Word and Objects

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The aim of this essay is to show that the subject-matter of ontology is richer than one might have thought. Our route will be indirect. We will argue that there are circumstances under which standard first-order regimentation is unacceptable, and that more appropriate varieties of regimentation lead to unexpected kinds of ontological commitment.

Quine has taught us that ontological inquiry—inquiry as to what there is—can be separated into two distinct tasks.[1] On the one hand, there is the problem of determining the ontological commitments of a given theory; on the other, the problem of deciding what theories to accept. The objects whose existence we have reason to believe in are then the ontological commitments of the theories we have reason to accept. Regarding the former of these two tasks, Quine maintains that a first-order theory is committed to the existence of an object satisfying a certain predicate if and only if some object satisfying that predicate must be admitted among the values of the theory's variables in order for the theory to be true. Quine's criterion is extremely attractive, but it applies only to theories that are couched in first-order languages. Offhand this is not a serious constraint, because most of our theories have straightforward first-order regimentations. But here we shall see that there is a special kind of tension between regimenting our discourse in a first-order language and allowing our quantifiers to range over absolutely everything.[2] We will proceed on the assumption that absolutely unrestricted quantification is possible, and show that an important class of English sentences resists first-order regimentation. This will lead us to develop alternate languages of regimentation, languages containing *plural*

[1] See Quine (1948).

[2] For an excellent discussion on quantifying over everything see Cartwright (1994). See also Dummett (1981) chapters 14-16, Parsons (1974), Boolos (1998b) and McGee (2000).

quantifiers and predicates. It will also lead us to set forth a more inclusive criterion of ontological commitment.

1 Regimentation

I use 'regimentation' in Quine's sense.[3] Ordinary discourse is plagued with unclarities and ambiguities. Usually they are harmless. But under special circumstances—such as the practice of scientists and philosophers—they may interfere with our goals. When we regiment, we paraphrase the sentences of our original discourse into sentences with fewer unclarities or ambiguities. There is no presupposition of synonymy, or of sameness of 'logical form'. It is only required that, to our satisfaction, whatever we hoped to achieve by way of our original sentences can be achieved closely enough by way of their paraphrase. In many cases, this means that truth-conditions must be preserved, but we needn't assume this in general. No questions about our original sentences are settled with regimentation: the old discourse is surrendered in favor of the new.

Regimentation is important for the purposes of ontological inquiry because our theories are not always expressed in ways that allow us to assess their ontological commitments. But we may be able to regiment them using languages for which some criterion of ontological commitment is available. In doing so no light is shed on the commitments of the original theories, but as long as we are willing to surrender them in favor of their regimentations, we will be in a position to determine what our ontological commitments are.

A language of regimentation needn't be a fragment of natural language. It is sufficient that it be well understood. For instance, we may attempt to eliminate ambiguity by adding subscripts to the pronouns of some suitable fragment of English. The resulting language is not itself a fragment of English, but it will presumably be well understood by any English speaker. Formalisms such as first-order logic can also be used for regimentation. We may regard '$\exists x_i$', '$x_i = x_j$', '$\neg$' and '$\wedge$' as abbreviating the expressions 'there is an object$_i$ such that', 'it$_i$ is identical to it$_j$', 'it is not the case that ...' and 'it is both the case that ... and ...' (respectively). Not all of the latter are part of English, but they will be well understood by any English speaker familiar with the relevant subscripting conventions.

First-order languages naturally suggest themselves as languages of regimentation. Besides enabling the application of Quine's criterion of ontological commitment, they provide us with grammatical simplicity, notational

[3] See Quine (1960), section 33.

perspicuity and considerable expressive power. So much so that there is some pull towards thinking that we should *always* choose a first-order language as our language of regimentation. Quine and others seem to have adopted just this view.[4] Nonetheless, the choice of a language of regimentation should be made on the basis of its ability to further our goals. And, depending on the circumstances, first-order languages may not turn out to be the best candidates for the job.

The adequacy of regimentation is constrained only by our needs. Not so for the adequacy of a criterion of ontological commitment. Once we have settled upon a language of regimentation and accepted a theory couched in that language, ontological commitments are forced upon us. They cannot be chosen on the basis of their ability to further our goals.

2 Critics

Suppose we agree that our language of regimentation is to be first-order. How might we regiment the Geach-Kaplan sentence?

> Some critics admire only one another.

One option is to introduce the first-order predicate 'P(...)' as an abbreviation for the English '... is such that some critics admire only one another' and go on to paraphrase the Geach-Kaplan sentence as '$\forall x$ P(x)'. But normally we expect a paraphrase to preserve some of the logical connections of the original sentence. For instance, we might want the existence of critics to be derivable from our paraphrase of the Geach-Kaplan sentence. And, of course, '$\exists x$ CRITIC(x)' isn't derivable from '$\forall x$ P(x)'.

Is it possible to find a first-order paraphrase of the Geach-Kaplan sentence which preserves all the logical connections we might be interested in? Kaplan answered part of this question in the negative.[5] He proved that there is no first-order sentence that is true in precisely the same models as the following second-order sentence, which is reminiscent of the Geach-Kaplan sentence when 'Axy' is read 'x admires y' and all quantifiers are restricted to critics:

(1) $\exists X(\exists x Xx \wedge \forall x \forall y[Xx \wedge Axy \rightarrow x \neq y \wedge Xy])$.

But now suppose we were to agree that a sentence φ can only be a regimentation of the Geach-Kaplan sentence if it meets the following condition:

[4] See, for instance, Quine (1948) and Davidson (1967).

[5] See Boolos (1984), p. 57.

(∗) φ is true in a model just in case (1) is.

Then, by Kaplan's proof, there is no way of regimenting the Geach-Kaplan sentence in a first-order language.

However, there is usually no reason to impose conditions as stringent as (∗) on our regimentations. We might, for example, paraphrase the Geach-Kaplan sentence as a first-order version of the following:

(2) There is a (non-empty) *set* of critics z such that, for any x and y, if $x \in z$ and x admires y, then $x \neq y$ and $y \in z$.

A first-order version of (2) does not meet condition (∗) because there are models that verify (1) with domains containing no sets. But it is possible for a sentence to serve as a paraphrase even if it doesn't preserve all the original's logical properties. All that is required is that, to our satisfaction, whatever we hoped to achieve by way of the original can be achieved closely enough by way of the paraphrase. Thus, if not all the logical properties of the Geach-Kaplan sentence are important for our present purposes, there needn't be an obstacle for paraphrasing it as (2). Moreover, solid intuitions about the logical properties of the Geach-Kaplan sentence run out well before forcing anything like (∗) upon us, and some of the intuitions we do have easily fade away in the presence of an otherwise attractive paraphrase.

3 Collective Predicates

There is a well-known distinction between collective and distributive readings of English predicates. For instance, 'The children carried the piano' can be taken to mean either that the children carried the piano *together*, or that *each* of the children is such that she carried the piano. In the former case, the predicate '... carried the piano' is understood collectively; in the latter, it is understood distributively. In general, we shall say that an occurrence of the predicate '... (are) P' is understood distributively in 'they (are) P' just in case 'they (are) P' can be paraphrased as 'each of them (is) P'. Otherwise, we shall say that '... (are) P' is understood collectively.[6]

Throughout this paper it will be convenient to eliminate the sort of ambiguity that afflicts sentences containing predicates which are open to both collective and distributive readings. We shall do so by stipulating

[6] By extending this characterization in the obvious way, we can speak of an n-adic predicate being understood distributively or collectively with respect to each of its argument-places.

that predicates are to be understood according to their *collective* readings whenever there is any risk of ambiguity. Also, we shall sometimes speak of collective and distributive *predicates* instead of collective and distributive *readings* of predicates.

Attention to collective predicates sheds light on the reason why finding a first-order paraphrase for the Geach-Kaplan sentence is not entirely straightforward. The following is presumably an uncontroversial *non*-first-order paraphrase:

(3) There are some critics such that, for any x and y, if x is one of them and x admires y, then $x \neq y$ and y is one of them.

Note that each of the three occurrences of the predicate '... is one of ...' in (3) must be understood collectively with respect to its second argument-place. But there is no *direct* way of paraphrasing collective English predicates into first-order languages because first-order predicates do not admit plural arguments. In order to find a first-order paraphrase for (3) some deviousness is required. One possibility is to replace plural quantification over critics with singular quantification over *sets* of critics, and replace '... is one of ...' with the set-theoretic '... ϵ ...'. What we get is (2) (from section 2).

More generally, we could replace plural quantification over critics with singular quantification over objects that serve as *surrogates* for critics: sets of critics, or classes of critics, or 'plural objects' composed of critics, or events involving critics. To ensure that the result is firstorderizable, we shall require of our surrogates that they admit of a 'membership' relation, that is, whenever s is a surrogate for the Fs we shall require that the Fs be all and only the 'members' of s.[7] This allows us to replace '... is one of ...' by the 'membership' relation in our paraphrases. For instance, (3) might be paraphrased as:

(4) There is a *surrogate* z with only critics as 'members' such that z has at least one 'member' and, for any x and y, if x is a 'member' of z and x admires y, then $x \neq y$ and y is a 'member' of z.

And, as one would expect, (4) is equivalent to (2) (from section 2) when our surrogates of choice are sets.

[7] Needless to say, 'membership' does not, in general, correspond to set-theoretic membership. Our requirement is equivalent to a *uniqueness* condition according to which, if s is a surrogate for the Fs and the Gs are not the Fs, then s is not a surrogate for the Gs. Accordingly, we cannot have it that s is a surrogate for the Fs just in case s is the *mereological sum* of the Fs, since the Fs and Gs might not be the same objects but share a mereological sum. Requirements on surrogates are liberalized in section 6.

In many cases, the surrogate-method (as we shall call it) is an extremely effective way of producing first-order paraphrases for sentences containing '...is one of ...'. But George Boolos has shown that it cannot always be made to work.[8] He noted that although the following is obviously true,

> There are some sets such that, for any y, y is one of them just in case $y \notin y$;

it gets assigned a necessarily false paraphrase by the surrogate method when surrogates are taken to be sets:

> There is a set x such that, for every y, $y \in x$ just in case y is a set such that $y \notin y$.

The problem generalizes. For, given any non-trivial[9] choice of surrogates σ, the following is true:

(5) There are some σ-surrogates such that, for any y, y is one of them just in case y is a σ-surrogate which is not a 'member' of itself.

But it gets assigned a necessarily false paraphrase by the surrogate method when our choice of surrogates is σ:

(6) There is a σ-surrogate x such that, for every y, y is a 'member' of x just in case y is a σ-surrogate which is not a 'member' of itself.

Thus, for any non-trivial choice of surrogates, we can find a sentence that cannot be paraphrased by appeal to those surrogates.

Friends of the surrogate method have a way of avoiding this conclusion. They can claim that the quantifier 'for every y' in (6) doesn't really range over *all* surrogates, and that x is outside this range. They might add that the quantifiers in (6) are systematically ambiguous, or that their range is indeterminate. But this move is blocked if we are allowed to assume that the domain of (5) consists unequivocally of everything there is.

[8]See Boolos (1984), Boolos (1985a) and Boolos (1985b).

[9]On the assumption that there are at least four individuals, say that a choice of surrogates σ is *non-trivial* only if every two individuals have a surrogate.

4 Bernays's Principle

The conclusions of the preceding section are not as strong as one might have hoped. We saw that, for any non-trivial choice of surrogates, there is a sentence that cannot be paraphrased by appeal to those surrogates. But this is compatible with the view that every sentence can be paraphrased by appeal to some choice of surrogates. Moreover, a sentence might have a first-order paraphrase even if the surrogate-method fails. Thus, (5) can be paraphrased as 'there exists a nonselfmembered σ-surrogate', which is certainly firstorderizable.

In this section we shall make a stronger case for the view that there are sentences resisting first-order paraphrase.

Cantor's Theorem is well-known. It states that there is no function from a set onto its power-set. Less well-known is a certain kind of extension of this result. Intuitively, the thought is that there is no function from the objects there are onto the 'pluralities' of objects there are. This goes beyond Cantor's Theorem because there is no set containing all objects.

So far, however, our proposition has not been properly expressed. To begin with, we have said nothing about what a 'plurality' is supposed to be.[10] Moreover, functions are normally taken to be sets, so it is unclear just what one might mean by 'function' in the present context. In order to express our proposition properly, we need a piece of notation. If the Gs are some ordered pairs, we shall say that the Gs *map* x onto the Fs if, for every y, $\langle x, y\rangle$ is one of the Gs just in case y is one of the Fs. Our proposition is then this:

(BP) Given any ordered pairs, there are some things onto which those ordered pairs map nothing.

As far as I know, Paul Bernays was the first to set-forth this kind of result,[11] so I shall refer to it as *Bernays's Principle*.

I submit that, when our domain consists of everything there is, Bernays's Principle has no first-order paraphrase. Unfortunately, I have no proof that this is so. In fact, I haven't the slightest idea what such a proof would look like. Note, for example, that a Kaplan-style nonfirstorderizability result is not what we are looking for. The lesson of section 2 is that a sentence may have an acceptable first-order paraphrase even if it is nonfirstorderizable in

[10] See, however, section 12.

[11] See Bernays (1942), pp. 137-8. Theorem 5.3 of Shapiro (1991) is a second-order version of (BP).

Kaplan's sense.[12] The best I can do to forward my claim is show that, when our domain consists of absolutely everything, the surrogate-method does not succeed in firstorderizing Bernays's Principle, no matter what surrogates we chose.

If we attempt to paraphrase Bernays's Principle in accordance with the surrogate-method what we get is the following:

(7) $\forall\alpha\exists\beta\forall x\neg\forall y(\langle x, y\rangle$ is a 'member' of $\alpha \leftrightarrow y$ is a 'member' of $\beta)$;

where Greek letters range over the surrogates of our choice, σ-surrogates say. Let an ordered pair be one of the Ss just in case its first component is a σ-surrogate and its second component is a 'member' of that σ-surrogate. On the assumption that our domain consists of everything there is, it is an instance of Bernays's Principle that there are some things onto which the Ss map nothing. If there is to be a corresponding instance of (7), we must assume that there is a σ-surrogate for the Ss—call it ρ.[13] What we get is then:

(8) $\forall x\neg\forall y(\langle x, y\rangle$ is a 'member' of $\rho \leftrightarrow y$ is a 'member' of $\gamma)$,

for some surrogate γ. But, if ρ exists, the following is a consequence of our definition of the Ss:

(9) $\forall y(\langle\gamma, y\rangle$ is a 'member' of $\rho \leftrightarrow y$ is a 'member' of $\gamma)$.

And, of course, (8) and (9) are in contradiction.

When our domain consists of absolutely everything, the surrogate-method does not succeed in firstorderizing Bernays's Principle, no matter what surrogates we chose. Perhaps it has some other first-order paraphrase. If so, I have been unable to find it. As far as I can tell, Bernays's Principle cannot be regimented in a first-order language.

[12]In fact, a Kaplan-style result isn't available for Bernays's Principle. In order to prove such a result, we would have to select some second-order sentence χ and insist that a formula φ can only be a first-order paraphrase of Bernays's Principle if it meets the following condition:

(∗) φ is true in a model just in case χ is.

To conclude the proof we would then have to show that there is no first-order formula true in precisely the same models as χ. But presumably (∗) is only plausible if we chose χ to be something along the following lines,

(1) $\neg\exists X\forall Y\exists u\forall v(X(u, v) \leftrightarrow Yv)$;

Unfortunately, (1) is a *theorem* of second-order logic, so there are plenty of first-order formulas which are true in precisely the same models as (1).

[13]In fact, this assumption may fail, as it does when we choose sets as our surrogates.

5 PFO Languages

An alternative language of regimentation suggests itself. Let a *plural first-order* language (PFO for short) be the result of enriching a first-order language with plural quantifiers and variables, and a dyadic predicate '$\prec$', which takes a singular variable in first argument-place and a plural variable in its second. Our plural variables are 'xx_1', 'xx_2', etc. '$\exists xx_i$' is to be interpreted as 'there are some objects$_i$ such that', and '$x_i \prec xx_j$' is to be interpreted as 'it$_i$ is one of them$_j$'. Thus, for instance, '$\exists x_i \exists xx_j(x_i \prec xx_j)$' is to be read

> there is an object$_i$ and some objects$_j$ such that it$_i$ is one of them$_j$.

(A formal characterization of PFO languages is provided in the appendix.)

PFO languages are an excellent means for regimenting English sentences containing the collective predicate '... is one of ...'. For instance, Bernays's Principle can be paraphrased as:

$$\forall xx \exists yy \forall u \exists v \neg(\langle u, v\rangle \prec xx \leftrightarrow v \prec yy).$$

The Geach-Kaplan sentence also has a natural PFO paraphrase:

$$\exists xx \forall y \forall z[(y \prec xx \wedge \textsc{Admires}(y, z)) \rightarrow (y \neq z \wedge z \prec xx)];$$

(where our domain of discourse consists of critics).

Our interpretation of PFO languages makes use of a convention that was introduced in Boolos (1984) and is now standard in the literature. There is some pull towards thinking that the English 'there are some Fs such that so-and-so' is only true if there are at least *two* Fs such that so-and-so. But it is by no means evident that this should be so. One could argue, for example, that an utterance of 'there are some Fs such that so-and-so' is *pragmatically inappropriate*, but true, when it is known that there is only one F such that so-and-so, in much the same way that the utterance of a disjunction can be pragmatically inappropriate, but true, when it is known of one of the disjuncts that it is true. Boolos's convention is to side-step this controversy altogether and stipulate that 'there are some Fs such that so-and-so' is to be true just in case there are *one or more* Fs such that so-and-so. I will assume that Boolos's convention is in place throughout the remainder of this essay.

6 Beyond PFO

The question now arises whether PFO languages can be used to regiment English sentences involving collective predicates *other* than '... is one of

. . . '. In this section I will make a case for the view that, under certain circumstances, they cannot. Consider the following sentences,

(A) The seashells *are scattered*;

(B) The Peano Axioms *imply Fermat's Last Theorem*;

(C) The mechanics *repaired the car*;

(D) The musicians *will perform the symphony*;

(E) The philosophers *mingled with* the mathematicians;

(F) The seashells are *mixed together with* the rocks;

(G) The soldiers *are between* the students *and* the administrators.

How might one paraphrase (A)–(G) into a PFO language? A natural thing to do is paraphrase (A) as:

The set of seashells is scattered;

which, in turn, has an straightforward first-order paraphrase and *a fortiori* a straightforward PFO paraphrase:

$$\exists x(\textsc{Set}(x) \wedge \forall y(y \in x \leftrightarrow \textsc{Seashell}(y)) \wedge \textsc{Scattered}(x)).$$

More generally, one might claim that there is an object which serves as a *surrogate* for the seashells: a set of seashells, or a class of seashells, or a 'plural object' composed of seashells, or an event involving seashells. When discussing first-order paraphrases in section 3, we required that surrogates admit of a 'membership' relation. But now we can be more generous. All we shall require is that 'x is a surrogate for the Fs' have a PFO paraphrase. This allows us to treat the mereological sum of the Fs as a surrogate for the Fs.[14]

With the machinery of surrogates at hand, one might hold that talk of the seashells being scattered is not to be paraphrased by predicating something of the seashells themselves. Rather, it is to be paraphrased by predicating something of their surrogate. In the case of (A) we get:

[14]Specifically, we can take 'x is a surrogate for the Fs' to abbreviate 'for every y, if y is one of the Fs then y is a part of x; and, for every y, if y is a part of x then, there is a z such that z is a part of y and z is a part of one of the Fs'. On the more restrictive conception of surrogates this cannot be done because of the presence of the collective predicate '. . . is one of . . . '. See footnote 7.

The seashells' surrogate is scattered.

The surrogate-method faces an important difficulty. Suppose that our domain consists of absolutely everything. Then it follows from Bernays's Principle that, no matter what surrogates we chose, at least one of the following must be the case:

(α) There are some objects with no surrogate.

(β) There are some objects—the Fs—and some objects—the Gs—such that the Fs are not the Gs but the Fs have the same surrogate as the Gs.

A couple of examples should make clear why this causes trouble for friends of the surrogate-method.

First, suppose we chose sets as our surrogates. That is, whenever we have some things, we will let their surrogate be the *set* containing precisely those things. Then set-extensionality guarantees that (β) will never be the case, so (α) follows from Bernays's Principle: there must be some things with no surrogate. In particular, it turns out that the cardinals have no surrogate (since there is no set of all cardinals). This is a problem because, although the following sentence is intuitively true,

The cardinals are scattered among the ordinals;

the surrogate-method would paraphrase it as something necessarily false:

There is a *set* with precisely the cardinals as members and it is scattered among the ordinals.

Second, suppose we decide to let mereological sums be our surrogates. That is, whenever we have some things, we let their surrogate be their mereological sum. Since any objects whatsoever have a mereological sum, (α) cannot be the case, so (β) follows from Bernays's principle: there are some objects and some other objects such that the former have the same mereological sum as the latter. That this is a problem can be illustrated as follows. Suppose that there are a few scattered plies of sand on a table. Then it is true of the piles of sand, but false of the grains of sand which make up the piles, that they are scattered. But, if we take mereological sums to be our surrogates, this fact cannot be captured by the surrogate-method, since the mereological sum of the piles is precisely the same object as the mereological sum of the grains of sand.

Given a choice of surrogates σ, let us say that the Fs are a *problem case* for σ if either the Fs have no σ-surrogate, or there are some things which are distinct from the Fs but have the same σ-surrogate. As it turns out, problem cases are far from scarce: it is easy to verify that there are 'more' problem cases than non-problem cases. So, no matter what surrogates we choose, we are at risk of coming across sentences—such as our examples—that get the wrong truth-value when paraphrased in accordance with the surrogate-method. Of course, we may sometimes be able chose our surrogates in such a way that problem cases turn out not to be important in the relevant context. But it is not possible in general. Unless a non-trivial Reflection Principle is assumed to hold, the formal semantics for second-order languages described in section 13.2 is an example of a case in which it cannot be done.[15]

A particularly sophisticated version of the surrogate-method, developed by James Higginbotham and Barry Schein,[16] deserves special attention. Their method of paraphrase uses *events* as surrogates. For example, they paraphrase 'Those boys built a boat' as:

> There is an event E such that (*a*) E is a boat-building and (*b*) for every x, x is an *agent* of E just in case x is one of those boys.

This proposal works nicely for many special cases. But it cannot be made to work generally, on pain of generating a version of Russell's Paradox. The problem emerges if we predicate something collectively of the events that are not 'agents' of themselves. For instance,

> The events that are not agents of themselves have little in common.

Higginbotham and Schein would have us paraphrase this sentence as:

> There is an event E such that (*a*) E is a having-little-in-common,[17] and (*b*) for every x, x is an agent of E just in case x is not an agent of itself.

[15]If an appropriate Reflection Principle holds—a principle ensuring that any true sentence of second-order set-theory is true in some standard model—then an adequate formal semantics for second-order languages can be carried out entirely within first-order logic. For further discussion, see section 13.2. See also Rayo and Uzquiano (1999).

[16]See Higginbotham (1998), Higginbotham and Schein (1989) and Schein (1993).

[17]A having-little-in-common is an awkward event indeed. This speaks against Higginbotham and Schein's account, not against my argument.

But clause (b) of the paraphrase implies a contradiction.[18]

It might be replied that it is somehow illegitimate to speak of events that are 'agents' of other events. Unfortunately, this restriction also undermines the generality of Higginbotham and Schein's proposal. For it would be unable to account for sentences like 'Events of this magnitude are very rare'.

We have seen that the surrogate-view faces important difficulties. Other ways of paraphrasing English sentences with collective predicates into PFO languages might do better. I can only report that I have been unable to find any.[19]

7 Plural Predicates

There is a natural way of extending PFO languages to accommodate collective English predicates such as those considered in the preceding section. Let PFO^+ languages be the result of extending PFO languages with *atomic plural predicates*, that is, atomic predicates taking plural variables in some of their argument places. Atomic plural predicates are interpreted in terms of collective English predicates. Thus, '$\textbf{Scattered}(xx_i)$' might be interpreted as 'they$_i$ are scattered', and '$\textbf{Surrounded}(x_i, xx_j)$' might be interpreted as 'it$_i$ is surrounded by them$_j$'. (A formal characterization of PFO^+ languages is provided in the appendix. Yi (unpublished) examines languages of this kind at some length.)

PFO^+ languages turn out to be enormously useful. In this section we shall see that they provide us with a natural way of regimenting English sentences containing plural definite descriptions, such as (A)–(G) from section 6.

There is a familiar procedure for regimenting sentences with *singular* definite descriptions. As an example, consider 'The sailor carried John home'. If we follow Russell's advice, this sentence can be formalized in a first-order language as:

(10) $\exists x[\forall y(\textsc{Sailor}(y) \leftrightarrow x = y) \ \wedge \ \textsc{CarriedJ}(x)]$.[20]

The following definitional equivalence is frequently introduced:

[18] Byeong-Uk Yi makes a similar point in footnote 34 of Yi (1999).

[19] The resilience of collective predicates gives rise to a curious puzzle about *de rebus* belief. See McGee and Rayo (2000).

[20] Here and elsewhere, I assume that the context makes clear what the variables should be taken to range over and how the non-logical predicates should be interpreted.

$$\psi(\iota_x[\varphi(x)]) \;\equiv_{def}\; \exists x[\forall v(\varphi(v) \leftrightarrow x = v) \;\wedge\; \psi(x)].$$[21]

Thus, (10) is equivalent to:

$$\textsc{Carried}\text{J}(\iota_x[\textsc{Sailor}(x)]).$$

Something similar can be done for *plural* definite descriptions. Here we shall focus on the simplest case: sentences of the form 'the Fs are G', where 'F' is a count noun. Richard Sharvy has set forth a general account of plural definite descriptions which can be naturally framed in a PFO$^+$ language.[22]

Consider 'The sailors carried John home'. When '...carried John home' is understood collectively, it might be paraphrased as:

(11) $\exists yy[\forall x(x \prec yy \leftrightarrow \textsc{Sailor}(x)) \wedge \mathbf{CarriedJ}(yy)];$

where '**CarriedJ**(...)' abbreviates the collective reading of '...carried John home'. This suggests the following notation:

$$\psi(\pi_x[\varphi(x)]) \;\equiv_{df}\; \exists yy[\forall x(x \prec yy \leftrightarrow \varphi(x)) \;\wedge\; \psi(yy)].$$

Thus, (11) is equivalent to

$$\mathbf{CarriedJ}(\pi_x[\textsc{Sailor}(x)]).$$

When '...carried John home' is understood distributively in 'The sailors carried John home', a slightly different paraphrase suggests itself:

(12) $\forall y(y \prec \pi_x[\textsc{Sailor}(x)]) \rightarrow \textsc{Carried}\text{J}(y));$

where '$\textsc{Carried}$J(...)' is the singular counterpart of '**CarriedJ**(...)'.[23] It is easy to verify that (12) amounts to nothing more than:

$$\forall x\;(\textsc{Sailor}(x) \rightarrow \textsc{Carried}\text{J}(x)).$$

Nonetheless, we may introduce the following piece of notation:

$$\mathbf{CarriedJ}^D(xx) \;\equiv_{df}\; \forall y(y \prec xx \rightarrow \textsc{Carried}\text{J}(y));$$

(and similarly for other predicates). Thus, (12) may be rewritten as:

[21] In this and other definitions I ignore differences of scope for the sake of simplicity.

[22] See Sharvy (1980).

[23] See section 7. When singular and plural PFO$^+$ predicates are spelled the same, I shall assume that they are counterparts.

$$\mathbf{CarriedJ}^D(\pi_x[\textsc{Sailor}(x)]).$$

The machinery we have set forth allows us to give PFO$^+$ paraphrases for a substantial class of English sentences containing plural definite descriptions. For instance, 'The seashells are scattered' can be paraphrased as:

$$\mathbf{Scattered}(\pi_x[\textsc{Seashell}(x)]);$$

and 'The fugitives crossed the border' can be paraphrased as:

$$\mathbf{CrossedBorder}^D(\pi_x[\textsc{Fugitive}(x)]).$$

8 Generalized Quantifiers

PFO$^+$ languages also provide us with the resources for regimenting English sentences with generalized quantifiers. Consider the following examples:

(a) *Almost half of* the monkeys became infected;

(b) *Many of* the bills are counterfeit;

(c) *Few of* the students have any patience left.

They can be paraphrased as:

(a') The monkeys who became infected are *almost half of* the monkeys;

(b') The counterfeit bills are *many of* the bills;

(c') The students who have any patience left are *few of* the students.

Accordingly, (a)–(c) can be formalized in a suitable PFO$^+$ language as:

(a'') $\mathbf{AlmostHalfOf}(\pi_x[\textsc{Monkey}(x) \wedge \textsc{Inf}(x)], \pi_x[\textsc{Monkey}(x)]);$

(b'') $\mathbf{ManyOf}(\pi_x[\textsc{Bill}(x) \wedge \textsc{Counterfeit}(x)], \pi_x[\textsc{Bill}(x)]);$

(c'') $\mathbf{FewOf}(\pi_x[\textsc{Student}(x) \wedge \textsc{Patience}(x)], \pi_x[\textsc{Student}(x)]);$

where all the non-logical predicates in (a'')–(c'') are to be understood in the obvious way; in particular, '$\textbf{AlmostHalfOf}(xx_i, xx_j)$' is interpreted as 'they$_i$ are almost half of them$_j$', '$\textbf{ManyOf}(xx_i, xx_j)$' is interpreted as 'they$_i$ are many of them$_j$', and '$\textbf{FewOf}(xx_i, xx_j)$' is interpreted as 'they$_i$ are few of them$_j$'.

The possibility of formalizing generalized quantifiers in terms of plural predicates is to be expected. In Barwise and Cooper's influential article on the subject, a determiner such as 'Many of' is interpreted as a binary relation taking a set S as its first argument and one of S's subsets as its second.[24] Thus, 'Many of the Fs are G' is true just in case 'Many of' holds between the set of Fs and its subset consisting of the F-and-Gs. But Barwise and Cooper's assumption that the Fs form a *set* is uncalled for. It would be better to think of 'Many of' as a two-place plural predicate, and say that 'Many of the Fs are G' is true just in case 'Many of' holds between the Fs and the F-and-Gs. That is what the present proposal amounts to.[25]

If it is along the right lines, then every formula of a first-order language with generalized quantifiers can be transformed into an equivalent PFO$^+$ formula.[26] The relevant transformation is stated formally in the appendix.

[24]See Barwise and Cooper (1981). In fact, Barwise and Cooper say that determiners are to be interpreted as functions that take a set S as an argument and deliver a subset of the power-set of S, but the two formulations are equivalent.

[25]I am ignoring two complications. First, note that 'Not all of the Fs are G' is true when none of the Fs are G. On Barwise and Cooper's proposal, this is accounted for by letting the determiner 'not all', understood as a two-place singular predicate, hold between the set of Fs and the empty set. But a similar move is not available when 'not all' is formalized as a plural predicate, because plural predicates do not admit of 'empty' arguments. Instead, one might formalize 'Not all of the Fs are Gs' as a PFO$^+$ version of 'Either there are Fs but no F-and-Gs, or the F-and-Gs are not all of the Fs'. A limit case is 'None of the Fs are G', which is true *only* when no Fs are G. It can still be formalized as 'either there are Fs but no F-and-Gs, or the F-and-Gs are none of the Fs', but the second clause is redundant.

The second complication arises because we have only considered quantifiers of the form 'Q of the Fs', whose definite description ensures the existence of Fs. What about quantifiers of the form 'Q Fs'? These come in two different flavors, depending on whether the absence of Fs makes 'Q Fs are G' true or false. If the latter, 'Q Fs are G' may be paraphrased as 'Q of the Fs are G'. (For instance, 'Many Fs are G' can be paraphrased as 'Many of the Fs are G'.) If the former, 'Q Fs are G' may be paraphrased as 'Either there are no Fs, or Q of the Fs are G'. (For instance, 'All Fs are G' can be paraphrased as 'either there are no Fs, or all of the Fs are G'.) In any case, quantifiers of the form 'Q Fs' reduce to quantifiers of the form 'Q of the Fs'. They may therefore be ignored with no loss of generality.

[26]It is noted in Heck (2000) that a uniform method for paraphrasing first-order talk of types as first-order talk of tokens brakes down when the language is enriched with generalized quantifiers. Although this is true, it is also true that the problem could be

For illustration, consider ‘Many of the bills are counterfeit’. In a first-order language with generalized quantifiers it may be formalized as:

$$[\textsc{ManyOf}\ x : \textsc{Bill}(x)]\ (\textsc{Counterfeit}(x)),$$

for which our transformation yields

$$\mathbf{ManyOf}(\pi_x[\textsc{Bill}(x) \wedge\ \textsc{Counterfeit}(x)], \pi_x[\textsc{Bill}(x)]).$$

Our transformation also works for sentences with iterated generalized quantifiers. Consider, for instance, ‘Most of the candidates alienate many of the voters’:

$$[\textsc{MostOf}\ x : \textsc{Candidate}(x)]\ [\textsc{ManyOf}\ y : \textsc{Voter}(y)]\ (\textsc{Alienate}(x, y));$$

it is transformed into:

$$\mathbf{MostOf}(\pi_x[\textsc{Candidate}(x) \wedge \mathcal{A}(x)], \pi_x[\textsc{Candidate}(x)]),$$

where $\mathcal{A}(x)$ is:

$$\mathbf{ManyOf}(\pi_y[\textsc{Voter}(y) \wedge \textsc{Alienate}(x, y)], \pi_y[\textsc{Voter}(y)]).$$

9 Truth and Satisfaction

In this section we will set forth definitions of truth and satisfaction for PFO$^+$ languages.

The most natural way to proceed is to expand upon the standard definition of satisfaction for first-order languages. As before, we let variable assignments associate an object in our domain with each singular variable, but we also let variable assignments associate *multiple* objects in our domain with each plural variable. Variable assignments are therefore treated as *relations* rather than functions. If σ is an assignment and ‘vv’ a plural variable, both ‘$\sigma(‘vv’, x)$’ and ‘$\sigma(‘vv’, y)$’ may be true even if $x \neq y$ (though, of course, when ‘v’ is a singular variable, σ behaves like a total function, so that ‘$\sigma(‘v’, x)$’ holds for precisely one x).

avoided by further enriching the language. For instance, in a PFO$^+$ language with a domain consisting of all and only the word-tokens, ‘Many word-types are short’ can be paraphrased as

$\forall xx\ [\forall x \exists! y\ (\textsc{SameType}(x, y) \wedge y \prec xx) \longrightarrow \mathbf{ManyOf}(\pi_x[x \prec xx \wedge \textsc{Short}(x)], \pi_x[x \prec xx])]$.

As far as I can see, this does not make covert use of a representation function—a function differing only in name from an assignment of tokens to their corresponding types.

Relations are standardly taken to be *sets* of ordered pairs. But this will not do for our purposes. Problems arise when our domain of discourse consists of too many objects to form a set. Since we want the sentence '$\exists xx \forall y (y \prec xx)$' to turn out to be true, we need a variable assignment that associates every object in our domain with the plural variable 'xx'. But such an assignment would contain an ordered pair $\langle$'xx'$, y\rangle$ for every object y in our domain, and would therefore have too many members to be a set.

Fortunately, Boolos has found a way out of this difficulty.[27] Instead of taking a variable assignment to be a certain *set* of ordered pairs, we shall consider the ordered-pairs themselves, and have *them* play the role of assigning values to our variables. Thus, we shall say of some ordered pairs that they form a plural variable assignment just in case a certain plural predicate '**Assignment**(xx)' is true of them. And, instead of treating the satisfaction relation as a first-order predicate '$\textsc{Sat}(\varphi, \sigma)$', which holds between a formula and a set of ordered-pairs, we shall take satisfaction to be a two-place plural predicate, '$\mathbf{Sat}(\varphi, xx)$', which holds between a formula and the ordered pairs forming a plural variable assignment. Once Boolos's modification is in place, the definitions of truth and satisfaction proceed along familiar lines (see appendix for details).

Our formal semantics yields an important *stability* result: the satisfaction predicate for a PFO^+ language can always be defined within another PFO^+ language. First-order languages are also stable in this sense, but PFO languages are not. In general, the satisfaction predicate for a PFO language can only be defined within a PFO^+ language.[28] This suggests that if the realm of first-order regimentation is to be left behind, PFO^+ languages are a more natural stopping point than their PFO counterparts.

[27]See Boolos (1985a).

[28]Matters would be otherwise if there existed a set w with the feature that an arbitrary set of PFO formulas is satisfied by the ordered-pairs of a given plural variable assignment just in case it is satisfied by the restriction of that assignment to w. This would allow us to regard variable assignments as *sets* of ordered pairs, and to frame our definition of satisfaction in a first-order language. Unfortunately, the existence of w is provably inconsistent with the axioms of second-order ZFC. A partial result holds if, *given the ordered pairs of a plural variable assignment*, there is always a set w' with the feature that an arbitrary set of PFO formulas is satisfied by the ordered-pairs of a plural variable assignment differing from the former in at most the values of singular variables just in case it is satisfied by the restriction of the latter assignment to w'. This would require the existence of a Π^2_0 indescribable cardinal, which is independent from the axioms of set-theory (if consistent with them). For an excellent discussion of these issues see Shapiro (1987).

10 Ontological Commitment

The goal of ontology may be regarded as more or less ambitious. The less ambitious goal is to discover, of each predicate, whether there are objects it is true of. The more ambitious goal requires us to start by dividing our predicates into those that pick out 'basic' ontological properties and those that do not. It might be argued, for instance, that '...is an abstract object' and '...is an electron' pick out basic ontological properties, but that '...is owned by my uncle Hector' and '...is such that all whales are mammals' do not. We must then discover which of the predicates that pick out basic properties are instantiated and, if possible, go on to give an account of how they are related to predicates that do not pick out basic properties.

In what follows we will conceive of ontology in the more modest sense. When we speak of a theory's being ontologically committed to objects satisfying a certain predicate, there will be no presupposition that the predicate expresses a basic ontological property.

The formal semantics we set forth in the preceding section allows us to introduce a useful piece of notation. Let us say that x is the possible value of a singular PFO$^+$ variable v just in case x is the object which the ordered-pairs forming some plural variable assignment associate with v. Similarly, we shall say that the Fs are the possible values of a plural PFO$^+$ variable vv just in case the Fs are the objects which the ordered-pairs forming some plural variable assignment associate with vv. To forestall any ambiguities, we shall always use 'the Fs are the possible values of a variable' to mean that the Fs are *together* the possible values of a variable.[29]

With this machinery on board, we may set forth a criterion of ontological commitment for PFO$^+$ languages. We begin by emulating Quine's original proposal:

> A theory couched in a PFO$^+$ language is committed to the existence of an object satisfying a certain singular predicate if and only if, some object satisfying that predicate must be admitted as a possible value of one of the theory's singular variables in order for the theory to be true;

but to this we add:

[29] It is worth noting that the formal semantics for PFO$^+$ languages which we set forth in the appendix has the result that the range of the plural variables is *abundant*, in the sense that, given any objects whatsoever, those objects are the possible values of a plural variable.

> the theory is committed to the existence of objects satisfying a *plural* predicate if and only if some objects satisfying that predicate must be admitted as the possible values of one of the theory's plural variables in order for the theory to be true.

A PFO$^+$ theory might be committed to the existence of elephants. This will be the case whenever some object satisfying the singular predicate 'ELEPHANT(x)' must be admitted as the possible value of a singular variable in order for the theory to be true. But it could also be committed to the existence of children who together carried the piano. This will be the case whenever some objects satisfying the plural predicate '**CarriedPiano**(xx)' must be admitted as the possible values of a plural variable in order for the theory to be true—which is not to say that '**CarriedPiano**(xx)' picks out a 'basic' ontological property, irreducible to properties expressed by singular predicates.

An especially interesting case of plural ontological commitment is cardinality. For instance, a theory is committed to the existence of infinitely many things if some objects satisfying the plural predicate '**InfiniteInNumber**(xx)' must be admitted as the possible values of a plural variable in order for the theory to be true.[30]

We have seen that there is more to the task of assessing the ontological commitments of a PFO$^+$ theory than the question of what *singular* predicates must be satisfied in order for a the theory to be true: there is also the question of what *plural* predicates must be satisfied in order for the theory to be true. In this sense, we have found that the subject-matter of ontology is richer than expected. It is not that a PFO$^+$ theory might be committed to unexpected objects. Rather, it is that PFO$^+$ theories might bear commitments of an unexpected kind—i.e. plural commitments—to familiar objects.

11 A One-Sorted Language

The predicates of PFO$^+$ languages are sharply divided into those which are plural and those which are not. In this respect, PFO$^+$ languages are a poor mirror of English. For consider the following sentences,

(13) The children carried the piano;

[30] For more on the use of plural predicates to describe cardinality see my 'Frege's Unofficial Arithmetic'.

(14) John carried the piano

Intuitively, we are employing the *same* English predicate in (13) and (14), first to say something about the children (collectively) and then to say something about John. For instance, we expect it to follow logically from both (13) and (14) that the piano was carried. This intuition is not preserved when we paraphrase (13) and (14) into a PFO$^+$ language as

(13′) $\exists xx(\forall y(y \prec xx \leftrightarrow \textsc{Child}(y)) \wedge \mathbf{CarriedPiano}(xx))$; and

(14′) $\textsc{CarriedPiano}(John)$.

For '$\mathbf{CarriedPiano}(xx)$' and '$\textsc{CarriedPiano}(x)$' are two different PFO$^+$ predicates: the former is plural and the latter is not.

In order to do better justice to the intuition that (13) and (14) have a predicate in common we may appeal to Boolos's convention,[31] and paraphrase (14) as:

(14″) $\mathbf{CarriedPiano}(\pi_x\ [x = \text{John}]))$;

Now we get what we wanted because (14″) shares the plural predicate '$\mathbf{CarriedPiano}(xx)$' with (13′).

This sort of move can be carried out quite generally. Whenever we have singular and a plural PFO$^+$ predicates which correspond to the same English predicate, we can eliminate the former and have the latter do the work of both. In fact, singular predicates with no corresponding plural can also be eliminated. If '$\mathrm{P}(x)$' is a singular predicate, we may introduce a plural predicate '$\mathbf{P}^*(xx)$' to play its role. All we need to do is pick '$\mathbf{P}^*(xx)$' so that the following is true:

$$\forall x \forall yy[\forall z(z \prec yy \leftrightarrow z = x) \longrightarrow (\mathbf{P}^*(yy) \leftrightarrow \mathrm{P}(x))].^{32}$$

Whenever this condition holds, we shall say that '$\mathbf{P}^*(xx)$' is a *plural counterpart* of '$\mathrm{P}(x)$', and that '$\mathrm{P}(x)$' is the *singular counterpart* of '$\mathbf{P}^*(xx)$'.

By bringing in plural counterparts, we have the option of eliminating all singular variables and predicates from the language. We introduce the plural predicate '$xx_i \precsim xx_j$' as an abbreviation for 'they$_i$ are some of them$_j$', and set forth the following definition:

[31] See section 5.

[32] Besides requiring that it satisfy this condition, we may let '$\mathbf{P}^*(xx)$' behave as we please, but the following seems like a natural further constraint:

$$\forall xx[(\mathbf{P}^*(xx) \leftrightarrow \forall y(y \prec xx \rightarrow \mathrm{P}(y))].$$

$$\mathbf{1}(xx) \ \equiv_{df} \ \forall yy(yy \precsim xx \rightarrow xx \precsim yy).$$

This guarantees that '$\mathbf{1}(\ldots)$' is true of some objects just in case there is only one of them—recall Boolos's convention! We then apply the following transformation:

- $Tr(\neg\varphi) = \neg Tr(\varphi)$;
- $Tr(\varphi \wedge \psi) = Tr(\varphi) \wedge Tr(\psi)$;
- $Tr(\exists xx_i(\varphi)) = \exists xx_{2i}\ Tr(\varphi)$
- $Tr(\exists x_i(\varphi)) = \exists xx_{2i+1}\ (\mathbf{1}(xx_{2i+1}) \wedge Tr(\varphi))$;
- $Tr(x_i \prec xx_j)) = (xx_{2i+1} \precsim xx_{2j})$;
- $Tr(x_i = x_j) = (xx_{2i+1} \precsim xx_{2j+1}) \ \wedge \ (xx_{2j+1} \precsim xx_{2i+1})$;
- if $\mathbf{P}^*$ is the plural counterpart of P, $Tr(\mathrm{P}(x_{i_1}, \ldots, x_{i_m}, xx_{j_1}, \ldots, xx_{j_n}))$ $=$ $\mathbf{P}^*(xx_{2i_1+1}, \ldots, xx_{2i_m+1}, xx_{2j_1}, \ldots, xx_{2j_n})$.

It is therefore possible to formulate PFO$^+$ languages as one-sorted languages consisting solely of logical connectives, parenthesis, and *plural* variables and predicates. But one can continue to use *singular* variables and predicates by employing the definitional equivalencies induced by our transformation:

- $\exists x_i(\varphi) \ \equiv_{df} \ \exists xx_i\ (\mathbf{1}(xx_i) \wedge \varphi)$;
- $x_i \prec xx_j \ \equiv_{df} \ (xx_i \precsim xx_j)$;
- $x_i = x_j \ \equiv_{df} \ (xx_i \precsim xx_j) \ \wedge \ (xx_j \precsim xx_i)$;
- if $\mathbf{P}^*$ is the plural counterpart of P, $\mathrm{P}(x_{i_1}, \ldots, x_{i_m}, xx_{j_1}, \ldots, xx_{j_n}) \ \equiv_{df} \mathbf{P}^*(xx_{i_1}, \ldots, xx_{i_m}, xx_{j_1}, \ldots, xx_{j_n})$.

In English, it is natural to think of 'something' as a generic quantifier, and of 'someone' as specialized: 'someone' is a variant of 'something' which we use to indicate that the objects we are quantifying over are persons. In a one-sorted PFO$^+$ language, we treat plural quantifiers as generic and singular quantifiers as specialized in much the same way. Singular quantifiers are variants of plural quantifiers which we use to indicate that the possible values of a variable are always a single object. Similarly, in a one-sorted PFO$^+$ language we treat plural predicates as generic and singular predicates as specialized. Singular predicates are variants of plural predicates which are used to indicate that the possible values of admissible arguments are always a single object.

Thinking of PFO^+ languages as one-sorted therefore eliminates the need for an account of the relation between *singular* quantification and predication and *plural* quantification and predication. The former are simply a special case of the latter. It also eliminates the need for separating the ontological commitments of PFO^+ theories in two. Our criterion reduces to the following:

> A theory couched in a PFO^+ language is committed to the existence of objects satisfying a plural predicate if and only if some objects satisfying that predicate must be admitted as the possible values of one of the theory's plural variables in order for the theory to be true.

Singular ontological commitments are now a special case of plural ontological commitments: a one-sorted PFO^+ theory is committed to the existence of an elephant if objects satisfying the plural predicate '$\mathbf{1}(xx) \wedge \mathbf{Elephants}(xx)$' must be admitted as the possible values of a plural variable in order for the theory to be true.

12 Back to First-Order Languages

In this section we will consider the prospects of treating one-sorted PFO^+ languages as first-order languages.[33]

As far as syntactic structure is concerned, one-sorted PFO^+ languages are no different from their first-order counterparts—the fact that bold fonts are used for predicates and double letters for variables is of no importance whatsoever. But there is an important semantic difference. Whereas a plural variable assignment may associate several objects with a PFO^+ variable, a first-order variable assignment always associates a single object with a first-order variable.

If we are to treat one-sorted PFO^+ languages as first-order languages, we must therefore modify our formal semantics so that, with respect to an assignment, each variable is associated with precisely one 'plurality'. This is the easy part. The hard part is elucidating the status of 'pluralities'.

Intuitively, what we want is a relation '$\lhd$' which holds of the plurality of Fs and the plurality of Gs just in case each of the Fs is one of the Gs. This

[33] The basic insight is due to Vann McGee.

allows us to say of a plurality x that it is *atomic* just in case '$x \lhd y$' holds of every plurality y such that '$y \lhd x$'. We can therefore have it that the role that was played by the Fs on our original semantics is played on the new semantics by the plurality with precisely the Fs as atoms. (The same goes for the special case in which there is only one F: the role that was played by an object x on the original semantics is played on the new semantics by the plurality with x as its unique atom). And, on the new semantics, '$\precsim$' is interpreted as $\lhd$.

In order for pluralities to meet these constraints, they must form an *atomic mereology* over $\lhd$, in which the atoms are the objects in our original domain of discourse.[34] It follows that there must be more pluralities in our new domain of discourse than objects in our original domain.[35] If our original domain consists of absolutely everything, this gives rise to a serious difficulty: there must be more pluralities in our new domain than there are objects in existence.

This conclusion might be regarded as a *reductio* for the view that it is possible to quantify over absolutely everything. Such a perspective is forced upon proponents of the view that any language displaying the inferential behavior of a one-sorted PFO$^+$ language must have the semantic features of a first-order language.[36] Alternatively, one may retain the thesis that it is possible to quantify over everything and abandon the idea of treating one-sorted PFO$^+$ languages as first-order languages.

Friends of the latter approach can still enjoy some of the benefits of treating one-sorted PFO$^+$ languages as first-order languages. For they can regard singular quantifiers ranging over pluralities as syntactical abbreviations for plural quantifiers ranging over objects, and predicates applying singularly to pluralities as syntactical abbreviations for predicates applying plurally to objects.

Doing so allows us to avoid the awkwardness of plural idioms in prac-

[34]In general, the Fs form an atomic mereology over R just in case they meet the following conditions: (a) R is transitive, (b) if x is one of the Fs, then some atomic F bears R to x (as before, we say that an F (call it y) is *atomic* just in case every F (call it z), such that z bears R to y is also such that y bears R to z), and (c), if each of the As is an atomic F, then the As have a unique *R-fusion* (that is, there is a unique F, x, such that each of the As bears R to x and any F, y, which bears R to x is such that some A bears R to y).

[35]Provided our original domain contains more than one object.

[36]It is important to note, however, that the *reductio* is not generally forced upon proponents of the view that any language with the inferential behavior of a first-order language must have the semantic features of a first-order language. There is room for arguing that, on account of '$\precsim$', one-sorted PFO$^+$ languages do not have the inferential behavior of first-order languages.

tice, while retaining the expressive richness of PFO^+ languages. It is also provides us with a useful way of picturing the ontological commitments of PFO^+ theories. We concluded in section 10 that there is more to the task of assessing the ontological commitments of a PFO^+ theory than the question of what *singular* predicates must be satisfied in order for a the theory to be true: there is also the question of what *plural* predicates must be satisfied in order for the theory to be true. In this sense, we found that the subject-matter of ontology is richer than expected. By regarding singular talk of pluralities as shorthand for plural talk of objects, we can represent this unexpected richness in a different way. Instead of speaking of a new (plural) kind of commitment to the inhabitants of our ontology—the realm of objects—we may speak of a familiar (singular) kind of commitment to the inhabitants of our *plethology*—the realm of pluralities.[37]

It is worth emphasizing that our terminological manoeuvre does not license the use of plural quantification over non-atomic pluralities. *Singular* quantification over pluralities is a relabelling of English plural quantification. What *plural* quantification over pluralities would call for is a relabelling of *super-plural* English quantification, and no reason has been given here to think that super-plural English quantifiers exist.

Throughout the remainder of this essay we will consider a family of applications for PFO^+ languages. Our discussion will not depend on whether they receive a plural two-sorted interpretation (as in section 7), a plural one-sorted interpretation (as in section 11), or a *faux* first-order interpretation (as in the present section). We assume the former for the sake of concreteness.

13 Applications

13.1 Second-Order Logic

A second-order formulation of standard (Zermelo-Fraenkel) set theory is highly desirable. It enables us to express the general principles underlying the first-order schemes of separation and replacement.[38] In addition, Vann McGee has shown that, when our domain of discourse consists of absolutely everything, there is an extension of second-order set theory that charac-

[37] I use 'plethology' instead of 'plethynticology', which is introduced in **?**, mainly because it is easier on the English speaker's ear, but also because 'πλῆθος' is a noun, whereas 'πληθυντικός' lends itself to adjectival uses.

[38] For more on the expressive limitations of first-order languages and the role of second-order languages in overcoming them, see chapter 5 of Shapiro (1991).

terizes the set-theoretic universe up to isomorphism.[39] Yet there is some debate as to whether it is legitimate to use second-order languages for the study of set theory. The reason is that, on one standard interpretation—Quine's interpretation—second-order languages are nothing but 'set-theory in sheep's clothing'.[40] More precisely, they are two-sorted first-order languages in which variables of the first sort range over the elements of a certain *set* S, and variables of the second sort range over the subsets of S.[41] Second-order languages are therefore useless when our variables range over objects which are too many to form a set. And this is certainly the case in the intended interpretation of set theory.

It is tempting to overcome this difficulty by taking second-order languages to be *class*-theory in sheep's clothing, that is, by understanding them as two-sorted first-order languages in which variables of the first sort range over the elements of a certain *class* C, and variables of the second sort range over the sub-classes of C. Doing so, however, only postpones our difficulties. For, in making second-order languages available for the study of set-theory, we have made them unavailable for the study of *class*-theory, which now takes center-stage.

Fortunately, Boolos has given us a new way of interpreting second-order logic, one which does not run into trouble when our variables range over objects which are too many to form a set. Boolos's original proposal involves a translation method from second-order formulas into English,[42] but we may obtain identical results by introducing the following definitional equivalencies into PFO languages:

- $X_i(x_j) \ \equiv_{df} \ x_j \prec xx_i$;
- $\exists X_i(\varphi) \ \equiv_{df} \ \exists xx_i(\varphi) \vee \varphi^*$,
  where φ^* is the result of substituting $\ulcorner x_j \neq x_j \urcorner$ everywhere for $\ulcorner X_i(x_j) \urcorner$ (or its notational variants).

The complication in our second definition is needed to accommodate the fact that, although '$\exists X \forall y \neg X(y)$' is a theorem of second-order logic, '$\exists xx \forall y \neg y \prec xx$' is necessarily false (since it is impossible for there to be some objects such that no object is one of them).

[39] See McGee (1997)

[40] See Quine (1986).

[41] However, the *model-theory* for second-order languages must differ from that of two-sorted first-order languages. I owe this observation to Gabriel Uzquiano.

[42] See Boolos (1984).

On this interpretation, second-order formulas are definitional variants of PFO formulas. Hence, in contexts where the expressive power of second-order ZFC is important, PFO languages turn out to be excellent languages of regimentation.

A point is worth mentioning. So far we have accounted only for *monadic* second-order variables. But, as Boolos points out, relation variables can be incorporated into his scheme by appealing to ordered pairs.[43]

13.2 Model Theory for Second-Order Languages

A standard model for the first- or second-order language of set theory is an ordered pair $\langle D, I \rangle$. Its domain, D, is a non-empty set, and its interpretation function, I, assigns a binary relation on D to the two-place predicate letter 'ϵ'. A sentence is then said to be *valid* if it is true in all standard models.

It is a familiar point that this does not correspond to our intuitive notion of validity. What we would really like to say is, roughly, that a sentence is valid if it is true no matter what its domain of discourse is, and no matter how its non-logical vocabulary is interpreted. But there are no standard models corresponding to certain domains of discourse and interpretations of 'ϵ'. For instance, there is no model $\langle D, I \rangle$ such that D contains all sets and I assigns to 'ϵ' the set of all pairs $\langle x, y \rangle$ for x a member of y, because it is a theorem of ZFC that there is no set of all sets and that there is no set of all pairs $\langle x, y \rangle$ for x a member of y. Among other things, this opens the alarming possibility of a false sentence which is true in all standard models.

There is therefore no immediate guarantee of the adequacy of standard model theory. If it does turn out to be adequate it will be in virtue of non-trivial set-theoretic principles, not merely in virtue of our definitions.

In fact, it is possible to improve upon the standard model theory. By building upon Boolos's work,[44] Gabriel Uzquiano and I have set forth a formal semantics for second-order set theory that is intuitively adequate.[45] We proceed by rejecting the idea that the domain of a model must be a *set* of objects. Instead we focus attention on the objects themselves, and let *them* function as our domain. Accordingly, we reject the idea that the interpretation function of a model must be a *set* of ordered-pairs. We let the

[43]Treat $\ulcorner \exists R^n_i \varphi \urcorner$ as a notational variant for $\ulcorner \exists X_i \varphi \urcorner$, and $\ulcorner R^n_i(x_1, \ldots, x_n) \urcorner$ as a notational variant for $\ulcorner X_i(\langle x_1, \ldots, x_n \rangle) \urcorner$ (where '$\langle \ldots \rangle$' is the ordered n-tuple function). For more on polyadic second-order logic, see Rayo and Yablo (2001).

[44]See Boolos (1985a).

[45]See Rayo and Uzquiano (1999). Similar ideas have been set forth in unpublished manuscripts by Josep Macià Fabrega and Byeong-Uk Yi. See also section 6.1 of Shapiro (1991).

ordered-pairs themselves provide an interpretation for 'ϵ'. To accommodate these changes, we take the satisfaction predicate to be a *plural* predicate '$\mathbf{Sat}(x, yy, zz)$'.[46] Thus, although our formal semantics cannot be formulated within a PFO language, it can easily be captured within a PFO^+ language.

With an intuitively adequate model theory at hand, it is natural to ask whether every intuitively satisfiable set of second-order formulas is satisfied by some standard model. A version of this proposition was first set forth by Georg Kreisel, and is commonly referred to as *Kreisel's Principle*.[47]

Alternative formulations of Kreisel's Principle—often under the guise of Reflection Principles—have played a significant role in the development of set theory. Nonetheless, the literature suggests that (without the aid of proper classes) there is no way of expressing Kreisel's Principle within a PFO (or second-order) language.[48] On the other hand, it is easily captured within PFO^+ languages. All we need is the plural predicate '$\mathbf{Sat}(x, yy, zz)$', from our novel formal semantics.

If true, Kreisel's Principle guarantees the adequacy of standard model theory. But only its restriction to first-order formulas is provable within standard set theory. In its unrestricted second-order form, it is demonstrably independent from the standard axioms of second-order set theory (if consistent with them).[49]

We may therefore rest assured that standard first-order model theory is adequate. In particular, the first-order version of Kreisel's Principle guarantees that every first-order sentence which is true in all standard models is true.[50] However, without the unrestricted version of Kreisel's Principle, we have no assurance that standard second-order model theory is adequate. For all we know, there is a false second-order sentence which is true in all standard models. Because of this, our novel formal semantics is a significant improvement over standard second-order model theory.

Without further logical resources, it is not possible to extend our formal semantics to encompass PFO^+ languages.[51] Intuitively, the problem is that there are 'too many' possible semantic values for plural predicates.

[46] Variable assignments are dealt with as in section 9.

[47] See Kreisel (1967) pp. 152-7.

[48] An excellent discussion is provided in chapter 6.3 of Shapiro (1991).

[49] See Shapiro (1991), chapter 6.3.

[50] This also follows from the first-order Completeness Theorem, which is not available in the second-order case.

[51] Yi (unpublished) provides a formal semantics for PFO^+ languages. Unfortunately, Yi appeals to an 'interpretation function' that exceeds the resources of PFO^+ languages.

PFO$^+$ regimentation might therefore turn out to be unstable in the strong sense that it may not be generally possible to formulate the notion of *truth-in-a-model* for a given PFO$^+$ language in another PFO$^+$ language.[52] In contrast, we know that first-order languages are stable in this respect. We have just seen that the first-order version of Kreisel's Principle legitimizes the use of standard first-order model theory, which can be formalized in a first-order language. To attain strong stability, a friend of PFO$^+$ regimentation might be tempted to postulate a strengthened version of Kreisel's Principle. But such a move would presumably require some sort of independent motivation.

Fortunately, we have seen in section 9 that PFO$^+$ languages turn out to be stable in the weaker sense that a Tarskian definition of *truth* for a given PFO$^+$ language can be defined in another PFO$^+$ language.

14 Conclusions

We have assumed that it is possible to quantify over absolutely everything, and found that certain English sentences containing collective predicates resist both first-order and PFO paraphrase. To capture such sentences we introduced PFO$^+$ languages, which may contain arbitrary plural predicates.

PFO$^+$ languages turn out to be tremendously fruitful. They allow us to give a formal semantics for second-order languages and state important set theoretic propositions; they also provide us with natural formalizations for English plural definite descriptions and generalized quantifiers. I believe this makes a solid case for the use of PFO$^+$ languages as languages of regimentation.

In leaving first-order regimentation behind, we were led to enrich Quine's criterion of ontological commitment. It emerged that PFO$^+$ theories can bear commitments of an unexpected kind—i.e. plural commitments—to familiar objects. In this sense, we discovered that the subject-matter of ontology is richer than one might have thought.

We noted that the unexpected ontological richness can be accounted for in different ways. On one construal, the singular is regarded as a special case of a plural and, accordingly, plural ontological commitments are taken to be the only kind of ontological commitments a PFO$^+$ theory can have.[53]

[52] This is not to be confused with the fact that a (sufficiently strong) PFO$^+$ language cannot be used to formulate its own formal semantics.

[53] For their many helpful comments, I would like to thank Richard Cartwright, Roy Cook, Matti Eklund, Adam Elga, Michael Glanzberg, Richard Heck, Øystein Linnebo,

Appendix

1 Formal Characterization of PFO languages

Let a PFO language (short for *plural first-order*) consist of these symbols: (a) *logical connectives:* '$\exists$', '$\neg$' and '$\wedge$'; (b) *singular variables:* 'x_1', 'x_2', etc.; (c) *plural variables:* 'xx_1', 'xx_2', etc.; (d) *logical predicates* '=' and '$\prec$'; (e) *singular non-logical predicates:* 'P^1_1', 'P^1_2', ..., 'P^2_1', 'P^2_2', ..., etc.; and (f) *auxiliaries:* '(' and ')'. The formulas of PFO languages are defined as follows:

- $\ulcorner x_i = x_j \urcorner$, $\ulcorner x_i \prec xx_j \urcorner$ and $\ulcorner P^n_i(x_{j_1}, \ldots, x_{j_n}) \urcorner$ are formulas;
- if $\ulcorner \varphi \urcorner$ and $\ulcorner \psi \urcorner$ are formulas then so are $\ulcorner \exists x_i(\varphi) \urcorner$, $\ulcorner \exists xx_i(\varphi) \urcorner$, $\ulcorner \neg\varphi \urcorner$ and $\ulcorner (\varphi \wedge \psi) \urcorner$;
- nothing else is a formula.

If φ is a PFO formula, we shall let it abbreviate the (subscripted) English sentence $Tr(\varphi)$, where $Tr(\ldots)$ is defined as follows:

- $Tr(\ulcorner \neg\varphi \urcorner)$ = 'it is not the case that' $\frown$ $Tr(\ulcorner \varphi \urcorner)$;
- $Tr(\ulcorner \varphi \wedge \psi \urcorner)$ = 'it is both the case that' $\frown$ $Tr(\ulcorner \varphi \urcorner)$ $\frown$ 'and' $\frown$ $Tr(\ulcorner \psi \urcorner)$;
- $Tr(\ulcorner \exists x_i(\varphi) \urcorner)$ = $\ulcorner$there is an object$_i$ such that$\urcorner$ $\frown$ $Tr(\ulcorner \varphi \urcorner)$;
- $Tr(\ulcorner \exists xx_i(\varphi) \urcorner)$ = $\ulcorner$there are some objects$_i$ such that$\urcorner$ $\frown$ $Tr(\ulcorner \varphi \urcorner)$;
- $Tr(\ulcorner x_i = x_j \urcorner)$ = $\ulcorner$it$_i$ is identical to it$_j\urcorner$;
- $Tr(\ulcorner x_i \prec xx_j \urcorner)$ = $\ulcorner$it$_i$ is one of them$_j\urcorner$;

in addition, non-logical predicates are to be translated into English in accordance with their intended interpretations. As it might be, $Tr(\ulcorner P^1_1(x_i) \urcorner)$ = $\ulcorner$it$_i$ is red$\urcorner$, and $Tr(\ulcorner P^2_1(x_i, x_j) \urcorner)$ = $\ulcorner$it$_i$ is bigger than it$_j\urcorner$.

As an example, note that (15) turns out to abbreviate something equivalent to (16),

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(15) $\exists x_i \exists xx_j(x_i \prec xx_j)$;

(16) there is an object$_i$ and some objects$_j$ such that it$_i$ is one of them$_j$.

The expressions '$\rightarrow$', '$\leftrightarrow$', '$\forall$' and '$\vee$' are defined as usual. Also, we will sometimes use variables 'x', 'y', 'z', ... instead of 'x_1', 'x_2', ..., and variables 'xx', 'yy', 'zz', ... instead of 'xx_1', 'xx_2', ... Finally, we may use predicates such as 'Red' and 'Bigger' in place of the non-logical predicate letters such as 'P_1^1' or 'P_1^2', and define constants and non-logical function letters out of relations in the ordinary way. A formal semantics for PFO languages is provided in section 13.2.

2 Formal Characterization of PFO$^+$ languages

Let PFO$^+$ languages be the result of extending PFO languages with *plural predicate letters:* $\ulcorner \mathbf{P}_1^{\langle m,n \rangle} \urcorner$, $\ulcorner \mathbf{P}_2^{\langle m,n \rangle} \urcorner$, ... (for $0 \leq m$ and $0 < n$). We then add the following clause to our characterization of formulas:

- $\ulcorner \mathbf{P}_i^{\langle m,n \rangle}(x_1, \ldots x_m, xx_1, \ldots, xx_n) \urcorner$ is a formula.

Plural predicates are to be understood as collective English predicates, in accordance with their intended interpretations. As it might be, $Tr(\ulcorner \mathbf{P}_1^{\langle 0,1 \rangle}(xx_i) \urcorner)$ = $\ulcorner$they$_i$ are scattered$\urcorner$, and $Tr(\ulcorner \mathbf{P}_1^{\langle 1,2 \rangle}(x_i, xx_j, xx_k) \urcorner)$ = $\ulcorner$it$_i$ is between them$_j$ and them$_k \urcorner$. In practice we shall use predicates such as '**Scattered**' and '**Between**' (in bold font) in place of plural predicates such as '$\mathbf{P}_1^{\langle 0,1 \rangle}$' and '$\mathbf{P}_1^{\langle 1,2 \rangle}$'.

3 Generalized Quantifiers

Let $Tr(\varphi)$ be the following transformation, from a first-order language with generalized quantifiers to an appropriate PFO$^+$ language:

- $Tr(\ulcorner x_i = x_j \urcorner) = \ulcorner xx_i \precsim xx_j \wedge xx_j \precsim xx_i \urcorner$;
- $Tr(\ulcorner \mathrm{P}(x_1, \ldots, x_n) \urcorner) = \ulcorner \mathrm{P}(x_1, \ldots, x_n) \urcorner$;
- $Tr(\ulcorner \psi \wedge \xi \urcorner) = Tr(\ulcorner \psi \urcorner) \wedge Tr(\ulcorner \xi \urcorner)$;
- $Tr(\ulcorner \neg \psi \urcorner) = \neg Tr(\ulcorner \psi \urcorner)$;
- $Tr(\ulcorner \exists x_i(\psi) \urcorner) = \ulcorner \pi_{x_i}[Tr(\psi)] \precsim \pi_{x_i}[x_i = x_i] \urcorner$;

- $Tr(\ulcorner[\mathrm{Q}x_i : \xi](\psi)\urcorner) = \ulcorner\mathbf{Q}^*(\pi_{x_i}[Tr(\xi) \wedge Tr(\psi)], \pi_{x_i}[Tr(\xi)])\urcorner$
(where $\ulcorner[\mathrm{Q}x_i : \xi]\urcorner$ is the basic quantifier $\ulcorner$ Q of the things$_i$ that are $\xi\urcorner$, and $\ulcorner\mathbf{Q}^{*}\urcorner$ is the plural predicate corresponding to the determiner $\ulcorner\mathrm{Q}\urcorner$).

A rather cumbersome induction on the complexity of formulas shows that, if φ is a sentence from a first-order language with generalized quantifiers, then φ and $Tr(\varphi)$ are equivalent.

4 Definitions of Truth and Satisfaction for PFO$^+$ Languages

We work within a PFO$^+$ language. For the sake of simplicity, we assume that the domain of discourse of the metalanguage is the same as the domain of discourse of the object language. Let '**Assignment**(xx)' abbreviate the following:

$\forall y(y \prec xx \rightarrow \exists z(y = \langle v, z\rangle$ for v a variable$)) \wedge$
$\forall v$ (v is a singular variable $\rightarrow \exists! z(\langle v, z\rangle \prec xx)) \wedge$
$\forall v$ (v is a plural variable $\rightarrow \exists z(\langle v, z\rangle \prec xx))$

Next, define the satisfaction predicate '$\mathbf{Sat}(\varphi, yy)$' implicitly, by way of the following axioms:

- $\mathbf{Sat}(\ulcorner\neg\psi\urcorner, yy) \leftrightarrow \neg\mathbf{Sat}(\ulcorner\psi\urcorner, yy)$;
- $\mathbf{Sat}(\ulcorner\psi \wedge \theta\urcorner, yy) \leftrightarrow \mathbf{Sat}(\ulcorner\psi\urcorner, yy) \ \wedge \ \mathbf{Sat}(\ulcorner\theta\urcorner, yy)$;
- $\mathbf{Sat}(\ulcorner\exists x_i\psi\urcorner, yy) \leftrightarrow \exists tt\ [\mathbf{Assignment}(tt) \ \wedge \ \forall v$ ((v is a variable $\wedge\ v \neq \ulcorner x_i\urcorner) \rightarrow$
$\forall w(\langle v, w\rangle \prec yy \leftrightarrow \langle v, w\rangle \prec tt)) \ \wedge \ \mathbf{Sat}(\ulcorner\psi\urcorner, tt)]$;
- $\mathbf{Sat}(\ulcorner\exists xx_i\psi\urcorner, yy) \leftrightarrow \exists tt\ [\mathbf{Assignment}(tt) \ \wedge \ \forall v$ ((v is a variable $\wedge\ v \neq \ulcorner xx_i\urcorner) \rightarrow$
$\forall w(\langle v, w\rangle \prec yy \leftrightarrow \langle v, w\rangle \prec tt)) \ \wedge \ \mathbf{Sat}(\ulcorner\psi\urcorner, tt)]$;
- $\mathbf{Sat}(\ulcorner x_i \prec xx_j\urcorner, yy) \leftrightarrow \forall z(\langle\ulcorner x_i\urcorner, z\rangle \prec yy \rightarrow \langle\ulcorner xx_i\urcorner, z\rangle \prec yy)$;
- $\mathbf{Sat}(\ulcorner\mathrm{P}(x_i)\urcorner, yy) \leftrightarrow \forall z(\langle\ulcorner x_i\urcorner, z\rangle \prec yy \rightarrow \mathrm{P}^*(z))$, where '$\mathrm{P}^*(\ldots)$' is a translation of '$\mathrm{P}(\ldots)$' into the metalanguage;
- $\mathbf{Sat}(\ulcorner\mathbf{P}(xx_i)\urcorner, yy) \leftrightarrow \forall zz\forall w((\langle\ulcorner xx_i\urcorner, w\rangle \prec yy \leftrightarrow w \prec zz) \rightarrow \mathbf{P}^*(zz))$, where '$\mathbf{P}^*(\ldots)$' is a translation of '$\mathbf{P}(\ldots)$' into the metalanguage.

Finally, truth is defined in terms of satisfaction in the usual way:

$\textsc{True}(\varphi) \equiv_{def} \forall yy\ (\mathbf{Assignment}(yy) \rightarrow \mathbf{Sat}(\varphi, yy))$.

References

Barwise, J., and R. Cooper (1981) "Generalized Quantifiers and Natural Language," *Ling. Phil.* 4, 159–220.

Bernays, P. (1942) "A System of Axiomatic Set Theory, IV," *Journal of Symbolic Logic* 7:4, 133– 145.

Boolos, G. (1984) "To Be is to Be a Value of a Variable (or to be Some Values of Some Variables)," *The Journal of Philosophy* 81, 430–49. Reprinted in George Boolos, *Logic, Logic and Logic.*

Boolos, G. (1985a) "Nominalist Platonism," *Philosophical Review* 94, 327–44. Reprinted in George Boolos, *Logic, Logic and Logic.*

Boolos, G. (1985b) "Reading the *Begriffsschrift*," *Mind* 94, 331–334. Reprinted in George Boolos, *Logic, Logic and Logic.*

Boolos, G. (1998a) *Logic, Logic and Logic*, Harvard, Cambridge, Massachusetts.

Boolos, G. (1998b) "A Reply to Parsons' "Sets and Classes"." Reprinted in George Boolos, *Logic, Logic and Logic.*

Cartwright, R. (1994) "Speaking of Everything," *Noûs* 28:1, 1–20.

Davidson, D. (1967) "The Logical Form of Action Sentences." Reprinted in Donald Davidson, *Essays on Actions and Events.*

Davidson, D. (1980) *Essays on Actions and Events*, Clarendon Press, Oxford.

Dummett, M. (1981) *Frege: Philosophy of Language*, Harvard, Cambridge, MA, second edition.

Heck, R. (2000) "Syntactic Reductionism," *Philosophia Mathematica* 8:3, 124–149.

Higginbotham, J. (1998) "Higher-Order Logic and Natural Language," *Proceedings of the British Academy* 95, 1–27.

Higginbotham, J., and B. Schein (1989) "Plurals," *Nels XIX Proceedings, GLSA University of Massachusetts, Amherst*.

Kreisel, G. (1967) "Informal Rigour and Completeness Proofs." In Irme Lakatos, *Problems in Philosophy of Mathematics.*

Lakatos, I., ed. (1967) *Problems in the Philosophy of Mathematics*, North Holland, Amsterdam.

Macià-Fabrega, J. (unpublished) "Plural Quantification and Second-Order Quantification."

McGee, V. (1997) “How We Learn Mathematical Language,” *Philosophical Review* 106:1, 35–68.

McGee, V. (2000) “Everything.” In Gila Sher and Richard Tieszen, *Between Logic and Intuition*.

McGee, V., and A. Rayo (2000) “A puzzle about *de rebus* belief,” *Analysis* 60:4, 297–299.

Parsons, C. (1974) “Sets and Classes,” *Noûs* 8, 1–12. Reprinted in Charles Parsons, *Mathematics in Philosophy*.

Parsons, C. (1983) *Mathematics in Philosophy*, Cornell University Press, Ithaca, NY.

Quine, W. V. (1948) “On what there is.” Reprinted in Willard V. Quine, *From a Logical Point of View*.

Quine, W. V. (1953) *From a Logical Point of View*, Harvard, Cambridge, Massachusetts.

Quine, W. V. (1960) *Word and Object*, MIT Press, Cambridge, Massachusetts.

Quine, W. V. (1986) *Philosophy of Logic, Second Edition*, Harvard, Cambridge, Massachusetts.

Rayo, A. (forthcoming) “Frege’s Unofficial Arithmetic,” *The Journal of Symbolic Logic*.

Rayo, A., and G. Uzquiano (1999) “Toward a Theory of Second-Order Consequence,” *The Notre Dame Journal of Formal Logic* 40:3.

Rayo, A., and S. Yablo (2001) “Nominalism Through De-Nominalization,” *Noûs* 35:1, 74–92.

Schein, B. (1993) *Plurals and Events*, MIT Press, Cambridge, Massachusetts.

Shapiro, S. (1987) “Principles of Reflection and Second-Order Logic,” *Journal of Philosophical Logic* 16, 309–333.

Shapiro, S. (1991) *Foundations Without Foundationalism: A Case for Second-Order Logic*, Clarendon Press, Oxford.

Sharvy, R. (1980) “A More General Theory of Definite Descriptions,” *The Philosophical Review* 89:4, 607–624.

Sher, G., and R. Tieszen, eds. (2000) *Between Logic and Intuition*, Cambridge University Press, New York and Cambridge.

Yi, B.-U. (1999) “Is Two a Property?” *Journal of Philosophy* 96:4, 163–190.

Yi, B.-U. (unpublished) “The Language and Logic of Plurals.”