Spacetime or Quantum Particles: The Ontology of Quantum Gravity? *

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1. Introduction

What is called a law of nature tends to depend on the historical circumstances in which the law was discovered or inferred. In the mathematical sciences generally, many laws can be expressed as an equation (or set of equations) which hold within a specific sphere of applicability. Our best confirmed theory of gravity is general relativity which is applicable to large scale phenomena. Quantum theory, on the other hand, is the most empirically successful theory at microscopic scales. However, we do not tend to speak of individual laws of general relativity or of quantum theory in the same way as we speak of Newton's Laws or the Laws of Phenomological Thermodynamics.

What results when theories/laws that have been developed in one sphere of applicability are applied to another (very different) sphere? The domains of quantum theory and general relativity overlap in situations where quantum mechanical effects cannot be ignored, such as might exist near (or inside) a black hole, or in the early universe. In order to deal with this overlap of theoretical domains, there has been a tendency to apply the rules of quantum field theory to the classical gravitational field equations and without much regard for the implications of the whole enterprise. These implications will be the primary concern of this essay and not the status of physical law in general relativity and/or quantum theory.[1]

Since its inception by Einstein in 1915, the general theory of relativity has been interpreted in realist terms. General relativity describes a classical (i.e., not quantum) gravitational field by means of its field equations, viz.:

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\kappa T_{\mu\nu} \qquad (1)$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, R is the Ricci scalar, $T_{\mu\nu}$ is the stress-energy-momentum tensor and κ is Einstein's constant. Classical general relativity is a theory about the *geometry of spacetime*.[2] The presence of matter (represented by $T_{\mu\nu}$) curves spacetime and the motion of matter is determined (in part) by the curvature of the spacetime (given by $g_{\mu\nu}$) through which the matter moves.

Quantum field theory, on the other hand, does not lend itself to a strongly realist interpretation, unlike general relativity. Despite the fact that quantum field theory does give accurate predictions for the interactions of 'elementary' particles, it is not difficult to construct convincing arguments to the effect that much of quantum field theory is a theoretical fiction.[3] This makes the realist assessment of quantum field theory far more problematic.[4] However, due to the large amount of experimental evidence that has been collected, most quantum particles are accepted by physicists as real entities and are classified into families according to their physical characteristics, e.g. mass, charge, spin, etc.

Researchers in quantum gravity, for the most part, can be divided into two broad groups which we shall denote as 'relativists' and 'particle theorists'. The former adhere to the geometric view of gravity, whilst the latter favour approaches in which gravity is a force mediated by one (or more) quantum particles.[5] Our best scientific theories when interpreted realistically ought to inform us about the sort of entities we should have ontological commitments to. Indeed, as Nerlich has remarked, "scientific realism ... what is that if not ontology?" [6]. Why do the particle theorists take such a different line to the relativists in regard to gravity when classical general relativity is so well confirmed? The primary reasons would seem to be as follows:[7]

(I) The right-hand side of the field equations (1) describes matter sources, the behaviour of which is governed by quantum theory. The left-hand side of the field equations describes gravitation as a classical field. If the right-hand side represents quantized matter then the field equations as they stand are *inconsistent*.

(II) The Uncertainty Principle implies that the gravitational field cannot be measured to arbitrary accuracy. The measured strength can only be given as an average over a spacetime region and not at individual spacetime points.

(III) The unification of all fundamental interactions under a single theoretical edifice has been a long-standing goal in physics. Recent successes (e.g. the Electroweak theory) have provided additional impetus for further unification endeavours (including gravity).

(IV) The removal of physical singularities. Classical general relativity predicts the existence of spacetime singularities. These are considered a blemish on spacetime models because all physical laws are assumed to break down at a singular point.

Although there are sound grounds for (I) - (IV), it should be noted that none of these reasons are *logically* compelling.

The dichotomy between geometric and particle pictures of gravity is evident especially in the case of gravitational waves. Consider the example of plane gravitational waves emanating from a weak source. Since the gravitational field is weak, any self-interaction may be ignored and the metric tensor can be written as:

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} \qquad (2)$$

where $\eta_{\alpha\beta}$ is the metric tensor for Minkowski (i.e., uncurved or 'flat') spacetime. Plane gravitational waves are represented by the tensor equation:

$$h_{\alpha\beta} = b_{\alpha\beta} \exp(ik_\mu x^\mu) + b^*_{\alpha\beta} \exp(-ik_\mu x^\mu) \qquad (3)$$

where $b_{\alpha\beta}$ is called the polarisation tensor, $b^*_{\alpha\beta}$ is its complex conjugate and k_μ is the wave vector. In the geometric view, these plane waves represent *ripples* in the 'fabric' of spacetime. If, however, equations (3) are quantized, the result is quantum fields of the form:

$$h_{\alpha\beta} = \sum_\sigma \int \{a(\mathbf{k},\sigma)\, b_{\alpha\beta}(\mathbf{k},\sigma) \exp(ik_\mu x^\mu) + a^\dagger(\mathbf{k},\sigma)\, b^*_{\alpha\beta}(\mathbf{k},\sigma) \exp(-ik_\mu x^\mu)\}\, d^3k$$

where $a(\mathbf{k},\sigma)$ and $a^\dagger(\mathbf{k},\sigma)$ are annihilation and creation operators respectively.[8] In the particle picture so obtained, the gravitational wave is considered to be made up of large numbers of quantum particles called gravitons of momentum $\hbar\mathbf{k}$ and helicity σ, where $\hbar$ is Planck's Constant divided by 2π. (The graviton is the particle that is responsible for mediating the gravitational force.) In this formalism, gravitons are created by application of the creation operator $a^\dagger(\mathbf{k},\sigma)$ to states described in a (mathematical) Hilbert space. It should be fairly obvious here that both particle and spacetime pictures cannot be (literally) correct. In the context of the realist assessment of theories of gravitation, the pivotal question surely is: *Which picture* (quantum particle or spacetime) *is warranted?*

2. Gravitational Time Dilation and Classical General Relativity

Gravitational time dilation is the phenomenon where time intervals are shorter when measured close to the source of a strong gravitational field than when measured further away from the source, i.e., at a 'higher' gravitational potential. (This is sometimes inaccurately referred to as the 'slowing down of clocks' in a gravitational field.) The gravitational redshift effect, where light travelling away from the source of a gravitational field suffers a change in wavelength towards the red end of the spectrum (and therefore a decrease in frequency) usually is associated with gravitational time dilation. Gravitational redshift has been the subject of empirical studies and is experimentally confirmed.[9]

The change in the frequency of light as it travels from one gravitational potential to another can be calculated using Newtonian gravitation and Einstein's photon hypothesis. Consider a photon of frequency f and kinetic energy $K = h\,f$, where h is Planck's Constant. The photon is emitted in a region with gravitational potential Φ and travels to another region with ('higher') gravitational potential Φ', e.g. from the Sun to the Earth. When this occurs the photon undergoes a redshift and increases its gravitational potential energy. As a consequence, it suffers a decrease in frequency and a loss of kinetic energy to $K' = h\,f'$, where f' is the frequency of the photon when observed at the 'higher' gravitational potential. The ratio of the two frequencies can be calculated by equating the total energies (kinetic plus potential) of the photon in the two regions: $(f'/f) = [(1 + \Phi/c^2) / (1 + \Phi'/c^2)] \approx (1 - \Delta\Phi/c^2)$, where the change in gravitational potential is $\Delta\Phi = \Phi' - \Phi$ (> 0) and c is the speed of light in vacuum. In deriving this result we have used the weak principle of equivalence and the assumption that both the emitter and the absorber of the light are of a non-relativistic nature.[10] The increase in the potential energy of the photon is due to the change in gravitational potential, but since its total energy remains constant the decrease in frequency is accountable *entirely* in terms of the loss of kinetic energy.

If we now compare the photon frequency f' with the frequency f^* of another photon emitted from an identical light source which is situated in the 'higher' gravitational potential (e.g. on the Earth), then we find (as expected) that $f' < f^*$, i.e., the light that has travelled from 'lower' to 'higher' gravitational potentials has been redshifted. These two frequencies are related by:

$$f^* = (1 + \Delta\Phi/c^2)\, f' \qquad (4)$$

This equation also may be derived using classical general relativity. Suppose that the components of the metric tensor are given by equation (2), i.e., $g_{\alpha\beta}$ differs only slightly from its Minkowski counterpart by the $h_{\alpha\beta}$. The tensor $h_{\alpha\beta}$ represents the deviation from 'flat' spacetime with $|h| << 1$. If we assume a static gravitational field and a frame of reference at rest (or moving with relative velocity $|\mathbf{v}| << c$) with respect to the field, then the temporal component of the metric tensor $g_{\alpha\beta}$ is given by:

$$g_{oo} = (1 + 2\Phi/c^2) \qquad (5)$$

where Φ (> 0) is the (Newtonian) gravitational potential of the field and terms of order h^2 have been neglected as being too small to contribute physically.[11] The metric of spacetime is given by $ds = (g_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta})^{½}$, so that a clock stationary with respect to and outside the gravitational field will record a time:

$$d\tau = (g_{oo})^{½}\, (dx^{o}) \qquad (6)$$

where dx^{o} is the coordinate time registered on a clock inside the field. Substituting equation (5) into equation (6) yields: $d\tau = (1 + 2\Phi/c^2)^{½}\, (dx^{o})$. If $\Phi << c^2$, then $(1 + 2\Phi/c^2)^{½} \approx (1 + \Phi/c^2)$ and the time intervals recorded by these two clocks will be related by the equation: $\Delta\tau = (1 + \Phi/c^2)\, \Delta t$, where Δt is the time interval recorded inside the gravitational field. In the case of two clocks that both are situated in the field but at different distances from its source, the time intervals measured will be related by:

$$\Delta\tau = (1 + \Delta\Phi/c^2)\, \Delta t \qquad (7)$$

where $\Delta\Phi$ is the change in gravitational potential and $\Delta\tau$ is the time interval recorded at the greater distance (i.e. in the 'higher' gravitational potential). Equation (7) is the equation of gravitational time dilation.

Since the time intervals measured in two different gravitational potentials are related by equation (7), the frequency of light measured from a light source further away from the field's origin will be greater than the frequency of light from an identical source which is close to the origin of the gravitational field. The frequencies of the light sources situated at different distances from the field's origin are related in the same manner as the time intervals $\Delta\tau$ and Δt, i.e., $f^* = (1 + \Delta\Phi/c^2)\, f'$, in agreement with equation (4). Indeed, the same result can be shown to follow from curved spacetime metrics without recourse to equation (2).[12] Thus, in classical general relativity, the gravitational redshift of light results solely from gravitational time dilation, i.e., the redshift is due only to differences in coordinate times found in curved spacetimes. With Newtonian mechanics and gravitation, the decrease in the frequency of light is attributable to a loss of kinetic energy. In classical general relativity, however, gravitational potential energy is not well defined and is not required to explain gravitational redshift.

Consider now the gravitational version of the asymmetric ageing of twins. It follows from equation (7) that if instead of clocks we have two identical biological specimens, one located in a 'higher' gravitational potential than the other, then the one in the 'higher' potential will physically 'age faster'. This could be demonstrated by initially having the specimens at the same spacetime location, separating them off to locations with different gravitational potentials and then bringing them back together. The discrepancy in the time intervals measured in the different gravitational potentials will be directly correlated with the lesser physical ageing of one of the pair of otherwise identical biological specimens. This phenomenon is unexplained in the context of a 'flat' spacetime as long as low velocities are used for the transport of the specimens. It is the curvature of spacetime that provides a natural explanation of the twin situation.

3. Gravitational Time Dilation and the Particle Approach to Quantum Gravity in Minkowski Spacetime

Most particle theorists see gravity as another classical (force) field to be quantized just like the electromagnetic field has been. However, general relativity can be differentiated in several respects from field theories that have been quantized. In particular we may point to the following distinguishing features: the non-linearity of the field equations (1); the dynamic nature of the metric tensor; and the dimensionality of the coupling constant κ. These aspects pose formidable problems to quantization, problems not encountered when quantizing other classical fields. All attempts to produce a fully quantized theory of gravity have failed, but not for lack of effort. Despite the methods having been many and varied, none have proved successful notwithstanding the exhausting labours of theoretical physicists.[13]

In conventional quantum field theory, a given classical field is quantized on a 'flat' background (i.e., Minkowski spacetime). Likewise, the covariant quantization of the gravitational field can be attempted on a 'flat' spacetime background. This 'flat' background is obtained by assuming that the metric tensor differs from the Minkowski form by the factors $h_{\alpha\beta}$ only, i.e., the metric tensor is split as per equation (2). In this case though, the $h_{\alpha\beta}$ are interpreted as representing a self-interacting, massless, spin-2 field propagating in Minkowski spacetime, similar to the weak gravitational wave case. The quanta of this field are gravitons. The gravitational force, in this context, is explained by a continual exchange of 'virtual' gravitons in much the same way as electromagnetic forces are mediated by an exchange of 'virtual' photons. (The particles are 'virtual' because the limits of the Uncertainty Principle do not allow for their experimental detection.)

All theories of quantum gravity must yield the same macroscopic predictions as classical general relativity does, including the same quantitative result for gravitational redshift. In the covariant method, computations are made by specifying an action functional, as is routinely performed in quantum field theory. Where a photon moves from 'lower' to 'higher' gravitational potentials, this corresponds to an interaction between gravitational and electromagnetic fields. The action functional is found from the Einstein Lagrangian density $\mathcal{L}_G$ in conjunction with a Lagrangian density for the electromagnetic field $\mathcal{L}_{EM}$.[14] The total action then is defined by the integral $\int(\mathcal{L}_G + \mathcal{L}_{EM})\, d^4\boldsymbol{x}$. Graviton-photon interactions of low order are calculated by expanding the Lagrangian densities as power series and constructing the scattering matrix elements from the resulting action. (Such calculations are extremely tedious and will not be reproduced here.[15]) The *crucial* point, however, is that the explanation for the redshift of light in covariant quantum gravity is essentially the same as when Newtonian gravitation is applied to the photon hypothesis, i.e., the change in frequency of a photon is due to a loss of kinetic energy.

Whilst it is the case that the assumption of a coupling of gravitons to photons can be used to calculate the size of the gravitational redshift, the usual account of this redshift is that the decrease in photon frequency implies a difference in coordinate times. This inference is incorrect because the change in photon frequency results, not from time dilation, but from a loss of kinetic energy. Such energy loss does not explain the lesser physical ageing of one of a pair of otherwise identical biological specimens situated in different gravitational potentials. Therefore, the phenomenon of gravitational asymmetric ageing is not only unexplained, it has become *utterly mysterious*.

There are further rationales that can be made in favour of this conclusion:

(1) The loss of kinetic energy of the photon is a consequence of the law of conservation of total energy. Physical processes by which we measure time may be affected by the operation of this law, but time itself is not.

(2) Whilst it may be the case that some physical processes may be affected by a loss of kinetic energy from the system in question, this (in isolation) does not warrant the further inference that the actual ageing processes of biological organisms would be altered.

(3) If the physical ageing of objects were affected by a loss of kinetic energy then this would be apparent in other situations, which it is not. Consider a simple example - the removal of heat from an object (as its heat energy ultimately is the sum of the individual molecular kinetic energies). Suppose that the object is a pure chemical system, a vat of liquid reactants say. A reduction of the heat content of the vat would produce slower and slower chemical reactions. If time intervals were to be judged by the 'rate of reaction' in the vat, then an inference about time similar to that made about frequency decrease in the 'flat' spacetime case might be made. Clearly though, any conclusion about time based on rates of reaction would be nonsense, for we know from the study of chemical kinetics that reaction rates vary enormously and depend on all sorts of different factors (e.g. temperature, pressure, chemical concentrations, surface areas, catalysts, etc.).

(4) If the particle theorists were correct, then the loss of a large amount of kinetic energy from one of a pair of otherwise identical biological organisms would produce a difference in their respective physical ages. Since heat energy is just the sum of molecular kinetic energies, this loss of energy could be achieved by progressively cooling one of the pair. Suppose this were to be done, what would happen? The biochemical reactions of the organism that is being cooled certainly will slow down. This, however, is a situation no different in principle from that depicted in point (3) above.

No sound inference about time itself follows from any of these considerations.

The twin situation demonstrates that gravitational time dilation is not explained by graviton interactions on a 'flat' spacetime background. This conclusion has (at least) two important consequences:

(i) spacetime curvature cannot be dispensed with merely by postulating gravitational exchange quanta; and

(ii) theories of quantum gravity need to incorporate a (geometrically interpreted) curved spacetime.

Theoreticians have pursued conventional quantization methods on 'flat' spacetime backgrounds since the early 1930s. However, even at an intuitive level, it should have been realised that these methods are unsuitable because they cannot account properly for gravitational time dilation. These 'flat' spacetime methods finally are being given away for the reason that they produce non-renormalizable theories (i.e., theories with infinities that cannot be removed).[16] Yet, the failure to account properly for gravitational time dilation always has been a sufficient reason to dismiss any relativistic theory of gravity with an uncurved spacetime.

4. The Particle Approach to Quantum Gravity in Curved Spacetime

In theories of quantum gravity, the continual exchange of 'virtual' particles accounts for the force of gravity. In order to account for gravitational asymmetric ageing, theories of quantum gravity need to incorporate a (geometrically interpreted) curved spacetime, as noted above. Does it really make sense to postulate gravitational exchange quanta in a theory that already is endowed with a curved spacetime? Such theories have a somewhat confusing, and perhaps incompatible, mix of basic entities (i.e., gravitons in a curved spacetime). What is important to note here is that, in addition to accounting for gravitational time dilation, curved spacetime readily explains deviations from inertial motion due to the influence of massive bodies. In other words, spacetime curvature can account, of itself, for both the 'force' of gravity and asymmetric ageing without the need for exchange quanta. (Gravitational force, as such, does not exist in a curved spacetime - it is fictitious, like centrifugal force.) Therefore, gravitational exchange quanta are *superfluous* in a curved spacetime for these purposes.

Another result of taking a particle view of gravity in curved spacetime has been shown by von Borzeszkowski and Treder who use a standard covariant technique. We shall reproduce part of their argument. Let the metric tensor be decomposed as follows:

$$g_{\mu\nu} = \delta_{\mu\nu} + eh_{\mu\nu} \tag{8}$$

where $\delta_{\mu\nu}$ is a fixed, but not necessarily 'flat' background metric and $e \ll 1$. (Negligible back reaction of the $eh_{\mu\nu}$ fields on the curvature of $\delta_{\mu\nu}$ is assumed.) Let L and Ω be characteristic lengths over which the δ and h parts of the field might (conceivably) alter in a substantial manner. The Ricci tensor $R_{\alpha\beta}$ formed from this metric (equation (8)) then can be expanded in powers of e:

$$R_{\alpha\beta}(\delta_{\mu\nu} + eh_{\mu\nu}) = R_{\alpha\beta}(\delta_{\mu\nu}) + eR'_{\alpha\beta}(h_{\mu\nu}) + e^2 R''_{\alpha\beta}(h_{\mu\nu}) + e^3 R'''_{\alpha\beta}(h_{\mu\nu}) + \ldots \quad (9)$$

where $R_{\alpha\beta}(\delta_{\mu\nu})$ is the Ricci tensor for the background metric.[17] The terms R', R'' and R''' have orders of magnitude L^{-2}, $e^2 \Omega^{-2}$, $e^3 \Omega^{-2}$ respectively. The powers of e will estimate each of the these terms only when $L \geq \Omega$. It follows that only for low frequency fields $h_{\mu\nu}$ will the background metric be governed by the equation $R_{\alpha\beta}(\delta_{\mu\nu}) = 0$. If $R_{\alpha\beta}(\delta_{\mu\nu})$ is made equal to zero for all Ω, then equations (9) will not yield the field equations (1). If we are to remain within the framework of general relativity, as distinct from another theory of gravity, then this approach can be applicable only to low frequency (i.e., classical) fields.[18]

Since the quanta of the $h_{\mu\nu}$ fields are gravitons, it is inferred from the above considerations that the quantum representation is an approximate one which is valid only when low frequency gravitational phenomena are involved. The conclusion reached on this basis is that the production of field quanta by covariant quantization does not correspond to any real, physical process.[19] Whilst it is obvious that the covariant method does not provide a full theory of quantum gravity, this result does raise the important question of whether the process of quantization in particle approaches will correspond to anything real (and measurable). Also, the problem of divergent terms in perturbation expansions does not go away when the background spacetime is curved.[20]

It has been the belief that classical gravity is a field *on par* with other classical fields that has lead particle theorists to assert that the gravitational field not only ought to be, but indeed must be quantizable as other fields have been. This view is mistaken, as classical force fields play out their interactions within a spacetime structure. The gravitational field should be considered a classical field for mathematical purposes only. Treating it otherwise has been an open invitation to all sorts of technical and conceptual problems. Part of the explanation for the existence of these problems is what was earlier referred to as the application of theories/laws developed in one sphere of applicability being applied to a different sphere without due consideration of the appropriateness of such application and its consequences.

In sum, given that the graviton representation is only an approximate one which does not lead to any new measurable effects, is superfluous in a curved spacetime and remains perturbatively non-renormalizable, then there would seem to be little advantage in pursuing particle approaches to quantum gravity.

5. Other Theories of Quantum Gravity

There's not much joy to be found in other quantum gravity theories either. We shall look briefly at the major problems found in two main theoretical contenders. Canonical theories of quantum gravity are those based on a Hamiltonian formulation of the classical theory. In these canonical approaches, the metric tensor itself is quantized! But if such a procedure is performed, how is the quantized metric tensor to be interpreted? What will be its resulting ontological status? Does it remain a geometrical entity? A quantum 'particle' of some sort? Some bizarre hybrid of geometrical and particle natures? Or something altogether different again? The conceptual 'pictures' that emerge from canonical quantum gravity theories are *not unambiguous* and require more precise renderings than have been advanced by particle theorists.

One of the most serious difficulties in canonical theories of quantum gravity is the problem of time (as it is called). In these theories, time is 'eliminated' from the governing equation of evolution of the relevant system, i.e., the Schrödinger equation is modified to one not involving time. However, in the low energy limit, it is required that state functions do evolve in time. Thus some internal degree of freedom must be used in place of parameter time against which state functions can 'evolve'.[21] Moreover, such an internal variable would have to result in the proper time parameter in the low energy limit. It is not always clear what this internal variable should be or how it is to be chosen. Ashtekar summed up the problem of time when he wrote:

> If there is no classical time, however, how can one do physics? After all, the business of physics is to make predictions. How can one predict if one does not have access to the familiar notion of time? [22]

String theory is the extension of covariant methods to ultra-small one-dimensional objects called (not surprisingly) strings. In some versions of string theory spacetime is not taken as a basic feature at all. The different modes of vibration of the string are identified as quantum particles, but the string itself does *not exist in* anything akin to a classical spacetime. Instead, classical spacetime is (somehow) supposed to emerge from a more fundamental realm as a low energy approximation.[23] What this fundamental realm is supposed to be has not been defined. The loss of a familiar notion of spacetime as both an arena of and participant in events has the potential to lead to serious problems, especially those associated with time. For example, such theories face a major problem in providing an *explanatory* account of gravitational asymmetric ageing, as distinct from being able to predict the correct numerical value for gravitational reshift. It is generally agreed that there is something different about spacetime structure at and below scales of the order of the Planck Length ($\approx 10^{-33}$ cm.). This, however, does not require the removal of spacetime. Rather it would tend to suggest that the classical concept of an infinitely differentiable manifold with its defined metric as a model of spacetime is inappropriate at this scale and that an some other *geometrical* structure is required. We shall return to this point in the next section.

String theory suffers from a complaint that might be called 'ontological proliferation'. First, most versions require the number of spatial dimensions to be greater than three. This is dictated by the requiements of conformal invariance, i.e., the invariance of the null-cone structure. These unseen dimensions (which may be as many as twenty-six!) are concealed by being 'compacted' into circles with radii of the order of the Planck Length. Second, the types and numbers of hypothetical particles also are increased markedly in string theory. The reasons for embarking on such complexities are purely theoretical and are justified by appeal to certain desiderata: the cancellation of infinities; the unification of fundamental particles and interactions; and the production of aesthetically satisfying theories.[24] However, theories displaying such 'ontological overkill' cannot be entertained seriously for an *indefinite* period without very convincing experimental evidence. There would appear to be very little likelihood of ever gaining this evidence due the enormous energies required to test such theories.[25] Given these factors, one is inclined to side with the physics Nöbel laureate, Sheldon Glashow, who has stated that he is "waiting for the superstring to break" !

6. Can Gravity be Fully Quantized?

There are severe problems of both a conceptual and a technical nature encountered in trying to construct a complete, physically viable, self-consistent, empirically adequate theory of quantum gravity. This point is not disputed. Purely in view of the scope and magnitude of these problems one might be tempted to conclude that the gravitational field cannot be quantized rigorously. Perhaps general relativity and quantum mechanics really are (to use Isham's phrase) "intrinsically incompatible".[26] After all, there is no *a priori* reason for asserting that gravity must be a quantum field. An alternative view is that the failure to solve the obstacles of quantum gravity is due to a lack of ingenuity on the part of theoreticians. Regardless of one's opinion, the reasons for seeking a theory of quantum gravity (points (I) - (IV) in the introduction) still need to be properly answered (or perhaps 'disarmed' is a more appropriate description).

Consider now a tentative (if not somewhat speculative) response. Equation (1) does demand a better form of 'accord' between gravitational and matter fields. The simplest unity is a form of semi-classical quantum gravity. The Einstein tensor can be equated with the expectation value of the quantum operator formed from the stress-energy-momentum tensor, viz.:

$$G_{\alpha\beta} = - \kappa \langle \mathbf{T}_{\alpha\beta} \rangle \qquad (10)$$

Difficulties with this semi-classical approach have been indicated in a number of articles.[27] The alleged problems include violation of the Uncertainty Principle, the possibility of faster-than-light communication and violation of momentum conservation. Some solutions to problems connected with semi-classical quantum gravity have been dealt with. In particular, if a version of quantum mechanics in which the wavefunction does not collapse is accepted then these problems do not occur.[28] This suggests that at least part of these difficulties might lie with 'orthodox' quantum theory. Non-collapse versions would not only deal with the above problems, but would go a long way in avoiding most of the standard quantum paradoxes.

In any case, equations (10) are still formally inconsistent. Dirac pointed out that it is mathematically incorrect merely to equate the Einstein tensor with the expectation value of the stress-energy-momentum operator. $G_{\alpha\beta}$ is a Hilbert space scalar, whereas $\mathbf{T}_{\alpha\beta}$ depends on the choice of the state vector in a Hilbert space. In order to avoid inconsistencies, such as those mentioned above, the latter quantity must be equated with the *average* value of the Einstein tensor:[29]

$$\langle G_{\alpha\beta}\rangle = - \kappa \langle \mathbf{T}_{\alpha\beta}\rangle \qquad (11)$$

There is no actual quantization of the gravitational field needed here and the classical geometric flavour is preserved. This deals with quantization reason (I) in the introduction. Also, given the problem of measurement in quantum gravity, it is highly probable that the above semi-classical treatment will be epistemically equivalent to any other. This is not to suggest, however, that a version of semi-classical quantum gravity is the complete picture or that all the problems of quantum gravity are solvable within such a framework. Semi-classical quantum gravity, for example, does not resolve the question of the existence of spacetime singularities.

A fundamentally geometric nature for gravitation would mean that a completely rigorous unification of all fields is *not* possible. Evidence in favour of the geometric nature of gravity and for the validity of the strong principle of equivalence continues to mount.[30] Acceptance of the geometric nature disposes of quantization reason (III). A geometric basis is not only consistent with the arguments presented in this essay, but also would account for the *insurmountable* difficulties encountered by particle theorists in attempting to quantize the classical gravitational field equations in artificial ways. Given that all attempts to create a full quantum theory of gravity have failed, it might prove more productive to concentrate on strategies that preserve the geometric character of gravity rather than on mathematical methods of 'quantizing' the field equations.

Consider what it would signify if spacetime naturally existed in a form which has a smallest unit, i.e., a minimum four-dimensional hypervolume. Then, instead of spacetime being modelled as a manifold with a metric ranging over the real numbers, we would have a discrete spacetime with a 'metric' that ranges over the integers only. The size of these spacetime units would be so minute that even at moderately small distances, spacetime would have the appearance of a manifold and the semi-classical equations (11) would make an excellent approximation. The existence of a fundamental spacetime hypervolume also implies that there should be some minimum spacetime interval that would be an invariant quantity from one frame of reference to another.

Moreover, if this notion of discrete spacetime is correct then the traditional mathematical tool of differential geometry would have to be replaced. It is interesting to note that some theoreticians are starting to come around to such a view, as the following statement by C.J. Isham indicates:

> Perhaps the entire paraphernalia of differential geometry is only appropriate at scales greater than the Planck length. But if so, with what should it be replaced ...? [31]

One possible answer to Isham's question is as follows: the spacetime continuum could be replaced with a metrically discrete one and the field equations (1) with finite difference equations. Indeed, the famous theoretican T.D. Lee, has made similar suggestions. He argues that difference equations are more fundamental than differential ones and that the latter should be seen only as approximations.[32]

The idea that spacetime might be discrete is not new. Various attempts have been made to construct viable equations that represent phenomena in a (metrically) non-continuous spacetime, usually with the stated aim of removing the infinities that plague field theories.[33] There have been a number of problems found, but the most serious would appear to be the violation of Lorentz invariance.[34] Whilst it may be the case that continuous Lorentz transformations only hold approximately if spacetime is discrete, their violation is cause for concern and if possible, should be avoided. A discrete spacetime (of the sort envisaged) would eliminate the problem of measurements to arbitrary accurary (quantization reason (II)). If a minimum four-dimensional hypervolume exists it would not be physically possible to make measurements to an accuracy less than the size of this hypervolume, for that would imply further divisibility. It should be noted, however, that a discrete approach to gravitation would bring its own assortment of difficulties, but ones no more problematic than those experienced thus far in the search for a correct theory of quantum gravity.

In conclusion, we note that geometrically interpreted curved spacetimes cannot be done away with by postulating gravitational exchange quanta and that particle based interpretations of quantum gravity bring forth a host of technical, conceptual and philosophical problems, which indicate their unsuitability. It is fair to say that there still remains much to investigate in the search for a complete theory of quantum gravity and discrete geometric models offer a promising alternative to the standard quantization ones.

Notes

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1. The reader is referred to the essays by Forge and Heathcote in this volume.

2. Cf. Rindler (1977), p.21; Wald (1984), p.68; Doughty (1990), pp.482-4.

3. McMullin (1984), p.14.

4. Cushing (1982), pp.43 & 78.

5. Isham (1988), p.82.

6. Nerlich and Westwell-Roper (1985), p.128.

7. Cf. Penrose (1982) and Isham (1984).

8. Weinberg (1972), p.288.

9. For example, see Will (1986), pp.54-64.

10. Weinberg (1972), pp.84-85.

11. Angel (1980), p.200.

12. See Wald (1984), p.137; or Weinberg (1972), pp.79-85.

13. The reader interested in these issues is referred to: Duff (1975); Duff and Toms (1984); Isham (1981, 1984, 1991); Kuchar (1981); Taylor (1989); Weingard (1989); Ashtekar and Stachel (1991).

14. $\mathcal{L}_G = (1/k)\,(-g)^{1/2}\, R$ and $\mathcal{L}_{EM} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}$, where $F^{\alpha\beta}$ is the electromagnetic energy-momentum tensor, $g = |g_{\alpha\beta}|$, R is the Ricci scalar and k is a constant.

15. For technical details, see Duff (1975), pp.87-90.

16. Ashtekar (1991a), pp.7-8.

17. $R'_{\alpha\beta}(h_{\mu\nu}) = \delta^{\theta\tau}(h_{\theta\tau|\alpha\beta} + h_{\alpha\beta|\theta\tau} - h_{\tau\alpha|\beta\theta} - h_{\tau\beta|\alpha\theta})$ with (|) being covariant differentiation with respect to the background metric. Similar expressions apply for R" and R"' terms, etc.

18. von Borzeszkowski and Treder (1988), pp.58-61.

19. The full non-linearity also can be taken into account. See von Borzeszkowski and Treder (1988), pp.62-66, for details.

20. Ashtekar (1991a), p.5.

21. Ashtekar (1991b), p.413.

22. Ashtekar (1991c), p.195.

23. For example, see Horowitz (1991), p.299.

24. Green (1985).

25. 't Hoof (1979), p.328.

26. Isham (1981), p.2.

27. For example, Eppley and Hannah (1977).

28. Page and Geilker (1982), p.13.

29. von Borzeszkowski and Treder (1988), pp.1-2.

30. See Will (1981), chap. 8.

31. Isham (1991), p.359.

32. Lee (1986), p.198.

33. For example, Snyder (1947); Flint (1948); Hellund and Tanaka (1954).

34. Cf. Schild (1948); Hill (1955); and Pavlopoulos (1967).

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