Beyond Plurals

Agustín Rayo
philosophy.ucsd.edu/arayo

July 10, 2008

I have two main objectives. The first is to get a better understanding of what is at issue between friends and foes of higher-order quantification, and of what it would mean to extend a Boolos-style treatment of second-order quantification to third- and higher-order quantification. The second objective is to argue that in the presence of absolutely general quantification, proper semantic theorizing is essentially unstable: it is impossible to provide a suitably general semantics for a given language in a language of the same logical type. I claim that this leads to a trilemma: one must choose between giving up absolutely general quantification, settling for the view that adequate semantic theorizing about certain languages is essentially beyond our reach, and countenancing an open-ended hierarchy of languages of ever ascending logical type. I conclude by suggesting that the hierarchy may be the least unattractive of the options on the table.

1 Preliminaries

1.1 Categorial Semantics

Throughout this paper I shall assume the following:

Categorial Semantics

Every meaningful sentence has a semantic structure,[1] which may be represented as a certain kind of tree.[2] Each node in the tree falls under a particular semantic category (e.g. 'sentence', 'quantifier', 'sentential connective'), and has an intension that is appropriate for that category. The semantic category and intension of each non-terminal node in the tree is determined by the semantic categories and intensions of nodes below it.

Although I won't attempt to defend Categorial Semantics here,[3] two points are worth emphasizing. First, the claim that meaningful sentences are endowed with some sort of semantic structure is not optional. It is forced upon us by considerations of compositionally. (It is hard to understand how a sentence could have different semantic 'constituents' in the absence of some kind of semantic structure.) Second, the notions of semantic structure and semantic category should be distinguished from the notions of *grammatical* structure and *grammatical* category. Whereas the former are chiefly constrained by a theory that assigns *truth-conditions* to sentences, the latter are chiefly constrained by a theory that delivers a criterion of *grammaticality* for strings of symbols. (The two sets of notions are nonetheless interrelated, since we would like to have a transformational grammar that specifies a class of legitimate transformations linking the two.)

1.2 An example

Let L_0 be an (interpreted) propositional language. It consists of the following symbols: the sentential-letters 'p', 'q', and 'r'; the one-place connective-symbol '$\neg$'; the two-place connective-symbols '$\vee$' and '$\wedge$'; and the auxiliary symbols '(' and ')'. Well-formed formulas are defined in the usual way.

Here is an example of a categorial semantics for L_0. There are three semantic categories: 'sentence', 'one-place connective' and 'two-place connective'. To each of these categories corresponds a different kind of intension: the intension of a sentence is a set of possible worlds, the intension of a one-place connective is a function that takes each set of possible worlds to a set of possible worlds, and the intension of a two-place predicate is a function that takes each pair of sets of possible worlds to a set of possible worlds. We let the basic semantic lexicon of L_0 consist of 'p', 'q', 'r', '$\neg$', '$\vee$' and '$\wedge$'. The lexical items 'p', 'q' and 'r' fall under the 'sentence' category and have the following intensions:

$$I(\text{‘}p\text{’}) = \{w : \text{snow is white according to } w\}$$
$$I(\text{‘}q\text{’}) = \{w : \text{roses are red according to } w\}$$
$$I(\text{‘}r\text{’}) = \{w : \text{violets are blue according to } w\}$$

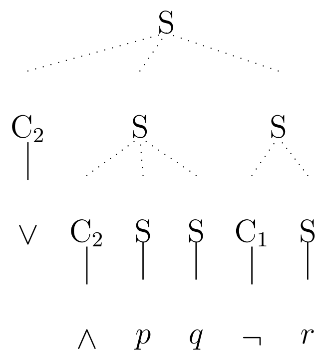
The lexical item '$\neg$' falls under the 'one-place connective' category and has the following intension:

$$I(\text{‘}\neg\text{’}) = \text{the function taking } W \text{ to its set-theoretic complement } \overline{W}$$

The lexical items '$\vee$' and '$\wedge$' fall under the 'two-place connective' category and have the following intensions:

$$I(\text{‘}\vee\text{’}) = \text{the function taking each pair } \langle W, V\rangle \text{ to } W \cup V$$
$$I(\text{‘}\wedge\text{’}) = \text{the function taking each pair } \langle W, V\rangle \text{ to } W \cap V$$

The semantic structure a formula of L_0 mirrors its syntax. For instance, the semantic structure of '$(p \wedge q) \vee \neg r$' (or, in Polish notation, '$\vee \wedge p\ q\ \neg r$') is given by the following tree:



(‘S’, ‘C_1’ and ‘C_2’ stand for ‘sentence’, ‘one-place connective’ and ‘two-place connective’, respectively.) Each terminal node in the tree is assigned the intension and semantic category of the lexical item displayed underneath. The intensions and semantic categories of non-terminal nodes are determined by the intensions and semantic categories of nodes below them, in the obvious way (e.g. the intension of base node is $(I(\text{‘}p\text{’}) \cap I(\text{‘}q\text{’})) \cup \overline{I(\text{‘}r\text{’})}$).

Since the intensions of sentences are taken to be sets of possible worlds, the semantics immediately delivers a characterization of truth for sentences in L_0: sentence S is true in world w if and only if w is a member of the intension assigned to the base node of S’s semantic structure.

1.3 Legitimacy

I shall say that a semantic category $\mathcal{C}$ is *legitimate* just in case it is in principle possible to make sense of a language whose semantic properties are accurately described by a categorial semantics employing $\mathcal{C}$.

One can certainly make sense of a propositional language. So ‘sentence’, ‘1-place connective’ and ‘2-place connective’ are all legitimate semantic categories. One can also make sense of a first-order language. So ‘name’, ‘n-place predicate’ and ‘first-order quantifier’ (with suitable intensions), are also legitimate semantic categories. But where do the limits lie? Might a purported semantic category be ruled out by the very nature of language?

An example might bring the matter into sharper focus. Let L_1 be an (interpreted) first-order language, and suppose it is agreed on all sides that the individual constant 'c' of L_1 falls under the semantic category 'name', and that the predicate '$P(\ldots)$' of L_1 falls under the semantic category 'one-place predicate'.[4] To fix ideas, think of the intension of a name as a function that takes each world to an individual in that world, and of the intension of a one-place predicate as a function that takes each world to a set of individuals in that world. Now suppose we tried to enrich L_1 with the new item 'ξ', in such a way that '$\xi(c)$' and '$P(\xi)$' are both sentences. It is tempting to think that there is no way of carrying out the extension without lapsing into nonsense. If this is right, then 'ξ' could not fall under a legitimate semantic category.

This is something of an extreme case, since the possibility of a semantic category corresponding to 'ξ' is ruled out by the category-formation rules of standard implementations of categorial semantics.[5] But the philosophy of logic is peppered with cases that are more difficult to adjudicate. Consider, for example, the debate that is sometimes labeled 'Is second-order logic really logic?'.[6] Quine has famously argued that the only way of making sense of second-order quantifiers is by understanding them as first-order quantifiers ranging over set-like entities. If this is right, second-order quantifiers cannot fall under a legitimate semantic category, at least not insofar as it is insisted that they not be thought of as first-order quantifiers.

Contrary claims are made by Quine's rivals. Boolos (1984) argues that it is possible to make sense of the Geach-Kaplan Sentence:

> Some critics admire only one another.

even though it is "a sentence whose quantificational structure cannot be captured by first order logic". If this is right, plural quantifiers fall under semantic category that is both legitimate and distinct from that of (singular) first-order quantifiers.

Boolos's work on plural quantification is an important contribution to the debate on second-order logic *not* because it shows that second-order quantifiers are plausibly understood as plural quantifiers. (Although plural quantifiers can be used to play the same role as second-order quantifiers for certain purposes, they should not be identified with second-order quantifiers because plural terms such as 'they' and 'them' do not take predicate positions.[7]) Rather, it is important because it makes a convincing case for the view that quantifiers other than the standard (singular) first-order quantifiers can fall under a legitimate semantic category, and this opens the door for thinking that it might be possible to understand second-order quantifiers in such a way that they too belong to a semantic category that is both legitimate and distinct from that of singular (first-order) quantifiers.

Needless to say, the concession that second-order quantifiers fall under a legitimate and *sui generis* semantic category would leave a number of important issues unresolved. The *logicality* of second-order quantifiers would not automatically be settled, since a single semantic category might include both logical and non-logical items (e.g. '$\ldots = \ldots$' and '... is a parent of ...'). The *ontological commitments* of second-order quantifiers would not automatically be settled, since the standard criterion of ontological commitment—Quine's criterion—applies only to first-order languages.[8] And the *determinacy* of second-order quantifiers would not automatically be settled, since an expression can be indeterminate even if its semantic category is fixed.[9]

2 Beyond Plurals

In this section I will introduce an infinite hierarchy of predicates, terms and quantifiers. I will claim without argument that each element in the hierarchy falls under a legitimate semantic category. Later in the paper I will try to motivate this claim.

2.1 First-level predicates

A first-level predicate is a predicate that takes a singular term in each of its argument places. It is tempting to think that the semantic value of the monadic first-level predicate '... is an elephant' is the set of elephants. More generally, it is tempting to think that the semantic value of the monadic first-level predicate '$P^1(\ldots)$' is the set of individuals '$P^1(\ldots)$' is true of. But now assume—as I will throughout the remainder of the paper—that it is possible to quantify over absolutely everything.[10] Then the thought is unsustainable. For it leads to the unwelcome conclusion that predicates such as '... is self-identical' or '... is a set' lack a semantic value.

(The problem cannot be solved by appealing to a non-standard set theory, or by employing entities other than sets. Suppose, for example, that one takes the semantic value of a monadic first-level predicate to be a set* rather than a set. It follows from a generalization of Cantor's Theorem that at least one of the following must be the case:[11] either (*a*) there are some things—the Fs—such that there is no set* consisting of all and only the Fs, or (*b*) there are some things—the Fs—and some things—the Gs—such that the Fs are not the Gs but the set* of the Fs is identical to the set* of the Gs. So we are left with the unsettling conclusion that either there are some things such that a predicate true of just those things would lack a semantic value, or there might be two predicates that share a semantic value but are not true of the same things.)

Rather than taking '... is an elephant' to stand for the set of elephants, I would like to suggest that one should take it to stand for the elephants themselves. It is grammatically infelicitous to say that the semantic value of '... is an elephant' is the elephants. So I shall say state the view by saying that '... is an elephant' *refers* to the elephants. Formally,

$$\exists xx(\forall y(y \prec^{1,2} xx \leftrightarrow \textsc{Elephant}^1(y)) \wedge \textsc{Ref}^{1,2}(\text{`}\ldots \text{ is an elephant'}, xx))$$

which is read:

> There are some things—the xxs—such that: (a) for every y, y is one of the xxs if and only if y is an elephant, and (b) '. . . is an elephant' refers to the xxs.

Double variables are used for plural terms and quantifiers,[12] and superscripts indicate the type of variable that the relevant predicate takes in each of its argument places. Predicates are interpreted in the obvious way: '$y \prec^{1,2} xx$' means 'y is one of the xxs' (or 'it$_y$ is one of them$_{xx}$'), '$\textsc{Elephant}^1(y)$' means 'y is an elephant' and '$\textsc{Ref}^{1,2}(y, xx)$' means '$y$ refers to the xxs' (or 'it$_y$ refers to them$_{xx}$'). The proposal therefore makes use of *second-level predicates*. (A second-level predicate is a predicate that takes a plural term in one of its argument places, and either a singular term or a plural term in the rest; more will be said about second-level predicates below.)

A snappy way of stating my claim is by saying that the reference of a monadic first-level predicate is a *plurality*. (One could say, for instance, that the reference of '. . . is an elephant' is the plurality of elephants.) But it is important to be clear that apparently singular quantification over 'pluralities' is a syntactic abbreviation for plural quantification over individuals; and that plural quantification is not the standard sort of (first-order) quantification over a new kind of 'item' ('plurality'); it is a new kind of quantification over individuals, which are the only kind of 'item' there is.

2.2 First-level terms

Let us now turn our attention to plural terms (or *first-level terms*, as I shall call them).[13] I argued above that it implausible to think that the first-level predicate '. . . is an elephant' refers to the set of elephants. It is similarly implausible to think that the first-level term 'the elephants' refers to the set of elephants, since the general view that 'the Fs' refers to the set of Fs leads to the unwelcome result that first-level terms such as 'the self-identical things' or 'the sets' are without reference. (And, for the same reasons as before, the problem cannot be avoided by appealing to a non-standard set-theory, or by employing entities other than sets.)

I would like to suggest that first-level terms—like monadic first-level predicates—refer to pluralities. Thus, 'the elephants' refers, not to the set of elephants, but to the elephants themselves. Formally,

$$\exists xx(\forall y(y \prec^{1,2} xx \leftrightarrow \text{E}\textsc{lephant}^1(y)) \wedge \text{R}\textsc{ef}^{1,2}(\text{`the elephants'}, xx))$$

which is read:

> There are some things—the xxs—such that: (a) for every y, y is one of the xxs if and only if y is an elephant, and (b) 'the elephants' refers to the xxs.

2.3 The saturation operator

Let the *saturation operator* 'σ' be such that, given any monadic first-level predicate '$P^1(\ldots)$', '$\sigma[P^1(\ldots)]$' is a first-level term for which the following holds:

$$\forall xx(\text{R}\textsc{ef}^{1,2}(\text{`}P^1(\ldots)\text{'}, xx) \leftrightarrow \text{R}\textsc{ef}^{1,2}(\text{`}\sigma[P^1(\ldots)]\text{'}, xx))$$

The term '$\sigma[P^1(\ldots)]$' may therefore be thought of as something along the lines of the plural definite description 'the P^1s'. (See, however, section 3.4.)

There are interesting similarities between our saturation operator and the abstraction operator 'λ'. But, of course, they are distinct. Whereas the saturation operator transforms predicates into terms, the abstraction operator transforms sentences into predicates.

One may wish to insist that it is inappropriate to use a single reference relation for predicates and terms. One might think that, strictly speaking, there are two different kinds of reference: predicate-reference and term-reference. Nothing I shall say is incompatible with such a view. If one wanted, one could characterize the saturation-operator as follows:

$$\forall xx(\text{P-R}{\scriptstyle\text{EF}}^{1,2}(\text{`}P^1(\ldots)\text{'}, xx) \leftrightarrow \text{T-R}{\scriptstyle\text{EF}}^{1,2}(\text{`}\sigma[P^1(\ldots)]\text{'}, xx))$$

But the real difference between '. . . is an elephant' and 'the elephants' is that they fall under distinct semantic categories. And, since that is a difference that is already reflected in the syntax, I won't bother distinguishing between predicate-reference and term-reference.

2.4 Second-level predicates

There is a case to be made for the view that English predicates such as '. . . are scattered on the table' in 'The seashells are scattered on the table' or '. . . are surrounding the building' in 'the students are surrounding the building' are best understood as genuine second-level predicates.[14] Suppose such a view is correct. One might then be tempted to think that the reference of '. . . are scattered on the table' is the set of all and only sets whose members are scattered on the table (or, alternatively, the plurality consisting of all and only sets whose members are scattered on the table). More generally, one might be tempted to think that the second-level predicate '. . . are P' refers to the set of all and only sets whose members are collectively P (or, alternatively, to the plurality consisting of all and only sets whose members are collectively P). But one would then be forced to withhold reference from, e.g. a second-level predicate true of all and only pluralities whose members are too many to form a set. (And, as before, the problem cannot be avoided by appealing to a non-standard set-theory, or by employing entities other than sets.)

I propose instead that the reference of '. . . are scattered on the table' should be characterized as follows:

$$\exists xxx(\forall yy(yy \prec^{2,3} xxx \leftrightarrow \textsc{Scattered}^2(yy)) \wedge \textsc{Ref}^{1,3}(\text{`}\ldots \text{ are scattered'}, xxx))$$

where treble variables are used for *super-plural* terms and quantifiers. There are, of course, no super-plural terms or quantifiers in English, but I would like to suggest the relevant semantic category is nonetheless legitimate: super-plural quantifiers are to third-order quantifiers what plural quantifiers are to second-order quantifiers.

Since I cannot use English to state my proposal, I shall state it by saying that the reference of a monadic second-level predicate is a *super-plurality*. The reference of '. . . are scattered on the table', for example, is the super-plurality to which all and only pluralities scattered on the table belong. But it is important to be clear that apparently singular quantification over 'super-pluralities' is a syntactic abbreviation for super-plural quantification over individuals. Super-plural quantification is not singular (first-order) quantification over a new kind of 'item' ('super-plurality'), nor is it plural quantification over a new kind of 'item' ('plurality'). Super-plural quantification is a new kind of quantification altogether. And like its singular and plural counterparts, it is quantification over individuals, which are the only kind of 'item' there is.

I would like to insist that thinking of super-plural quantification as an iterated form of plural quantification—plural quantification over pluralities—would be a serious mistake. Plural quantification over pluralities can only make sense if pluralities are taken to be 'items' of some kind or other. And a plurality is not an 'item': apparently singular quantification over pluralities is a syntactic abbreviation for plural quantification over individuals.

It is one thing to have a general understanding of the sort of role super-plural quantifiers are supposed to play. But acquiring a genuine *grasp* of super-plural quantification—making sense of a language containing super-plural quantifiers—is a very different matter. The remarks in this section are intended to help with the former, but certainly not the latter. The best way of attaining a genuine grasp of super-plural quantification is presumably by mastering the use of super-plural quantifiers. (A suitable deductive system is discussed in section 4.2.)

2.5 Second-level terms

Debatable examples such as 'the couples' or 'the collections' aside, English appears to contain no second-level terms. But I submit that the relevant semantic category is nonetheless legitimate.[15] One can use the saturation operator, 'σ', to form second-level terms by stipulating that, for any monadic second-level predicate '$P^2(\ldots)$', '$\sigma[P^2(\ldots)]$' is a second-level term for which the following holds:

$$\forall xxx(\textsc{Ref}^{1,3}(\text{`}P^2(\ldots)\text{'}, xxx) \leftrightarrow \textsc{Ref}^{1,3}(\text{`}\sigma[P^2(\ldots)]\text{'}, xxx))$$

Thus, '$\sigma[\textsc{Scattered}^2(\ldots)]$' is to '…are scattered' what '$\sigma[\textsc{Elephant}^1(\ldots)]$' is to '…is an elephant'. In each case, there is a difference in semantic category without a difference in reference.

2.6 Beyond

A third-level predicate is a predicate that takes a second-level term in one of its argument places, and either a second-level term, a first-level term or singular term in the rest. It seems clear that English contains no third-level predicates. But I submit that the relevant semantic category is nonetheless legitimate. In analogy with the above, the reference of a monadic third-level predicate '$P^3(\ldots)$' may be characterized as follows:

$$\exists xxxx(\forall yyy(yyy \prec^{3,4} xxxx \leftrightarrow \mathrm{P}^3(yyy)) \wedge \textsc{Ref}^{1,4}(`P^3(\ldots)', xxxx))$$

where quadruple variables are used for *super-duper-plural* terms and quantifiers. There are, of course, no super-duper-plural terms or quantifiers in English, but, again, I submit that the relevant semantic category is nonetheless legitimate: super-duper-plural quantifiers are to fourth-order quantifiers what super-plural quantifiers are to third-order quantifiers and plural quantifiers are to second-order quantifiers.

And, of course, one can use the saturation operator, 'σ', to form third-level terms by stipulating that, for any monadic third-level predicate '$P^3(\ldots)$', '$\sigma[P^3(\ldots)]$' is a third-level term for which the following holds:

$$\forall xxxx(\textsc{Ref}^{1,4}(`P^3(\ldots)', xxxx) \leftrightarrow \textsc{Ref}^{1,4}(`\sigma[P^3(\ldots)]', xxxx))$$

A similar story can be told about n-th level terms and predicates for any finite n.

3 Fine-tuning

3.1 Improving the notation

Consider the first-level predicate '... is an ancestor of Clyde'. The first level-term

σ[... is an ancestor of Clyde]

might be read 'the ancestors of Clyde' (or, more idiomatically, 'Clyde's ancestors'). We can therefore write

(1) $\textsc{VeryNumerous}^2$(σ[... is an ancestor of Clyde]),

(read: 'Clyde's ancestors are very numerous').

Now consider the result of deleting 'Clyde' from (1). Since 'Clyde' is a singular term and (1) is a sentence, what we should get is a first-level predicate, true of all and only individuals whose ancestors are very numerous. But it is be infelicitous to write

$\textsc{VeryNumerous}^2(\sigma[\ldots \text{is an ancestor of} \ldots]),$

because it is unclear which of the two empty argument places in '... is an ancestor of ...' the saturation operator is germane to. We need to improve our notation. One possibility is to add indices to 'σ' and each of the empty argument places in '... is an ancestor of ...'. This allows us to distinguish between

$\textsc{VeryNumerous}^2(\sigma_1[\ldots_1 \text{ is an ancestor of } \ldots_2])$

and

$\textsc{VeryNumerous}^2(\sigma_2[\ldots_1 \text{ is an ancestor of } \ldots_2]).$

Both are first-level predicates. The first is true of all and only individuals whose ancestors are very numerous; the second is true of all and only individuals whose descendants are very numerous. Accordingly, one can construct the following sentences:

(2) $\textsc{FormClub}^2(\sigma_2[\textsc{VeryNumerous}^2(\sigma_1[\ldots_1 \text{ is an ancestor of } \ldots_2])]),$

(roughly: the individuals whose ancestors are very numerous form a club);

and

(3) $\textsc{FormClub}^2(\sigma_1[\textsc{VeryNumerous}^2(\sigma_2[\ldots_1 \text{ is an ancestor of } \ldots_2])]),$

(roughly: the individuals whose descendants are very numerous form a club).

Maintaining the dotted-line notation turns out to be somewhat inconvenient, however. I shall therefore forego the use of '$\ldots_i$' in favor of 'v_i^n' (where n is the level of terms taking the place of '$\ldots_i$'). Thus, (2) and (3) become (2′) and (3′), respectively:

(2′) $\textsc{FormClub}^2(\sigma_2^0[\textsc{VeryNumerous}^2(\sigma_1^0[v_1^0 \text{ is an ancestor of } v_2^0])])$

(3′) $\textsc{FormClub}^2(\sigma_1^0[\textsc{VeryNumerous}^2(\sigma_2^0[v_1^0 \text{ is an ancestor of } v_2^0])])$

3.2 The reference of polyadic predicates

I offered a proposal about the reference of monadic nth level predicates in section 2. But nothing has been said so far about the reference of polyadic predicates, such as '$\textsc{Ancestor}^{1,1}(\ldots,\ldots)$', or '$\ldots \prec^{1,2} \ldots$'.

One possibility is to take the reference of '$\textsc{Ancestor}^{1,1}(\ldots,\ldots)$' to be a plurality of ordered-pairs. Alternatively, one can take it to be a super-duper-plurality. (Specifically: the super-duper-plurality consisting of all and only super-pluralities that consist of two pluralities, one of them consisting of an individual and her ancestor and the other consisting of the ancestor alone.[16]) Such proposals generalize naturally to polyadic predicates of any arity and any finite level. A generalization of the first proposal is supplied in the appendix.

3.3 Intensions

Section 2 focused on the notion of reference. But the proposal can easily be generalized to provide a characterization of the *intensions* of nth-level predicates and terms.

Consider the monadic first-level predicate '$\textsc{Elephant}^1(\ldots)$' as an example. One possibility is to take its intension to be the plurality of ordered-pairs $\langle w, x\rangle$ such that w is a possible world and x is an elephant in w. Alternatively, one can take the intension of '$\textsc{Elephant}^1(\ldots)$' to be a super-duper-plurality. (Specifically: the super-duper plurality consisting of all and only super-pluralities that consist of two pluralities, one of them consisting of an possible world and an elephant in that world and the other consisting of the world alone.[17]) Such proposals generalize naturally to polyadic predicates of any arity and any finite level.

To keep things simple, I will focus on reference rather than intension throughout the remainder of the paper. But the view can be extended to accommodate intensions if need be.

3.4 Empty predicates

The second-order sentence

(4) $\exists X \forall y \neg (Xy)$

is true, since it can be derived from the true sentence '$\forall y \neg(\textsc{Unicorn}^1(y))$' by existential generalization. By contrast, the structurally analogous plural sentence

(5) $\exists xx \forall y \neg (y \prec^{1,2} xx)$

is false, since it is to be interpreted as 'there are some things such that nothing is one of them'.[18] In some respects, the difference in truth-value between (4) and (5) is of little importance: Boolos (1984) has shown that there is a systematic way of paraphrasing second-order sentences as sentences a first-level language enriched with plural quantifiers (and no second-level predicates other than '$\prec^{1,2}$'). We will see, however, that the falsity of (5) is not without implications in the present context.

One cannot say of the predicate '$\textsc{Unicorn}^1(\ldots)$', which is satisfied by no object, that it has a reference—not if the reference of a monadic first-level predicate is to be a plurality. For saying of '$\textsc{Unicorn}^1(\ldots)$' that it refers to an 'empty' plurality amounts to saying:

(6) $\exists xx(\textsc{Ref}^{1,2}(\text{'}\textsc{Unicorn}^1(\ldots)\text{'}, xx) \wedge \forall y \neg (y \prec^{1,2} xx));$

and the falsity of (6) is an immediate consequence of the falsity of (5). Happily, the claim that '$\text{U}\textsc{nicorn}^1(\ldots)$' has no reference is distinct from the claim that it is meaningless.[19]

To characterize the compositional behavior of empty predicates, let us begin by stipulating that an atomic predication based on an empty predicate is always false. Thus, for the case of '$\text{U}\textsc{nicorn}^1(\ldots)$', we have:

(7) $\neg\text{T}\textsc{rue}^1(\ulcorner \text{U}\textsc{nicorn}^1(t^0)\urcorner)$,

for $\ulcorner t^0 \urcorner$ an arbitrary singular term. This does not, however, settle the question of how to deal with '$\text{U}\textsc{nicorn}^1(\ldots)$' when it occurs within the scope of the saturation operator, as in '$\sigma^0_1[\text{U}\textsc{nicorn}^1(v^0_1)]$'. We know from section 2.3 that:

$$\forall xx(\text{R}\textsc{ef}^{1,2}(`\text{U}\textsc{nicorn}^1(\ldots)', xx) \leftrightarrow \text{R}\textsc{ef}^{1,2}(`\sigma^0_1[\text{U}\textsc{nicorn}(v^0_1)]', xx)).$$

Since '$\text{U}\textsc{nicorn}^1(\ldots)$' is referenceless, it follows that '$\sigma^0_1[\text{U}\textsc{nicorn}^1(v^0_1)]$' must be referenceless as well. So the question we face is that of characterizing the compositional behavior of empty terms. For instance, under what circumstances should one say that the following sentence is true?

$$\text{P}^2(\sigma^0_1[\text{U}\textsc{nicorn}^1(v^0_1)]).$$

Think of the matter like this. An atomic *first*-level predicate $\ulcorner \text{P}^1(\ldots)\urcorner$ is used to say of an individual that it is thus-and-so. Thus, when $\ulcorner t^0 \urcorner$ is an empty singular term, such as 'Zeus', $\ulcorner \text{P}^1(t^0)\urcorner$ is false; for one cannot truthfully say of *nothing* that it is thus-and-so. Similarly, an atomic *second*-level predicate $\ulcorner \text{P}^2(\ldots)\urcorner$ is used to say of some individuals that they are thus-and-so. Thus, when $\ulcorner t^1 \urcorner$ is an empty singular term, such as '$\sigma^0_1[\text{U}\textsc{nicorn}^1(v^0_1)]$', '$\ulcorner \text{P}^2(t^1)\urcorner$' is false; for one cannot truthfully say of *no things* that they are thus-and-so.[20]

Let us therefore stipulate that an atomic predication that applies to an empty term is always false. For the case of '$\sigma^0_1[\text{U}\textsc{nicorn}^1(v^0_1)]$', we have:

(8) $\neg\textsc{True}(\ulcorner P^2(\sigma^0_1[\textsc{Unicorn}^1(v^0_1)])\urcorner)$,

for $\ulcorner P^2(\ldots)\urcorner$ an an arbitrary atomic monadic second-level predicate. (The polyadic case is analogous). A semantics based on (7) and (8) is developed in the appendix.

3.5 Collapse

Say that Socrates is the one and only Socratizer. It would still be incorrect to the say the following:

(∗) Socrates is the same individual as the Socratizers

But this is not because (∗) is false. The problem with (∗) is that it is *ungrammatical*. It is ungrammatical in just the way that each of the following is ungrammatical:

The Socratizers is the same individual as Socrates *

Socrates are the same individuals as the Socratizers *

It is important to be clear, however, that this is a point about grammar, not metaphysics. Just because mixed identity statements are ungrammatical, it doesn't follow that the world contains additional items—the 'pluralities'—over and above individuals. Individuals are the only 'items' there are.

One could, if one wished, extend the formation rules of one's language so as to admit mixed identities as well-formed, and extend the semantics for one's language to assign mixed identities suitable truth conditions. More generally, one could, if one wanted, allow nth-level predicates to take mth-level terms as arguments for any $m < n$. The most natural way of doing so is by identifying the truth-conditions of $\ulcorner \mathrm{P}^n(v^k_l)\urcorner$ $(k+1 < n)$ with those of

$$\exists v^{k+1}_i \forall v^k_j ((v^k_j \prec^{k+1,k+2} v^{k+1}_i \leftrightarrow v^k_j =^{k+1,k+1} v^k_l) \wedge \mathrm{P}^n(v^{k+1}_i));$$

where $\ulcorner v_i^n =^{n+1,n+1} v_j^n \urcorner$ $(0 < n)$ is a syntactic abbreviation of

$$\forall v_s^{n-1}(v_s^{n-1} \prec^{n,n+1} v_i^n \leftrightarrow v_s^{n-1} \prec^{n,n+1} v_j^n);$$

(and similarly for polyadic predicates). With the extended conception of grammaticality in place, $(*)$ can be formalized as something which is both well-formed and true.

This revised conception of grammaticality will be ignored in what follows. But as long as one is prepared to resist the temptation of drawing metaphysical conclusions from terminological maneuvering, I can see no objections to adopting it.

3.6 Higher-*order* predicates

A monadic $(n+1)$th-*level* predicate should be distinguished from a monadic $(n+1)$th-*order* predicate: whereas the former takes an nth-level *term* in its argument-place, the latter takes a nth-level *predicate* in its argument-place.

On its most natural interpretation, the hierarchy of higher-order predicates is not structurally analogous to the hierarchy of higher-level predicates we have considered here. One important difference was emphasized in the preceding section. If '$E^1(\ldots)$' is a first-order predicate satisfied by nothing, the standard semantics for higher-order languages allows for the atomic second-order predication '$P_x^2(E^1(x))$' to be either true or false. But, on the semantics sketched above, the atomic second-level predication '$\mathrm{P}^2(\sigma_i^0[E^1(v_i^0)])$' is always false.

In this paper I give no reason for favoring a hierarchy of higher and higher level predicates over a hierarchy of higher and higher order predicates. I have chosen to focus on the former because it seems to me that second-level predicates deliver a more natural regimentation English predicates with collective readings than their second-order counterparts. But either hierarchy will do, as far as the purposes of this paper are concerned.[21]

4 Higher-level languages

4.1 Limit$_\omega$ languages

Let a limit$_\omega$ language consist of the following symbols:

1. The logical connectives '$\wedge$' and '$\neg$';

   ('$\vee$', '$\supset$' and '$\leftrightarrow$' are characterized in terms of '$\neg$' and '$\wedge$' in the usual way);

2. for $n \geq 0$ and $i \geq 1$, the placeholder $\ulcorner v_i^n \urcorner$;

3. for $i \geq 1$, the individual constant symbol $\ulcorner c_i^0 \urcorner$;

   (in practice, we will sometimes write, e.g. 'Clyde' or 'c' in place of $\ulcorner c_i^0 \urcorner$);

4. for s a finite sequence of positive integers and $i \geq 1$, the non-logical predicate-letter $\ulcorner P_i^s \urcorner$;

   (in practice, we will sometimes write, e.g. '$\textsc{Ancestor}^{1,1}$' and '$\textsc{Scattered}^2$' in place of $\ulcorner P_i^{1,1} \urcorner$ and $\ulcorner P_j^2 \urcorner$);

5. for $n \geq 2$, the logical predicate-letters '$=^{1,1}$', $\ulcorner \prec^{n-1,n} \urcorner$ and $\ulcorner \mathrm{Ex}^n \urcorner$;

   (in practice, we will sometimes write '$=$' in place of '$=^{1,1}$', and '$\prec$' in place of $\ulcorner \prec^{n-1,n} \urcorner$);

6. for $n \geq 0$ and $i \geq 1$, an instance of the saturation-symbol $\ulcorner \sigma_i^n \urcorner$;

7. the auxiliaries '(', ')', '[' and ']'.

Terms and formulas are characterized simultaneously, as follows:

1. $\ulcorner c_i^0 \urcorner$ is a term of level 0;

2. $\ulcorner v_i^n \urcorner$ is a term of level n;

3. if s is the sequence $n_1, \ldots, n_m$ and $\ulcorner t_1 \urcorner, \ldots, \ulcorner t_m \urcorner$ are terms of level $n_1 - 1, \ldots, n_m - 1$ (respectively), then $\ulcorner P_i^s(t_1, \ldots, t_m) \urcorner$ is a formula;

4. if $\ulcorner t_1 \urcorner$ and $\ulcorner t_2 \urcorner$ are terms of level 0, then $\ulcorner t_1 = t_2 \urcorner$ is a formula;

5. if, for $n \geq 2$, $\ulcorner t_1 \urcorner$ and $\ulcorner t_2 \urcorner$ are terms of level $n-2$ and $n-1$ (respectively), then $\ulcorner t_1 \prec^{n-1,n} t_2 \urcorner$ is a formula;

6. if, for $n \geq 2$, $\ulcorner t \urcorner$ is a term of level $n-1$, then $\ulcorner \mathrm{Ex}^n(t) \urcorner$ is a formula;

7. if φ is a formula, then $\ulcorner \sigma_i^n[\varphi] \urcorner$ is a term of level $n{+}1$;

8. if φ and ψ are formulas, $\ulcorner \neg\varphi \urcorner$ and $\ulcorner (\varphi \wedge \psi) \urcorner$ are formulas;

9. nothing else is a term or a formula.

Finally, we say that formula φ is a *sentence* if every occurrence of a placeholder $\ulcorner v_i^n \urcorner$ in φ is within a subformula of the form $\ulcorner \sigma_i^n[\psi] \urcorner$.

It is worth emphasizing that limit$_\omega$ languages contain no primitive quantifier-symbols. Instead, we introduce the following syntactic abbreviations:

$$\exists v_i^n(\varphi) \equiv_{df} \mathrm{Ex}^{n+2}(\sigma_i^n[\varphi])$$

$$\forall v_i^n(\varphi) \equiv_{df} \neg\exists v_i^n(\neg\varphi)$$

On the intended interpretation of $\ulcorner \mathrm{Ex}^n \urcorner$, this has the result that '$\exists v_i^0$' may be used to play the role of singular quantifiers, '$\exists v_i^1$' may be used to play the role of plural quantifiers, '$\exists v_i^2$' may be used to play the role of super-plural quantifiers, and so forth. Thus, '$\exists v_1^0(\textsc{Elephant}^1(v_1^0))$', which abbreviates

$\mathrm{Ex}^2(\sigma_1^0[\textsc{Elephant}^1(v_1^0)])$

(roughly: the plurality of elephants exists), may be paraphrased as

$\exists x(\textsc{Elephant}^1(x))$

(there is something that is an elephant);

and '$\exists v_1^1 \exists v_1^0 (v_1^0 \prec v_1^1)$', which abbreviates

$\mathrm{Ex}^3(\sigma_1^1[\mathrm{Ex}^2(\sigma_1^0[v_1^0 \prec v_1^1])])$

(roughly: the super-plurality *xxx* exists, where *xxx* consists of all and only pluralities *xx* such that the plurality *yy* exists, where *yy* consists of all and only individuals *y* such that *y* is one of the *xx*),

may be paraphrased as

$\exists xx \exists y(y \prec xx)$

(there are some things such that something is one of them).

To improve readability, I shall sometimes write, e.g. $\ulcorner \exists x^n \urcorner$ and $\ulcorner \exists y^m \urcorner$ in place of $\ulcorner \exists v_i^n \urcorner$ and $\ulcorner \exists v_j^m \urcorner$.

4.2 A deductive system

I this section I will specify a deductive system for limit$_\omega$ languages.[22] It is sound with respect to the semantics supplied in the appendix. But Gödel's Incompleteness Theorem implies that one cannot hope for completeness.

We begin with a standard deductive system for the propositional calculus:

(P1) $\varphi \supset (\psi \supset \varphi)$

(P2) $(\varphi \supset (\psi \supset \chi)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \chi))$

(P3) $(\neg\varphi \supset \neg\psi) \supset (\psi \supset \varphi)$

(MP) *Modus Ponens*

$$\frac{\varphi, \varphi \supset \psi}{\psi}$$

Next, we introduce an axiom-schema governing the identity-sign:

(I) $t_i^0 = t_j^0 \supset (\varphi(t_i^0) \supset \varphi(t_j^0))$

(where t_j^0 is free for t_i^0 in φ)[23]

A universally quantified version of reflexivity, '$\forall v_1^0(v_1^0 = v_1^0)$', is an immediate consequence of (UI^1) below. But our deductive system does not include the reflexivity axiom-schema '$t^0 = t^0$'. This is because we wish to allow for empty singular terms. When 't^0' is empty, the semantics sketched in section 3.4 makes any sentence of the form $\ulcorner P^1(t^0)\urcorner$ false (and its negation true), for $\ulcorner P^1 \urcorner$ atomic. It makes, for instance, 'Zeus = Zeus' false (and its negation true)—though, of course, if the language includes an atomic non-identity-symbol '$\neq$', then 'Zeus $\neq$ Zeus' will be false (and its negation true).

The next step is to introduce axioms and rules governing quantification. But care must be taken. For, whereas a limit$_\omega$ language may involve empty terms of any level, there is no such thing as an 'empty' individual, or an 'empty' plurality, or an 'empty' super-plurality (and so forth). So each of the axioms and rules includes a provision disallowing empty terms:

(EI^1) *Existential Introduction – first-level version*

$$t^0 = t^0 \supset (\varphi(t^0) \supset \exists v_i^0(\varphi(v_i^0)))$$

(where t^0 is free for v_i^0 in φ)

(EE^1) *Existential Elimination – first-level version*

$$\frac{\Gamma}{c = c \supset (\varphi(c) \supset \psi)} \rightarrow \frac{\Gamma}{\exists v_i^0(\varphi(v_i^0)) \supset \psi}$$

(where c does not occur in Γ or ψ)

(EI^H) *Existential Introduction – higher-level version*

$$\exists v_j^n(\psi(v_j^n)) \supset (\varphi(\sigma_j^n[\psi(v_j^n)]) \supset \exists v_i^{n+1}(\varphi(v_i^{n+1})))$$

(where $\sigma_j^n[\psi(v_j^n)]$ is free for v_i^{n+1} in φ)

(EE^H) *Existential Elimination – higher-level version*

$$\frac{\Gamma}{\exists v_j^n(P_k^{n+1}(v_j^n)) \supset (\varphi(\sigma_j^n[P_k^{n+1}(v_j^n)]) \supset \psi)} \rightarrow \frac{\Gamma}{\exists v_i^{n+1}(\varphi(v_i^{n+1})) \supset \psi}$$

(where P_k^{n+1} does not occur in Γ or ψ)

Finally, we include an axiom-schema that simultaneously governs the behavior of '$\prec$' and the behavior of the saturation-operator:

(S) $\forall v_i^n(v_i^n \prec \sigma_j^n[\varphi(v_j^n)] \leftrightarrow \varphi(v_i^n))$

And one could, if one wished, add some version or other of the Axiom of Choice.

On the basis of these axioms and rules, it is straightforward to prove a suitable version of the Deduction Theorem. One can also derive an axiom-schema of comprehension and principles governing universal quantification:

(C) *Comprehension*

$$\exists v_i^n(\varphi(v_i^n)) \supset \exists v_i^{n+1}\forall v_j^n(v_j^n \prec v_i^{n+1} \leftrightarrow \varphi(v_j^n))$$

(where v_i^{n+1} does not occur in φ)

(UE[1]) *Universal Elimination – first-level version*

$$t^0 = t^0 \supset (\forall v_i^0(\varphi(v_i^0)) \supset \varphi(t^0))$$

(where t^0 is free for v_i^0 in φ)

(UI[1]) *Universal Introduction – first-level version*

$$\frac{\Gamma}{c = c \supset (\psi \supset \varphi(c))} \longrightarrow \frac{\Gamma}{\psi \supset \forall v_i^0(\varphi(v_i^0))}$$

(where c does not occur in Γ or ψ)

(UE^H) *Universal Elimination – higher-level version*

$$\exists v_j^n(\psi(v_j^n)) \supset (\forall v_i^{n+1}(\varphi(v_i^{n+1})) \supset \varphi(\sigma_j^n[\psi(v_j^n)]))$$

(where $\sigma_j^n[\psi(v_j^n)]$ is free for v_i^{n+1} in φ)

(UI^H) *Universal Introduction – higher-level version*

$$\frac{\Gamma}{\exists v_j^n(P_k^{n+1}(v_j^n)) \supset (\psi \supset \varphi(\sigma_j^n[P_k^{n+1}(v_j^n)]))} \longrightarrow \frac{\Gamma}{\psi \supset \forall v_i^{n+1}(\varphi(v_i^{n+1}))}$$

(where P_k^{n+1} does not occur in Γ or ψ)

4.3 nth-level languages

Some additional notation will be useful in what follows:

- A *basic first-level* language is what one might recognize as a first-order language with no non-logical vocabulary. It is the fragment of a limit$_\omega$ language containing no placeholders other than those of the form $\ulcorner v_i^0 \urcorner$, no non-logical predicates, no logical predicates other than '=' and 'Ex^2', and no occurrences of the saturation-symbol other than those of the form $\ulcorner \sigma_i^0 \urcorner$.

- A *full first-level language* is the result of enriching a basic first-level language with non-logical first-level predicates.

- A *first-level language* is a full or basic first-level language.

And correspondingly for finite levels greater than 1:

- A *basic* $(n+1)$*th-level language* is the result of enriching a full nth-level language with placeholders of the form $\ulcorner v_i^n \urcorner$, the logical predicates '$\prec^{n,n+1}$' and 'Ex^{n+2}', and occurrences of the saturation-symbol of the form $\ulcorner \sigma_i^n \urcorner$.

- A *full* $(n+1)$*th-level language* is the result of enriching a basic $(n+1)$th-level language with non-logical $(n+1)$th-level predicates.

- An $(n+1)$*th-level language* is a full or basic $(n+1)$th-level language.

5 Motivating the hierarchy

5.1 Preliminaries

Let me begin with a warm-up case. Consider a skeptic who doubts that the standard first-level quantifiers fall under a legitimate semantic category. How might one respond to such skepticism?

The first thing to note is that the skeptic's doubts might take two different forms. A radical skeptic would deny that there is any sense to be made of sentences involving quantifier-symbols (as used by logicians). A moderate skeptic, on the other hand, would concede that the sentences make sense but contend that their semantic properties are best described by semantic categories other than 'first-level quantifier' (or, more generally, 'second-level predicate'). Each type of skepticism calls for a different kind of response.

Let us consider the more radical position first. In responding to the radical skeptic, appeals to introspection are unlikely to be very effective. One might claim, for instance, that one gets a certain 'feeling of understanding' when one considers, e.g. '$\exists x$ ELEPHANT(x)'. But it is open to the skeptic to counter by arguing that speakers are not always reliable judges of what they do and do not understand.

An alternative approach is to set forth a theory of what linguistic understanding consists in, and respond to the skeptic by arguing that the relevant speakers satisfy the constraints of the theory when it comes to the relevant sentences. But even this more sophisticated strategy is likely to be of limited effectiveness. Suppose one held the view that to understand a sentence is, at least in part, to know its truth conditions. It would be useless to respond to the skeptic by claiming that speakers know that '$\exists x$ ELEPHANT(x)' is true just in case there is an individual such that it is an elephant. For rather than conceding that there is sense to be made of '$\exists x$ ELEPHANT(x)', the skeptic would protest that one has begged the question by attributing to speakers knowledge whose intelligibility is under dispute.

A more promising strategy would be to bracket the question of what linguistic understanding consists in and attempt to come to an agreement with the skeptic about the sorts of things that would count as *evidence* of linguistic understanding. Here are some natural candidates:

1. that speakers have the ability to use assertions of sentences containing the disputed vocabulary to update their beliefs about the world;

2. that speakers have the ability to use their beliefs about the world to regulate their assertions of sentences containing the disputed vocabulary;

3. that speakers have the ability to use sentences involving the disputed vocabulary as part of a robust and consistent inferential practice.

One can then go on to produce a non-question-begging argument for the intelligibility of sentences involving quantifier-symbols by showing that the relevant constraints are met.

As long as something along the lines of 1–3 is admitted as evidence of linguistic understanding, there will be a route for silencing the radical skeptic. But nothing has been done so far to address the moderate skeptic's concerns. The moderate skeptic can agree that 1–3 provide evidence for the view that speakers understand '$\exists x$ Elephant(x)', and go on to insist that the semantic properties of such sentences are best described by appeal to semantic categories other than 'first-level quantifier'. She might insist, for example, that the semantic structure of '$\exists x$ Elephant(x)' is best described as the infinite disjunction

$$\textsc{Elephant}(n_1) \vee \textsc{Elephant}(n_2) \vee \textsc{Elephant}(n_3) \vee \ldots$$

where the $\ulcorner n_i \urcorner$ are the singular terms in the language. How might the issue be resolved?

As before, linguistic introspection is unlikely to help. For in response to the claim that one's intuitions suggest that 'first-level quantifier' is the right semantic category, the skeptic can claim that her intuitions suggest otherwise (and add that she is as competent a logician and English speaker as you). And, as before, one shouldn't expect much progress from the suggestion that a speaker's understanding of '$\exists x$ Elephant(x)' consists, at least in part, of knowing that that '$\exists x$ Elephant(x)' is true just in case there is an individual such that it is an elephant. For the skeptic will immediately grant the point, and go on to claim that the semantic properties of 'there is an individual such that ...' are best described by appeal to semantic categories other than 'first-level quantifier'.

It seems to me that the best way of addressing the moderate skeptic's concern is to argue that relevant mental and linguistic phenomena are best explained by a semantic theory that makes use of 'first-level quantifier' (or 'second-level predicate') as a semantic category. Here are some examples of considerations that might be set forth on behalf of the standard semantics:

- on the skeptical semantics, but not the standard semantics, the semantic structure of quantified sentences has infinitely many semantic constituents;
- on the skeptical semantics, but not the standard semantics, the meaning of '$\exists x\ F(x)$' is in constant flux, as some singular terms are introduced to the language and others are dropped;
- on the skeptical semantics, but not the standard semantics, there is a risk of being left with the result that speakers cannot fully grasp '$\exists x\ F(x)$' until they learn every singular term in the language.

If, as one might expect, it turns out that the best semantic theory (all things considered) is the standard semantic theory, one will be in a position to answer the moderate skeptic. It is worth noting, in particular, that there is a certain kind of argument that it would be illegitimate for the skeptic to employ. She should not defend her position by arguing that until one has *independent* evidence for the view that first-level quantifiers fall under a legitimate semantic category, the relevant linguistic practice cannot be accounted for by a categorial semantics that mentions first-level quantifiers. That would be to put the cart before the course. The best evidence one could have for the legitimacy of a semantic category is its presence in our best—simplest, most fruitful, best integrated—semantic theorizing.

5.2 The argument

In preceding sections I claimed without argument that, for any finite n, nth-level predicates and terms belong to legitimate semantic categories. (I also claimed that plural quantifiers, super-plural quantifiers and beyond fall under legitimate semantic categories, but we saw in section 4.1 that the such quantifiers needn't be taken as primitive once one has higher-level predicates and terms.) In this section I will try to supply the missing justification. My argument will be similar in form to that of the warm-up example, but this time proponents of the view that nth-level predicates and terms do not fall under legitimate semantic categories will take the place of the skeptic. The argument will not be conclusive, but I hope it is enough to show that the legitimacy of higher-level predicates and terms can be taken seriously.

I begin with the following observation:

> NO PARAPHRASE
>
> When an all-encompassing domain of discourse is allowed, it is not generally possible to paraphrase a basic second-level language as a first-order language.

(We say that a basic second-level language $\mathfrak{L}^2$ can be *paraphrased* as a first-order language just in case there is a range of individuals—the 'classes', say—such that, for any sentence in $\mathfrak{L}^2$, the following transformation preserves truth-value:

- $(\exists v_i^0(\varphi))^{Tr} \rightarrowtail \exists x_i(\varphi^{Tr})$
- $(\exists v_i^1(\varphi))^{Tr} \rightarrowtail \exists \alpha_i(\varphi^{Tr})$
- $(v_i^0 \prec v_j^1)^{Tr} \rightarrowtail x_i \in \alpha_j$
- $(v_i^0 = v_j^0)^{Tr} \rightarrowtail x_i = x_j$
- $(P(v_{i_1}^0, \ldots, v_{i_n}^0))^{Tr} \rightarrowtail P(x_{i_1}, \ldots, x_{i_n})$

- $(\varphi \wedge \psi)^{Tr} \rightarrowtail \varphi^{Tr} \wedge \psi^{Tr}$

- $(\neg\varphi)^{Tr} \rightarrowtail \neg(\varphi^{Tr})$

where $\ulcorner x_i \urcorner$ ranges over the individuals in the domain of discourse of $\mathcal{L}^2$, $\ulcorner \alpha_i \urcorner$ ranges over (non-empty) 'classes' of these individuals, and '$\in$' expresses a 'membership' relation appropriate for 'classes'.)

To see that NO PARAPHRASE holds, assume for *reductio* that it is generally possible to paraphrase a second-level language as a first-order language. Let the domain of discourse of $\mathcal{L}^2$ consist of absolutely everything, and let $\mathcal{L}^2$ contain a predicate, 'MEMBER', which is true of x and y just in case x is a 'member' of 'class' y. Then the following must be true:

(9) $\forall v_1^1 \exists v_1^0 \forall v_2^0 (v_2^0 \prec v_1^1 \leftrightarrow \text{MEMBER}(v_2^0, v_1^0))$,

since the result of applying Tr to (9) is:

(10) $\forall \alpha_1 \exists x_1 \forall x_2 (x_2 \in x_1 \leftrightarrow \text{MEMBER}(x_2, \alpha_1))$,

(which is true because 'MEMBER' and '$\in$' must be coextensive in light of the fact that the domain of discourse of $\mathcal{L}^2$ is absolutely unrestricted). But, on the assumption that there are at least two objects, (9) entails a contradiction. To see this, note that one can derive the following from (9) by applying (UE^H) and (S):

(11) $\exists v_2^0 (\neg\text{MEMBER}(v_2^0, v_2^0)) \supset \exists v_1^0 \forall v_2^0 (\neg\text{MEMBER}(v_2^0, v_2^0) \leftrightarrow \text{MEMBER}(v_2^0, v_1^0))$.

A tedious but straightforward proof shows that the antecedent of (11) can be derived from (9) (together with the assumption that there are at least two objects). So we are left with:

$$\exists v_1^0 \forall v_2^0 (\neg\text{MEMBER}(v_2^0, v_2^0) \leftrightarrow \text{MEMBER}(v_2^0, v_1^0)),$$

from which one can derive a contradiction by applying (EE^1) and (UE^1). This concludes the *reductio*.

I would like to suggest that No Paraphrase provides some evidence for the view that second-level predicates and first-level terms fall under legitimate semantic categories. The argument runs as follows.

Consider a community of speakers that sets out to speak a second-level language. They let their syntax be governed by (suitable restrictions of) the rules in section 4.1, and let their deductions be constrained by (suitable restrictions of) the axioms and rules in section 4.2. The new conventions eventually take hold, and speakers come to engage in a successful linguistic practice. In particular, conditions 1–3 from section 5.1 are all satisfied.

As long as it is conceded that such a scenario is possible, one will be in a position to counter radical skepticism: one will be in a position to argue that members of the community succeed in speaking some language or other and, accordingly, that there is sense to be made of the relevant sentences. But this is not yet to concede that second-level predicates and first-level terms fall under legitimate semantic categories. A moderate skeptic would concede that there is sense to be made of the relevant sentences but doubt that the language is best described by a semantic theory employing the semantic categories 'second-level predicate' and 'first-level term'. In particular, the moderate skeptic might endorse a *firstorderist* view whereby members of the community speak the first-order language induced by Tr, and supply a semantics accordingly.

The lesson of No Paraphrase is that when an all-encompassing domain of discourse is allowed, proponents of the firstorderist position must make a concession. They must concede that some of the rules and axioms that speakers take their inferences to be constrained by are not, in fact, logically valid. For the proof of No Paraphrase entails that the following is a *theorem* of the deductive system in section 4.2:

(12) $\exists v_1^0 \exists v_2^0 (\neg(v_1^0 = v_2^0)) \rightarrow \neg \forall v_1^1 \exists v_1^0 \forall v_2^0 (v_2^0 \prec v_1^1 \leftrightarrow \mathrm{P}_1^{1,1}(v_2^0, v_1^0))$

But from the perspective of a firstorderist, (12) must be false when '$P_1^{1,1}$' expresses the 'membership' relation appropriate for 'classes' and the domain of discourse is absolutely unrestricted.

This puts the firstorderist position under some pressure. Suppose, for example, that speakers resolve to enrich their language with the first level predicate 'MEMBER', which, as before, is to be true of x and y just in case x is a 'member' of 'class' y (and that speakers continue to take their syntax to be governed by the rules in section 4.1, and their deductions be constrained by the axioms and rules in section 4.2). Firstorderists will then face an awkward decision. On the one hand, they might concede that some of the community's fundamental axioms are not merely not logically valid but outright false (or that some of the community's fundamental rules are not merely not logically valid but have a true premise and a false conclusion). For firstorderists must regard the result of replacing '$P_1^{1,1}$' with 'MEMBER' in (12) as false, even though it is a deductive consequence of the community's fundamental rules and axioms. Alternatively, firstorderists might claim that enriching the language with 'MEMBER' leads to a change in the way quantification works: whereas in the original language the $\ulcorner \alpha_i \urcorner$ range over 'classes', in the enriched language they range over 'classes*', which are such that (12) is true. This would certainly forestall any breaches in charity, but at the cost of complicating one's semantic theory, since the semantic behavior of the quantifiers will have to depend on what predicates the language happens to contain. And there would appear to be little independent motivation for the additional complexity.

Of course, the firstorderist position might still be vindicated at the end of the day. For all I have argued here, it might be possible to make a case for the view that, e.g. the gain in parsimony that is achieved by limiting one's stock of legitimate semantic categories is significant enough to outweigh firstorderism's less palatable consequences. But, as in the warm up case considered earlier, it is important to keep in mind that the there is a certain kind of argument that it is important to resist. One should not argue for the firstorderist position by claiming that, unless one has *independent* evidence for the view that higher-level predicates and terms fall under legitimate semantic categories, the relevant linguistic practice cannot be accounted for by a categorial semantics that mentions higher-level predicates and terms. The best evidence one could have for the legitimacy of a semantic category is its presence in our best—simplest, most fruitful, best integrated—semantic theorizing. By insisting that higher-level predicates and terms remain unavailable to semantic theorizing until the relevant semantic categories have been shown to be legitimate on independent grounds, firstorderists would be begging the question against their opponents.

In sum, my argument is this. Should it turn out that a community's linguistic practice is best accounted for by a semantic theory that makes use of the categories 'second-level predicate' and 'first-level term', we would be justified in thinking that second-level predicates and first-level terms fall under legitimate semantic categories. But when an all-encompassing domain of discourse is allowed, No Paraphrase suggests that firstorderism—the most salient alternative—will be subject to certain kinds of difficulties. So, when an all-encompassing domain of discourse is allowed, we have some preliminary evidence for the view that second-level predicates and first-level terms fall under legitimate semantic categories.

I have focused on second-level languages for expository purposes, but the argument is quite general. For each finite n, one can prove a version of No Paraphrase for $(n+1)$th-level languages:[24]

> When an all-encompassing domain of discourse is allowed, it is not generally possible to paraphrase a basic $(n+1)$th-level language as an nth-level language.

If one then considers a community of speakers who have set out to speak an $(n+1)$th-level language, one can replicate the argument above to make a preliminary case for the view that the relevant linguistic practice is best accounted for by a semantic theory that mentions

$(n+2)$th level predicates and $(n+1)$th level terms, and therefore a preliminary case for the view that $(n+2)$th-level predicates and $(n+1)$-level terms fall under legitimate semantic categories. All of this on the assumption that an all-encompassing domain of discourse is allowed.[25]

6 Model-theory

A model-theory for a language L is *strictly adequate* just in case it agrees with one's categorial semantics for L in the following sense: any reference a (non-logical) predicate might take by the lights of one's categorial semantics corresponds to the semantic value the predicate gets assigned by some model of one's model-theory. Thus, given a categorial semantics whereby the reference of a first-level predicate is a plurality, a model-theory for the relevant language can only be strictly adequate if, for any plurality, there is a model on which a given first-level predicate is assigned a semantic value corresponding to that plurality.

When quantification over absolutely everything is allowed, it is easy to show that there must be 'more' pluralities than there are individuals.[26] So, on the assumption that the reference of a monadic first-level predicate is a plurality, there must be 'more' ways of assigning reference to a monadic first-level predicate than there are individuals. It follows that a model-theory for a full first-level language can only be strictly adequate if it appeals to 'more' models than there are individuals.

An immediate consequence of this result is that no model-theory according to which a model is a *set* can be strictly adequate. More generally, one cannot give a strictly adequate model-theory for a full first-level language in a first-level language, since a model-theory requires quantification over models, and the only kind of quantification available in a first-level language is singular quantification over individuals. Fortunately, this does not mean that it is impossible to give a strictly adequate model-theory for full first-level languages. By taking a model to be a *plurality*, one can give a strictly adequate model-theory for first-level languages in a basic second-level language.[27] (To fix ideas, think of a model m^1 as a plurality consisting of ordered-pairs of the form $\langle$'$\forall$', $x^0\rangle$ and ordered-pairs of the form $\langle$'P_i^1', $x^0\rangle$, for 'P_i^1' a predicate in the language. Intuitively, $\langle$'$\forall$', $x^0\rangle \prec m^1$ just in case x^0 is in the 'domain' of m^1, and $\langle$'P_i^1', $x^0\rangle \prec m^1$ just in case x^0 is in the reference of 'P_i^1' according to m^1.) For reasons relating to Tarski's Theorem, it is impossible to give a strictly adequate model-theory for a basic second-level language in another basic second-level language.[28] But one can give a strictly adequate model-theory for a basic second-level language in a full second-level language.[29]

These results can be generalized for $n \geq 1$.[30] Thus:

Semantic Ascent

(*a*) It is impossible to give a strictly adequate model-theory for a full nth-level language in an nth level language.

(*b*) It is possible to give a strictly adequate model-theory for full nth-level languages in a basic $(n+1)$th-level language.

(*c*) It is impossible to give a strictly adequate model-theory for a basic $(n+1)$th-level language in a basic $(n+1)$th level language.

(*d*) It is possible to give a strictly adequate model-theory for basic $(n+1)$th-level languages in a full $(n+1)$-th level language.

(A strictly adequate model-theory for nth-level languages is developed in the appendix.)

A famous argument of Kreisel's can be used to show that any first-level sentence that is true according to some model of a strictly adequate model-theory is also true according to some model of a standard model-theory (in which models are *sets* rather than pluralities).[31]

This result—which I shall refer to as *Kreisel's Principle*—guarantees that an extensionally adequate characterization of logical consequence for a first-level language can be given within another first-level language. In light of Kreisel's Principle, it is tempting to conclude that a thorough understanding of first-level languages can be attained by appeal to a model-theory that is not strictly adequate, and hence that the requirement of strict adequacy is unnecessarily strong.

The temptation should be resisted. For there is more to model-theory than a characterization of logical consequence. Conspicuously, model-theory might be thought to deliver a generalized notion of reference, which is concerned not just with the assignment of reference an expression actually takes, but with any possible assignment of reference the expression might take. Suppose, for example, that one wished to record the fact that, by the lights of ones categorial semantics, a monadic first-level predicate could be assigned a reference that consists of too many objects to form a set. A strictly adequate model-theory immediately delivers the resources to do so:

$$\exists mm \exists xx[\textsc{Model}^2(mm) \wedge \textsc{Ref}^{2,1,2}(mm, `P^1(\ldots)', xx) \wedge \neg\exists y \forall z(z \in y \leftrightarrow z \prec^{1,2} xx)]$$

(where '$\textsc{Model}^2(mm)$' is a second-level formula stating that the mms form a model, and '$\textsc{Ref}^{2,1,2}(mm, `P^1(\ldots)', xx)$' is a second-level formula stating that the reference assigned by the mms to '$P^1(\ldots)$' is the plurality consisting of all and only the xxs). But it is hard to see how one could make a similar statement within the confines of a standard model-theory.

This limitation of standard model-theory also affects its ability to produce extensionally adequate characterizations of logical consequence in certain special cases. Consider, for instance, the result of enriching a first-level language with a quantifier '$\exists^{AI}$', as in McGee (1992). The sentence '$\exists^{AI} x(\phi(x))$' is to be true just in case the individuals satisfying '$\phi(x)$' are too many to form a set. So '$\exists^{AI} x(x = x)$' is true (and therefore consistent).

But it would be deemed false by any model of a standard model-theory. The lesson is clear. Kreisel's Principle shows that strictly adequate model-theories can be supplanted by standard model theories for the purposes of one particular application, but not that they can be supplanted in general.

Two further points are worth emphasizing. First, the benefits of Kreisel's Principle can only be claimed by those who have already ventured beyond first-level languages. For although the principle is often stated informally, it cannot be formulated properly within a first-level language. Within a second-level language, on the other hand, it has a straightforward formulation.

Second, the status of higher-level versions of Kreisel's Principle is increasingly problematic. A version of Kreisel's Principle for a basic second-level language, for instance, is provably independent of the standard axioms of set theory (if consistent with them).[32] So friends of plural quantification cannot make use of Kreisel's Principle to avoid giving a strictly adequate model-theory without making substantial set-theoretic presuppositions.

7 An open-ended hierarchy

Since a limit$_\omega$ language contains an nth-level language as a part for each finite n, the following is a consequence of Semantic Ascent (a) and (c):[33]

Semantic Ascent

(e) It is impossible to give a strictly-adequate model-theory for a limit$_\omega$ language in a limit$_\omega$ language.

I would like to consider two different ways of dealing with this result. The first strategy is to settle for what I shall call *semantic pessimism*: the view that it is impossible to provide a strictly adequate model-theory for some language built up from legitimate semantic categories. Philosophers have grown accustomed the fact that any given language must suffer from important expressive limitations. The Liar Paradox, for instance, has taught us that (on appropriate assumptions) the truth predicate for a given language cannot be expressed in the language itself, even though it can be expressed in a different language of the same logical type. But semantic pessimism is pessimism of a much more radical kind. For one is forced to countenance the view that a language might have features whose investigation is ruled out by the nature of language itself.[34]

The second strategy is to try to avoid semantic pessimism by claiming that the *legitimate* languages—the languages it is in principle possible to make sense of—form an open-ended hierarchy such that any language in the hierarchy can be given a strictly adequate model-theory in some other language higher-up in the hierarchy. So there is no legitimate language with respect to which semantic pessimism would threaten. The simplest way of setting forth such a hierarchy is by claiming that nth-level languages are legitimate for any finite n but denying that languages of transfinite level (including limit$_\omega$ languages) are legitimate. An alternative is to take the hierarchy into the transfinite by treating limit$_\omega$ languages as legitimate and postulating the legitimacy of a language $L^\star$ of transfinite-level in which a strictly adequate model-theory for limit$_\omega$ languages can be given, postulating the legitimacy of a language $L^{\star\star}$ in which a strictly adequate model-theory for $L^\star$ can be given, and so forth.[35] Whatever the details of the hierarchy, what is crucial is that there be no such thing as an *absolute*-level language: a language combining the resources of all legitimate languages. For a suitable generalization of Semantic Ascent would imply that it is impossible to give a strictly adequate model-theory for an absolute-level language. So the result of making room for an absolute-level language is that one would be left with semantic pessimism after all.

A potential difficulty for the postulation of such hierarchies emerges from the observation that the legitimacy of absolute-level languages follows from two seemingly plausible principles: the first is a Principle of Union according to which the result of combining the resources of legitimate languages is itself a legitimate language; the second is a principle to the effect that it make sense to talk about all languages in the hierarchy. For, by the first principle, the hierarchy must be closed under unions, and, by the second, one of the unions must be maximal.

Denying the second of these principles seems especially problematic in the present context. For we began our investigation by assuming that one can quantify over absolutely everything and used this assumption to argue for the view that higher-level predicates and terms fall under legitimate semantic categories. But now, in an attempt to study the semantic properties of higher-level resources, we are under pressure to countenance the idea that one cannot talk about all legitimate languages. Is this not a *reductio* of the original assumption? If it is possible to quantify over absolutely everything, and if 'F(...)' is a predicate in good standing, shouldn't it be possible to quantify over all Fs?

It is not clear, however, that '...is a language' is a predicate in good standing in the relevant respect. To see the problem, observe that there are at least as many (interpreted) first-level languages as there are assignments of reference to a first-level predicate. Since the reference of a first-level predicate is a plurality, and since there are 'more' pluralities than there are objects, this means that there are 'more' first-level languages than individuals, and therefore that a first-level language cannot, in general, be an individual. It is best to think of a first-level language as a certain kind of *plurality*. For analogous reasons, it is best to think of a second-level language as a certain kind of super-plurality, and best to think of a third-level language as a super-duper-plurality, and so forth. Accordingly, the predicate '...is an nth-level language' must be (at least) of level $n+1$. And one can expect a similar result to hold for languages of infinite level: the predicate '...is an αth-level language' must be (at least) of level $\alpha + 1$. So what type of predicate could '...is a language' be? Any predicate falling under a legitimate semantic category must be of some level or other.

But '. . . is a language' cannot be an αth-level predicate, lest one be left with the unintended result that α-level languages are not languages. The lesson, I would like to suggest, is that '. . . is a language' is best understood as ambiguous between various legitimate predicates of the form '. . . is a language of at most level α'. And if this is right, one cannot go from quantification over absolutely everything to quantification over all languages.

> (*Parenthetical remark:* The preceding remarks suggest a novel way of making sense of the idea that the classes form an *indefinitely extensible* totality.[36] For if one thinks of a class, not as an *individual* of a certain kind, but as the reference of a predicate in some language, then the predicate '. . . is a class' will have a status similar to that of '. . . is a language'. The reference of a first-level predicate is a plurality; the reference of a second-level predicate is a super-plurality; the reference of a third-level predicate is a super-duper-plurality; and so forth. So what type of predicate could '. . . is a class' be? Any predicate falling under a legitimate semantic category must be of some level or other. But '. . . is a class' cannot be an αth-level predicate, lest one be left with the unintended result that the reference of an α-level predicate is not a class. Accordingly, '. . . is a class' is best understood as ambiguous between various legitimate predicates of the form '. . . is a class of at most level α'.)

The postulation of an open-ended hierarchy of languages faces a familiar difficulty: it leads to the result that statements of the form 'the hierarchy is so-and-so' are, strictly speaking, nonsense.[37] (This does not, of course, imply that the state of affairs under discussion fails to obtain; what it shows is that there are important limits in the sorts of statements that can be made about it.)

In spite of its problems, the postulation of an open-ended hierarchy of languages may turn out to be the least unattractive of the options on the table. It may very well be part of the nature of language and thought that matters cannot be improved upon. Note, for example, that there is a striking parallel between the open-ended hierarchy of *ideology* that we have considered here and the open-ended hierarchy of *ontology* that defenders of the view that it is impossible to quantify over absolutely everything have sometimes set forth.[38] (This is not to say, of course, that the two pictures are equivalent: whereas proponents of the ideological hierarchy consider only a fragment of their logical resources at a time, and are thereby able to supply a strictly adequate model-theory for the language under consideration, proponents of the ontological hierarchy consider only a fragment of their ontology at a time, and are thereby *unable* to supply a strictly adequate model-theory for any language complex enough to be interesting.) From the present perspective, one might think of the ontological hierarchy as the first-order 'projection' of the ideological hierarchy that results from objectifying pluralities, super-pluralities and beyond.[39]

Appendix

I provide a model-theory for a full nth-level language $\mathfrak{L}$ $(n > 1)$ in a basic $(n+1)$th-level language.

The first step is to characterize a generalized notion of n-tuple membership:

- $z^0 \ll^{1,1}_{i,n} x^0 \ \equiv_{df} \ \exists y^0_1 \ldots \exists y^0_i \ldots \exists^0_n (x^0 = \langle y^0_1, \ldots, y^0_i, \ldots, y^0_n \rangle \wedge z^0 = y^0_i)$

  (z^0 is the ith member of zeroth-level n-tuple x^0)

- $z^1 \ll^{2,2}_{i,n} x^1 \ \equiv_{df} \ \forall w^0 (w^0 \prec z^1 \leftrightarrow \exists y^0 (y^0 \prec x^1 \wedge w^0 \ll^{1,1}_{i,n} y^0))$

  (z^1 is the ith member of first-level n-tuple x^1)

  $\vdots$

- $z^m \ll^{m+1,m+1}_{i,n} x^m \ \equiv_{df} \ \forall w^{m-1} (w^{m-1} \prec z^m \leftrightarrow \exists y^{m-1} (y^{m-1} \prec x^m \wedge w^{m-1} \ll^{m,m}_{i,n} y^{m-1}))$

  (z^m is the ith member of mth-level n-tuple x^m)

  $\vdots$

Next, we characterize a same-level pseudo-identity relation:

- $x^0 \approx y^0 \ \equiv_{df} \ x^0 = y^0$

- $x^{n+1} \approx y^{n+1} \ \equiv_{df} \ \forall z^n (z^n \prec x^{n+1} \leftrightarrow z^n \prec y^{n+1})$

and a cross-level pseudo-identity relation:

- $x^n \approx y^{n+1} \ \equiv_{df} \ \forall z^n (z^n \prec y^{n+1} \rightarrow x^n \approx z^n)$

- $x^n \approx y^{n+k+1} \ \equiv_{df} \ \exists z^{n+1} \ldots \exists z^{n+k} (x^n \approx z^{n+1} \wedge \ldots \wedge z^{n+k} \approx y^{n+k+1})$

By using the pseudo-identity relation we can extend our characterization of generalized n-tuple membership as follows:

$$z^r \ll^{r+1,m+1}_{i,n} x^m \;\equiv_{df}\; \exists z^m(z^r \approx z^m \wedge z^m \ll^{m+1,m+1}_{i,n} x^m)$$

(where $r < m$)

Some additional pieces of preliminary notation:

$$x^m \approx \langle y_1^{r_1}, \ldots, y_k^{r_k} \rangle \;\equiv_{df}\; (y_1^{r_1} \ll^{r_1+1,m+1}_{1,k} x^m \wedge \ldots \wedge y_k^{r_k} \ll^{r_k+1,m+1}_{k,k} x^m)$$

(where $r_i \leq m$)

$$\langle y_1^{r_1}, \ldots, y_k^{r_k} \rangle \prec x^{m+1} \;\equiv_{df}\; \exists z^m(z^m \approx \langle y_1^{r_1}, \ldots, y_k^{r_k} \rangle \wedge z^m \prec x^{m+1})$$

(where $r_i \leq m$ and 'z^m' is an unused variable)

$$y^r \prec x^{m+1} \;\equiv_{df}\; \exists z^m(y^r \approx z^m \wedge z^m \prec x^{m+1})$$

(where $r \leq m$)

We may now characterize the notion of an assignment function. Intuitively, an assignment function maps an object to each zeroth-level placeholder, a plurality to each first-level placeholder, a super-plurality to each second-level placeholder, and so forth, for every placeholder in $\mathfrak{L}$. Formally, an nth-level predicate 'A(x^{n-1})' (read 'x^{n-1} is an assignment') may be characterized as follows:

$$
\begin{aligned}
\mathrm{A}(x^{n-1}) \ \equiv_{df}\ \{&\forall y^{n-2}(y^{n-2} \prec x^{n-1} \rightarrow \\
&\quad [\exists w^0 \exists z^0(w^0 \text{ is a zeroth-level place-holder} \wedge y^{n-2} \approx \big\langle w^0, z^0 \big\rangle) \vee \\
&\quad \exists w^0 \exists z^0(w^0 \text{ is a first-level place-holder} \wedge y^{n-2} \approx \big\langle w^0, z^0 \big\rangle) \vee \\
&\quad \exists w^0 \exists z^1(w^0 \text{ is a second-level place-holder} \wedge y^{n-2} \approx \big\langle w^0, z^1 \big\rangle) \vee \\
&\quad \vdots \\
&\quad \exists w^0 \exists z^{n-2}(w^0 \text{ is an } (n-1)\text{th-level place-holder} \wedge y^{n-2} \approx \big\langle w^0, z^{n-2} \big\rangle)]) \wedge \\
&\forall w^0(w^0 \text{ is a place-holder} \rightarrow \exists z^{n-2}(\big\langle w^0, z^{n-2} \big\rangle \prec x^{n-1})) \wedge \\
&\forall w^0(w^0 \text{ is a zeroth-level place-holder} \rightarrow \\
&\quad \exists z^0(\big\langle w^0, z^0 \big\rangle \prec x^{n-1} \ \wedge \forall t^{n-2}(\big\langle w^0, t^{n-2} \big\rangle \prec x^{n-1} \rightarrow z^0 \approx t^{n-2})))\}
\end{aligned}
$$

When $\mathrm{A}(x^{n-1})$ and v is a place-holder of $\mathcal{L}$, it will sometimes be useful to employ the following notational abbreviation:

$$\Phi(\alpha_{x^{n-1}}(v)) \ \equiv_{df}\ \exists y^{n-1} \forall z^{n-2}(((\langle v, z^{n-2} \rangle \prec x^{n-1} \leftrightarrow z^{n-2} \prec y^{n-1}) \wedge \Phi(y^{n-1}))$$

were 'y^{n-1}' is an unused variable.

(Intuitively, '$\Phi(\alpha_{x^{n-1}}(v))$' says that the value that placeholder v is assigned by assignment x^{n-1} is Φ.)

We shall also use the following notation:

- $x^0 \prec^{1,2}_{trans} y^1 \equiv_{df} x^0 \prec y^1$
- $x^0 \prec^{1,3}_{trans} y^2 \equiv_{df} \exists z^1(x^0 \prec^{1,2}_{trans} z^1 \wedge z^1 \prec y^2)$

  $\vdots$

- $x^0 \prec^{1,k+1}_{trans} y^k \equiv_{df} \exists z^{k-1}(x^0 \prec^{1,k}_{trans} z^{k-1} \wedge z^{k-1} \prec y^k)$

$\vdots$

The next step is to characterize the notion of a model. Intuitively, a model might be thought of as codifying four distinct things. Firstly, it codifies information about a domain (in the form of a plurality of ordered-pairs $\langle$'$\forall$', $x\rangle$); secondly, it a codifies a function mapping an object (or nothing at all) to each individual constant symbol, a plurality (or nothing at all) to each non-logical monadic first-level predicate-letter, a super-plurality (or nothing at all) to each non-logical monadic second-level predicate-letter, and so forth for every non-logical monadic predicate-letter in $\mathfrak{L}$ (and similarly for non-logical polyadic predicate-letters); thirdly, it codifies information about the denotations of terms of $\mathfrak{L}$ (relative to an assignment function a^{n-1}); finally it codifies information about the satisfaction of formulas of $\mathfrak{L}$ (relative to an assignment function a^{n-1}). Formally, the notion of a model can be characterized as follows. (In each clause, I omit initial universal quantifiers for the sake of readability.)

$\mathrm{M}(x^n) \ \equiv_{df}$

$\{[c$ is an individual constant $\rightarrow (\langle c, a^{n-1}, z^{n-1}\rangle \prec x^n \leftrightarrow$

$$\exists w^0(w^0 \approx z^{n-1} \wedge \langle c, w^0\rangle \prec x^n \wedge \forall y^{n-1}(\langle c, y^{n-1}\rangle \prec x^n \rightarrow w^0 \approx y^{n-1})))] \wedge$$

(*Intuitive gloss: the reference assigned to individual constant c by the model relative to assignment* a^{n-1} *is the reference assigned to c by the model.*)

$[v$ is a place-holder $\rightarrow (\langle v, a^{n-1}, z^{n-1}\rangle \prec x^n \leftrightarrow$

$$\exists y^{n-2}(y^{n-2} \approx z^{n-1} \wedge y^{n-2} \prec \alpha_{a^{n-1}}(v)))] \wedge$$

(*Intuitive gloss: the reference assigned to placeholder v by the model relative to assignment* a^{n-1} *is the reference assigned to v by* a^{n-1}.)

$[\ulcorner t_1 = t_2 \urcorner$ is a formula $\rightarrow (\langle \ulcorner t_1 = t_2 \urcorner, a^{n-1}\rangle \prec x^n \leftrightarrow$

$$\exists z^0(\langle t_1, a^{n-1}, z^0\rangle \prec x^n \wedge \langle t_2, a^{n-1}, z^0\rangle \prec x^n))] \wedge$$

(*Intuitive gloss:* $\ulcorner t_1 = t_2 \urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case* $\ulcorner t_1 \urcorner$ *and* $\ulcorner t_2 \urcorner$ *are assigned the same reference by the model relative to* a^{n-1})

$[\ulcorner t_1 \prec^{k-1,k} t_2 \urcorner$ is a formula $\rightarrow (\langle \ulcorner t_1 \prec^{k-1,k} t_2 \urcorner, a^{n-1}\rangle \prec x^n \leftrightarrow$

$$\exists z^{n-1}(\forall w^{n-2}(w^{n-2} \prec z^{n-1} \leftrightarrow \langle t_1, a^{n-1}, w^{n-2}\rangle \prec x^n) \wedge \langle t_2, a^{n-1}, z^{n-1}\rangle \prec x^n))] \wedge$$

(*Intuitive gloss:* $\ulcorner t_1 \prec^{k-1,k} t_2 \urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case the reference assigned by the model to* $\ulcorner t_1 \urcorner$ *relative to* a^{n-1} *is 'among' the reference assigned by the model to* $\ulcorner t_2 \urcorner$ *relative to* a^{n-1}.)

$[\ulcorner P_j^{r_1,\ldots,r_k}(t_1, \ldots, t_k)\urcorner$ is a formula $\rightarrow (\langle \ulcorner P_j^{r_1,\ldots,r_k}(t_1, \ldots, t_k)\urcorner, a^{n-1}\rangle \prec x^n \leftrightarrow$

$(\exists y^{n-1}[\langle \ulcorner P_j^{r_1,\ldots,r_k} \urcorner, y^{n-1}\rangle \prec x^n \ \wedge$ for each $\ulcorner t_i \urcorner$ $(1 \leq i \leq k)$ $\exists z^{n-1}($

$\forall w^{n-2}(w^{n-2} \prec z^{n-1} \leftrightarrow \langle t_i, a^{n-1}, w^{n-2}\rangle \prec x^n) \wedge$

$\exists r^{n-1}(\langle i, k, z^{n-1}, r^{n-1}, y^{n-1}\rangle \prec x^n))] \wedge$

$\forall z^{n-1}\forall r^{n-1}\forall y^{n-1}(\langle 1, k, z^{n-1}, r^{n-1}, y^{n-1}\rangle \prec x^n \leftrightarrow (z^{n-1} \ll_{1,2}^{n,n} y^{n-1} \wedge r^{n-1} \ll_{2,2}^{n,n} y^{n-1})) \wedge$

$\forall i(1 \leq i \leq k)\forall z^{n-1}\forall r^{n-1}\forall y^{n-1}(\langle i+1, k, z^{n-1}, r^{n-1}, y^{n-1}\rangle \prec x^n \leftrightarrow$

$\forall w^{n-1}\forall u^{n-1}(\langle i, k, w^{n-1}, u^{n-1}, y^{n-1}\rangle \prec x^n \rightarrow (z^{n-1} \ll_{1,2}^{n,n} u^{n-1} \wedge r^{n-1} \ll_{2,2}^{n,n} u^{n-1})))))] \wedge$

(*Intuitive gloss:* $\ulcorner P_j^{r_1,\ldots,r_k}(t_1,\ldots,t_k)\urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case the (generalized) k-tuple consisting of the references assigned by the model to each of the* $\ulcorner t_i \urcorner$ *relative to assignment* a^{n-1} *is 'among' the reference assigned by the model to* $\ulcorner P^{r_1,\ldots,r_k}\urcorner$ *relative to* a^{n-1}*. The clause is cumbersome because it makes use of a coding system to attain the effect of quantifying over k-tuple positions: '*$\langle i,k,z^{n-1},r^{n-1},y^{n-1}\rangle \prec x^n$*' might be thought of as enconding the information that according to the model,* z^{n-1} *(which is the reference assigned by the model to* $\ulcorner t_i \urcorner$ *relative to* a^{n-1}*) is the ith component of* y^{n-1} *(which is 'among' the reference assigned by the model to* $\ulcorner P^{r_1,\ldots,r_k}\urcorner$ *relative to* a^{n-1}*). The last three lines of the clause are a specification of how the coding is to work.)*

[ϕ is a formula $\rightarrow (\left\langle \ulcorner \sigma_i^k[\phi]\urcorner, a^{n-1}, z^{n-1}\right\rangle \prec x^n \leftrightarrow$

$(\forall s^0(s^0 \prec_{trans}^{1,n} z^{n-1} \rightarrow \left\langle \text{'}\forall\text{'}, s^0\right\rangle \prec x^n) \wedge$ the assignment $\hat{a}^{n-1}$ (which is just like a^{n-1}

except that $\alpha_{\hat{a}^{n-1}}(\ulcorner v_i^k \urcorner) \approx z^{n-1}$) is such that $\left\langle \phi, \hat{a}^{n-1}\right\rangle \prec x^n)$] $\wedge$

(*Intuitive gloss: 'among' the reference assigned by the model to* $\ulcorner \sigma_i^k[\phi]\urcorner$ *relative to assignment* a^{n-1} *are all and only those* z^{n-1} *such that:* (*a*) *the 'transitive closure' of* z^{n-1} *consists entirely of individuals in the domain, and* (*b*) ϕ *is true in the model relative to the* a^{n-1}*-variant assigning* z^{n-1} *to* $\ulcorner v_j^k \urcorner$*.)*

[$\ulcorner \mathrm{Ex}^k(t)\urcorner$ is a formula $\rightarrow (\left\langle \ulcorner \mathrm{Ex}^k(t)\urcorner, a^{n-1}\right\rangle \prec x^n \leftrightarrow \exists z^{n-1}(\left\langle t, a^{n-1}, z^{n-1}\right\rangle \prec x^n))] \wedge$

(*Intuitive gloss:* $\ulcorner \mathrm{Ex}^k(t)\urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case the model assigns a reference to* t *relative to* a^{n-1}*.)*

[ϕ is a formula $\rightarrow (\left\langle \ulcorner \neg\phi\urcorner, a^{n-1}\right\rangle \prec x^n \leftrightarrow \neg(\left\langle \ulcorner \phi\urcorner, a^{n-1}\right\rangle \prec x^n))] \wedge$

(*Intuitive gloss:* $\ulcorner \neg\phi \urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case* ϕ *is not.)*

[ϕ and ψ are formulas $\rightarrow (\left\langle \ulcorner \phi\wedge\psi\urcorner, a^{n-1}\right\rangle \prec x^n \leftrightarrow$

$(\left\langle \ulcorner \phi\urcorner, a^{n-1}\right\rangle \prec x^n \wedge \left\langle \ulcorner \psi\urcorner, a^{n-1}\right\rangle \prec x^n))]\}$

(*Intuitive gloss:* $\ulcorner \phi\wedge\psi \urcorner$ *is true in the model relative to assignment* a^{n-1} *just in case* ϕ *and* ψ *are.)*

It is then straightforward to characterize logical consequence for $\mathfrak{L}$:

ϕ is a logical consequence of $\Gamma \ \equiv_{df}$

$$\begin{aligned}
&\forall x^n[M(x^n) \rightarrow \\
&\quad (\forall \psi(\psi \in \Gamma \rightarrow \forall a^{n-1}(A(a^{n-1}) \rightarrow \langle \psi, a^{n-1} \rangle \prec x^n)) \rightarrow \\
&\qquad \forall a^{n-1}(A(a^{n-1}) \rightarrow \langle \phi, a^{n-1} \rangle \prec x^n))]
\end{aligned}$$

By using the technique in Rayo and Uzquiano (1999), this *explicit* characterization of logical consequence for a full nth-level language in a basic $(n+1)th$-level langauge can be transformed into an *implicit* characterization of logical consequence for a basic $(n+1)$th-level language in a full $(n+1)$th-level language.

Notes

[1]If a sentence is ambiguous, it might have more than one semantic structure. I will henceforth ignore ambiguity of this kind to simplify my presentation. For present purposes, ambiguity may be thought of as a matter of homophonic but distinct expressions.

[2]More precisely, as a *finite tree with ordered nodes*. A *finite tree* is an ordered-pair $\langle N, \leq \rangle$, where N is a finite set of 'nodes' and $\leq$ is a binary relation on N with the following properties: (*i*) $\leq$ is reflexive, transitive and antisymmetric; (*ii*) N has a $\leq$-minimal element, which we call 'base node'; (*iii*) every node x in N other than the base node has an immediate $\leq$-predecessor (i.e. there is a y in N such that $y \leq x$ and there is no z in N such that $y \leq z \leq x$); and (*iv*) for any x in N there is a unique path back to the base node (i.e. for any y and z, if $y \leq x$ and $z \leq x$ then either $y \leq z$ or $z \leq y$). We say that x is a *terminal node* if, for every y, $x \leq y$ only if $x = y$. If $x \leq y$ and $x \neq y$ we say that y is *below* x in the tree. If y is below x and no z is such that z is below x and y is below z, then we say that y is *immediately* below x. Finally, a finite tree *with ordered nodes* is a pair $\langle T, F \rangle$ where T is a finite tree and F is a one-one function from nodes in T to natural numbers. If y and z are both immediately below x, we say that y is *to the left* of z just in case $F(y) < F(z)$.

[3]For a defence, see Lewis (1970) and Montague (1970).

[4]Strictly, one should distinguish between the expressions of a first-order language, and the members of the language's semantic lexicon. Only the latter can be properly said to fall under semantic categories. I shall fudge this distinction—here and throughout the remainder of the paper—for presentational purposes.

[5] According to Lewis (1970), for example, the semantic category of a predicate is $\langle S/N \rangle$

and the semantic category of a name is N (or, alternatively, $\langle S/(S/N)\rangle$). So, in order for '$\xi(c)$' to be a sentence, the semantic category of 'ξ' would have to be $\langle S/N\rangle$ (or, alternatively, either $\langle S/N\rangle$ or $\langle S/(S/(S/N))\rangle$), and in order for '$P(\xi)$' to be a sentence, the semantic category of 'ξ' would have to be either N or $\langle S/(S/N)\rangle$. And it is impossible to fulfill both of these conditions at once.

[6]For the Quinean side of the debate see Quine (1986) ch. 5, Resnik (1988), Parsons (1990) and Linnebo (2003) (among others). For the Boolosian side of the debate see Boolos (1984), Boolos (1985a), Boolos (1985b), McGee (1997), Hossack (2000), McGee (2000), Oliver and Smiley (2001), Rayo and Yablo (2001), Rayo (2002) and Williamson (2003) (among others).

[7]See Simons (1997), Rayo and Yablo (2001) and Williamson (2003).

[8]For an expanded criterion of ontological commitment, see Rayo (2002).

[9]For discussion of determinacy, see Jané (2003).

[10]For more on absolutely unrestricted quantification see Parsons (1974b), Dummett (1981) chapters 14-16, Cartwright (1994), Boolos (1998b), Williamson (1999), McGee (2000), the postscript to Field (1998) in Field (2001), Rayo (2003), Rayo and Williamson (2003), Glanzberg (2004), and Williamson (2003).

[11]See Rayo (2002).

[12]In using this notation I follow Burgess and Rosen (1997).

[13]Although a cleaner example of an English first-level term would be 'they' in "Some elephants passed by; they were generally nice to each other", I treat expressions like 'the elephants' as if they were uncontroversially first-level terms for ease of exposition.

[14]See Rayo (2002).

[15]In this connection, see Black (1970) and Hazen (1997).

[16] Formally,

$$\exists xxxx \forall yyy[yyy \prec^{3,4} xxxx \leftrightarrow \forall zz(zz \prec^{2,3} yyy \leftrightarrow$$
$$\exists w \exists u(\textsc{Ancestor}^{1,1}(w,u) \wedge (\forall t(t \prec^{1,2} zz \leftrightarrow$$
$$(t = w \vee t = u)) \vee \forall t(t \prec^{1,2} zz \leftrightarrow (t = w)))))\wedge$$
$$\textsc{Ref}^{1,4}(\text{`}\textsc{Ancestor}^{1,1}(v_i^0, v_j^0)\text{'}, xxxx)].$$

[17] Formally,

$$\exists xxxx \forall yyy[yyy \prec^{3,4} xxxx \leftrightarrow \forall zz(zz \prec^{2,3} yyy \leftrightarrow$$
$$\exists w \exists u(\textsc{World}^{1}(w) \wedge \textsc{ElephantIn}^{1,1}(w,u) \wedge (\forall t(t \prec^{1,2} zz \leftrightarrow$$
$$(t = w \vee t = u)) \vee \forall t(t \prec^{1,2} zz \leftrightarrow (t = w)))))\wedge$$
$$\textsc{Int}^{1,4}(\text{`}\textsc{Elephant}^{1}(v_i^0)\text{'}, xxxx)].$$

[18]See, however, Schein (forthcoming).

[19]This way of thinking of empty predicates yields the result that a predicate not-F might be referenceless even though F is not, and that a predicate F-and-G might be referenceless even though F and G are not. One must also take special care in dealing with generalized quantifiers (see Rayo (2002)). Empty predicates may not be the only case of meaningful but referenceless predicates. Predicates such as 'is taller than him' in a context in which 'him' hasn't been assigned a reference might constitute another example. (Thanks here to

Tim Williamson.)

[20]Compare with the treatment of empty names in Oliver and Smiley (typescript).

[21]This subsection and the last have benefited greatly from discussion with Øystein Linnebo.

[22]With some modifications, I follow the presentation in Shapiro (1991) §3.2.

[23]The required notion of freedom-for is an exact analogue of the notion of freedom-for in a standard first-order language: t_j is free for t_i in φ just in case no occurrence of t_i in φ lies within a formula of the form $\ulcorner \sigma^m_j[\psi] \urcorner$, where $\ulcorner v^m_j \urcorner$ is a placeholder occurring in t_j. Thus, 'v^0_0' is free for 'v^0_1' in '$G^2(\sigma^0_1[\mathrm{F}^1(v^0_1)])$' or '$G^2(\sigma^0_2[\mathrm{F}^1(v^0_1)])$' but not in '$G^2(\sigma^0_0[\mathrm{F}^1(v^0_1)])$'.

[24]In analogy with the above, we say that a basic $(n+1)$th-level language $\mathfrak{L}^{n+1}$ $(n \geq 2)$ can be *paraphrased* as an nth-level $\mathfrak{L}^n$ language just in case there is a range of individuals—the 'classes'—such that, for any sentence in $\mathfrak{L}^{n+1}$, the following transformation into $\mathfrak{L}^n$ preserves truth-value (certain clauses are omitted for the sake of brevity):

- for $m < (n-1)$, $(\exists v^m_i(\varphi))^{Tr} \rightarrowtail \exists v^m_i(\mathrm{D}^{m+1}(v^m_i) \wedge \varphi^{Tr})$
- $(\exists v^{n-1}_i(\varphi))^{Tr} \rightarrowtail \exists v^{n-1}_{2i-1}(\mathrm{D}^n(v^{n-1}_{2i-1}) \wedge \varphi^{Tr})$
- $(\exists v^n_i(\varphi))^{Tr} \rightarrowtail \exists v^{n-1}_{2i}(\mathrm{C}^n(v^{n-1}_{2i}) \wedge \varphi^{Tr})$
- $(v^{n-1}_i \prec v^n_j)^{Tr} \rightarrowtail v^{n-1}_{2i-1} \ll v^{n-1}_{2j}$

  where 'D^m', 'C^m' and $v^m_i \ll v^m_j$ are characterized as follows:

  - 'D^1' is true of all and only individuals in the domain of discourse of $\mathfrak{L}^{n+1}$
  - $\mathrm{D}^{k+2}(v^{k+1}_s) \leftrightarrow \forall v^k_t(v^k_t \prec v^{k+1}_s \supset \mathrm{D}^{k+1}(v^k_t))$

– 'C^1' is true of all and only (non-empty) 'classes' of individuals in the domain of discourse of $\mathfrak{L}^{n+1}$

– $C^{k+2}(v_s^{k+1}) \leftrightarrow \forall v_t^k(v_t^k \prec v_s^{k+1} \supset C^{k+1}(v_t^k))$

– $v_i^0 \ll v_j^0 \leftrightarrow v_i^0 \in v_j^0$

– $v_i^{k+1} \ll v_j^{k+1} \leftrightarrow \exists v_s^k(v_s^k \prec v_j^{k+1} \wedge \forall v_t^k(v_t^k \prec v_i^{k+1} \leftrightarrow v_t^k \ll v_s^k))$,

and '$\in$' expresses a 'membership' relation appropriate for 'classes'.

The higher-level version of No Paraphrase can then be established by focusing on the following sentence of $\mathfrak{L}^{n+1}$:

$$\forall v_1^n \exists v_1^{n-1} \forall v_2^{n-1}(v_2^{n-1} \prec v_1^n \leftrightarrow \text{Member}(v_2^{n-1}, v_1^{n-1}))$$

where 'Member' is characterized just like '$\ll$'.

[25]This section benefited greatly from discussion with Gabriel Uzquiano and Crispin Wright.

[26]The basic idea is due to Bernays (1942); for a formal statement of the result see Rayo (2002). The proof is analogous to that of theorem 5.3 of Shapiro (1991).

[27]See McGee (forthcoming). For present purposes, it is best to think of the dyadic second-order quantifier in McGee's construction as a monadic quantifier ranging over ordered-pairs.

[28]See Rayo and Williamson (2003), footnote 7.

[29] See Rayo and Uzquiano (1999).

[30]One might worry that the my arguments for Semantic Ascent rely on unwarranted assumptions about predicate-reference. For I used the assumption that, e.g. the reference

of a monadic first-level predicate is a plurality. But what if one held a view such as the following?

$$\forall x(\text{Ref}^{1,1}(\text{`}\ldots\text{ is an elephant'}, x) \leftrightarrow \text{Elephant}^{1}(x))$$

In fact, it makes no difference whether one chooses to say that something is a referent of '...is an elephant' just in case it is an elephant, rather than saying that the reference of '...is an elephant' is the plurality of elephants. The arguments for Semantic Ascent go through just the same.

[31]See Kreisel (1967).

[32]It implies, for example, the existence of inaccessible cardinals. See Shapiro (1991), §6.3.

[33]*Proof:* Suppose for *reductio* that some formula φ in a limit$_\omega$ language captures the notion of truth-in-a-model. Since φ contains finitely many symbols, it is also a formula in some nth-level language. And this contradicts the conjunction of (a) and (c), since a limit$_\omega$ language includes nth-level languages as proper parts for any finite n.

[34]See Williamson (2003).

[35]The matter of giving a strictly adequate model-theory for languages of transfinite-level is non-trivial. Andrews (1965) develops a strictly adequate model-theory for limit$_\omega$ languages by allowing quantification over types.

[36]For more on indefinite extensibility, see Russell (1906), Dummett (1963), Parsons (1974a), Parsons (1974b), Dummett (1991) pp. 316-319, Dummett (1993b), Hazen (1993), Williamson (1998), Glanzberg (2004), Shapiro (2003a) and Shapiro (2003b).

[37]For a detailed discussion of these matters, see Priest (2003).

[38]See, for instance, Parsons' contribution to this volume. For further discussion of this point, see Linnebo and Weir's contributions.

[39]Many thanks to Kit Fine, Øystein Linnebo, Tom McKay, Charles Parsons, Marcus Rossberg, Barry Schein, Gabriel Uzquiano, Tim Williamson, Crispin Wright, an anonymous referee for this volume, and audiences at MIT's MATTI Reading Group, La Universidade de Santiago de Compostela and *Arché* the AHRC Research Centre for the Philosophy of Logic, Language, Mathematics and Mind.

References

Andrews, P. B. (1965) *A Transfinite Type Theory with Type Variables*, North-Holland Publishing Company, Amsterdam.

Beall, J., ed. (2003) *Liars and Heaps*, OUP, Oxford.

Bernays, P. (1942) "A System of Axiomatic Set Theory, IV," *Journal of Symbolic Logic* 7:4, 133– 145.

Black, M. (1970) "The elusiveness of sets," *Review of Metaphysics* 24, 614–636.

Boolos, G. (1984) "To Be is to Be a Value of a Variable (or to be Some Values of Some Variables)," *The Journal of Philosophy* 81, 430–49. Reprinted in Boolos (1998a).

Boolos, G. (1985a) "Nominalist Platonism," *Philosophical Review* 94, 327–44. Reprinted in Boolos (1998a).

Boolos, G. (1985b) "Reading the *Begriffsschrift*," *Mind* 94, 331–334. Reprinted in Boolos (1998a).

Boolos, G. (1998a) *Logic, Logic and Logic*, Harvard, Cambridge, Massachusetts.

Boolos, G. (1998b) "Reply to Parsons' "Sets and Classes"." In Boolos (1998a), 30-36.

Burgess, J., and G. Rosen (1997) *A Subject With No Object*, Oxford University Press, New York.

Cartwright, R. (1994) "Speaking of Everything," *Noûs* 28, 1–20.

Dales, H. G., and G. Oliveri, eds. (1998) *Truth in Mathematics*, Oxford University Press, Oxford.

Dummett, M. (1963) "The philosophical significance of Godel's theorem," *Ratio* 5, 140–155. Reprinted in Dummett (1978).

Dummett, M. (1978) *Truth and other enigmas*, Duckworth, London.

Dummett, M. (1981) *Frege: Philosophy of Language*, Harvard, Cambridge, MA, second edition.

Dummett, M. (1991) *Frege: Philosophy of Mathematics*, Duckworth, London.

Dummett, M. (1993a) *The seas of language*, Clarendon Press, Oxford.

Dummett, M. (1993b) "Wittgenstein on Necessity: Some Reflections." In Dummett (1993a).

Field, H. (1998) "Which Undecidable Mathematical Sentences have Determinate Truth Values?" In Dales and Oliveri (1998). Reprinted in Field (2001).

Field, H. (2001) *Truth and the Absence of Fact*, Oxford University press, Oxford.

Glanzberg, M. (2004) "Quantification and Realism," *Philosophy and Phenomenological Research* 69, 541–572.

Hazen, A. P. (1993) "Against Pluralism," *Australasian Journal of Philosophy* 71:2, 132–144.

Hazen, A. P. (1997) "Relations in Lewis's Framework Without Atoms," *Analysis* 57, 243–248.

Hossack, K. (2000) "Plurals and Complexes," *The British Journal for the Philosophy of Science* 51:3, 411–443.

Jané, I. (2003) "Higher-order Logic is not Logic." In Shapiro (2005).

Kreisel, G. (1967) "Informal Rigour and Completeness Proofs." In Lakatos (1967).

Lakatos, I., ed. (1967) *Problems in the Philosophy of Mathematics*, North Holland, Amsterdam.

Lepore, E., and B. Smith, eds. (forthcoming) *Handbook of Philosophy of Language*, Oxford University Press, Oxford.

Lewis, D. (1970) "General Semantics," *Synthese* 22, 18–67. Reprinted in Lewis (1983).

Lewis, D. (1983) *Philosophical Papers, Volume I*, Oxford.

Linnebo, Ø. (2003) "Plural Quantification Exposed," *Noûs* 37, 71–92.

McGee, V. (1992) "Two Problems With Tarski's Theory of Consequence," *Proceedings of the Aristotelian Society* 92, 273–292.

McGee, V. (1997) "How We Learn Mathematical Language," *Philosophical Review* 106, 35–68.

McGee, V. (2000) "'Everything'." In Sher and Tieszen (2000).

McGee, V. (forthcoming) "Universal Universal Quantification." In *Liars and Heaps*.

Montague, R. (1970) "Universal Grammar," *Theoria* 36, 373–398.

Oliver, A., and T. Smiley (2001) "Strategies for a Logic of Plurals," *Philosophical Quarterly* 51, 289–306.

Oliver, A., and T. Smiley (typescript) "A Modest Logic of Plurals."

Parsons, C. (1974a) "The Liar Paradox," *The Journal of Philosophical Logic* 3, 381–412. Reprinted in Parsons (1983).

Parsons, C. (1974b) "Sets and Classes," *Noûs* 8, 1–12. Reprinted in Parsons (1983).

Parsons, C. (1983) *Mathematics in Philosophy*, Cornell University Press, Ithaca, NY.

Parsons, C. (1990) "The Structuralist View of Mathematical Objects," *Synthese* 84, 303–346.

Priest, G. (2003) *Beyond the Limits of Thought*, Oxford University Press, Oxford, second edition.

Quine, W. V. (1986) *Philosophy of Logic, Second Edition*, Harvard, Cambridge, Massachusetts.

Rayo, A. (2002) "Word and Objects," *Noûs* 36, 436–464.

Rayo, A. (2003) "When does 'everything' mean *everything*?" *Analysis* 63, 100–106.

Rayo, A., and G. Uzquiano (1999) "Toward a Theory of Second-Order Consequence," *The Notre Dame Journal of Formal Logic* 40, 315–325.

Rayo, A., and T. Williamson (2003) "A Completeness Theorem for Unrestricted First-Order Languages." In Beall (2003).

Rayo, A., and S. Yablo (2001) "Nominalism Through De-Nominalization," *Noûs* 35:1, 74–92.

Resnik, M. (1988) "Second-Order Logic Still Wild," *Journal of Philosophy* 85:2, 75–87.

Russell, B. (1906) "On some difficulties in the theory of transfinite numbers and order types," *Proceedings of the London Mathematical Society* 4, 29–53.

Schein, B. (forthcoming) "Plurals." In Lepore and Smith (forthcoming).

Shapiro, S. (1991) *Foundations Without Foundationalism: A Case for Second-Order Logic*, Clarendon Press, Oxford.

Shapiro, S. (2003a) "All Sets Great and Small: And I do Mean All" 467–490. In Hawthorne, J. and D. Zimmerman, eds. *Philosophical Perspectives 17: Language and Philosophical Linguistics*, Blackwell, Oxford.

Shapiro, S. (2003b) "Prolegomenon To Any Future Neo-Logicist Set Theory: Abstraction and Indefinite Extensibility," *British Journal for the Philosophy of Science* 54, 59–91.

Shapiro, S., ed. (2005) *The Oxford Handbook for Logic and the Philosophy of Mathematics*, Clarendon Press, Oxford.

Sher, G., and R. Tieszen, eds. (2000) *Between Logic and Intuition*, Cambridge University Press, New York and Cambridge.

Simons, P. (1997) "Higher-order quantification and ontological commitment," *Dialectica* 51, 255–271.

Williamson, T. (1998) "Indefinite Extensibility," *Grazer Philosophische Studien* 55, 1–24.

Williamson, T. (1999) "Existence and Contingency," *Proceedings of the Aristotelian Society, Supplementary Volume* 73, 181–203. Reprinted with printer's errors corrected in *Proceedings of the Aristotelian Society* 100 (2000), 321–343 (117–139 in unbounded edition).

Williamson, T. (2003) "Everything" 415–465. In Hawthorne, J. and D. Zimmerman, eds. *Philosophical Perspectives 17: Language and Philosophical Linguistics*, Blackwell, Oxford.