Information Flow and Relevant Logics

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1 Introduction

John Perry, one of the two founders of the field of situation semantics, indicated in an interview in 1986 that there is some kind of connection between relevant logic and situation semantics.

> I do know that a lot of ideas that seemed off the wall when I first encountered them years ago now seem pretty sensible. One example that our commentators don't mention is relevance logic; there are a lot of themes in that literature that bear on the themes we mention. (Barwise and Perry 1985)

In 1992, in *Entailment* volume 2, Nuel Belnap and J. Michael Dunn hinted at similar ideas. Referring to situation semantics, they wrote

> ... we do not mean to claim too much here. The Barwise-Perry semantics is clearly independent and its application to natural-language constructions is rich and novel. But we like to think that at least first degree (relevant) entailments have a home there. (Anderson et al. 1992)

In this paper I show that these hints and gestures are true. And perhaps truer than those that made them thought at the time.

2 Routley-Meyer Frames

Relevant logics[1] are interesting things. One motivating principle for these logics is the requirement that if a conditional of the form $\phi \to \psi$ is to be true, there must be a connection between ϕ and ψ. The antecedent ϕ must somehow be *relevant* to the consequent ϕ. This is simple to chart proof-theoretically, and for years, this was how relevant logics were studied. (A great deal of that work is charted in Anderson and Belnap 1975.)

It was quite a deal harder to give relevant logics a *semantics* like the possible-worlds semantics for modal logics. There is a good reason for this. In possible worlds talk, a conditional $\phi \to \psi$ is true at a world w (say $w \models \phi \to \psi$) if and only if for each world v accessible from w (say wRv) if ϕ is true at v then ψ is also true at v. In other words, a conditional is true at a world if and only if it is truth preserving at all accessible worlds.

[1] I am an Australian, and Australians tend to call these logics "relevan*t* logics," as opposed to the American "relevan*ce* logics." There is a long and involved story behind this geographical bifurcation of terminology. A story into which we will not go at the moment.

On this story a tautologous conditionals, like $\phi \to \phi$, are true at every world. This means that any conditional with a tautologous consequent, like $\psi \to (\phi \to \phi)$, is *also* true at every world. But this is a paradigmatic case of an irrelevant conditional (when ϕ has nothing to do with ψ). This happens *whatever* the accessibility relation R is like. So a possible-worlds semantics for relevant logics is out of the question.

Well, *not quite*. The answer to the problem is to liberate the accessibility relation. Routley-Meyer frames use a *ternary* relation to model the conditional. The clause for conditionals is this:

$w \models \phi \to \psi$ if and only if for each u, v where $Ruwv$, if $u \models \phi$ then $v \models \psi$.

And this does the trick.[2] We can have worlds where $w \not\models \phi \to \phi$, simply by having $Ruwv$ for some worlds u and v where $u \models \phi$ and $v \not\models \phi$. This is not hard to do, formally speaking. More bells and whistles are needed to get the semantics up and running. Details can be found in many places (Anderson et al. 1992, Priest and Sylvan 1992, Restall 1993, Restall 1994a, Routley and Meyer 1973 and Routley et al. 1982) but all presentations are *something* like this.

Definition 1 A *Routley-Meyer frame* is a 4-tuple $\langle g, W, R, \sqsubseteq\rangle$ satisfying the following conditions:

- $\sqsubseteq$ is a partial order on W.
- If $Rwuv$ and if $w' \sqsubseteq w, u' \sqsubseteq u$ and $v \sqsubseteq v'$ then $Rw'u'v'$ also.
- $g \in W$. $Rwgv$ if and only if $w \sqsubseteq v$.

Definition 2 An *evaluation* $\models$ on a Routley-Meyer frame $\langle g, W, R, \sqsubseteq\rangle$ is a relation between worlds and formulae such that

- If $w \sqsubseteq v$ then if $w \models p$ then $v \models p$ for atomic propositions p.
- $w \models \phi \wedge \psi$ if and only if $w \models \phi$ and $w \models \psi$.
- $w \models \phi \vee \psi$ if and only if $w \models \phi$ or $w \models \psi$.
- $w \models \phi \to \psi$ if and only if for each u, v where $Ruwv$, if $u \models \phi$ then $v \models \psi$.

In the semantics a theorem is simply something true at g in all evaluations on all frames. (There's nothing true *everywhere* in all frames, so we need a special world g to record logical consequence, where logical consequence is still preservation across all worlds.) It is simple to show that these

$$\phi \to \phi \vee \psi \quad \phi \wedge \psi \to \phi \quad (\phi \to \psi) \wedge (\phi \to \theta) \to (\phi \to \psi \wedge \theta)$$

$$(\phi \to \theta) \wedge (\psi \to \theta) \to (\phi \vee \psi \to \theta) \quad \phi \wedge (\psi \vee \theta) \to (\phi \wedge \psi) \vee (\phi \wedge \theta)$$

are theorems, whereas these

$$\phi \to (\psi \to \psi) \qquad \phi \to (\psi \to \phi)$$

[2]In the literature, the first two places of the relation R are often swapped. (So, it would be '$Rwuv$' and not '$Ruwv$' in the clause displayed.) We use this arrangement for a smooth transition to what follows.

are not. In fact, many more things are not theorems as the semantics stands. There are counterexamples to each of these

$$\phi \to \big((\phi \to \psi) \to \psi\big) \quad \phi \wedge (\phi \to \psi) \to \psi$$

$$(\phi \to \psi) \to \big((\psi \to \theta) \to (\phi \to \theta)\big) \quad (\phi \to \psi) \to \big((\theta \to \phi) \to (\theta \to \psi)\big)$$

So, the logic is quite weak insofar as theorems relating to the conditional. This is as one would expect, for there are few conditions on the relation R. If we add more conditions on R, we get more theorems.

There are a number of unanalysed concepts in this presentation of the semantics for relevant logics. It is hard to understand what the worlds are, what a 'logic' world could be, what the relation R is grounded in, and so on. There are many different interpretations offered, but none have very widespread support. This is a thorny problem for practitioners of relevant logic, and it is one place where recent work in situation semantics can help.

3 Information Structures

In Barwise's recent work on information flow in situation semantics, he constructs a formalism which we can use to describe information flow (Barwise 1993). The crucial components are *situations* (with which you are familiar) and *channels*. Channels relate situations to one another, and they ground the flow of information. We classify situations with *types*, and we classify channels with *constraints*, which are of the form $\phi \to \psi$ where ϕ and ψ are types. The details of the definition of an information structure are given below.

Definition 3 An *information structure* $\langle S, T, C, \models, \mapsto, \|, ; \rangle$ is a structure made up of a set S of *situations*, T of *types*, C of *channels*, a binary relation $\models$ which relates both pairs of situations and types and pairs of channels and constraints and a ternary relation $\mapsto$ relating channels to pairs of situations. (Here $s_1 \overset{c}{\mapsto} s_2$ is read as "c is a channel from s_1 to s_2.") This structure must satisfy a number of further conditions:

- Types are closed under binary operations $\wedge$ and $\vee$. Furthermore, for each $s \in S$ and each $\phi, \psi \in T$, $s \models \phi \wedge \psi$ if and only if $s \models \phi$ and $s \models \psi$, and $s \models \phi \vee \psi$ if and only if $s \models \phi$ or $s \models \psi$.
- For every $\phi, \psi \in T$, the object $\phi \to \psi$ is a *constraint*. The relation $\models$ is extended to channels and constraints in the way indicated: $c \models \phi \to \psi$ if and only if for each $s_1, s_2 \in S$ where $s_1 \overset{c}{\mapsto} s_2$, if $s_1 \models \phi$ then $s_2 \models \psi$. (So, a channel supports a constraint, just when for each pair of situations s_1, s_2 related by the channel, if s_1 supports the antecedent, s_2 supports the consequent. This is the crux of information flow.)
- There is a *logic channel*, channel 1. It relates each situation s to all situations that contain it. In other words, $s \overset{1}{\mapsto} s'$ if and only if $s \sqsubseteq s'$, where $s \sqsubseteq s'$ if and only if whenever $s \models \phi$, $s' \models \phi$ too.

- Every pair of channels c_1 and c_2 has a unique *sequential composition* $c_1; c_2$ (such that $s_1 \overset{c_1;c_2}{\mapsto} s_2$ if and only if there is a situation s such that $s_1 \overset{c_1}{\mapsto} s$ and $s \overset{c_2}{\mapsto} s_2$). In addition, $c_1; (c_2; c_3) = (c_1; c_2); c_3$. Sequential composition enables us to link together long chains of information transfers.
- Every pair of channels c_1 and c_2 has a unique *parallel composition* $c_1 \| c_2$ (such that $s_1 \overset{c_1 \| c_2}{\mapsto} s_2$ if and only if $s_1 \overset{c_1}{\mapsto} s_2$ and $s_1 \overset{c_2}{\mapsto} s_2$). In addition, parallel composition is commutative, associative, and idempotent. Parallel composition gives us a way to add together information transfers from the same signal (antecedent) to the same target (consequent).
- Channel c_1 is a *refinement* of channel c_2, written $c_1 \preceq c_2$ iff $c_1 = c_1 \| c_2$. So, a channel c_1 is a refinement of c_2 just when it relates *fewer* pairs. This means it will support more constrains by being more discriminating in what it relates.
- Sequential composition preserves refinement. That is, if $c_1 \preceq c_2$ then $c_1; d \preceq c_2; d$ and $d; c_1 \preceq d; c_2$.

Recall the treatment of declarative utterances on Barwise's approach. A declarative utterance has both a demonstrative and a declarative content — respectively, a situation and a type. We pick out a situation and classify it with a type whenever we make a declarative utterance. According to Barwise's work on conditionals, whenever we utter a conditional, we classify a *channel* with a *constraint*. This makes sense, because when I utter a conditional, like

If white exchanges knights on $d5$ she will lose a pawn.

in the course of a game of chess, I am stating that there is some kind of relation between antecedent and consequent situations. There is a channel, relating next-move situations to situations which follow from it in the course of the game. I am saying that in all of the antecedent situations in which white exchanges knights on $d5$, in the consequent situations, she loses a pawn. This is exactly what the channel-constraint evaluation clause dictates.

In Barwise's initial account of information flow, he shows how a range of model structures in logic, computer science and information theory can each be seen as models of information flow. We'll show that a large class of Routley-Meyer frames also count as models of information flow.

4 Frames Model Information Flow

Recall the condition for a channel to support a conditional type.

> The channel $c \models \phi \to \psi$ if and only if for all situations s_1, s_2, if $s_1 \overset{c}{\mapsto} s_2$ and $s_1 \models \phi$ then $s_2 \models \psi$.

Clearly this is reminiscent of the modelling condition of conditionals in frame semantics. If we take it that channels *are* situations, then the condition *is* that of the conditional in the frame semantics, where $\mapsto$ is R.

In frame semantics $x \overset{y}{\mapsto} z$ means that the conditional information given by y applied to x results in no more than z. This grounds the monotonicity condition

$$\text{If } x' \sqsubseteq x,\ y' \sqsubseteq y \text{ and } z \sqsubseteq z' \text{ and } x \overset{y}{\mapsto} z \text{ then } x' \overset{y'}{\mapsto} z'.$$

It is natural to take the serial composition $x;y$ to be contained in situation z just when $x \overset{y}{\mapsto} z$. This is because $x \overset{y}{\mapsto} z$ is read as "applying the information in y to that in x gives information in z." But serially composing x and y is just applying the information from y to that in x in order to get a new channel. So, if the application of y to x is bounded above by z, we must have $x;y$ contained in z (given the identification of channels and situations). And *vice versa*. So, from now we will read $x \overset{y}{\mapsto} z$ as $x;y \sqsubseteq z$ and *vice versa*.

What does associativity of channels mean in this context? We simply require that $(x;y);z \sqsubseteq u$ if and only if $x;(y;z) \sqsubseteq u$ for each u. But this comes out as follows. $(x;y);z \sqsubseteq u$ if and only if for some v, $x;y \sqsubseteq v$ and $v;z \sqsubseteq u$. In other words, for some v, $x \overset{y}{\mapsto} v$ and $v \overset{z}{\mapsto} u$. Conversely, $x;(y;z) \sqsubseteq u$ if and only if for some w, $x;w \sqsubseteq u$ and $y;z \sqsubseteq w$, which can be rephrased as $x \overset{w}{\mapsto} u$ and $y \overset{z}{\mapsto} w$. Given our rewriting of sequential channel composition in terms of the channel relation $\mapsto$ we have an associativity condition in terms of $\mapsto$ alone. This will be enough to start our definition of a frame modelling information flow.

Definition 4 A *bare frame* is a quadruple $\langle g, S, \mapsto, \sqsubseteq\rangle$, where S is a set of *situations*, $\mapsto$ is a ternary relation on S, $g \in S$, is the *logic situation* and $\sqsubseteq$ is a partial order on S. The objects satisfy the following further conditions.

- If $x' \sqsubseteq x$, $y' \sqsubseteq y$ and $z \sqsubseteq z'$ and $x \overset{y}{\mapsto} z$ then $x' \overset{y'}{\mapsto} z'$
- $x \overset{g}{\mapsto} y$ if and only if $x \sqsubseteq y$.
- $(\exists v)(x \overset{y}{\mapsto} v \text{ and } v \overset{z}{\mapsto} u)$ if and only if $(\exists w)(x \overset{w}{\mapsto} u \text{ and } y \overset{z}{\mapsto} w)$.

Now that we have the structures defined, we need to show that these structures really model the axioms, by defining parallel and serial composition.

Take situations a and b. Their serial composition ought to be the 'smallest' situation x (under $\sqsubseteq$) such that $a \overset{b}{\mapsto} x$ given our motivation of identifying $a \overset{b}{\mapsto} c$ with $a;b \sqsubseteq c$. However, nothing assures us that such a minimal situation exists. There may be two candidate situations which agree with regard to all conditionals, but disagree with regard to a disjunction $p \vee q$. As situations are prime, neither of these is minimal. Instead of requiring that such a situation exist, we will model the serial composition of these two situations as the *set* $\{x : a \overset{b}{\mapsto} x\}$. If we take a set to support the type ϕ just when *each* of its elements supports ϕ, the set $\{x : a \overset{b}{\mapsto} x\}$ will work as the serial composition of a and b. It may be considered to be a 'non-prime situation,' or merely as the information shared by a collection of situations. From now on we take our channels to be sets of situations like this. A channel can be taken to be part of a situation just when the situation is an element of the channel. Let's make things formal with a few definitions.

Definition 5 Given a bare frame $\langle g, S, \mapsto, \sqsubseteq\rangle$

- $X \subseteq S$ is a *cone* iff for each $x \in X$, if $x \sqsubseteq y$ then $y \in X$.
- If X is a cone, $X \models \phi$ iff $x \models \phi$ for each $x \in X$.
- If X, Y and Z are cones, $X \overset{Y}{\mapsto} Z$ if and only if for every $z \in Z$ there are $x \in X$ and $y \in Y$ where $x \overset{y}{\mapsto} z$.
- If X and Y are cones, $X \sqsubseteq Y$ if and only if $Y \subseteq X$. In addition, $X;Y = \{z : X \overset{Y}{\mapsto} z\}$, $X \parallel Y = \{z : X \sqsubseteq z \text{ and } Y \sqsubseteq z\}$.
- For each situation x, $\uparrow x$ is the principal cone on x. In other words, $\uparrow x = \{x' : x \sqsubseteq x'\}$.

Given these definitions, it is not difficult to prove the following results.

Lemma 1 *Given a bare frame $\langle g, S, \mapsto, \sqsubseteq\rangle$ with an evaluation $\models$*

- *$X \models \phi \to \psi$ iff for each pair of cones Y, Z, where $Y \overset{X}{\mapsto} Z$, if $Y \models \phi$ then $Z \models \psi$.*
- *$X \models \phi \to \psi$ iff for each pair of situations y, z, where $\uparrow y \overset{X}{\mapsto} \uparrow z$, if $y \models \phi$ then $z \models \psi$.*
- *$X \models \phi \wedge \psi$ iff $X \models \phi$ and $X \models \psi$.*
- *$X \models \phi \vee \psi$ iff for each $x \in X$, either $x \models \phi$ or $x \models \psi$.*
- *$\uparrow x \sqsubseteq Y$ iff for each $y \in Y$, $x \sqsubseteq y$.*
- *$X \sqsubseteq \uparrow y$ iff $y \in X$.*
- *$(\exists v)(X \overset{Y}{\mapsto} v$ and $v \overset{Z}{\mapsto} U)$ iff $(\exists w)(X \overset{w}{\mapsto} U$ and $Y \overset{Z}{\mapsto} w)$.*
- *$\uparrow x \sqsubseteq \uparrow y$ iff $x \sqsubseteq y$.*
- *$\uparrow x \overset{\uparrow y}{\mapsto} \uparrow z$ iff $x \overset{y}{\mapsto} z$.*
- *$\uparrow x \models \phi$ if and only if $x \models \psi$.*

Proof. Straight from the definitions. We leave them as an exercise. □

Because of the last three results in that lemma, principal cones will do for situations whenever they occur. From now, we will slip between a principal cone and its situation without mentioning it.

The significant result is that $X;Y$ really is the serial composition of X and Y. In other words, we can prove the following:

Lemma 2 *For all cones X and Y, and for all situations a and c, $a \overset{X;Y}{\longmapsto} c$ iff there is a situation b such that $a \overset{X}{\mapsto} b$ and $b \overset{Y}{\mapsto} c$.*

Proof. Suppose that $a \overset{X;Y}{\longmapsto} c$. Then for some $d \in X;Y$, $a \overset{d}{\mapsto} c$. However, if $d \in X;Y$ we must have an $x \in X$ and a $y \in Y$ where $x \overset{y}{\mapsto} d$. So, $x \overset{y}{\mapsto} d$ and $a \overset{d}{\mapsto} c$. This means that for some b, $a \overset{x}{\mapsto} b$ and $b \overset{y}{\mapsto} c$ by one half of the associativity condition. This means that $a \overset{X}{\mapsto} b$ and $b \overset{Y}{\mapsto} c$ as desired.

The converse merely runs this proof backwards and we leave it as an exercise. □

Then serial composition is associative because of our transitivity condition on modelling conditions.

Lemma 3 *In any bare frame, for any cones* X, Y *and* Z, $X;(Y;Z) = (X;Y);Z$.

Proof. If $w \in X;(Y;Z)$ then $X \overset{Y;Z}{\longmapsto} w$, which means that for some $x \in X$ and $v \in Y;Z$, $x \overset{v}{\mapsto} w$. Similarly, $v \in Y;Z$ means that for some $y \in Y$ and $z \in Z$, $y \overset{z}{\mapsto} v$. But this means that for some u, $x \overset{y}{\mapsto} u$ and $u \overset{z}{\mapsto} w$, which gives $u \in X;Y$ and so, $w \in (X;Y);Z$ as desired. The converse proof is the proof run backwards, as usual. □

As things stand, parallel composition may not work as we intend. We may have cones X and Y for which the parallel composition is empty (that is, there is no z where $X \sqsubseteq z$ and $Y \sqsubseteq z$), but this means that $a \overset{X \| Y}{\longmapsto} b$ does not hold (look at the clauses). However, we may still have $a \overset{X}{\mapsto} b$ and $a \overset{Y}{\mapsto} b$. We need an extra modelling condition.

Definition 6 A bare frame $\langle g, S, \mapsto, \sqsubseteq \rangle$ is an *information frame* if and only if for each a, b, x, y where $a \overset{x}{\mapsto} b$ and $a \overset{y}{\mapsto} b$, there is a z where $x \sqsubseteq z$ and $y \sqsubseteq z$ and $a \overset{z}{\mapsto} b$. Parallel composition in an information frame is defined as you would expect. $X \| Y = \{z : X \sqsubseteq z \text{ and } Y \sqsubseteq z\}$.

Lemma 4 *In any information frame, for all cones* X *and* Y*, and for all situations* a *and* b, $a \overset{X}{\mapsto} b$ *and* $a \overset{Y}{\mapsto} b$ *iff* $a \overset{X \| Y}{\longmapsto} b$. *Furthermore, parallel composition then does what we would expect of it:* $X \| X = X$, $X \| Y = Y \| X$ *and* $X \|(Y \| Z) = (X \| Y) \| Z$. *In fact* $X \| Y = X \cap Y$.

Proof. Trivial. Clearly, if $a \overset{X \| Y}{\longmapsto} b$ then $a \overset{X}{\mapsto} b$ and $a \overset{Y}{\mapsto} b$ by monotonicity. The converse holds by the definition of an information frame. The properties of information frames follow from the fact that $\sqsubseteq$ is a partial order. The fact that parallel composition turns out to be intersection follows from the fact that it is defined on cones on the partial order. □

The relation $\preceq$ of refinement is simply $\sqsupseteq$; $X \preceq Y$ iff $X \| Y = X$, iff $X \cap Y = X$ iff $X \subseteq Y$ iff $X \sqsupseteq Y$.

The last thing to show is that serial composition preserves refinement. We need to show the following:

Lemma 5 *In any information frame, if* $X_1 \sqsubseteq X_2$ *and* $Y_1 \sqsubseteq Y_2$ *then* $X_1;Y_1 \sqsubseteq X_2;Y_2$.

Proof. This follows immediately from the hereditary conditions on $\mapsto$. □

So, our frame structures, with a few extra bells and whistles, are models of information flow. The resulting logic is weaker than the relevant logic **R** (and it even invalidates some theorems of **E**), but it is nonetheless quite an interesting system in its own right. Elsewhere (Restall 1994b) I show it (and several a close cousin) to be decidable, and naturally motivated from other points of view.

5 Frames Are the Best Model of Information Flow

Barwise's paper shows that model structures like those of classical logic and intuitionistic logic also model information flow. If we were only able to put relevant logics into this class, this would not say very much them. Why favour systems like relevant logics? Why favour he distinctions that frame semantics can draw (between serial and parallel composition) and the identifications it makes (such as identifying channels with situations)?

5.1 Why Frames?

Firstly, frames are the sensible model of information flow, because they remain faithful to the original intuitions about the meanings of conditional utterances. If we say that situations and channels are totally distinct, then we have a problem. We decided that a conditional statement "if S_1 then S_2" has the constraint $\phi \to \psi$ as descriptive content, and a particular *channel* c as demonstrative content. But we already decided that other statements have *situations* as their demonstrative content. Why are conditional statements any different? To avoid such an arbitrary distinction, we must admit a relationship between situations and channels. If the conditional "if S_1 then S_2" has a situation s as its demonstrative content, then we may take s itself to be a channel between situations. This is admitted in the frame semantics. Each situation is a channel and arbitrary *cones* of situations are also channels — chiefly to deal with channel composition.

In other words, once we recall a major motivating application of information flow (modelling conditionals in situation semantics in terms of regularities grounded in the world) the frame semantics identification of channels with situations is the natural conclusion.

5.2 Why So Many Frames?

We know that frames model information flow. It doesn't follow that every information frame is relevant to our cause. Barwise specified that serial composition be associative, leaving open whether it was symmetric, idempotent or whatever. Our formalism leaves it open for us to construct frames in which serial composition fails to be symmetric or idempotent. Is this justified, or should there be more conditions on frames?

To deal with a part of this question, let's see how a failure of the idempotence of composition can be motivated.[3] We want to find a counterexample to $x; x \sqsubseteq x$. (It is simple to show that in all of Barwise's examples in his paper, this condition holds. Yet, it does not hold in all information struc-

[3] I am especially fond of this example, for a great deal of my research has been into logics in which this rule fails (Restall 1994a). It is equivalent to rejecting $\phi \wedge (\phi \to \psi) \to \psi$ and $\big(\phi \to (\phi \to \psi)\big) \to \big(\phi \to \psi\big)$.

tures.) But this is simple. For many channels, you may eke out more information by repeated application. We need only find a domain in which using a channel twice (serially) yields more information than using it once. We'll just sketch one such application, and leave the reader to think of more.

Take situations to be mathematical proofs, and we will take the information content to be the *explicit content* of the proof.[4] That is, the things that are stated in the proof. We can model the information flow from a proof to another proof (which may contain the first proof as a part) by way of information links that relate proof-situations by means of the deductive principles of mathematics. For example, one such rule is this:

If n is even, so is $n + 2$.

If this rule is warranted by a channel x, then if y and z are proof situations where $y \overset{x}{\mapsto} z$, and y supports the claim '6 is even,' then z supports the claim '8 is even.' The proof z may have been produced from the proof y by applying the rule of inference we've seen, and the channel x indicates (or warrants) that application.

Now a proof may have '8 is even' as a part of its explicit content without '10 is even' also being a part of its content. To get that from the initial proof situation y, we need to apply the rule *twice*, and so, use the channel x twice. The channel $x;x$ therefore warrants more than x. We may have $y \overset{x;x}{\longmapsto} z$ without $y \overset{x}{\mapsto} z$.

6 Putting the Account to Work

Now that we have the formalism in place, we can describe the relationship between channels (or situations) and conditionals. This will also give the wherewithal to give an account of some seemingly paradoxical arguments using conditionals. This is important, because while relevant logics are good at blocking paradoxes such as $\phi \to (\psi \to \psi)$ and so on, they are not so good at blocking other difficulties with conditionals. Common examples are failures of transitivity (from $\phi \to \psi$ and $\psi \to \theta$ to deduce $\phi \to \theta$) and strengthening of the antecedent (from $\phi \to \psi$ to deduce $\phi \wedge \theta \to \psi$). The formal system we consider has

$$(\psi \to \theta) \to \big((\phi \to \psi) \to (\phi \to \theta)\big) \qquad (\phi \to \psi) \to (\phi \wedge \theta \to \psi)$$

as theorems. How can we avoid being nailed by the counterexamples people give?

The situated semantics of natural language provides an answer: We must recall that all declarative utterances have a demonstrative content (a situation) and a descriptive content (a type). This is no different for conditional utterances, which have a situation as demonstrative content,

[4]And its real logical consequences, as defined in the situation-theoretic account. So, if $A \wedge B$ is a part of the information content of a proof, so are A, B and $A \vee C$.

and a type of the form $\phi \to \psi$ as descriptive content. The truth or otherwise of the conditional depends on whether the situation described is of the type or not. This situation relativity gives us the means to give an account of odd-sounding arguments.

The first is a putative counterexample to transitivity. Consider the two conditionals:[5]

If an election is held on December the 25th, it will be held in December.
If an election is held in December, it will not be held on December the 25th.

By a naïve application of transitivity we could argue from these two claims that if an election is held on December the 25th, it will not be held on December the 25th. This is an odd conclusion to draw. The analysis in terms of channels and constraints can help explain the puzzle without rejecting transitivity.

Firstly, consider the situation described by the first conditional. This situation — in its job as a channel — pairs election-on-December-the-25th situations with election-in-December situations. Presumably this channel arises by means of logical truth, or our conventions regarding dates and months. Quite probably, it pairs each election situation with itself. If the antecedent situation has the election on December the 25th, then the consequent situation (the same one) has the election in December. There is little odd with this scenario.

The second situation pairs antecedent election-in-December situations with consequent election-not-on-December-the-25th situations. Given the plausible assumption that it pairs antecedent situations only with identical consequent situations (or at least, consequent situations not incompatible with the antecedent situations — so it will not pair an antecedent situation with a consequent situation in which the election occurs at a *different* date) it will 'filter out' antecedent situations in which the election is held on Christmas Day. In Barwise's words, these situations are *pseudo-signals* for the constraint. These aberrant situations are not related (by our channel) with any other situation at all. The channel only relates 'reasonably likely' election situations with themselves, and so, it supports the constraint that elections in December are not held on Christmas Day just because it doesn't relate those (unlikely) situations in which an election is held on that day.

Given these two channels, it is clear that their composition supports the constraint 'if there is an election on December the 25th, then the election is not on December the 25th' simply because there are no signal/target pairs for that channel in which the signal is a situation in which the election is on December the 25th. The composition of the channels filters out these antecedent situations by construction. That channel supports other odd

[5]In Australia the timing of elections is decided by the government of the day.

constraints like 'if there is an election on December the 25th, then Queensland has won the Sheffield Shield for the last twenty years.' This does not tell us anything enlightening about what would happen were an election to actually occur on December the 25th — it only tells us that this particular channel has ruled that possibility out.

Composition of channels may 'filter out' possibilities (like elections held on Christmas day) that will later become relevant. Then we typically broaden the channels to admit more possibilities. (This is akin to expanding the set of 'nearby possible worlds' used to evaluate counterfactuals on the Lewis-Stalnaker accounts.) Typically when we utter a counterfactual conditional we mean that situations classified by the antecedent will feature in signal/target pairs of the channel being utilised. (Otherwise, why utter the conditional?) In cases like this argument, the composed channel is not like this. The antecedent situations being described do not feature as in signal-target pairs of the channel being classified. So, the conditional given by the Xerox principle is not the same as the conditional you would typically be expressing had you said

If an election is held on December the 25th, it will not be held
on December the 25th.

Had you said that (and it is a strange thing to say) then most likely, the channel being classified would have as signal/target pairs some situations in which the election is held on December the 25th. (Otherwise, why call attention to their possibility?) And if this is so, the conditional you express by asserting the sentence will differ from that arising from the Xerox principle but this only points out the channel relativity of conditionals. This is parallel to the situation-theoretic fact that propositions are situation relative. So, the principle itself is sound, but difficulties like these must keep us alert to the way it is used.

As was the case with the transitivity, we can use the channel-theoretic account to explain the oddity of certain 'deductions' using conjunction. For example, given a number of background conditions

If it snows, I won't be surprised.
If it hails, I won't be surprised.

could be both true. It may be quite cold, and both snow and hail could be possible given the current weather conditions. Furthermore, my expectations are attuned to the relevant facts about the weather, and so, I won't be surprised under either of those conditions. However, in this situation, there is no guarantee that

If it both snows and hails, I won't be surprised.

because the combination of snow and hail is a very rare thing indeed — and I may be aware of this. This appears to be a counterexample to the principle of addition of information (collapsing the two conjuncts in the consequent to one). Yet, as with transitivity, this is not a real counterexample. What's more, the account in terms of channels can help us explain the surprising nature of the 'deduction.'

Consider the channels supporting the original two claims. Obviously they do not relate all snowing or hailing signal-target pairs with consequent mental states, because for some snowing or hailing situations (ones that are combined) I *am* surprised. So, the channels supporting these claims must ignore the possible (but rare) situations in which snow and hail coincides. In other words, snow-and-hail situations are pseudo-signals for this constraint. This is understandable, because it is a rare situation to encounter. Now when we consider the parallel composition of the two channels, it is simple to see that it has no signal-target pairs where it is snowing and hailing in the signal situation. In each of the original channels, these possibilities were filtered out, so they cannot re-emerge in their parallel composition. The third conditional is supported by the parallel composition of channels only vacuously. The composed channel does not relate any snowy-and-haily situations.

Were we to say 'if it both snows and hails, I won't be surprised' the channel classified would (usually) not be one that filters out odd snowy-and-haily situations, because we have explicitly mentioned that situation as a possibility. Again, we must be careful to not identify the conclusion of the addition of information principle with a claim we may express ourselves. For each declarative utterance, there is a corresponding situation or channel that is classified. Different utterances could well classify different situations or channels.

In this way we can use the formalism to explain the oddity in certain conditional argument forms. They are sensitive to the situations being described, which can vary from premise to conclusion, without this fact being explicit.

7 Metaphysical Issues

The semantic structure makes essential use of non-actual situations. Consider false conditionals with false antecedents. To make

If Queensland win the Sheffield Shield, grass will be blue

untrue at a situation x we need a y and z where $y \overset{x}{\mapsto} z$, $y \models$ Queensland win the Sheffield Shield and $z \not\models$ grass is blue. This requires the existence of a situation y that supports the claim that Queensland win the Sheffield Shield. This is patently false, so no actual situation supports it. It follows that the conditional is true (which is an unsavoury conclusion) or there are some non-actual situations.

It follows that a decent account of conditionals in this formalisation must involve non-actual situations — so these don't qualify as bits of the world (in the same way that actual situations do, anyway). Barwise agrees. His answer to the question is that situations ought to be seen as

> mathematical objects for modelling possibilities, and not as real but not actual situations. (Barwise 1993)

But this won't suffice as an explanation — at least if this semantics is to work as an account of anything. Because if a *model* is to have any explanatory force, the things in the model must correspond to *something* real. If we have a mathematical model for a possibility, then if it does any explanatory work (which it does) there must be something *real* that corresponds to it, and that grounds the explanation in the Way Things Are. If these things are not "real but not actual situations" it would be interesting to hear what they are. Giving an account of these that doesn't amount to realism about (particular) non-actual situations is exactly parallel to the task of giving a non-realist account of possible worlds. Calling them "mathematical models" is honest, but it only pushes the question back to those who want to know what the model actually *models*. For this approach to have any chance of working without committing us to modal realism, you must explicate the notion of "modelling possibilities." At face value this does seem to involve a relationship between models and the possibilities they purport to model. However, there may be another way to cash out the conception: we can say that x models a possibility (or *represents* a possibility) if x *would* model (or represent) a situation, were things different in some relevant respect. (Chris Menzel (Menzel 1990) has worked out this approach in the context of modal logic.) To make this work in our context we need to spell out what way things are allowed to vary, and be more specific about the representation relation. However, it is plausible that some explanation like this might work.

Taking this line would result in the analysis being circular in one sense. Cashing out the notion of representation requires using conditionality or possibility — so we will not be giving a reductive account of conditionals. On this semantics, possibility or conditionality will be primitives. However, they will be primitives that are closely associated with other concepts such as the channeling relation between situations, and this, as in all formal semantics, will give us a helpful regimentation of our intuitions about conditionals, and it will give us a new way to analyse their semantic content.[6]

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