What Price Determinism?
A Hole Other Story![*]

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Abstract

In their modern classic "What Price Substantivalism? The Hole Story" Earman and Norton argued that substantivalism about spacetime points implies that general relativity is indeterministic and, for that reason, must be rejected as a candidate ontology for the theory. More recently, Earman has cottoned on to a related argument (in fact, related to a *response* to the hole argument) that arises in the context of canonical general relativity, according to which the enforcing of determinism along standard lines—using the machinery of gauge theory—leads to a 'frozen universe' picture (grounded in an absence of changes in values of general relativity's observables). *Prima facie* this would seem to land the anti-substantivalist in waters at least as deep as those that Earman and Norton argued troubled substantivalism. In this paper I introduce the argument in what I think are clearer terms than Earman's, and assess his treatment of the problem. For the most part I agree with Earman about the nature of the problem, but I find aspects of his discussion wanting, especially as regards his proposed ontology. I argue that ontological sense can be made of the changelessness if a structuralist stance is adopted with respect to a natural class of observables.

1 Introduction.

In a recent examination of the concept of gauge freedom in relation to the constrained Hamiltonian formulation of theories John Earman writes that

> ...one key motivation for seeking gauge freedom is to take up the slack that would otherwise constitute a breakdown of determinism: taken at face value, a theory which admits 'local' gauge symmetries is indeterministic because the initial value problem does not have a unique solution; but the apparent breakdown is to be regarded as merely apparent because the allegedly different solutions for the same initial data are to be regarded as merely different ways of describing the same evolution. Putting the point in slightly different terminology, the evolution of the genuine or gauge-invariant quantities (or 'observables') is manifestly deterministic. ([2003], p. 143)

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This 'gauge interpretation' has been applied to general relativity, and conceived of along these lines the hole argument (discussed in §2.2) is easily evaded: the kinds of quantities that are targeted by the hole argument—absolute quantities, such as the value of the metric field at some spacetime point (a quantity whose evolution is supposed to be indeterministic when spacetime is conceived along substantivalist lines)—are simply deemed to be unphysical. In Earman's words:

> The apparent breakdown of determinism is explained away by saying that what the proponent of the container view [the manifold substantivalist] took to be two different...histories is just a single history represented in two different ways in terms of gauge dependent quantities. ([2002a], p. 14)

The diffeomorphism symmetry, on which the hole argument rests, is, according to this this account, viewed as a gauge freedom of general relativity. In the framework of the constrained Hamiltonian formalism the diffeomorphisms of spacetime are encoded (albeit in a rather bastardized way, as we shall see) in the constraints. Crucially, (1) the Hamiltonian, the generator of motion, is a sum of the constraints; (2) the observables must commute with these constraints, and, therefore, with the Hamiltonian; (3) the constraints generate gauge transformations which, following Dirac, we are to understand as "transformations ... corresponding to no change in the physical state" ([1964], p.23). Given this, in evading the hole argument by adopting the gauge interpretation general relativity is thrown from the frying pan in to the fire. The reason is that the gauge interpretation seemingly implies that general relativistic worlds are 'frozen'

> ...since the Hamiltonian constraints generate the motion, motion is pure gauge, and the observables of the theory are constants of the motion in the sense that they are constants along the gauge orbits. Taken at face value, the gauge interpretation of GTR implies a *truly* frozen universe: not just the 'block universe' that philosophers endlessly carp about – that is, a universe stripped of A-series change or shifting 'nowness' – but a universe stripped of its B-series change in that no genuine physical magnitude (= gauge-invariant quantity) changes its value with time. (Earman [2003], p. 152)

The logic is pretty much ineluctable. To repeat: if diffeomorphism-invariance comprises a gauge freedom; the diffeomorphism-invariance is encoded in the constraints; the Hamiltonian is a sum of constraints; and the observables commute with the constraints (the Hamiltonian), then it follows that the observables do not change. Why? Because time-evolution is a diffeomorphism and the observables must be invariant under such transformations; they must commute with those changes (evolutions) in the data that are generated by the constraints. Yet such evolutions correspond to the unfolding of a gauge transformation. General relativity's observables do not, then, evolve at all: they are frozen in time!

Since the original motivation for the gauge interpretation was to "sop up the non-uniqueness in temporal evolution" (Earman [2003], p.143) the gauge resolution might seem to be about as effective as chopping off one's whole arm to cure a sore thumb! For one has apparently cured the non-uniqueness temporal evolution, indeterminism—and an indeterminism of a rather 'mild' and peculiar sort at that (but, nonetheless, in need of an explanation)—by disposing of temporal evolution *tout court*. It is, then, perhaps not surprising that many are deeply suspicious of the problem of the frozen formalism, and that some have suggested that we simply abandon the gauge interpretation, or at least restrict it (to 'non-temporal' diffeomorphisms, for example).

The central question I wish to consider in this paper is this: Is this eradication of time and change worth the restoration of determinism? I will argue that the price is right, providing we can come up with a sensible way of making conceptual sense of the frozen formalism, and come up with some way of making it compatible with the apparent profusion of change we certainly do see around us. In the final section of this paper I attempt to provide an ontology that satisfies this requirement. Before I get to that, I shall begin by reviewing the details of the hole argument, *qua* problem of indeterminism. I then explain how the gauge interpretation works and how it is supposed to resolve the difficulty posed by the hole argument, before going on to consider how this interpretation leads to the problem of change.

2 The Price of Substantivalism.

In this section we see how general covariance coupled with manifold substantivalism leads to a form of indeterminism (manifold substantivalism's supposed 'price') in the context of general relativity. Since this argument (the 'hole argument') is so well known, I shall be very brief, simply presenting the necessary details in capsule form. I conclude this section with a concise taxonomy of responses. I begin with a review of some basic background material.

2.1 Models, Symmetries, and General Covariance.

The hole argument is generally couched in the language of model theory. Recall that in the model theoretic view of scientific theories, one is interested in mathematical structures and in the set of models that are associated with them. In the context of general relativity, the relevant models are of the form $\langle \mathcal{M}, g, T \rangle$, where $\mathcal{M}$ is a 4-dimensional differentiable manifold, g is a Lorentz signature metric tensor representing the chronogeometrical structure of spacetime (and, of course, the gravitational field), and T is the stress-energy tensor representing the flow of energy and momentum through each point $x \in \mathcal{M}$. Einstein's field equations couple g and T so that the *dynamically possible* models of general relativity are those for which the pair $\langle g, T \rangle$ satisfies $R_{ab} - (1/2) g_{ab} R^c_c = 8\pi G T_{ab}$ everywhere on $\mathcal{M}$—where R_{ab} and R are the Ricci tensor and Ricci scalar constructed from g_{ab}, and G is Newton's constant.

These field equations are generally covariant. General covariance is a symmetry of the laws powered by diffeomorphism-invariance. It says that if $\langle \mathcal{M}, g, T \rangle$ is a dynamically possible model then so is $\langle \mathcal{M}, \phi^* g, \phi^* T \rangle$ ($\forall \phi \in \mathrm{Diff}(\mathcal{M})$, i.e. the group of diffeomorphisms of $\mathcal{M}$, where a diffeomorphism is a smooth one-to-one map between manifolds that possesses a smooth inverse). In other words, diffeomorphism-invariance tells us that the application of a diffeomorphism to a model gives us an equivalently structured model—one satisfying the field equations—back, and this underwrites general covariance.[1]

We will see in §4.1 how the adoption of the view that diffeomorphism-invariance is a gauge symmetry of general relativity, so that general covariance becomes a principle of gauge-invariance,

[1] Of course, I am skipping over many, many *nuances* here, and there are complications regarding the understanding of both general covariance and diffeomorphism invariance; however, they are not germane to the theme tackled in this paper.

leads both to a natural resolution of the hole argument and, when couched within the Hamiltonian formulation general relativity, to an even deeper conceptual problem known as the problem of 'frozen dynamics.' First, let us quickly review the essential details of the hole argument.

2.2 Earman and Norton's Hole Argument.

In this subsection I present the hole argument in a way that will make the transition to the subsequent sections much easier; none of the essential details of Earman & Norton's ([1987]) original argument are erased by taking this slight liberty.

Suppose we have a dynamically possible model of general relativity $\langle \mathcal{M}, g, T\rangle$—$\mathcal{M}$ is, for reasons that will become clear, a compact 4-manifold diffeomorphic to (i.e. with topology) $\Sigma \times \mathbb{R}$, with Σ a compact 3-manifold representing space and $\mathbb{R}$ is the real line representing (unphysical) time; g is a pseudo-Riemannian tensor on $\mathcal{M}$; and T will encode any structures representing matter (i.e. scalar fields, systems of particles, dust clouds, or whatever). The natural interpretation is that this model represents a certain possible world: a spacetime on which a gravitational field is defined in relation to a certain distribution of matter—this might be our own world, for example. Now, we know from the general covariance of the theory that we can apply a diffeomorphism $\phi \in \mathrm{Diff}(\mathcal{M})$ to $\mathcal{M}$, and thereby carry along the fields defined on it, to generate another dynamically possible model. The complete set of diffeomorphic models generated in this way gives an equivalence class (under $\mathrm{Diff}(\mathcal{M})$) of models known as an 'orbit' of $\mathrm{Diff}(\mathcal{M})$. Let's take two models from this orbit, $\langle \mathcal{M}, g, T\rangle$ and $\langle \mathcal{M}, d^*_{\mathcal{H}}g, d^*_{\mathcal{H}}T\rangle$, related by a certain type of diffeomorphism known as a 'hole diffeomorphism.' Let us now suppose that the matter distribution is such that there is a region in which $T = 0$, i.e. a matter-free 'hole.' Outside of the hole and on its boundary the two models agree (they clearly agree with respect to T *everywhere*). Since $\langle \mathcal{M}, g, T\rangle$ is a dynamically possible model, general covariance implies that $\langle \mathcal{M}, d^*_{\mathcal{H}}g, d^*_{\mathcal{H}}T\rangle$ is too. Now choose a slice Σ through spacetime such that the hole is to its future. Clearly the two models have the same initial data on this slice since the diffeomorphism was chosen so that the models agree everywhere but the hole. But they differ within the hole. For example, according to $\langle \mathcal{M}, g, T\rangle$ there might be a ripple of gravity at the point $x \in \mathcal{M}$, whereas according to $\langle \mathcal{M}, d^*_{\mathcal{H}}g, d^*_{\mathcal{H}}T\rangle$ the ripple occupies the point $y = d_{\mathcal{H}}(x) \in \mathcal{H} \subset \mathcal{M}$. Since the hole is to the future then it is apparent that Einstein's equations cannot uniquely determine the future behaviour of the data.[2] We thus have a stark violation of determinism: worlds described by identical initial conditions plus identical laws may diverge.

The key interpretive question is 'Does each model represent a distinct situation, or do the orbits as a whole represent a single situation?' Or is there, perhaps, some other relation between models and situations? Clearly, if the models *do* represent distinct situations then the full force of the indeterminism will have to be faced. Earman and Norton argue that substantivalists[3] must be thus committed to such a view, and they then use this commitment to hang the

[2] One needn't just put the hole to the future. One might put the hole to the past of the slice too; then, depending on whether we put the slice to the past or future of the hole, we see, respectively, that either the data to the future is *underdetermined* by the equations plus the initial data (so we get a case of indeterminism), or the data set on the chosen slice is *overdetermined* by the equations plus the past data.

[3] Earman and Norton direct their argument against *manifold* substantivalism, a view they say is the most defensible form of substantivalism in the context of modern spacetime theories. A manifold substantivalist is, roughly, one who

substantivalist with the noose of indeterminism. The commitment flows from the fact that the points of spacetime are taken to have their existence and identity independently of what's going on with the dynamical fields, therefore "shifting those fields [i.e. by applying d^*] will produce different physical states of affairs" (Earman [1989], p. 180). Substantivalists are thus led into an unacceptable indeterministic corner.[4]

Prima facie this indeterminism may not seem to present much of a problem. Since the models in question differ only in how the dynamical fields are spread out over the points of the manifold there will be no observable difference between them. The worlds represented by such models will, therefore, only differ haecceitistically, in virtue of which point plays which role in the worlds (models)—*cf.* Lewis ([1986], p. 221). However, if we are straightforward realists about our theory and interpret the points of the manifold as spacetime points *independently* of the fields, in the way the manifold substantivalist is supposed to, then this is, in principle, as serious a violation of determinism as one in which the non-uniqueness in temporal evolution manifested itself in terms of observable differences between the models.

2.3 Taxonomy of Interpretive Options.

Following Rynasiewicz [(1994)] we can distinguish three interpretive options as regards the relationship between the relevant diffeomorphic models (hole diffeomorphs) and possible worlds[5]:

1. Hole diffomorphs represent genuinely distinct worlds, so that models and worlds stand in a one-to-one correspondence.
2. Hole diffomorphs represent one and the same world, so that models and worlds are related in a many-to-one fashion.
3. Only some subset of the models represent genuinely possible worlds.

Rynasiewicz calls these "model literalism" [ML], "Leibniz equivalence" [LE] (following Earman and Norton), and "model selectivism" [MS], respectively. We can formalize these responses as follows:

- ML: $\forall d^*_{\mathcal{H}}$, $\langle \mathcal{M}, g, T\rangle$ and $\langle \mathcal{M}, d^*_{\mathcal{H}}g, d^*_{\mathcal{H}}T\rangle$ represent distinct physical possibilities.
- LE: $\forall d^*_{\mathcal{H}}$, $\langle \mathcal{M}, g, T\rangle$ and $\langle \mathcal{M}, d^*_{\mathcal{H}}g, d^*_{\mathcal{H}}T\rangle$ represent one and the same physical possibility.

adopts the view that spacetime is like a 'container' and is represented by the manifold. This position involves a realism about the points of the manifold, along with their differential and topological properties and relations, such that the points exist (with these properties and relations) *independently* of any 'contents' and they have their identities fixed independently of any contents.

[4]It is unacceptable to Earman and Norton since it is an indeterminism derived from metaphysics rather than physics. One should not, say they, be able to dictate such matters from the comfort of one's armchair. Note that some responses work by arguing that determinism is a *formal* property of theories, so that it is quite independent of such abstruse matters of interpretation such as the issue of the ontological status of spacetime points.

[5]I use 'possible worlds,' 'situations,' and 'possibilities' as if they were synonyms in this paper. They are not in general, of course, but nothing hinges on this here.

- MS: $\forall d^*_{\mathcal{H}}$, if $\langle \mathcal{M}, g, T\rangle$ represents a physical possibility then $\langle \mathcal{M}, d^*_{\mathcal{H}} g, d^*_{\mathcal{H}} T\rangle$ does not represent a physical possibility.

The crucial claim of Earman and Norton's argument against manifold substantivalism is that the adherent to such a conception of spacetime is necessarily committed to ML. This is false for two reasons. Firstly, there's more than one way to deny LE: one can choose ML or MS, where the latter does not lead to indeterminism.[6] Secondly, LE can in fact be endorsed by substantivalists—see for example Hoefer ([1996]). It is not my purpose to review these options here, we need simply to be aware of how the indeterminism is generated and how the endorsement of LE cures it. The gauge interpretation that we consider in the next section works in much the same way, though it gives us both a principled reason for adopting it and a formal framework for making sense of it.

3 The Gauge Interpretation.

To give an interpretation of a theory is to answer the questions "Under what conditions is this theory true?" and "What does it say the world is like?" (van Fraassen [1991], p. 242). In other words it amounts to providing an ontology for a theory; namely, a set of possible worlds that make the theory true. As Earman says, this involves "specifying which quantities the theory takes to be "observables" in the sense of genuine physical magnitudes" ([2002a], p. 15). In fact, the gauge interpretation provides only a partial answer to this problem: it says that the observables of general relativity must be gauge-invariant.[7] In other words, rather than allowing *any* real-valued functions $\mathcal{O}$ on the phase space to represent physical observables, one simply chooses those that are constant on gauge orbits, such that if x and y are gauge related states then $\mathcal{O}(x) = \mathcal{O}(y)$. Such quantities are said to be *gauge-invariant*. But we still need to know what *kinds* of quantity satisfy this characterization, what they are like, what kind of world they describe, and so on: this is the job of ontology. Hence, rather than providing us with a set of observables, the gauge interpretation imposes a restriction on what counts as observable (and, therefore on what form an ontology can take). There are a variety of options open to us as regards the filling in of the details. Before we get to these, the subject matter of §5, let us first gain some familiarity with the formalism within which the gauge interpretation works. We then see how this interpretation secures determinism. I present only the very barest of details, and, in the interests of clarity and brevity, I must ask the reader to take many of the results on trust.[8]

[6] Both Jeremy Butterfield ([1989]) and Tim Maudlin ([1988]) espouse variants of MS. Butterfield bases his version (a version he calls "One") on a general denial of haecceitism based in counterpart theory, while Maudlin bases his on an essentialism about the metrical properties of spacetime points.

[7] In the context of a gauge theory, only those quantities or variables that are left unchanged by gauge transformations are deemed observable and of physical significance.

[8] A more thorough presentation of this material can be found in Rickles ([2005b]). Belot and Earman give an excellent account of this material in their papers "From Metaphysics to Physics" ([1999]) and "Pre-Socratic Quantum Gravity" ([2001]). I am much indebted to this pair of articles.

3.1 The Hole Argument Gauged.

The standard position of physicists as regards the status of general covariance (e.g., as one finds in textbooks on general relativity) is to interpret the general covariance of the field equations as expressing the gauge freedom of general relativity; general relativity is then taken to be a gauge theory with gauge group $\mathrm{Diff}(\mathcal{M})$. One can find this viewpoint voiced very explicitly in the following well-quoted passage from Wald:

> If a theory describes nature in terms of a spacetime manifold, M, and tensor fields, $T^{(i)}$, defined on the manifold, then if $\Phi : M \to N$ is a diffeomorphism, the solutions $(M, T^{(i)})$ and $(N, \Phi^* T^{(i)})$ have physically identical properties. Any physically meaningful statement about $(M, T^{(i)})$ will hold with equal validity for $(N, \Phi^* T^{(i)})$...Thus, the diffeomorphisms comprise the gauge freedom of any theory formulated in terms of tensor fields on a spacetime manifold. In particular, the diffeomorphisms comprise the gauge freedom of general relativity. ([1984], p. 438)

The real root of the problem of indeterminism—so well exposed by the hole argument—according to this gauge theoretic account, is to be understood as follows:

> In a gauge theory, one cannot expect that the equations of motion will determine all the dynamical variables for all times if the initial conditions are given because one can always change the reference frame in the future, say, while keeping the initial conditions fixed. A different time evolution will then stem from the same initial conditions. Thus it is a key property of a gauge theory that *the general solution of the equations of motion contains arbitrary functions of time.* (Henneaux & Teitelboim [1992], p. 3)

And again, this time from Dirac's *Lectures on Quantum Mechanics*—the work which laid the mathematical foundations of the gauge interpretation:

> We have arbitrary functions of the time occurring in the general solution of the equations of motion with given initial conditions. These arbitrary functions of time must mean that we are using a mathematical framework containing arbitrary features, for example, a coordinate systems which we can choose in some arbitrary way, or the gauge in electrodynamics. As a result of this arbitrariness in the mathematical framework, the dynamical variables at future times are not completely determined by the initial dynamical variables, and this shows itself up through the arbitrary functions appearing in the general solution. ([1964], p. 17)

Conceived in this way, the indeterminism issuing from the hole argument is simply a natural consequence of the underdetermination resulting from the gauge freedom of the theory, something to be found in *any* gauge theory. Clearly, the argument strikes manifold substantivalism because that view is seen as underwriting a naive realism about the underdetermined future states, whereby diffeomorphic solutions (models; states) are held to be physically distinct and gauge dependent quantities (such as the value at some specific spacetime point of some metrical construction, say the scalar curvature $R(x)$) are thereby invested with physical reality. Again, I don't want to enter here into the debate between substantivalism and relationalism *vis-à-vis* the hole argument. I simply want to expose the mechanisms that allow the hole argument to function, and to show how the gauge interpretation avoids the problem of indeterminism. The simple reason can be discerned from Wald's remarks above: since the properties associated to diffeomorphic models are physically identical, one should adopt LE, and in this way preserve determinism. The gauge theoretical framework, understood as a subset of the theory of constrained Hamiltonian systems, provides the machinery for giving LE a firm technical grounding in such a way that it is the observables that do the work.[9]

Setting general relativity up as an Hamiltonian system allows us to expose the dynamical content of the theory; it lets us see general relativity as a theory about the evolution over time of the geometry of space. The great irony is, as we shall see, that the most natural interpretation seems to rule against any observationally significant evolution! But that story has to wait until the next section. Let us begin by developing the hole argument in the same framework as that in which the problem of frozen dynamics appears. This will allow us to quickly see the connections between the two problems, and understand how the latter flows from a particular way of dealing with the former. We can then go on to introduce the gauge interpretation itself, which corresponds to a particular way of defining the theory's observables.

The framework is that of constrained Hamiltonian systems.[10] On this approach we consider a 'splitting' of spacetime $\mathcal{M}$ into 'space' Σ and 'time' $t \in \mathbb{R}$, and then go on to construct the phase space relative to this splitting. We begin by choosing an arbitrary splitting of $\mathcal{M}$ by means of a diffeomorphism $\phi : \mathcal{M} \to \mathsf{S} \times \mathbb{R}$ (where S is a 3-dimensional manifold). This topology allows us to foliate $\mathcal{M}$ into slices (i.e. hypersurfaces), Σ_t (where $t \in \mathbb{R}$ will play the role of time). Cutting across many preliminary moves, we then construct a phase space Γ relative to the slicing, which we shall take to be the cotangent bundle $T^*\mathrm{Riem}(\Sigma)$ defined over the space of Riemannian metrics on Σ.[11] Points in phase space are then given by pairs (q_{ab}, p^{ab}), with q_{ab} a 3-metric on Σ (induced by the foliation) and p^{ab} a symmetric tensor on Σ (given by $\sqrt{\mid q \mid}(K^{ab} - K^c q_c^{ab})$, where $\mid q \mid$ is the determinant of q).

However, the diffeomorphism invariance of the spacetime covariant theory manifests itself as constraints—of the general form $\phi_m(q,p) = 0$, $m = 1, ..., M$—on (q_{ab}, p^{ab}), so that not all points in Γ represent dynamically possible states. Those states that *are* dynamically possible are given by points in the subspace (the "constraint surface" $\mathcal{C} \subset \Gamma$) on which the constraints are satisfied. The constraints of general relativity come in two flavours, the 'diffeomorphism' or 'vector' constraint $\mathcal{H}_a$ and the 'Hamiltonian' or 'scalar' constraint $\mathcal{H}_\perp$ defined by:

[9] Earman makes the claim that "the [constrained Hamiltonian] apparatus helps to clarify the classic form of the dispute about the ontological status of spacetime and to make precise the connection of this dispute with the fortunes of determinism" (2002, p. 13). Earman is right that this apparatus connects up to the taxonomy of §2.3, as I show below. However, it isn't clear that this taxonomy itself carves up distinct ontological positions about spacetime for the reasons I gave in §2.3.

[10] See Dirac ([1964]) for a very clear general exposition, and Arnowitt *et al.* ([1962]) for an early application to general relativity. The constrained systems 'bible' is Henneaux & Teitelboim ([1992]).

[11] Here I follow the seemingly standard philosophers' procedure of couching my discussion in terms of the metric variables approach. However, I should point out that the canonical approach based on these variables is now largely defunct and has been replaced by the connection representation (using Ashtekar variables: *cf.* Asktekar [1986]) and the loop representations (a nice introduction is Ashtekar & Rovelli [1992]). These result in 'simpler' expressions for the constraints and solutions for the Hamiltonian constraint (none were known for the metric variables!). The justification for sticking with the metric variables is simply that the hole argument and the problem of change afflicts any canonical approach and takes on much the same form regardless of which variables one chooses to map out the phase space with.

$$\mathcal{H}_\perp = \frac{1}{\sqrt{\mid q \mid}}[q_{ac}q_{cd} - \frac{1}{2}q_{ab}q_{cd}]p^{ab}p^{cd} - \sqrt{\mid q \mid}R \tag{1}$$

$$\mathcal{H}_a = -2q_{ac}\nabla_b p^{bc} = 0 \tag{2}$$

These two constraints allow data to be evolved by taking Poisson brackets. Thus, in general, for an observable $\mathcal{O} \in C^\infty\Gamma$, $\{\mathcal{O}, \mathcal{H}_a\}$ changes it by a Lie derivative *tangent* to Σ and is generated by a spatial diffeomorphism, while $\{\mathcal{O}, \mathcal{H}_\perp\}$ changes $\mathcal{O}$ in the direction *normal* to Σ. The Hamiltonian for the theory is given by $\mathcal{H} = \int_\Sigma d^3x \ N^a\mathcal{H}_a + N\mathcal{H}_\perp$, where N^a and N are Lagrange multipliers called the shift vector and lapse function respectively. The dynamics are thus entirely generated by (first-class) constraints.[12] Dirac's 'conjecture' for such constraints is that they generate gauge transformations; *viz.* "transformations ... corresponding to no change in the physical state, are transformations for which the generating function is a first class constraint" ([1964], p. 23). This gives us a 'taster' of the next section's business, for the implication is that the evolution of states (i.e. the motion) is pure gauge!

The hole argument can be reproduced in phase space terms as follows (*cf.* Belot & Earman ([2001], p. 228)). Firstly, we note that the constraints generate Hamiltonian vector fields $X_\mathcal{H}$ on the constraint surface $\mathcal{C}$, such that the vectors are null with respect to the geometry of $\mathcal{C}$ (given by a presymplectic form)—these are the 'gauge orbits.' Now, consider two points (that is, two states of the gravitational field) $a = (q, p)$ and $b = (q', p')$ lying in the same gauge orbit $[a] = [b]$ on $\mathcal{C}$, and such that they can be joined by an integral curve of the *diffeomorphism* constraint. That means there is a diffeomorphism $d : \Sigma \to \Sigma$ such that $d^*p = p'$ and $d^*q = q'$, implying that a and b agree on the geometrical structure of Σ. Since they differ by a diffeomorphism, we know that difference amounts to a disagreement over which points of Σ play which roles—i.e. as to the geometrical properties assigned to the points $x \in \Sigma$. Hence, the diffeomorphism constraint generates gauge transformations that act by permuting the points of a spatial slice, simply rearranging their geometrical properties. (The Hamiltonian constraint is more complicated since it generates transformations normal to an initial slice that can roughly be understood as pushing data 'forward in time,' we save this for the next section).

This relates back rather nicely to the taxonomy presented in §2.3. We can simply recast the options in the following terms. Given a pair of states $a = (q, p)$ and $b = (q', p')$, such that $d^*p = p'$ and $d^*q = q'$ (where $d \in \mathrm{Diff}(\Sigma)$), so that $[a] = [b]$:

- ML: $\forall d \in \mathrm{Diff}(\Sigma)$, a and b represent distinct physical possibilities.
- LE: $\forall d \in \mathrm{Diff}(\Sigma)$, a and b represent one and the same physical possibility.
- MS: $\forall d \in \mathrm{Diff}(\Sigma)$, if a represents a physical possibility then b does not represent a physical possibility.

[12] A constraint ϕ_k is said to be "first-class" if its Poisson bracket with any other constraints is given as a linear combination of the constraints: $\{\phi_k, \phi_i\} = C^j_{ki}\phi_j \forall i$. The appearance of such constraints in a theory implies that the dynamics is restricted to the constraint surface.

It is obvious that Earman and Norton's manifold substantivalist will be forced into considering the different points along a gauge orbit as representing distinct states of affairs since the points of space will have different geometrical properties according to each such state. For example, according to a it is the point x that has the maximal scalar curvature, whereas according to b it is the point $y = d(x)$ (where $x, y \in \Sigma$). If this is the case, then it is indeed true that general relativity is indeterministic for it can, at best, determine the geometrical structure of Σ, it cannot determine how this structure is distributed over the points. Thus, as is the case in any gauge theory, for some initial state, there will generally be multiple, though gauge-related, evolutions compatible with the equations of motion. The indeterministic conclusion follows only if we accept the premise that a and b represent distinct physically possible worlds, i.e. ML. The gauge interpretation adopts LE and it does so by endorsing the gauge invariant definition of observables associated to the constrained Hamiltonian formulation of general relativity. In this way, a principled avoidance of the hole argument's indeterminism is effected. Let us now look at this how this achieved by looking at how observables are defined in the constrained Hamiltonian formalism (and, by comparison, in canonical theories in general).

3.2 Determinism Regained.

The perspective of most physicists is that there is a direct connection between constrained systems and gauge systems: certain constraints (the first-class ones) generate gauge transformations on the (constrained) phase space. In particular, it is often assumed that any constraint surface contains 'redundancies,' and these are understood in terms of gauge freedom.[13] Domenico Giulini exposes these connections very clearly in the following passage:

> For systems with gauge redundancies the original phase space P does not directly correspond to the set of (mutually different) classical states. First of all, only a subset $\hat{P} \subset P$ will correspond to classical states of the system, i.e. the system is *constrained* to $\hat{P}$. Secondly, the points of $\hat{P}$ label the states of the system in a redundant fashion, that is, one state of the classical system is labeled by many points in $\hat{P}$. The set of points which label the same state form an orbit of the group of gauge transformations which acts on $\hat{P}$. 'Lying in the same gauge orbit' defines an equivalence relation (denoted by '$\sim$') on $\hat{P}$ whose equivalence classes form the space $\overline{P} := \hat{P}/\sim$ which is called the *reduced phase space*. Its points now label the classical states in a faithful fashion. ([2003], p. 32)

The problem of indeterminism in the hole argument was found to be related to these "gauge redundancies," where the equivalence relation $\sim$ is given by the diffeomorphisms. However, merely clarifying how the indeterminism arises has not yet provided us with a solution; we saw in the previous subsection, that much the same interpretive options face us again in this context.[14] The following does secure determinism. If we have it that an initial data set can evolve onto multiple data sets *while respecting the equations of motion of the theory*, then the

[13] Indeed, Earman ([2003], p. 153) speaks of the constrained Hamiltonian formalism as an "apparatus ... used to detect gauge freedom."

[14] Giulini mentions in the above quote that we can form the reduced phase space, and this would indeed eliminate the indeterminism, by eliminating the gauge freedom. But the construction of the reduced space is often complicated and still leaves the task of explicating what the theory is about untouched. In other words, the observables must be found before we can label the points of the reduced space.

gauge theoretic approach will say that the multiple data sets are gauge related. In other words, as represented on the phase space, they will lie in the same gauge orbit and can be mapped to each other by means of a gauge transformation. The next step, the gauge interpretation, secures determinism within this setup: we say that only those variables that are *constant* on the gauge related data sets are 'true' or 'physical' observables—these are the gauge invariant 'Dirac observables.' In other words, Dirac observables will give the same value along a gauge orbit, and there will only be differences between Dirac observables if there is a difference in gauge orbits. Let us spell this out in a little more detail.

In general, the *physically measurable* properties (the observables) of an Hamiltonian system are described by simple functions $\mathcal{O}(q,p) : \Gamma \to \mathbb{R}$ in terms of a canonical basis (a set of canonical variables), with position q_i and momenta p_i, satisfying the Poisson bracket relations:

$$\{q_i, p_j\} = \delta_{ij} \tag{3}$$

Systems described in such terms are, in general, rather simple to interpret: each point (p, q) in the phase space represents a distinct physically possible state that the system can occupy. In such 'unconstrained' systems, there is a *unique* curve—determined by the Hamiltonian function for the system—through each point of phase space, so that a simple one-to-one understanding of the representation relation is possible that does not lead to indeterminism or underdetermination. However, 'weakening' the geometry of the phase space, and moving to gauge systems, puts pressure on this simple direct interpretation precisely because determinism breaks down and the canonical variables are underdetermined—and a suitably chosen initial slice will, of course, lead to hole argument style indeterminism. When one considers systems with redundant variables and symmetries the formulation contains constraints holding between the canonical variables. Such constraints are a byproduct of the Legendre transform taking one from a Lagrangian to a Hamiltonian description of a system.[15]

The first change to note in the shift from a Hamiltonian system to a constrained Hamiltonian system is that the former's 'symplectic form' ω is replaced by a 'presymplectic form' σ, so that the phase space $\mathcal{C}$ of a gauge system inherits its geometrical structure from this. The presymplectic form induces a partitioning of the phase space into the gauge orbits mentioned previously, such that each point $x \in \mathcal{C}$ lies in exactly one orbit $[x] \subset \mathcal{C}$. In this case, given the weaker geometrical structure induced by the presymplectic form, the Hamiltonian is not able to determine a unique curve through the phase points. Instead, there are infinitely many curves through the points. However, the presymplectic form *does* supply the phase space with sufficient structure to determine which gauge orbit a point representing the past or future state will lie in. Hence, for two curves $t \to x(t)$ and $t \to x'(t)$ intersecting the same initial phase point $x(0)$, we find that the gauge orbit containing $x(t)$ is the same as that containing $x'(t)$: i.e., $[x(t)] = [x'(t)]$.

[15] The idea of gauge freedom manifests itself at the level of the Lagrangian formalism too. A theory's action principle $\delta \int \mathcal{L}(q, \dot{q}) dt = 0$ allows us to derive its Euler-Lagrange equations. Sometimes—in general relativity, for example—these equations will be non-hyperbolic, they can't be solved for all accelerations. This results in a *singular* Lagrangian, revealing itself in the singularity of the Hessian matrix $\partial^2 \mathcal{L} / \partial \dot{q}^k \partial \dot{q}^h$. This implies that when we Legendre transform to the Hamiltonian formulation, the canonical momenta are not independent, but will satisfy a set of relations (constraints) related to the identities of the Lagrange formalism. See Earman ([2003], pp. 144-145) for a clear explanation of this material.

Now, if we treat observables of constrained, gauge systems in the same way as for unconstrained systems—i.e. as a real-valued function $\mathcal{O}(q,p) : \mathcal{C} \rightarrow \mathbb{R}$ on the phase space—we face an obvious problem: given that the future phase point of an initial phase point is underdetermined, it will be impossible to uniquely predict the future value of the observables. Hence, there appears to be a breakdown of determinism. The initial-value problem does not appear to be well posed, as it is for standard Hamiltonian systems. The reason is clear enough: there is a unique curve through each phase point in a Hamiltonian system but there are infinitely many curves through the phase points of a gauge system. The trick for restoring determinism and recovering a well posed initial-value problem is to be *restrictive* about what one takes the observables to be. Rather than allowing *any* real-valued functions on the phase space to represent physical observables, one simply chooses those that are *constant* on gauge orbits, such that if $[x] = [y]$ then $\mathcal{O}(x) = \mathcal{O}(y)$. Such quantities are said to be *gauge-invariant*; these are our Dirac observables.

The initial-value problem is well posed for such quantities since for an initial state $x_{t=0}$, and curves $x(t)$ and $x'(t)$ through $x_{t=0}$, $\mathcal{O}[x(t_1)] = \mathcal{O}[x'(t_1)]$. These will be the 'first-class' variables, where a dynamical variable $\mathcal{O}$ (still a function of the ps and qs) is first-class iff it has *weakly vanishing* (i.e. on the constraint surface) Poisson bracket with *all* of the first-class constraints:

$$\{\mathcal{O}, \phi_m\} \approx 0,\ m = 1, ..., M. \tag{4}$$

In other words, Dirac observables are constant along gauge orbits generated by the constraints, and there will only be differences between Dirac observables if there is a difference in gauge orbits. This view is motivated by the fact that the gauge related data sets will be *indistinguishable* from the point of view of the theory's laws. Relating this back to the hole argument, note that quantities that are gauge *dependent* are such that they are altered by gauge transformation. Recall that the constraints generate such transformations, and one of these generates diffeomorphisms of the spatial manifold. Hence, any quantity that is dependent upon the manifold is thereby rendered gauge dependent. This rules out precisely those quantities that the hole argument worked against. Thus, there is no indeterminism as far as Dirac observables are concerned, for they treat each gauge related state as one and the same, thus encoding LE.[16] However, this way of restoring determinism has a heavy price, as we chart in the next section.

4 The Price of Determinism: A Deeper Hole.

The hole argument showed how a certain interpretation, ML, led to an indeterminism in the context of general relativity, in the sense that models with the same set of initial data are able

[16]Note that we haven't had to 'eliminate' degrees of freedom from the phase space here; by choosing the class of observables in this way, the gauge symmetries are retained. However, it is possible to shift to the 'reduced phase space' by factoring out these symmetries, so that a 'smaller' space is obtained. In this case, the standard definition of an observable, as a real-valued function on the space, is equivalent to the Dirac definition. More technically, gauge-invariant observables naturally induce a function $\mathcal{O}_{[x]} : \Gamma_{red} \rightarrow \mathbb{R}$ (under a submersion map π^*), which is just to say that such functions $\mathcal{O}_{\Gamma_{red}}$ on the reduced phase space are automatically gauge-invariant, corresponding as they do to gauge-invariant functions on the constraint surface. The fact that we needn't eliminate gauge from the phase space to achieve a gauge invariant account informs much of Rovelli's response to the problem of change—see §5.2.

to evolve (under suitable diffeomorphisms) onto different sets of (gauge related) data at a later time. ML is in trouble since it has to say that the models represent distinct possibilities even though these differences are opaque to the laws of the theory and are solely haecceitistic. The gauge interpretation restores determinism by restricting what is observable to gauge invariant quantities, so that local quantities like the Ricci scalar $R(x)$ at a point $x \in \mathcal{M}$ are not what the theory is about. Since the non-uniqueness in temporal evolution concerns gauge related data sets, gauge invariant quantities will be entirely insensitive to such differences and will evolve uniquely into the future. The condition for being a physical observable O (where, you will recall, $\mathcal{H}$ is the Hamiltonian formed from the constraints) is:

$$\mathsf{O} \in \mathcal{O}_i \text{ iff } \{\mathsf{O}, \mathcal{H}\} \approx 0 \tag{5}$$

From this gauge invariant vantage point, the 'frozen formalism' problem (determinism's 'price') is well nigh ineluctable. I mentioned above that the dynamics is generated by constraints; or, in other words, the dynamics takes place on the constraint surface, and evolution is along the Hamiltonian vector fields $X_{\mathcal{H}}$ generated by the constraints on this surface (i.e. along the gauge orbits). For any observable O (a function of the canonical variables), the time-evolution is given by $\dot{\mathsf{O}} = \{\mathsf{O}, H\}$. In a constrained system, the dynamics takes place on the constraint surface, so that the equality becomes a 'weak equality'. But on the constraint surface the observables must commute with the constraints, as in eq.(5), and the Hamiltonian is a sum of the constraints. Therefore, the observables are constants of the motion: $\frac{d\mathsf{O}}{dt}(q(t), p(t)) \approx 0$ (where t is associated to some foliation given by a choice of lapse and shift). Since such quantities are supposed to form the basis of the ontology of the theory, it follows that no genuine physical quantities change over time. This is the problem of frozen dynamics, or the frozen formalism. Let us unpack it a little more.

4.1 The Frozen Formalism.

From the previous section, we can see that we have the following setup to work with. The spacetime manifold $\mathcal{M}$ is a background structure with the topological structure $\mathcal{M} = \mathbb{R} \times \Sigma$, with Σ a spatially compact 3-manifold. The configuration space $\mathcal{Q}$ for the gravitational field is the space of Riemannian metrics $\mathrm{Riem}(\Sigma)$ on Σ.[17] The (extended) phase space Γ is the cotangent bundle, $T^*\mathcal{Q}$, over $\mathrm{Riem}(\Sigma)$. And the dynamics takes place on the constraint surface $\mathcal{C} \supset \Gamma$ given by $\mathcal{H}_\perp = 0$ and $\mathcal{H}_a = 0$, where $\mathcal{H}_\perp$ is the Hamiltonian constraint and $\mathcal{H}_a$ is the momentum constraint, and where the 'full' Hamiltonian is $\mathcal{H} = \int_\Sigma d^3x \ N^a\mathcal{H}_a + N\mathcal{H}_\perp$. The observables of the theory have vanishing Poisson brackets with these constraints and the Hamiltonian.

In the constrained Hamiltonian version of general relativity described in the previous section, the constraints were shown to generate gauge transformations. This means that phase points

[17] As I mentioned above, there are alternative formulations according to which a connection is used as a configuration variable instead of the metric. Most physics research now carried in the canonical approach to general relativity is discussed mostly in the connection version since it lends itself better to quantization (leading, after another shift of variables, to the loop representation and loop quantum gravity). However, the conceptual problems discussed in this paper are quite insensitive to which variable is chosen: so long as we stick to spacetimes with the specified topology, all are targets.

get shunted along the orbits of the gauge group under their action, where states represented by phase points lying on the same gauge orbit are observationally indistinguishable.[18] Thus, any transformations that are instigated by the constraints will, from this point of view, result in *equivalent* physical states; the different phase points represent one and the same physical state differently described. Since this ability to redescribe the state is unphysical, it should not be the case that the theory's observables should depend upon the specific descriptions used. That is, an observable must be the same (i.e. give the same value) for all points on a gauge orbit; it must be insensitive to any differences between them, for the differences are merely the result of gauge freedom in the mode of representation. Sensitivity should be at the level of whole gauge orbits, rather than their elements. We can sum this up more formally by saying that the Poisson bracket of an observable with the constraints should vanish: observables must commute with the constraints. Therefore they must commute with the Hamiltonian.

The problem we have is that in (Hamiltonian) general relativity all of the physical content, the dynamics, is given by the constraints. Applying the gauge interpretation to this formalism leads to the view that the observable $\mathcal{O}_i$ are those functions satisfying $\{\mathcal{O}_i, \mathcal{H}(x)\} = 0$. Since one of these constraints, the Hamiltonian constraint, generates the dynamical evolution of the canonical variables from one hypersurface to the next, the commutation condition implies that dynamical variables must take on the same value on each hypersurface: the *dynamical* variables must be *constants* of the motion. However, intuitively, observables cannot be constants of the motion; how do we explain the appearance of a changing world if they are? As Earman rightly pointed out ([2003], p. 152), this is incompatible with a B-series conception of change since that account requires that there are things that have different incompatible properties at different times. Hence, it is much more radical than the claims of 'eternalism' that are made on the basis of the lack of becoming in special relativity's Minkowski spacetime.

4.2 Taxonomy of Interpretive Options.

This subsection aims to provide a rough roadmap of the options we have in responding to the problem of change. The account I give is not intended to be exhaustive, and I consider only those responses with relevance to the issues considered in this paper. Broadly, then, there are three basic stances we can adopt:

1. Accept the gauge interpretation along with the consequences.

2. Restrict the gauge interpretation in some way.

3. Scrap the gauge interpretation.

Earman is very much taken with the Hamiltonian formalism, and believes that the frozen dynamics is something that must be accommodated by any sound interpretation of general relativity. That is, he opts for response 1. I agree. Tim Maudlin and Karel Kuchař do not. I consider option 1 in the next section, in the remainder of this subsection we consider options 2 and 3, as defended by Kuchař ([1992], [1993]) and Maudlin ([2002]) respectively.

[18]Note that this intended to be a very strong sense of "observationally indistinguishable." It is not that there is no procedure that *we* could use to detect a difference; rather, there is no procedure imaginable that could achieve this.

Both Maudlin and Kuchař are in agreement that something has gone awry in the gauge interpretation. Maudlin says we should abandon it altogether; Kuchař, however, has his eye firmly on quantization, and thinks that we should retain the gauge interpretation on account of its utility in this context, but that we should restrict it to $\mathrm{Diff}(\Sigma)$, and the transformations generated by $\mathcal{H}_a$. In other words, Kuchař thinks that we should *restrict* the gauge interpretation to the diffeomorphism constraint, and not view the Hamiltonian constraint as a generator of gauge. Maudlin goes further; he thinks the frozen formalism involves "some Alice-in-Wonderland logic" ([2002], p. 13). As he explains:

> Any interpretation which claims that the deep structure of the theory says that there is no change at all – and that leaves completely mysterious why there *seems* to be change and why the merely apparent changes are correctly predicted by the theory – so separates our experience from physical reality as to render meaningless the evidence that constitutes our grounds for believing the theory. ([2002], p. 12)

Prima facie, Maudlin is quite right, of course; on a surface reading of the formalism there appears to be no scope for change, therefore, given the apparent existence of change, something is wrong. However, there *is* scope for interpreting the formalism so as to introduce change, as I show in §5. We can, on these interpretations, say why the surface reading is "yielding nonsense," as Maudlin puts it (*ibid.*). The answer is related to the gauge invariant response to the hole argument (the response that is supposed to cause the problems of change in the first place): only change with respect to the manifold is ruled out, if we focus on those quantities that are independent of the manifold we can restore change by considering the 'evolving' relationships between these quantities.[19]

In his response to Earman ([2002b]), Maudlin claims to be against the gauge interpretation, yet he rather oddly appears simply to regurgitate what is the gauge theoretical lesson of general relativity; namely, that local quantities cannot be genuine observables. Thus—referring to the problem of frozen dynamics as "the observables argument"—he writes that

> ...the Observables Argument gets any traction only by considering candidates for observables (values at points of the bare manifold) which are neither the sorts of things one actually uses the GTR to predict nor the sorts of things one would expect – quite apart from diffeomorphism invariance – to be observables. ([2002], p. 18)

Thus Maudlin charges Earman with making "a bad choice of logical form of an observables" ([2002], p. 18). However, Maudlin has in fact simply accepted the gauge-invariance interpretation seemingly without realizing it. That values at the points of the bare manifold are not the things one predicts cannot be separated from the issues of diffeomorphism invariance, for it's precisely this that results in the problems for local field quantities that we have seen in the hole argument. Thus, we can agree, and Earman will agree, that the observables argument (the frozen formalism), like the hole argument, gets off the ground by considering the 'wrong' type of observables, but this is to adopt a substantive response that buys into the gauge theoretical interpretation! Nevertheless, the *right* observables will be frozen. Indeed, the observables

[19] In §5.3 I suggest that we should view the correlations themselves as observables, following Rovelli's evolving constants of motion interpretation; strictly speaking, on this account there is no evolution as such, one merely has the 'appearance' of evolution.

he suggests as kosher are of the 'frozen' type—e.g. "the amount by which light from the sun is redshifted when it reaches the Earth" ([2002], p. 13). Since there is no dependence on the manifold, this will be gauge invariant and, therefore, will not change from slice to slice. Indeed, another example Maudlin gives of a good observable bears striking similarity to Earman's chosen type (the "coincidence quantities" that we shall meet in the next section). Thus, he writes that "[w]hat we *can* identify by observation are the points that satisfy definite descriptions such as "the point where these geodesics which originate here meet", and against *these* sorts of [local] quantities Earman's diffeomorphism argument has exactly zero force" (*ibid.*). Presumably Maudlin has made a slip here, for points are not quantities; yet if we were to define a quantity, like scalar curvature, at the so-defined point, then we have a quantity that fits the bill. Indeed so, but here Maudlin is essentially *gauge-fixing* spacetime points and then constructing gauge invariant quantities by attaching them to the physically defined points—the reasoning is that for some quantity 'F', physical object 'thing', and space point x: $F(\text{thing})$ is gauge invariant but $F(x)$ is not (*cf.* Isham [1993], p.15). If Maudlin is willing to go this far, then why not allow that change is accounted for with just such observables: the evolution and change concerns the relations between things or quantities, rather than the having and losing of properties at times? One can form of chain of values for F by using the values of 'thing' as the 'ticks' of a clock—this is essentially what Rovelli proposes (see §5). Moreover, all of this is perfectly possible in the context of the Hamiltonian formulation, the "surface reading" simply doesn't go deep enough.

Rather than ruling the gauge interpretation out *tout court*, Kuchař maintains that we should make an ontological distinction between the constraints. The spatial constraint should be viewed as a generator of gauge transformations, but not so the Hamiltonian constraint, for that "tells us how the state evolves" ([1993], p. 21). The simple reason is that if we do choose to view the Hamiltonian constraint as generating gauge, then we would face the frozen formalism picture, something Kuchař views as a *reductio* of the 'unrestricted' gauge interpretation:

> [$\mathcal{H}_\perp$] generates the dynamical change of data from one hypersurface to another. The hypersurface itself is not directly observable, just as the points $x \in \Sigma$ are not directly observable. However, the collection of the canonical data $(q_{ab}(1), p^{ab}(1))$ on the first hypersurface is clearly distinguishable from the collection $(q_{ab}(2), p^{ab}(2))$ of the evolved data on the second hypersurface. If we could not distinguish between those two sets of data, we would never be able to observe dynamical evolution. ([1993], p. 20)

> I would say that the state of the people in this room now, and their state five minute ago should not be identified. These are not merely two different descriptions of the same state. They are physically distinguishable situations. (Kuchař in Ashtekar & Stachel (eds.) [1991], p. 139)

This is simply the problem of frozen dynamics again: if the Hamiltonian constraint generates gauge and observables must commute with it then observables must be constants of motion, but, says Kuchař, "if we could observe only constants of motion, we could never observe any change" (*ibid.*). On this basis he distinguishes between two types of variable: *observables* and *perennials*. The former class are dynamical variables that have vanishing Poisson bracket with the diffeomorphism constraint but that *do not* commute with the Hamiltonian constraint; while the latter are dynamical variables that commute with both types of constraint (i.e. with the full Hamiltonian). Kuchař's key claim is that one can observe dynamical variables that are *not* perennials, and that we *have* to in order to observe change. Thus, Kuchař restricts the gauge interpretation to the spatial constraint (we might call this move 'Kuchař restriction'). But there is a problem. Kuchař's claim that observables should not have to commute with the Hamiltonian constraint leads him to the *internal time* strategy, where an attempt is made to construct a time variables T from the classical phase space variables. This strategy conceives of general relativity (as described by Γ) as a parametrized field theory. The idea is to find a notion of time hidden amongst the phase space variables. I shan't go into the technical details, which are in any case directed at quantization; suffice it to say, however, that general relativity cannot be conceived in this way since it is not a parametrized field theory. To see that this is so, we need to test whether or not the identification between the phase space Γ of general relativity and the phase space Υ of a parameterized field theory goes through. The proposal requires that there is a canonical transformation $\Phi : \Upsilon \rightarrow \Gamma$ such that $\Phi(\overline{\Upsilon}) = \overline{\Gamma}$. However, there can be no such transformation because $\overline{\Upsilon}$ is a manifold while $\overline{\Gamma}$ is not (*cf.* Torre [1993]). Hence, there are serious, basic technical issues standing in the way of this approach. A further problem is that Kuchař essentially begs the question against the unrestricted gauge interpretation by saying that in order to observe change observables mustn't be constants of the motion, for there are proposals that *are* able to account for observed change within this framework. We turn to these in the next section.

4.3 A Cost *Versus* Benefit Analysis.

As Earman points out, not everyone is taken with the constraint Hamiltonian formalism and the gauge interpretation:

> Some philosophers and physicists [i.e. Maudlin and Kuchař] have found this "frozen dynamics" so bizarre that they think it shows that the constraint apparatus, which is otherwise so fruitful and successful in other domains, has gone haywire when applied to GTR. ([2002a], p. 14)

In his defense Earman notes that we have often had to accommodate in the scientific image ideas "that initially shock our intuitions" (ibid.). That might be so, but in those other cases it has always been necessary to find some way of making sense of the manifest image out of the scientific image. We know that the frozen picture, the alleged price of determinism, is one cost with the gauge interpretation, but what exactly do we gain by invoking the constrained Hamiltonian formalism, along with Dirac's treatment of the constraints? The most obvious benefit, of course, is the recovery of determinism from the jaws of gauge dependency.[20] But Earman gives us another.

Earman proposes "that one way of testing an interpretational stance for classical GTR is to see how well the stance lends itself to promoting a marriage of GTR and quantum physics that

[20]In fact, it isn't entirely clear that the restoration of determinism *is* at the root of the problem of change. After all, we identify gauge related states, by choosing gauge invariant observables, because those states are thought of as being *indistinguishable* (at least relative to the laws of the theory). Hence, it is possible that a more general belief in anti-haecceitism might underpin the frozen formalism too. However, a discussion of this point would take me too far off track, so I leave it hanging in the air.

issues in a successful quantum theory of gravity" ([2002a], p.16). I would, however, contest this as a test of interpretational fitness. To my mind, one should seek an interpretation that is consistent and cogent in itself, an 'internal' criterion of interpretive fitness. If an interpretation of some classical theory satisfies this, then it will most likely lead to a successful translation into the quantum realm. Indeed, if it has conceptual problems at the classical level, then more likely than not, these will be reflected in the quantum theory. This is indeed shown to be the case with the present frozen formalism example in the context of the canonical quantization approach known as loop quantum gravity in which the gauge interpretation is adopted. In loop quantum gravity, and many other canonical quantizations, the frozen formalism problems leads to a quantum variation known as "the problem of time," according to which the quantum states of the theory do not evolve on account of the zero Hamiltonian. Even on Earman's own terms this seems like a bad test in the present scenario, for he appears to equivocate. He says elsewhere that we should look at canonical general relativity and the frozen formalism problem on its own ground, and should consider them as separate and serious independently of quantum considerations (*cf.* e.g. ([2002b], p.6)). If this is all there is to prop up the benefits side, then we are in danger of a collapse to Maudlin's 'anti-gauge' perspective.

Certainly this is all Earman gives us, but there is more to be said. For one, in giving general relativity a gauge interpretation we are forging a connection with our other best physical theories; that is, we achieve a 'formal unity' with these other theories, i.e. those involved in the standard model of particle physics. Furthermore, and related to the previous point, it fits a successful and well established research programme going back to Dirac. However, most importantly (at least as far as I am concerned) it meets certain *expectations* for what a background independent theory (one without a *fixed* metric or connection on spacetime) like general relativity should look like. Thus, we should expect such problems as the frozen formalism to arise given the independence from the manifold that is suggested by diffeomorphism invariance. My view is that the problem of change is a bomb we should learn to stop worrying about and love.

5 Correlational Ontologies.

As I mentioned in §3, the gauge interpretation is, as it stands, not much of an interpretation: it does not provide an ontology, though it does place restrictions on what form an ontology might take. It tells us, for example, that there can be no local observables, observables given as the value of a field, or some quantity or other, at an (independently specified) manifold point. The reason is that points are not diffeomorphism invariant, so neither are quantities defined at points—that for me is the lesson of the hole argument. At the core of the gauge interpretation is, then, simply a formal characterization of the observables (namely that of Dirac): sensitivity at the level of gauge orbits, but not their elements. There are several types of quantity that satisfy the basic characterization. For example, there are very 'non-local' or 'global quantities,' such as the spacetime volume of a compact universe, or the spacetime average of some scalar function of the dynamical fields. Or there are 'relational' or 'coincidence' quantities that are of the form '$F(thing)$' or, more complicatedly, 'value of F when and where value of G is n' (where F and G are gauge dependent quantities) rather than '$F(x)$,' '$F(t)$,' or '$F(x,t)$.' I shall call these latter complex observables 'correlational observables.' The idea is that the elements of the correlations are not observables, but the correlation itself is an observable. For example, the values of the four invariant scalar fields constructed from the metric taken at a certain point of the manifold is not an observable, nor is the value of the electromagnetic field at a certain point of the manifold. However, the correlation formed by the electromagnetic field's having a certain value when and where the four scalar fields take on a certain value is an observable. The question is, How are we to make ontological sense of this? The proposals I consider in this section, of John Earman and Carlo Rovelli, are intended to answer this question in such a way as to explain the appearance of change. I argue that neither succeeds in providing a satisfactory account: both fall prey to the same problem, concerning the nature of the relation between the correlations and the elements of the correlation. I then show how a simple structuralist gloss on their basic position resolves the problem in a flash.

5.1 Earman's D-Series of Coincidence Events.

Earman's response to the problem of frozen dynamics is to argue that although general relativity is incompatible with both the A-series and B-series conceptions of change, it *is* compatible with a revised conception change. To this end he introduces what he refers to as a "D-series" ontology consisting of a "time ordered series of occurrences or events, with different occurrences or events occupying different positions in the series" ([2002b], p. 3). These are events formed from the coincidence quantities familiar from Einstein's 'point-coincidence argument' (see Howard [1999]), though it is rather more general. As an example he gives the "the *Komar state*," represented by the functional relationship (the correlation) $g^{\mu\nu}(\phi^{\lambda})$ involving the coincidence such that the metric has a certain value where the four scalar invariants of the Riemann tensor take on a certain value ([2002b], pp. 13-4). Earman writes that "[t]he occurrence or non-occurrence of a coincidence event is an observable matter [in the technical sense of *observable*]...and that one such event occurs earlier than another such event events is also an observable matter...Change now consists in the fact that different positions in the D-series are occupied by different coincidence events" (*ibid.*, p. 14). Thus, Earman maintains that his D-series is temporally ordered.

This is not equivalent to the B-series, consisting of a string of events which are either earlier-then, later-than, or simultaneous with each other, because, according to Earman, that "can be described in terms of the time independent correlations between gauge dependent quantities which change with time" (*ibid.*, p. 15). B-series change, says Earman, is an artifact of the local representations (the elements of the equivalence class of metrics) rather than a real feature of the world, that associated with the equivalence class itself (to which his D-series is supposed to apply).[21] This is a strange way of viewing the content of B-series time, and I have never seen any philosophers of time dabbling with such concepts before: why does the B-series depend on gauge-dependent quantities? Perhaps it is a way to understand the B-series given an ontology that sticks by the gauge dependent quantities, but for different ontologies it needn't follow. If, for example, we adopt a standard ontology of events then it seems that Earman has simply constructed a B-series all over again.

Earman claims that a coincidence event "floats free of the points of $\mathcal{M}$" and "captures the

[21] What Earman means by a "local representation" in this context is, I think, what Rovelli calls a "local universe" ([1992]): a world in which properties are 'attached' to spacetime points. In other words, the elements of the orbits.

intrinsic, gauge-independent state of the gravitational field" (*ibid.*). General covariance implies that if this state is represented by one spacetime model it is also represented by any model from a diffeomorphism class of its copies. Now, Earman's interpretation of this, and his resolution of the problem of change, is to claim that the notion of spacetime points, properties localized to points, and change couched in terms of relationships between these, is to be found "in the representations" and not "in the world" (*ibid.*). This conclusion is clearly bound to the idea that in order to have any kind of change, a *subject* is required to undergo the change and *persist* through the change. In getting rid of the notion of a subject (i.e. spacetime points or objects), Earman sees the only way out as abolishing change in this sense. The idea that change is a matter of representation is one way (not a particularly endearing one, say I) of accounting for the psychological impulse to believe that the world itself contains changing things, though I think it needs spelling out in much more detail than Earman has given us.

As Earman is quick to note, there is something weird about an ontology based on such coincidence quantities since

> [w]e are used to thinking of an event as the taking on (or losing) of a property by a subject, whether that subject is a concrete object or an immaterial spacetime point or region. But the coincidence events in question are apparently subjectless. Note also that one doesn't verify the occurrence of a coincident event by first measuring the values of the electromagnetic and the scalar fields in question, and then verifying that the required coincidence of the value of the former with the latter does indeed hold; for by themselves none of these fields are gauge invariant quantities and so cannot be measured. The verifying measurement has to respond directly to the coincidence. ([2002b], p. 15-6)

Earman is not entirely forthcoming on how this idea might be cashed out. However, I think an approach developed by Rovelli provides a suitable formal underpinning. Moreover, given a structuralist gloss this framework is sufficient to provide a sensible ontology for the theory that is well equipped to avoid the problem of the frozen formalism, and it avoids a further problem, presented in §5.3, with the correlation strategies of Earman and Rovelli. Let us begin by looking at Earman's tentative response, before introducing an idea of Rovelli's that looks suspiciously like realism about gauge dependent quantities (quantities defined on the extended phase space). The structuralist gloss I give the correlation interpretation avoids this kind of realism.

5.2 Rovelli's Evolving Constants and Partial Observables.

Rovelli's *evolving constants of motion* proposal is made within the framework of a gauge-invariant interpretation. Like Earman, he accepts the conclusion that quantum gravity describes a fundamentally frozen reality, but argues that sense can be made of dynamics and change within such a framework. Take as a naive example of an observable m = 'the mass of the rocket'. This cannot be an observable of the theory since it changes over (coordinate) time; it fails to commute with the constraints, $\{m, \mathcal{H}\} \neq 0$, because it does not take on the same value on each Cauchy surface reached by applying the constraints. Rovelli's idea is to construct a one-parameter family of observables (constants of the motion) that can represent the sorts of changing (evolving) magnitudes we observe. Thus, instead of speaking of, say, 'the mass of the rocket' or 'the mass of the rocket at t', which are both gauge dependent quantities,[22] one speaks instead of 'the mass of the rocket when it entered the asteroid belt', $m(0)$, and 'the mass of the rocket when it reached Venus', $m(1)$, and so on up until $m(n)$. These quantities *are* gauge-invariant, and, hence, constants of the motion; but, by stringing them together in an appropriate manner, we can explain the appearance of change in a property of the rocket. The change we normally observe taking place is to be described in terms of a one-parameter family of constants of motion, $\{m(t)\}_{t\in\mathbb{R}}$, an *evolving* constant of motion.[23]

However, technically, it is hard to construct such families of constants of motion as phase functions on the phase space of general relativity. For this, and other reasons, Rovelli has recently shifted to an earlier view of his involving a distinction between what he calls 'partial observables' and 'complete observables,' where the former is defined as a quantity to which we can associate a measurement leading to a number, and the latter is defined as a quantity whose value can be *predicted* by the relevant theory, i.e. a (gauge-invariant) Dirac observable (Rovelli [1992] and [2002]). The formal setting for this view is the unreduced phase space, where we let the observables enforce the gauge invariance (it is, in other words, a gauge interpretation). The theory describes relative evolution of gauge-dependent variables (i.e. partial observables) as functions of each other. No variable is privileged as an independent one (*cf.* Montesinos *et al.* [1992], p. 5). How does this resolve the problem of change? The idea is that coordinate time evolution and physical evolution are entirely different beasts. To get physical evolution, all one needs is a pair $\langle\mathcal{C}, C\rangle$ consisting of an extended configuration space (coordinated by partial observables) and a function on $T^*\mathcal{C}$ giving the dynamics. The dynamics concerns the relations between elements of $\mathcal{C}$, and though the individual elements do not have a well defined evolution, relations between them (i.e. correlations) do, though they are independent of coordinate time.

Partial observables can be measured but not predicted, and complete observables are correlations between partial observables that can be both measured and predicted. The key question is: 'how can a pair of partial observables make a complete observable?' (*cf.* [2002], p. 124013-5); or, in other words, "how can a correlation between two nonobservable [gauge-dependent] quantities be observable?" ([2002], p. 124013-1). A simplified answer goes like this. Consider two non-gauge-invariant (i.e. gauge dependent) functions α and β. These are our partial observables; we can suppose that α is the matter density of a compact hypersurface and that β is the volume of a compact hypersurface. Recall that neither of these quantities is predictable, for their evolution will be gauge-dependent. We want to construct from this pair of partial observables a complete observable $\mathcal{E}^{\tau}_{\alpha|\beta}$ (where τ will be understood to be a 'clock' variable). To do this we consider the relational quantity that is formed by correlating the values of the two partial observables. We arbitrarily take one of the partial observables to be the 'clock' whose values will parameterize the evolution of the other. Let β be the clock. $\mathcal{E}^{\tau}_{\alpha|\beta}$ then gives the quantity that gives the value of α when, under the flow generated by the constraints, the value of β is τ. Thus, a partial observable is evolved along a gauge flow, such that the evolution is

[22]Unless, of course, t is itself a physical variable, in which case we have an example of a gauge invariant correlation observable.

[23]Rovelli's approach has a certain appeal from a philosophical point of view. It bears similarities to four-dimensionalist, temporal parts views on time and persistence (see, for example Sider ([2001])). The basic idea of both of these views is that a changing individual can be constructed from essentially unchanging parts. I think philosophers of time might perhaps profit from a comparison of Rovelli's proposal with these views.

a gauge-transformation, and is to be understood as a clock 'ticking' along the gauge orbit. On its own, of course, this is an expression of the problem of change since evolution along a gauge orbit is just the problem! But when we correlate another partial observable with the values at which $\beta = \tau$ we form a time-independent observable since the value of α when $\beta = \tau$ does not change. Variation in τ allows for the formation of a 1-parameter family of complete observables that correspond to empirically observable change. The evolution does not occur with respect to some background time parameter, but with respect to the values of the arbitrary clock; the complete observables will *predict* the value of α at the 'time' $\beta = \tau$. More precisely, the evolution will be a map $\mathfrak{E}^\tau : \mathcal{E}^{\tau_0}_{\alpha|\beta} \to \mathcal{E}^{\tau+\tau_0}_{\alpha|\beta}$, taking complete observables into complete observables. The fact that the clock β is arbitrary (since it can be chosen from $C^\infty(\mathcal{C}) \subset C^\infty(\Gamma)$) implies that the theory is a multi-fingered time formalism: there are numerous (infinitely many) choices that one can make for the clocks, and so there are numerous times—though not all choices will be 'good' clocks physically speaking. We might speculate that Rovelli's framework of partial and complete observables provides the technical grounding for Earman's D-series. In other words, the D-series gives us a 'picture' of $\mathcal{E}^{\tau_i}_{\alpha|\beta}$.

5.3 An Ontology of Structural Correlations.

As we have seen, both Earman's and Rovelli's responses belong in the category of 'correlation strategies,' according to which space, time and change are captured by relationships (correlation observables) holding between things or quantities. But such strategies, as implemented by Earman and Rovelli at least, face the following simple problem:

> ...one could [try to] define an instant of time by the correlation between Bryce DeWitt talking to Bill Unruh in front of a large crowd of people, and some event in the outside world one wished to measure. To do so however, one would have to express the sentence "Bryce DeWitt talking to Bill Unruh in front of a large crowd of people" in terms of physical variables of the theory which is supposed to include Bryce DeWitt, Bill Unruh, and the crowd of people. However, in the type of theory we are interested in here, those physical variables are all time independent, they cannot distinguish between "Bryce DeWitt talking to Bill Unruh in front of a large crowd of people" and "Bryce DeWitt and Bill Unruh and the crowd having grown old and died and rotted in their graves." ... The subtle assumption [in the correlation view] is that the individual parts of the correlation, e.g. DeWitt talking, are measurable when they are not. (Unruh [1991], p. 267)

A similar criticism to Unruh's comes from Kuchař ([1993], p. 22), specifically targeting Rovelli's evolving constants approach. Kuchař takes Rovelli to be advocating the view that observing "a changing dynamical variable, like Q [a particle's position, say], amounts to observing a one-parameter family $Q'(\tau_1) := Q' + P'\tau = Q - P(T - \tau)$, $\tau \in \mathbb{R}$ of perennials" (*ibid.*, p. 22)—where a perennial is a variable that commutes with *all* of the constraints. By measuring $Q'(\tau)$ at τ_1 and τ_2 "one can infer the change of Q from $T = \tau_1$ to $T = \tau_2$" (*ibid.*). So the idea is that a changing observable can be constructed by observing correlations between two dynamical variables T and Q, so that varying τ allows one a notion of 'change of Q with respect to T'. Kuchař objects that one has no way of observing τ that doesn't smuggle in non-perennials (i.e. non-gauge invariant quantities, or partial observables).

Belot and Earman question Unruh's and Kuchař's interpretation of the correlation view here, and suggest that it might be better understood "as a way of explaining the illusion of change in a changeless world" ([2001], p. 234). The basic idea is just the kernel of the correlation strategy; namely, that one should deal in quantities of the form "clock-1-reads-t_1-when-and-where-clock-2-reads-t_2". We get the illusion of change by (falsely) taking the elements of these relative (correlation) observables to be capable of being measured independently of the correlation. (They suggest that Rovelli's notion of evolving constants of motion is a good way of "fleshing out" the relative observables view. Presumably they would include the partial observables programme too.) However, if it is false to take the individual elements of the relational quantity to be measured independently, then we are owed an account of the sense in which this is false. The problem remains, as Unruh writes, "[o]ne cannot try to phrase the problem by saying that one measures the gauge dependent variables, and then looks for time independent correlations between them, since the gauge dependent variables are not measurable within the context of the theory" ([1991], p. 266).

In fact, I think both Unruh and Kuchař are guilty of the same *non sequitur*, though not for the reason given by Belot and Earman. One doesn't need to observe τ independently of Q: we can simply *stipulate* that the two are a 'package deal,' inseparable, or else that the correlations are *primary* with the individual elements being 'structurally constituted' by the correlation. In other words, we suppose the correlations are 'undecomposable' or 'non-factorizable;' the elements of the correlation have whatever properties they have in virtue of the correlation. In this way, I think both Unruh's and Kuchař's objections can be successfully dealt with. However, neither Earman nor Rovelli are able to get around this problem with their responses. Both Earman and Rovelli appear to want to cling to the notion that the 'elements' of the correlations (the partial observables or coinciding elements) have some *independent* physical reality. This is most explicit is Rovelli claiming that the partial observables (the elements of correlations) "are the quantities with the most direct physical interpretation in the theory" (*ibid*., p. 124013-7). I say, this is the wrong way around.

If the correlations (coincidence events, complete observables, or whatever) are subjectless, as indeed they seem to be and as Earman mentioned, then the components only have their individuality in virtue of the correlation that binds them together. But note that this does not tell us anything about *priority*: the correlation does not necessarily come before the fields (or 'correlata' as I shall refer to them). Thus, this is a structuralist position—Rovelli, Smolin, and others appear to see this as supporting relationalism; however, that position requires the independent reality of the relata, which leads to problems in this case (likewise for substantivalism). The ontology is structural. Once we take this on board, we have an ontology capable of dealing with the frozen formalism: it is the correlations that evolve—the appearance of change in gauge dependent quantities is due to the fact that we happen to decompose correlations. But that we do tend to do this does not imply that the decomposition must be a genuine possibility. Just as one can 'imagine' splitting a Democritean atom, in a possible world with such atoms this is not a real possibility at all. We can also escape Unruh and Kuchař's objection since, on this account, it is not the case that the correlation observables can, ontologically speaking, be 'broken down' into individual parts.

I shall call the overall structure formed from such correlations a *correlational network*, and the correlates I shall call *correlata*. Change is at the level of the correlational structure (a variation in correlations), but there is a derivative 'trickle down' effect from the correlations to the correlata. There are two core ways of understanding the structuralist position I have described: one in which neither the correlation nor the correlata is given ontological priority; and one in which the correlations are given priority over the correlata. The former option can be understood along the lines of Skyrms' 'Tractarian Nominalism.' The idea here is to understand individuals, properties, and relations as 'abstractions' from the structure of the world (i.e. the correlational network) but not as existing independently of that structure: "We may conceive of the world not as a world of individuals or as a world of properties and relations, but as a world of facts - with individuals and relations being equally abstractions from the facts" ([1981], p. 199). Likewse, the 'totality of facts' (the structure of the world) itself is 'composed' of such facts. As regards the question of ontological priority, then, we see that relations and relata (correlations and correlata) share the same status: "the Tractarian Nominalist ... takes both objects and relations quite seriously, and puts them on par. Neither is reduced to the other" ([1981], p. 202). The second option, the one I favour, can be understood by analogy with Armstrong's 'states of affairs'—this is, I think, more applicable to the 'decomposition problem' of Unruh and Kuchař. Thus, speaking in terms of 'states of affairs' rather than 'facts,' Armstrong writes that "while by an act of selective attention they [individuals, properties, and relations] may be *considered* apart from states of affairs in which they figure, they have no existence outside states of affairs" ([1986], p. 578). Likewise, the correlations are the fundamental things; they are things that can be measured and predicted: the stuff out of which reality is made. The correlata are measurable only in virtue of their position in the correlation, and have no independence outside of this. However, the correlata are our access point to the correlations, and this is why, I think, Rovelli imbues his partial observables with fundamental significance. If his, and Earman's, position is to escape the interpretive troubles highlighted by Unruh and Kuchař, however, the primacy needs to be reversed and shifted to the complete observables. By taking these seriously, as an ontological basis, those difficulties are easily resolved, along with the problem of frozen dynamics. In this way, we see that the claims about the demise of change is overcooked. Change is possible, but it is a *revised* conception not involving any background spatial and temporal structure. This is a natural notion from the perspective of a structuralist ontology.

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