

# A Puzzle About Ineffable Propositions\*

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February 22, 2010

I will argue for *localism* about credal assignments: the view that credal assignments are only well-defined relative to suitably constrained sets of possibilities. I will motivate the position by suggesting that it is the best way of addressing a puzzle devised by Roger White.

## 1 Ineffable propositions

You are deciding where to get a philosophy PhD, and you've narrowed down your options to Harvard and Oxford. All you care about in a philosophy career is the amount of money you'll make. So you wish to evaluate the truth of  $H$ : the proposition that you'd make more money if you went to Harvard. There is plenty of relevant information to be had. You compile faculty lists and placement statistics, you gauge the level of engagement of current graduate students, you assess the compatibility of your own philosophical proclivities with the problems that have been receiving the most attention at each of the universities. When all the evidence is finally in, you find yourself unable to consolidate it into anything like an assignment of probability to  $H$ . The evidence is too complex, and has too many dimensions.

As a result, you find yourself unable to decide where to go to graduate school on the

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\*For their many helpful comments, I am grateful to Juan Comesaña, Adam Elga, Dan Greco, Caspar Hare, Alejandro Pérez Carballo, Susanna Rinard, Brad Skow, Bob Stalnaker, Scott Sturgeon, and especially Roger White. I would also like to thank audience members at the 2009 Mexican Philosophers' Conference, and seminar participants at MIT.

basis of your evidence. And, of course, your predicament is not limited to that particular decision. Suppose that rather than deciding where to go to graduate school you are deciding which of two bets to accept. If you bet that  $H$  you'll receive a good if  $H$  and nothing if not- $H$ ; if you bet that not- $H$  you'll receive the good if not- $H$  and nothing if  $H$ . Again, you will find yourself unable to make a decision on the basis of your evidence.<sup>1</sup>

Whenever you face this sort of predicament with respect to a proposition, I shall say that you regard the proposition as *unresolved*:

Suppose you are offered one of two bets. If you bet that  $p$  you'll receive a good if  $p$  and nothing if not- $p$ ; if you bet that not- $p$  you'll receive the good if not- $p$  and nothing if  $p$ . I shall say that you regard  $p$  as *unresolved* if and only if you are unable to decide which of the bets to take on the basis of your evidence.

By saying that you regard it as unresolved, we have not fully characterized your epistemic attitude towards  $H$ . Note, for example, that any proposition to which you assign credence 0.5 is also a proposition you regard as unresolved. But there is a world of difference between your epistemic attitude towards  $H$  and your epistemic attitude towards a proposition to which you assign credence 0.5. Suppose, for example, that there is a certain coin you know to be fair, and that you are offered a choice between betting Heads and betting Tails. You should be indifferent between the two bets on the basis of your evidence. But your betting dispositions should be highly sensitive to new information. Any piece of independent evidence that the coin landed Heads should tip the balance, and lead you to prefer the bet that Heads

Your betting dispositions regarding  $H$ , in contrast, are not highly sensitive to new

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<sup>1</sup>Suppose you are offered a \$10 bet on  $H$ . What are the highest odds you should be willing to take? My own view is that this is an issue one ought to remain silent about. It is tempting to think otherwise if one assumes that when one's evidence is too complex to be consolidated into a probability assignment one should accept some sort of Indifference Principle. It seems to me that one should reject such an assumption. But this is a delicate matter which won't be discussed here.

evidence. Suppose you learn that Prof. E is to join the faculty at Harvard. You take this to be a positive development for Harvard, and think it improves your chances of making more money with a Harvard PhD. Prof. E's move, in other words, is something you regard as evidence for the proposition that you'll make more money if you go to Harvard. But your *total* body of evidence may still be too complex and have too many dimensions for you to be in a position to decide whether to bet  $H$  or not- $H$  on the basis of your evidence. You may, in other words, continue to regard  $H$  as unresolved. (This is not to say, of course, that no evidence could ever tip the balance. Should you learn, for example, that the entire Oxford faculty intends to resign, you should certainly prefer the bet that  $H$  to the bet that not- $H$ .)

Here is a more precise characterization of the relevant notion of sensitivity to evidence:

Let  $p$  be a proposition you regard as unresolved, and let  $E$  be a proposition to which you do not assign a credence of 1, and which you would regard as independent evidence for  $p$ , but not as independent evidence for not- $p$ . You regard  $p$  as *sensitive* (with respect to  $E$ ) if and only if, were you to learn that  $E$ , you would prefer the bet that  $p$  over the bet that not- $p$  on the basis of your total evidence. (Insensitivity is the contradictory of sensitivity.)

The focus of this paper will be on propositions like  $H$ , which are regarded as both unresolved and insensitive. I shall refer to them as *ineffable*.

## 2 White's Puzzle

Roger White devised the following puzzle. (See White (forthcoming).)

Start with a proposition you regard as ineffable, such as  $H$ .

On Sunday you learn that a coin-toss has taken place, and that the coin was

fair.

On Monday you learn that the Oracle has known all along whether  $H$  is true. Before the coin-toss took place, she painted a gold star on one side of the coin. If  $H$  is true, she painted the star on the Heads side of the coin; if  $H$  is false she painted it on the Tails side.

On Tuesday you learn that the coin landed with the star facing up. So any world compatible with your Tuesday knowledge is either a world in which  $H$  obtains and the coin landed Heads, or a world in which  $\neg H$  obtains and the coin landed Tails. In other words: relative to the space of worlds compatible with your Tuesday knowledge, the proposition that the coin landed Heads is precisely the proposition  $H$ . What should your Tuesday credence in this proposition be?

*Thought:* The only relevant information you had on Sunday was that the coin was fair. So on Sunday your credence that the coin landed Heads should have been 0.5. You certainly shouldn't change your credences on the basis of what you learned on Monday. And the only way your Tuesday knowledge should affect your credences about whether the coin landed Heads is by having some independent reason for accepting or rejecting  $H$ . But before to the exercise took place you regarded  $H$  as unresolved. So your Tuesday credence that the coin landed Heads should continue to be 0.5. Equivalently: your Tuesday credence in  $H$  should be 0.5.

*Counterthought:* On Sunday you regarded  $H$  as ineffable. But in the course of the exercise you have gained no evidence as to whether you would make more money if you went to Harvard.  $H$  is the proposition that you'd make more money if you went to Harvard. So on Tuesday you should continue to regard  $H$  as ineffable. And if you regard  $H$  as ineffable you cannot assign it a credence of 0.5.

### 3 Against *Thought*

I urge you to reject the reasoning in *Thought*. It entails that your credences should be affected by the order in which evidence is received. To see this, let  $E$  be a piece of evidence such that, on Sunday, you regard  $H$  as insensitive with respect to  $E$ . (As it might be: the proposition that Prof. E is moving to Harvard.)

*Scenario 1:* you learn that  $E$  before reasoning in accordance with *Thought*. Because you regarded  $H$  as unresolved before learning that  $E$ , you should continue to regard it as unresolved after learning that  $E$ . But all that the reasoning in *Thought* presupposes about  $H$  is that it be regarded as unresolved. So, by *Thought*, your credence in  $H$  should be 0.5.

*Scenario 2:* you learn that  $E$  after reasoning in accordance with *Thought*. By *Thought*, your Tuesday credence in  $H$  should be 0.5. But any proposition to which you assign a credence of 0.5 is a proposition you should regard as sensitive. So your credence in  $H$  after learning that  $E$  should not be 0.5.

The two scenarios differ only with respect to the order in which evidence was received.

### 4 Evidential Relativity

The right response, it seems to me, is to embrace the conclusion that, on Tuesday, you should refrain from assigning a particular credence to the proposition that the coin landed Heads.

Here is what I think is going on. We are accustomed to treating information about the relative merits of the Harvard and Oxford philosophy departments as *evidentially irrelevant* to the question of whether a given coin landed Heads. What the White-setup does is force

us to treat it as relevant. (Should you learn, for example, that Prof. E will be joining the faculty at Harvard, you should treat it as evidence that the coin landed Heads.) The problem is that you come to see information about Harvard and Oxford as evidentially relevant without learning how to consolidate it into a probability assignment. The new evidence is too complex, and has too many dimensions.

Brining all of one's evidence to bear can be a good way of protecting oneself from falsity. But it is not necessarily the road to truth. For it can have a paralyzing effect. And the opportunity of hitting on the truth may be well worth the risk of landing in falsity. So it should not be taken for granted that considering additional evidence is always the right thing to do, all things considered. When a career in philosophy is at stake, or an enormous bet is in the works, it might be worth availing oneself of every single piece of evidence at one's disposal, and thinking about it carefully, even if there is a risk of paralysis. But when less is at stake, or when one needs to limit the resources one devotes to the decision-making process, or when the risk of paralysis is too great, the costs of taking account of additional evidence may well outweigh the benefits.

On Tuesday, information about the relative merits of the Harvard and Oxford philosophy departments is evidentially relevant to the proposition that the coin landed Heads. But it may or may not be a good idea, all things considered, to take this evidence into account. It should certainly be taken into account for the purposes of deciding where to go to graduate-school. So, for the purposes of deciding where to go to graduate-school, one should, all things considered, refrain from assigning a particular credence to the proposition that the coin landed Heads.

On the other hand, it would be a mistake, all things considered, to take such evidence into account for the purposes of deciding whether to bet Heads when there is relatively little money at stake and you need to make a decision relatively quickly. The cost of engaging with the evidence would be too great. So, for the purposes of taking such a bet,

one should, all things considered, assign a credence of 0.5 to the proposition that the coin landed Heads.

Perhaps you think there is a purely epistemic sense of ‘should’ with respect to which it would be true to say that one should always take all available evidence into account. It seems to me that such a notion would be of limited interest as a guide to rational decision-making. For in insisting that all evidence be taken into account one privileges the avoidance of falsity over the attainment of truth. And, as we have seen, it is sometimes rational to increase the risk of falsity in order to increase one’s chances of hitting on the truth.

However that may be, it is a consequence of the picture I am suggesting here that—in a sense of ‘should’ that requires all evidence to be considered—one should, on Tuesday, refrain from assigning a particular credence to the proposition that the coin landed Heads, regardless of one’s aims. This is the right conclusion. The upshot of White’s Puzzle is that one can make just about any piece of evidence relevant to the proposition that the coin landed Heads. And when the evidence is sufficiently complex, and sufficiently multi-dimensional, one may not be able to consolidate it into a particular assignment of credence.

## 5 Modeling ineffability

The discussion above does not take a stand on the issue of how the doxastic state of a subject who regards propositions as ineffable should be modeled.

One possibility is to claim that it is best modeled by using a *range* of probability functions, rather than a single one. (For details see Levi (1974), Kaplan (1998) or Joyce (2005); for criticism see Sturgeon (2008), Sturgeon (forthcoming) and White (forthcoming).) Credal ranges are useful in other contexts, but my suspicion is that they are not the

key to understanding ineffability.

The way forward, it seems to me, is to give up the assumption that one can say, once and for all, what value (or range of values) represents a subject's credence that  $p$ . Such assignments are only well-defined *relative to a space of possibilities*: the possibilities that are treated as open for the purposes at hand.

This 'localist' approach has two advantages. The first is that it gives us a straightforward way of modeling evidential relativity. As we have seen, a subject's attitudes towards  $H$  can vary significantly, depending on whether she is assessing  $H$  for the purposes of deciding where to go to graduate school, or for the purposes of deciding whether to accept a low-stakes bet. I suggested that, in the former case, the subject takes a broad view of the available evidence, and that relative to such evidence her Tuesday credences in  $H$  are undefined. In the latter case, on the other hand, she takes a narrow view of the available evidence, and relative to such evidence her Tuesday credences in  $H$  are 0.5.

A localist can model different degrees of sensitivity to evidence by using more or less fine-grained possibility-spaces. To capture the fact that evidence is being ignored, the localist can use worlds that take no stand as to what the relevant evidence is like. When  $H$  is assessed for the purposes of deciding whether to accept a low-stakes bet, the subject ignores evidence concerning faculty-lists and placement-statistics. So the subject's doxastic state will be modeled by a set of worlds each of which specifies how the coin landed but says nothing about who's on the Oxford or Harvard faculty, or about where students have been placed. Relative to this possibility space, the subject can be said to distribute half of her credence on Heads-worlds and half on Tails-worlds. So, relative to this possibility space,  $H$  is assigned a credence of 0.5.

If, on the other hand, the localist wishes to capture the fact that a piece of evidence is being taken into account, she can use worlds that take a stand as to what the relevant evidence is like. When  $H$  is assessed for the purposes of deciding where to go to graduate



school, the subject takes into account such evidence as faculty lists and placement-records. So the subject's doxastic state will be modeled by a set of worlds each of which specifies how the coin landed and specifies who's on the Oxford and Harvard faculty, and where they've placed their students. Relative to such a possibility-space, the subject will be said not to know how to assign credences (or credal ranges). So, relative to this space, the subject's subjective probability in  $H$  is undefined.

The second advantage of the localist approach is that it supplies a way of capturing the fact that one might have views about the relative merits of Harvard and Oxford even if one is unable to consolidate them into an assignment of credences to  $H$ . Perhaps you believe that a good ranking according to the *Philosophical Gourmet Report* is evidence that a philosophy program will land its graduate students high-paying jobs. This doxastic state can be represented by a probability-distribution (or a set of probability-distributions) over a space of possibilities that differ only with respect to the *Gourmet*-rankings of Oxford and Harvard, and with respect to which of the two programs would land you a higher-paying job. Again, you might believe that a good placement-record is evidence that a philosophy program will land its graduate students in high-paying jobs. This second doxastic state can be represented by a probability-distribution (or a set of probability-distributions) over a space of possibilities that differ only with respect to the placement-records of Oxford and Harvard, and with respect to which of the two programs would land you a higher-paying job. But a localist leaves room for the view that you may nonetheless be unable to balance considerations of the two kinds. This can be modeled by claiming that your credences are undefined relative to a less restricted space of worlds, one in which there is variation both with respect to *Gourmet*-rankings and with respect to placement-records.

It is also worth mentioning that localist accounts of mental representation are not an *ad hoc* response to the phenomenon of ineffability. They find independent motivation in the phenomenon of vagueness. (See Rayo (2008).)

## References

- Joyce, J. (2005), 'How probabilities reflect evidence', *Philosophical Perspectives* **19**, 153–178.
- Kaplan, M. (1998), *Decision Theory as Philosophy*, Cambridge University Press, Cambridge.
- Levi, I. (1974), 'On indeterminate probabilities', *The Journal of Philosophy* **71**, 391–418.
- Rayo, A. (2008), 'Vague representation', *Mind* **117**, 329–373.
- Sturgeon, S. (2008), 'Reason and the grain of belief', *Noûs* **42**, 139–165.
- Sturgeon, S. (forthcoming), 'Confidence and coarse-grained attitudes'. *Oxford Studies in Epistemology*.
- White, R. (forthcoming), 'Evidential symmetry and mushy credence'. *Oxford Studies in Epistemology*.