Time and Structure in Canonical Gravity.[1]

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Abstract

In this paper I wish to make some headway on understanding what *kind* of problem the "problem of time" is, and offer a possible resolution—or, rather, a new way of understanding an old resolution.[2] The response I give is a variation on a theme of Rovelli's *evolving constants of motion* strategy (more generally: correlation strategies). I argue that by giving correlation strategies a *structuralist* basis, a number of objections to the standard account can be blunted. Moreover, I show that the account I offer provides a suitable ontology for time (and space) in both classical and quantum canonical general relativity.

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[2] I am referring to the problem of time that appears in *canonical* formulations of both classical and quantum GR, and also in certain diffeomorphism-invariant covariant QFTs (e.g. topological quantum field theories: see Baez (this volume) for a clear and elementary account). More generally, though I cannot demonstrate the fact here, *any* theory that is independent of a fixed metric (or connection) on space or spacetime will be subject to the problems considered here. Since it is likely that the 'final' theory of quantum gravity will be of this form, the problem of time will almost inevitably be a problem for that theory, or, at least, will play a role in its development and eventual formulation.

1 Introduction.

Interpreting modern day fundamental physical theories is hard. Our four best theories—three quantum field theories (describing the strong, electro-weak, and electromagnetic forces) and one classical field theory describing gravity— are *gauge* theories.[3] Interpreting these theories is complicated by the presence of a special class of symmetries (gauge symmetries) whose action does not 'disturb' any 'qualitative' properties and relations; only non-observable, non-qualitative features of a theory (or family of models) are affected.[4] This leads to empirically superfluous elements—"surplus structure" in Redhead's sense (Redhead 1975); 'gauge freedom' in physicists' jargon—in the description of such theories that must be be dealt with in some way, either by 'elimination' or 'accommodation'. While classically inert, the decision regarding how to deal with the gauge freedom can lead to non-trivial differences at the quantum level (i.e. inequivalent quantizations). The root cause of interpretive headaches in the context of gauge theories is, then, the gauge freedom; the problem facing philosophers (and physicists!) is to explicate and provide some account of both the gauge symmetries and the elements that are acted upon by those symmetries.

The interpretive problems of gauge theory take on what is arguably their most pathological form in the context of the problem of space (better known as the 'hole argument') and the problem of time.[5] I will argue that the latter problem is essentially just a recapitulation of the former, although focused upon the Hamiltonian rather than the diffeomorphism constraint. Therefore, I think that one should respond to the problems in the same way: I favour a non-reductive gauge-invariant conception of observables coupled with a kind of structuralism. My main aims in this paper are as follows: (1) to explain the problem of time in a way that is accessible to philosophers; (2) to provide a critique of the usual responses; (3) to disentangle the debate between substantivalists and relationalists from the problem of time; and (4) to defend a structuralist resolution of the problem of time.

[3] I should point out that this claim is not entirely uncontentious. Weinstein (2001) has argued that certain features of general relativity—namely, the fact that the the gauge groups of the first three theories are Lie groups and can be viewed as acting at spacetime points whereas in general relativity the candidate for the gauge group (the diffeomorphism group) acts on the points themselves and is not a Lie group—debar it from being classified as a gauge theory proper. See Earman (2003) for a defense of the contrary view based on the Hamiltonian formulation of general relativity.

[4] Belot (2003) offers a detailed philosophical survey of gauge theories, I refer the reader unacquainted with the basic details of the concept of 'gauge' to this insightful article. Redhead (2003) is an exceptionally clear, and more elementary, guide to the interpretation of gauge theories. Earman (2003) examines the concepts of gauge theory from the perspective of the constrained Hamiltonian formalism – indeed, Earman (2003, 153) speaks of the constrained Hamiltonian formalism as an "apparatus ... used to detect gauge freedom".

[5] The best places to learn about the problem of time are (still) Isham (1993) and Kuchař (1992). Belot and Earman (1999, 2001) give two excellent philosophical examinations of the problem; the latter is more comprehensive and technically demanding than the former. I am much indebted to this quadruplet of articles.

2 Constraints, Gauge, and Holes.

In their recent survey of the problem of time in quantum gravity, Belot and Earman note that there is a "sentiment - which is widespread among physicists working on canonical quantum gravity - that there is a tight connection between the interpretive problems of general relativity and the technical and conceptual problems of quantum gravity" (2001: 214). Belot and Earman share this sentiment, and go even further in claiming that certain proposals for understanding the general covariance of general relativity *underwrite* specific proposals for quantizing gravity. These proposals are then seen as being linked to "interpretive views concerning the ontological status of spacetime" (*ibid*.). I agree with their former claim but strongly disagree with the latter: such proposals cannot be seen as linked with stances concerning the ontological status of spacetime *vis à vis* relationalism vs substantivalism (for reasons I will discuss more properly in §7).

The crucial claim they make, for the purposes of this paper, is that the *gauge invariance* reading of the general covariance of general relativity "seems to force us to accept that change is not a fundamental reality in classical and quantum gravity" (*ibid*.). I agree with Belot and Earman that, like the hole argument, the problem of time is an aspect of the more general problem of interpreting gauge theories. I also agree with Earman's claim that the problems do not only have teeth in the quantum context, but bite in the classical context too (see Earman (2003: 6)) – indeed, I don't find all that much to distinguish the two cases. In order to fully appreciate this problem, we need to take a brief detour to introduce a variety of concepts: gauge and constraints; phase spaces and possible worlds; and the interpretive problems and and options in gauge theory, including the hole argument.

2.1 Hamiltonian Systems: Constraints and Gauge.

In this section I introduce the Hamiltonian formalism of theories, and show how the constraints arise in systems whose description possesses surplus structure.[6] I relate the presence of a certain class of constraints (those that are first class) to the presence of gauge freedom. Finally, I outline, in broad strokes, how one tackles the problem of interpreting the theories considered. This brief primer should provide enough of the technical apparatus required to understand the classical and quantum problems of time and change.

A Hamiltonian system is represented by a triple $\langle \Gamma, \omega, H\rangle$ consisting of a manifold Γ (the cotangent bundle $T^*\mathcal{Q}$, where $\mathcal{Q}$ is the configuration space of a system), a tensor ω (a symplectic, closed, non-degenerate 2-form), and a function H (the Hamiltonian $H : \Gamma \rightarrow \mathbb{R}$).

[6] The presentation I give here relies heavily upon Dirac (1964), Henneaux & Teitelboim (1992), and the articles in Ehlers and Friedrich (eds.) (1994).

These elements interact to give the kinematical and dynamical structure of a classical theory. The manifold inherits its structure from the tensor, making it into a phase space with a symplectic geometry. The points of this space are taken to represent physically possible states of some classical system (i.e., set of particles, a system of fields, a fluid, etc...). Finally the Hamiltonian function selects a class of curves from the phase space that are taken to represent physically possible histories of the system (given the symplectic structure of the space). Any system represented by such a triple will be deterministic in the sense that knowing which phase point represents the state of the system at an initial time, there will be a *unique* curve through that point whose points represent the past and future states of the system.[7] The physical interpretation of this framework is as follows. Recall that the phase space is given by the cotangent bundle of the configuration space, where points of the configuration space represent possible instantaneous configurations of some system (relative to an inertial frame). The cotangent bundle is the set of pairs (q, p), where q is an element of the configuration space and p is a covector at q. Thinking of q as representing the position of a system leads to the view that p represents that system's momentum. The value of the Hamiltonian at a point of phase space is the energy of the system whose state is represented by that point. The *physically measurable* properties of an Hamiltonian system are described by functions $A(q, p) : \Gamma \to \mathbb{R}$ in terms of a canonical basis (a set of canonical variables), with position q_i and momenta p_i, satisfying Poisson bracket relations:

$$\{q_i, p_j\} = \delta_{ij} \tag{1}$$

Systems described in such terms are rather simple to interpret: each point, (p, q), in the phase space represents a distinct physically possible world. Furthermore, since there is a unique curve through each point of phase space, one can interpret the phase space as *directly* representing the physically possible states of a system, and the curves as *directly* representing the physically possible histories of a system. A simple one-to-one understanding of the representation relation is possible that does not lead to indeterminism or underdetermination as regards the canonical variables, the possibilities, or the possible worlds.

Weakening the geometry of the phase space, and moving to gauge systems, however, puts pressure on this simple direct interpretation[8], precisely because indeterminism breaks down and the canonical variables are underdetermined. When one considers systems

[7]In a little more detail: Hamilton's equations determine a map $f \to X_f$ between smooth functions f on Γ and vector fields X_f on Γ. Integrating a vector field X_f associated to the smooth function f gives a unique curve through each point of Γ. The symplectic structure gives the set $C^\infty(\Gamma)$ of smooth function on Γ the structure of a Poisson algebra by means of the Poisson bracket $\{f, g\}$ between pairs of functions f and g. $\{f, g\}$ is interpreted as giving the rate of change of g with respect to the set of curves generated by f such that g is constant along the curves generated by f just in case $\{f, g\} = 0$. For any observable A (a function of the canonical variables), the time-evolution is given by $\dot{A} = \{A, H\}$.

[8]Note that I don't say that such an interpretation isn't possible. It is, provided one either accepts the consequence of indeterminism and underdetermination, or else finds another way to deal with them.

with redundant variables and symmetries - such as Maxwell's theory and general relativity - the formulation contains constraints, where the constraints are relations of the form $\phi_m(p_i, q_i) = 0$ ($i = 1, ..., m$) holding between the canonical variables. Such constraints are a byproduct of the Legendre transform taking one from a Lagrangian to a Hamiltonian description of a system.[9] These are known as *primary constraints*. If these constraints should be preserved by evolution a new set of constraints is generated to carry out this job. These are called *secondary constraints*. One may wish to repeat the procedure on these, resulting in *tertiary constraints*, and so on.

The first change to note in the shift from a Hamiltonian system to a constrained Hamiltonian system is that the symplectic form is replaced by a *presymplectic form* σ, so that the phase space $\mathcal{C}$ of a gauge system inherits its geometrical structure from this. The presymplectic form induces a partitioning of the phase space into subspaces (not necessarily manifolds) known as *gauge orbits*, such that each point x in the phase space lies in exactly one orbit $[x]$. Once again we choose a Hamiltonian function on phase space, such that the value at a phase point represents the energy. However, in this case, given the weaker geometrical structure induced by the presymplectic form, the Hamiltonian is not able to determine a unique curve through the phase points. Instead, there are infinitely many curves through the points. However, the presymplectic form *does* supply the phase space with sufficient structure to determine which gauge orbit a point representing the past or future state will lie in. Hence, for two curves $t \to x(t)$ and $t \to x'(t)$ intersecting the same initial phase point $x(0)$, we find that the gauge orbit containing $x(t)$ is the same as that containing $x'(t)$: i.e., $[x(t)] = [x'(t)]$.

In a constrained system, each classical observable is represented by a function $P : \mathcal{C} \to \mathbb{R}$ on the phase space. But given that the future phase points of an initial phase point is underdetermined, it will be impossible to uniquely predict the future value of the observables. Hence, there appears to be a breakdown of determinism; the initial-value problem does not appear to be well posed, as it is for standard Hamiltonian systems. The reason is clear enough: there is a unique curve through each phase point in a Hamiltonian system but infinitely many curves through the phase points of a gauge system (see fig.1).

Yet there are many theories that are gauge theories and that are evidently *not* indeterministic in any pathological sense. The trick for restoring determinism and recovering a well posed initial-value problem is to be *restrictive* about what one takes the observables to be. Rather than allowing *any* real-valued functions on the phase space to represent physical

[9] The idea of gauge freedom manifests itself at the level of the Lagrangian formalism too. The action principle $\delta \int \mathcal{L}(q, \dot{q})dt = 0$ allows us to derive Euler-Lagrange equations. Sometimes—in general relativity, for example—these equations will be non-hyperbolic, they can't be solved for all accelerations. This results in a *singular* Lagrangian, revealing itself in the singularity of the Hessian $\partial^2\mathcal{L}/\partial\dot{q}^k\partial\dot{q}^h$. This implies that when we Legendre transform to the Hamiltonian formulation, the canonical momenta are not independent, but will satisfy a set of relations called primary constraints, related to the identities of the Lagrange formalism. As I mention below, preserving these under evolution may require the imposition of higher-order constraints. Once one has a situation where all the constraints are preserved by the motion, one will have defined a submanifold where all of the constraints are satisfied - this is the "constraint surface" $\mathcal{C}$. See Earman (2003: 144-145) for a clear explanation of these constraints and their relation to the singularity of the Hessian.

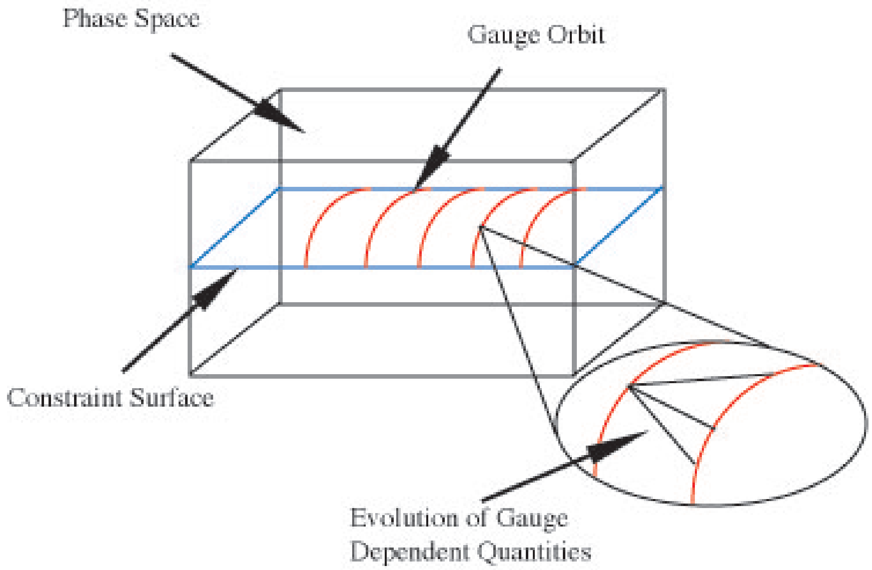

Figure 1: The underdetermination ubiquitous in gauge systems.

observables, one simply chooses those that are *constant* on gauge orbits, such that if $[x] = [y]$ then $f(x) = f(y)$. Such quantities are said to be *gauge-invariant*. The initial-value problem is well posed for such quantities since for an initial state $x_{t=0}$, and curves $x(t)$ and $x'(t)$ through $x_{t=0}$, $f[x(t_1)] = f[x'(t_1)]$.

Another important distinction—perhaps the most important as far as the problem of time goes—between constraints is that holding between *first class* and *second class constraints*. A constraint ϕ_k is said to be first class if its Poisson bracket with any other constraints is given as a linear combination of the constraints:

$$\{\phi_k, \phi_i\} = C^j_{ki}\phi_j, \ \forall i. \tag{2}$$

Any constraint not satisfying these relations is second class. Our sole concern is with the first class constraints. The appearance of such constraints in a theory implies that the dynamics is restricted to a submanifold $\mathcal{C}$ of the full phase space Γ, this is known as the *constraint surface*. Dynamical evolution on $\mathcal{C}$ has a representation in terms of an infinite family of physically equivalent trajectories. This is how the appearance of gauge freedom is represented in the constrained Hamiltonian formalism. Projecting out from $\mathcal{C}$ to Γ results in ambiguity, for any quantities that differ only by a combination of constraints come out as equal on $\mathcal{C}$. This ambiguity is a formal counterpart of the 'many-one' problems encountered in both electrodynamics formulated in terms of the vector potential and the hole argument (touched upon below); it can be seen, as such, as the origin of one kind of surplus structure; namely, that associated with gauge freedom.

A dynamical variable P (a function of the $\mathsf{p}s$ and $\mathsf{q}s$: $\mathsf{P}(\mathsf{q}, \mathsf{p})$) is first class iff it has *weakly vanishing* Poisson bracket with all of the constraints:[10]

$$\{\mathsf{P}, \phi_{\mathsf{j}}\} \approx 0, \ \mathsf{j} = 1, ..., \mathsf{j}. \tag{3}$$

These quantities comprise the observables of the classical theory. They are defined by their invariance under the symmetries generated by the constraints. These symmetries are the gauge symmetries of the theory; thus, in a gauge theory the observables are defined by gauge-invariance.

The constraints occurring in general relativity are all first class, implying that they generate gauge transformations. Crucially, the constraints also make up the Hamiltonian of general relativity: it is a sum of first class constraints. In a constrained Hamiltonian system, the observables must commute with the Hamiltonian since it is a constraint (or, rather, a linear combination of such) – in a gauge theory this translates into the condition that the observables must be gauge-invariant. As always, the Hamiltonian generates motion *via* Poisson brackets of observables with the Hamiltonian. In this case, since the Hamiltonian vanishes on $\mathcal{C}$, this implies that motion is 'pure gauge'. Already we see a potential problem for the evolution of the theory's observables if the observables are defined to be the gauge-invariant quantities. The problem is this: the constraints of the theory pick out a submanifold (the constraint surface) on which observables must have vanishing Poisson bracket with the constraints. In the case of the Hamiltonian constraint (on which more below), the different points of this manifold correspond to states of the system at different times (indexed by parameter time τ). Since the constraints generate gauge transformations (i.e. along a gauge orbit) this implies that time evolution is itself a gauge transformation! This, in capsule form, is the problem of the frozen formalism of the classical theory. Let me say a little more about the kinds of constraints that appear in general relativity and how the concept of gauge freedom arises in this context.

2.2 Constraints and Gauge in General Relativity.

The Lagrangian for general relativity contains a number of variables appearing without their corresponding velocities.[11] This implies that when we define the canonical momenta $\mathsf{p}_{\mathsf{i}} = \partial\mathcal{L}/\partial\dot{\mathsf{q}}_{\mathsf{i}}$ of the Hamiltonian formulation, we find that they vanish. This is a sure sign that the Hamiltonian formulation will possess constraints. Two families of constraints are picked up when we perform the Legendre transform from the Lagrangian to the Hamilto-

[10]The condition of weak vanishing refers to equality on the constraint surface embedded in the phase space. I say more about this in §2.2.

[11]Such terms become Lagrange multipliers in the Hamiltonian formulation. There are two types: the lapse function N and the shift vector N^{i}. These two expressions tell us how much a slice Σ is to be 'pushed forwards in time': the former acts *normally* and the latter *tangentially*.

nian formulation of general relativity: diffeomorphism constraints and Hamiltonian constraints - three diffeomorphism constraints per space point and one Hamiltonian constraint per space point.[12] The diffeomorphism constraints generate infinitesimal transformations (3-dimensional diffeomorphisms) of Σ onto itself; they have the effect of 'sliding' Cauchy data along Σ in the direction of the shift vector N^i. The Hamiltonian constraints generate infinitesimal transformations of Σ onto some slice Σ' displaced normally to Σ in $\mathcal{M}$; hence, data is 'pushed' orthogonal to Σ in the direction of the lapse function N. The Hamiltonian of general relativity is a sum of these constraints such that setting lapse to zero gives a Hamiltonian that is identical to the diffeomorphism constraint and setting the shift to zero gives a Hamiltonian that is identical to the Hamiltonian constraint.

Recall that in geometrodynamics (*cf.* Arnowitt, et al. (1962)) the points in the phase space of GR are given by pairs (q, p) — where q is a Riemannian metric on a 3-manifold Σ and p is related to the extrinsic curvature K of Σ describing the way it is embedded in a four dimensional Lorentzian manifold. In GR, the pair must satisfy the four constraint equations, and this condition picks out a surface in the phase space called the constraint surface. The observables of the theory are those quantities that have vanishing Poisson Bracket with all of the constraints.[13] According to the geometrodynamical program, each point on the constraint surface represents a physically possible (i.e., by the lights of general relativity) spacelike hypersurface of a general relativistic spacetime. Points lying on the complement of this surface are also 3-manifolds, but they do not represent physically possible spacetimes; they have metric and extrinsic curvature tensors that are incompatible with those needed to qualify as a 3-space imbedded in a general relativistic spacetime: they represent physically *impossible* states.

The constraint surface comes equipped with a set of transformations $\mathcal{C} \rightarrow \mathcal{C}$ that partition the surface into subspaces known as "gauge orbits" (the transformations are the gauge transformations). The natural interpretation of the gauge orbits is as representing equivalence classes of isometric models of general relativistic spacetimes. We face the problem we faced in interpreting electrodynamics: do we take the points of the orbits to represent the same state of affairs or does each point represent a distinct possibility?

[12] In the connection formalism a further constraint is picked up, namely the Gauss constraint. This generates infinitesimal (global) gauge transformations. It is the only constraint that Yang-Mills theories possess, and, since these are taken to be gauge theories *par excellence*, this might provide further motivation for gauge theoretical interpretations of general relativity.

[13] Much has been made of the fact that the Poisson bracket algebra of the constraints does not close, and, therefore, does not form a Lie algebra. Steven Weinstein, for one, argues that this feature mitigates against viewing general relativity as a gauge theory. This leads him to the view that diffeomorphisms should not be viewed as gauge transformations (*cf.* (2001: 88). In fact, a more general structure called a Dirac algebra is formed that has the group of spatial diffeomorphisms, Diff(Σ), as a subgroup. This has been interpreted as implying that general relativity is not, properly speaking, a gauge theory, since it lacks a feature of Yang-Mills theories - the term 'gauge theory' commonly being reserved for Yang-Mills theories (*cf.* Earman 2003: 151). I agree with Earman that this is largely "label mongering" (*loc. cit.*: 151-2). We can use 'gauge' to refer to Yang-Mills theories or we can use it to refer to theories containing arbitrary functions of time. We might even use the term more generally to refer to theories containing 'redundancy' of a certain specified type. However, it might still be instructive to see what feature is missing from GR that supposedly robs it of gauge theory status.

This leads us into the general problem of interpreting gauge theories (and, in particular, *gauge freedom*). In the case of general relativity the gauge freedom concerns the points of the spatial manifold and how the metric field (and other fields) are to be spread out over them: the intrinsic geometry of the metric is indifferent as to which points play which role in the overall relational structure determined by the fields. Satisfaction of the constraints by a solution gives a class of 'spreadings' that are compatible with Einstein's equation and some—those related by gauge transformations—may differ *only* in how the fields are spread about over the points. The hole argument uses general covariance (active diffeomorphism invariance) to demonstrate that a manifold substantivalist conception of spacetime—i.e. the view that spacetime points are real and have their identities fixed independently of any fields defined with respect to them—implies that general relativity is indeterministic. The conclusion follows by applying a diffeomorphism to any dynamical fields to the future of an initial slice through spacetime; general covariance implies that the resulting pair of diffeomorphic models (differing in how the metric is distributed over the points) solve Einstein's equation; therefore, if the points are real then the equations of motion cannot determine how the metric will evolve into the future. This procedure is essentially reapplied in the case of the problem of time: since the data on an initial slice is gauge-equivalent to that on a later slice (i.e. time-evolution is a gauge transformation - a diffeomorphism) they must describe a *qualitatively* identical state of affairs, differing only in which points lie under which bits of the fields. However, a substantivalist will, on the above view, have to keep them apart, giving a peculiar indeterministic world in which nothing observable (qualitative) changes! However, the prospects are no better for a relationalist, who will generally have to identify gauge-equivalent states, for the time-evolved slices will have to be identified, thus freezing out *any* kind of evolution and eradicating change.

2.3 Interpreting Gauge Theories.

From what I have said so far we can see that there are two competing interpretations of a gauge theory: on the one hand there is a one-to-one interpretation of the phase points, such that each point (curve) represents a distinct possible state (history) of a system; on the other hand there is a many-to-one interpretation according to which many phase points (namely, those within the same gauge orbit) represent a single possible state of a system.[14] The former leads to indeterminism and (if not supplemented by a gauge-invariant account of the observables) an ill-posed initial-value problem, while the latter involves surplus structure that can be eradicated, but only in a way that violates such

[14]This option is available because the phase points lying within the same gauge orbit are related by a gauge transformation: if they represent real possibilities then they represent qualitatively indistinguishable possibilities differing solely with respect to which individuals get which properties. Hence, the one-to-one interpretation of the representation relation if interpreted simplistically will lead to haecceitistic differences between the worlds represented by the solutions.

things as locality and (manifest) covariance.[15] Hence, though the interpretations will be empirically equivalent (at least, they will at the classical level) the choice of is, ontologically speaking, a non-trivial matter.

The key problem in trying to interpret gauge theories, is knowing what to do with the gauge freedom, the surplus that results from the equivalence of the points within the same gauge orbits (ontologically: the indistinguishability of the worlds represented by such points). There are multiple options, and hence, multiple ways of interpreting gauge theories. Let us call an interpretation that takes each phase point as representing a distinct physically possible state of a system a *direct* interpretation. Hence, each point x^i in a gauge orbit $[x]$ represents a distinct possibility. However, such a direct interpretation leads to a form of indeterminism for the reasons outlined in §2.1. But, since each of the phase points represents a distinct physical possibility, there is (strictly speaking) no surplus structure according to such an interpretation: each bit of the formalism plays a role in representing reality. Recall also that the indeterminism is of a very peculiar kind: the multiple futures that were compatible with an initial state were physically (read 'qualitatively') indistinguishable, for they are represented by points lying within the same gauge orbit. Hence, the indeterminism concerns haecceitistic differences. However, for realists the indeterminism will still constitute a problem, though it is not insurmountable. As Belot notes (1998: 538):

> if we supplement this account of the ontology of the theory with an account of measurement which implies that its observable quantities are gauge-invariant, then the indeterminism will not interfere with our ability to derive deterministic predictions from the theory.

Using this method one can help oneself to gauge-invariance at the level of observable ontology and remain neutral about the rest (spacetime points, quantum particles, shifted worlds, vector potentials, etc...).

Let us call an interpretation that takes many phase points (from within the same gauge orbit) as representing a single physically possible state of a system an *indirect* interpretation. There are two ways of achieving such an interpretation. The first method one simply takes the representation relation between phase points from within the same gauge orbit and physically possible states to be many-to-one. Since the points of a gauge orbit represent physically indistinguishable possibilities, there is no indeterminism on this approach. Redhead suggests that "the 'physical' degrees of freedom [i.e. the fields] at [a future] time t are being multiply represented by points on the gauge orbit ... in terms of

[15] Locality is lost since the points of gauge orbits represent states that differ in how a catalogue of properties gets distributed over a domain of points; since such points are identified in many-to-one accounts, the notion of properties attaching to points is lost — though this has been contested (quite rightly, in my opinion) on the grounds that the properties can be seen as (dynamically) 'individuating' the points (*cf*. Pooley (this volume)). Covariance is seen to be put under pressure by the fact that the original symmetry is removed in some many-to-one accounts.

the ‘unphysical’ degrees of freedom” (2003: 130).[16] The gauge freedom is simply an artifact of the formalism. There are superficial similarities between this approach and the modified direct approach mentioned by Belot above. However, the stance taken on this approach that not all of the phase points represent distinct possibilities. Even on the modified direct approach this is false. The latter approach simply says that the question of whether or not all of the phase points represent distinct possibilities is irrelevant to the observable content of the theory, the observables are indifferent as to what state underlies them provided the states are physically indistinguishable.

The second method involves treating the gauge orbits rather than phase points as the fundamental objects of one’s theory. By taking the set of gauge orbits as the points of a new space, and endowing this set with a symplectic structure, one can construct a phase space for a Hamiltonian system - this new space is known as the *reduced phase space*,[17] and the original is the enlarged phase space.[18] Hence, the procedure amounts to giving a *direct* interpretation of the reduced phase space – i.e. one that takes each gauge orbit as representing a distinct physically possible state – but an *indirect* interpretation of the enlarged phase space. The resulting system is deterministic since real-valued functions on the reduced space correspond to gauge-invariant functions on the enlarged space. In effect, the structure of the reduced space *encodes* all of the gauge-invariant information of the enlarged space even though no gauge symmetry remains (i.e. there is no gauge freedom). Note, however, that complications can arise in reduced space methods: the reduced space might not have the structure of a manifold, and so will not be able to play the role of a phase space; or some phenomena might arise that requires the gauge freedom to be retained, such as the Aharonov-Bohm effect (*cf.* Earman (2003: 158-9) and Redhead (2003: 132)). If these complications do arise, one can nonetheless stick to the claim that complete gauge orbits represent single possible worlds, as *per* the above method.[19]

[16] Redhead’s analysis seems to suggest that this is the *only* way to interpret the direct formulation (speaking in terms of vector potentials) - though he mentions that a gauge-invariant or gauge-fixing account can resolve the indeterminism. But clearly, it is open to us to give a direct interpretation and accept the qualitatively indistinguishable worlds that are represented by the isomorphic futures (points within the gauge orbit).

[17] In order to distinguish this approach from the previous one, let us call it a *reductive* interpretation. Note that this matches Leibniz’s form of relationalism since it can be seen as enforcing the Principle of Identity of Indiscernibles ($\forall F \forall xy : Fx \equiv Fy \rightarrow x = y$) on phase points within the same gauge orbit. Thus, to complete the analogy, an enlarged phase space Γ would correspond to that containing phase points related by the symmetries associated with $\mathcal{G}_N$ (the Galilean group of Newtonian mechanics representing indistinguishable shifted, rotated, and boosted worlds) and the reduced phase space $\tilde{\Gamma}$ would correspond to the space with the symmetries removed: $\tilde{\Gamma} = \Gamma / \mathcal{G}_N$.

[18] Thus the points of the reduced space correspond to gauge orbits of the original enlarged space. Curves in the reduced space contain information about which gauge orbits the system (as represented by the enlarged space) passes through.

[19] One can even help oneself to haecceitistic notions on this interpretation by utilizing Lewis’ idea of “cheap quasi-haecceitism” (1983: 395): as long as one distinguishes between *possibilities* and *possible worlds* one can view each gauge orbit as the sum total of possibilities compatible with a single world. On the reduced account this option is not available: hence, the desire to accommodate certain modal talk and concepts may be called upon to play a role in the choice of representational geometric space.

There is another method that involves taking only a single phase point from each gauge orbit as representing a physically possible state of a system. To do this one must introduce *gauge fixing* conditions that pick out a subset of phase points (a *gauge slice*) such that each element of this subset is a unique representative from each gauge orbit (*cf.* Govaerts (2001: 63)). Gauge fixing thus 'freezes out' the gauge freedom of the enlarged phase space.[20] This method leads to an interpretation that is neither direct nor indirect, I shall call it a *selective* interpretation. There is a serious problem - known as a *Gribov obstruction* (*ibid.*: 64) - facing certain gauge fixing procedures, for some lead to different coverings of the space of gauge orbits that, while being gauge-invariant, are not physically equivalent. The obstruction implies that the gauge conditions do not result in a unique 'slicing' of phase space, but may result in the selection of two or more points from within the same gauge orbit.[21]

Each of these interpretive options is seen to be applicable in both general relativity and quantum gravity; indeed, they are seen to play a crucial role in both their technical and philosophical foundations, though not, I say, to the extent that Belot and Earman suggest. Recall that the hole argument is based upon a direct, local interpretation of the models of general relativity. The argument is connected to the nature of spacetime since the gauge freedom is given by (active) diffeomorphisms of spacetime points (or by 'drag-alongs' of fields over spacetime points). What we appear to have in the hole argument, is an expression of the old Leibniz shift argument couched in the language of the models of general relativity (*qua* gauge theory), with diffeomorphisms playing the role of the translations. Earman and Norton (1987) see a direct, local interpretation as being implied by spacetime (manifold) substantivalism (i.e. the view that spacetime points, as represented by a differentiable manifold, exist independently of material objects). Clearly, this view is then going to be analogous to the interpretation of Maxwell's theory that takes the vector potential as a physically real field. Such an interpretation is indeterministic: the time-evolution of the potential can only be specified up to a gauge transformation. Earman and Norton extract a similar indeterminism from the direct interpretation in the spacetime case, and use this conclusion to argue against substantivalism. The "problem of time" applies the reasoning of the hole argument (as broadly catalogued in my direct, indirect, reductive, and selective interpretations) to the evolution of data off an initial spatial slice. One's interpretation of the gauge freedom then has an impact on the question of whether or not time and change exist! However, the problems will remain in some form on *any* account that views the diffeomorphism invariance of general relativity as a gauge freedom in the theory.

[20] With reference to the hole argument, the present interpretive move would correspond to imposing a condition such that exactly one localization of the metric field relative to the points was chosen. However, in this case, it is difficult to see what could be gained by such a move; there is no symmetry or geometrical structure available to explain the various invariance principles and conservation laws.

[21] As Redhead notes (2003: 132), in the case of non-Abelian gauge theories, the application of the gauge fixing method leads to a breakdown of unitarity (in perturbative field theory) that has to be dealt with by the ad hoc introduction of "fictitious" ghost fields - thus replacing one type of surplus structure with another.

3 What is the Problem of Time?

There are two ways of understanding the problem of time: (1) in terms of states and (2) in terms of observables. These lead to quite distinct conceptual problems: the former leads to a problem of time and the latter leads to a problem of change.[22] The first problem concerns the fact that distinct Cauchy surfaces of the same model will be connected by the Hamiltonian constraint, and therefore will be gauge related. The gauge-invariant view demands that we view them as representing the same state of affairs. The second problem concerns the observables: no gauge invariant quantity will distinguish between Cauchy surfaces of the above sort. Together, these problems constitute the *frozen formalism* problem of classical general relativity. Each of these classical problems transforms into a quantum version.

Let us fix some formalism so we can see how these two problems arise. We are working in the Hamiltonian formulation so we start by splitting spacetime into a space part and a time part. Thus, the spacetime manifold $\mathcal{M}$ is a background structure with the topological structure $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a spatially compact 3-manifold. We begin with a phase space Γ, which we shall take to be the cotangent bundle defined over the space of Riemannian metrics on Σ.[23] Points in phase space are then given by pairs (q_{ab}, p^{ab}), with q_{ab} a 3-metric on Σ and p^{ab} a symmetric tensor on Σ. The physical (instantaneous) states of the gravitational field are given by points $x \in \tilde{\Gamma} \subset \Gamma$, where $\tilde{\Gamma}$ is the constraint surface consisting of points that satisfy the diffeomorphism (vector) and Hamiltonian (scalar) constraints: $\mathcal{H}_a = \mathcal{H}_\perp = 0$. These two constraints allow data to be evolved by taking the Poisson bracket of the latter with the former; thus $\{O, \mathcal{H}_a\}$ changes $O \in C^\infty\Gamma$ by a Lie derivative tangent to Σ and is generated by a spatial diffeomorphism, while $\{O, \mathcal{H}_\perp\}$ changes O in the direction normal to Σ. The Hamiltonian for the theory is given by $\mathbf{H} = \int_\Sigma d^3x \; N^a\mathcal{H}_a + N\mathcal{H}_\perp$, where N^a and N are Lagrange multipliers called the shift vector and lapse function respectively. The dynamics are thus entirely generated by (first class) constraints.[24] The

[22] If one believes that change is a necessary condition for time then the second problem will naturally pose a problem of time too, and vice versa. The necessity of time for 'real' (i.e. non-illusory) change is fairly obvious, but the (Aristotelian) converse, that time requires change, has been questioned in the philosophical literature (e.g. Shoemaker (1969)).

[23] I follow 'standard procedure' of couching my discussion in terms of the metric variables. However, I should point out that the canonical approach based on these variables is now defunct and has been replaced by the connection (Ashtekar variables: *cf.* Asktekar (1986)) and loop representations (a nice introduction is Ashtekar & Rovelli (1992)). These result in simpler expressions for the constraints and solutions for the Hamiltonian constraint (none were known for the metric variables!). The justification for sticking with the metric variables is simply that the problem of time afflicts any canonical approach and takes on much the same form regardless of which variables one coordinatizes the phase space with. Generally, one can simply imagine replacing any expression involving functionals of the metric with functionals of these other variables. I should also note that the relation between the connection and metric representations of general relativity is one of a canonical transformation on the phase space. The idea is that we 'change basis' from one set of variables to a new set of variables such that the Poisson bracket relations are preserved by these new variables. It can happen that a new set of variables simplifies certain situations, and can even help with conceptual problems. This is just what happened in the 'connection-variable turn'.

[24] Dirac's 'conjecture' for such constraints is that they generate gauge transformations: "transformations

implication is that the evolution of states (i.e. motion) is pure gauge!

What I have described above is general relativity as a constrained Hamiltonian system. The observables $\mathcal{O}_i$ for such $\mathbf{H} = 0$ systems are defined as follows:

$$O \in \mathcal{O}_i \text{ iff } \{O, \mathbf{H}\} \approx 0 \tag{4}$$

This condition states that observables must have weakly vanishing Poisson brackets with all of the constraints; i.e. they must vanish on the constraint surface. From this vantage point, the observables argument is well nigh ineluctable. I mentioned above that the dynamics is generated by constraints; or, in other words, the dynamics takes place on the constraint surface, and evolution is along the Hamiltonian vector fields $X_{\mathbf{H}}$ generated by the constraints on this surface (i.e. along the gauge orbits). Therefore, the observables are constants of the motion: $\frac{dO}{dt}(q(t), p(t)) = 0$ (where t is associated to some foliation given by a choice of lapse and shift). This much gives us our two problems in the classical context. As Earman sums it up: "the Hamiltonian constraints generate the motion, motion is pure gauge, and the observables of the theory are constants of the motion in the sense that they are constant along the gauge orbits" (2003: 152). Now to the quantum problems.

Depending upon one's interpretive strategy with regard to the constraints at the classical level, there will be distinct quantization methods for the classical theory, and these correspond to different strategies for tackling the problem of time.[25] Quantization along such lines splits into two types: one can either quantize on the extended phase space or on the reduced phase space. The former method, "constrained quantization", is due to Dirac (1964): classical constraints are imposed as operator constraints on the physical states of the quantum theory. The latter method reduces the number of degrees of freedom of the extended phase space by factoring out the action of the symmetries generated by the constraints. Hence, the reduced space is the space of orbits of the extended space; it is a (quotient) manifold and inherits a symplectic structure (see Marsden and Weinstein (1974)): gauge invariance is automatic on the reduced phase space, for observables on the reduced space will correspond to gauge-invariant functions on the unreduced space. The extended and reduced phase spaces are equivalent on a classical level, but generally they will be inequivalent on a quantum level (*cf.* Gotay (1984)), so the choice is non-trivial.

In brief, and papering over a number of technical subtelties, the constrained (extended phase space) quantization method runs as follows:

... corresponding to no change in the physical state, are transformations for which the generating function is a first class constraint " (Dirac, 1964: 23).

[25]Since they associate methods of dealing with the constraints (to eliminate the gauge freedom or not) with particular interpretational stances on spacetime ontology, it is in just this way that Belot and Earman claim that quantization methods are linked to the substantivalism/relationalism debate and, therefore, that quantum gravity is also implicated in the grand old debate.

- Choose quantum states (representation space $\mathcal{F}$):

$$\psi[\mathsf{q}] \in \mathsf{L}^2(\mathrm{Riem}(\Sigma, \mu)) \tag{5}$$

- Represent the canonical variables $\mathsf{q}_{ab}, \mathsf{p}^{ab}$ on $\mathcal{F}$ as:

$$\hat{\mathsf{q}}_{ab}(\mathsf{x})\psi[\mathsf{q}] = \mathsf{q}_{ab}\psi[\mathsf{q}] \tag{6}$$

$$\hat{\mathsf{p}}^{ab}(\mathsf{x})\psi[\mathsf{q}] = \mathsf{i}(\frac{\partial}{\partial \mathsf{q}_{ab}})\psi[\mathsf{q}] \tag{7}$$

- Impose the diffeomorphism and Hamiltonian constraints:

$$\hat{\mathcal{H}}_a\psi[\mathsf{q}] = {}^3\hat{\nabla}_b\hat{\mathsf{p}}^b_a\psi[\mathsf{q}] = 0 \tag{8}$$

$$\hat{\mathcal{H}_\perp}\psi[\mathsf{q}] = \mathcal{G}_{abcd}\,\frac{\partial^2}{\partial \mathsf{q}_{ac}\partial \mathsf{q}_{bd}}\psi[\mathsf{q}] -^3 \mathsf{R}(\mathsf{q})\psi[\mathsf{q}] = 0^{26} \tag{9}$$

- Find a representation of a subset of classical variables on the physical state space, such that the operators commute with all of the quantum constraints.[27]

The classical observables argument filters through into this quantum setup since, by analogy with the classical observables, the quantum observables $\hat{\mathcal{O}}_i$ are defined as follows:

$$\hat{\mathsf{O}} \in \hat{\mathcal{O}}_i \text{ iff } [\hat{\mathsf{O}}, \hat{\mathbf{H}}] \approx 0 \tag{10}$$

Note that the weak equality '$\approx$' is now defined on the solution space of the quantum constraints; i.e. $\mathcal{F}_0 = \{\Psi : \hat{\mathbf{H}}\Psi = 0\}$. Clearly, if eq. (10) did not hold, then there could be possible observables whose measurement would 'knock' a state Ψ out of $\mathcal{F}_0$. The state version of the problem then follows simply from the fact that the quantum Hamiltonian annihilates physical states: $\hat{\mathbf{H}}\Psi = 0$. What motivates this view is the idea common to gauge theories that if a pair of classical configurations q and q' are gauge related then, for any observable O you could care to choose, $\mathsf{O}(\mathsf{q}) = \mathsf{O}(\mathsf{q}')$; so we should impose gauge invariance at the level of quantum states too: thus, $\psi(\mathsf{q}) = \psi(\mathsf{q}')$. The diffeomorphism constraint, eq.8, is particularly easy to comprehend along such lines; it simply says that for any diffeomorphism $\mathsf{d} : \Sigma \to \Sigma$, and state $\Psi[\mathsf{q}]$, $\Psi[\mathsf{q}] = \Psi[\mathsf{d}^*\mathsf{q}]$—in other words, no quantum state should be able to distinguish between gauge-related metrics. Were this

[26] $\mathcal{G}_{abcd}$ is the DeWitt supermetric defined by $[|\det \mathsf{q}|^{1/2}[(\mathsf{q}_{ab}\mathsf{q}_{cd} - \frac{1}{2}\mathsf{q}_{ac}\mathsf{q}_{bd})]$, and ${}^3\mathsf{R}(\mathsf{q})$ is the scalar curvature of q. The equation (9) is known as the 'Wheeler-DeWitt equation'.

[27] One must also find an inner product making these self-adjoint – no easy matter when there is no background metric or connection!

not the case, one could use the quantum theory to distinguish between classically indistinguishable states. The Hamiltonian constraint is more problematic, for it generates changes in data 'flowing off' Σ, and is seen as generating evolution. If we forbid quantum states to distinguish between states related by the Hamiltonian constraint, then there is cleary no evolution, for we must identify the 'evolved' slices Σ_0 and Σ_{t+d^*t} because evolution is a gauge motion (a diffeomorphism).

According to the alternative method, reduced phase space quantization, the constraints are solved for *prior* to quantization (i.e. at the classical level). To solve the constraints, one divides $\tilde{\Gamma}$ by its gauge orbits $[x]^i_\Gamma$. This yields a space $\tilde{\Gamma}_{\rm red}$ equipped with a symplectic form $\tilde{\omega}$. The resulting symplectic geometry $(\tilde{\Gamma}_{\rm red}, \tilde{\omega})$ is the reduced phase space, and in the case of general relativity corresponds to the space of non-isometric (vacuum) spacetimes. Thus, the symmetries generated by the constraints are factored out and one is left with an *intrinsic* geometrical structure of standard Hamiltonian form. In this form the canonical quantization is carried out as usual, and the observables are automatically gauge-invariant when considered as functions on the enlarged space. However, since one of the constraints (the Hamiltonian constraint) was associated with time evolution, in factoring it's action out the dynamics is eliminated, since time evolution unfolded along a gauge orbit (i.e. instants of time correspond to the points 'parametrizing' a gauge orbit). Thus, on this approach, states of general relativity are given by points in the reduced phase space, as opposed to the enlarged phase space used in constrained quantization approach.[28]

Of course, one can *completely* remove the ambiguity associated with gauge freedom by imposing gauge conditions, thus allowing for an unproblematic direct interpretation. However, in the case of general relativity (and other non-Abelian gauge theories) the geometrical structure of the constraint surface and the gauge orbits can prohibit the implementation of gauge conditions, so that some gauge slices will intersect some gauge orbits more than once, or not at all. If the former occurs then some states will be multiply represented (i.e. surplus remains); if the latter occurs, some genuine possibilities will not be represented in the phase space and, therefore, will not be deemed possible. One frequently finds that the reduced phase space method is mixed with gauge fixation methods, so that one has a partially reduced space, with the remaining gauge freedom frozen by imposing gauge conditions. Such an approach is used by a number of *internal time* responses to the problem of time. The idea is that one first solves the diffeomorphism constraint and then imposes gauge conditions on the gauge freedom generated by the Hamiltonian constraint. This is essentially the position of Kuchař (see below), and *constant mean curvature* approaches (see Carlip (1998) for a clear and thorough review).

[28] Little is known about the structure of the space of 3-geometries; the (Wilson) loop variables offer the best hope of carrying out the proposed reduction, or, rather, coordinatising the reduced space. The diffeomorphism constraint is solved by stipulating that the quantum states be knot invariants. The Gauss constraint that is picked up in the loop representation is easily solved since the Wilson loops are gauge invariant. However, the Hamiltonian constraint is still problematic, though at least *some* solutions can be found. See Brügmann (1994) for more details on these points. Thiemann has done more than anyone to make the Hamiltonian constraint respectable. However, there are problems even with his version. (REFS)

Before we consider the technical proposals for dealing with the problem of time, let us first review what little there is of the philosophical debate concerning the nature of the problem.

4 A Snapshot of the Philosophical Debate.

The philosophical debate on the problem of time has, I think, tended to misunderstand the kind of problem it is; often taking it to be nothing more than a result of eradicating indeterminism by applying the quotienting procedure for dealing with gauge freedom. This point of view can be seen quite clearly in action in a recent mini-debate between John Earman (2002) and Tim Maudlin (2002), where both authors see the restoration of determinism via hole argument type considerations as being the ultimate culprit. Thus, Earman writes: "In a constrained Hamiltonian system the intrinsic dynamics ... is obtained by passing to the reduced phase space by quotienting out the gauge orbits. When this is done for a theory in which motion is pure gauge, there is an "elimination of time" in that the dynamics on the reduced phase space is frozen" (*ibid*.: 14).[29] Before I outline some of the 'standard' responses, it will prove instructive to examine Maudlin's views; I will argue that Maudlin seriously misunderstands the nature of the problem of time.[30]

Maudlin distinguishes two separate arguments in Earman's paper that appear to lead to the frozen formalism: the "Hamiltonian Argument" and the "Observables Argument" - corresponding, more or less, to my states and observables arguments. He takes the crux of the Hamiltonian Argument to consist in the following observation:

> Applying this standard method ["quotenting out"] to the GTR does indeed restore the determinism of the theory-but at a price. The price is that the dynamics of the theory becomes "pure gauge"; that is, states of the mathematical model which we had originally taken to represent physically different conditions occurring at different times

[29]Maudlin (2002: 7) asks why one would want to cast GR in Hamiltonian form. He assumes that the sole reason is a desire to quantize the theory. I think this rather clouds his opinion as regards the analysis of the conceptual problems of this formalism. However, in addition to its utility in quantization, it also allows one to uncover the dynamical structure of the theory, it tells us that the Einstein field equations describe how the geometry of space evolves as time passes (*cf.* Baez & Munian 1994: 413). However, it is not entirely clear from the text whether Earman endorses the view that it is *only* when reduction is carried out that there is a problem of time. As I explain below, the problem of time is a problem of dynamics generated by gauge transformations (themselves generated by constraints); whether or not one 'solves' these, the gauge freedom will cause a problem of time if by "gauge transformation" we follow Dirac in taking them not to correspond to a physical operation.

[30]As I just mentioned, Earman might be interpreted as agreeing with the claim that it is quotienting in a bid to restore determinism that leads to the eradication of time evolution. This is false, as I argue below; however, I think the resolution Earman gives is along the right lines (as I explain in §5.2). I should point out that both Earman and Maudlin do, however, give the correct presentation of the observables argument as a problem of *change*.

> are now deemed equivalent since they are related by a "gauge transformation". We find that what we took to be an "earlier" state of the universe is "gauge equivalent" to what we took to be a "later" state. If gauge equivalent states are taken to be physically equivalent, it follows that *there is no physical difference between the "earlier" and the "later" states:* there is no real physical change. [*ibid*.: 2]

Maudlin's claim is that "the key to the Hamiltonian Argument" is based "in the freedom to foliate" (*ibid*.: 7). A specific foliation is an essential ingredient of any Hamiltonian formulation, for we need an initial data slice on a hypersurface. However, in relativistic theories there are many ways to slice up the spacetime manifold $\mathcal{M}$. Given an arbitrary foliation, a phase space can be constructed so that points of this space represent instantaneous states (in this case 3-geometries). The complete four-dimensional solution (i.e., a model of general relativity) is given by a trajectory through the phase space. One and the same solution can be represented by many different trajectories depending upon the foliation that one chooses. He then claims that this yields an indeterminism of the kind that the quotienting procedure is used for. But, he claims that it is a *faux* indeterminism. The quotienting is unnecessary, and not only is it unnecessary it leads to "silly" claims such as "change is not real, but merely apparent" (*ibid*.: 11). Claims, says Maudlin, that Earman thinks are revealed about the deep structure of general relativity by the constrained Hamiltonian formalism. For Maudlin, any such interpretation is absurd; as he explains:

> Any interpretation which claims that the deep structure of the theory says that there is no change at all – and that leaves completely mysterious why there *seems* to be change and why the merely apparent changes are correctly predicted by the theory – so separates our experience from physical reality as to render meaningless the evidence that constitutes our grounds for believing the theory. So the only real question is not *that* the constrained Hamiltonian formalism is yielding nonsense in this case, but *why* it is yielding nonsense. And the freedom to foliate provides the perfectly comprehensible answer. [12]

As regards the observables argument, he is equally negative:

> the Observables Argument gets any traction only by considering candidates for observables (values at points of the bare manifold) which are neither the sorts of things one actually uses the GTR to predict nor the sorts of things one would expect – quite apart from diffeomorphism invariance – to be observables. [*ibid*.: 18]

Maudlin concludes from this double debunking that the frozen formalism problem is simply a result of a "bad choice of formalism or a bad choice of logical form of an observables" (*ibid*.: 18). As regards his assessment of the Hamiltonian argument, it suffices to note that Maudlin sees the quotienting procedure as responsible for the eradication of time and this is patently false. The simple reason is that the gauge interpretation of the constrained

Hamiltonian formalism firstly does not require quotienting to be carried out and secondly, even when it is not carried out, the Cauchy slices represented by points within the same gauge orbit will still represent the same physical state and so time evolution will be frozen out (given that this amounts to the unfolding of a gauge transformation). Thus, Maudlin can claim that he is willing to accept the indeterminism that follows from such gauge transformations rather than quotienting if he likes[31], but the fact that the indeterminism is unobservable is tantamount to saying that the time-evolution is unobservable, which simply lets the problem in through the back door. As regards the observables argument it seems to me that far from showing it to be "broken-backed", Maudlin has simply taken a stance (and a highly non-trivial one at that) with respect to the observables argument. Specifically, he opts for the view that the 'proper' observables of general relativity are relational quantities involving intersections of quantities.[32] Thus, he writes that "[w]hat we *can* identify by observation are the points that satisfy definite descriptions such as "the point where these geodesics which originate here meet", and against *these* sorts of quantities Earman's diffeomorphism argument has exactly zero force" (*ibid*.). But Earman would agree with this! Indeed, the observables Maudlin mentions sound suspiciously like Earman's coincidence quantities. This is just what many physicists take to be the 'lesson' of the hole argument and the problem of time: the proper observables are independent of the manifold and, therefore, independent of time. The problem remains: how do we reconcile this with the manifest change we seem to observe? I review some options in the next section.

5 Catalogue of Responses.

Those approaches to classical and quantum gravity that attempt to understand these theories without change and time existing at a fundamental level I shall call *timeless*, and those that disagree I call *timefull*. An alternative pair of names for these views, suggested by Kuchar, are "Parmenidean" and "Heraclitean" respectively (1993b). But it is important to note that the debate here is not directly connected to the debate in the philosophy of time between 'A-theorists' and 'B-theorists' (or 'tensers' and 'detensers', if you prefer). Both of these latter camps agree that time exists, but disagree as to its nature. By contrast, the division between timefull and timeless interpretations concerns whether or not time exists *simpliciter*! I begin by reviewing several timefull responses.

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[31]Something he is willing to do on the grounds that the indeterminism is "*completely phoney*" (*ibid*.: 9; see also p.16).

[32]Note that Maudlin gives no account as to the nature of the individual elements participating in these intersections. The standard line is to take these elements as having some physical reality independently of the relation; but this leads to serious problems as we shall see in §5.2.

5.1 Timefull Stratagems.

Recall that the observables argument required that in order to class as kosher, the relevant observables must have vanishing Poisson brackets with *all* of the constraints. This idea filtered through into the quantum version, modified appropriately. Kuchař has been a vociferous opponent of this 'liberal' gauge invariant approach to observables. He agrees with the plan to the level of the diffeomorphism constraint, so that $\{O, \mathcal{H}_a\} \approx 0$, $[\hat{O}, \hat{\mathcal{H}}_a] \approx 0$ and $\mathcal{H}_a\Psi = 0$; but does not agree that we should apply the same reasoning to the Hamiltonian constraint, $\mathcal{H}_\perp\Psi \neq 0$. Thus, neither states nor observables should distinguish between metrics connected by $\mathrm{Diff}(\Sigma)$: only the 3-geometry ${}^3\mathcal{G}$ counts. But the alterations generated by the Hamiltonian constraint are a different matter says Kuchař:

> $[\mathcal{H}_\perp]$ generates the dynamical change of data from one hypersurface to another. The hypersurface itself is not directly observable, just as the points $x \in \Sigma$ are not directly observables. However, the collection of the canonical data $(q_{ab}(1), p^{ab}(1))$ on the first hypersurface is clearly distinguishable from the collection $(q_{ab}(2), p^{ab}(2))$ of the evolved data on the second hypersurface. If we could not distinguish between those two sets of data, we would never be able to observe dynamical evolution. [1993b: 20]

Ditto for states: the Wheeler-DeWitt equation does not say that an evolved state is indistinguishable from some initial state – as the diffeomorphism constraint does – rather, it "tells us how the state evolves" (*ibid*.: 21). More colourfully:

> I would say that the state of the people in this room now, and their state five minute ago should not be identified. These are not merely two different descriptions of the same state. They are physically distinguishable situations. [Ashtekar & Stachel (eds.), 1991: 139]

Thus, Kuchař concludes that "if we could observe only constants of motion, we could never observe any change" (*ibid.*). On this basis he distinguishes between two types of variable: *observables* and *perennials*. The former class are dynamical variables that remain invariant under spatial diffeomorphisms but *do not* commute with the Hamiltonian constraint; while the latter are observables that *do* commute with the Hamiltonian constraint. Kuchař's key claim is that one can observe dynamical variables that are not perennials.[33]

In their assessment of Kuchař's proposal, Belot and Earman (1999: 183) claim that he "endeavours to respect the spirit of general covariance of general relativity without

[33]He goes further than this, arguing that perennials are in fact hard to come by. I do not deal with this aspect of his argument here. In fact, I think that relational observables show that they are not at all hard to come by. How one makes a quantum theory out of these is, of course, quite another matter. The hard task is to find quantum operators that correspond to such classical observables without facing operator ordering ambiguities, and so on.

treating it as a principle of gauge invariance." For this reason they see his strategy as underwritten by substantivalism. I argue against the connection between the denial of gauge-invariance and substantivalism in §7; for now I note that Kuchař *does* treat general covariance as a principle of gauge invariance as far as the diffeomorphisms of Σ are concerned (and, in the connection representation, as far as the $SO(3)$ Gauss constraint goes). Observables are gauge-invariant quantities on his approach; the crucial point is simply that the Hamiltonian constraint should not be seen as generating gauge transformations. Viewed in this light, according to Belot and Earman's own taxonomy (*ibid.*: §2), Kuchař's position should more properly be seen as underwritten by a *relationalist* interpretation of space coupled with a *substantivalist* interpretation of time! Let me spell out some more of the details of Kuchař's idea.

Kuchař's claim that observables should not have to commute with the Hamiltonian constraint leads almost inevitably to the conclusion that the observables do not act on the space of solutions; or, as he puts it "if $\Psi \in \mathcal{F}_0$ and $\hat{F}$ is an observable, $\hat{F}\Psi \notin \mathcal{F}_0$" (1993b: 26). This, amongst other things, motivates the *internal time* strategy, where an attempt is made to construct a time variables T from the classical phase space variables. This strategy conceives of general relativity (as described by Γ) as a parametrized field theory. The idea is to find a notion of time *before* quantization hidden amongst the phase space variables so that a time-dependent Schrödinger equation can be constructed; the quantum theory's states then evolve with respect to the background time picked out at the classical level. Kuchař's method involves finding four (scalar) fields $X^A = (\mathcal{T}(x;q,p], \mathcal{Z}^a(x;q,p])$ (where $A = 0, 1, 2, 3$ and $a = 1, 2, 3$) from the full phase space Γ that when defined on $\overline{\Gamma}$ represents a spacelike embedding $X^A : \Sigma \rightarrow \mathcal{M}$ of a hypersurface Σ in the spacetime manifold $\mathcal{M}$ (without metric). These kinematical variables are to be understood as position at the manifold and the dynamical variables (separated out from the former variables within the phase space) are observables evolving along the manifold. The constraints are then understood as conditions that identify the momenta P_A conjugate to X^A with the energy-momenta of the remaining degrees of freedom: they thus determine the evolution of the true gravitational degrees of freedom between hypersurfaces.

There are two broadly 'technical 'ways of dealing with Kuchař's arguments. The first involves demonstrating that general relativity is not a parametrized field theory; the second involves showing that *observing* change is compatible with the view that all observables are constants of the motion. I deal with the second when I get to the timeless responses; the first I outline now. Clearly, we need to test whether or not the identification between the phase space Γ of general relativity and the phase space Υ of a parameterized field theory goes through. The proposal requires that there is a canonical transformation $\Phi : \Upsilon \rightarrow \Gamma$ such that $\Phi(\overline{\Upsilon}) = \overline{\Gamma}$. However, there can be no such transformation because $\overline{\Upsilon}$ is a manifold while $\overline{\Gamma}$ is not (*cf.* Torre (1994)). Hence, there are serious, basic technical issues standing in the way of this approach: general relativity is not a parameterized field theory!

Along more 'philosophical' lines, one might perhaps question the line of reasoning that led Kuchař to deny that observables commute with only some of the constraints in the first place. Is it an empirical input that determines the break, or is it something internal?

I think that it is neither, but is instead an intuitive belief that change is a real feature of the world. He takes the fact that the liberal gauge-invariance position entails that observables are constants of the motion as providing a reductio of that view, and as providing a counterexample to Dirac's conjecture that first class constraint generate gauge transformations. But we might question this. Indeed, analogous reasoning might lead one to deny the principles of relativity on the grounds that it grinds against common sense: sometimes our intuitions are wrong. Indeed, I think that the timeless proposals of the next subsection demonstrate that sense can be made of the idea that all observables commute with all of the constraints.

An alternative (internal) timefull approach uses *matter* variables coupled to spacetime geometry instead of (functionals) of the gravitational variables as above. Thus, one might consider a space filling dust field, each mote of which is considered to be a clock (i.e. the proper time of the motes gives a preferred time variable and, therefore, amounts to fixing a foliation). These variables are once again used to 'label' spacetime points. This includes an internal time variable against which systems can evolve, and which can function as the fixed background for the construction of the quantum theory. Another internal approach, *unimodular gravity*, amounts to a modification of general relativity, according to which the cosmological constant is taken to be a dynamical variable for which the conjugate is taken to be 'cosmological' time.[34] The upshot of this is that the Hamiltonian constraint is augmented by a cosmological constant term $\lambda + q^{-\frac{1}{2}}(x)$, $x \in \Sigma$, giving the super-Hamiltonian constraint $\lambda + q^{-\frac{1}{2}}(x)\,\mathcal{H}_{\perp}(x) = 0$. The presence of this extra term (or, rather, its conjugate τ) unfreezes the dynamics, thus allowing for a time-dependent Schrödinger equation describing dynamical evolution with respect to τ. The conceptual details of this approach are, however, more or less in line with gauge fixation methods like that mentioned above.[35] Another popular, but now aged approach is that which takes surfaces of constant mean curvature $\tau = q_{ab}p^{ab}/\sqrt{\det q} = \text{const.}$ as providing a time coordinate by providing a privileged foliation of spacetime.[36]

The basic idea underlying each of these approaches is to introduce some preferred *internal* time variable so that general relativity can be set up as a time-dependent system describing the evolution in time of a spatial geometry (possibly involving the extrinsic curvature and possibly coupled to matter or some reference fluid). With this background time parameter in hand, the quantization proceeds along the lines of other quantum field theories since there will be a non-zero Hamiltonian for the theory. Naturally, the selection of a preferred time coordinate breaks the general covariance of the theory, for it is tantamount to accepting that there is a preferred reference frame. One would have to

[34] The idea to use unimodularity as a response to the problem of time was originally suggested by Unruh (1989). For a nice philosophical discussion of unimodular gravity see Earman (2003b) - §6 of his paper focuses the discussion on the problem of time. See also Isham (1992: 63).

[35] Isham (*ibid*.: 62) goes so far as to say that it is in line with reference fluid methods since it amounts to the imposition of a coordinate condition (on the metric γ_{ab}): $\det\gamma_{ab}(x^i) = 0$. See (*ibid*.: 60-62) for more details on the notion of a reference fluid and how it might offer a solution to the problem of time.

[36] This approach was first suggested by York (1971). See Beig (1994) for a nice discussion.

demonstrate that the resulting quantum theory is independent of the choice.[37]

Suffice it to say that I do not think that these timefull approaches are the correct direction to go. Aside from the technical difficulties, they either represent a step backwards towards unphysical, ad hoc or arbitrary background structures, or else they point to the idea that a robust notion of time is required to get a quantum theory up and running. The proposals in the next subsection show that this is simply false.

Before I leave the 'timefull' methods, I should first mention one more related approach: Hájiček's *perennial formalism* (1996, 1999), according to which the dynamics is constructed solely from the geometry of phase space, and no reference is made to spacetime. The idea is to begin with some system whose time evolution is well understood, like a Newtonian system, and transform the spacetime structure into a phase space structure so that a quantum time evolution can be reconstructed from phase space objects. Then one attempts to find similar phase spaces for systems without background spacetimes, effectively 'guessing' a theory. This approach links technically to Kuchař's scheme, but conceptually it links up to the timeless approaches – especially Rovelli's evolving constants scheme. However, questions need to be asked about the way the phase space is constructed, for it is not intrinsically done, but is parasitic on what we know of phase spaces for systems with background spacetime structure (fixed metrics and connections). If the virtue of this approach is that it retains background independence, then we would surely like the formalism to reflect this property.[38]

5.2 Timeless Stratagems.

We come now to the timeless strategies; the most radical of which is surely Barbour's. I deal with this first, and then outline the view I favour. Butterfield (2001) has written a fine account of Barbour's timelessness as outlined in the latter's book *The End of Time* (2003); he describes the resulting position as "a curious, but coherent, position which combines aspects of modal realism *à la* Lewis and presentism *à la* Prior" (*ibid*.: 291). I agree that these aspects do surface; however, I disagree with his account on several key substantive points. In particular, I will argue – *contra* Butterfield – that Barbour's brand of timelessness is connected to a denial of persistence, and as such is not timeless at all; rather, it is changeless. I go further: far from denying time, Barbour has in fact *reduced* it (or, rather, the instants of time) to the points of a relative configuration space!

The central structure in Barbour's vision is the space of Riemannian metrics mod the spatial diffeomorphism group (known as "superspace"): $\mathrm{Riem}(\Sigma)/\mathrm{Diff}(\Sigma)$. Choosing this

[37]Note that Kuchař's approach escapes this objection since it quantizes the 'multi-time' formalism according to which dynamical evolution takes place along deformations of *arbitrary* hypersurfaces embedded in $\mathcal{M}$ (see Isham, *op. cit.*.: 46).

[38]Compare this with Earman's point that the relationalist should be able to construct his theories in relationally pure vocabulary, rather than 'piggy backing' on the substantivalists formulations (1989: 135).

space as the configuration space of the theory amounts to solving the diffeomorphism constraint; this is Barbour's *relative configuration space* that he labels "Platonia" (*ibid*.: 44). The Hamiltonian constraint (i.e. the Wheeler-DeWitt equation, eq.9) is then understood as giving (once solved, and "once and for all" (Barbour, 1994: 2875)) a *static* probability distribution over Platonia that assigns amplitudes to 3-geometries (Σ, q) in accordance with $\mid \Psi[\mathrm{q}] \mid^2$. Each 3-geometry is taken to correspond to a "possible instant of experienced time" (*ibid*.) This much is bullet biting and doesn't get us far as it stands; there remains the problem of accounting for the appearance of change. This he does by introducing his notion of a 'time capsule,' or a 'special Now', by which he means "any fixed pattern that creates or encodes the appearance of motion, change or history" (Barbour, 2003: 30). Barbour *conjectures* that the relative probability distribution determined by the Wheeler-DeWitt equation is peaked on time capsules; as he puts it "the timeless wavefunction of the universe concentrates the quantum mechanical probability on static configurations that are time capsules, so that the situations which have the highest probability of being experienced carry within them the appearance of time and history" (*ibid*.). What sense are we to make of this scheme?

Barbour's approach is indeed timeless in a certain sense: it contains no reference to a background temporal metric in either the classical or quantum theory. Rather, the metric is defined by the dynamics, in true Machian style. Butterfield mentions that Barbour's denial of time might sound (to a philosopher) like a simple denial of temporal becoming – i.e. a denial of the A-series conception of time. He rightly distances Barbour's view from this B-series conception. Strictly speaking, there is neither an A-series nor a B-series on Barbour's scheme. Barbour believes that space is fundamental, rather than spacetime.[39] This emerges from his Machian analysis of general relativity. What about Butterfield's mention of presentism and modal realism? Where do they fit in?

Presentism is the view which says that only presently existing things actually exist.[40] The view is similar in many respects to modal actualism, the view that only actually existing things exist simpliciter. Yet Butterfield claims that Barbour's view blends with modal realism. What gives? We can make sense of this apparent mismatch as follows: Barbour believes that there are *many* Nows that exist 'timelessly', even though we happen to be confined to *one*. The following passage brings the elements Butterfield mentions out to the fore:[41]

[39] I might add that Belot writes that he does "not know of any philosopher who entertains, let alone advocates, substantivalism about space as an interpretive option for GR" (1996: 83). I think that Barbour's proposal ends up looking like just such an interpretive option; a position recently defended by Pooley (2002).

[40] The consensus amongst philosophers seems to be that special and general relativity are incompatible with presentism (*cf.* Callender (2000), Savitt (2000), and Saunders (1996, 2000)). I think that special relativity allows for presentism in a certain sense - we simply need to modify what we mean by 'present' in this context, distinguishing it from what we mean in Newtonian mechanics-, and that general relativity (classical and quantum) too allows for presentism in the canonical formulation (a view recently defended by Monton (2001) in the context of timefull, 'fixed foliation' strategies). But we need to distinguish the kind of presentism that classical and quantum general relativity allows for from that which special relativity allows for, and that Newtonian mechanics allows for. But this is not the place to argue the point.

[41] Fans of Lewis' *On The Plurality of Worlds* (1986) will notice a remarkable similarity to a certain famous

> All around NOW ... are other Nows with slightly different versions of yourself. All such nows are 'other worlds' in which there exist somewhat different but still recognizable versions of yourself. [*ibid*.: 56]

Clearly, given the multiplicity of Nows, this cannot be presentism conceived of along Priorian lines, though we can certainly see the connection to modal realism; talk of other nows being "simultaneously present" (*ibid.*) surely separates this view from the Priorian presentist's thesis. That Barbour's approach is not a presentist approach is best brought out by the lack of temporal flow; there is no A-series change. Such a notion of change is generally tied to presentism. Indeed, the notion of many nows existing simultaneously sounds closer to eternalism than presentism; i.e. the view that past and future times exist with a much ontological robustness as the present time. These points also bring out analogies with the'many-worlds' interpretation of quantum mechanics; so much so that a more appropriate characterization might be a 'many-Nows' theory.[42] Thus, I don't think that Butterfield's is an accurate diagnosis. What is the correct diagnosis?

There is a view, that has become commonplace since the advent of special relativity, that objects are four-dimensional; objects are said to 'perdure', rather than 'endure': this latter view is aligned to a three-dimensionalist account according to which objects are wholly present at each time they exist, the former view is known as 'temporal part theory'. The four-dimensionalist view is underwritten by a wide variety of concerns: for metaphysicians these concerns are to do with puzzles about change; for physics-minded philosophers they are to do with what physical theory has to say. Change over time is characterized by differences between successive temporal parts of individuals. Whichever view one chooses, the idea of *persisting* individuals plays a role; without this, the notion of change is simply incoherent, for change requires there to be a *subject* of change. Although Barbour's view is usually taken to imply a three-dimensionalist interpretation (by Butterfield for one), I think it is also perfectly compatible with a kind of temporal parts type theory. However, rather than the structure of time being linear (modeled by $\mathbb{R}$), it is non-linear (modeled by relative configuration space) and the 'temporal evolution' is probabilistic (governed by a solution to the Hamiltonian constraint). We see that the parts themselves do not change or endure and they cannot perdure since they are three-dimensional items and the parts occupying distinct 3-spaces (and, indeed, the 3-spaces themselves) are not genidentical; rather, the quantum state 'jumps' around from Now to Now in accordance with the Hamiltonian constraint in such a way that the parts contain records that 'appear' to tell a story of linear evolution and persistence. Properly understood, then, Barbour's views arise from a simple thesis about identity over time, i.e., a denial of persistence:

> We think things persist in time because structures persist, and we mistake the structure for substance. But looking for enduring substance is like looking for time. It slips

passage from that work. Hence the suggested link to modal realism.

[42] Indeed, Barbour himself claims that his approach suggests what he calls a "many-instants ... interpretation of quantum mechanics" (*ibid*.). However, it seems clear that the multiplicity of Nows is as much a classical as a quantum feature.

through your fingers. [*ibid.*, 49]

In denying persisting individuals, Barbour has given a philosophical grounding for his alleged timelessness. However, as I mentioned earlier, the view that results might be seen as not at all timeless: the relative configuration space, consisting of Nows, can be seen as providing a *reduction* of time, in much the same way that Lewis' plurality of worlds provides a reduction of modal notions.[43] The space of Nows is given once and for all and does not alter, nor does the quantum state function defined over this space, and therefore the probability distribution is fixed too. But just like modality lives on in the structure of Lewis' plurality, so time lives on in the structure of Barbour's Platonia. However, also like Lewis' plurality, believing in Barbour's Platonia requires substantial imagination stretching. Of course, this isn't a knock down objection; with a proposal of this kind I think we need to assess its cogency on a cost versus benefit basis. As I show below, I think that the same result (a resolution of the problem of time) can be gotten on a tighter ontological budget. However, I think there is real value in Barbour's analysis of the problem of time, and philosophers of time would do well to further consider the connections between Lewis' and Barbour's reductions, and the stand alone quality of the view of time that results.[44]

Not quite as radical as Barbour's are those timeless views that accept the fundamental timelessness of general relativity and quantum gravity that follows from the gauge-invariant conception of observables, but attempt to introduce a *thin* notion of time and change into this picture. A standard approach along these lines is to account for time and change in terms of time-independent correlations between gauge-dependent quantities. The idea is that one never measures a gauge-dependent quantity, such as position of a particle; rather, one measures 'position at a time', where the time is defined by some *physical* clock.[45] Thus, in the general relativistic context, we might consider the spatial volume of the universe, $V = \int_\Sigma \sqrt{-\det g}\, d^3x$; this is gauge-dependent (for compact Σ) and, therefore, is not an observable. Now suppose we wish to measure some quantity defined over Σ, say the total matter density $\rho(x)$, $\forall_i x_i \in \Sigma$. Of course, this too is a gauge-dependent quantity; but the *correlation* between V and ρ when they take on a certain value is gauge-*independent*. In this way, one can define an instant of time; one can write $\tau = \rho(V)$ or $\tau = V(\rho)$. One can then use these correlations to function as a clock giving a monotonically increasing time parameter τ against which to measure some other quantities. Unruh objects to this method along the following lines:

one could [try to] define an instant of time by the correlation between Bryce DeWitt talking to Bill Unruh in front of a large crowd of people, and some event in the outside

[43]Roughly, Lewis' idea is that the notions of *necessity* and *possibility* are to be cashed out in terms of holding at all or some of a class of 'flesh and blood' worlds.

[44]I expect that the view of most philosophers of time would be that Barbour has simply outlined a variation of eternalism, albeit a peculiar one.

[45]See the exchange between DeWitt, Rovelli, Unruh, and Kuchař in Asktekar & Stachel (eds.) (p.137-140) for a nice quick introduction to the timeless vs timefull views: Rovelli and DeWitt are firmly in favour of the correlation view, while Unruh and Kuchař are firmly against it. I outline Unruh's and Kuchař's objections below.

> world one wished to measure. To do so however, one would have to express the sentence “Bryce DeWitt talking to Bill Unruh in front of a large crowd of people” in terms of physical variables of the theory which is supposed to include Bryce DeWitt, Bill Unruh, and the crowd of people. However, in the type of theory we are interested in here, those physical variables are all time independent, they cannot distinguish between “Bryce DeWitt talking to Bill Unruh in front of a large crown of people” and “Bryce DeWitt and Bill Unruh and the crowd having grown old and died and rotted in their graves.” ... The subtle assumption [in the correlation view] is that the individual parts of the correlation, e.g. DeWitt talking, are measurable when they are not. [1991: 267]

Belot and Earman question Unruh’s interpretation of the correlation view, and suggest that it might be better understood “as a way of explaining the illusion of change in a changeless world” (2001: 234). The basic idea is that one deals in quantities of the form “clock-1-reads-t_1-when-and-where-clock-2-reads-t_2”. We get the illusion of change by (falsely) taking the elements of these relative (correlation) observables to be capable of being measured independently of the correlation. They suggest that Rovelli’s notion of evolving constants of motion is a good way of “fleshing out” the relative observables view.

Rovelli’s *evolving constants of motion* proposal is made within the framework of a gauge-invariant interpretation. He accepts the conclusion that quantum gravity describes a fundamentally timeless reality, but argues that sense can be made of dynamics and change within such a framework. Take as a naive example of an observable m = ‘the mass of the rocket’. This cannot be an observable of the theory since it changes over (coordinate) time; it fails to commute with the constraints, $\{m, \mathbf{H}\} \neq 0$, because it does not take on the same value on each Cauchy surface. Rovelli’s idea is to construct a one-parameter family of observables (constants of the motion) that can represent the sorts of changing magnitudes we observe. Instead of speaking of, say, ‘the mass of the rocket’ or ‘the mass of the rocket at t’, which are both gauge dependent quantities (unless t is physical), one speaks instead of ‘the mass of the rocket when it entered the asteroid belt’, $m(0)$, and ‘the mass of the rocket when it reached Venus’, $m(1)$, and so on up until $m(n)$. These quantities are gauge-invariant, and, hence, constants of the motion; but, by stringing them together in an appropriate manner, we can explain the appearance of change in a property of the rocket. The change we normally observe taking place is to be described in terms of a one-parameter family of constants of motion, $\{m(t)\}_{t\in\mathbb{R}}$, an *evolving* constant of motion.[46]

A similar criticism to Unruh’s comes from Kuchař (1993b: 22), specifically targeting Rovelli’s approach. Kuchař takes Rovelli to be advocating the view that observing “a chan-

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[46] Rovelli, in collaboration with Connes (1994), has argued that the ‘flow’ of time can be explained as a “thermodynamical” effect, and is state dependent. The thermal time is given by the state dependent flow generated by the statistical state s over the algebra of observables: $\frac{dq}{dt} = -\{q, \log s\}$. Hence, the Hamiltonian is given by $-\log s$, so that the (statistical) state that a system occupies determines the Hamiltonian *and* the associated flow. Rovelli connects this idea up to his evolving constants proposal by identifying the thermal time flow with the one-parameter group of automorphisms of the algebra of observables (as given by the Tomita flow of a state).

ging dynamical variable, like Q [a particle's position, say], amounts to observing a one-parameter family $Q'(\tau_1) := Q' + P'\tau = Q - P(T - \tau)$, $\tau \in R$ of perennials" (*ibid*.: 22). By measuring $Q'(\tau)$ at τ_1 and τ_2 "one can infer the change of Q from $T = \tau_1$ to $T = \tau_2$" (*ibid*.). So the idea is that a changing observable can be constructed by observing correlations between two dynamical variables T and Q, so that varying τ allows one a notion of 'change of Q with respect to T'. Kuchař objects that one has no way of observing τ that doesn't smuggle in non-perennials. But this is a *non sequiter*; one doesn't need to observe τ independently of Q: we can simply *stipulate* that the two are a 'package deal', inseparable. In this way, I think both Unruh's and Kuchař's objections can be successfully dealt with. I outline this view further in the next section, where I attempt to strengthen the correlation solution.

Rovelli's approach has a certain appeal from a philosophical point of view. It bears similarities to four-dimensionalist views on time and persistence. The basic idea of both of these views is that a changing individual can be constructed from unchanging parts. Change over time is conceptually no different from variation over a region of space. (I think philosophers of time might perhaps profit from a comparison of Rovelli's proposal with four-dimensionalist views.) However, technically, it is hard to construct such families of constants of motion as phase functions on the phase space of general relativity. To the extent that they can be constructed at all, they result in rather complicated functions that are hard to represent at the quantum level (i.e. as quantum operators on a Hilbert space: *cf.* Hájiček (1996: 1369)), and face the full force of the factor ordering difficulties (*cf.* Ashtekar & Stachel (eds.), 1991: 139).[47] For this, and other reasons, Rovelli has recently shifted to something more like the original correlation view I outlined above (see Rovelli (2002); his earlier paper (1991) contains much the same view).

As with the evolving constants of motion program, Rovelli believes that the observables of general relativity and quantum gravity are *relative* or *relational* quantities expressing correlations between dynamical variables. The problem Rovelli sets himself in his *partial observables* program, as if in answer to Unruh's complaint, is this: "how can a correlation between two nonobservable quantities be observable?" (*ibid*.: 124013-1). He distinguishes between *partial* and *complete* observables, where the former is defined as a physical quantity to which we can associate a measurement leading to a number, and the latter is defined as a quantity whose value (or probability distribution) can be predicted by the relevant theory. The above question can then be rephrased in these terms: 'how can a pair of partial observables make a complete observable?' (see pages 124013-5). His answer is somewhat surprising, for he argues that this question is just as applicable to classical non-relativistic theories as it is to relativistic theories. However, there is a further distinction to be made, within the class of partial observables, that only holds in non-general relativistic (more generally: background dependent) theories: *dependent* and *independent*. These can be understood as follows: take two partial observables, $\mathtt{q}$ and $\mathtt{t}$ (position and time); if we can write $\mathtt{q}(\mathtt{t})$ but not $\mathtt{t}(\mathtt{q})$ then we say that $\mathtt{q}$ is a dependent partial observable and $\mathtt{t}$ is an independent partial observable. He then traces

[47] But see Montesinos et al. (1999) for a construction of such a family for a simple $SL(2, \mathbb{R})$ model.

the confusion in Unruh's objection to the notion of localization in space and time and, in particular, that this makes no sense in the context of general relativistic physics. The absolute localization admitted in non-relativistic theories means that the distinction can be disregarded in such quantum theories since "the space of observables reproduces the fixed structure of spacetime" (p. 124013-1). However, where the structure of spacetime is dynamical t and q are partial observables for which we cannot assume that an external clock or spatial reference frame exists. Going back to Unruh's example, we see that Unruh, DeWitt and the crowd of people are analogs of partial observables. Unruh assumes that the dependent/independent distinction must hold. However, this is just what Rovelli denies:

> A pre-GR theory is formulated in terms of variables (such as q) evolving as functions of certain distinguished variables (such as t). General relativistic systems are formulated in terms of variables ... that evolve with respect to each other. General relativity expresses relations between these, but in general we cannot solve for one as a function of the other. Partial observables are genuinely on the same footing. [Rovelli, *ibid*.: 124013-3]

The theory describes relative evolution of (gauge-dependent) variables as functions of each other. No variable is privileged as the independent one (*cf*. Montesinos, *et al.*, 1992: 5).[48] How does this resolve the problem of time? The idea is that coordinate time evolution and physical evolution are entirely different beasts. To get physical evolution, all one needs is a pair $\mathcal{C}, \mathrm{C}$ consisting of an extended configuration space (coordinated by partial observables) and a function on $T^*\mathcal{C}$ giving the dynamics. The dynamics concerns the relations between elements of $\mathcal{C}$, and though the individual elements do not have a well defined evolution, relations between them (i.e. correlations) do: they are independent of coordinate time.

However, both Earman and Rovelli appear to want to cling to the notion that the elements of the relations (the partial observables or coinciding elements) have some independent physical reality.[49] This is most explicit is Rovelli who takes the extended configuration space (physically impossible states and all!) to have physical significance as the space of the partial observables. I agree that, without empirical evidence to the contrary, the extended space should be retained since it gives us more conceptual elbow room; but I favour a view whereby gauge-invariance itself picks out the physical parts of this space. The interpretation then follows the correlation view, but with the correlates and the correlations

[48] Earman appears to endorse this view, and claims that the events (he calls the "Komar events") formed by such coincidences between gauge-dependent variables can be strung together to give a temporal evolution, generating a "D-series". However, I think that *coincidences* narrow the class of observables down to much.

[49] Note that Rovelli reads the gauge-fixation methods involving dust variables, curvature scalars, and the like as partial observables. What occurs in these strategies is that the partial observables are taken to be independent so that they are able to function as coordinate systems. However, as Rovelli notes, since the dependent and independent players can have their roles permuted, the distinction collapses (*ibid*.: 124013-4).

understood as simply different aspects of one and the same basic structure. The natural interpretation of Rovelli's view is that there is no physical distinction between gauge dependent and independent quantities. This implies that there are physically real quantities that are not predictable, even though we can associate a measurement procedure with them; indeed, Rovelli claims that these variables "are the quantities with the most direct physical interpretation in the theory" (*ibid*.: 124013-7).

It is interesting to note how this links up to Belot and Earman's interpretive taxonomy regarding constraints and spacetime ontology. Since Belot and Earman equate the view that there are physically real quantities that do not commute with the constraint with (straightforward) substantivalism, it appears that Rovelli would have to class as such, for his partial observable are just such quantities! Combined with the role reversal of Kuchař given earlier, this makes something of a mockery of their taxonomy, for they have Kuchař and Rovelli as the archetypical substantivalist and relational respectively. This, I would urge, is yet another aspect of my claim that the relationalist/substantivalist controversy doesn't get any support from those problems with their roots in the interpretation of gauge symmetries.

6 Enter Structuralism.

Rovelli, and other defenders of the correlations view[50], are of the opinion that the observables of general relativity and quantum gravity are *relative* quantities that express correlations between dynamical, and hence gauge-dependent, variables. The problems posed to the correlation-type timeless strategies are based upon an understanding that is couched in terms of relationalism. The fact that correlations between *material* systems are required to define instants of time (and points of space) does indeed look, superficially, to entail relationalism. I suspect that this entailment is what was motivating the objections of Unruh and Kuchař. The assumption was that if it is relations doing the work, then the relata must have some physical significance independently of these relations. This is just what I deny: the distinction between material systems and space and time simply amounts to different aspects of one and the same physical structure (*cf.* Stein (1967)). It is not that relations can be free standing; maybe they can, but in this case we have clear relata entering into the relations: DeWitt, Unruh, and a crowd of people! The question concerns the relative ontological priority of these relata over the relations. Relationalists will argue that the relations supervene upon the relata so that the relata are fundamental. Substantivalists will argue that the relata enter into their relations only in virtue of occupy-

[50]Others include DeWitt (see Ashtekar & Stachel (eds.), 1991: 137), Marolf (1994), Page and Wooters (1983), and, on the philosophical side, Earman (2003c: see below). Page and Wooters' idea is that one deal with conditional probabilities for outcomes of pairs of obervables. One then takes one the observables as defining an instant of time (*qua* the value of a physical clock variable) at which the other observable is measured. A notion of evolution emerges in terms of the dependence of conditional probabilities on the values of the (internally defined) clock variables.

ing a position in some underlying spatiotemporal structure that exists independently of both the relations and relata. An alternative position will see the relata as being some kind of epiphenomena or 'by-product' resulting from intersections occurring between the relations. But there is a middle way between these two extremes: neither relations nor relata have ontological priority. The relata are individuated in vitue of the relations and the relations are individuated by the relata.[51] Thus, the idea is to understand the correlation view structurally: one cannot decompose or factor the relative observables in to their relata, since the relata have no physical significance outside (independently) of the correlations. But one need not imbue the relations themselves with ontological primacy either. Thus, one can evade the objection that gauge dependent quantities are independently measurable by taking the correlations and correlates to be interdependent.

I shall call the overall structure formed from such correlations a *correlational network*, and the correlates I shall call *correlata*. It is important to note that the correlata need not be material objects, and we can find suitable items from the vacuum case. One is able to use (any) four invariants of the metric tensor to provide an intrinsic coordinate system that one can use to set up the necessary correlational network.[52] Thus, this approach does not imply relationalism; but it does not imply substantivalism either (neither sophisticated nor straightforward). The reason is, of course, that those interpretations require a stance to be taken with regard to the primacy of some category of object (points, fields, or whatever). Each of these other positions is problematic in the context of the problem of time since they both require that some set of objects take the ontological burden to function as a clock or a field of clocks.

Earman too seems to defend a version of the correlation view. His account is based on his notion of *coincidence events*; thus, he writes:

> The occurrence or non-occurrence of a coincidence event is an observable matter ... and that one such event occurs earlier than another such event is also an observable matter. ... Call this series of coincidence events the *D-series* ... Change now consists in the fact that different positions in the D-series are occupied by different coincidence events. [2002: 14]

Earman claims that the coincidence event (represented by the functional relationship $g^{\mu\nu}(\phi^\lambda)$: "the *Komar state*") "floats free of the points of $\mathcal{M}$" and "captures the intrinsic, gauge-independent state of the gravitational field" (*ibid*.). General covariance implies that if this state is represented by one spacetime model it is also represented by any model

[51]Thus, though admittedly similar, this should be distinguished from Teller's brand of *relational holism* (see his 1991). Teller argues that in some cases—entanglement is the example he focuses on—we should view relations as being primitive (non-supervenient).

[52]This is, of course, the method developed by Bergmann & Komar (1972). They used the four eigenvalues of the Riemann tensor. Dorato & Pauri (this volume) use this method, and these 'Weyl scalars' to argue for a form of structuralism they call "spacetime structural realism". This is a far cry from what I have in mind since they retain fairly robust notions of independent object (the metric field) in their approach.

from a diffeomorphism class of its copies. Now, Earman's interpretation of this, and his resolution of the problem of time, is to claim that the notion of spacetime points, properties localized to points, and change couched in terms of relationships between these, is to be found "in the representations" and not "in the world" (*ibid*.). This conclusion is clearly bound to the idea that in order to have any kind of change, a *subject* is required to undergo the change and *persist* under the change. In getting rid of the notion of a subject (i.e. spacetime points), Earman sees the only way out as abolishing change. The idea that change is a matter of representation is one way (not a particularly endearing one, say I) of accounting for the psychological impulse to believe that the world itself contains changing things, though I think it needs spelling out in much more detail than Earman has given us. But - quite aside from the fact that I don't think the existence of spacetime points is ruled out[53] - I don't see why Earman needs to go to this extreme; there is *variation* in the structure formed from the various correlations. True, we don't get any notion or account of time *flow* from this variation, but that is a hard enough problem outside of general relativity and quantum gravity anyway (but see Rovelli & Connes (1994)).

However, some other remarks of Earman's show that he doesn't have in mind the same view as mine. For instance, Earman (2002: 16-17) makes the following observations:

> [T]he gauge interpretation of diffeomorphism invariance ... calls into question the traditional choices for conceiving the subject vs. attribute distinction. The extremal choices traditionally on offer consist of taking individuals to be nothing but bundles of properties vs. taking individuals to have a 'thisness' (*haecceitas*) that is not explained by their properties. The gauge interpretation of GTR doesn't provide any grounds for *haecceitas* of spacetime points. Nor does it fit well with taking spacetime points as bundles of properties since it denies that the properties that were supposed to make up the bundle are genuine properties. The middle way between the *haecceitas* view and the bundles-of-properties view takes individuals and properties to require each other, the slogan being that neither exists independently of the states of affairs in which individuals instantiate properties.

As Earman goes on to explain, in the context of general relativity this middle way fares no better than the bundle-of-properties view since the gauge interpretation of general covariance "implies that the state of affairs composed of spacetime points instantiating, say, metrical properties do not capture the literal truth about physical reality; rather, these states of affairs are best seen as representations of a reality ... that itself does not have this structure." What Earman means by "representation" in this context, is, I think, what Rovelli calls a "local universe" (1991): a physically possible world in which properties are 'attached' to spacetime points. However, as Earman and Rovelli point out, this is not how general relativity represents the world; it does so by means of an equivalence class of such local universes, yielding a very 'non-local' description. However, if we extend the account Earman gives to include *relations* rather than simply properties (which clearly *do*

[53] For example, Saunders' account of identity allows that spacetime points exist as individual objects while respecting diffeomorphism symmetry.

require subjects of some sort) then we can in fact get directly at the structure Earman mentions.

This way of understanding the correlation view avoids Unruh's and Kuchař's objections, and it sidesteps Earman's worry. Not only does it resolve these objections, and the problem of time, it also provides a suitable ontological framework for classical and quantum gravity. I defend this latter claim in §7. Before I leave the problem of time I should just mention one more interesting aspect of the conception I have outlined. Rovelli mentions that a consequence of the correlation view is that there will be no notion of a *global* time, "[a] clock time ... generally behaves as a clock only in certain states or for a limited amount of time" (2004: 59). Thus, the correlation I mentioned earlier concerning the volume of the universe will be unable to ground physical evolution if the volume for some reason becomes fixed on some value. This appears to be a classical analogue of the quantum no-go theorem of Unruh & Wald (1989) according to which there can be no quantum observable that can function as an absolute, global time parameter (*cf.* Weinstein (1999)). These results are seen to be a problem for relationally construed correlation interpretations since it is required there that some physical system fix just such a notion. However, on the structuralist view I sketched above there is no separation between clocks and systems, and so no such quasi-external clock is needed.

Of course, avoidance of the problem of time can hardly be said to provide an adequate *defense* of the structuralist conception of the correlation view; as we have seen, there are other alternatives that are also compatible with both the correlation view and the problem of time. For this reason, I expect to be charged with ad hocness at this point. However, the structuralist conception does allow one to sidestep difficult problems with the relationally construed correlation view, and it remains in line with the gauge-invariance conception of observables, unlike the timefull responses. Furthermore, it offers a unifying perspective of the gauge-invariance view of observables, since it treats the problems of space and time on an equal footing. But the charge is well taken, and I shall attempt to defend the view more directly in the next section.

7 Quantum Gravity and Spacetime Ontology.

As with the hole argument—and, indeed, the Leibniz shift argument and permutation symmetry of quantum mechanics (see my (2004) and mine and French's (2001))— there have been many grand proclamations about of the impact of quantum gravity on the issue of spacetime ontology and the debate between substantivalists and relationalists. I think it is fair to say that the received view amongst physicists working in the field of canonical quantum gravity is that the theory supports some form of relationalism (or, at least, anti-substantivalism). The most explicit defender of this view has surely been Rovelli (most explicit in: 1992 and 1997) — Smolin (2000, 2001), Baez (2001, 2004), and Crane (1993, 1995) paint similar philosophical stances. This has been largely backed up by philosophers who have taken an interest in the subject. Belot and Earman line up gauge-invariant and non-gauge-invariant interpretations with relationalism and substantivalism respectively; and, as we have seen, Belot sees reduced phase space and unreduced phase space quantizations respectively as similarly aligned — these spaces are themselves linked to solving and not solving the constraints respectively. I wish to argue against these claims in this final section. My key point is that the methods for dealing with gauge freedom (or not, as the case may be) do not bear any relation to spacetime ontology (as charted in the substantivalism vs relationalism debate), and either side of the debate can help themselves to any of the methods. Since these methods are central to the conclusions drawn in the quantum gravity context, we see that quantum gravity does not have the bearing on spacetime ontology that is often thought to hold. My conclusion is that the methods, although central to our understanding of the *structure* of space and time, cannot in fact allow us to draw deeper metaphysical morals about the nature of this structure.

Towards the end of their review of the problem of time, Belot and Earman make the following rather metaphysically weighty claims:

> It would require considerable ingenuity to construct an (intrinsic) gauge-invariant substantivalist interpretation of general relativity. And if one were to accomplish this, one's reward would be to occupy a conceptual space already occupied by relationalism. Meanwhile, one would forgo the most exciting aspect of substantivalism: it's link to approaches to quantum gravity, such as the internal time approach. To the extent that such links depend upon the traditional substantivalists' commitment to the existence of physically real quantities which do not commute with the constraints, such approaches are clearly unavailable to relationalists. [2001: 248-9]

Their argument is based on the following line of reasoning: if spacetime points were real, then quantities like 'the curvature at point x' would be real too; but such quantities do not commute with the constraints, so spacetime points cannot be real after all. Substantivalists are then seen as being committed to the view that there are physically real quantities that do not commute with the constraints, and relationalists are committed to the denial of this.[54] Hence, they have Kuchař occupying the first position and Rovelli occupying the latter. I already argued against the first alignment on the grounds that Kuchař *is* committed to the view that all physical quantities commute with the diffeomorphism constraint. It is true that Rovelli sees himself as occupying a relationalist position, and he sees this as following from *complete* gauge-invariance. However, there are a number of reasons why

[54]Belot connects the substantivalist/relationalist debate to the treatment of symmetries in Hamiltonian systems and their retention or removal respectively (2000: 571). Likewise for other philosophical stances related to similar symmetry arguments. The idea is that 'substantivalism' and 'relationalism' are linked to a certain treatment of the symmetries in *any* theory formulated in a phase space description. Thus, one could be substantivalist or relationalist about vector potentials, for example; and this would simply correspond to endorsing an unreduced (direct) or reduced phase space formulation respectively. In my (2004) I argued that these links can be severed.

this is problematic—Rovelli and Kuchař can in fact be ‘permuted’ over relationalist and substantivalist positions according to their taxonomy!

Let me first detach substantivalism from the internal time approaches. Belot (1996: 241) claims that “substantivalism is ... a necessary condition for loyalty to the sort of approach to quantum gravity that Kuchar advocates”; namely an approach according to which observables commute with the diffeomorphism but not the Hamiltonian constraint. But although Kuchař might claim that his position is substantivalist (see Belot, *ibid*.: 238), it is quite clear that a relationalist could just as well adopt it. Indeed, given that the diffeomorphism constraint is solved, Kuchař’s position will come out as relationalist according to the received view - a view that Belot elsewhere endorses (see, e.g., Belot (2001)). According to Kuchař the lesson of the hole argument is that it is the geometry of a spatial manifold that has physical content: the diffeomorphism constraint should be solved for. If this *is* substantivalist, then it is clearly of the ‘sophisticated’ sort; Belot characterizes these as “crypto-relationalist” (2000: 576, fn. 36).

Next, let me disentangle the view that relationalists cannot adopt the view that there are some observables that do not commute with the constraints. I grant Belot and Earman’s point that the *reductive* relationalist will be barred from those strategies that outlaw commutation with *all* of the constraints. However, as I hinted at above, the relationalist (even the reductive one) can help himself to Kuchař’s position. The phase space there is a *partially* reduced one, with the gauge freedom generated by the diffeomorphisms of space modded out. This is a reasonable object for the relationalist even by Belot and Earman’s lights. The fact that the observables are not to commute with the Hamiltonian constraint is no problem: the relationalist too might want to deny that the geometries related by the Hamiltonian constraint are to be identified for exactly the reasons outlined by Kuchař. Thus, it is perfectly possible for a relationalist to deny Belot & Earman’s condition.

Belot and Earman are agreed that the best (easiest) way to avoid the indeterminism that arises in the hole argument, and gauge from gauge freedom in general, is to adopt a gauge-invariant interpretation. However, they make the mistake of assuming that the way to achieve this is by giving a direct interpretation of the reduced phase space. They take such interpretations as showing, in the context of general relativity, there could not “be two possible worlds with the same geometry which differ only in virtue of the way this geometry is shared out over the existent spacetime points” (2001: 228). This, they say, leads to relationalism (in the absence of “an attractive form of sophisticated substantivalism). They list several problems facing the reduced space accounts: the singular points corresponding to symmetric models; non-differentiability; and the unavailability of a set of coordinates able to separate out the space’s points. For these reasons they conclude that “a dark cloud hangs over the programme of providing gauge-invariant interpretations of general relativity ... the present state of ignorance concerning the structure of the reduced phase space ... - and the lingering worry that this structure may be monstrous - should give pause to advocates of gauge-invariant interpretations of the theory” (ibid.: 228-9). Perhaps this is a fair comment as far as the reduced space methods go; but such methods are not necessary for gauge-invariant interpretations. One can accommodate gauge-invariance without removing the gauge freedom by giving a many-to-one interpretation of the unreduced space.

Thus, what I am denying here is that the various strategies used in responding to the problem of time and the hole argument (the analogous problem for *space*) are related to interpretive stances regarding the nature of spacetime in general relativity. The strategies do not definitively support any such stance, nor do any such stances definitively support the strategies. Thus, what we have is an underdetermination of the various strategies and stances with respect to each other. Whatever it is that pushes one towards a particular stance as regards the nature of spacetime, it cannot be the hole argument or the problem of time. The best these arguments can do is to tell us about the *structure* or spacetime, not its nature. However, as I argued in the previous section, for a structuralist, this is all one needs: nature just is structure![55] Furthermore, given the radical underdetermination between the various possibility counting schemes and the physics, it would seem to be somewhat foolhardy to base one's metaphysical positions on such schemes.

The high degree of symmetry that is found in general relativity, and will be most likely be found in its quantization, is a result of the theory's background independence. In sharp contrast to the other field theories we have classical and quantum descriptions for, general relativity has no background structure above the differentiable manifold, and this structure has a characteristic group of diffeomorphism symmetries. This automatically makes any theory defined with respect to the manifold generally covariant (diffeomorphism invariant), provided, of course, no other background fields are introduced to gauge fix the symmetry. The fact that there is no background metric or connection means that the conceptual structure of the theory is going to be very different from any theory with such structure. In particular, the idea of gauge symmetry is going to have somewhat deeper implications; as Isham observes: "Yang-Mills transformations occur at a fixed spacetime point whereas the diffeomorphism group moves points around. Invariance under such an active group of transformations robs the individual points in $\mathcal{M}$ of any fundamental ontological significance" (1993: 13). Isham is here voicing a fairly common view among physicists that diffeomorphism invariance is sufficient to show that spacetime points do not exist, and that some form of relationalism is forced upon us by it. One might just as well think, however, that moving a thing around would endow that thing with "ontological significance". My view is that diffeomorphism invariance shows that spacetime *localization* is problematic.[56] A relationalist will view this is grist to their mill, but we saw that substantivalists can accommodate diffeomorphism invariance too. Thus, neither view is given unique support. The structuralist position I have presented above looks to the structure of the observables and reads this structure as ontologically neutral with respect to the exact nature of spacetime. Furthermore, without any grounds for opting for a reduced phase space description

[55]Note, however, that this is not structuralism of the Ladyman stripe (1998). The underdetermination brought out here includes the eliminative structural realism that Ladyman espouses; hence, one of the original motivations for that approach is undercut.

[56]I hesitate to say that it rules against it on the grounds that Saunders' analysis (2002 and 2003) appears to allow for some such notion. This is simply to restate my point that diffeomorphism invariance and the hole argument are inert or the question of the ontological status of spacetime points.

the neutrality goes deeper still, affecting the possibility structure too. But in adopting the anti-reductionist view one is able to encompass both haecceitistic and anti-haecceitistic possibility sets and both relationalism and substantivalism.

Now, assuming that we still wish to adopt some substantivalist or relationalist approach; what problems might they be expected to face in the light of quantum gravity? The primary problem, as I see it, is essentially the same for both interpretations. Either approach requires a primitive set of objects of some kind: space(time) points or material objects (here understood as parts (or excitations) of a field). In either case the objects are used to set up a notion of locality, so that states and observables of quantum fields can be referred to points or regions of space(time) as determined materially (relationalism) or assuming a substantival (but, nonetheless, dynamical) background. Now, as regards substantivalism, I agree with Hoefer (1996) that the most defensible form will involve the metric field; the bare manifold cannot function as spacetime.[57] Thus, either a material field or a metric field will play the role of individuating the points of spacetime, thus enabling local operators to be defined and a quantum theory to be constructed. However, if the individuating fields are physical then we expect them to be quantized like any other field. This, of course, means that they will be subject to the uncertainty relations and, therefore, will fluctuate in general. But if they are allowed to fluctuate then it isn't at all clear how they are supposed to perform their individuating function. One way of understanding this situation is, of course, to consider the individuation as 'fuzzy', leading to some kind of non-commutative notion of geometry. However, while the picture may constitute a possible interpretation of quantum spacetime, it does not give us what we were after in the first place; namely, local quantum field operators (be they at spacetime or material points). This, I think, is *the* sticky point for both relationalist and substantivalist interpretations in the new context of quantum gravity. Structuralism, as I understand it, evades the problem: there are neither primitive points nor objects to be individuated. Rather, one has a correlational network that fluctuates quantum mechanically as a whole. This, I suggest, is a safe and sane ontological basis from which to view space and time in both classical and quantum (canonical) gravity.

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[57]Though, I should point out that the manifold substantivalist isn't completely out of the game; the 3-manifold is a central part of the canonical quantization approaches. However, the usual take on this piece of background structure is that it is merely an auxiliary device (i.e. a 'heuristic crutch' in French's sense (1998)); this seems to be Rovelli's view (1992, 1999).

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