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PARTS OF SINGLETONS*

Here is a view that we like: Cary is a part—not just a *member*, but a *part*—of his singleton, the set whose sole member is Cary. Here is another view that we like: the mereological difference between Cary and his singleton—what you get when you subtract Cary from his singleton, so to speak—is the empty set. Putting these views together, we get the view that Cary's singleton is composed of Cary and the empty set: or, more suggestively, that {Cary} is composed of Cary and {}. We like this view, too.

On this view, you get Cary's singleton by adding the empty set to Cary. If in general you get the singleton of something by adding the empty set to it, then you get the singleton of Cary's singleton—the set whose sole member is Cary's singleton—by adding the empty set to Cary's singleton. But the empty set is already a part of Cary's singleton. So, it turns out, Cary's singleton and *its* singleton are both composed of Cary and the empty set. This violates a principle of classical mereology, according to which no two things are ever composed of the same parts. So, if, like us, you like the view that Cary's singleton is composed of Cary and the empty set, then you are going to need a nonclassical mereology.

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In this paper, we develop a view on which Cary's singleton is composed of Cary and the empty set.[1] In section I, we present a nonclassical mereology, due to Kit Fine, and explain how it can be used to vindicate the claim that Cary's singleton is composed of Cary and the empty set.[2] In section II, we compare this view, which we call *the Fine view*, with David Lewis's view, according to which neither Cary nor the empty set is a part of Cary's singleton, since Cary's singleton has no proper parts.[3] In the appendix, we recover Zermelo set theory from the Fine view.

I. FINE

I.1. Compounds, Aggregates, and Other Fusions. One way of doing mereology, the theory of wholes and their parts, is to take the *parthood* relation as primitive and use it to define other mereological relations.[4] Parthood is reflexive (everything is a part of itself), transitive (if one thing is a part of a second thing, which in turn is a part of a third thing, then the first thing is a part of the third thing), and anti-symmetric (if two things are parts of each other, then they are identical). Given parthood, we can define the *overlap* relation: two things overlap just in case something is a part of both of them. To take an everyday example, suppose that Cary is making bruschetta and Kate is making buttered toast. Things go comically awry in the dark, and it turns out that they have used the same piece of bread. In that case, Cary's bruschetta and Kate's buttered toast overlap, since the piece of bread is a part of both. Given parthood and overlap, we can define the *composition* relation: some things compose a thing just in case each of those things is a part of it and every part of it overlaps one of them. Suppose that, after the toast debacle, Cary makes a rudimentary ham sandwich out of two pieces of rye bread and a slice of ham. As a first pass, the bread and the ham compose the ham sandwich, because they are all parts of it and every part of it overlaps one of them. Given composition, we can define the *fusion* relation: something is a fusion of some things just in case they compose it. If the bread and

[1] We are inclined to take sets to be *sui generis* entities. The view that we develop in this paper is our second-favorite view. For a defense of the view that sets are *sui generis* entities, see Alexander Paseau, "Motivating Reductionism about Sets," *Australasian Journal of Philosophy*, LXXXVI, 2 (June 2008): 295–307.

[2] See Fine, "Things and Their Parts," *Midwest Studies in Philosophy*, XXIII, 1 (1999): 61–74.

[3] See Lewis, *Parts of Classes* (Cambridge: Blackwell, 1991), and "Mathematics is Megethology," *Philosophia Mathematica*, I, 3 (1993): 3–23, the latter of which is reprinted in his *Papers in Philosophical Logic* (New York: Cambridge, 1997), pp. 203–29. A proper part of something is a part of it that is not identical to it.

[4] For a different way of doing mereology, one that takes mereological operations to be more fundamental than mereological relations, see Fine, "Towards a Theory of Part," forthcoming in this JOURNAL.

the ham compose the ham sandwich, then the ham sandwich is a fusion of them. And, finally, we can define what it is for something to be a fusion *tout court*: something is a fusion *tout court* just in case there are some things such that it is a fusion of them. If the ham sandwich is a fusion *of* the bread and the ham, then the ham sandwich is a fusion *tout court*.[5]

In "Compounds and Aggregates," Fine distinguishes two kinds of fusion: namely, *compounds* and *aggregates*.[6] Compounds and aggregates are distinguished by their existence conditions. A compound of some things is a fusion of those things that exists at a time if and only if *all* of them exist then, whereas an aggregate of some things is a fusion of those things that exists at a time if and only if *any* of them exists then. If you are a fusion of temporal parts and you exist right now even though only some of your temporal parts do, then you are an aggregate rather than a compound of those temporal parts.[7] By contrast, if a quantity of gold is a fusion of two sub-quantities of gold, and if it exists only when both sub-quantities exist, then it is a compound rather than an aggregate of those sub-quantities.[8]

[5] The definitions in this paragraph come from Lewis, *Parts of Classes*, pp. 72–74, and Peter van Inwagen, "Can Mereological Sums Change Their Parts?" this JOURNAL, CIII, 12 (December 2006): 614–30, at pp. 618–20. In "Towards a Theory of Part," Fine denies the connection between *fusion of* and *fusion tout court*. On his view, something is a fusion *tout court* only if its identity is explained by its being generated by a nonstructural method of composition. (See also his "Compounds and Aggregates," *Noûs*, XXVIII, 2 (June 1994): 137–58, at p. 137.) But all sides can distinguish *fusions in the broad sense* (where *x* is a fusion in the broad sense if there are one or more *y*s such that *x* is a fusion of the *y*s, say) and *mere fusions* (where *x* is a mere fusion only if there are one or more *y*s such that *x*'s identity is explained by *x*'s being generated from the *y*s by a nonstructural method of composition, say). The disagreement is about whether 'fusion (*tout court*)' picks out fusions in the broad sense or mere fusions. (For a parallel distinction between "unloaded" and "loaded" senses of 'fusion', see Theodore Sider, "Replies to Gallois, Hirsch and Markosian," *Philosophy and Phenomenological Research*, LXVIII, 3 (May 2004): 674–87, at pp. 676–77.) We suspect that nothing hangs on this. In the text, we follow van Inwagen rather than Fine and use 'fusion (*tout court*)' to pick out fusions in the broad sense. On this use, everything is a fusion *tout court*, since everything is a fusion of itself.

[6] Judith Jarvis Thomson's distinction between all-fusions and some-fusions is a temporally relativized analogue of Fine's distinction between compounds and aggregates. See her "The Statue and the Clay," *Noûs*, XXXII, 2 (June 1998): 149–73, at pp. 158–61, 166–67.

[7] See Fine, "Compounds and Aggregates," p. 138. (But Fine doubts that there are any aggregates. See *ibid.*, pp. 154–57.) On the view that you are a fusion of temporal parts, see, for example, Lewis, "Survival and Identity," in Amelie O. Rorty, ed., *The Identities of Persons* (Berkeley: California UP, 1976), pp. 17–40, and *On the Plurality of Worlds* (Malden, MA: Blackwell, 1986), pp. 202–04, the former of which is reprinted in his *Philosophical Papers*, vol. 1 (New York: Oxford, 1983), pp. 55–77. But not everyone who holds that view says that, strictly speaking, you exist now. See, for example, Mark Heller, *The Ontology of Physical Objects: Four-Dimensional Hunks of Matter* (New York: Cambridge, 1990), pp. 12–14.

[8] See Fine, "Compounds and Aggregates," pp. 139–45.

But, it seems, compounds and aggregates do not exhaust all the fusions there are. For example, the ham sandwich is a fusion of the bread and the ham, but it does not seem to be either a compound or an aggregate of them. If the ham sandwich were an aggregate of the bread and the ham, then it might be the case that it came into existence as soon as the bread came into existence—even if ham had yet to be discovered. And, if the ham sandwich were a compound of the bread and the ham, then it would continue to exist as long as the bread and the ham continue to exist—even if they were scattered to three of the four corners of the earth. Insofar as the existence of the bread and the ham at a time seems to be necessary for the existence of the ham sandwich then, the ham sandwich does not seem to be an aggregate of the bread and the ham. But, insofar as the existence of the bread and the ham at a time does not seem to be sufficient for the existence of the ham sandwich then, the ham sandwich does not seem to be a compound of the bread and the ham either. For the ham sandwich to exist at a time, it seems that it is not enough for the bread and the ham to exist then; it seems that they also need to be arranged ham-sandwich-wise, as it were.[9]

In "Things and Their Parts," Fine proposes that the ham sandwich is a new kind of fusion, one that exists at a time if and only if the bread and the ham stand in a suitable relation then.[10] He denies that this new kind of fusion is a state of affairs. He says that "it is not a fact or state. It is a whole, whose components are linked by the relation, rather than the fact or state of the components being so linked."[11] Fine proposes that the relation is a further part of the ham sandwich.[12]

[9] Some philosophers think that the object that is now a ham sandwich existed before the bread and the ham were arranged ham-sandwich-wise; it is just that it was not a ham sandwich then. (See, for example, Achille C. Varzi, "Perdurantism, Universalism, and Quantifiers," *Australasian Journal of Philosophy*, LXXXI, 2 (June 2003): 208–15.) On their view, the ham sandwich might well be a compound of the bread and the ham. But this is not our view, since we think that everything—quantifying unrestrictedly—that is a ham sandwich now is a ham sandwich at every time at which it exists.

[10] See also Fine, "Acts, Events and Things," in Werner Leinfellner, Eric Kraemer, and Jeffrey Schank, eds., *Language and Ontology: Proceedings of the Sixth International Wittgenstein Symposium, 23–30 August 1981* (Vienna: Hölder-Pichler-Tempsky, 1982), pp. 97–105, and "Coincidence and Form," *Proceedings of the Aristotelian Society Supplementary Volume*, LXXXII, 1 (June 2008): 101–18. Also see Mark Johnston, "Hylomorphism," this JOURNAL, CIII, 12 (December 2006): 652–98; and Kathrin Koslicki, *The Structure of Objects* (New York: Oxford, 2008).

[11] Fine, "Things and Their Parts," p. 65.

[12] In "Things and Their Parts," Fine says that "[i]ntuitively an attribute—a property or relation—is a part of a whole that is generated when one or more objects instantiate that attribute" (p. 65). But he seems more hesitant about this claim than he is about the claim that the objects that instantiate that attribute are parts of the whole. For example, he says that we "certainly want" to adopt an axiom according to which the objects are

For example, if you list the bread and the ham, you have not listed all the ingredients in the ham sandwich: you have forgotten the *being arranged ham-sandwich-wise* relation.[13]

I.2. Perdurantism. Perdurantists—those who think that ordinary physical objects persist by having different temporal parts located at different times—who do not want to countenance Fine's new kind of fusion might say that the ham sandwich exists at a time if and only

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parts of the whole, whereas he says merely that we "may also wish" to adopt an axiom according to which the attribute is a part of the whole (*ibid.*, p. 66). In the text, we follow Fine's official view, on which attributes are parts. If this aspect of Fine's official view were dropped, we would no longer be able to say, as we do in the text, that the mereological difference between Cary and his singleton is the empty set. (Thanks to David Sanson here.) But we suspect that we could still say that Cary is a part of his singleton and that we could still provide an account of the membership relation. (Even if the attributes that are used to generate wholes from certain parts are not themselves parts of the wholes that are thereby generated, we could still distinguish those parts from other parts. Along these lines, see Johnston, "Hylomorphism.") We leave this as an exercise, either for the reader or for the authors.

On properties and relations as parts, see also Koslicki, *op. cit.* And see D. C. Williams, "On the Elements of Being," *Review of Metaphysics*, VII, 1 (September 1953): 3–18, at pp. 8–10 and VII, 2 (December 1953): 171–92, and "Necessary Facts," *Review of Metaphysics*, XVI, 4 (June 1963): 601–26, at pp. 603–05, the former of which is reprinted as "The Elements of Being" in his *Principles of Empirical Realism: Philosophical Essays* (Springfield, IL: Thomas, 1966), pp. 74–109; Douglas Ehring, "Temporal Parts and Bundle Theory," *Philosophical Studies*, CIV, 2 (May 2001): 163–68; and Kris McDaniel, "Tropes and Ordinary Physical Objects," *Philosophical Studies*, CIV, 3 (June 2001): 269–90. But Williams et al. take the properties and relations to be tropes rather than universals. (Lewis discusses but does not endorse such a view. See *On the Plurality of Worlds*, pp. 64–68.)

[13] Might the ham sandwich be a compound of the bread, the ham, and the relation? Not if their existence is not sufficient for the existence of the ham sandwich, and not if the bread and the ham have to stand in that relation for the ham sandwich to exist. If polyadic tropes have their relata essentially and exist only if instantiated, then the ham sandwich might be a compound of the ham, the bread, and a polyadic trope. (See Fine, "Things and Their Parts," pp. 63–64. Thanks to Kris McDaniel here.) But it is not clear that polyadic tropes have their relata essentially (although, for a view on which at least some do, see Anna-Sofia Maurin, *If Tropes* (Boston: Kluwer, 2002), p. 164). And it is often assumed that, if there were tropes, they would be sparse. (See, for example, Lewis, *On the Plurality of Worlds*, pp. 66–67; D. M. Armstrong, *Universals: An Opinionated Introduction* (Boulder: Westview, 1989), p. 119; and Keith Campbell, *Abstract Particulars* (Cambridge: Blackwell, 1990), pp. 24–26.) There might be sparse tropes at various levels: microphysical, chemical, biological, psychological. (See Jonathan Schaffer, "Two Conceptions of Sparse Properties," *Pacific Philosophical Quarterly*, LXXXV, 1 (March 2004): 92–102.) But it is not clear whether *being arranged ham-sandwich-wise* tropes would be among the sparse tropes at any level.

Incidentally, in "Things and Their Parts," Fine suggests that the relevant attribute is the *betweenness* relation (p. 67); but this is a toy aspect of the example. If you slide a package of ham between two sliced loaves of bread, you have not thereby made a ham sandwich, even if the slice of ham stands in the *betweenness* relation to the pieces of bread. (We owe this counterexample to Cynthia Read and Donovan Hulse.) Fine speaks of "simplifying assumptions" (*ibid.*, p. 67); presumably his choice of the *betweenness* relation is one such simplification.

if it has a temporal part at that time, and something is a temporal part of the ham sandwich at that time if and only if it is a ham-sandwich stage: that is, if and only if it is appropriately related, via the continuity relation for ham sandwiches, to other temporal parts of the ham sandwich.[14] Not every fusion of temporal parts of the bread and the ham need be a ham-sandwich stage, so the ham sandwich need not exist at a time when the bread and the ham are massively scattered, say.

But perdurantism cannot capture ordinary parthood claims. Ordinarily, one would say that the bread is a part of the ham sandwich. But, according to perdurantism, that claim is, strictly speaking, false. For the bread is too big to be a part of the ham sandwich: if the bread exists before or after the ham sandwich, then the bread has temporal parts that are not parts of the ham sandwich, in which case the bread is not a part of the ham sandwich. And, although we realize that not all metaphysicians will agree with us on this methodological point, we think that, other things being equal, it is a good thing for a theory to be able to capture such ordinary parthood claims.

Perdurantists who want to capture ordinary parthood claims could reply that, although the bread is not a part of the ham sandwich *tout court*, the bread is a part of the ham sandwich *now*, and that is enough to capture the ordinary claim that the bread is a part of the ham sandwich. To derive the claim that the bread is a part of the ham sandwich *now*, perdurantists can help themselves to the following principle linking the two-place, atemporal parthood relation (call it $part_2$) to the three-place, temporally relativized parthood relation (call it $part_3$).

> *Link*: For any things x and y and any time t, x is a part_3 of y at t if and only if x's temporal part at t is a part_2 of y.[15]

Since the bread's current temporal part is a part_2 of the ham sandwich, Link entails that the bread is now a part_3 of the ham sandwich.

Fine anticipates something like this reply.[16] In reply to the reply, he says something like the following. It is true that, with Link, perdurantists can say that the bread is now a part_3 of the ham sandwich, but, with Link, perdurantists also have to say that the fusion of the bread and Cleopatra is now a part_3 of the ham sandwich. For, given that Cleopatra does not exist now, the current temporal part of the fusion of the bread and Cleopatra just is the current temporal part of the bread, and that temporal part is a part_2 of the ham sandwich. If the claim that the

[14] On perdurantism, see note 7.

[15] For a similar principle, see Sider, *Four-Dimensionalism: An Ontology of Persistence and Time* (New York: Oxford, 2001), p. 57.

[16] Fine, "Things and Their Parts," p. 63.

bread is now a $part_3$ of the ham sandwich captures the ordinary claim that the bread is a part of the ham sandwich, then the claim that the fusion of the bread and Cleopatra is now a $part_3$ of the ham sandwich would capture the ordinary claim that the fusion of the bread and Cleopatra is a part of the ham sandwich. But that is not a claim that we want to capture.

Perdurantists might reply that, in most ordinary contexts, the domain of quantification is restricted to what we ordinarily think about, so the domain includes familiar things like the bread but excludes unfamiliar things like the fusion of the bread and Cleopatra.[17] Link, understood in this restricted way, licenses the conclusion that the bread is now a $part_3$ of the ham sandwich, but it does not similarly license the conclusion that the fusion of the bread and Cleopatra is now a $part_3$ of the ham sandwich.

But suppose that one asks ordinary speakers, "Is the fusion of the bread and Cleopatra a part of the ham sandwich?" In that case, since one is explicitly discussing the fusion of the bread and Cleopatra, the domain of quantification is sufficiently unrestricted to include that fusion. Ordinary speakers are likely to answer, "No, it is not." And, once the domain of quantification is sufficiently unrestricted to include the fusion of the bread and Cleopatra, Link licenses the conclusion that the fusion of the bread and Cleopatra is now a $part_3$ of the ham sandwich. So perdurantists cannot appeal to Link and quantifier domain restriction to capture the ordinary claim that the bread is a part of the ham sandwich but the fusion of the bread and Cleopatra is not.[18]

I.3. Fine Fusions. So we think that perdurantists who want to capture ordinary parthood claims cannot escape the need for Fine's new kind of fusion. But what can we say about fusions of this new kind? Suppose that some things, the *x*s, instantiate an attribute (a property or relation), *R*. On Fine's view, a new object—the *x*s' instantiating *R*—might come into existence. Fine uses 'the *x*s/*R*' to refer to this new object.[19] On his view, the *x*s and *R* are parts of

[17] See, for example, Lewis, *On the Plurality of Worlds*, p. 213, and *Parts of Classes*, p. 80.

[18] For a similar reply to Lewis's defense of Unrestricted Composition, see Ned Markosian, "Restricted Composition," in John Hawthorne, Sider, and Dean Zimmerman, eds., *Contemporary Debates in Metaphysics* (Malden, MA: Blackwell, 2008), pp. 341–63, at pp. 344–45. This point was first made to us, independently, by Kristian Olsen. (On Unrestricted Composition, see below in the text at note 29.)

[19] See Fine, "Things and Their Parts," p. 65. We use plural expressions (for example, 'the *x*s') where Fine uses a list ('*a*, *b*, *c*, ...'). On Fine's view, order can matter: for example, *a*, *b*/*loving* ≠ *b*, *a*/*loving*. (This follows from (R3). See *ibid.*, p. 66.) But, in the cases discussed in the text, the attribute is more of a group property or a neutral relation (for example, *being arranged ham-sandwich-wise*) than a relation with a direction (for example, *loving*); so order does not matter; so there is no harm in using plural

the *x*s/*R*.[20] The *x*s/*R* exists at a time *t* if and only if the *x*s instantiate *R* at *t*.[21] (As a limiting case, there is only one of the *x*s and *R* is a one-place attribute: namely, a property.[22] In this case, *x*/*F* exists at a time *t* if and only if *x* instantiates *F* at *t*.[23]) Let us say that the *x*s/*R* is a *Fine fusion*.[24] The *x*s and *R* are privileged parts of the *x*s/*R*; let us call them *Fine parts* of the *x*s/*R*.[25] Let us say that the *x*s are the *material* Fine parts of the *x*s/*R* and that *R* is the *formal* Fine part of the *x*s/*R*.[26] (Given Fine parts, we can define *Fine composition*: the *x*s Fine-compose *y* if and only if *y* is a Fine fusion and the *x*s are the Fine parts of *y*.)

The ham sandwich is a Fine fusion: namely, the bread and the ham/*being arranged ham-sandwich-wise*. The ham sandwich exists at a time if and only if the bread and the ham stand in the *being arranged ham-sandwich-wise* relation then. The bread, the ham, and the relation are Fine parts of the ham sandwich: the relation is a formal Fine part, and the bread and the ham are material Fine parts. Not every part of a

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expressions. On neutral relations, see Fine, "Neutral Relations," *Philosophical Review*, CIX, 1 (January 2000): 1–33. (In correspondence, Fine has indicated that, although neutral relations are not an official part of the theory of "Things and Their Parts," he is sympathetic to this use of them.) Although in the text we discuss Fine fusions whose formal Fine parts are neutral relations, we are open to there being Fine fusions whose formal Fine parts are non-neutral relations. In addition to helping ourselves to plural expressions, we are also helping ourselves to plural comprehension. On plural expressions (and plural comprehension), see, for example, George Boolos, "To Be Is to Be a Value of a Variable (Or to Be Some Values of Some Variables)," this JOURNAL, LXXXI, 8 (August 1984): 430–49, which is reprinted in his *Logic, Logic, and Logic*, ed. Richard Jeffrey (Cambridge: Harvard, 1998), pp. 54–72; Lewis, *Parts of Classes*, pp. 62–71; and Thomas McKay, *Plural Predication* (New York: Oxford, 2006).

[20] This is (R4) and (R5). See Fine, "Things and Their Parts," pp. 66–67.

[21] This is (R1). See *ibid.*, p. 66. The claim in the text might need to be qualified. See the discussion of Unrestricted Fine Composition below in the text at note 45.

[22] See *ibid.*, p. 67. A variant on this limiting case, one that is important for our purposes later, would have the attribute be a multigrade attribute that can be instantiated by a single thing. On such attributes, see note 52.

[23] Fine's examples include airline passengers, mayors, and judges. See *ibid.*, pp. 67–68. One might add to this list the structures-in-use with which Robert Howell and Robert Stecker identify musical works. See Howell, "Types, Indicated and Initiated," *British Journal of Aesthetics*, XLII, 2 (April 2002): 105–27, at pp. 115–26, and Stecker, *Interpretation and Construction: Art, Speech, and the Law* (Malden, MA: Blackwell, 2003), pp. 81–92. In correspondence, Fine has indicated that at the time he also had in mind the application to musical works. See also Fine, "The Problem of Non-Existents: I. Internalism," *Topoi*, I, 1–2 (December 1982): 97–140 on fictional characters.

[24] Fine calls it a "rigid embodiment." See "Things and Their Parts," p. 65.

[25] Fine parts are nice, in Lewis's sense: Fine fusions uniquely decompose into their Fine parts. See Lewis, *Parts of Classes*, p. 22.

[26] Fine speaks, hylomorphically, of the *x*s as the "matter" and of *R* as the "form." See "Things and Their Parts," p. 65.

Fine fusion (in the ordinary sense of 'part') is a Fine part of it (in the technical sense of 'Fine part' introduced above). For example, various molecules that are parts of the bread or the ham are also parts of the ham sandwich (since parthood is transitive and the bread and the ham are parts of the ham sandwich), but they are not Fine parts of it.

Fine lays down some axioms for Fine fusions.[27] Among other things, they are located where their material Fine parts are.[28] The ham sandwich is not where the *being arranged ham-sandwich-wise* relation is (wherever that is); rather, it is where the bread and the ham are.

I.4. Axioms. Fine's mereology contains axioms governing fusions and parthood. It also contains axioms governing Fine fusions and the *being a Fine part of* relation. But which axioms does it contain? In addressing this question, we take Lewis's mereology, which is equivalent to classical extensional mereology, as our starting point.[29]

Lewis's mereology has the following three axioms governing fusions and parthood.

> *Transitivity*: If one thing is a part of a second thing, which in turn is a part of a third thing, then the first thing is a part of the third thing.
>
> *Unrestricted Composition*: Any things have at least one fusion.
>
> *Uniqueness of Composition*: Any things have at most one fusion.[30]

The other formal properties of parthood that we mentioned above—namely, reflexivity and anti-symmetry—are consequences of these axioms.[31]

Fine's mereology does not contain all of these axioms. In "Towards a Theory of Part," Fine accepts Transitivity. But Unrestricted Composition and Uniqueness of Composition are not axioms of his mereology. As we will see, the falsehood of Uniqueness of Composition follows from another axiom that Fine should accept. And Fine suggests that, to unify mereology and set theory, we might have to reject Unrestricted Composition in favor of something like the following axiom.

[27] Fine, "Things and Their Parts," pp. 66–67.

[28] This is (R2). See *ibid.*, p. 66.

[29] See *Parts of Classes,* p. 74. On classical extensional mereology, see Peter Simons, *Parts: A Study in Ontology* (New York: Oxford, 1987), pp. 37–41.

[30] Uniqueness of Composition is somewhat misnamed: by itself, it does not entail that, for any things, those things have exactly one fusion, since it allows that some things do not have any fusion at all. But, together, Unrestricted Composition and Uniqueness of Composition entail that, for any things, those things have exactly one fusion.

[31] See Gabriel Uzquiano, "Unrestricted Unrestricted Quantification," in Agustín Rayo and Uzquiano, eds., *Absolute Generality* (New York: Oxford, 2006), pp. 305–32, at p. 312n18.

> *Iterative Composition*: Any things have at least one fusion if and only if there is some level in the cumulative hierarchy in which they all appear.[32]

The cumulative hierarchy is a series of levels. In this case, mereological atoms appear in Level 0; fusions of things that appear in Level 0 appear in Level 1; fusions of things that appear in Levels 0 and 1 appear in Level 2; and so on.[33] (The standard objection to restrictions on composition is that they are vague and hence lead to vague existence.[34] But this objection does not apply to Iterative Composition, since it is not vague whether there is some level in the cumulative hierarchy in which some things all appear.) Without Unrestricted Composition and Uniqueness of Composition, we cannot derive the reflexivity or the anti-symmetry of parthood. If we want those properties, then we have to get them some other way.[35]

Lewis's axioms are formulated for fusions and parthood. Parallel axioms could be formulated for Fine fusions and the *being a Fine part of* relation.

> *Fine Transitivity*: If one thing is a Fine part of a second thing, which in turn is a Fine part of a third thing, then the first thing is a Fine part of the third thing.

> *Unrestricted Fine Composition*: For any things and any attribute, if those things instantiate that attribute, then there is a Fine fusion that has those things as its material Fine parts and that has that attribute as its formal Fine part.[36]

[32] See Fine's replies to critics of *The Limits of Abstraction* (New York: Oxford, 2002) in *Philosophical Studies*, CXXII, 3 (February 2005): 367–95, at pp. 367–74. The axiom is formulated in Uzquiano, "Unrestricted Unrestricted Quantification," p. 323. Alternatively, we could replace Unrestricted Composition with the following axiom, which is based on limitation of size principles in set theory:

> *Limitation of Composition*: Any things have at least one fusion if and only if they are few.

Things are few if and only if there is no one-one mapping from their mereologically atomic parts to everything there is. (A mereological atom is something that has no proper parts.) This formulation of the axiom comes from Uzquiano, "The Price of Universality," *Philosophical Studies*, CXXIX, 1 (May 2006): 137–69, at p. 153, and "Unrestricted Unrestricted Quantification," p. 322n40. For a prior formulation, one based on Lewis, *Parts of Classes*, pp. 88–91, see Gideon Rosen, "Armstrong on Classes as States of Affairs," *Australasian Journal of Philosophy*, LXXIII, 4 (December 1995): 613–25, at pp. 622–23n18. Rosen attributes the idea to John Bigelow.

[33] For a more careful statement of the cumulative hierarchy (albeit for Fine fusions rather than fusions more generally), see the appendix.

[34] See Lewis, *On the Plurality of Worlds*, pp. 212–13; and Sider, *Four-Dimensionalism*, pp. 120–32.

[35] For example, in "Towards a Theory of Part," Fine derives the transitivity and the reflexivity of parthood from his definition of part (in terms of mereological operations), and he derives the anti-symmetry of parthood from that definition and an axiom (Anti-Cyclicity) governing those operations.

[36] A cursory reading of Fine's axiom (R1) suggests something like Unrestricted Fine Composition. See "Things and Their Parts," p. 66.

> *Uniqueness of Fine Composition*: For any things and any attribute, if those things instantiate that attribute, then there is at most one Fine fusion that has those things as its material Fine parts and that has that attribute as its formal Fine part.[37]

Uniqueness of Fine Composition is an axiom of Fine's mereology.[38] But Fine Transitivity and Unrestricted Fine Composition are not. First, Fine Transitivity is false. Suppose that the ham has Fine parts, for example, the *being arranged slice-of-ham-wise* relation. Although the ham is a Fine part of the ham sandwich, the *being arranged slice-of-ham-wise* relation is not. So Fine Transitivity is false.[39]

Some think that this denial of Fine Transitivity is conceptually incoherent. For example, Kris McDaniel holds that, as a matter of conceptual necessity, any fundamental parthood relation must be transitive and obey some kind of remainder principle.[40] Following McDaniel, we will say that a relation is a *fundamental* parthood relation if and only if (i) it is a parthood relation, and (ii) it is not analyzable in terms of some other parthood relation.[41] We hold that the *being a Fine part of* relation is a fundamental parthood relation in this sense. But, although we hold that parthood is transitive, we hold that the *being a Fine part of* relation is not. If McDaniel is right, this is conceptually incoherent, since, as a matter of conceptual necessity, no fundamental parthood relation can fail to be transitive.

But we are suspicious of such claims of conceptual necessity. Parthood is transitive, and it might even be so as a matter of conceptual necessity. But it does not follow that, as a matter of conceptual necessity, every fundamental parthood relation is transitive. In particular, even if we are right that the *being a Fine part of* relation is a fundamental parthood

[37] Uniqueness of Fine Composition is somewhat misnamed for the same reason that Uniqueness of Composition is. See note 30.

[38] Uniqueness of Fine Composition follows from (R3). See Fine, "Things and Their Parts," p. 66.

[39] The *being a material Fine part of* relation is not transitive either. If the slice of ham is some molecules/*being arranged slice-of-ham-wise*, then each of those molecules is a material Fine part of the slice of ham; and the slice of ham is a material Fine part of the ham sandwich; but none of the molecules is a material Fine part of the ham sandwich.

[40] See McDaniel, "Structure-Making," *Australasian Journal of Philosophy*, LXXXVII, 2 (June 2009): 251–74, at p. 264. For similar claims about conceptually necessary conditions on parthood, see Simons, *Parts*, p. 26, cf. pp. 10–11; Koslicki, *The Structure of Objects*, pp. 167–68; and Varzi, "The Extensionality of Parthood and Composition," *Philosophical Quarterly*, LVIII, 230 (January 2008): 108–33, at p. 110, and "Universalism Entails Extensionalism," *Analysis*, LXIX, 4 (October 2009): 599–604, at p. 601.

[41] See "Structure-Making," p. 254. In McDaniel's sense, a parthood relation does not fail to be fundamental merely by being analyzable in terms of other mereological relations. So, for instance, if classical extensional mereology is correct, parthood does not fail to be fundamental merely because it is analyzable in terms of proper parthood, or overlap, or fusion, or what have you. Thanks here to Cody Gilmore.

relation, it does not follow that, as a matter of conceptual necessity, the *being a Fine part of* relation is transitive.

So we are prepared to deny something that McDaniel takes to be conceptually necessary. This position is not as embarrassing for us as it might seem, since we are not alone. To see how someone might be led to reject a seemingly innocent requirement that McDaniel takes to be conceptually necessary, consider the requirement that any fundamental parthood relation must obey some kind of remainder principle. Suppose that endurantism—the view that ordinary physical objects persist by being multiply located at distinct space-time regions—is true.[42] If time travel is possible, then it is presumably also possible for a single time-travelling brick to compose a brick wall.[43] Such a wall would have the brick as a proper part, but it would not have any other part that does not overlap that very brick. (On the view under consideration, the wall is composed entirely of a single, multiply located brick.) So, on this view, there is no remainder if we subtract one of its proper parts—that is, its one proper part: the brick—from the wall.[44] So endurantists who believe in time travel should believe that parthood does not obey some kind of remainder principle. But endurantists who believe in time travel and who think that parthood

[42] See, for example, Thomson, "Parthood and Identity Across Time," this JOURNAL, LXXX, 4 (April 1983): 201–20; van Inwagen, "Four-Dimensional Objects," *Noûs*, XXIV, 2 (April 1990): 245–55, and "Temporal Parts and Identity Across Time," *The Monist*, LXXXIII, 3 (July 2000): 437–59, which are reprinted in his *Ontology, Identity, and Modality: Essays in Metaphysics* (New York: Cambridge, 2001), pp. 111–21, 122–43; and Fine, "In Defense of Three-Dimensionalism," this JOURNAL, CIII, 12 (December 2006): 699–714.

[43] This case is originally discussed in Nikk Effingham and Jon Robson, "A Mereological Challenge to Endurantism," *Australasian Journal of Philosophy*, LXXXV, 4 (December 2007): 633–40. For a case that raises similar issues, see Cody Gilmore, "Time Travel, Coinciding Objects, and Persistence," in Zimmerman, ed., *Oxford Studies in Metaphysics*, vol. 3 (New York: Oxford, 2007), pp. 177–98.

[44] The usual remainder principles are the following.

Strong Supplementation: If *y* is not a part of *x*, then there is some *z* that is a part of *y* but does not overlap *x*.

Weak Supplementation: If *x* is a part of *y* but is distinct from *y*, then there is some *z* that is a part of *y* but does not overlap *x*.

The case considered is a counterexample to both principles. The wall is not a part of the brick. The brick is a part of the wall and is distinct from it. But there is no part of the wall that does not overlap the brick. Effingham and Robson conclude from this that endurantism is false. (But see Effingham, "Mereological Explanation and Time Travel," *Australasian Journal of Philosophy*, LXXXVIII, 2 (June 2010): 333–45.) For a reply, see Donald Smith, "Mereology without Weak Supplementation," *Australasian Journal of Philosophy*, LXXXVII, 3 (September 2009): 505–11. For a different sort of reply, see Gilmore, "Why Parthood Might Be a Four-Place Relation, and How It Behaves if It Is," in Ludger Honnefelder, Edmund Runggaldier, and Benedikt Schick, eds., *Unity and Time in Metaphysics* (New York: de Gruyter, 2009), pp. 83–133, and "Parts of Propositions," in Shieva Kleinschmidt, ed., *Mereology and Location* (New York: Oxford, forthcoming).

is not analyzable in terms of some other parthood relation are not conceptually confused about parthood. We are happy to keep them company in rejecting one of McDaniel's strictures.

Second, Unrestricted Fine Composition is false.[45] Take all the Fine fusions that are not material Fine parts of themselves. They instantiate some attribute (*being Fine fusions*, say). So, if Unrestricted Fine Composition were true, then there would be a Russell Fine fusion: a Fine fusion that has as its material Fine parts all and only the Fine fusions that are not material Fine parts of themselves. But, on pain of paradox, there can be no Russell Fine fusion. (Is a Russell Fine fusion a material Fine part of itself? If it is, then it is not, because it has as material Fine parts only those Fine fusions that are not material Fine parts of themselves; and, if it is not, then it is, because it has as material Fine parts all those Fine fusions that are not material Fine parts of themselves.) So, instead, we need to restrict Fine composition in some way. If we replace Unrestricted Composition with Iterative Composition, then it would be natural to replace Unrestricted Fine Composition with the following axiom.

> *Iterative Fine Composition*: For any things and any attribute, there is at least one Fine fusion that has those things as its material Fine parts and that has that attribute as its formal Fine part if and only if those things instantiate that attribute and there is some level in the cumulative hierarchy in which all of those things appear.[46]

As before, the cumulative hierarchy is a series of levels. In this case, things that have no proper Fine parts appear in Level 0; Fine fusions of things that appear in Level 0 appear in Level 1; Fine fusions of things that appear in Levels 0 and 1 appear in Level 2; and so on. (And, as before, vagueness is not a worry here, since it is not vague whether there is some level in the cumulative hierarchy in which some things all appear.[47])

[45] Here we are indebted to Mark Brown and Gabriel Uzquiano.

[46] The final conjunct is redundant if some things instantiate an attribute only if there is some level in the cumulative hierarchy in which all of those things appear. But we are happy to be needlessly explicit. Thanks to Kit Fine here. Alternatively, we could replace Unrestricted Fine Composition with the following axiom.

> *Limitation of Fine Composition*: For any things and any attribute, there is at least one Fine fusion that has those things as its material Fine parts and that has that attribute as its formal Fine part if and only if those things instantiate that attribute and those things are few.

If we replace Unrestricted Composition with Limitation of Composition (see note 32), then it would be natural to replace Unrestricted Fine Composition with Limitation of Fine Composition.

[47] If it is vague whether some things instantiate an attribute, then it might be vague whether a Fine fusion exists. But this is no objection to the restriction, since Unrestricted Fine Composition would face the same problem.

One reason for rejecting Unrestricted Fine Composition in favor of Iterative Fine Composition is that Unrestricted Fine Composition has the vice of being inconsistent. But that is not the only reason. A hierarchical view of composition is a natural view to adopt for those who think that ordinary objects are composed of form and matter.[48] Robert Pasnau, for example, interprets Aristotle as endorsing a hierarchical view of composition in *Physics* II.1:

> On this conception, there will often be hierarchies of matter, with the most basic stuff, *prime matter*, at the bottom, and various form-matter composites at higher levels, which may themselves be conceived as the matter for some further form. Wood, for example, is a form-matter composite that can itself serve as the matter of a bed.[49]

Another reason, then, for rejecting Unrestricted Fine Composition in favor of Iterative Fine Composition is that Iterative Fine Composition has the virtue of being an instance of such a hierarchical view of composition.

We can now see why Fine should reject Uniqueness of Composition: given Iterative Fine Composition, Uniqueness of Composition is false. Suppose that a thing x that appears in some level in the cumulative hierarchy has a property F. In that case, Iterative Fine Composition entails that there is a y that appears in some level and is the following Fine fusion: x/F. (Recall that something instantiating a property is a limiting case of some things instantiating an attribute.) Suppose that y also has F. In that case, Iterative Fine Composition entails that there is a z that appears in some level and is the following Fine fusion: y/F. y and z are both fusions of x and F, contrary to Uniqueness of Composition.[50]

This concludes our discussion of axioms governing parthood and composition and also *being a Fine part of* and Fine composition. In the rest of this paper, we assume that Fine's mereology includes an axiom governing parthood—namely, Transitivity—and two axioms governing Fine composition: namely, Iterative Fine Composition and Uniqueness of Fine Composition.

[48] Schaffer defends an ontology that is both hierarchical and neo-Aristotelian. (See "On What Grounds What," in David Chalmers, David Manley, and Ryan Wasserman, eds., *Metametaphysics: New Essays on the Foundations of Ontology* (New York: Oxford, 2009), pp. 347–83.) He does not endorse hylomorphism or a hierarchical view of composition, though.

[49] "Form and Matter," in Robert Pasnau, ed., *The Cambridge History of Medieval Philosophy*, vol. 2 (New York: Cambridge, 2010), pp. 635–46, at p. 636.

[50] Fine rejects Weak Supplementation (see note 44) for similar reasons. (See "Response to Kathrin Koslicki," *Dialectica*, LXI, 1 (March 2007): 161–66, at pp. 161–62.) Weak Supplementation is a consequence of Unrestricted Composition and Uniqueness of Composition.

I.5. Sets. Sets can be identified with Fine fusions. In particular, a set that has the *x*s as its members can be identified with a Fine fusion that has the *x*s as its material Fine parts. For example, on this view, the set that has the bread and the ham as its members is a Fine fusion that has the bread and the ham as its material Fine parts. But which Fine fusion? The ham sandwich is a Fine fusion that has the bread and the ham as its material Fine parts, but it is not the only such Fine fusion. The bread and the ham instantiate all sorts of attributes and so are the material Fine parts of all sorts of Fine fusions, which differ only in their formal Fine parts. Let us call any Fine fusion that has the bread and the ham as its material Fine parts a *candidate* Fine fusion (for being the set that has the bread and the ham as its members). The problem is that there are multiple candidates.[51]

But we think that not all candidate Fine fusions are equally eligible. The bread and the ham instantiate any number of attributes. As a result, they also instantiate a higher-order multigrade attribute: the *instantiating some attribute or other* attribute.[52] Our official solution to the problem is to maintain that the Fine fusion the bread and the ham/*instantiating some attribute or other* is the most eligible candidate for being the set that has the bread and the ham as its members, because that Fine fusion captures what matters—namely, the material

[51] For a similar worry about Armstrong's view (in "Classes Are States of Affairs," *Mind*, C, 398 (April 1991): 189–200), see Alex Oliver, "The Metaphysics of Singletons," *Mind*, CI, 401 (Jan 1992): 129–40, at p. 132.

[52] Let us fix on some terminology. Following Henry S. Leonard and Nelson Goodman, let us say that a relation of one or more places that has a fixed adicity—that takes a fixed number of arguments—is a *unigrade relation.* (See Leonard and Goodman, "The Calculus of Individuals and Its Uses," *Journal of Symbolic Logic*, V, 2 (June 1940): 45–55, at p. 50n11.) A one-place unigrade relation is a *unigrade property.* A unigrade relation of two or more places is a *unigrade proper relation.* In addition to unigrade relations, there are relations that have variable adicities; that is, they take a variable number of arguments. (See, for example, *ibid.*, pp. 50–53; Adam Morton, "Complex Individuals and Multigrade Relations," *Noûs*, IX, 3 (September 1975): 309–18; Fine, "Neutral Relations," p. 22; Oliver and Timothy Smiley, "Multigrade Predicates," *Mind*, CXIII, 452 (October 2004): 609–81; Fraser MacBride, "The Particular-Universal Distinction: A Dogma of Metaphysics?" *Mind*, CXIV, 455 (July 2005): 565–614, at pp. 568–95; and Joop Leo, "Modeling Relations," *Journal of Philosophical Logic*, XXXVII, 4 (August 2008): 353–85, and "Relational Complexes," unpublished manuscript.) Following Leonard and Goodman again, let us say that these are *multigrade relations.* (See "The Calculus of Individuals and Its Uses," p. 50.) Among multigrade relations are those that take one or more arguments. It is perhaps a bit of a strain to call them 'relations', since 'relation' is sometimes used for unigrade proper relations, which cannot take one argument. So, instead, we call them 'attributes'. We thus use 'attribute' broadly to cover, not only unigrade properties and unigrade proper relations, but also multigrade relations. The *instantiating some attribute or other* attribute is such a multigrade relation. It is instantiated by one thing (that instantiates a unigrade property or a multigrade relation) and also by many things (that instantiate a unigrade proper relation or a multigrade relation). Thanks to Kit Fine here.

Fine parts—and comes as close as possible to washing away distinctions that do not matter: namely, the distinctions between various formal Fine parts.[53]

The view in question, then, is that the set that has the *x*s as its members is the following Fine fusion: the *x*s/*instantiating some attribute or other*. We can easily extend this view to singletons and the empty set. Cary's singleton is the following Fine fusion: Cary/*instantiating some attribute or other*. And the empty set is what you are left with if you subtract Cary from his singleton, so to speak: namely, the *instantiating some attribute or other* attribute.

> *Sets*: A nonempty set is a Fine fusion that has *instantiating some attribute or other* as its formal Fine part, and the empty set is the *instantiating some attribute or other* attribute.

If Cary's singleton is Cary/*instantiating some attribute or other*, and if *instantiating some attribute or other* is the empty set, then Cary's singleton is the following Fine fusion: Cary/the empty set. That is, Cary's singleton is a Fine fusion that has Cary as its material Fine part and the empty set as its formal Fine part. On this view, {Cary} is composed of Cary and {}, as we desired. This view tells us what sets, including singletons and the empty set, are. We can also say what the membership relation is.

> *Membership*: For any x and y, x is a member of y if and only if y is a set and x is a material Fine part of y.

Let us call the conjunction of Sets and Membership *the Fine view*. The Fine view seems to us to be a fine view to have.[54]

[53] Another solution to the problem would be to maintain that the most eligible Fine fusion has as a formal Fine part a primitive set-theoretic attribute. Although using a primitive set-theoretic attribute is more in the spirit of the primitivism to which we ultimately are more sympathetic (see note 1), in the text we opt for *instantiating some attribute or other* rather than a primitive set-theoretic attribute because we suspect that many—including many Lewisians—might be readier to accept higher-order attributes and eligibility facts than to accept primitive set-theoretic attributes. (We discuss Lewis's views in section II.)

[54] Fine, Koslicki, Johnston, Armstrong, Peter Forrest, and Jerrold J. Katz all discuss views in the vicinity of Sets. Of these, only Katz's view is one on which Cary's singleton is composed of Cary and the empty set. We take that to be a reason to prefer the Fine view to all but Katz's view, and we take Katz's view to have other drawbacks.

On Katz's view, sets are composed of their members and the empty set. But Katz does not say what the empty set is. (See *Realistic Rationalism: Representation and Mind* (Cambridge: MIT, 1998), pp. 138–52.) And he does not explicitly discuss Membership. (Also, Katz says that a set comes into existence in virtue of its members and the empty set standing in the containment relation. See *ibid.*, p. 142. But containment—the disjunction of the converses of membership and subsethood—is a relation that holds between a set, on the one hand, and its members and the empty set, on the other; it is not a relation that holds between the members and the empty set themselves.)

In the appendix, we show that, from the Fine view, the axioms of Fine's mereology, and some axioms governing the cumulative hierarchy, we can recover Zermelo set theory (Z): namely, Empty Set, Pair Set, Union Set, Power Set, Infinity, Separation, Foundation, and Extensionality.[55] But we cannot recover *ZF* or *ZFC*. *ZF* is *Z* together

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The views that Fine, Koslicki, and Johnston discuss are closest to Sets. Fine considers (but does not endorse) a view on which the formal Fine part of a nonempty set is "the method of forming a set from its members." (See "Response to Kathrin Koslicki," p. 162; see also "Towards a Theory of Part.") Koslicki considers (but does not endorse) a view on which a nonempty set is a Fine fusion that has one or more formal Fine parts. (See *The Structure of Objects*, p. 180n20.) And Johnston proposes a view on which the formal Fine part of a nonempty set is *existence*, and he considers (but does not endorse) the view that *existence* is something like *instantiating some attribute or other.* (See "Hylomorphism," pp. 687–96.)

But, on the view that Fine discusses, the formal Fine part of a nonempty set might be a relation between the material Fine parts and the Fine fusion rather than simply a relation among the material Fine parts; Koslicki does not say what the formal Fine parts of a nonempty set are; Fine and Koslicki do not discuss the empty set; and the view of the empty set that Johnston suggests—one on which the empty set is the singleton of "an arbitrary item, an item that is no item in particular" (*ibid.*, p. 694n21)—is not particularly in the vicinity of Sets. In addition, Fine, Koslicki, and Johnston do not explicitly discuss Membership. (And all of these views, including Sets, seem to be at odds with Fine's view elsewhere, where he considers rejecting the iterative conception of sets and the view that sets are less fundamental than their members. See "Class and Membership," this JOURNAL, CII, 11 (November 2005): 547–72, especially pp. 571–72.)

On Armstrong's view, Cary's singleton is Cary's having some unit-making property or other. (See "Classes Are States of Affairs.") And Armstrong endorses something in the vicinity of Membership. But he has a different, Lewisian view about nonempty sets that are not singletons (for Lewis's view, see section II); he does not discuss the empty set; and he does not think that material Fine parts are, strictly speaking, parts. (Also, Armstrong's view faces Russell's paradox. But we think that Unrestricted Composition is the culprit. See Rosen, "Armstrong on Classes as States of Affairs"; Armstrong, *Truth and Truthmakers* (New York: Cambridge, 2004), pp. 120–23; and Hud Hudson, "Confining Composition," this JOURNAL, CIII, 12 (December 2006): 631–51, at pp. 631–38.)

On Forrest's view, sets have their members as parts. (See "Nonclassical Mereology and Its Application to Sets," *Notre Dame Journal of Formal Logic*, XLIII, 2 (2002): 79–94.) But Forrest provides a different account of membership. And, on his view, either there is no empty set or it is "the fictitious null thing"; he provides too many mereological surrogates (that is, more than one) for some sets, like Cary's singleton; and he provides too few mereological surrogates (that is, none) for other sets, like {Cary, {Cary, Kate}}. See *ibid.*, pp. 81, 88–89. (Elsewhere, Forrest proposes a different account of sets, one on which sets do not have their members as parts. See "Sets as Mereological Tropes," *Metaphysica*, III, 1 (2002): 5–9.)

[55] Zermelo set theory is not Ernst Zermelo's set theory from 1908; Zermelo's set theory from 1908 includes Choice but not Foundation. See "Untersuchungen über die Grundlagen der Mengenlehre I," *Mathematische Annalen*, LXV (1908): 261–81, which is reprinted as "Investigations in the Foundations of Set Theory I" in Jean van Heijenoort, ed., *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (Cambridge: Harvard, 1967), pp. 199–215; and Boolos, "The Iterative Conception of Set," this JOURNAL, LXVIII, 8 (April 22, 1971): 215–31, which is reprinted in his *Logic, Logic, and Logic*, pp. 13–29, at p. 26n11.

with the axiom of Replacement. (Roughly, Replacement says that, if you have a set whose members can be mapped one-one onto some *y*s, then the *y*s form a set.) And *ZFC* is *ZF* together with the axiom of Choice. (Roughly, Choice says that, if you have a set of disjoint nonempty sets, then there is a set that contains exactly one member from each of those disjoint nonempty sets.) We cannot derive Replacement or Choice from the Fine view, the axioms of Fine's mereology, and the axioms governing the cumulative hierarchy. But this is not surprising, since Replacement and Choice are not consequences of the iterative conception of set.[56]

The Fine view, on which the set that has the *x*s as its members is the *x*s/*instantiating some attribute or other*, might well require that properties and relations be abundant rather than sparse.[57] And, if the Fine view is right, then—on pain of circularity—we cannot identify properties and relations with sets of particulars (or set-theoretical constructions thereof). But that is okay, because the most viable version of that view presupposes Lewisian modal realism. (Otherwise, there would be no plausible way to identify distinct but contingently coextensive properties, like *being a renate* and *being a cordate*, with distinct sets.[58]) And

[56] See Boolos, "The Iterative Conception of Set," pp. 24–29, and "Iteration Again," *Philosophical Topics*, XVII, 2 (Fall 1989): 5–21, the latter of which is reprinted in his *Logic, Logic, and Logic*, pp. 88–104, at pp. 96–97. Replacement and Choice are consequences of a limitation-of-size conception of set, but Power Set and Infinity are not. (See "Iteration Again," pp. 96–103. On limitation of size axioms, see notes 32 and 46.) Lewis gets Replacement and Choice—but only by assuming plural versions of them at the outset. (See *Parts of Classes*, pp. 71–72, 90–91, 102, 103–04.) If we wanted to, we could get Replacement and Choice in the same way. Alternatively, we could get Replacement and Choice from what Paseau calls the "liberalised iterative conception," which includes Reflection and Combinatorialism. (See "Boolos on the Justification of Set Theory," *Philosophia Mathematica*, XV, 1 (February 2007): 30–53, at pp. 33–35.)

[57] Even if there are sparse properties and relations at various levels (see note 13), *instantiating some attribute or other* is probably not among the sparse properties and relations at any level. We are tempted by the idea that a primitive set-theoretic attribute (see note 53) would be among the sparse properties and relations. But we will not pursue that outlandish idea here.

[58] See Lewis, *On the Plurality of Worlds*, pp. 50–69. Lewis takes properties to be sets of actual and merely possible individuals. But there are reasons for Lewisian modal realists to take properties to be functions from worlds to sets of (perhaps merely possible) individuals. See Andy Egan, "Second-Order Predication and the Metaphysics of Properties," *Australasian Journal of Philosophy*, LXXXII, 1 (March 2004): 48–66, which is reprinted in Frank Jackson and Graham Priest, eds., *Lewisian Themes: The Philosophy of David K. Lewis* (New York: Oxford, 2004), pp. 49–67. Those who adopt the Fine view and who are Lewisian modal realists can say that there are set-theoretical objects that *represent* the attribute *instantiating some attribute or other*, but none of these set-theoretical objects *is* that attribute. Those who adopt the Fine view and who are not Lewisian modal realists cannot say that there are such set-theoretical objects. Thanks to an anonymous referee here.

giving up Lewisian modal realism is a cost that most of us are prepared to bear anyway.

In a way, the Fine view is not particularly surprising. If you are a realist about abundantly many properties and relations and do not identify them with sets, then you should be able to come up with some account or other of sets.[59] Still, we think that the Fine view has some virtues. For one thing, the Fine view vindicates the claim that Cary's singleton is composed of Cary and the empty set. For another, the Fine view goes some way towards answering the questions about singletons with which Lewis's view has trouble. In particular, the Fine view offers a nonstructuralist account of why Cary is a member of his singleton. We discuss this and other virtues of the Fine view in the next section.

II. LEWIS

II.1. Lewisian Mysteries. On Lewis's view, the parts of a nonempty set are its nonempty subsets.[60] With this view in place, Lewis says what nonempty sets, the empty set, and membership are.[61]

> *Lewis Sets*: A nonempty set is the fusion of some singletons (provided that that fusion itself has a singleton), and the empty set is the fusion of everything that does not have a singleton as a part.[62]

> *Lewis Membership*: For any x and y, x is a member of y if and only if y is the fusion of some singletons and x's singleton is a part of y.

[59] See George Bealer, *Quality and Concept* (New York: Oxford, 1982), pp. 101–19; and Michael Jubien, "On Properties and Property Theory," in Gennaro Chierchia, Barbara H. Partee, and Raymond Turner, eds., *Properties, Types, and Meaning* (Boston: Kluwer, 1989), pp. 159–75, and "Straight Talk about Sets," *Philosophical Topics*, XVII, 2 (Fall 1989): 91–107. Although Bealer and Jubien eliminate sets and replace them with properties, one could easily eliminate their elimination-and-replacement strategy and replace it with an identification one.

[60] See Lewis, *Parts of Classes*, pp. 3–10, and "Mathematics is Megethology," pp. 206–10. For objections to Lewis's arguments for this view, see Oliver, "Are Subclasses Parts of Classes?" *Analysis*, LIV, 4 (October 1994): 215–23. For a similar view, see Harry C. Bunt, *Mass Terms and Model-Theoretic Semantics* (New York: Cambridge, 1985), pp. 53–72, 233–301. x is a subset of y just in case every member of x is a member of y.

[61] See Lewis, *Parts of Classes*, pp. 10–16, and "Mathematics is Megethology," pp. 210–12.

[62] For objections to Lewis's view of the empty set, see Michael Potter's critical notice of *Parts of Classes* in *Philosophical Quarterly*, XLIII, 172 (April 1993): 362–66; and Michael Hand, "What the Null Set Could Not Be," *Australasian Journal of Philosophy*, LXXIII, 3 (September 1995): 429–31. We suspect that these objections can be met if one adopts counterpart theory. (See Lewis, "Counterpart Theory and Quantified Modal Logic," this JOURNAL, LXV, 5 (March 7, 1968): 113–26, which is reprinted in his *Philosophical Papers*, vol. 1, pp. 26–46; "Counterparts of Persons and Their Bodies," this JOURNAL, LXVIII, 7 (April 8, 1971): 203–11, which is reprinted in *Philosophical Papers*, vol. 1, pp. 47–54; and *On the Plurality of Worlds*, chapter 4.) But we suspect that one can do just about anything with counterpart theory.

It is a consequence of Lewis's view that singletons are mereological atoms.[63] (Since singletons have no nonempty proper subsets, they have no proper parts.) So, on Lewis's view, neither Cary nor the empty set is a part of Cary's singleton. This, we think, is a vice of Lewis's theory.

In any case, there are further problems with Lewis's view. Since singletons are mereological atoms, three questions are said to be particularly vexing.[64] The first two are about singletons; the third is about the relation between singletons and their members. First, there is a question about *location*: namely, where are singletons? Are they located where their members are? (This is the answer that Lewis gives in *On the Plurality of Worlds*.[65]) Or are they not located anywhere at all? (This is the answer that Lewis gives in a postscript to "Counterpart Theory and Quantified Modal Logic."[66]) We cannot appeal to the location of their proper parts to answer this question, because they have no proper parts. Second, there is a question about *intrinsic character*: namely, do singletons all have the same intrinsic properties, or do they differ intrinsically? Are Cary's singleton and Kate's duplicates? Again, we cannot appeal to the intrinsic character of their proper parts to answer this question, because they have no proper parts. Third, there is a question about *membership*: namely, why do singletons have the members they do? Cary is a member of his singleton, and Kate is a member of hers. Why is it not the other way around? It is not that Cary is a part of his (but not of Kate's) and that Kate is a part of hers (but not of Cary's), because their singletons are mereological atoms. Because of questions like these, Lewis admits that singletons are "profoundly mysterious."[67] He says that our talk of singletons makes sense somehow, but, he admits, he does not know how.[68]

In response to these questions, Lewis suggests a kind of structuralism, according to which there is no such thing as *the* singleton function (that is, *the* function from things to their singletons).[69] Rather,

[63] See Lewis, *Parts of Classes*, p. 15, and "Mathematics is Megethology," p. 211.

[64] See Lewis, *Parts of Classes*, pp. 29–35, and "Mathematics is Megethology," pp. 215–17; Armstrong, "Classes Are States of Affairs," pp. 189–90, 199–200; Oliver, "The Metaphysics of Singletons," pp. 130, 137; and Rosen, "Armstrong on Classes as States of Affairs," p. 615.

[65] See Lewis, *On the Plurality of Worlds*, p. 83.

[66] See Lewis, *Philosophical Papers*, vol. 1, p. 40.

[67] Lewis, *Parts of Classes*, p. 57, and "Mathematics is Megethology," p. 217.

[68] See Lewis, *Parts of Classes*, p. 59.

[69] See Lewis, *Parts of Classes*, pp. 45–54, and "Mathematics is Megethology," pp. 218–22. For a similar view, see Paul Fitzgerald, "Meaning in Science and Mathematics," *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association* (1974): 235–69, at pp. 246–51.

there are many functions that have "the right formal character" and satisfy "any 'unofficial' axioms we see fit to accept."[70] On this structuralist view, one singleton function *f* maps Cary onto a mereological atom *a*, whereas another singleton function *g* maps Cary onto a distinct mereological atom *b*. There is no fact of the matter, once and for all, whether *a* or *b* is Cary's singleton. According to *f*, *a* is; according to *g*, *b* is.

The structuralist view might answer the question about membership. Perhaps what it is for Cary to be a member of a singleton *x* according to a singleton function *f* is just for *f* to map Cary onto *x*. And, for any singleton function *f*, there will be some *x* such that *f* maps Cary onto *x*. So, no matter which singleton function we pick, 'Cary is a member of his singleton' will come out true. Perhaps this is enough to answer the question about membership. But one might worry about how well the structuralist view fits with mathematical practice.[71] And the structuralist view does not answer the questions about location and intrinsic character. We can still ask where, if anywhere, *a* or *b* or anything else that some singleton function maps something onto is located, just as we can ask about its intrinsic character, if it has one. Lewis concedes that, "even if we could remove all mystery about the nature of the member-singleton relation, we would still have a mystery about the nature of the singletons."[72]

II.2. Fine Answers. The Fine view goes some way towards answering questions about singletons with which Lewis's view has trouble. In answer to the question about location, the Fine view says that singletons—and nonempty sets more generally—are located where their members are. That is because, by Sets, nonempty sets are Fine fusions; by Membership, the members of a nonempty set are the material Fine parts of the Fine fusion that it is; and Fine fusions are located where their material Fine parts are. For example, the set that has the bread and the ham as its members is located where the bread

[70] Lewis, "Mathematics is Megethology," p. 219. See also Lewis, *Parts of Classes*, p. 46.

[71] See Lewis, *Parts of Classes*, pp. 53–54, and "Mathematics is Megethology," pp. 221–22.

[72] Lewis, *Parts of Classes*, p. 48. See also pp. 48–49 and "Mathematics is Megethology," p. 221. Rosen suggests that, on the structuralist view, the questions about singletons have false presuppositions and hence are not genuine questions. (See "Armstrong on Classes as States of Affairs," p. 616.) The "no genuine question" response might be right when it comes to questions like 'Where is Cary's singleton?' and 'What is Cary's singleton like?' since on the structuralist view there is no single thing that, once and for all, is Cary's singleton. But the "no genuine question" response does not seem right when it comes to questions like 'Where are all the *x*s such that there is a *y* and a singleton function *f* such that *f* maps *y* onto *x*?' and 'What are those *x*s like?' since on the structuralist view there are such *x*s.

and the ham are, because the set that has the bread and the ham as its members is the Fine fusion the bread and the ham/*instantiating some attribute or other*, and that Fine fusion is located where the bread and the ham are. In this respect, the set that has the bread and the ham as its members is like the ham sandwich, which is the Fine fusion the bread and the ham/*being arranged ham-sandwich-wise*; it, too, is located where the bread and the ham are. Singletons are also located where their members are, for the same reason. So, for example, Cary's singleton is located where he is. And *its* singleton is located where it is, which is to say where Cary is.[73]

But the empty set is a special case, since it has no members and hence no material Fine parts. So the Fine view does not immediately entail an answer to the location question in this case. Still, by Sets, the empty set is the *instantiating some attribute or other* attribute, and, if we knew where that is, then we would know where the empty set is. Perhaps it is nowhere, or perhaps it is wherever its instances are: namely, everywhere. (If you think that 'Everywhere!' is a bad answer to the question 'Where is the empty set?', then you will not like Lewis's answer to that question either.[74])

In answer to the question about intrinsic character, the Fine view tells us that certain sets differ intrinsically. For example, Cary's singleton and Kate's singleton differ intrinsically because they have different proper parts, and those proper parts differ intrinsically: Cary and Kate are not duplicates. (There is a tie between parthood and intrinsicness: a whole inherits, in some sense, the intrinsic properties of its proper parts.[75]) The Fine view does not tell us that other sets—for example, Cary's singleton and its singleton—differ intrinsically, for they are composed of the same parts: namely, Cary and the *instantiating some attribute or other* attribute. Still, this gives us a grip on which sets differ intrinsically and which sets might not.

In answer to the question about membership, Membership says that x is a member of y if and only if y is a set and x is a material Fine part of y. The reason that Cary is a member of his singleton but not of Kate's is that he is a material Fine part of Cary/*instantiating some attribute or other* but not of Kate/*instantiating some attribute or other*. Conversely,

[73] The Fine view is thus committed to massive co-location. For a similar point, see Lewis's review of Bigelow's *The Reality of Numbers: A Physicalist's Philosophy of Mathematics* (New York: Oxford, 1988) in *Australasian Journal of Philosophy*, LXVII, 4 (December 1989): 487–89, at pp. 488–89.

[74] See Lewis, *Parts of Classes*, p. 14, and "Mathematics is Megethology," p. 211.

[75] See Sider, "Parthood," *Philosophical Review*, CXVI, 1 (January 2007): 51–91, at pp. 70, 74–75.

the reason that Kate is a member of her singleton but not of Cary's is that Kate is a material Fine part of Kate/*instantiating some attribute or other* but not of Cary/*instantiating some attribute or other.*

II.3. Fine Mysteries. The Fine view appeals to Fine fusions and the *being a material Fine part of* relation. Lewisians—those who adopt Lewis's mereology—might object that Fine fusions and the *being a Fine part of* relation are just as mysterious as sets and membership and hence that the Fine view does nothing to dispel the mysteries that surround sets and membership.[76] We suspect that Lewis, for example, would find Fine fusions and the *being a material Fine part of* relation mysterious. He objects to Armstrong's states of affairs, which are an awful lot like Fine fusions, on the grounds that they require what he calls "unmereological 'composition'":

> I find unmereological 'composition' profoundly mysterious. After expelling it from set theory, I scarcely want to welcome it back via the anatomy of facts [or states of affairs].[77]

Lewis would no doubt object to Fine fusions on the grounds that they, too, require such mysterious "unmereological 'composition'."

One might think that what makes Fine composition, or the composition that yields Armstrong's states of affairs, deserving of the name 'unmereological' or less than fully deserving of the name 'composition' is that it violates Lewis's axioms, particularly Uniqueness of Composition.[78] One might think that this is what makes Fine fusions mysterious: how could there be things that violate Uniqueness of Composition? And one might think that this is precisely what makes sets mysterious: how can we have distinct sets—for example, Cary's singleton and its singleton—that are both generated from one thing: namely, Cary? If that is what makes Fine fusions mysterious and what makes sets mysterious, then the Fine view would be appealing to some

[76] For parallel worries about Armstrong's view, see Oliver, "The Metaphysics of Singletons," pp. 133–35.

[77] Lewis, *Parts of Classes,* p. 57. Fine denies that Fine fusions are Armstrongian states of affairs. (See "Things and Their Parts," p. 65.) But, even if Fine fusions are Armstrongian states of affairs, we do not think that would be so bad for the Fine view. For example, we do not think that it would undermine the answer that the Fine view provides to the question about location. For Armstrong allows that states of affairs might be located in space-time. (See "Classes Are States of Affairs," p. 195.) Thanks to an anonymous referee here.

[78] See, for example, Lewis, *On the Plurality of Worlds,* pp. 94–95, and his critical notice of Armstrong's *A Combinatorial Theory of Possibility* (New York: Cambridge, 1989) in *Australasian Journal of Philosophy,* LXX, 2 (June 1992): 211–24, which is reprinted as "Armstrong on Combinatorial Possibility" in his *Papers in Metaphysics and Epistemology* (New York: Cambridge, 1999), pp. 196–214, at p. 200.

things—namely, Fine fusions—that are mysterious for one reason to give us a view about something else—namely, sets—that are mysterious for precisely the same reason. One might reasonably think any such enterprise is doomed to fail.

But Lewis distinguishes what makes sets mysterious from what makes what he calls "unmereological 'composition'" mysterious. Nelson Goodman objects to sets precisely on the grounds that distinct sets are generated from the same elements.[79] But Lewis makes it clear that this is not what he has in mind when he says that sets are mysterious.[80] When he mentions what makes sets mysterious, Lewis does mention once that, with set theory, we can get many out of one.[81] But the rest of the time either he mentions that the informal characterization of sets as groups of things does not apply to singletons,[82] or he mentions the questions about singletons: about their location, intrinsic character, and members.[83] These are mysteries that the Fine view addresses, and they are not mysteries that Fine fusions face.

But, even if Fine fusions do not face those mysteries, Fine fusions violate Uniqueness of Composition, and Lewis finds it profoundly mysterious that any things could violate Uniqueness of Composition. Indeed, he finds it "unintelligible":

> ...how can different things be composed of exactly the same parts? I know how two things can be made of parts that are qualitatively just the same—that is no problem—but this time, the two things are supposed to be made not of duplicate parts, but of numerically identical parts. That, I submit, is unintelligible.[84]

Charges of unintelligibility are a little like incredulous stares: there is often little that one can do to reply to them (other than report that one has not noticed the alleged unintelligibility). If you think that denying Uniqueness of Composition is unintelligible, we will not say anything further to try to convince you otherwise.

Although we will not try to convince you that denying Uniqueness of Composition is intelligible, we will say two things to try to convince

[79] See Goodman, "A World of Individuals," in I. M. Bochenski, Alonzo Church, and Goodman, eds., *The Problem of Universals: A Symposium* (Notre Dame: University Press, 1956), pp. 15–31, and "On Relations that Generate," *Philosophical Studies*, IX, 5/6 (October–December 1958): 65–66.

[80] See Lewis, *Parts of Classes*, p. 38.

[81] *Ibid.*, pp. 5–6.

[82] *Ibid.*, pp. vii, 29–31.

[83] *Ibid.*, pp. 31–38, 41–42, 45, 48–50, 53, 54–57.

[84] Lewis, "Against Structural Universals," *Australasian Journal of Philosophy*, LXIV, 1 (March 1986): 25–46, which is reprinted in his *Papers in Metaphysics and Epistemology*, pp. 78–107, at p. 94.

you that denying Uniqueness of Composition might be worthwhile. First, as we saw in section I, the falsehood of Uniqueness of Composition follows from Iterative Fine Composition, which is an axiom in Fine's mereology. And Fine's mereology (or something like it) seems required to capture our ordinary parthood claims, like the claim that the bread is a part of the ham sandwich but the fusion of the bread and Cleopatra is not.

Second, consider a statue and a piece of clay out of which it is made. Suppose that they come into and go out of existence at the same time and that they do not gain or lose molecules throughout their careers. Fine's mereology aside, we think that the right thing to say about this case is that, contrary to Uniqueness of Composition, the statue and the clay are distinct objects that are composed of the same molecules.[85] Lewis, of course, denies this.[86] But his view has two unpleasant consequences. (We do not find these consequences unintelligible, but we would rather avoid them all the same.) One is that, although the statue and the clay are identical, they would not be if only one of them had come into or gone out of existence a millisecond earlier or later than it actually did. Identity does not seem that contingent to us. The other is that, if in the distant future the clay were to go out of existence a millisecond before the statue or vice versa, it would *now* be the case the statue and the clay are distinct because of facts about the statue and the clay in the distant future. Present distinctness does not strike us as being best explained by such facts about the distant future.[87] These points do not establish

[85] See Johnston, "Constitution Is Not Identity," *Mind*, CI, 401 (January 1992): 89–105; Thomson, "The Statue and the Clay"; and Fine, "A Counterexample to Locke's Thesis," *The Monist*, LXXXIII, 3 (July 2000): 357–61, "The Non-Identity of a Material Thing and Its Matter," *Mind*, CXII, 446 (April 2003): 195–234, and "Arguing for Non-Identity: A Response to King and Frances," *Mind*, CXV, 460 (October 2006): 187–210. Those who accept Fine's mereology need not accept this as a counterexample to Uniqueness of Composition; they can say that the statue and the clay have different formal Fine parts. See Fine, "Coincidence and Form"; and Koslicki, *The Structure of Objects*, pp. 179–83. But, of course, those who accept Fine's mereology are committed to other counterexamples to Uniqueness of Composition. Koslicki denies this. (See *ibid.*, pp. 182–83.) But we think that she is wrong, because of examples like the one given above in the text at note 50. See also Johnston, "Hylomorphism," pp. 672–75; and Trenton Merricks's review of *The Structure of Objects* in this JOURNAL, CVI, 5 (May 2009): 301–07, at pp. 304–05.

[86] See Lewis, "Counterparts of Persons and Their Bodies," and *On the Plurality of Worlds*, pp. 248–63.

[87] Sider says that "the natural reaction" is that "the statue should not be distinguished from the lump today just because 'they' will differ tomorrow." (See *Four-Dimensionalism*, p. 200.) We agree, although unlike Sider we think that the statue and the clay should be distinguished now for other reasons, for example, because they now differ in modal and kind properties. (Sider's view allows him to maintain Uniqueness of Composition and to deny that the statue and the lump are distinct today "because 'they' will differ

that Lewis's view is false; nor do they say anything about non-Lewisian views on which the statue and the clay are identical. Still, we think, they suggest that denying Uniqueness of Composition might be worthwhile.[88]

Armstrong and Forrest deny Uniqueness of Composition to get states of affairs, which they use as truthmakers.[89] (The conjunctive states of affairs *Fa & Gb* and *Fb & Ga* are distinct, but they are both composed of *a*, *b*, *F*, and *G*.) Lewis concludes his reply to them as follows:

> ...we're left with a stark clash of principles: a truthmaker for every truth, versus uniqueness of composition. If that's the choice we face, I say it's no contest. I expect Armstrong and Forrest would say the same. But there I fear our agreement gives out.[90]

Similarly, we are left with a stark clash: a mereology that captures ordinary parthood claims versus Uniqueness of Composition. We are not saying that it is no contest. But we are not betting on Uniqueness of Composition.

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APPENDIX: ZERMELO

Let us be a little more precise about the structure of the cumulative hierarchy.[91] The cumulative hierarchy is a series of levels. These levels are ordered by a *lower than* relation, which is irreflexive, transitive, and connected.

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tomorrow." But, to give the same answer to "How many statues are there now?" and "How many statues have been there over the last minute?" his view requires a non-uniform semantics for 'statue'. See *Four-Dimensionalism*, p. 197.)

[88] For a reply to this sort of argument against Uniqueness of Composition, see Varzi, "The Extensionality of Parthood and Composition."

[89] See Armstrong, "In Defence of Structural Universals," *Australasian Journal of Philosophy*, LXIV, 1 (March 1986): 85–88; and Forrest, "Neither Magic nor Mereology: A Reply to Lewis," *Australasian Journal of Philosophy*, LXIV, 1 (March 1986): 89–91.

[90] Lewis, "Comment on Armstrong and Forrest," *Australasian Journal of Philosophy*, LXIV, 1 (March 1986): 92–93, which is reprinted as "A Comment on Armstrong and Forrest" in his *Papers in Metaphysics and Epistemology*, pp. 108–10, at p. 110.

[91] Our discussion of the cumulative hierarchy follows Boolos, "The Iterative Conception of Set," pp. 16–22. (See also "Iteration Again," pp. 88–94.)

Irreflexivity (of lower than): No level is lower than itself.

Transitivity (of lower than): If one level is lower than a second level, which in turn is lower than a third level, then the first level is lower than the third level.

Connectedness (of lower than): For any two distinct levels, either the first is lower than the second or the second is lower than the first.

As a result, the *lower than* relation is anti-symmetric: for any levels x and y, if x is lower than y, then y is not lower than x. (Otherwise, if x is lower than y, and y is lower than x, then by Transitivity (of *lower than*) x is lower than x. But, by Irreflexivity (of *lower than*), no level is lower than itself.)

The cumulative hierarchy begins with a lowest level.

Lowest Level: Some level is lower than every other level.

Let us call this lowest level 'Level 0'. Let us say that one level is *immediately below* another just in case the second level is not Level 0 and any level that is lower than the second level is either lower than, or identical to, that first level.

Next Level: Every level is immediately below some level.

And there is a limit level.

Limit Level: Some level other than the lowest level is such that there is no level immediately below it.

By Lowest Level, there is a lowest level, Level 0. By Next Level, Level 0 is immediately below some level. Call it 'Level 1'. And so on for Levels 2, 3, 4, ... By Limit Level, there is a level—call it 'ω'—that is distinct from Level 0 and that is such that there is no level immediately below it.

Lemma I: For any natural number n, Level n is lower than ω.

Proof. Level 0 is lower than ω. By Limit Level, ω is distinct from Level 0. By Connectedness, either ω is lower than Level 0 or Level 0 is lower than ω. But, by Lowest Level, no level is lower than Level 0. So Level 0 is lower than ω. Suppose, for induction, that Level i (for some natural number i) is lower than ω.[92] Suppose, for *reductio*, that Level $i+1$ is not lower than ω. By Connectedness, ω is either lower than

[92] We help ourselves to mathematical induction (in two equivalent forms). See Boolos, "The Iterative Conception of Set," pp. 21–22. We think that, however mathematical induction is justified, it is not the case that it is justified by the axioms of Zermelo set theory.

or identical to Level $i+1$. ω is not identical to Level $i+1$, since Level i is immediately below Level $i+1$ (by Next Level), and no level is immediately below ω (by Limit Level). So ω is lower than Level $i+1$. Since Level i is immediately below Level $i+1$, ω is either lower than or identical to Level i. ω is not identical to Level i. (By assumption, Level i is lower than ω. If ω is identical to Level i, and Level i is lower than ω, then Level i is lower than Level i. But that violates Irreflexivity.) So ω is lower than Level i. But, by assumption, Level i is lower than ω. So ω is lower than Level i, and Level i is lower than ω; but this violates anti-symmetry. So Level $i+1$ is lower than ω.

Fine fusions and their Fine parts appear in various levels of the cumulative hierarchy.

Upward Mobility: Anything that appears in any level also appears in any higher level.

Let us say that anything that appears in any level is an *inhabitant* of the cumulative hierarchy. And let us say that an inhabitant of the cumulative hierarchy *makes its debut* in some level if and only if that level is the lowest level in which that inhabitant appears.

Unique Debut: Every inhabitant makes its debut in exactly one level.

Fine Debut: Every Fine fusion makes its debut in some level that is higher than any of the levels in which any of its Fine parts makes its debut.

Let us say that an inhabitant makes its debut *swiftly* just in case, for any level that is higher than all of the levels in which at least one of its Fine parts makes its debut, that inhabitant makes its debut in that level or in some lower level.

Swift Debut: Every inhabitant makes its debut swiftly.

From these axioms, the Fine view, and the axioms of Fine's mereology, we can recover the axioms of Zermelo set theory: namely, Empty Set, Pair Set, Union Set, Power Set, Infinity, Separation, Foundation, and Extensionality.[93]

Lemma II: The empty set makes its debut in Level 0.

Proof. By Sets, the empty set is the *instantiating some attribute or other* attribute, which has no material Fine parts. Level 0 is higher than all (none) of the levels in which at least one of the empty set's material Fine parts makes its debut. Assuming that the empty set is an inhabitant of

[93] Our proofs follow Boolos, "The Iterative Conception of Set," pp. 22–26, and Lewis, *Parts of Classes*, pp. 100–07.

the cumulative hierarchy, it follows, by Swift Debut, that the empty set appears in Level 0. By Lowest Level, no level is lower than Level 0. Level 0 is the lowest level in which the empty set appears.

Empty Set: Some set has no members.

Proof. By Sets, the *instantiating some attribute or other* attribute is a set (since it is the empty set). By Membership, something is a member of *instantiating some attribute or other* only if it is a material Fine part of that attribute. That attribute has no material Fine parts. So it has no members. So *instantiating some attribute or other* is a set that has no members.

Pair Set: For any two things, some set has those two things as its only members.

Proof. Consider any two objects a and b. Assuming that a and b are inhabitants of the cumulative hierarchy, it follows, by Unique Debut, that there is a unique level $L(a)$ in which a makes its debut and that there is a unique level $L(b)$ in which b makes its debut. By Connectedness, either $L(a)$ is lower than $L(b)$, $L(b)$ is lower than $L(a)$, or $L(a)$ is identical to $L(b)$. Pick whichever of the two levels is higher than the other (or, if a and b make their debuts in the same level, pick that one). By Next Level, that level is immediately below some level. Call it '$L(a+b)$'. By Upward Mobility, a and b both appear in $L(a+b)$. a and b instantiate some attribute (for example, *being distinct* if they are distinct and *being identical* if they are identical). So, by Iterative Fine Composition, there is a Fine fusion that has a and b as its material Fine parts and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the set whose members are the (perhaps identical) objects a and b.

Union Set: For any set x, there is a set whose members are the members of the members of x.

Proof. Suppose that x has members that have members. (If x does not have members that have members, then the empty set is a set whose members are the members of the members of x.) Call x's members 'the ys', and call the ys' members 'the zs'. Consider one of the zs; call it 'z'. z is a member of one of the ys; call it 'y'. So, by Sets and Membership, y is a Fine fusion that has z as one of its material Fine parts; by Fine Debut, the level in which z makes its debut is lower than the level in which y makes its debut. And y is a member of x. So, by Sets and Membership, x is a Fine fusion that has y as one of its material Fine parts; and, by Fine Debut, the level in which y makes its debut is lower than the level in which x makes its debut. By Transitivity (of *lower than*), the level in which z makes its debut is lower than the level in which x makes its debut. So, by Upward Mobility, each of the *z*s appears in the level in which x makes its debut. And the *z*s instantiate some attribute or other (for example, *being members of members of x*). So, by Iterative Fine Composition, there is a Fine fusion that has the *z*s as its material Fine parts and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the set whose members are the members of the members of x.

> *Power Set*: For any set x, there is a set whose members are the subsets of x.

Proof. Consider a set a. Call its subsets 'the *b*s'. By Unique Debut, there is a unique level $L(a)$ in which a makes its debut. Suppose, for *reductio*, that one of the *b*s—call it 'b'—makes its debut in a level that is higher than $L(a)$. b is not the empty set. (By Lemma II, the empty set makes its debut in Level 0. And, by Lowest Level, Level 0 is not higher than $L(a)$, since Level 0 is not higher than any level.) Since b is a nonempty set, by Sets and Membership b is a Fine fusion whose material Fine parts are some (but perhaps not all) of the members of a. Call the members of b 'the *c*s'. By Swift Debut, one of the *c*s—call it 'c'—is such that

> (i) c makes its debut in a level $L(c)$ that is higher than or identical to $L(a)$.

(Otherwise, b would have made its debut in $L(a)$ or in some level that is lower than it; but, by hypothesis, b makes its debut in a level that is higher than $L(a)$.) c is a member of a. So, by Membership, c is a material Fine part of a. So, by Fine Debut,

> (ii) c makes its debut in a level $L(c)^*$ that is lower than $L(a)$.

By (ii), $L(c)^*$ is lower than $L(a)$, and by (i), $L(a)$ is lower than $L(c)$, so by Transitivity (of *lower than*) $L(c)^*$ is lower than $L(c)$. By Unique Debut, $L(c)$ is identical to $L(c)^*$. So $L(c)$ is lower than itself; but this violates Irreflexivity. So none of the *b*s makes its debut in a level that is higher than $L(a)$. So, by Connectedness, each of the *b*s makes its debut in a level that is lower than or identical to $L(a)$. So, by Upward Mobility, each of the *b*s appears in $L(a)$. And the *b*s instantiate some attribute or other (for example, *being subsets of a*). So, by Iterative Fine Composition, there is a Fine fusion that has all of the *b*s as its material Fine parts and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the set whose members are a's subsets.

> *Infinity*: Some set x has the empty set as a member and is such that, for every member of x, that member's singleton is also a member of x.

Proof. We know that Level 0 is immediately below Level 1, which is immediately below Level 2, and so on. Let us say that the empty set is the 0th singleton of itself; that the singleton whose sole member is the empty set is the first singleton of the empty set; that the singleton whose sole member is that singleton is the second singleton of the empty set; and so on. We start by showing that, for each natural number n, the nth singleton of the empty set makes its debut in Level n.

By Lemma II, the empty set makes its debut in Level 0. Suppose, for induction, that the ith singleton of the empty set makes its debut in Level i. The ith singleton of the empty set instantiates some attribute or other (for example, *being self-identical*). And, we are supposing, the ith singleton of the empty set makes its debut in Level i. So, by Iterative Fine Composition, there is a Fine fusion that has the ith singleton of the empty set as its material Fine part and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the $(i+1)$th singleton of the empty set: the singleton whose sole member is the ith singleton of the empty set. By Fine Debut, the level in which that Fine fusion makes its debut is higher than the level in which the ith singleton of the empty set makes its debut, which we are assuming is Level i. By Swift Debut, that Fine fusion appears in Level $i+1$, since Level $i+1$ is higher than Level i, and all of that Fine fusion's material Fine parts—namely, the ith singleton of the empty set—appear in Level i. Since the $(i+1)$th singleton of the empty set appears in Level $i+1$ but not in Level i, the $(i+1)$-th singleton of the empty set makes its debut in Level $i+1$.

Now consider the xs such that each x is the nth singleton of the empty set (for some natural number n). Each of them makes its debut in Level 0, or Level 1, or .... Call these levels 'the ls'. By Lemma I, ω is higher than any of the ls. By Upward Mobility, all of the xs appear in ω. And the xs instantiate some attribute or other (for example, *being inhabitants of the cumulative hierarchy*). So, by Iterative Fine Composition, there is a Fine fusion that has all of the xs as its material Fine parts and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the set whose members are the empty set, the singleton whose sole member is the empty set, the singleton whose sole member is that singleton, and so on. The empty set is a member of that set, and every singleton of every member of that set is itself a member of that set.

> *Separation*: For any set x and any things the ys, there is a set whose members are the members of x that are among the ys.

Proof. Suppose that x has at least one member that is among the ys. (If x has no members that are among the ys, then the empty set is a set whose members are the members of x that are among the ys.) Call the members of x 'the zs'. By Sets and Membership, x is a Fine fusion whose material Fine parts are the zs. By Fine Debut, each of the zs makes its debut in some level that is lower than the level $L(x)$ in which x makes its debut. By Upward Mobility, all of the zs appear in $L(x)$, in which case so do all of the zs that are among the ys. And the zs that are among the ys instantiate some attribute or other (for example, *being members of* x *that are among the* ys). So, by Iterative Fine Composition, there is a Fine fusion that has all of the zs that are among the ys as its material Fine parts and that has *instantiating some attribute or other* as its formal Fine part. By Sets and Membership, that Fine fusion is the set whose members are the members of x that are among the ys.

> *Foundation*: Every nonempty set x has at least one member that does not intersect x.[94]

Proof. Let us say that something is *grounded* if and only if it is not a member of a set that violates Foundation: that is, if and only if it is not a member of a nonempty set x every member of which intersects x. We can show Foundation by showing that everything is grounded. And we can show that everything is grounded by showing that everything that makes its debut in a level is grounded if everything that makes its debut in any lower level is grounded.[95] Suppose, for *reductio*, that a makes its debut in Level $L(a)$ but is not grounded even though everything that makes its debut in any level lower than $L(a)$ is grounded. Since a is not grounded, a is a member of a nonempty set b such that

> (i) every member of b intersects b.

Since a is a member of b, a intersects b. So there is a c such that c is a member of both a and b. By Membership, c is a material Fine part of a. By Fine Debut, c makes its debut in a level that is lower than Level $L(a)$. So, by assumption, c is grounded. So c is not a member of a nonempty set x every member of which intersects x. And c is a member of b. So b is not a nonempty set every member of which intersects b. So (i) is false. Contradiction. So a is grounded.

> *Extensionality*: Two sets are identical if and only if they have the same members.

[94] x intersects y just in case there is a z such that z is a member both of x and of y.

[95] On the form of mathematical induction used in this proof, see Boolos, "The Iterative Conception of Set," pp. 21–22.

Proof. Left-to-right is trivial: if a is identical to b, then by Leibniz's law every member of a is a member of b and vice versa. For right-to-left, suppose that a and b have the same members; call those members 'the xs'. (If a and b have no members, then by Sets a and b are both identical to the *instantiating some attribute or other* attribute and hence are identical to each other.) By Sets and Membership, a and b are Fine fusions that have the xs as their material Fine parts and that have *instantiating some attribute or other* as their formal Fine part. By Uniqueness of Fine Composition, there is only one such Fine fusion; so a is identical to b.