

Nominalism, Trivialism, Logicism

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May 1, 2014

This paper is an effort to extract some of the main theses in the philosophy of mathematics from my book, *The Construction of Logical Space*. I show that there are important limits to the availability of nominalistic paraphrase-functions for mathematical languages, and suggest a way around the problem by developing a method for specifying nominalistic *contents* without corresponding nominalistic *paraphrases*.

Although much of the material in this paper is drawn from the book—and from an earlier paper (Rayo 2008)—I hope the present discussion will earn its keep by motivating the ideas in a new way, and by suggesting further applications.

I Nominalism

Mathematical Nominalism is the view that there are no mathematical objects. A standard problem for nominalists is that it is not obvious that they can explain what the point of a mathematical assertion would be. For it is natural to think that mathematical sentences like ‘the number of the dinosaurs is zero’ or ‘ $1 + 1 = 2$ ’ can only be true if mathematical objects exist. But if this is right, the nominalist is committed to the view that such sentences are untrue. And if the sentences are untrue, it not immediately obvious why they would be worth asserting.

*For their many helpful comments, I am indebted to Vann McGee, Kevin Richardson, Bernhard Salow and two anonymous referees for *Philosophia Mathematica*. I would also like to thank audiences at Smith College, the Università Vita-Salute San Raffaele, and MIT’s Logic, Language, Metaphysics and Mind Reading Group. Most of all, I would like to thank Steve Yablo.

A nominalist could try to address the problem by suggesting nominalistic *paraphrases* for mathematical sentences. She might claim, for example, that when one asserts ‘the number of the dinosaurs is zero’ one is best understood as making the (nominalistically kosher) claim that there are no dinosaurs, and that when one asserts ‘ $1 + 1 = 2$ ’ one is best understood as making the (nominalistically kosher) claim that any individual and any other individual will, taken together, make two individuals.¹

Such a strategy faces two main challenges. The first is to explain why mathematical assertions are to be understood non-standardly. One way for our nominalist to address this challenge is by claiming that mathematical assertions are set forth ‘in a spirit of make-believe’ (Yablo 2001). She might argue, in particular, that when one makes a mathematical assertion one is, in effect, claiming that the asserted sentence is true in a *fiction*, and more specifically a fiction according to which: (a) all non-mathematical matters are as in reality, but (b) mathematical objects exist with their standard properties. This proposal leads to the welcome result that fictionalist assertions of mathematical sentences can convey information about the real world. For instance, one can use a fictionalist assertion of ‘the number of the dinosaurs is zero’ to convey the information that there are no dinosaurs, since the only way for ‘the number of the dinosaurs is zero’ to be true in a fiction whereby mathematical objects have all the standard properties is for the fiction to entail that there are no dinosaurs, and the only way for such a fiction to agree with reality in all non-mathematical respects is for it to be the case that there are no dinosaurs.

The second main challenge is that of specifying nominalistic paraphrases for arbitrary mathematical sentences. It is perfectly straightforward to come up with plausible nominalistic paraphrases for toy sentences like ‘the number of the dinosaurs is zero’ or ‘ $1 + 1 = 2$ ’. But we need is a method that will work in general.

Say that a paraphrase-function (for language L with output-language L^N) is an effectively specifiable function that assigns to each sentence in L a paraphrase in L^N . One of the main

¹More carefully: $\forall x \forall y (x \neq y \rightarrow \exists!_2 z (z = x \vee z = y))$.

objectives of this paper is to show that finding a nominalist paraphrase-function is not as easy as one might have thought.

2 Constraints

What would it take for a paraphrase-function for the language of (applied) arithmetic to count as a *nominalist* paraphrase-function? I suggest the following three constraints:

1. *The Counting Constraint*

The paraphrase assigned to $\ulcorner \text{The number of the } F\text{s} = n \urcorner$ should have the same truth-conditions as $\ulcorner \exists!_n x(Fx) \urcorner$.²

2. *The Inferential Constraint*

Suppose that ϕ and ψ are arithmetical sentences, and that the truth-conditions of ϕ are at least as strong as the truth-conditions of ψ (for short: ϕ *entails* ψ). Then the paraphrase assigned to ϕ should entail the paraphrase assigned to ψ .

3. *The Triviality Constraint*

- (a) The paraphrase assigned to any true sentence of pure arithmetic should have *trivial* truth-conditions (that is, truth-conditions that would be satisfied regardless of how the world turned out to be).
- (b) The paraphrase assigned to any false sentence of pure arithmetic should have *impossible* truth-conditions (that is, truth conditions that would fail to be satisfied regardless of how the world turned out to be).

It seems to me that a paraphrase-function satisfying these three constraints (for short: a *trivialist* paraphrase-function) should be thought of as the ‘gold standard’ of nominalist

²As usual, we let $\ulcorner \exists!_0 x(\phi(x)) \urcorner$ be short for $\ulcorner \neg \exists x(\phi(x)) \urcorner$, and $\ulcorner \exists!_{n+1} x(\phi(x)) \urcorner$ be short for $\ulcorner \exists z(\phi(z) \wedge \exists!_n x(\phi(x) \wedge x \neq z)) \urcorner$.

paraphrase-functions. It is hard to see how a nominalist paraphrase-function could deserve the label ‘nominalist’ if it failed to respect the Counting Constraint, and it is clear that something important would be left out if it failed to respect the Inferential Constraint.

What about the Triviality Constraint? Although some nominalists might be willing to settle for a paraphrase-function that failed to satisfy it, I hope it can be agreed on all sides that the nominalist should *prefer* a paraphrase-function that satisfies the Triviality Constraint over one that does not. Notice, for example, that the fictionalist proposal we considered above presupposes that the Triviality Constraint ought to be satisfied. For a truth of pure mathematics will count as true in the relevant fiction *regardless of how matters stand in reality*. So our fictionalist is committed to thinking that a truth of pure mathematics can be correctly asserted ‘in a spirit of make believe’ regardless of how matters stand in reality.

It is also worth noting that the Triviality Constraint was satisfied by the nominalistic reading that I had earlier suggested for ‘ $1 + 1 = 2$ ’. For the paraphrase I suggested is a logical truth, and it is reasonable to assume that the truths of pure logic have trivial truth-conditions. One might be inclined to think that it would be desirable if the nominalistic reading of *any* truth of pure mathematics turned out to be a logical truth, and therefore had trivially satisfiable truth-conditions.

3 The Bad News

The bad news is that it is *impossible* to specify a trivialist paraphrase-function for the language of arithmetic. A little more carefully: there is a formal result that suggests that it is impossible to specify a paraphrase-function for the language of arithmetic that is *uncontroversially* trivialist.

Here ‘uncontroversial’ means three different things: (1) no controversial linguistic assumptions, (2) no controversial metaphysical assumptions, and (3) no controversial subtraction-assumptions. I will say a few words about each kind of assumption before turning to the

formal result itself.

3.1 Controversial Linguistic Assumptions

If the expressive resources of one's output-language—i.e. the language in which nominalistic paraphrases are given—are sufficiently powerful, it is straightforward to define a trivialist paraphrase-function.

There is, for example, a method for paraphrasing each sentence of the language of arithmetic as a sentence of an $(\omega + 3)$ -order language.³ Would this count as a trivialist paraphrase-function? Yes: *if* one assumes that $(\omega + 3)$ -order logic is 'genuine logic' (if one assumes, in other words, that any truth of $(\omega + 3)$ -order logic has trivial truth-conditions). That is, however, a highly controversial assumption.

The view that *second*-order logic is 'genuine logic' is increasingly popular amongst philosophers. But most philosophers seem to think that languages of high finite order—to say nothing of languages of transfinite-order—can only be made sense of as 'set theory in sheep's clothing' (Quine 1986), and many philosophers would conclude on that basis that the truths of higher-order logic have non-trivial truth-conditions.

My own view is that $(\omega + 3)$ -order logic is, in fact, 'genuine logic',⁴ and that our $(\omega + 3)$ -paraphrase-function is, in fact, a trivialist paraphrase function. But even I must concede that it is not *uncontroversially* a trivialist paraphrase-function. The linguistic assumptions one would need to justify such a claim are just too great.

3.2 Controversial Metaphysical Assumptions

There is a nominalistic paraphrase-function that I find very attractive. It draws its inspiration from Frege's *Grundlagen*, so I will refer to it as the *Fregean* paraphrase-function. The basic

³An $(\omega + 3)$ -order language has variables of all finite types, plus three levels of variables of transfinite type. For further details, see (Linnebo & Rayo 2012). For more on the relevant paraphrase-method, see (Rayo 2013, ch. 7).

⁴I believe, in other words, that the truths of $(\omega + 3)$ -order logic have trivial truth-conditions. I also believe, however, that the truths of pure set-theory have trivial truth conditions. See (Rayo 2013, ch. 3).

idea is that a sentence of the form ‘the number of the F s = the number of the G s’ is to be paraphrased as:

the F s are just as many as the G s

What about a *quantified* arithmetical sentence, such as ‘there is an $n > 0$ such that: $n =$ the number of the planets’? We first paraphrase the sentence as:

there are some things, the F s, such that: the number of the F s = the number of the planets

We then eliminate arithmetical-terms altogether, and say:

there are some things, the F s, such that: the F s are just as many as the planets.

It is easy to show that similar transformations can be applied to every sentence in the language of applied arithmetic (excluding mixed-identity statements such as ‘Caesar = 17’).⁵ The result is a nominalistic paraphrase-function that assigns to each arithmetical sentence a second-order sentence.⁶

Suppose we concede that second-order logic is ‘genuine logic’, and that the truths of second-order logic have trivial truth-conditions. Is this enough to conclude that the Fregean Paraphrase-Function is a *trivialist* paraphrase-function? No—at least not if what we’re looking for is an *uncontroversially* trivialist paraphrase-function. For consider ‘any number greater than 0 has a successor’, which is a truth of pure arithmetic. Its Fregean paraphrase is:

For any things, the F s, there are some things, the G s, such that: for some g amongst the G s, the F s are just as many as the G s distinct from g .

⁵I spell out the details in (Rayo 2002); for a similar proposal, see (Fine 2002, II.5).

⁶Alternatively, one could think of the paraphrase-function as assigning to each arithmetical sentence a sentence of a *plural language* which has been enriched with the atomic plural predicate ‘they are just as many as them’. Here I fudge the distinction between the two for expositional purposes. For more on plural languages, see (Boolos 1984) and (Linnebo 2004).

which will only be true if there are infinitely many objects (or none). So we have a violation of the Triviality Constraint. More guardedly: we have a violation of the Triviality Constraint *unless* we are prepared to accept the following (highly controversial) metaphysical thesis:

Trivialist Infinitarianism

Not only is it the case that the world contains infinitely many objects, it is *trivially* the case that the world contains infinitely many objects. In other words: to assume that the world is finite is to assume something worse than false, it is to assume something *absurd*.

I myself think that Trivialist Infinitarianism is true.⁷ So I believe that the Fregean Paraphrase-Function is, in fact, a trivialist paraphrase-function. But even someone like me, who thinks that Trivialist Infinitarianism is true, must concede that the Fregean Paraphrase-Function is not *uncontroversially* a trivialist paraphrase-function.

It is perhaps worth mentioning that the Fregean Paraphrase-Function is in good company when it comes to infinity assumptions. Many attractive nominalist paraphrase-functions will only count as trivialist paraphrase-functions in the presence of Trivialist Infinitarianism.⁸

3.3 Controversial Subtraction-Assumptions

Joseph Melia (2000) has argued for a satisfyingly straightforward nominalistic paraphrase-function. One is simply to paraphrase the mathematical sentence ϕ as:

ϕ , except for the part about mathematical objects

⁷Why think that Trivialist Infinitarianism is true? Because it follows from [NUMBERS]—see section ref-sec:logicism.

⁸This is true, in particular, of (Hodes 1984) and (Yablo 2002). It is also true of a form of if-then-ism whereby a sentence ϕ is paraphrased as the universal closure of $\ulcorner(A \rightarrow \phi)^*\urcorner$, where A is the conjunction of the second-order Dedekind-axioms and ϕ^* is the result of uniformly replacing arithmetical vocabulary for variables of appropriate type.

A potential worry about this paraphrase-method is that it relies on a non-trivial *subtraction*-assumption. Suppose, for example, that ϕ is a complex physical theory couched in a mathematical language—quantum theory, as it might be. Melia’s method presupposes that the operation of subtracting away the ‘mathematical part’ from the content of quantum theory yields a result which is both well-defined and non-empty. But it is not immediately obvious that this is so: it is not immediately obvious that extricating the mathematical part from quantum theory leaves an interesting remainder.

Mark Colyvan (2010) has a nice example to illustrate why extricability might be a worry:

J. R. R. Tolkien could not, for example, late in the Lord of the Rings trilogy, take back all mention of hobbits; they are just too central to the story. If Tolkien did retract all mention of hobbits, we would be right to be puzzled about how much of the story prior to the retraction remains, and we would also be right to demand an abridged story—a paraphrase of the hobbitless story thus far.

The worry here is not necessarily that the result of subtracting all mention of hobbits from *The Lord of the Rings* is ill-defined—it may well not be. The point is that even if the result is well-defined, one shouldn’t expect much of a narrative. It would be a bit like *Harry Potter* without the wizards: what we’re left with just isn’t unified enough to be much of a story.⁹ Similarly, a skeptic might worry that even if the result of subtracting the mathematical part from the content of quantum theory turns out to be well-defined, what we’re left won’t be unified enough to tell us anything very interesting about the physical world. (Field’s (1984) ‘Heavy Duty Platonist’ is presumably one such skeptic.)

Another way to see that extricability claims can be problematic is to consider the question of what would be left if one subtracted *someone is thirsty* from *I’m thirsty* (Yablo 2012); or the question of what would be left if one subtracted *the tomato is red* from *the tomato is scarlet* (Searle & Körner 1959, Woods 1967, Kraemer 1986, Yablo 2012)? It’s not clear that there are well-defined answers to be given—unless, of course, one is prepared to say ‘nothing’.

⁹Thanks here to Kevin Richardson.

A further example, which I find especially illuminating, concerns the notion of *narrow content*. Narrow contents are supposed to be the result of subtracting away certain kinds of environmental facts from the contents of our beliefs (Brown 1992). The narrow content corresponding to my belief that water is wet, for example, is supposed to be the result of subtracting from what I believe when I believe that water is wet the fact that items in my environment playing a certain theoretical role are composed of H₂O. Since the claim that narrow-contents are both well-defined and non-empty is a highly controversial philosophical thesis, one can use the debate between friends and foes of narrow content to underscore the fact that the operation of subtracting particular ‘environmental factors’ from the contents of our beliefs shouldn’t be assumed to deliver the intended results. (For illuminating discussion, see Yablo forthcoming.)

The most straightforward way of justifying the claim that mathematical content can be usefully extricated from mathematical claims would be to set-forth a nominalist paraphrase-function—one that does not itself rely on subtraction-assumptions. For one would then be in a position to claim that the result of subtracting away the mathematical part from the content of a mathematical sentence is simply the content of the sentence’s paraphrase. But it is not immediately obvious that a suitable paraphrase-function can be found. For although we want the result of subtracting away the mathematical part from quantum theory, say, to deliver a non-empty content, we presumably want the result of subtracting away the mathematical part from a truth of *pure* mathematics to be an empty content: a content that would be satisfied however the world turned out to be. So the relevant paraphrase-function had better be a *trivialist* paraphrase-function. And, as we have seen, it is not easy to find an uncontroversial example of a trivialist paraphrase-function. It is therefore not immediately obvious that the operation of subtracting mathematical content can be defined in a way that delivers interesting results.¹⁰

¹⁰Yablo (2012, forthcoming) has as sophisticated treatment of these issues, which yields an illuminating characterization of the circumstances under which the subtraction operation delivers results which are both well-defined and non-empty. On its own, however, Yablo’s account does not settle the question of whether

My own view is that Melia’s use of the subtraction-operation can, in fact, be defined so as to deliver the right results. But I don’t think that such a claim can be simply taken for granted: a substantial argument is required. (I will attempt to provide the missing argument in section 4.)

Melia’s paraphrase-function has the advantage of wearing its subtraction-assumption on its sleeve. But it is worth noting that similar assumptions are required by other nominalist paraphrase-methods:

Method	Paraphrase ϕ as ...	Subtraction-Assumption
Fictionalism	ϕ is true according to a fiction which is accurate in all non-mathematical respects but in which mathematical objects exist with the standard properties (Yablo 2001).	One would impose an interesting condition on the world by insisting that it agree in non-mathematical respects with a fiction that makes, say, quantum theory true.
Modalism	ϕ is true at the closest possible world which agrees with the actual world in non-mathematical respects but in which mathematical objects exist with the standard properties (Hellman 1989, Dorr 2007).	One would impose an interesting condition on the world by insisting that it agree in non-mathematical respects with the closest possible world that makes, say, quantum theory true.
Subject-Matterism	ϕ is true as far as its non-mathematical subject-matter is concerned, where a claim’s non-mathematical subject-matter is defined as the set of worlds which agree in all non-mathematical respects with a world at which the claim is literally true (Yablo 2012).	One would impose an interesting condition on the world by insisting that it agree in non-mathematical respects with quantum theory’s non-mathematical subject-matter.

It is instructive to note that each of these paraphrase-methods could be easily modified so as to deliver a ‘narrowist’ paraphrase-method, in which paraphrases are meant to capture the *narrow contents* of the original claims, rather than their nominalistic contents. (One could claim, for example, that the *narrow part* of ‘water is wet’ is the claim that ‘water is subtracting ‘the mathematical part’ from, say, quantum theory delivers results which are both well-defined and non-empty.

wet' is true according to a fiction that is accurate in all respects, except perhaps for the nature of the substance that plays the theoretical role that water currently plays in our cognitive lives.) Just as the resulting 'narrowist' paraphrase-functions would do nothing to move a foe of narrow contents, the nominalistic paraphrase-functions in the table above should do nothing to move a skeptic of Melia's subtraction-assumption.

3.4 The Formal Result

Earlier I claimed that it is impossible to specify a paraphrase-function for the language of arithmetic that is uncontroversially trivialist. I am now in a position to give a precise statement of the underlying formal result.

First some assumptions:

A₁ We shall assume that the logical resources of our output-language do not go beyond those of the simple theory of types (which is a language with n th-order quantifiers for each finite n).¹¹

A₂ We shall assume that our output-language contains no intensional operators.

A₃ Any trivialist paraphrase-function must, by definition, satisfy the Triviality Constraint of section 2, and therefore preserve truth-values when applied to sentences of pure arithmetic. (In other words: every truth of pure mathematics must get paraphrased as a truth of the output-language, and every falsity of pure mathematics must get paraphrased as a falsehood of the output-language.)

We shall assume that this condition can be met even if the output-language has a finite domain.

Think of A₁–A₃ as stating, respectively, that our paraphrase-function is to make no controversial linguistic assumptions, that it is to make no controversial subtraction-assumptions,

¹¹Since paraphrase-functions are, by definition, effectively specifiable, we may assume with no loss of generality that our output-language has a *finite* stock of non-logical predicates and terms.

and that it is to make no controversial metaphysical assumptions. More precisely:

- A_1 places an upper bound on the expressive resources of the output-language. Such a bound is justified by the fact that the use of a language more powerful than the simple theory of types would be a sure sign of a controversial linguistic assumption in the sense of section 3.1. (As I noted above, the use of a *third*-order language would be pretty controversial already, but I'm trying to keep my assumptions as weak as possible.)
- A_2 is a ban on intensional operators, and is intended to ensure that our paraphrase-function makes no controversial subtraction-assumptions. I have certainly not shown that any sensible paraphrase-method based on intensional operators will require controversial subtraction-assumptions. But as the table in section 3.3 illustrates, such assumptions are required by the most natural methods for supplying intensional paraphrases for language of applied arithmetic.
- A_3 is meant to ensure that our paraphrase-method does not rely on infinity assumptions. This is important because—unless one embraces Trivialist Infinitarism, which is a decidedly controversial metaphysical thesis—one should think that no trivialist paraphrase-function can presuppose an infinite domain.

Now that our assumptions are in place, it is easy to state the formal result:

Impossibility Theorem

No paraphrase-function for the language of arithmetic can satisfy A_1 – A_3 .

The proof is totally straightforward. By assumptions 1 and 2, our output-language has a finite lexicon and quantifiers of finite type; by assumption 3, our paraphrase-function can preserve truth-value over pure sentences even if the output-language is assumed to have a finite domain. But the set of truths of a language with a finite lexicon and quantifiers of

finite type on a finite domain is effectively specifiable. So our paraphrase-function would deliver a decision procedure for arithmetical truth, which we know to be impossible from Gödel's Theorem.¹²

To the extent that one is prepared to think of A_1 – A_3 as capturing the idea that there are to be no controversial assumptions, one can think of this result as showing that no paraphrase-function for the language of arithmetic can be uncontroversially trivialist.

4 A Way Forward

Let us take stock. We started out by noting that the nominalist faces a challenge: she needs to explain what the point of making a mathematical assertion might be. We then noted that the challenge might be addressed by offering nominalistic paraphrases for mathematical sentences, and going on to claim that the point of asserting a mathematical sentence can be to convey the content of its paraphrase.¹³

What should a nominalist paraphrase-function look like? I listed three constraints in section 2, and suggested that paraphrase-functions satisfying those constraints—i.e. *trivialist* paraphrase-functions—should be thought of as the ‘gold standard’ of nominalist paraphrase. We have seen, however, that that the Impossibility Theorem suggests that there is no way of specifying a trivialist paraphrase function for the language of arithmetic without making controversial assumptions.

What is the nominalist to do? She could embrace one of the nominalist paraphrase-functions we discussed above, and insist that it is a trivialist paraphrase-function by making a controversial assumption. Or she could settle for a nominalist paraphrase-function that falls short of the gold standard.

Here I will propose an alternative. It seems to me that the *real* reason to be interested

¹²Thanks to Vann McGee for pointing out a strengthening of the original result.

¹³I say ‘can be’ rather than ‘is’ because one might think that the truths of *pure* arithmetic have trivially satisfiable contents, and it is not obvious that conveying such contents would be particularly interesting. For further discussion of the point of mathematical assertions, see (Rayo 2013, Chapter 4).

in nominalistic paraphrases is that one can use them to claim that the *nominalistic content* of a mathematical sentence is the literal content of the sentence’s nominalist paraphrase—where the nominalistic content of a sentence is the requirement that the world would have to satisfy in order for a given sentence to be true ‘as far as the non-mathematical facts are concerned’.

What I propose to do here is cut out the middle man. I will argue that there is a method for specifying the nominalistic contents of arithmetical sentences that does not proceed via paraphrases. This alternative method has an advantage and a disadvantage. The advantage is that it delivers *trivialist* contents (i.e. contents satisfying analogues of the three conditions in section 2), and does so without making controversial philosophical assumptions of the kind discussed in section 3. The disadvantage is that the method is couched in mathematical language, and is therefore only available to someone who is prepared to engage in mathematical practice. As we will see in section 4.2, this places certain limits on the purposes for which the proposal can be deployed.

4.1 Outscoping

On the view I would like to discuss, one assigns nominalistic contents to mathematical sentences by way of a *compositional semantics*: an assignment of semantic values to basic lexical items, together with a set of rules for assigning semantic values to a complex expression on the basis of the semantic values of its constituent parts.

I will assume that the semantic value of a sentence is a set of possible worlds. Accordingly, a compositional semantics should allow us to prove a statement of the following form for each sentence ϕ of the object language:

ϕ is true at world w if and only if w is such that ...

The usual way of interpreting such a clause is as a specification of ϕ ’s *truth-conditions*, that is, as a specification of the condition that a world w would need to satisfy in order for ϕ to

count as true at w . Here, however, we will be using the compositional semantics to specify *nominalistic contents*, rather than truth-conditions. Accordingly we will interpret the clause above as supplying a specification of the condition that a world w would need to satisfy in order for ϕ to count as true ‘as far as the non-mathematical facts are concerned’.

Suppose, for example, that a compositional semantics delivers the following clause for ‘the number of the dinosaurs is 0’ (where ‘ $[\dots]_w$ ’ is read ‘at w , it is the case that ...’).

‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’ is true at w if and only if $[\text{there are no dinosaurs}]_w$

The right-hand-side of this clause specifies the following condition on w : that it represent reality as being such that there are no dinosaurs.¹⁴ Accordingly, if the relevant semantics is thought of as a specification of nominalistic contents, one should interpret the clause as stating that what it takes for ‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’ to count as true ‘as far as the non-mathematical facts are concerned’ is for the world to satisfy the condition that there be no dinosaurs.

The example above has the form:

ϕ is true at w if and only if $[p]_w$

where ‘ ϕ ’ is a sentence of the object-language and ‘ p ’ is a nominalistic *paraphrase* of that sentence in the metalanguage. A compositional semantics that only outputs sentential clauses of this form will be severely limited in its ability to specify nominalistic contents, since it presupposes that one is in a position to specify a nominalist paraphrase-function for the object-language in the metalanguage. And, as we have seen, there are good reasons for thinking that it is impossible to specify a trivialist paraphrase-function for the language of arithmetic without making controversial assumptions.

Fortunately, a compositional semantics need not be restricted to outputs of the above form. It can deploy *outscoping*. To see what outscoping is all about, it is useful to contrast the following semantic clauses:

¹⁴What is it for a possible world w to *represent* reality as being such that p ? It is simply for it to be the case that p at w .

[WIDE]

‘ $\exists x(\text{President}(x) \wedge \text{Mustache}(x))$ ’ is true at world w if and only if [there is an x such that x is the president of the United States and x wears a mustache] $_w$

[NARROW]

‘ $\exists x(\text{President}(x) \wedge \text{Mustache}(x))$ ’ is true at world w if and only if there is an x such that x is the president of the United States and [x wears a mustache] $_w$

The only difference between [WIDE] and [NARROW] is the scope of the ‘[. . .] $_w$ ’ operator. But one can see that the difference is significant by considering the following question: How must a world w represent reality if it is to satisfy the right-hand-sides of each of the two clauses?

In the case of [WIDE] the answer is straightforward: w must represent reality as being such that there is an x such that x president of the United States and x wears a mustache. Accordingly, [WIDE] might be thought of as associating the following (unsurprising) condition with ‘ $\exists x(\text{President}(x) \wedge \text{Mustache}(x))$ ’: that there be an x such that x is president of the United States and x wears a mustache.

In the case of [NARROW], however, we get significantly different results. In the actual world, the president of the United States is Barack Obama. So, in order for a world w to satisfy the right-hand-side of [NARROW], it must represent reality as being such that Barack Obama wears a mustache, whether or not he happens to be president. Accordingly, [NARROW] might be thought of as associating the following condition with ‘ $\exists x(\text{President}(x) \wedge \text{Mustache}(x))$ ’: that Obama—the man himself—wear a mustache.

So, whereas [WIDE] specifies a content whereby the president—whoever that may be—wears a mustache, [NARROW] specifies a content whereby Obama—whatever his occupation—wears a mustache.

Let us now consider an arithmetical example. The following two semantic clauses for ‘the number of the dinosaurs is zero’ differ only in the scope of the ‘[. . .] $_w$ ’ operator:

[WIDE]

‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’ is true at w if and only if [the number of x s such that x is a dinosaur = 0] $_w$

[NARROW]

‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’ is true at w if and only if the number of x s such that [x is a dinosaur] $_w = 0$.

How must w represent reality in order to satisfy the right-hand-sides of each of these clauses? In the case of [WIDE], w must represent reality as being such that there is a *number* which numbers the dinosaurs and is identical to zero. So [WIDE] associates a *Platonist* content with ‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’: a content whose satisfaction requires the existence of numbers. In the case of [NARROW], on the other hand, all it takes for the right-hand-side of the clause to be satisfied is for nothing to be such that w represents *it* as being a dinosaur. So [NARROW]—unlike [WIDE]—remains *neutral* on the question of whether w must represent reality as containing numbers.¹⁵

The crucial feature of [NARROW] is, of course, that all arithmetical vocabulary has been *outsourced*: it has been removed from the range of ‘ $[\dots]_w$ ’. So although one uses arithmetical vocabulary in the metalanguage to *characterize* a requirement on w , the requirement itself brings in no specifically arithmetical constraints: it is simply the requirement that nothing be counted by w as a dinosaur. The role of arithmetical vocabulary is to impose a metatheoretical *test* that will ensure that such a requirement is met: one asks, in the metalanguage, for the number of the objects that are counted as dinosaurs by w and demands that that number be zero. But because the test is performed while working *outside* the scope of ‘ $[\dots]_w$ ’, one doesn’t have to presuppose that the resources one uses to perform the test are present in w .

¹⁵Here I assume, for simplicity, that the domain of the metalanguage includes merely possible objects. Without this assumption—or, alternatively, without the assumption of necessitism (Williamson 2013)—[NARROW]’s right-hand-side will be satisfied by worlds which represent reality as containing dinosaurs but don’t represent *of any actually existing individual* that it is a dinosaur, and it therefore won’t succeed in associating with ‘ $\#_x(\text{Dinosaur}(x)) = 0$ ’ the condition that there be no dinosaurs. Happily, there is a technical trick that allows one get the right results without surrendering modal actualism (or contingentism); see (Rayo 2008), (Rayo 2012) and (Rayo 2013, Chapter 6) for details.

As it turns out, it is possible to give a compositional semantics that delivers suitably outscoped semantic clauses for *every sentence in the language of arithmetic* (see appendix). Not just that: the resulting clauses succeed in delivering the gold standard. They specify *trivialist* nominalistic contents, and do so with no need for controversial philosophical assumptions of the kind we discussed in section 3. (They do not, however, deliver a *paraphrase-function*, since our semantic clauses do not assign a non-mathematical sentence to each mathematical sentences. But, of course, the Impossibility Theorem suggests that that would be too much to hope for.)

Our semantics assigns every truth of pure arithmetic a trivial semantic clause (i.e. a clause whose right-hand-side will be satisfied by a world w regardless of how reality is represented by w), and it assigns every falsehood of pure arithmetic is assigned an impossible semantic clause (i.e. a clause whose right-hand-side will fail to be satisfied by w regardless of how reality is represented by w). The reason we get this result is that, when it comes to sentences of pure arithmetic, *everything* gets outscoped. The clause for ‘ $1 + 1 = 2$ ’, for example, will be:

‘ $1 + 1 = 2$ ’ is true at w if and only if $1 + 1 = 2$

in which *nothing* remains in the scope of ‘ $[\dots]_w$ ’. Since the right-hand-side of this biconditional is true (and contains no free variables), it will be satisfied by w regardless of how the world is represented by w . So our semantics will assign a trivial nominalistic content to ‘ $1 + 1 = 2$ ’.

4.2 What Outscoping Can and Cannot Do

Our trivialist semantics is couched in an arithmetical language. So use of the theory presupposes that one is able to understand arithmetical vocabulary. Not just that: in order to extract illuminating results from an outscoped semantic clause, one usually needs to prove an arithmetical claim in the metatheory. (In arguing that the semantic clause for ‘ $1 + 1 = 2$ ’

delivers a trivial content, for example, I made use of the fact that $1 + 1 = 2$.) So illuminating use of our semantic theory presupposes that one is able to prove arithmetical results. Either of these presuppositions would be utterly uncontroversial in a non-philosophical context. But it is worth considering how they play out in the present discussion.

It will be useful to start by seeing things from the point of view of a *mathematical Platonist*: someone who thinks that mathematical objects exist. Suppose, for example, that our Platonist is interested in the project of understanding which nominalistic contents a nominalist would wish to associate with arithmetical sentences. Since the Platonist feels comfortable using arithmetical vocabulary, she is in a position to set forth the trivialist semantics we have been discussing, and read off the nominalistic content of arithmetical sentences from the outscoped semantic clauses that are delivered by the semantics.

In doing so, our Platonist will have found a way around the Impossibility Theorem of section 3. For even if she lacks a general method for characterizing trivialist *paraphrases* for arithmetical sentences, our Platonist will have succeeded in finding a general method for characterizing trivialist *contents* for arithmetical sentences. It is true that she will have *used* arithmetical vocabulary in the process. But this is no threat to the project because the contents themselves will involve no specifically arithmetical constraints.

We started out assuming that the Platonist is interested in the project of characterizing nominalistic contents in order to better understand the nominalist, but she might also be interested in the project for a different purpose. Suppose she wishes to understand how arithmetical claims can be *relevant* to one's knowledge of the natural world (Steiner 1998, Yablo 2001). She might hypothesize that the answer is partly to do with the fact that an arithmetical claim like 'the number of the dinosaurs is zero' can impose non-trivial demands on the natural world, and see her outscoped semantic clauses as supplying a precise statement of the relevant demands.

Relatedly, our Platonist might wish to know whether the operation of *content-subtraction* delivers interesting results in the arithmetical case. Our Platonist will see the trivialist se-

mantic theory as decisive proof that the operation is well-defined, and delivers results of the right kind. For she will see the outscoped semantic clause corresponding to each arithmetical sentence as a precise statement of the result of subtracting ‘the mathematical part’ from the relevant arithmetical claim. (This is what I meant in section 3.3 when I reported thinking that Melia’s use of the subtraction-operation was, in fact, well-defined.)

We have been seeing things from the perspective of a Platonist. But what would a nominalist make of our trivialist semantics? It seems to me that the important issue is not whether one is a nominalist, but whether one is prepared to *engage in mathematical practice*. Suppose, for example, that our nominalist is also a fictionalist, and that she is happy to engage in mathematical practice: she proves mathematical theorems and uses mathematical vocabulary in making claims about the world; in her more philosophical moments, however, she insists that her mathematical assertions are set forth ‘in a spirit of make believe’ and that there really are no mathematical objects. A nominalist of this kind should have no difficulty working with the trivialist semantics that we have been discussing, and using outscoped semantic clauses to give a precise statement of the nominalistic content of her mathematical assertions.

A nominalist who *would* be barred from employing the trivialist semantics is what might be called a *nominalistic zealot*: someone who thinks that one cannot engage in normal mathematical practice. The zealot would not be prepared to assert ‘the number of the dinosaurs is zero’ in describing the world, even if she thought there were no dinosaurs; similarly, she would not be prepared to use a mathematically formulated semantic clause to characterize a nominalistic content. Our trivialist semantics is also unavailable to a mathematical *novice*: someone who is not competent in the use of mathematical vocabulary.

We set out to give a precise characterization of the nominalistic contents of arithmetical sentences. Had we been able to do so by way of an uncontroversially trivialist paraphrase-function, we might have been in position to satisfy both the zealot and the novice.¹⁶ But the

¹⁶Paraphrase-functions, like other functions, are mathematical objects. So whether or not a given paraphrase-

Impossibility Theorem shows that there are real limits to what can be done when it comes to giving paraphrases. Our trivialist semantics allows us to circumvent the theorem, but it is only available to someone who is prepared to engage in ordinary mathematical practice. It seems to me, however, that such an exclusion is not a particularly high price to pay, because neither the zealot nor the novice represent serious philosophical positions.

4.3 Beyond Arithmetic

We have seen that it is possible to give a compositional semantics that delivers suitably outscoped semantic clauses for every sentence in the language of (pure and applied) arithmetic. As it turns out, it is also possible to use the outscoping technique to characterize a trivialist semantics for the language of (pure and applied) *set-theory*. Full details are supplied in the appendix, but the basic idea is straightforward. In place of a standard homophonic semantic clause such as:

[WIDE]

‘Socrates \in $\{x : x \text{ is a philosopher}\}$ ’ is true at w if and only if [Socrates \in $\{z : z \text{ is a philosopher}\}$] $_w$

one uses an outscoped semantic clause such as:¹⁷

[NARROW]

‘Socrates \in $\{x : x \text{ is a philosopher}\}$ ’ is true at w if and only if Socrates \in $\{z : [z \text{ is a philosopher}]_w\}$

Although the outscoping technique happens to be available both in the case of arithmetic and in the case of set-theory, it is important to be clear that these results are not automatic:

function could actually be used to satisfy the zealot or the novice might depend on just how the function is presented to them. Consider, for example, the Fregean paraphrase-function of section 3.2. If such a function were to be described as a set of order-pairs, neither the zealot nor the novice would be moved. But one might get better results if one presents it a finite list of syntactic rules for transforming any given arithmetical sentence into the target second-order sentence.

¹⁷Here and below, I retain the simplifying assumption that the domain of the metalanguage includes merely possible objects.

there is no general reason to think that outscoping will be available whenever abstracta are used to describe features of the concrete world.

The best way to see this is to consider an example. Suppose that a mass of one kilogram is defined as the mass of N carbon-12 atoms (where ‘ N ’ is replaced by some particular numeral),¹⁸ and suppose that one wishes to specify a nominalist content for ‘Oscar’s mass-in-kilograms is 72’. One might suggest an outscoped semantic clause such as the following:

‘Oscar’s mass-in-kilograms is 72’ is true at w if and only if

$$\exists X((\#_x(Xx) = N \times 72) \wedge \forall x(Xx \rightarrow [^{12}\text{C-atom}(x)]_w) \wedge [\text{SameMassAs}(\text{Oscar}, X)]_x)$$

Although this clause has the right kind of flavor, it would presumably need to be refined in a number of ways. Notice, to begin with, that it presupposes that second-order quantification is nominalistically unproblematic, since a second-order variable occurs within the scope of ‘ $[\dots]_w$ ’. (It is possible to outscope the relevant variable, by making suitable mereological assumptions.¹⁹) Notice, further, that our clause presupposes that w contains enough carbon-12 atoms to establish an equal-mass comparison with Oscar. This won’t be a problem in this particular case, if Oscar is an ordinary-sized human and if w is a world roughly like our own. But it will be a problem if one wants to generalize the proposal to talk about, e.g. the mass of the entire universe. Perhaps one could amend the clause so as to allow for mass

¹⁸As of the time of this writing, a mass of one kilogram is officially defined as the mass of the International Prototype Kilogram, a particular artifact which is kept in a vault in the outskirts of Paris. An unhappy consequence of this definition is that every time the Prototype loses an atom, the mass of everything else in the world increases. The definition has nonetheless been kept in place because, until recently, we lacked the technology to produce more precise measurements using alternate definitions. It is likely that a new definition will be adopted soon, however. The definition I consider above is the simplest of the proposals under consideration.

¹⁹If one is prepared to countenance mereological sums, for example, then one can replace the right-hand-side of the original clause with the following:

$$\begin{aligned} &\exists X(\#_x(Xx) = N \times 72 \wedge \forall x(Xx \rightarrow [^{12}\text{C-atom}(x)]_w) \wedge \\ &\exists z([\text{SameMassAs}(\text{Oscar}, z)]_x \wedge \forall x(Xx \rightarrow [\text{PartOf}(x, z)]_w) \wedge \forall y([\text{Overlaps}(y, z)]_w \rightarrow \exists v(Xv \wedge [\text{Overlaps}(y, v)]_w)))) \end{aligned}$$

comparisons with different kinds of fundamental particles.²⁰ Even if that works, however, other problems might emerge. What should one do, for example, if one thinks that there are worlds in which the masses of fundamental particles differ from their actual masses?

The lesson of our example is the availability of outscoping is not automatic. Each new way of using abstracta to describe features of the concrete world calls for new type of outscoped semantic clause, and it is by no means obvious that suitable semantic clauses will always be available.²¹ On the other hand, the problem of finding suitable clauses is closely linked to the problem of better understanding the ways in which talk of abstracta conveys information about the way the world is. So limitations in our ability to outscope may sometimes reflect limitations in our understanding of the relevant subject-matter.

5 Logicism

When we discussed outscoping earlier in the paper, we were thinking of it as a means for specifying the nominalistic contents of arithmetical sentences rather than their literal truth-conditions. But there is room for arguing that our trivialist compositional semantics is, in fact, an accurate statement of *literal* truth-conditions. On such a view, all that is required of the world for ‘the number of the dinosaurs is zero’ to be literally true is that there be no dinosaurs, and *nothing* is required of the world for ‘ $1 + 1 = 2$ ’ to be literally true.

Should one conclude from this that arithmetical sentences don’t carry commitment to numbers? Not according to the version of the proposal I wish to consider here. I would like to consider a view whereby it is *both* the case that ‘the number of the dinosaurs is zero’ is

²⁰Here is a modified right-hand-side, where P_1, \dots, P_k is a list of all types of fundamental properties possessing mass, and, for each $i \leq k$, N_i particles of type P_i have a mass of one kilogram:

$$\exists X_1 \dots \exists X_k \left(\frac{(\#_x(X_1(x)))}{N_1} + \dots + \frac{(\#_x(X_k(x)))}{N_k} = 72 \wedge \right. \\ \left. \forall x \left(([\text{FundParticle}(x) \wedge \text{PartOf}(x, \text{Oscar})]_w) \leftrightarrow (X_1(x) \wedge [P_1(x)]_w) \vee \dots \vee (X_k(x) \wedge [P_k(x)]_w) \right) \right)$$

²¹For relevant discussion, see (Williams 2010).

committed to the number zero, *and* that all that is required of the world for ‘the number of the dinosaurs is zero’ to be literally true is that there be no dinosaurs.

The proposal escapes incoherence by endorsing the following claim:

For the number of the dinosaurs to be zero *just is* for there to be no dinosaurs.

and, more generally,

[NUMBERS]

For the number of the F s to be n *just is* for it to be the case that $\exists!_n x(Fx)$.

A friend of [NUMBERS] thinks that there is *no difference* between there being no dinosaurs and their number’s being zero, in the same sort of way that there is no difference between drinking a glass of water and drinking a glass of H_2O . More colorfully: when God created the world and made it the case that there was water to be drunk, there was nothing *extra* she needed to do or refrain from doing to make it the case that there was H_2O to be drunk. She was already done. Similarly, a friend of [NUMBERS] thinks that when God created the world and made it the case that there would be no dinosaurs in 2013, there was nothing *extra* she needed to do or refrain from doing to make it the case that the number of the dinosaurs in 2013 would be zero. She was already done.

An immediate consequence of [NUMBERS] is that a world without numbers would be *inconsistent*:

Proof: Assume, for *reductio*, that there are no numbers. By [NUMBERS], for the number of numbers to be zero *just is* for there to be no numbers. So the number of numbers is zero. So zero exists. So a number exists. Contradiction.

One might therefore think of [NUMBERS] as delivering a *trivialist* form of mathematical Platonism—the number zero exists, but its existence is a trivial affair. And, of course, it is not just the existence of the number zero that is a trivial affair: one can use [NUMBERS] to

show that each of the natural numbers must exist, on pain of contradiction, and to show that they are distinct from one another. (That is what I had in mind in section 3.2 when I reported thinking that Trivialist Infinitarianism is true.)

Someone who accepts [NUMBERS] can claim *both* that the truth-conditions of ‘the number of the dinosaurs is zero’ consist entirely of the requirement that the number of the dinosaurs be zero, *and* that they consist entirely of the requirement that there be no dinosaurs. She can make both these claims because she thinks that the proposed requirements are one and the same: there is *no difference* between there being no dinosaurs and their number’s being zero.

The trivialist semantic theory we set forth in the preceding section can be used to generalize this idea to every sentence in the language of arithmetic. One can claim that the literal truth-conditions of an arithmetical sentence are accurately stated *both* by a standard (homophonic) compositional semantics *and* by our trivialist semantics with outscoped semantic clauses. But the two semantic theories do not contradict one another because the truth-conditions they associate with a given sentence are, in fact, one and the same: there is *no difference* between what would be required of the world to satisfy the truth-conditions delivered by one semantic theory and what would be required of the world to satisfy the truth-conditions delivered by the other.²²

Consider ‘ $1 + 1 = 2$ ’ as an example. A standard (homophonic) semantics tells us that the truth-conditions of ‘ $1 + 1 = 2$ ’ demand of the world that it contain numbers. Our trivialist semantics tells us that the truth-conditions are trivial—that they will be satisfied regardless of how the world happens to be. But the two claims are consistent with each other because the existence of numbers is a trivial affair. ‘ $1 + 1 = 2$ ’ carries commitment to numbers, but this is a commitment that will be satisfied regardless of how the world happens to be.

What does this tell us about *logicism*—the view that mathematics can be reduced to logic? The Impossibility Theorem of section 3 suggests that the formal systems that contemporary

²²For a detailed defense of this view see (Rayo 2013, Chapters 1 and 2).

philosophers tend to think of as ‘pure logic’ are not expressive enough to capture basic arithmetic. So one might think of the theorem as a refutation of logicism: in an interesting sense, mathematics cannot be reduced to logic. But one could also think of the view developed in the present section as a certain kind of *vindication* of logicism. For it delivers the result that the truths of pure arithmetic—like the truths of pure logic—have trivially satisfiable truth-conditions, and the result that the falsehoods of pure arithmetic—like the falsehoods of pure logic—have impossible truth conditions. Admittedly, one also gets the result that a truth of pure arithmetic can carry commitment to numbers. But because the existence of numbers is a trivial affair, there is room for thinking of numbers as ‘logical objects’, as in Frege’s *Grundgesetze* .

6 Concluding Remarks

In this paper I have tried to shed new light on mathematical nominalism.

I began with the observation that the nominalist is committed to answering a particular challenge. She must explain what the *point* of making a mathematical assertion could be, if there are no numbers. One way of addressing this challenge is to argue that the point of mathematical assertions is not to communicate the literal content of the sentence asserted, but to communicate its *nominalistic content*: the requirement that the world would need to satisfy in order to make the sentence true ‘as far as non-mathematical facts are concerned’.

It is natural to suppose that one can specify the relevant nominalist contents by setting forth a nominalistic *paraphrase function*: an effectively specifiable procedure that assigns to each mathematical sentence a non-mathematical paraphrase in such a way that the nominalistic content of the mathematical sentence matches the literal content of its paraphrase. We have seen, however, that there is a formal result that suggests that it is *impossible* to specify a suitable paraphrase-function for the language of arithmetic, in the absence of potentially controversial assumptions.

One might have been tempted to think of the Impossibility Theorem as a decisive blow to the nominalistic dream of specifying nominalistic contents for arbitrary mathematical sentences. But we have seen that nominalistic paraphrase-functions are not the only way of specifying nominalistic contents. The method of outscoping makes it possible to construct a *compositional semantics* that assigns the right nominalistic contents to arbitrary arithmetical (and set-theoretic) sentences.

This result sheds light on nominalism in two different ways. First, it allows us to discard the idea that the case for nominalism ought to be linked to the availability of a nominalistic paraphrase function—a bad idea from the start, since it tied the *metaphysical* thesis that there are no numbers to potentially controversial *linguistic* theses concerning the legitimacy of particular expressive resources.

Second, and more importantly, our outscoped semantics shows that the notion of nominalistic content can be rigorously defined, and is therefore suited for serious philosophical work. We noted, in particular, that it can be used to address the question of how mathematical claims can be relevant to one's knowledge of the natural world. But we also noted that it can be used to reassess nominalism, by allowing one to give a rigorous characterization of a subtle variety of Platonism: a view according to which there is *no difference* between what would be required of the world to satisfy the nominalistic content of a given arithmetical sentence and what would be required of the world to satisfy the truth-conditions that would be assigned to that sentence by a homophonic semantic theory.

From a purely mathematical point of view, there is no particular reason to prefer Subtle Platonism over its rivals. But Subtle Platonism is philosophically significant because it casts doubt on Benacerraf's Dilemma: the idea that one must choose between holding onto the claim that mathematical assertions carry commitment to mathematical objects, and making contentious claims about our cognitive relationship to a causally inert realm of abstract objects (Benacerraf 1973). The Dilemma is sometimes construed as an argument for nominalism, since it seems to suggest that only the nominalist could have a sensible epistemology

of mathematics. But when Subtle Platonism is treated as a live option, we can no longer take for granted that commitment to numbers comes with epistemological costs. (The Subtle Platonist would argue, for example, that someone who has verified that there are no dinosaurs is thereby in a position to know that the number of dinosaurs is zero, since the fact that there are no dinosaurs is *already* the fact that the number of dinosaurs is zero.²³) If this is right, then the notion of a nominalistic content—which we first introduced in an effort to help nominalists answer a challenge—can also be used to cause trouble for nominalism, by allowing for rigorous development of a rival view.²⁴

²³For further discussion, see (Rayo 2013, Chapters 3 and 4).

²⁴For their many helpful comments, I am grateful to Duilio Guerrero, Bernhard Salow and Steve Yablo, to participants at MIT's Logic, Language and Metaphysics Reading Group, and to audiences at the University of Missouri, Kansas City, the Università Vita-Salute San Raffaele and Smith College.

Appendix

The material in this appendix is drawn from (Rayo 2013, Chapter 3), where I discuss further technical details. (As noted in footnotes 15 and 17, I assume, for simplicity, that the domain of the metalanguage includes merely possible objects; but the assumption can be avoided by appeal to the technique described in (Rayo 2013, Chapter 6). In the case of arithmetic, the details are spelled out in (Rayo 2008).)

1. A Trivialist Semantics for the Language of Arithmetic

We work with a two-sorted first-order language with identity, L . Besides the identity-symbol ‘=’, L contains *arithmetical* variables (‘ n_1 ’, ‘ n_2 ’, ...), individual-constants (‘0’) and function-letters (‘S’, ‘+’ and ‘ \times ’), and *non-arithmetical* variables (‘ x_1 ’, ‘ x_2 ’, ...), constants (‘Caesar’) and predicate-letters (‘Dinosaur(...)'). In addition, L has been enriched with the function-letter ‘ $\#_v(\dots)$ ’ which takes a first-order predicate in its single argument-place to form a first-order arithmetical term (as in ‘ $\#_{x_1}(\text{Dinosaur}(x_1))$ ’), which is read ‘the number of the dinosaurs’).

Let σ be a variable assignment and w be a world. $\delta_{\sigma,w}(t)$ will be our denotation function, which assigns a referent to term t relative to σ and w ; $Sat(\phi, \sigma, w)$ will be our satisfaction predicate, which expresses the satisfaction of ϕ relative to σ and w ; and $[\phi]_w$ will be our true-at-a-world operator, which expresses the thought that ϕ is true at w . Denotation and satisfaction are defined simultaneously, by way of the following clauses:

Denotation of arithmetical terms:

1. $\delta_{\sigma,w}(\ulcorner n_i \urcorner) = \sigma(\ulcorner n_i \urcorner)$
2. $\delta_{\sigma,w}(\ulcorner 0 \urcorner) = \text{the number Zero}$
3. $\delta_{\sigma,w}(\ulcorner S(t) \urcorner) = \delta_{\sigma,w}(t) + 1$

4. $\delta_{\sigma,w}(\ulcorner t_1 + t_2 \urcorner) = \delta_{\sigma,w}(t_1) + \delta_{\sigma,w}(t_2)$
5. $\delta_{\sigma,w}(\ulcorner t_1 \times t_2 \urcorner) = \delta_{\sigma,w}(t_1) \times \delta_{\sigma,w}(t_2)$
6. $\delta_{\sigma,w}(\ulcorner \#_{x_i}(\phi(x_i)) \urcorner) =$ the number of zs such that $Sat(\ulcorner \phi(x_i) \urcorner, \sigma^{z/\ulcorner x_i \urcorner}, w)$
7. $\delta_{\sigma,w}(\ulcorner \#_{n_i}(\phi(n_i)) \urcorner) =$ the number of ms such that $Sat(\ulcorner \phi(n_i) \urcorner, \sigma^{m/\ulcorner n_i \urcorner}, w)$

Denotation of non-arithmetical terms:

1. $\delta_{\sigma,w}(\ulcorner x_i \urcorner) = \sigma(\ulcorner x_i \urcorner)$
2. $\delta_{\sigma,w}(\ulcorner \text{Caesar} \urcorner) = \text{Gaius Julius Caesar}$

Satisfaction:

1. $Sat(\ulcorner \exists n_i \phi \urcorner, \sigma, w) \leftrightarrow$ there is a number m such that $Sat(\phi, \sigma^{m/\ulcorner n_i \urcorner}, w)$
2. $Sat(\ulcorner \exists x_i \phi \urcorner, \sigma, w) \leftrightarrow$ there is a z such that $([\exists y(y = z)]_w \wedge Sat(\phi, \sigma^{z/\ulcorner x_i \urcorner}, w))$
3. $Sat(\ulcorner t_1 = t_2 \urcorner, \sigma, w) \leftrightarrow \delta_{\sigma,w}(t_1) = \delta_{\sigma,w}(t_2)$
4. $Sat(\ulcorner \text{Dinosaur}(t) \urcorner, \sigma, w) \leftrightarrow [\delta_{\sigma,w}(t) \text{ is a dinosaur}]_w$ (for t a non-arithmetical term)
5. $Sat(\ulcorner \phi \wedge \psi \urcorner, \sigma, w) \leftrightarrow Sat(\phi, \sigma, w) \wedge Sat(\psi, \sigma, w)$
6. $Sat(\ulcorner \neg \phi \urcorner, \sigma, w) \leftrightarrow \neg Sat(\phi, \sigma, w)$

2. A Trivialist Semantics for the Language of Set-Theory

We work with a two-sorted first-order language with identity, L . Besides the identity-symbol ‘=’, L contains the membership predicate ‘ \in ’, set-theoretic variables ($\alpha_1, \alpha_2, \dots$), urelement variables (x_1, x_2, \dots), and urelement predicate-letters (‘Philosopher(...)’).

As before, we let σ be a variable assignment and w be a world. $Sat(\phi, \sigma, w)$ will be our satisfaction predicate, which expresses the satisfaction of ϕ relative to σ and w ; and $[\phi]_w$ will be our true-at-a-world operator, which expresses the thought that ϕ is true at w . Satisfaction is defined as follows:

Satisfaction:

1. $Sat(\ulcorner \exists x_i \phi \urcorner, \sigma, w) \leftrightarrow$ there is a z such that $([\exists y(y = z)]_w \wedge Sat(\phi, \sigma^{z/\ulcorner x_i \urcorner}, w))$
2. $Sat(\ulcorner \exists \alpha_i \phi \urcorner, \sigma, w) \leftrightarrow$ there is a set β such that: (i) for any urelement z in the transitive closure of β , $[\exists y(y = z)]_w$, and (ii) $Sat(\phi, \sigma^{\beta/\ulcorner \alpha_i \urcorner}, w)$
3. $Sat(\ulcorner x = y \urcorner, \sigma, w) \leftrightarrow \sigma(x) = \sigma(y)$
4. $Sat(\ulcorner \alpha \in \beta \urcorner, \sigma, w) \leftrightarrow \sigma(\alpha) \in \sigma(\beta)$
5. $Sat(\ulcorner x \in \beta \urcorner, \sigma, w) \leftrightarrow \sigma(x) \in \sigma(\beta)$
6. $Sat(\ulcorner \text{Philosopher}(x) \urcorner, \sigma, w) \leftrightarrow [\sigma(x) \text{ is a philosopher}]_w$
7. $Sat(\ulcorner \phi \wedge \psi \urcorner, \sigma, w) \leftrightarrow Sat(\phi, \sigma, w) \wedge Sat(\psi, \sigma, w)$
8. $Sat(\ulcorner \neg \phi \urcorner, \sigma, w) \leftrightarrow \neg Sat(\phi, \sigma, w)$

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