)\Delta} \Bigg[a_i u_i (\theta, x) - \sum_{j=1}^{n} b_{ij} f_j \left(u_j (\theta, x)\right) - \sum_{j=1}^{n} c_{ij} f_j \left(u_j (\theta - \tau (\theta), x)\right) - \sum_{j=1}^{n} h_{ij} \int_{\theta-\rho(\theta)}^{\theta} f_j \left(u_j (s, x)\right) ds \Bigg] d\theta \Bigg\|^2 \Bigg\}$$

$$
\begin{aligned}
&\leqslant 4\mathbb{E}\left\| e^{\alpha t}\int_0^t e^{-q_i(t-\theta)\Delta}a_i u_i(\theta,x)\,d\theta\right\|^2 \\
&+4\mathbb{E}\left\| e^{\alpha t}\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^n b_{ij}f_j\left(u_j(\theta,x)\right)d\theta\right\|^2 \\
&+4\mathbb{E}\left\| e^{\alpha t}\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^n c_{ij}f_j\left(u_j(\theta\right.\right. \\
&\left.\left.-\tau(\theta),x)\right)d\theta\right\|^2 + 4\mathbb{E}\left\| e^{\alpha t}\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^n h_{ij}\right. \\
&\left.\cdot\int_{\theta-\rho(\theta)}^{\theta} f_j\left(u_j(s,x)\right)ds\,d\theta\right\|^2 \longrightarrow 0, \\
&\qquad \text{if } t\longrightarrow +\infty.
\end{aligned}
\tag{30}
$$

Burkholder-Davis-Gundy inequality and Hölder inequality derive

$$
\begin{aligned}
&\mathbb{E}\left(e^{\alpha t}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sigma_i\left(u_i(\theta,x)\right)dw_i(\theta)\right\|^2\right) \\
&\leqslant 8\mathbb{E}\left(M^2T_i^2e^{-(2\gamma-\alpha)t}t_*\max_{\theta\in[0,t^*]}e^{2\gamma\theta}\left\|u_i(\theta,x)\right\|^2\right) \\
&\quad + 8\varepsilon\mathbb{E}\left(M^2T_i^2\sqrt{\frac{1}{4\gamma(\gamma-\alpha)}}\right),
\end{aligned}
\tag{31}
$$

which together with the arbitrariness of ε implies

$$
\mathbb{E}\left(e^{\alpha t}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sigma_i\left(u_i(\theta,x)\right)dw_i(\theta)\right\|^2\right)\longrightarrow 0, \quad \text{if } t\longrightarrow +\infty.
\tag{32}
$$

Next, we may assume that $t_{l-1}<t_*\leqslant t_l$ and $t_{j-1}<t\leqslant t_j$.

In addition, one can deduce from (H1)

$$
\begin{aligned}
&\mathbb{E}\left\{e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{0<t_k\leqslant t_*}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right\} \\
&\leqslant \mathbb{E}\left[Me^{(\alpha/2)t}e^{-\gamma t}\left(\sum_{0<t_k\leqslant t_l}Me^{\gamma t_k}G_i\left\|u_i(t_k,x)\right\|\right)\right]^2 \\
&\longrightarrow 0,\quad \text{if } t\longrightarrow +\infty.
\end{aligned}
\tag{33}
$$

Besides, we can estimate by means of definite integral

$$
\begin{aligned}
&\mathbb{E}\left\{e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{t_*<t_k<t}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right\} \\
&\leqslant \varepsilon G_i^2M^4\mathbb{E}\left(e^{-(1/2)(2\gamma-\alpha)t}\sum_{t_l\leqslant t_k\leqslant t_{j-1}}e^{(1/2)(2\gamma-\alpha)t_k}\right)^2 \\
&\leqslant \varepsilon\mathbb{E}\left(M^2G_i\left(1+\frac{2}{\mu(2\gamma-\alpha)}\right)\right)^2.
\end{aligned}
\tag{34}
$$

Moreover, the arbitrariness of ε implies

$$
\mathbb{E}\left\{e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{t_*<t_k<t}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right\}\longrightarrow 0, \quad \text{if } t\longrightarrow +\infty.
\tag{35}
$$

Hence, if $t\rightarrow +\infty$,

$$
\begin{aligned}
&\mathbb{E}\left\{e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{0<t_k<t}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right\} \\
&\leqslant 2\mathbb{E}\left(e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{0<t_k\leqslant t_*}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right) \\
&\quad + 2\mathbb{E}\left(e^{\alpha t}\left\|e^{-q_i t\Delta}\sum_{t_*<t_k<t}e^{q_i t_k\Delta}g_i\left(u_i(t_k,x)\right)\right\|^2\right) \\
&\longrightarrow 0.
\end{aligned}
\tag{36}
$$

Combining (19), (20), (23), (30), (32), and (36) results in (18).

Step 3. Finally, we claim that Θ is a contractive mapping on $\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$.

Indeed, from the above two steps, we know $\Theta_i(\Gamma_i)\subset\Gamma_i$, and then $\Theta(\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n)\subset\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$.

On the other hand, for any $i\in\mathcal{N}$ and $u,v\in\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$,

$$
\begin{aligned}
&\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\Theta_i(u_i)(t,x)-\Theta_i(v_i)(t,x)\right\|^2 \\
&\leqslant 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}a_i\left(u_i(\theta,x)\right.\right. \\
&\left.\left.-v_i(\theta,x)\right)d\theta\right\|^2 + 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\right. \\
&\left.\cdot\sum_{j=1}^n b_{ij}\left(f_j\left(u_j(\theta,x)\right)-f_j\left(v_j(\theta,x)\right)\right)d\theta\right\|^2
\end{aligned}
$$

$$+ 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^{n}c_{ij}\cdot\left(f_j\left(u_j\left(\theta-\tau\left(\theta\right),x\right)\right)-f_j\left(v_j\left(\theta-\tau\left(\theta\right),x\right)\right)\right)d\theta\right\|^2$$
$$+ 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^{n}h_{ij}\cdot\left(\int_{\theta-\rho(\theta)}^{\theta}\left[f_j\left(u_j\left(s,x\right)\right)-f_j\left(v_j\left(s,x\right)\right)\right]ds\right)d\theta\right\|^2$$
$$+ 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\left[\sigma_i\left(u_i\left(\theta,x\right)\right)-\sigma_i\left(v_i\left(\theta,x\right)\right)\right]dw_i\left(\theta\right)\right\|^2$$
$$+ 6\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|e^{-q_it\Delta}\sum_{0<t_k<t}e^{q_it_k\Delta}\left[g_i\left(u_i\left(t_k,x\right)\right)-g_i\left(v_i\left(t_k,x\right)\right)\right]\right\|^2. \tag{37}$$

Besides, it follows by the Hölder inequality that

$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}a_i\left(u_i\left(\theta,x\right)-v_i\left(\theta,x\right)\right)d\theta\right\|^2 \leqslant M^2\cdot\frac{1}{\gamma^2}\left(\max_{i\in\mathcal{N}}a_i^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(\theta,x\right)-v_i\left(\theta,x\right)\right\|^2,$$
$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^{n}b_{ij}\cdot\left(f_j\left(u_j\left(\theta,x\right)\right)-f_j\left(v_j\left(\theta,x\right)\right)\right)d\theta\right\|^2 \leqslant nM^2\frac{1}{\gamma^2}\cdot\max_{i\in\mathcal{N}}\left(\sum_{j=1}^{n}\left|b_{ij}\right|^2L_j^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(t,x\right)-v_i\left(t,x\right)\right\|^2,$$
$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\sum_{j=1}^{n}c_{ij}\cdot\left(f_j\left(u_j\left(\theta-\tau\left(\theta\right),x\right)\right)-f_j\left(v_j\left(\theta-\tau\left(\theta\right),x\right)\right)\right)d\theta\right\|^2 \leqslant nM^2\frac{1}{\gamma^2}\max_{i\in\mathcal{N}}\left(\sum_{j=1}^{n}\left|c_{ij}\right|^2L_j^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(t,x\right)-v_i\left(t,x\right)\right\|^2,$$
$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\cdot\sum_{j=1}^{n}h_{ij}\left(\int_{\theta-\rho(\theta)}^{\theta}\left[f_j\left(u_j\left(s,x\right)\right)-f_j\left(v_j\left(s,x\right)\right)\right]ds\right)d\theta\right\|^2 \leqslant nM^2\tau^2\frac{1}{\gamma^2}\max_{i\in\mathcal{N}}\left(\sum_{j=1}^{n}\left|h_{ij}\right|^2L_j^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(t,x\right)-v_i\left(t,x\right)\right\|^2,$$
$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|e^{-q_it\Delta}\sum_{0<t_k<t}e^{q_it_k\Delta}\cdot\left[g_i\left(u_i\left(t_k,x\right)\right)-g_i\left(v_i\left(t_k,x\right)\right)\right]\right\|^2 \leqslant M^4\left(\max_{i\in\mathcal{N}}G_i^2\right)\cdot\left[2\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(t,x\right)-v_i\left(t,x\right)\right\|^2+2\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left(\frac{1}{\mu}e^{-\gamma t}\int_0^t e^{\gamma s}\left\|u_i\left(s,x\right)-v_i\left(s,x\right)\right\|ds\right)^2\right] \leqslant 2M^4\left(1+\frac{1}{\gamma^2\mu^2}\right)\left(\max_{i\in\mathcal{N}}G_i^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(t,x\right)-v_i\left(t,x\right)\right\|^2, \tag{38}$$

where we assume that $t_{j-1} < t \leqslant t_j$.

In addition, it follows from Burkholder-Davis-Gundy inequality that

$$\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|\int_0^t e^{-q_i(t-\theta)\Delta}\cdot\left[\sigma_i\left(u_i\left(\theta,x\right)\right)-\sigma_i\left(v_i\left(\theta,x\right)\right)\right]dw_i\left(\theta\right)\right\|^2 \leqslant \frac{2}{\gamma}\cdot M^2\left(\max_{i\in\mathcal{N}}T_i^2\right)\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}\left\|u_i\left(\theta,x\right)-v_i\left(\theta,x\right)\right\|^2. \tag{39}$$

Now, combining (37)–(39) gives

$$\operatorname{dist}\left(\Theta\left(u\right),\Theta\left(v\right)\right) \leqslant \sqrt{\kappa}\operatorname{dist}\left(u,v\right), \quad \forall u,v\in\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n, \tag{40}$$

where κ is defined as (6), satisfying $0 < \kappa < 1$. This implies that $\Theta : \Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n \rightarrow \Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$ is a contraction mapping such that there exists the fixed point $u \triangleq (u_1(t,x),u_2(t,x),\ldots,u_n(t,x))$ of Θ in $\Gamma_1\times\Gamma_2\times\cdots\times\Gamma_n$, which implies that u is a solution of CNN (1), satisfying $\mathbb{E}(e^{\alpha t}\|\Theta_i(u_i(t,x))\|^2) \rightarrow 0$, $t \rightarrow +\infty$ so that $\mathbb{E}\max_{i\in\mathcal{N}}\sup_{t\geqslant-\tau}(e^{\alpha t}\|\Theta_i(u_i(t,x))\|^2) \rightarrow 0$, $t \rightarrow +\infty$. Therefore, CNN (1) is stochastically exponentially mean square stable. □

Table 1: Comparison of the complexity of system models in the literature related to fixed point theory.

	Theorem 5	[16]	[15]	[23]	[24]
Using fixed point theory	Yes	Yes	Yes	Yes	Yes
Impulse model	Yes	No	No	Yes	Yes
Distributed delays	Yes	No	No	No	No
Reaction-diffusion model	Yes	No	No	No	No
Itô stochastic model	Yes	No	No	No	No
Equations type	Integrodifferential (partial) eq.	Differential eq.	Differential eq.	Differential eq.	Integrodifferential eq.
Stability type	Stochastically exponential	Exponential	Exponential	Exponential	Exponential

4. Numerical Example

Consider the following impulsive stochastic reaction-diffusion CNN with distributed delay:

$$\begin{aligned} du_i(t,x) &= -q_i \operatorname{div}\nabla u_i(t,x)\,dt - \Bigg[a_i u_i(t,x) \\ &\quad - \sum_{j=1}^{n} b_{ij}\sin\left(\frac{j}{10}u_j(t,x)\right) \\ &\quad - \sum_{j=1}^{n} c_{ij}\sin\frac{j}{10}\left(u_j(t-\tau(t),x)\right) \\ &\quad - \sum_{j=1}^{n} h_{ij}\int_{t-\rho(t)}^{t}\sin\left(\frac{j}{10}u_j(s,x)\right)ds\Bigg]dt \\ &\quad + \sin\left(0.05 i u_i(t,x)\right)dw_i(t), \\ &\qquad\qquad t \neq t_k,\ x \in \Upsilon,\ i \in \mathcal{N} \\ u(t_k^+,x) &= u(t_k^-,x) + 0.1 j \sin\left(u(t_k,x)\right), \\ &\qquad\qquad x \in \Upsilon,\ k = 1,2,\ldots \\ u_i(t,x) &= \zeta_i(t,x), \quad (s,x) \in [-\tau,0]\times\Upsilon \\ u(t,x) &= 0, \quad u \in [0,+\infty)\times\partial\Upsilon, \end{aligned} \tag{41}$$

where we suppose $\Upsilon = (0,\pi)$, $n = 2$, $\tau = 3$, $\mu = 1.5$, $a_i = 0.5i$, $b_{ij} = 0.01(i+j) = c_{ij} = h_{ij}$, and $q_i = -1$. Then, via computing the eigenfunctions of $-\Delta$, we can obtain that $\|e^{t\Delta}\| \leqslant e^{-\pi^2 t}$, $t \geqslant 0$, so that we can take $\gamma = \pi^2$, $M = 1$. In addition, differential mean value theorem yields

$$\begin{aligned} &\left|\sin\left(\frac{j}{10}u_j(t,x)\right) - \sin\left(\frac{j}{10}v_j(t,x)\right)\right| \\ &\quad \leqslant \frac{j}{10}\left|u_j(t,x) - v_j(t,x)\right|, \end{aligned} \tag{42}$$

and then we get $L_j = j/10$, $j = 1,2$. Similarly, we can compute that $G_i = 0.1i$, $T_i = 0.05i$, and $i = 1,2$. Finally, we can compute (6) on a computer running Matlab software, obtaining $\kappa = 0.8716 \in (0,1)$. Therefore, Theorem 5 tells us that CNN (41) is stochastically exponentially mean square stable.

Table 1 is presented to compare the complexity of neural networks investigated in various literatures via fixed point theorems and techniques.

Remark 6. Impulsive reaction-diffusion Itô-type stochastic model gives a lot of mathematical difficulties in deriving the stability criterion. Motivated by some methods and techniques of the above-mentioned literature ([3–31]), this is the first time for us to analyze such a complex model by way of fixed point theorem. Our model is closer to real engineering so that it is more complex than those of the previous literature, and we utilize Banach fixed point theorem, Hölder inequality, Burkholder-Davis-Gundy inequality, and the continuous semigroup of Laplace operators to overcome the difficulties. Besides, the distance defined in this paper satisfies the triangle inequality, which is another point different from those of previous related literatures.

5. Conclusions

Since our CNN model involves pulse and Laplacian operators, our model is different from the previous model ([15–22]), which also implies some difficulties in mathematical techniques. Motivated by the previous literature related to fixed point theory ([15–22, 25–31]), the authors employed Banach fixed point theorem, Hölder inequality, Burkholder-Davis-Gundy inequality, and the continuous semigroup of Laplace operators to derive the stochastically exponential stability criterion of impulsive stochastic reaction-diffusion cellular neural networks with distributed delay.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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