Plurals*

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English contains singular terms, quantifiers and predicates (e.g. 'it', 'something' and '... is an elephant'). But it also contains *plural* terms, quantifiers and predicates (e.g. 'they', 'some things' and '... are scattered on the floor').[1] Philosophers have become increasingly interested in plurals over the past couple of decades. The purpose of this paper is to explain why plurals might be thought to have philosophical importance, and why they have led to philosophical debate.

1 Introduction

There is a useful way of supplying quasi-English readings for the expressions of a standard first-order language. The first-order sentence '$\exists x$ ELEPHANT(x)', for example, is read 'something is such that it is an elephant'. More generally, a variable $\ulcorner x_i \urcorner$ is read $\ulcorner \text{it}_i \urcorner$ (with the subindex used to distinguish occurrences of 'it' corresponding to different variables); the existential quantifier $\ulcorner \exists x_i \urcorner$ is read $\ulcorner$something$_i$ is such that$\urcorner$ (with the subindex used to make clear which occurrences of 'it' are being bound); atomic predicates are read in accordance with their intended interpretations (e.g., 'ELEPHANT(...)' is read '... is an elephant'); and the connectives '$\wedge$' and '$\neg$' are read 'and' and 'it is not the case that', respectively. (The universal quantifier $\ulcorner \forall x_i \urcorner$ may be thought of as a syntactic abbreviation for $\ulcorner \neg \exists x_i \neg \urcorner$, and the remaining logical connectives may be defined in terms of '$\wedge$' and '$\neg$', in the usual way).

Boolos (1984) noted that an exactly analogous procedure can be used to supply quasi-English readings for the expressions of what might be called a *plural first-order* language. The plural first-order sentence '$\exists xx$ SCATTERED(xx)', for example, might be read $\ulcorner$some things are such that they are scattered on the floor$\urcorner$.[2] More generally, a variable $\ulcorner xx_i \urcorner$ is read $\ulcorner$they$_i \urcorner$ or $\ulcorner$them$_i \urcorner$ (with the subindex used to distinguish between occurrences of 'they' or 'them' corresponding to different plural variables); the plural existential quantifier $\ulcorner \exists xx_i \urcorner$ is read 'some things$_i$ are such that' (with the subindex used to to make clear which occurrences of 'they' or 'them' are being bound); and atomic plural predicates are read

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in accordance with their intended interpretations (e.g., 'SCATTERED(...)' is read '... are scattered on the floor', and 'ELEPHANTS(...)' is read '... are such that each of them is an elephant'). (As before, the plural universal quantifier $\ulcorner\forall xx_i\urcorner$ may be thought of as a syntactic abbreviation for $\ulcorner\neg\exists xx_i\neg\urcorner$.)

The focus of this paper will be on three claims that are often made in connection with plural first-order languages:

1. PLURALISM

   PLURALISM is best characterized by way of a dual contrast with SINGULARISM:

   (a) A plural predicate is a predicate taking a plural variable in one or more of its argument places. According to SINGULARISM, a plural predicate '$\mathrm{F}(xx)$' is satisfied or unsatisfied by a single individual. According to PLURALISM, '$\mathrm{F}(xx)$' can be *jointly* satisfied or unsatisfied by several individuals at once.

   (b) According to SINGULARISM, the truth of '$\exists xx\ \mathrm{F}(xx)$' requires that there be a 'plural entity' (a set, for instance) that satisfies '$\mathrm{F}(xx)$' and has individuals in the range of one's singular quantifiers as 'constituents'. According to PLURALISM, all that is required for the truth of '$\exists xx\ \mathrm{F}(xx)$' is that there be individuals in the range of one's singular quantifiers that jointly satisfy '$\mathrm{F}(xx)$'. As a result, pluralists believe that the use of plural quantifiers carries no commitment to entities outside the range of one's singular quantifiers.

2. LOGICALITY

   LOGICALITY is the claim that certain principles governing plural first-order languages are logically valid; specifically, introduction and elimination rules for the plural quantifiers,[3] and the following axiom schema (where the plural predicate '$\prec$' is read 'is one of'):

   PLURAL COMPREHENSION

   $$\exists x(\phi(x)) \rightarrow \exists xx \forall x(x \prec xx \leftrightarrow \phi(x))$$

   (Read: "assuming there is at least one ϕ, some things are such that something is one of them just in case it is an ϕ".)

3. FULLNESS

   Full and non-full interpretations of the plural quantifiers agree that every instance of PLURAL COMPREHENSION is true, but disagree about whether it could have false instances if the language was expanded with new atomic predicates. On a non-full interpretation, there are possible expansions of the language whereby PLURAL COMPREHENSION would have false instances. On a full interpretation, PLURAL COMPREHENSION could have no false instances no matter how the language was expanded.

> FULLNESS is the claim that our linguistic practice rules out non-full interpretations of the plural quantifiers.

There is an additional claim that does not immediately concern plurals but will turn out to be useful because of its intimate connections with PLURALISM, LOGICALITY and FULLNESS:

> ABSOLUTE GENERALITY
>
> One can get one's domain of discourse to consist of absolutely everything there is.[4]

The paper proceeds as follows. Section 2 explains why PLURALISM, LOGICALITY and FULLNESS have been thought to be important; section 3 gauges the extent to which the claims have been justified; section 4 offers some speculative remarks about the differences between SINGULARISM and PLURALISM.[5]

2 Why Bother?

In this section I will list some applications of plural first-order languages.

2.1 Metatheory

Tarski (1929) taught us how to give a characterization of truth for a large class of formal languages. Insofar as a Tarskian characterization of truth results in a specification of truth-conditions for every sentence in the language, it may be thought of as supplying an *interpretation* for the language. But a Tarskian characterization of truth gives us no way of quantifying over the possible interpretations a language might take.

One reason it would be desirable to be able to quantify over interpretations is that it would put us in a position to give a characterization of logical validity. (Specifically, one can say that a sentence is logically valid just in case it is true on every interpretation of the language that assigns logical terms their intended meanings.) But characterizing logical consequence is only the beginning. Metalogical investigation has yielded a wealth of results that make essential use of quantification over interpretations. (The Löwenheim-Skolem-Tarski Theorem, for example, entails that a (singular) first-order theory that is true on some interpretation with an infinite domain is true on interpretations with domains of arbitrary infinite cardinality.) In addition, contemporary approaches to semantics rely on quantification over interpretations of sentences and sub-sentential components (as when one says that the semantic value of a branching node is a function of the semantic values of its daughter nodes—a representative textbook is Heim and Kratzer (1998).)

The standard way of achieving the effect of quantification over interpretations is by quantifying over *models*.[6] In the form that is now standard, a model m may be thought of as a set of ordered pairs containing at least one pair of the form $\langle \text{`}\forall\text{'}, x\rangle$. An object z

can then be said to belong to the domain of m just in case $\langle$'$\forall$', $z\rangle \in m$, and an atomic predicate P can be said to apply to z according to m just in case $\langle \mathrm{P}, z\rangle \in m$. Truth in m is characterized recursively, as in Tarski's original characterization of truth.

With the addition of suitable bells and whistles, the standard notion of a model has proved extraordinarily fruitful as a stand-in for the notion of an interpretation in applications of the kind mentioned above. But—on the assumption that ABSOLUTE GENERALITY is true—it suffers from an important limitation. For ABSOLUTE GENERALITY entails that there are domains of discourse encompassing too many objects to form a set, and the standard notion of a model can only be used to capture interpretations with set-sized domains.

Kreisel (1967) famously showed that any set of (singular) first-order formulas that is true on some interpretation is true on some interpretation with a set-sized domain. So when it comes to the particular project of characterizing logical validity for singular first-order languages, the limitation to set-sized domains turns out not to get in the way. But Kreisel's result relies on special features of singular first-order languages, and breaks down when additional expressive resources are wheeled in.[7] Moreover, the standard notion of a model is unsuitable for stating certain metalogical results.[8] It is also unsuitable for supplying semantic theories in the style of Heim and Kratzer (1998) for languages whose domain of discourse encompasses too many objects to form a set.

By taking models to be *classes* rather than sets one would only postpone the problem. For although class-sized models can be used to capture interpretations in which the domain of discourse encompasses too many objects to form a set, they cannot be used to capture interpretations in which the domain of discourse contains classes. And—assuming ABSOLUTE GENERALITY—it is easy to show that a version of this difficulty generalizes to any conception of model whereby a model is an individual.[9]

When plural first-order languages are brought into the picture, however, the situation is quite different. The crucial observation is that rather than using a *set* of ordered pairs to capture an interpretation, one can use the ordered pairs themselves.[10] More precisely, one can say of some ordered pairs—the *mm*—that they form a *plural model* just in case at least one of them is of the form $\langle$'$\forall$', $x\rangle$. (As before, an object z can be said to belong to the domain of the *mm* just in case $\langle$'$\forall$', $z\rangle$ is one of the *mm*, and an atomic predicate P can be said to apply to z according to the *mm* just in case $\langle \mathrm{P}, z\rangle$ is one of the *mm*.) So one can get the effect of quantifying over interpretations by quantifying plurally over ordered-pairs.

If PLURALISM is true, then the individuals forming a plural model can consist of more ordered pairs than can form a set (or other set-like object). So they can capture an interpretation whose domain consists of too many individuals to form a set (or other set-like object). If, in addition, FULLNESS is true, then, for any interpretation a first-order language might take in which the domain consists entirely of existing objects, there are some things that form a plural model capturing that interpretation.

2.2 Categoricity

There are arithmetical truths that cannot be expressed in the language of first-order arithmetic. One cannot, for example, state the least number principle: that amongst any numbers there is a smallest one. A manifestation of this expressive deficiency is that the theory of arithmetic fails to be categorical. In other words, there are models that are not isomorphic to the standard model but verify every true sentence in the language of first-order arithmetic.

The least number principle can, however, be stated in a plural first-order language:

$$\forall xx \exists y(y \prec xx \wedge \forall z(z \prec xx \rightarrow y \leq z)).$$

And assuming Fullness is true, the result of adding the least number principle to the standard axioms of first-order arithmetic is categorical. In other words, any model verifying the axioms in which the plural quantifiers and '≺' receive their intended interpretations is isomorphic to the standard model.[11] (There are, however, arithmetical truths that are not deductively implied by the axioms, since the set of sentences deductively implied by the axioms is effectively enumerable, and Göde's Incompleteness Theorem entails that set of arithmetical truths is not effectively enumerable.[12])

The case of set theory is a bit more nuanced. As in the case of arithmetic, there are set-theoretic truths that cannot be expressed in the language of first-order set-theory. Crucially:

> Replacement
> Any objects that can be put in one-one correspondence with some of the members of a set themselves form a set.

Assuming Pluralism is true, this claim can be expressed in a plural first-order language. But, unlike the case of arithmetic, one doesn't get a categorical axiomatization of set-theory. In fact, it follows from a Global Reflection Principle[13] that there can be no such axiomatization: if Global Reflection is true, then given any recursive set of true sentences of the language of plural first-order set-theory, there is a (plural) model that verifies every sentence in the set but is not isomorphic to the intended (plural) model of set-theory.[14]

One can, however, get the next best thing. If Fullness is true, then the result of adding a plural version of Replacement to the standard axioms of first-order set-theory is *quasi-categorical*. In other words, any two (plural) models which (a) verify the axioms and (b) assign the plural quantifiers and '≺' their intended interpretations are such that the pure sets of one of them are isomorphic to an *initial segment* of the pure sets of the other.[15] If Absolute Generality is assumed to be true and if the axioms are further enriched with the claim that the non-sets form a set, then one gets the additional result that any two (plural) models which (a) verify the axioms, (b) assign the plural quantifiers and '≺' their intended interpretations and (c) have domains of discourse consisting of absolutely everything are such that the pure sets of the one are isomorphic to the pure sets of the other.[16]

2.3 Foundations of Set Theory

On the most natural way of thinking about sets, Absolute Generality holds, and any objects form a set. It is a sad consequence of Russell's Paradox that this naïve conception of sets is untenable. So one needs a different way of motivating the axioms of set theory.

One idea is to use an *iterative* conception of sets whereby sets are generated in stages and each stage introduces, for any objects occurring at previous stages, a set formed from those objects. As far as we know, the iterative conception is consistent. Moreover, it can be used to motivate most of the standard axioms of set theory. Unfortunately, it is not clear that it can be used to motivate Replacement (or the Axiom of Choice),[17] and standard set theory would be severely weakened in the absence of Replacement.[18]

An alternative conception of sets is based on the idea that the sets are so plentiful that anything that can be truly said about the sets would remain true if the quantifiers were restricted to the elements of some set. So, in particular, every instance of the following schema must hold:

Reflection

$\phi \rightarrow \exists t\ \phi^t$

(where ϕ^t is the result of restricting the quantifiers in ϕ to t)

And here plurals come into the picture. For if one works within a plural first-order language and Logicality is holds, then all of the standard axioms of set theory (except for Choice) can be derived from Reflection in the presence of suitable definitions.[19]

(It is also worth noting that without plural-talk—and without the tacit assumption that Pluralism, Fullness and Absolute Generality are true—it is hard to express the conceptions of sets we have considered in this section.)

2.4 Metaphysics

There are various metaphysical views with respect to which plural first-order languages might be put to use. Here are some representative examples:

1. Mereological Nihilism

   Mereological Nihilism is the view that only mereological atoms exist.[20] On the assumption that chairs and octopuses have proper parts if they exist, it is a consequence of Mereological Nihilism that there are no chairs or octopuses. A Mereological Nihilist may nonetheless wish to claim that that it would be *correct* to assert 'there is an octopus' (in some suitable sense of correctness) just in case there are some mereological atoms octopusly arranged.

   If Pluralism is true, the correctness-conditions of arbitrary sentences in a (singular) first-order language can be explicitly stated in a plural first-order language. (Specifically, one can take the correctness-conditions of a singular first-order sentence ϕ to be expressed by ϕ^*, where $\ulcorner \exists x_i \psi \urcorner^* = \ulcorner \exists xx_i(\psi^*) \urcorner$, $\ulcorner \psi \wedge \xi \urcorner^* = \ulcorner \psi^* \wedge \xi^* \urcorner$,

$\ulcorner \neg\psi \urcorner^* = \ulcorner \neg\psi^* \urcorner$ and $\ulcorner P(x_i) \urcorner^* = \ulcorner \hat{P}(xx_i) \urcorner$, where $\hat{P}$ is a suitable plural counterpart for P.) A limitation of this strategy is that it cannot be used to state the correctness-conditions of English sentences containing certain kinds of plural constructions (e.g. 'some critics admire only one another').[21]

2. Nominalism

   Nominalism is the view that there are no abstract objects (and, in particular, no numbers). A nominalist may nonetheless wish to claim that it would be *correct* to assert 'the number of the cats is identical to the number of the dogs' (in some suitable sense of correctness) just in case there are as many cats as there are dogs.

   If Pluralism is true, there is a way of stating correctness-conditions for arbitrary sentences in the language of first-order applied arithmetic in a plural first-order language containing the plural predicates $\ulcorner xx_i \approx xx_j \urcorner$ (read $\ulcorner$they$_i$ are just as many as them$_j\urcorner$), $\ulcorner \text{Finite}(xx_i) \urcorner$ (read $\ulcorner$they$_i$ are finitely many in number$\urcorner$) and $\ulcorner xx_i \prec xx_j \urcorner$ (read as before). (Specifically, one can take the correctness-conditions of a singular first-order sentence ϕ to be expressed by ϕ^*, where $\ulcorner \exists n_i \psi \urcorner^* = \ulcorner \exists zz_i(\text{Finite}(zz_i) \wedge \psi^*) \urcorner$, $\ulcorner n_i = n_j \urcorner^* = \ulcorner zz_i \approx zz_j \urcorner$, $\ulcorner \text{Num}(n_i, F) \urcorner^* = \ulcorner \exists yy(zz_i \approx yy \wedge \forall w(w \prec yy \leftrightarrow F(w))) \urcorner$, $\ulcorner \psi \wedge \xi \urcorner^* = \ulcorner \psi^* \wedge \xi^* \urcorner$, $\ulcorner \neg\psi \urcorner^* = \ulcorner \neg\psi^* \urcorner$, $\ulcorner \exists x_i \psi \urcorner^* = \ulcorner \exists x_i(\psi^*) \urcorner$ and $\ulcorner P_j(x_i) \urcorner^* = \ulcorner P_j(x_i) \urcorner$ for $\ulcorner P_j \urcorner$ a non-arithmetical predicate.[22]) A limitation of this strategy is that it does not appear to generalize beyond the special case arithmetic. In addition, it only delivers the standard truth-values on the assumption that the universe is infinite.

3. Truth-makers

   Some philosophers believe that there are *truth-makers*: entities in virtue of which truths are true.[23] It is sometimes part of the view that a true conjunction is true not in virtue of a 'conjunctive' truth-maker, but in virtue of truth-makers corresponding to the conjunction's conjuncts. If Pluralism is true, this position can be stated in a plural first-order language.

2.5 Second-order languages

Boolos (1984) famously observed that the sentences of a monadic second-order languages can be paraphrased as sentences of a plural first-order language containing the plural predicate '$\prec$' (read as above). The second-order formula $\ulcorner X_i(x_j) \urcorner$ can be paraphrased as the plural formula $\ulcorner x_j \prec xx_i \urcorner$, and, to a first approximation, the monadic second-order quantifier $\ulcorner \exists X_i \urcorner$ can be paraphrased as the plural quantifier $\ulcorner \exists xx_i \urcorner$. (The reason this is only an approximation is that second-order quantifiers take 'empty' values, but plural quantifiers do not; Boolos's suggestion is therefore to paraphrase $\ulcorner \exists X_i(\phi) \urcorner$ as $\ulcorner \exists xx_i(\phi) \vee \phi^* \urcorner$, where ϕ^* is the result of substituting $\ulcorner x_j \neq x_j \urcorner$ everywhere for $\ulcorner X_i(x_j) \urcorner$ in ϕ.)

If Pluralism is true, Boolos's paraphrase yields the result that the use of monadic second-order quantifiers carries no commitment to entities beyond the range of one's first-order quantifiers. If Fullness is true, Boolos's paraphrase yields the result that non-full

Henkin models for monadic second-order languages should be ruled out as unintended. If Logicality is true, the result of applying Boolos's paraphrase to any theorem of monadic second-order logic is a logical validity.

In some respects, it is a mistake to think of second-order quantifiers as plural quantifiers. (Conspicuously, there is no plural analogue of dyadic second-order quantification, and second-order variables, unlike plural variables, take predicate positions.[24]) But Boolos's work was of enormous importance, for the following reason. At the time of Boolos's writing, it was commonly assumed that insofar as second-order quantifiers are intelligible at all, they should be understood as (singular) first-order quantifiers ranging over set-like entities.[25] Boolos challenged this assumption by setting forth his paraphrase and arguing for Pluralism, and this led to two crucial insights. Firstly, it showed that there is room for the view that there is quantification other than (singular) first-order quantification. Secondly, it put the mighty expressive power of plural first-order languages on display.

3 Justification

The purpose of this section is to gauge the extent to which Pluralism, Fullness and Logicality have been justified.

3.1 Pluralism

I know of only two kinds of arguments for Pluralism that have any hope of dialectical effectiveness in an exchange with a singularist. They are the following:

1. *Arguments from* Absolute Generality

   Arguments from Absolute Generality proceed in two stages. The first is to argue that a certain plural sentence is true when one's domain of discourse consists of absolutely everything; the second is to show that the sentence would turn out to be false on a singularist reading. Boolos (1984) set forth the first such argument, and a number of refinements have been since proposed.[26]

   Boolos's version of the argument proceeds by suggesting that anyone who thinks that there is at least one nonselfmembered set will take the following plural sentence to be true when one's domain of discourse consists of everything there is:

   > Plural Russell
   > There are some things such that something is one of them just in case it is a nonselfmembered set.

   But a singularist who thinks that plural quantifiers ought to be understood as first-order quantifiers ranging over *sets* is committed to paraphrasing Plural Russell as:

Singular Russell
There is a set such that its members are all and only the nonselfmembered sets.

where the quantifiers range over absolutely everything. And Singular Russell leads to paradox, contradicting the claim that an adequate paraphrase for Plural Russell has been supplied.

A dialectical weakness of such arguments is that singularists often have reservations about Absolute Generality. According to one prominent line of thought, our conception of *set* is not fully determinate. It is refined as our set-theoretic practice develops, with successive refinements entailing the existence of more and more sets. And, crucially, the process is open-ended: any determinate conception of *set* could be used to generate a more inclusive one.[27] A singularist sympathetic to this line of thought could block Boolos's argument by claiming that it is a mistake to think that the quantifiers in Plural Russell can be taken to range over absolutely everything with no further ado: Plural Russell can only express a determinate content relative to a given conception of *set*. And so relativized it can be properly paraphrased as a non-paradoxical version of Singular Russell in which the range of the innermost quantifier only includes 'sets' entailed by the given conception of *set* but the range of the outermost quantifier includes 'sets' entailed by a suitable development of the given conception.

2. *The argument from truth-conditional specification*

The task of convincing a singularist to embrace Pluralism is beset by a special kind of difficulty. For part of what the pluralist would like to do is get the singularist to understand what the truth-conditions of plural sentences are supposed to be. But—when conjoined with Fullness and Absolute Generality—pluralism entails that certain plural fist-order languages are more expressive than any singular first-order language.[28] So a pluralist will typically think that it is *impossible* to find paraphrases for arbitrary plural sentences using only linguistic resources that the singularist is guaranteed to regard as unproblematic.

For the special case in which one's domain of discourse forms a set, however, there is a devious way of specifying truth-conditions for plural sentences. Given an arbitrary sentence ϕ of a plural first-order language, one can identify a formula $\psi(w)$ (where w is a free variable ranging over representations) which uses only singular quantifiers and is such that what the pluralist thinks is demanded of the world to make ϕ true is precisely what is demanded of a representation to satisfy $\psi(w)$. One can say, for example, that what the truth of '$\exists xx(\text{Susan} \prec xx)$' demands of the world is precisely what satisfaction of the following open formula demands of w:

> There is a nonempty set such that: (a) every member of the set is an object x such that, according to w, x exists, and (b) Susan is a member of the set.

In other words, what the truth of '$\exists xx(\text{Susan} \prec xx)$' demands of the world is that Susan exist.[29]

The fact that this technique works only in cases where one has a set-sized domain means that it it cannot be used without supplementation to justify some of the applications of plural languages described in sections 2.1 and 2.2. On the other hand, the limitation to set-sized domains is unlikely to trouble singularists who reject Absolute Generality.

3.2 Fullness

I know of no argument for Fullness that has any hope of convincing a skeptic.[30] There is, however, an argument that assumes a full interpretation of the plural quantifiers in the metalanguage and aims to explain how it is that the practice of speakers succeeds in ruling out non-full interpretations of the plural quantifiers in the object language.[31] The argument may be thought of on the model of scientific accounts of perception that explain why it is that perception is reliable while assuming that it is reliable: although not the sort of thing that would move a skeptic, such accounts can be used to reassure non-skeptics of the stability of their own position.

Here is one way of spelling out the argument. Working within a plural first-order metalanguage whose plural quantifiers are assumed to receive a full interpretation, one begins by characterizing a space of possible (singular) predicates. One claims, in particular, that for any things whatsoever, there might be a (singular) predicate which is satisfied by just those things. The assumption that the plural quantifiers of the metalanguage receive a full interpretation comes in at this point to guarantee that by using the plural locution 'any things whatsoever' one succeeds in characterizing a maximally inclusive space of possible predicates. The next step is to characterize the notion of open-endedness. An axiom schema of the object-language is said to be accepted open-endedly if speakers are committed not only to the truth of each of its instances in the language as it currently stands, but also to the truth of each of its instances in the result of expanding the language with an arbitrary predicate from the space of possible (singular) predicates. One then goes on to argue that the acceptance of a schema by speakers of the object-language may be assumed to be open-ended unless speakers' linguistic practice gives one positive reason for imposing some sort of restriction. In particular, acceptance of Plural Comprehension may be assumed to be open-ended, since speakers' linguistic practice gives one no positive reason for postulating restrictions. The final step of the argument is the observation that only a full interpretation of the plural quantifiers is compatible with open-ended acceptance of the schema of Plural Comprehension.

A limitation of this sort of argument is that even if it explains how the practice of speakers might succeed in ruling out non-full interpretations of the plural quantifiers, it doesn't help with the project of acquiring a better understanding of full interpretations. One can show, for example, that either a plural version of the Continuum Hypothesis is true on any full interpretation of the plural quantifiers or a plural version of the negation of the Continuum Hypothesis is true on any full interpretation of the plural quantifiers.[32] But the argument from open-endedness doesn't help us determine which.[33]

3.3 Logicality

There are (non-apodictic) arguments going from Logicality to the conjunction of Pluralism and Fullness, and back.

> Logicality ⇒ Pluralism
>
> If Singularism is true, Plural Comprehension has instances that aren't logically true (i.e. "if there is an elephant, then there is a set-like object with just the elephants as members"). So if Logicality is true, Singularism is false. In a context in which Pluralism is conceived as the only alternative to Singularism, this yields the result that Logicality entails Pluralism.
>
> Logicality ⇒ Fullness
>
> Since the truth of logical validities does not depend on the meaning of non-logical terms, it follows from Logicality that Plural Comprehension holds open-endedly (i.e. that it will have true instances no matter how the language is expanded). But Plural Comprehension can hold open-endedly only if Fullness is true.
>
> (Pluralism & Fullness) ⇒ Logicality
>
> When Pluralism and Fullness are both in place, the meanings of '$\prec$' and '$\exists xx$' (plus the semantic structures of the relevant sentences) suffice to guarantee that every instance of Plural Comprehension is true and the introduction and elimination rules for the plural quantifies are truth preserving. But now suppose that a sentence (or rule) is logically valid if its truth (or truth-perservingness) is guaranteed by the meanings of its logical terms (plus the semantic structures of the relevant sentences). Then all it takes for Plural Comprehension and the introduction and elimination rules to count as logically valid is for '$\prec$' and '$\exists xx$' to be logical notions. But it follows from Pluralism and Fullness that '$\prec$' and '$\exists xx$' are both invariant under permutations in the sense of Tarski (1986), which makes them candidates for counting as logical notions.[34]

The label 'Is second-order logic is really logic?' has sometimes been used in connection with the debate between singularists and pluralists. This is potentially misleading. As emphasized in section 2.5, there are important differences between plural quantifiers and second-order quantifiers. More importantly, the label makes the logical status of plural sentences sound more significant than it really is. As evidenced by the applications listed in section 2, Pluralism and Fullness are at least as significant as Logicality. Things would be otherwise if one were entitled to assume that logical truths generally enjoy special epistemic properties (e.g. *a priori* knowability), but it is not obvious that there are any real grounds for this assumption when it comes to languages as expressively resourceful and proof-theoretically intractable as plural languages.[35] Things would also be otherwise if LOGICALITY was able to play a role in grounding one's confidence in PLURALISM and FULLNESS, but I know of no good arguments for LOGICALITY that do not proceed via PLURALISM and FULLNESS.

An additional drawback of thinking of the debate between singularists and pluralists as a debate about whether second-order logic is really logic is that it encourages the thought that there is a philosophically significant divide between the plural predicate 'is one of' (which is used in Boolos's plural paraphrase of second-order sentences) and other plural predicates with non-distributive readings (e.g. 'are scattered on the floor' or 'are finitely many in number'). As far as I can tell, there are no real grounds for postulating such a divide.[36]

4 Two sides of the same coin?

It seems to me that SINGULARISM and PLURALISM have more in common than is usually supposed. The best way to see this is by considering the phenomenon of semantic instability.

Say that a model-theory is strictly adequate if it allows for quantification over models corresponding to each of the possible interpretations the object language might take. It was noted in section 2.1 that—when ABSOLUTE GENERALITY is in place—singular first-order languages are *semantically unstable* in the following sense: it is impossible to formulate a strictly adequate model-theory for a (singular) first-order language in another (singular) first-order language. But it was also noted that one can use the notion of a plural model to formulate a strictly adequate model-theory for an arbitrary (singular) first-order language in a plural first-order language. As it turns out, the situation generalizes. Like their singular counterparts, plural first-order languages are semantically unstable, but one can formulate a strictly adequate model-theory for an arbitrary plural first-order language in a language of the next-higher logical type (i.e. a language containing *super-plural* terms, variables and predicates).[37] In general, if ABSOLUTE GENERALITY holds, nth-level languages are semantically unstable, but one can give a strictly adequate model-theory for an arbitrary nth-level langauge in an $(n+1)$th-level language.[38]

The lesson is that unless pluralists are willing to give up ABSOLUTE GENERALITY or willing to countenance the existence languages for which a strictly adequate model-theory cannot be given—either of which would put a damper on the applications discussed in section 2.1—they must learn to live with an open-ended hierarchy of increasing logical types. And once such a hierarchy is in place there is room for wondering whether the differences between a pluralist who accepts ABSOLUTE GENERALITY and a singularist who rejects it haven't been overrated.[39] For what is there to stop one from thinking that singularists and pluralists have been using the term 'individual' differently, with the pluralist insisting that its application be limited to the lowest level of the hierarchy and the singularist insisting that it be applied throughout?

Notes

[1]I am told by my linguist friends that it is not uncommon for natural languages to lack singular markers, and that there are reasons for doubting that there is anything like a singular/plural distinction at the level of semantic structure in any natural language. When Anglo-American philosophers think of singulars as being basic and plurals as being special, this is most likely a reflection of their familiarity with formal logic and of parochial features of English, rather than a reflection of any deep features of natural language. From a natural language perspective, it is probably best to think of plurals as basic and singulars as a special case of plurals in which a certain kind of restriction is in place. (For a plural language with a single type of variable, see Rayo (2002b) and McKay (2006) ch. 6.)

[2] As far as I know, the use of double variables as plural variables was first introduced in Burgess and Rosen (1997).

[3] In the case of the existential plural quantifier, the introduction and elimination rules are as follows:

Existential Introduction

$$\phi(xx_i) \rightarrow \exists xx_i(\phi(xx_i))$$

Existential Elimination

$$\frac{\Gamma}{\phi(xx_i) \rightarrow \psi} \Rightarrow \frac{\Gamma}{\exists xx_i(\phi(xx_i)) \rightarrow \psi}$$

(where $\ulcorner xx_i \urcorner$ does not occur free in Γ or ψ).

[4] For the view that one's first-order variables might range over absolutely everything even if one's second-order variables are restricted, see Shapiro (2003).

[5] Discussion is guided largely by my own interests; for a more comprehensive treatment of plural quantification, see Linnebo (2004); for a discussion of plurals from a linguistic perspective see Link (1991) and Schein (forthcoming); three book-length treatments of plurals are Schein (1993), Yi (2002) and McKay (2006).

[6] The original idea is due to Tarski (1936), but the notion of model that is now standard diverges from Tarski's in significant respects. Firstly, the standard notion of model allows

for variations in the domain of discourse, but Tarski's did not (see Etchemendy (1990)); secondly—and crucially for present purposes—the standard notion of model is formulated in the language of first-order set theory, but Tarski's was formulated in the language of the theory of types.

[7] See McGee (1992).

[8] One example is Kreisel's result; section 2.2 and Rayo and Williamson (2003) contain additional examples.

[9] See Rayo (2002b) and Williamson (2003); for discussion of set-like objects that correspond neither to the standard conception of set nor to the standard conception of class see Forster (1995) and Linnebo (forthcoming).

[10] See Boolos (1985), Rayo and Uzquiano (1999)).

[11] The result is due to Dedekind; see Shapiro (1991), Theorem 4.8.

[12] See, however, Oliver and Smiley (2006).

[13] The relevant version of Global Reflection states that there is a set α such that every true sentence in the language of plural first-order set-theory is true when the quantifiers are restricted to the elements of α.

[14] See Rayo (2005).

[15] The original result is due to Zermelo.

[16] See McGee (1997).

[17] See Boolos (1971).

[18] To see the extent of the damage in the context of Zermelo set theory, see Uzquiano (1999). Matters are somewhat different when it comes to Scott set theory; see Potter (2004) (appendix A) and Uzquiano (2005).

[19] See Burgess (2005) §3.6. As Burgess points out, the Axiom of Choice can be motivated by taking the present conception of set to entail that any objects that cannot be put in one-one correspondence with everything there is form a set; see p. 201.

[20] See van Inwagen (1990), Hossack (2000) and Dorr and Rosen (2002).

[21] See Uzquiano (2004).

[22] See Fine (2002) II.5 and Rayo (2002a). To keep things simple, I assume that the number sequence begins with 1 rather than 0. The easiest way of avoiding this assumption is by employing the Boolosian trick described in section 2.5.

[23] See Mulligan et al. (1984). Some proponents of truth-makers draw inspiration from a certain reading of Wittgenstein's *Tractatus*.

[24] See Simons (1997) and Rayo and Yablo (2001).

[25] See Quine (1986), ch. 5.

[26] See Lewis (1991), Schein (1993), Yi (1999), Oliver and Smiley (2001), Rayo (2002b), Rumfitt (2005), Yi (20052006) and Rayo (2006).

[27] For discussion of related positions, see Dummett (1981) chapter 15, Parsons (1974), Dummett (1991) chapter 24, Hazen (1993), Linnebo (2003), Glanzberg (2004), Fine (forthcoming), Glanzberg (forthcoming), Linnebo (forthcoming) and Parsons (forthcoming); for criticism see Williamson (1998) and Williamson (2003).

[28] As noted in section 2.1, it is a consequence of PLURALISM and FULLNESS that a plural first-order language can be used to talk about the possible interpretations a first-order language might take, and it is a consequence of ABSOLUTE GENERALITY that singular first-order languages cannot.

[29] See my 'On Specifying Content'.

[30] For discussion of skeptical worries, see Jané (2005).

[31] See McGee (2000) and McGee (forthcoming); for criticism see Lavine (forthcoming).

[32] See Shapiro (1991), §5.1.

[33] For more on the sorts of considerations that might be useful in deciding the Continuum Hypothesis, see Dehornoy (2002 3) and Woodin (2004). Thanks here to Alejandro Pérez Carballo.

[34] For more on invariance as a criterion of logicality, see Sher (1991) and McGee (1996).

[35] For more on the properties of plural languages, see Shapiro (1991), especially chapters 4 and 5.

[36] See Yi (1999), Oliver and Smiley (2001), Rayo (2002b), Rumfitt (2005) and Yi (20052006).

[37] For discussion of super-plurals see Black (1970), Hazen (1997) and Rayo (2006).

[38] See Rayo (2006).

[39] See Linnebo (2003), Linnebo (forthcoming) and Rayo (2006).

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