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PARACONSISTENT LOGICS!

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Abstract In this note I respond to Hartley Slater's argument [12] to the effect that there is no such thing as paraconsistent logic. Slater's argument trades on the notion of contradictoriness in the attempt to show that the negation of paraconsistent logics is merely a subcontrary forming operator and not one which forms contradictories. I will show that Slater's argument fails, for two distinct reasons. Firstly, the argument does not consider the position of non-dialethic paraconsistency (which rejects the possible truth of any contradictions). Against this position Slater's argument has no bite at all. Secondly, while the argument does show that for dialethic paraconsistency (according to which contradictions *can* be true), certain other contradictions must be true, I show that this need not deter the dialethic paraconsistentist from their position.

Paraconsistent Logics!

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Comments are still welcome

— I —

In this journal, Hartley Slater [12] provides an interesting argument against paraconsistent logics. The argument goes like this: Contradictories cannot be true together. A proposition A and its negation $\sim A$ are contradictories. According to the semantics of paraconsistent logics, for instance Priest's **LP** [5], there are valuations of propositions A and $\sim A$ which assign them *both* a true value. (In **LP**, a valuation V is a function from propositions to values in $\{\{0\}, \{1\}, \{0, 1\}\}$, satisfying these conditions:

- $1 \in V(A \wedge B)$ iff $1 \in V(A)$ and $1 \in V(B)$.
- $0 \in V(A \wedge B)$ iff $0 \in V(A)$ or $0 \in V(B)$.
- $1 \in V(\sim A)$ iff $0 \in V(A)$.
- $0 \in V(\sim A)$ iff $1 \in V(A)$.

A proposition A is *true according to* V if $1 \in V(A)$. Clearly we can assign an atomic proposition p the value $\{0, 1\}$. In this case both p and $\sim p$ are true according to this valuation.)

Slater's argument continues: This shows that $\sim A$ cannot be the negation of A, as $\sim A$ and A can be true together (as there is some valuation according to which both $\sim A$ and A are true). So, paraconsistent logics are not, after all, a formalisation of the logic of negation. They might be a formalisation of something else, but since A and $\sim A$ are not contradictories, the concept formalised by $\sim$ in these logics cannot be negation.

In the rest of this paper I will show that Slater's argument need not deter the paraconsistent logician. To do this, I must define some terms. Firstly, *paraconsistency*. A paraconsistent logic is one for which the inference from $A \wedge \sim A$ to an arbitrary B (*ex falso quodlibet*, hereafter EFQ) is not valid [7]. Or equivalently, paraconsistency is the doctrine that there are inconsistent but non-trivial theories, where by a theory we mean a set of sentences closed under logical consequence. These are equivalent under standard assumptions, for if EFQ were valid, then there would be no non-trivial inconsistent theories, for any theory containing a contradiction would contain every sentence. And conversely, if EFQ were invalid in some instance (suppose that $A \wedge {\sim} A \not\vdash B$ for some particular A and B) then we could find a theory containing $A \wedge {\sim} A$ without it also containing B. Note that paraconsistency is a doctrine about logical consequence. It says nothing about what is possible or what is necessary.

Paraconsistency comes in a number of varieties. There have been different taxonomies of paraconsistent logics or theories [1, 7, 8], but none of these suit the needs of this argument, so I will introduce my own distinction between three sorts of paraconsistency. The first is *regular dialethism.* Dialethism is the doctrine that there are true contradictions [6, 7], and I will rename this view *regular* dialethism, for it has a close cousin, in *light dialethism* — the view that there are *possibly true* contradictions. Someone may be a 'light dialethist' without being a 'regular dialethist' by holding that some contradiction could be true, but actually isn't. The world could have been inconsistent, but for some reason or other, it happens to be consistent. I will call *dialethic paraconsistency* the view which combines paraconsistency with either light or regular dialethism.

Carving the paraconsistent pie like this makes it quite clear that there is one piece left. There is *non-dialethic paraconsistency.* This is the view which combines paraconsistency (not everything follows from each contradiction) with the rejection of both light and regular dialethism. For the non-dialethic paraconsistentist, contradictions can't be true.[1]

Slater does not make the distinction between dialethic and non-dialethic paraconsistency, and his argument does not work in the same way against both varieties of paraconsistency. In the next section I will consider how his argument fares against non-dialethic paraconsistency. In the last section I will consider how dialethic paraconsistency changes the evaluation of Slater's argument.

— II —

How does Slater's argument deal with non-dialethic paraconsistency? The non-dialethic paraconsistentist is free to agree with Slater that A and ${\sim} A$ are contradictories.[2] The non-dialethic paraconsistent rejects the possible truth of any contradiction. So, they can follow Slater with the first premise of his argument.

[1] I suppose there is also *agnostolethic paraconsistency*, which combines paraconsistency with agnosticism about the possibility of contradictions, but this does not seem to be an interesting position to hold. After all, each theorist in this particular debate thinks that if paraconsistency is true, then either dialethic or non-dialethic paraconsistency is true too.

[2] They are *free* to agree, not *forced* to agree. You could be a constructivist non-dialethic paraconsistentist. You may think that sometimes neither A nor ${\sim} A$ is true.

The non-dialethic paraconsistentist can also agree that there are valuations V which assign 'true' to both A and to $\sim A$. But this tells us nothing about the co-possibility of A and $\sim A$ until we until we know how to intepret valuations. And this matter is one in which there is a degree of flexibility. The non-dialethic paraconsistentist will note that the world could not be such as the valuation describes, for the valuation is an inconsistent one. Validity is a matter of preservation over all valuations (that is, $A \vdash B$ if and only if for every valuation V, if $1 \in V(A)$ then $1 \in V(B)$ too)[3] not merely over all the consistent ones. However, contradictoriness is a matter concerning only consistent valuations, as only these have anything to do with what is *possible*. It is not sufficient for B to be a consequence of A that necessarily, given that A is true, so is B (or more crudely, that B is true in all worlds in which A is true). That approach would take the paradoxes of entailment (like EFQ) to valid once we grant that contradictions are necessarily not true. So, the non-dialethic paraconsistentist pares those two notions apart. Slater's (hidden) premise that if there's a valuation V which assigns 'true' to both A and $\sim A$, then it is possible that both A and $\sim A$ be true, fails for the non-dialethic paraconsistentist.

Non-dialethic paraconsistency is not a merely theoretical position. There are a number of active proponents of this brand of paraconsistency. The canonical weak paraconsistentists are relevant logicians, such as the relevantist of Belnap and Dunn [3], Read [9, 10], and Meyer and Martin [4]. It is unclear whether Brazilian school of paraconsistency adheres to a non-daialethic or a dialethic parcaonsistency, as their writings do not indicate whether they think of inconsistencies as possibly true or not [1, 2]. I have also defended non-dialethic paraconsistency [11].

So, for the weak paraconsistentist, not all of the premises of Slater's argument go through unscathed. The premise that the valuations represent possibilities is rejected. The mere fact that a valuation may assign truth to both A and $\sim A$ does not commit you to taking A and $\sim A$ as possibly true together, any more than the fact that in some fiction someone trisects an angle with ruler and compass commits you to taking such a trisection to be possible. The non-dialethic paraconsistentist is free to say that this valuation represents a way that things *cannot* be, not a way that things *can* be. Non-dialethic paraconsistency cannot be defeated by Slater's argument, as not all of the premises are true.

— III —

For the dialethic paraconsistentist, the approach of the previous section is

[3] Most weak paraconsistentists will want the quantifier here to range over more than just **LP** valuations when testing for validity, particularly because $A \vee \sim A$ is true in all of them, so $A \vee \sim A$ will follow from every proposition. Allowing the value of a proposition to be empty helps here.

inappropriate. A dialethic paraconsistentist can agree with Slater that **LP** valuations represent possibilities, since it is possible for contradictions to be true. We cannot reject the conclusion of the argument in that way. In this section I want to show that the appropriate response of the dialethic paraconsistentist is not to reject the conclusion of the argument, but rather to accept it. However, I will also show that this is not in itself a reason for the dialethic paraconsistentist to reject paraconsistent logic as a formalisation of negation. In other words, Slater's argument draws our attention to contradictions which the dialethic paraconsistentist is bound to accept. While someone from a non-dialethic position would find this contradiction troubling, and reason to reject one of the premises of Slater's argument, the dialethic theorist is free to accept the contradiction (as long as she is a regular dialethist).

So let's consider Slater's argument. I will assume that all proponents in the debate agree that a proposition and its negation are contradictories. That is, they are all committed to the following proposition, for every instance of A.

$$\Box(A \vee \sim A) \wedge \sim \Diamond(A \wedge \sim A) \qquad (*)$$

Priest, for one, will agree with this proposition. However, a purely 'light dialethist' cannot agree with this proposition. For the light dialethist must hold $\Diamond(A \wedge \sim A)$ for some A, by virture of being a light dialethist, and this contradicts the second conjunct $\sim \Diamond(A \wedge \sim A)$. So the light dialethist is committed to the contradiction $\Diamond(A \wedge \sim A) \wedge \sim \Diamond(A \wedge \sim A)$. So, Slater's argument shows quite explicitly that a merely light dialethist cannot hold that A and $\sim A$ are contradictories (in the sense given here) without thereby accepting a contradiction, and hence, becoming a regular dialethist as well.

We can adapt the argument to show that if the theorist who holds to $(*)$ also holds to $\Diamond(A \wedge \sim A)$ then they must agree that A and $\sim A$ are contradictories (by virtue of $(*)$) but also that A and $\sim A$ are not contradictories (by agreeing to $\Diamond(A \wedge \sim A)$).

$\Diamond(A \wedge \sim A)$	by assumption;
$\sim \Box(A \vee \sim A) \vee \Diamond(A \wedge \sim A)$	disjunction introduction;
$\sim \Box(A \vee \sim A) \vee \sim\sim \Diamond(A \wedge \sim A)$	$B \vdash \sim\sim B$;
$\sim(\Box(A \vee \sim A) \wedge \Diamond(A \wedge \sim A))$	a de Morgan law.

The conclusion is the negation of $(*)$. Now not everyone will agree with the steps of this argument. The move $B \vdash \sim\sim B$ is disputed in Brazilian paraconsistent logics, for example. However, each move is valid in **LP** and similar paraconsistent logics. This means that contradictions in paraconsistent logics can *spread*. If you hold that it's possible that A and $\sim A$, then you also hold both that A and $\sim A$ are contradictories and that A and $\sim A$ are *not* contradictories.

Not only do contradictions spread in this way, but they also spread in other ways. I will examine just two before I consider what this means for dialethic paraconsistency. The next example is inspired by Slater's contention that in paraconsistent logics $\sim A$ is not the negation of A. In one sense he is right. This can be proved in Priest's favoured logic Δ of his monograph *In Contradiction* [6]. First we need to explain clearly what it is for Priest for B is the negation of A. As good a formalisation as any would be to take this to be $B \Leftrightarrow \sim A$ where $C \Leftrightarrow D$ is set as $(C \Rightarrow D) \wedge (D \Rightarrow C)$, and where $\Rightarrow$ is entailment. Priest has an entailment operator in his logic Δ, and we need not examine all of its properties. However, one property which it does have is the fact that it satisfies this rule:

$$\Diamond(A \wedge \sim B) \vdash \sim(A \Rightarrow B) \qquad (**)$$

If it is possible that $A \wedge \sim B$ then it follows that $\sim(A \Rightarrow B)$. It follows from this that if A is possibly contradictory — if $\Diamond(A \wedge \sim A)$ — then $\sim A$ is not the negation of A, in the sense which we have defined above. The argument is simple.

$\Diamond(A \wedge \sim A)$	by assumption;
$\Diamond(\sim A \wedge \sim\sim A)$	$A \vdash \sim\sim A$ and diddling;
$\sim(\sim A \Rightarrow \sim A)$	by $(**)$;
$\sim(\sim A \Leftrightarrow \sim A)$	by diddling.

So, by using moves Priest endorses, we have shown that $\sim A$ is not the negation of A. Of course, Priest is also committed to $\sim A$ being the negation of A, and it is indeed a theorem that $(\sim A \Leftrightarrow \sim A)$. So we have another contradiction, this time featuring entailment, and not necessity or possibility.

For our final argument, we will show that where L is a proposition which can plausibly be held to encode *logic*, then $\sim L$ is true. That is, for the dialethic paraconsistentist, logic is false. To do this, let L be the proposition

$$\bigwedge \{p \Rightarrow q \mid p \vdash q\}$$

That is, L is the conjunction of all true statements of the form 'p entails q'. This is as good an account as any of the content of 'logic', if logic is meant to be an account of what follows from what. Then the dialethic paraconsistentist must agree that L is false.

$\Diamond(A \wedge \sim A)$	by assumption;
$\sim(A \Rightarrow A)$	by $(**)$;
$\sim \bigwedge\{p \Rightarrow q \mid p \vdash q\}$	since $A \vdash A$;
$\sim L$	by the definition of L.

So, for the dialethic paraconsistentist, their very logic is false. But Priest said this years ago [5], none of this is particularly new. The dialethic paraconsistentist is committed to certain contradictions being true. Slater's argument shows just how far they go. The proper response of the dialethic paraconsistentist is to *accept* these contradictions. To a dialethic paraconsistentist, for some propositions A, $\sim A$ is the negation of A and it is not the negation of A; A entails A and A does not entail A; and the theory of inference itself is inconsistent. This is one of the distinctives of the position of dialethic paraconsistency, especially as practiced by Priest. The metalanguage of theorising is itself inconsistent, for Priest rejects any radical break between the metalanguage and the object language. The latter is inconsistent, and so is the former. Slater's arguments can each be formalised in Priest's formal systems, and they come out as valid. But the conclusions are ones Priest is prepared to accept, without thereby *rejecting* his dialethic paraconsistent position. To really get a grip on his position, you need to show that it entails something really repugnant — something Priest wants to *reject* — like $0 = 1$. That is much harder.

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