Rovelli on disharmony between the quantum arrows of time

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Draft of April 5, 2017

1. Introduction

Carlo Rovelli (2016) argues that the statistical predictions of quantum theory are in general time reversal invariant, while the evolution of the quantum state is not, and proposes this disharmony as evidence against a realist interpretation of the wave function. In a response note, Zeh (2015) agrees that the disharmony exists, but disputes its implications for realist interpretations. In this note I argue that the disharmony arises only out of Rovelli's adoption of a non-standard definition of time reversal, which in short reverses the 'little-t' in the dynamics but forgets the 'big-T' time reversal operator responsible for conjugating the wave function. Once we adopt the ordinary definition of time reversal, harmony between the two temporal arrows is restored.

2. A briefing on time reversal

If a quantum system begins in a state ψ and evolves to a state ϕ, the time-reverse of that process is an evolution that begins in the state $T\phi$ and ends at $T\psi$, where

Acknowledgements. Thanks to Carlo Rovelli for a stimulating paper and discussion on the topic of this note.

the 'big T' is a bijection on the state space called the 'time reversal operator'. This operator serves to reverse all instantaneous facts about the state that change under time reversal. For example, given a wave function $\psi = \psi(x)$ in the (irreducible) Schrödinger representation, the 'big T' is the conjugation operator $T\psi(x) := \psi(x)^*$. The function $\psi(x) = e^{ipx}$ associated with momentum p is transformed under this operation to the function $T\psi(x) = \psi^*(x) = e^{i(-p)x}$, which is associated with momentum $-p$ in the opposite direction as expected.[1]

Given a rule of dynamical evolution, we can say what it means for the evolution to be 'invariant' under a symmetry. For example, to check for rotational invariance, we consider a fixed potential, and ask whether rotation takes possible trajectories to possible trajectories. Time reversal works the same way: a 'time reversal invariant' system is one that takes possible trajectories to possible trajectories, for a given, fixed Hamiltonian. More precisely, let $\mathcal{H}$ be a separable Hilbert space, and let $t \mapsto U_t = e^{-itH}$ be the unitary dynamics associated with a self-adjoint operator H. Suppose ψ evolves to ϕ, in that, $\phi = e^{-itH}\psi$. Time reversal invariance says that the time-reversed trajectory from $T\phi$ to $T\psi$ is also possible, in that $T\psi = e^{-itH}T\phi$, for the same Hamiltonian H. With a little manipulation this expression is quickly seen to be equivalent to the statement,

$$T^{-1}(e^{-itH})T = e^{itH}. \tag{1}$$

Like unitary operators, time reversal operator satisfies $TT^* = T^*T = I$. But unlike unitary operators, T is also antilinear, in that, $T(a\psi + b\phi) = a^*T\psi + b^*T\phi$, and thus $\langle T\psi, T\phi\rangle = \langle \psi, \phi\rangle^*$. Using the former fact, we find that the expression of time reversal invariance in Equation (1) is equivalent to $e^{itT^{-1}HT} = e^{itH}$, which is in turn equivalent to,

$$T^{-1}HT = H. \tag{2}$$

[1] To guarantee this function is square-integrable, just assume that it is zero outside a compact region.

In this way we recover the well-known claim that the unitary evolution of a quantum system is time reversal invariant if and only if $[T, H] = 0$, where T is the antiunitary time reversal operator.

3. Rovelli's thought experiment

Rovelli (2016) motivates his argument with the following simple thought experiment. Consider a wave function in ordinary non-relativistic quantum mechanics that expands symmetrically around a point p in space for some time t, and then collapses to a small shell around a different point q. What is the time-reverse of this process? It would seem to be a wavefunction that begins in a small shell that is symmetric about q, then jumps to a large shell that is symmetric around p, before finally shrinking down to a smaller shell around p.

This latter process is not what quantum mechanics says will happen. A wave function that begins in a shell around q will continue to expand symmetrically around q. The quantum dynamics, described in this way, appears to exhibit a form of time-asymmetry.

Rovelli proceeds to argue for a sense in which the quantum statistics is time-symmetric, in spite of the time-asymmetry associated with the thought experiment above. I will argue that this 'disharmony' between two senses of time symmetry only arises when one ignores the 'big-T' appearing in the standard definition of time reversal. There are two cases to consider: the unitary aspect of quantum evolution, and more general non-unitary rules of evolution like the one Rovelli describes above. In each case, I will argue that the rule of evolution is time-symmetric if and only if the statistics is, thus escaping Rovelli's charge of disharmony.

4. Harmony for the unitary dynamics

How does the time symmetry of unitary evolution compare to that of the statistical 'Born rule' of quantum mechanics? There is one clear sense in which the Born rule is always time reversal invariant: the probability of a transition from ψ to ϕ is the same as the probability of a transition from $T\phi$ to $T\psi$, in that,

$$|\langle T\phi, T\psi\rangle|^2 = |\langle \psi, \phi\rangle|^2. \tag{3}$$

This follows immediately from the fact that T is antiunitary. However, Rovelli is concerned with the statistics associated with a different sort of process, writing:

> If we observe the value a of an observable, the probability of observing the value b of another observable after a time t is equal to the probability of observing the value a a time t after b was observed (because $|\langle a|e^{-iHT}b\rangle|^2 = |\langle b|e^{iHT}a\rangle|^2$). More precisely: given an ensemble of sequences of measurement outcomes, we cannot figure out the arrow of time from them. (Rovelli 2016, pp.1229-1230)

The 'big-T' in the ordinary definition of time reversal is notably lacking from this claim. So, let us restore the ordinary definition of time reversal with a 'big-T' to this description. Once we do this, I claim, Rovelli's statement here about time reversal invariance no longer holds.

Suppose, as Rovelli suggests, that we allow an initial state ψ to evolve unitarily for a time t before a measurement occurs. Let $\psi(t) = e^{-itH}\psi$ be the rule of unitary evolution, fixing H once and for all as in the discussion above. Then the probability of a transition to a state φ after a fixed time t is given by the Born rule,

$$\Pr(\varphi|\psi) = |\langle \varphi, e^{-itH}\psi\rangle|^2. \tag{4}$$

What is the time-reverse of this scenario? As above, we must apply the time reversal operator T to our initial and final states, in addition to reversing their order. So, the time-reversed process begins with the initially prepared state $T\varphi$, which we let evolve according to the same rule of dynamical evolution for a duration of time t, at which point it transitions to the final state $T\psi$. The probability of this transition is given by,

$$\Pr(T\psi|T\varphi) = |\langle T\psi, e^{-itH}T\varphi\rangle|^2. \tag{5}$$

Recalling that $T^* = T^{-1}$ for antiunitary operators, let us write the right-hand-side of (5) as $|\langle \varphi, T^{-1}e^{itH}T\psi\rangle|^2$. It follows that the probabilities associated with Equations

(4) and (5) are equal for all ψ, φ if and only if $T^{-1}e^{itH}T = e^{-itH}e^{i\theta}$ for some fixed phase factor $e^{i\theta}$. The latter can always be eliminated by redefining the Hamiltonian $H \mapsto H - \theta$ at no cost to the physical description. And we know (from the previous section) that $T^{-1}e^{itH}T = e^{-itH}$ holds if and only if $THT^{-1} = H$. That is, we have:

$$(6) \qquad |\langle T\psi, e^{-itH}T\varphi\rangle|^2 = |\langle \varphi, e^{-itH}\psi\rangle|^2 \quad \text{if and only if} \quad [T, H] = 0.$$

The former says that the quantum statistics are time reversal invariant. The latter says that the Schrödinger evolution is time reversal invariant. In other words, with the 'big-T' restored in the definition of time reversal, we find that the testable, statistical predictions of quantum theory are in perfect harmony with the law of unitary evolution with respect to the arrow of time: either both fail to be time reversal invariant, and thus both distinguish an arrow of time, or neither do. One cannot have one without the other.

Indeed, the fact that time-symmetry of unitary evolution is related to the time-symmetry of the quantum statistics in this way is what underpins the modern direct detections of T-violation, such as those produced by Angelopoulos et al. (1998) at CPLEAR and by Lees et al. (2012) at SLAC. Each found evidence of a decay mode that occurrs with different probability than its time reverse. From this it immediately follows that the corresponding unitary dynamics of each is T-violating, since the time-asymmetries of these two rules are in harmony.

5. Harmony for non-unitary laws of evolution

What of more general rules of evolution, such as one that includes 'collapse' of the kind described in Rovelli's thought experiment? To characterise such a rule, suppose the law of dynamical evolution is expressed more generally in terms of a one-parameter set of operators $t \mapsto D_t$ that is not necessarily unitary. Suppose moreover that this law takes one from an initial state ψ to a final $\varphi = D_t\psi$ after a duration of time t. For the Born probabilities to be the same before and after time reversal again means that,

$$(7) \qquad |\langle \varphi, D_t\psi\rangle|^2 = |\langle T\psi, D_tT\varphi\rangle|^2 = |\langle \varphi, T^*D_t^*T\psi\rangle|^2,$$

where the second equality just applies properties of the inner product. If D_t is linear, these two probabilities are equal for ψ, φ if and only if $T^* D_t^* T = D_t$ (up to a phase factor). Given an arbitrary dynamical law of this kind, this is statement a natural interpretation of what it means for the law to be time reversal invariant. Thus, if D_t is not time reversal invariant, it follows that the statistical law for this system is not either. On the other hand, if D_t is non-linear, then there is no obvious no assurance either way. However, one certainly cannot conclude from the fact that D_t is *not* time reversal invariant that the statistics *is* time reversal invariant; indeed, one would generally expect for non-linear rules D_t that equality of the time-reversed statistics expressed in the equation above would fail. Thus, Rovelli's conclusion that the two rules of evolution are in disharmony is not warranted in the case of more general laws of dynamical evolution, either.

6. Disharmony remains

Some philosophers of physics have proposed that it is conceptually correct to drop the 'big T' from the definition of time reversal, including Albert (2000) and Callender (2000). But such proposals have pathological features. The time reversal operator T is responsible for reversing all the instantaneous facts about the quantum state under time reversal: conjugating the wavefunction, reversing phases, reversing momentum eigenfunctions, and reversing angular momentum and spin, among other things. In the example above of a function $\psi(x) = e^{ipx}$ with momentum p, such a transformation fails to reverse p without the 'big-T' operator. This results in the pathological description of a system with momentum opposite to the direction of motion. The 'big T' is also needed in order to recover a Newtonian limit that has the same time reversal symmetries as the corresponding quantum theory. Eliminating these consequences from the definition of time reversal has been called "the symptom of a perverse view" by Earman (2002). In contrast, it is possible to derive the standard T from plausible first principles about the nature of time (Roberts 2017).

However, as supporters of dropping the 'big T' appear to remain (e.g. Castellani and Ismael 2016), perhaps an appropriate use of Rovelli's observation is as further evidence against them. Rovelli is right to say that disharmony between the arrows of time in the quantum statistics and in Schrödinger evolution would be pathological. As we will now see, this pathogy of temporal arrows is essentially forced on us if we ignore the 'big-T' by setting $T = I$ equal to the identity operator.

To establish this, let me refer to time reversal without the 'big T' as *order reversal*, so as to distinguish it from time reversal in the ordinary sense. The Born rule is in general invariant under order reversal, in the sense that,

$$|\langle\phi, e^{-itH}\psi\rangle|^2 = |\langle e^{itH}\phi, \psi\rangle|^2 = |\langle\psi, e^{itH}\phi\rangle|^2. \tag{8}$$

This is what Rovelli's observation shows. In contrast, Schrödinger evolution is never preserved under the order reversal $\psi(t) \mapsto \psi(-t)$ when the Hamiltonian is not the zero operator. For, given a state ψ that evolves to a state $\phi = e^{-itH}\psi$, the order-reversal of this evolution is a state ϕ that evolves to a state ψ. But invariance under this transformation would mean that the resulting trajectory follows the very same rule of dynamical evolution, which is to say that $\psi = e^{-itH}\phi$. This implies that $e^{-itH}\psi = e^{itH}\psi$. Since ψ is arbitrary, that is only possible if $H = -H$, which implies that H is the zero operator.

In other words, if H is *not* the zero operator, then Schrödinger evolution is *not* invariant under the order reversal transformation $\psi(t) \mapsto \psi(-t)$. The disharmony that Rovelli wishes to avoid is thus forced on us whenever ordinary time reversal is replaced with order reversal.

7. Conclusion

Rovelli's conclusion that "physics is blind to the direction of time" is incomplete: once the 'big-T' of ordinary time reversal is restored to his discussion, we find that the statistics is blind if and only if the continuous dynamics is. As a result, disharmony on the orthodox interpretation is no argument against wave function realism. However, Rovelli is still correct to identify the presence of disharmony amongst the arrows of time as pathological. Since this was found to be generic to the Albert-Callender 'order reversal' account of the arrow of time, Rovelli's proposal thus turns out to provide further evidence against their account.

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