A New Spin on the Hole Argument.*

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Abstract

This brief paper shows how an exact analogue of Einstein's original hole argument can be constructed in the loop representation of quantum gravity. The new argument is based on the embedding of spin-networks in a manifold and the action of the diffeomorphism constraint on them. The implications of this result are then discussed. I argue that the conclusions of many physicists working on loop quantum gravity—Rovelli and Smolin in particular—that the loop representation uniquely supports relationalism are unfounded.

1 Introduction.

Arguably the greatest unsolved puzzle of physics as we begin our march through the 21st century is that of constructing a quantum theory of gravity; i.e. the quantum theory that, minimally, has general relativity as a classical limit. There is, of course, something of a profusion of competing approaches each directed at resolving this puzzle.[1] A particularly promising approach

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[1]As Duff put it in the first Oxford quantum gravity symposium: "many and manifold and the methods of quantizing the gravitational field" ([Duf78]). Smolin refers to the contenders as "roads" in his recent popular account of the puzzle [Smo00]. There are several possible ways of taxonomising the various approaches using, e.g., the level of background dependence or the particular quantization method used. There are some—generally those who have worked on many approaches—who believe that the approaches are converging and can be seen as different aspects of the same underlying theory (*cf.* Baez [Bae96] and Smolin (*loc. cit.*)).

is the canonical quantization method known as *loop quantum gravity*.[2] This approach will be of most interest to philosophers of physics and of space and time both because it says novel things about the structure of space and time but also because it offers a quantization of the *vacuum* Einstein equations. For this latter reason it gives those who dabble in the substantivalism *vs* relationalism debate a nice vantage point from which to argue for their preferred conceptions of space and time.[3] However, a philosopher looking at the recent work in loop quantum gravity might easily be forgiven for thinking that the debate is settled firmly in favour of relationalism. I certainly think it is fair to say that the majority of physicists working on this approach see one of its 'lessons' as showing that a relationalist conception of space and time is the only way to make ontological sense of the theory, thus nestling under the wings of their heros 'papa Leibniz' and 'papa Einstein'. I suspect that the hole argument of Earman and Norton [EN87]—along with their particular way of characterizing relationalist and substantivalist positions via endorsements and denials of Leibniz equivalence respectively—has played no small role in this supposed vindication of relationalism.[4] This paper will attempt to show that this view is misguided. As regards the debate between substantivalists and relationalists, we are in precisely the same situation we were in with respect to the classical theory based on metric variables: both sides can get a firm foothold.

I begin by reviewing the central details of the hole argument using the canonical formalism with the metric as a 'position' variable and the extrinsic curvature as the canonically conjugate 'momentum' variable. I place especial emphasis on the importance of the diffeomorphism constraint in the functioning of the hole argument, and the importance of the treatment of these con-

[2]The level of pedagogy is already quite high with several textbooks and monographs available that are devoted specifically to the subject: see any of [AT96, GP96, Rov04]. A nice general review article is Rovelli [Rov99]— it can be found at `http://relativity.livingreviews.org/Articles/lrr-1998-1/`.

[3]Of course, string theory—the other 'big hitter'—adopts the stance that in order to bring together general relativity and quantum theory one has to deal with *all* of the other forces of nature too. I do not discuss this approach any further – I recommend Zweibach's new book on string theory to those who wish to learn more about it [Zwe04].

[4]This 'contamination' has recently spread into the problem of time in quantum gravity with the appearance of Belot and Earman's papers on the subject [BE99, BE01]. There they attempt to align gauge-invariant views with relationalism and non-gauge-invariant with substantivalism by arguing that the latter must countenance quantities that commute with the constraints, but not so the former. I argue against this taxonomy in [Ric05].

straints for matters of interpretation. I then show where spin-networks enter the picture by introducing general relativity in the 'new variables' discovered by Ashtekar and sketch the method of (constrained) quantization as applied to them.[5] Solving the quantum gauge-theoretic constraint associated with the new variables leads to a space of states whose points are naturally separated by spin-networks. I then show how a hole argument can be constructed for these states by considering the action of the diffeomorphism constraint on them. I then draw from this some philosophical morals concerning the ontological status of space and time.

2 Hole Arguments for General Relativity.

The hole argument is most easily couched in terms of models $\langle \mathcal{M}, \mathcal{D} \rangle$ (where $\mathcal{D}$ is a set of dynamical fields on $\mathcal{M}$ — any further background fields are, of course, absent in general relativity). Let us stick to the vacuum case and therefore assume that $\mathcal{D} = g$, so that the (Lorentzian) metric is the only dynamical field on $\mathcal{M}$. The models $\langle \mathcal{M}, g \rangle$ then minimally correspond to a 'bare' manifold possessing only topological and differential structure along with geometrical structure determined *dynamically* by g in accordance with the vacuum Einstein equation:

$$\mathbf{G}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R^{\alpha}_{\alpha} g_{\mu\nu} = 0. \tag{1}$$

The crucial property of these models (equivalently: Einstein's equations) for the hole argument is that they are *generally covariant*: if $\langle \mathcal{M}, g \rangle$ solves the field equations then so does $\langle \mathcal{M}, \phi^* g \rangle$, $\forall \phi \in \mathsf{Diff}(\mathcal{M})$. The 'pushed forward' field $\phi^* g$ will generally be different in the sense that, given a global chart on $\mathcal{M}$ with coordinates $\{x^i\}$, $\phi^* g(x) \neq g(x)$. However, it can happen that $\phi^* g = g$—i.e. that ϕ is a spacetime symmetry for g—even though $\{x^i\}$, $\phi^* g(x) \neq g(x)$. When this happens we have the beginnings of a hole argument; there will be many metrics that solve the equations that will give (locally) different results. Hence, choose a region of the manifold, $\mathcal{H} \subset \mathcal{M}$ (the hole), and suppose that we can solve completely for $\mathcal{M} - \mathcal{H}$ (i.e., we know $g(x)$, $\forall x \in \mathcal{M} - \mathcal{H}$). Now let $\phi_{\mathcal{H}}$ be a diffeomorphism that acts as the

[5] According to this formalism, an $\mathsf{SU}(2)$-connection takes the place of the metric and an orthonormal triad takes the place of the extrinsic curvature, thus making the phase space of general relativity resemble that of a Yang-Mills theory.

identity on $\mathcal{M} - \mathcal{H}$ but not within $\mathcal{H}$ (nor on the boundary $\partial\mathcal{H}$): then the field equations do not uniquely determine $g(x)$ for $x \in \mathcal{H}$ for both $g(x)$ and $\phi^* g(x)$ are solutions, and yet $\phi^* g(x) \neq g(x)$. If we put the hole to the future of some initial slice then this signals a violation of determinism!

In order to properly appreciate the workings of hole argument (*qua* problem of determinism, at any rate), one has to shift to the canonical (constrained Hamiltonian) formulation of general relativity and thus construct its phase space Γ. This move will allow us to connect the discussion up to the newer formulations of general relativity with different dynamical fields playing the role of the metric; to gauge theories; and it will also pave the way for the presentation of the quantum gravitational hole argument. Let us quickly review the basic form of a hole argument from this perspective. For now, I couch the discussion in the canonical formalism based on metric variables—as, I assume, was intended in Earman and Norton's original presentation and as is brought out in [BE99, BE01]. Thus, firstly we take spacetime to be a four-dimensional manifold $\mathcal{M}$ diffeomorphic to $\mathsf{S} \times \mathbb{R}$ – with S a (compact, orientable) 3-manifold taken to represent 'space' and where $\mathbb{R}$ is taken to represent 'time'. We choose S so that it is spacelike with respect to g and so that it is a Cauchy surface – let's call this surface Σ. Let t be the function on $\mathcal{M}$ associated with the foliation by Σ and whose level surfaces are the leaves of the foliation. A phase space is then constructed by taking the basic variables of the theory to be the 3-metric q_{ab} on Σ (playing the role of canonical 'position' variable) and p^{ab} (playing the role of canonical 'momentum' variable conjugate to q_{ab})[6] — both are induced by the 3+1 'split' together with g. Thus, an instantaneous state of the gravitational field is given by pairs $(q, p) \in \Gamma$. However, not any old pairs will do. The 4-dimensional diffeomorphism invariance of the covariant theory is translated into a pair of constraints on the initial data (Σ, q, p) so that they must satisfy:

$$\mathcal{D}_a(q,p) = -2q_{ac}\nabla_b p^{bc} = 0 \tag{2}$$

$$\mathcal{H}_\perp(q,p) = \mathsf{det}(q)^{-1/2}[q_{ac}q_{bd} - \frac{1}{2}q_{ab}q_{cd}]p^{ab}p^{cd} - \mathsf{det}(q)^{1/2}\,{}^3\mathsf{R} = 0 \tag{3}$$

[6]The momentum variable is related to the extrinsic curvature K^{ab} of Σ by $p^{ab} \equiv \mathsf{det}(q)^{1/2}(K^{ab} - K^c_c q^{ab})$, where K_{ab} describes the embedding of Σ in $(\mathcal{M}, g)$).

This analysis brings out the gauge theoretic aspects of general relativity. In phase space terms we see that the full phase space Γ does not correctly represent the physically possible worlds of general relativity, for not all points will satisfy the constraints. However, the points that *do* satisfy the constraints form a submanifold of Γ known as the constraint surface $\mathcal{C}$. The connection to the hole argument is pretty clear: the spacetime diffeomorphisms correspond to gauge motions generated by the constraints. The relevant constraint for the hole argument is $\mathcal{D}_a$ since this generates spatial diffeomorphisms of Σ. The gauge motions act on all points of Γ *including* those points lying within $\mathcal{C}$, but they have the property of leaving these latter points invariant since they generate gauge transformations on $\mathcal{C}$.

Recasting the hole argument in phase space terms allows us to connect up the interpretive difficulties it engenders to those generic to gauge theories (*cf.* Belot & Earman [BE99, BE01]). This way we see that the problem that was seen to be faced by substantivalists arises from a supposed one-to-one link between the points of phase space (in this case $\mathcal{C}$) and physical possibilities. The problem is that points will occupy gauge orbits and, therefore, those within the same orbit will represent physically indistinguishable states of affairs. If one wants to say that these points represent distinct possibilities then one faces an indeterminism, but it is an indeterminism that faces any one-to-one interpretation of a gauge theory. Aside from biting the bullet and living with it, there are several methods of escaping this indeterminism: (1) one can adopt a many-to-one interpretation, so that one and the same physical possibility is being multiply represented by the gauge-equivalent models; (2) one can select a single model from the equivalence class as the 'true' representative; or one can (3) factor out the gauge symmetry by forming the reduced space of orbits (option (3) is also known as 'solving the constraints'). Let us call the one-to-one interpretation a *direct* interpretation and call (1)-(3) *indirect*, *selective*, and *reductive* interpretations respectively. Of course, a live issue in recent discussions of the hole argument is the question of whether the substantivalist is necessarily committed to this *direct* interpretation. A group of philosophers—dubbed "sophisticated substantivalists" by Belot & Earman ([BE01]: 247)—have argued that the substantivalist is not necessarily thus committed and that, in fact, substantivalists can help themselves to interpretations that were thought to be unique to relationalists.[7] Let me

[7]For example, Maudlin's 'essentialist substantivalism' [Mau88] and Butterfield's 'counterpart-theoretic substantivalism' [But89] can, I think, be viewed as selectivist sub-

now introduce the barest outlines of the recent work conducted in the 'new variables' for general relativity.

3 Connections, Loops and Spin-Networks.

Loop quantum gravity is a non-perturbative, background-independent quantization of general relativity. In this paper am interested in the space of quantum states that is provided by the spin-network basis: elements of this space are taken to represent quantum excitations of the geometry of space. The loop representation of quantum gravity begins with the (classical) connection representation. Here, rather than using the metric on space as a configuration variable (as above), the trick is to use an $\mathsf{SU}(2)$-connection $\mathcal{A}_a^i$ on space (again represented by a compact 3-surface Σ). The phase space is then formed by taking orthonormal triads $\tilde{E}_i^a$ as the momenta—note that it is these rather than the connection that determine the geometry of space.[8] The fundamental degrees of freedom of the theory are then given by holonomies of $\mathcal{A}_a^i$ and $\tilde{E}_i^a$.

The two canonical variables are related to the geometrodynamical formalism as follows: $\tilde{E}_i^a(x)$ is equivalent to the square root of the 3-metric $g_{ab}(x)$ of constant time hypersurfaces. A_a^i is given by:

$$A_a^i(x) = \Gamma_a^i(x) + \gamma K_a^i(x) \tag{4}$$

where $\Gamma_a^i(x)$ is the spin-connection associated to a basis; γ is the so-called Immirzi parameter[9]; and $K_a^i(x)$ is the extrinsic curvature of a constant time hypersurface.

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stantivalist interpretations of the phase space of general relativity. Maidens [Mai93] and Hoefer's [Hoe96] 'primitive-identity denying substantivalists' can be viewed as giving indirect interpretations. And Pooley's 'anti-haecceitistic substantivalist' [Poo02] can be seen as offering a reductive interpretation (I would place Brighouse [Bri94] and Saunders [Sau02] in this category too). See Pooley (*ibid.*) for a recent detailed discussion.

[8]The tilde above the E-field refers to the fact that E is 'denisitized', i.e. $\rho(E) = 1$; $i = 1, 2, 3$ are 'internal' indices labelling the 3-axis of a local triad (a triad is a 3-dimensional orthonormal frame, related to the metric by $\eta_{ab}e_\mu^a e_\nu^b = g_{\mu\nu}$); and, $a = 1, 2, 3$ are spatial indices.

[9]This parameter functions like a Lagrange multiplier (like the Lapse function and the shift vector), its value determines the nature of the number field the theory is defined over. We work in the *real* (Barbero connection) theory where $\gamma = 1$; the Ashtekar formulation uses the complex connection defined by setting $\gamma = i$.

In contrast to the holonomies of the connection representation, the best candidates for the canonical variables of loop quantum gravity are Wilson loops. Given a connection $\mathcal{A}_a^i$, a Wilson loop is just the trace of the holonomy of $\mathcal{A}_a^i$ around a loop γ in space, written $\mathrm{tr}\,\mathcal{P}\,\exp\,\oint_\gamma \mathcal{A}_a^i$.[10] The holonomies contain all of the gauge-invariant information of the connection; as Gambini and Pullin point out (1996: 2), "[k]nowledge of the holonomy for any closed curve ... allows one ... to reconstruct the connection at any point of the base manifold up to a gauge transformation". However, the holonomies themselves are not gauge-invariant; the Wilson loops are and in consequence they solve the Gauss law constraint (from the $SU(2)$-connection) in a rather natural way.

When $\mathcal{A}_a^i$ is quantized the Wilson loops receive an interpretation in terms of operators on a Hilbert space spanned by spin-networks; these spin-networks yield a representation of the (kinematical) quantum geometry of space. Mathematically a spin-network S is a triple $\{\Gamma, s_i, v_i\}$, where Γ is a graph composed of i links and $i-2$ nodes; s is a half-integer (of course, this is where the "spin" in "spin-network" comes from); and v is a basis in some linear (though not necessarily Hilbert) space. To each link is assigned a 'colour' s_i; that is, an irreducible representation of $\mathsf{SU}(2)$, or, more simply, a half-integer (the half-integers 'label' the representation). To each node (i.e. the point at which the links meet) is assigned an intertwiner v (intertwiners are operators that are invariant under some group action, in this case $\mathsf{SU}(2)$ transformations). More precisely, the intertwiners form a (finite dimensional) vector space V the basis elements v_i of which correspond to the nodes.

The idea is that to each spin-network there is assigned a quantum state of the gravitational field, or, alternatively, a quantum state for the geometry of *space*. The 'spins' correspond to the units of area carried by the link and the 'intertwiners' correspond to units of volume: both quantities are quantized. The physical picture is based on *intersections* of spin-networks (links and nodes) with surfaces and regions: the more links and nodes there are, the more area and volume there is. Thus, Rovelli claims that we can view each spin-network "as an elementary 'quantum chunk of space"', where "the

[10]Formulated in this way, general relativity bears many mathematical similarities to the theories of the other forces of nature (as described by *gauge* theories). In each case the basic dynamical variables are Wilson loops of connections on a principle bundle. The crucial difference, of course, concerns the *background independence* of general relativity (i.e. the fact that in general relativity the connection in is viewed as living on a *differentiable* manifold, rather than a *metric* manifold).

links represent (transverse) surfaces separating quanta of space" (2001: 110). Moreover, he claims that "the spin-network[s] represent relational quantum states: they are not located in space"; rather "[l]ocalization must be defined in relation to them" (*ibid.*). However, below I argue that a very different interpretation of this state of affairs can be given; there is nothing 'intrinsically' relational about spin-networks, and the fact that they admit a hole argument highlights this fact.

4 Constraints in the Loop Representation.

As in the metric representation we find that general relativity is a constrained theory. However, in the connection and loop representations there are *three* constraints: (1) diffeomorphism, (2) Hamiltonian, and (3) Gauss. (1) generates infinitesimal displacements (spatial diffeomorphisms) of the initial data *tangentially* to the initial data slice Σ; (2) generates infinitesimal displacements of the data *normal* to the initial data slice; and (3) generates SO(3) gauge transformations. These constraints enforce the condition that the states of the theory be diffeomorphism-invariant, gauge-invariant functionals of the connection $\mathcal{A}$. As I mentioned above, a natural set of such states proceeds from first defining the holonomy of a connection and then forming the Wilson loop. In more detail, the holonomy $\mathsf{H}_\gamma(a_1, a_2)$ for a curve $\gamma : [0,1] \rightarrow \Sigma$ is given by:

$$\mathsf{H}_\gamma(a_1, a_2) = \mathcal{P}\, exp[-\int_{a_1}^{a_2} dl \frac{dx^i(l)}{dl}\mathcal{A}_i] \tag{5}$$

The Wilson loop $\mathrm{tr}[\mathsf{H}_\gamma(0,1)]$ is then gauge-invariant when γ is a closed curve. The diffeomorphism constraints acts on Wilson loops and spin-networks in much the same way that they act on the metric in the context of the original hole argument; namely, by reshuffling the points of the manifold so that the location of the spin-network is not preserved. Thus, one can drag a spin-network around in the manifold just as if it were a metric field.

The diffeomorphism invariance (as represented by the diffeomorphism constraint) of general relativity effectively functions as a constraint on the possible operators of the quantum theory. The notion of a correlation becomes an integral part of the definition of these operators. For example, the area operator associated to the classical phase function, $\mathsf{A}(\mathsf{S}) = \int_{\mathsf{S}} \mid E_i^3 E^{3i} \mid^{1/2} d^2x$ (with S a smooth 2-surface), is understood as a sum over *intersections* of a spin-network with a surface. One of the truly novel and exciting aspects of loop quantum gravity is the result that the spectra of these 'geometrical' operators is discrete. For example, the eigenvalues for the area operator are given by:

$$\hat{\mathsf{A}} = 8\pi G\,\hbar\,\gamma\;\Sigma_{i=1,n}\sqrt{j_i(j_i+1)} \tag{6}$$

where the terms j_i are the links from the spin-network. The physical picture is simply a quantized version of what we have for the classical picture: as I already pointed out, the links of the spin-networks are seen as carrying quanta of area and the nodes as carrying quanta of volume. However, as these expressions stand they are clearly not gauge-invariant; they are defined for surfaces and regions on a compact space. Thus, we need to be careful with the discrete geometry result. The operators corresponding to the classical expression for the area and volume of a spatial surface or region do not commute with the constraints; in particular, they are not invariant under the transformations generated by the diffeomorphism constraints. Hence, they cannot correspond to genuine physical observables; they cannot be measured. Thus, in the quantum theory, these operators do not admit a representation on the solution space. However, if we take the operators to correspond to the area and surface of some *physically* defined surface or region, then they are invariant, and they can be constructed as operators on the space of solutions. Though the introduction of 'material' variables might strike some as courting relationalism, there need be no such implication; a position can be set up that does not privilege matter over space.[11] Moreover, the sophisticated substantivalist options I mentioned above can be made to square with this too.

However, there remains the problem of *imposing* the constraints (now considered as quantum constraints) from the gauge freedom of the classical theory. These are imposed as operator constraints in the quantum theory (*á la* Dirac quantization), such that all physical states must be annihilated by them. The Gauss law constraint is satisfied by any loop state, since the formulation is intrinsically gauge invariant).[12] As for the diffeomorphism

[11] I outline and defend such a view in (Rickles [Ric05])) where I apply it to the problem of time. An alternative is to use the *invariants* of the gravitational field itself to ground the required intersections.

[12] This feature motivates the application of the loop description to Yang-Mills theories where the Gauss constraint is the only constraint to solve. See Brügmann [Bru94].

constraint, it is the case that a state $\Psi[\gamma]$ is annihilated by its action just in case $\Psi[\gamma] = \Psi[\gamma']$ whenever γ and γ' are connected by a diffeomorphism. Here a relation to knot theory becomes apparent, since a diffeomorphism (equivalence) class of loops is characterized by a knot. Thus, the physical states of the loop representation depend on loops in virtue of the knot class defined by the loops. In terms of the spin-networks, the diffeomorphism constraints can be seen as dragging the graph around in Σ. Solving the constraints amounts to a 'disregarding' of the specific points of the manifold in which the network is embedded. The states depend on knots (equivalence classes of spin-networks or 's-knots'). However, as I intimated at above, this is conceptually—i.e. as regards the ontology of space *vis-à-vis* the substantivalist/relationalist debate—no different to the case of a metric field on a manifold used in Earman and Norton's hole argument. The idea that the states depend on the knot class is, more or less, tantamount to a state's being dependent on an equivalence class of metrics; but that offered just one solution to the hole argument–there are many other resolutions that do not rely on the equivalence class; even in the case of those that do so rely, the determined substantivalist is at liberty to occupy it too.

5 A Hole Argument for Spin Networks.

The original motivation behind Penrose's introduction of spin networks was the desire to have an account of the quantum geometry of space that was both discrete and relational (*cf.* Smolin [Smo97]: 3). Spin-networks enter the picture of loop quantum gravity because they are found to label the states that span the space $\mathcal{L}^2(\mathcal{A}/\mathcal{G})$ of gauge-equivalent connections on a spatial manifold. Let me now attempt to connect this to the hole argument.

The treatment of the diffeomorphism constraint is really at the root of any hole argument for general relativity. The problem in the metric-variable case is most easily and most often resolved by shifting to the equivalence class of metrics related by diffeomorphism (as mentioned above). This amounts to solving the constraints; one removes the symmetry and the supposed redundancy in the description of the location of the metric relative to the manifold. Localization of fields is thus relativized and, the story goes, the location of embedding of the field in the manifold is a gauge freedom in the theory. A similar story can be told with regard to spin-networks, and, once again, the root is the diffeomorphism constraint. Thus, one solves the constraints—after quantization—by shifting to equivalence classes of spin-networks related by diffeomorphism so that the quantum states depend only upon the s-knot class (where an s-knot is an equivalence class of spin-networks that can be deformed into each other by a diffeomorphism).

Curiously, and, in terms of interpreting loop quantum gravity, *crucially*, all of the logic of the hole argument applies equally well here[13]: the diffeomorphisms act on spin-networks by dragging them around in the manifold. The spin-networks are diffeomorphism-invariant so they do not 'register' these draggings. Hence, the desire of many physicists to simply eradicate the manifold and claim a victory for relationalism. But this underlying motivation and excuse to dispense with the manifold is exactly the same as it was with in the classical theory based on the metric field. Consider the following passage:

> In going from the spin-network state $\mid \mathsf{S}\rangle$ to the s-knot state $\mid \mathsf{s}\rangle$, we preserve the information in $\mid \mathsf{S}\rangle$ except for it's location in Σ. This is the quantum analog to the fact that physically indistinguishable solutions of the classical Einstein equation are not fields but equivalence classes of fields under diffeomorphisms. It reflects the core of the conceptual revolution of general relativity: spatial localization concerns only the *relative* location of the dynamical fields, and their location in a background space. Accordingly, the s-knot states are not quantum excitations *in* space, they are quantum excitations *of* space. An s-knot does not reside "somewhere": the s-knot itself defines the "where". [[FR04]: 5]

The hole argument can be constructed simply by noting that Einstein's equation cannot determine where spin-networks are in the manifold. Thus, for any specification of initial data $\langle \Sigma_0, \mathsf{S}\rangle$, the Einstein equation will fail to determine the data at $\Sigma_{t>0}$ since many diffeomorphic solutions will solve the equation. But that this implies that the loop representation spells victory for relationalism is as much as a *non sequiter* as it is for the metric in the classical case, for precisely the same reasons given by sophisticated substantivalists: we need not necessarily shift to the equivalence classes; and even if we do, the substantivalist is able to accommodate this either by being

[13] To the best of my knowledge, no one has picked up on this yet; though it is implicitly suggested in a number of works (the paper by Fairburn & Rovelli [FR04]containing the passage below for example).

selectivist (roughly corresponding to a gauge fixing procedure) or else anti-haecceitistic (i.e. deny that the diffeomorphic possibilities brought about by the application of the diffeomorphism constraint correspond to distinct possible worlds).

Why might loop quantum gravity be seen as lending (unique) support to relationalism? I think that there are two lines of reasoning. The first goes as follows: Diff(Σ) acts on spin-networks s in such a way that $\phi(s) = s$. Thus, the diffeomorphism group is a symmetry and allows the formation of equivalence classes of diffeomorphic spin-networks $[\mathsf{s}]$, such that $\mathsf{s}' = \phi(\mathsf{s}) \rightarrow \mathsf{s}, \mathsf{s}' \in [\mathsf{s}]$. Now, Rovelli calls these diffeomorphism-equivalence classes of embedded spin-networks an "abstract spin-network" or "s-knot" and claims that it can be understood as a spin-network that has been 'smeared' out over space. This much is familiar from the old hole arguments where a standard response—indeed, the response of most physicists—is to shift to the *geometry*, the equivalence class of metrics modulo the diffeomorphism symmetry. Taking the geometry as the true representation of a state of spacetime amounts to *reduction* in phase space terms: one factors out the symmetry and thus reduces out those states related by diffeomorphism. This reduction is, of course, the mathematical analogue of Leibniz equivalence which is based on the idea that indiscernible states should be identified. Finally, this reduction is associated with relationalism about spacetime.

The second line of reasoning is based on the relationship between spin-networks and determinations of length, area, and volume of physical surfaces and regions of space. Thus, Rovelli claims that "the picture of quantum spacetime that emerges from loop quantum gravity" is characterized by "an abstract graph [that] can be seen as an elementary quantum excitation of space formed by "chunks" of space (the nodes) with quantized volume, separated by sheets (corresponding to the links), with quantized area ... [and the] key point is that the graph does not live on a manifold" ([GR01]: 312). The idea was that these objects (surfaces and regions of space) are *physical* only on account of intersections with physical objects. Mathematically speaking, the problem is caused by the spectrum of the area of a surface spec($\hat{A}[S]$) (or volume of a region). This will only take on a non-zero value when a spin-network (edge) *intersects* the *physically defined* surface. Now, unless the surface S is defined by some physical field (possibly the invariants of the metric field), the area of the surface will not be diffeomorphism invariant for it will simply be a coordinate patch in Σ. In this case the spectrum will be physically meaningless. Thus, the interpretation given to S is that it is a surface where a physical field takes on a non-vanishing value. However, the nature of the physical objects is not spelled out. It is open to the substantivalist to claim that the gravitational field and space are one and the same entity and use it in the definitions of the geometrical operators.[14]

6 Conclusion.

I hope to have shown that the hole argument is alive and well in quantum gravity and that what goes for the old argument goes, *mutatis mutandis*, for the new one. Both relationalism and substantivalism are workable and plausible ways of dealing with the hole argument: relationalism is not the only game in town and quantum gravity does nothing to compromise this.

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[14]This is more or less along the lines of Hoefer's "metric-field substantivalist" ([Hoe96]). I think it is fair to say that the majority of substantivalists think of the gravitational (metric) field in just this way.

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